MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.
   (a) Because printing money can cause inflation and devalue the currency.
   (b) Market participants are expecting inflation to decrease in long term, so the investors and traders are worried about the macroeconomic outlook.
   (c) Quantitative easing is a form of monetary policy in which a central bank purchases longer-term securities from the open market in order to increase the money supply and encourage lending and investment, which was applied during this pandemic to offset the financial depression caused by COVID-19.

2. We need to find 10 bond that each matures in every half-year starting from now for bootstrapping. Altogether we will get 10 bonds and use them for bootstrapping. The data was collected on 2/1/2021, so we need to collect bonds which matures between 8/1/2021 and 2/1/2026. When the time gaps cannot perfectly match half a year, we can get a bond that matures within 6 months from this point. When the gap between two bonds are more than 6 months, we can get the nearest bond. Both process should use interpolations. Finally, the bonds are: CAN 1.50 Aug 21, CAN 1.50 Feb 22, CAN 0.25 Aug 22, CAN 0.25 Feb 23, CAN 1.50 Jun 23, CAN 2.25 Mar 24, CAN 1.50 Sep 24, CAN 1.25 Mar 25, CAN 0.50 Sep 25, CAN 0.25 Mar 26.

3. We can rank the eigenpairs with the size of the eigenvector, from largest to the smallest. Each eigenvectors are perpendicular to each other. The ith eigenpair has an eigenvector that points to the direction with the ith largest variance of the points.

Empirical Questions - 75 points

4.
   (a) The way that I calculated the yield is to solve the equation for y, and repeat this process for each YTM values:

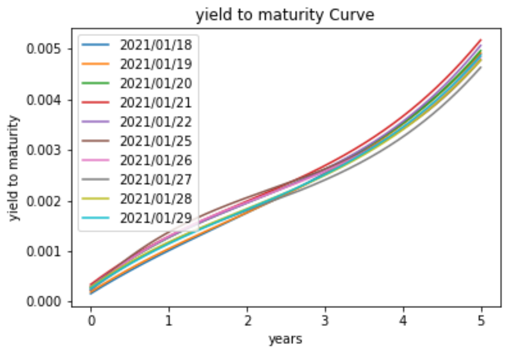
$$P = \sum_{i=1}^{n-1} \frac{C/2}{(1+y/2)^{-2t_i}} + \frac{N + C/2}{(1+y/2)^{-2t_n}}$$

   where:

   i. t_i = time from now in years
   ii. C = annual coupon
   iii. N = Notional
   iv. P = dirty price now

The yield has small differences between dates. On 2021/01/18, the yields are: 0.000828, 0.001102, 0.001226, 0.001891, 0.001782, 0.002534, 0.003158, 0.003722, 0.004311, 0.004955, for bonds mature earliest to latest respectively. (Due to limited pages, check the codes in github for YTM values on other days. The output is in variable ytm_data.) The interpolation technique I used is spline. A spline is a special function defined piecewise by polynomials. I used this technique in plotting the curves.

See Figure 1(a) for the 5-year yield curve plot.



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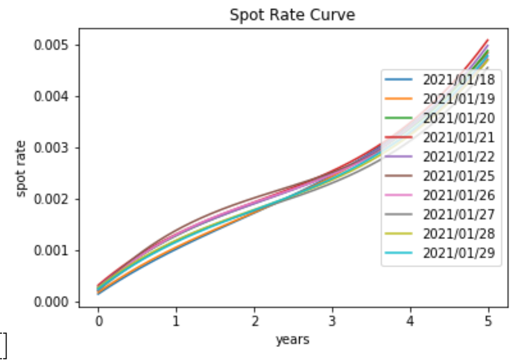

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Figure 1: (a). YTM Curves plot ;(b). Spot rate Curves plot

(b) Here is a psudo-code for spot curve:

i. Loop through the ten days for each spot curves.

ii. Assume the rate to be zero for now, so at $t_0 = 0$, $r_0 = 0$. $t_1, ..., t_10$ are the maturity dates of the 10 bonds.

iii. Calculate the rate on 2021-08-01, with the same technique as calculating YTM.

iv. Calculate spot rates from 2022-02-02 to 2023-02-01. Loop through $i \in [1, 2, 3]$ to solve the equation for r_i:

$$P = \sum_{j=0}^{i-1} \frac{C/2}{(1 + r_j/2)^{-2t_j}} + \frac{N + C/2}{(1 + r_i/2)^{-2t_i}}$$

v. Calculate the rate on 2023-06-01. Using spline interpolation to get rates $r_0, ..., r_3$. Solve the equation in (iv) for r_4.

vi. Calculate the rate on 2024-03-01. Use spline interpolation to get $r_0, ..., r_4$. Use linear interpolation to get $r_5 = (r_4 + r_6)/2$. Solve the equation in (iv) for r_6.

vii. Calculate spot rates from 2024-09-01 to 2026-03-01. Loop through $i \in [6, 7, 8, 9]$. Using the same technique as (v) to solve for $r_7, ..., r_{10}$.

viii. Use the spot rate $r_0, ..., r_{10}$ and time $t_0, ..., t_{10}$ and spline interpolation to plot the spot curve.

See Figure 1(b) for the spot rate curve plot.

(c) Here is a psudo-code for forward rates:

i. Interpolate the spot rates $r_0, ..., r_4$ for 1,2,3,4,5 years from now using spline.

ii. Loop in $i \in [0, 1, 2, 3]$, calculate the forward rates by:

$$f = \frac{(1 + r_{i+1})^{i+2}}{(1 + r_i)^{i+1}} - 1$$

iii. Use spline interpolation to draw the 1 year forward curve for 1 to 4 years from now.

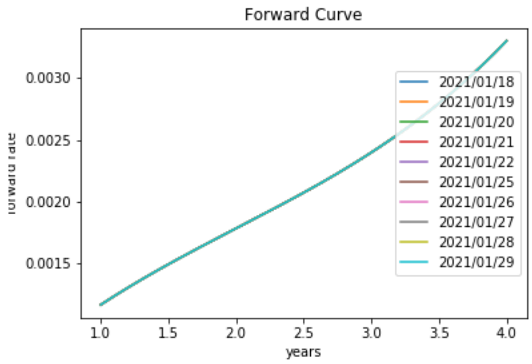

Figure 2: (c). Forward Curves plot

See Figure 2 for the forward curve plot.

5. The covariance matrix for yield is:

$$\begin{pmatrix} 0.00719 & 0.00453 & 0.00201 & 0.00045 & 0.00021 \\ 0.00453 & 0.00304 & 0.00155 & 0.00056 & 0.00032 \\ 0.00201 & 0.00155 & 0.00118 & 0.00089 & 0.00074 \\ 0.00045 & 0.00056 & 0.00089 & 0.0011 & 0.00107 \\ 0.00021 & 0.00032 & 0.00074 & 0.00107 & 0.00113 \end{pmatrix}$$

The covariance matrix for forward rates is:

$$\begin{pmatrix} 1.75 \times 10^{-03} & 4.00 \times 10^{-05} & -5.40 \times 10^{-04} & 4.00 \times 10^{-05} \\ 4.00 \times 10^{-05} & 1.32 \times 10^{-03} & 1.56 \times 10^{-03} & 8.90 \times 10^{-04} \\ -5.40 \times 10^{-04} & 1.56 \times 10^{-03} & 2.32 \times 10^{-03} & 1.61 \times 10^{-03} \\ 4.00 \times 10^{-05} & 8.90 \times 10^{-04} & 1.61 \times 10^{-03} & 1.71 \times 10^{-03} \end{pmatrix}$$

The code for calculating the two matrices are in the github link.

6. The following table includes the eigenvalues and their corresponding eigenvectors for the two matrices.

Yield		Forward	
Eigenvalues	Eigenvectors	Eigenvalues	Eigenvectors
1.089e-02	[-0.801, -0.251, -0.439, -0.313, 0.064]	4.649e-03	[0.119, 0.962, 0.236, -0.07]
2.540e-03	[-0.524, -0.003, 0.463, 0.622, -0.352]	1.815e-03	[-0.467, 0.155, -0.585, -0.644]
1.970e-04	[-0.263, 0.381, 0.526, -0.229, 0.676]	4.224e-07	[-0.702, -0.093, 0.692, -0.141]
8.220e-06	[-0.098, 0.626, 0.07, -0.475, -0.607]	6.279e-04	[-0.524, 0.205, -0.351, 0.748]
1.958e-08	[-0.065 , 0.632, -0.558, 0.487, 0.218]		

The eigenvector corresponding to the largest eigenvalue is weight of each time series for the 4 or 5 periods that can best explain the joint movement of the yield or forward rate movement in these ten days.

References and GitHub Link to Code

References:

1. Markets Insider. (n.d.). Retrieved February 08, 2021, from https://markets.businessinsider.com/bonds/

Github Link: https://github.com/Tristal25/APM466_Assignment1.git