Tristan Denning

ECE 310 – Microelectronics I

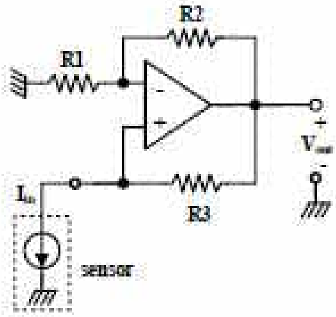
Dr. Suat Ay

Homework #8

Fall 2021

Due Date: 12/08/2021

1. (25pts) An OPAMP based sensor readout circuit is to be analyzed.

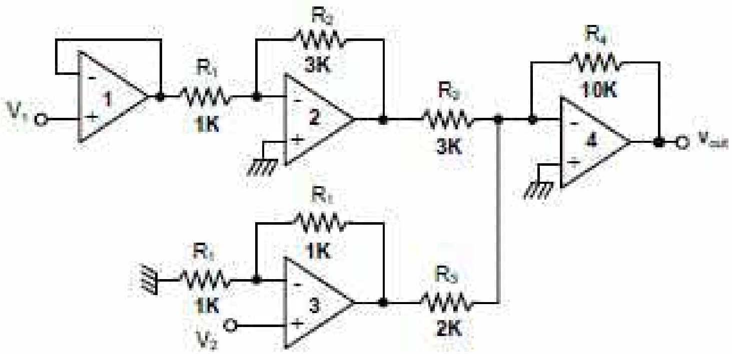


a) (10 pts) Find transresistance gain ($A_R=V_{out}/I_{in}$) of the circuit using <u>ideal OPAMP</u> model.

b) (10 pts) Find transresistance gain ($A_R=V_{out}/I_{in}$) of the circuit using <u>non-ideal OPAMP</u> model. Verify that your derivation is correct using part (a).

c) (5 pts) Find input impedance of the circuit (Rin) that the sensor sees using <u>non-ideal OPAMP</u> model.

2. (10 pts) Find V_{out} versus V_1 and V_2. Use <u>ideal OPAMP</u> model for the circuit shown below.

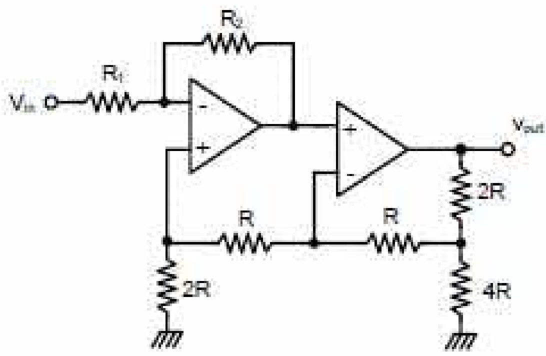


3. (25points) Use ideal Opamp model for the circuit.

a) (10 pts) Find V_{out} versus V_{in} expression

b) (10pts) Verify that your derivation is correct using two different ways (unit check is not allowed).

c) (5pts) Calculate transresistance gain if $R_1=2R_2$.



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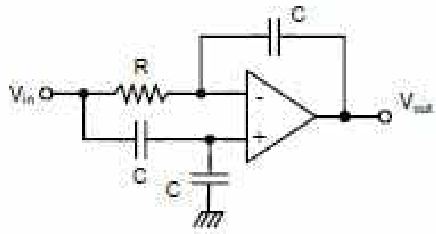
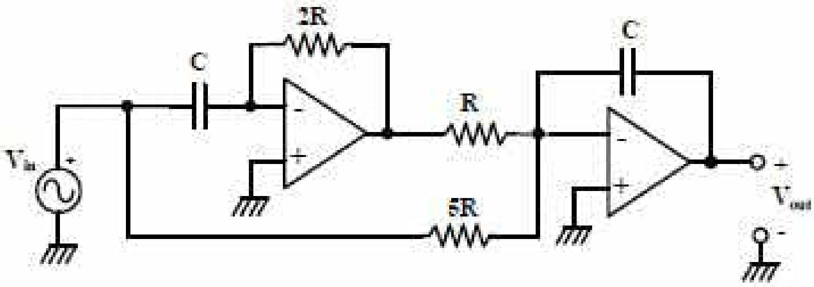
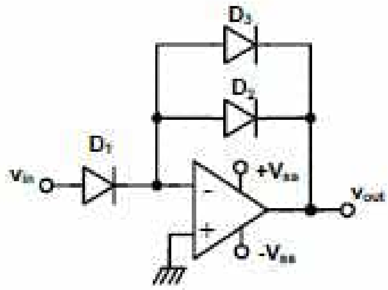
4. (10 pts) Find V_{out} versus V_{in} in **time-domain**. Use ideal OPAMP model.



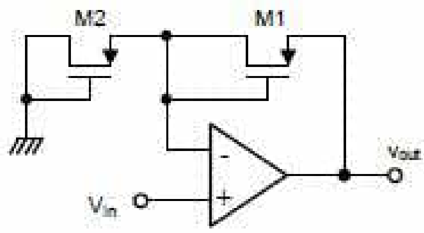
5. (10pts) Find the input-output relation of following circuit in time (t) domain (V_{out} versus V_{in}) using ideal OPAMP model.



6. (10 pts) Find the input-output relation of the following circuit. Use ideal OPAMP model, and assume all diodes are identical.

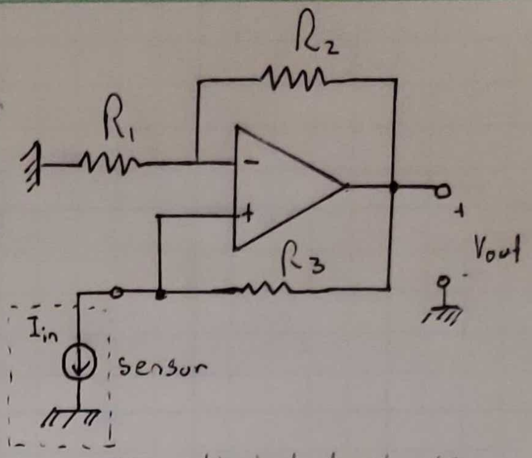


7. (10 pts) Find input output relation V_{out} versus V_{in}.Use ideal Opamp model and λ=0 for MOSFET in the circuit shown below.



8. (10 points) Design an OPAMP based circuit that divides two inputs. Derive your circuit's input output relation (i.e. Vout=A(Vin1/Vin2)+B) using ideal OPAMP model.

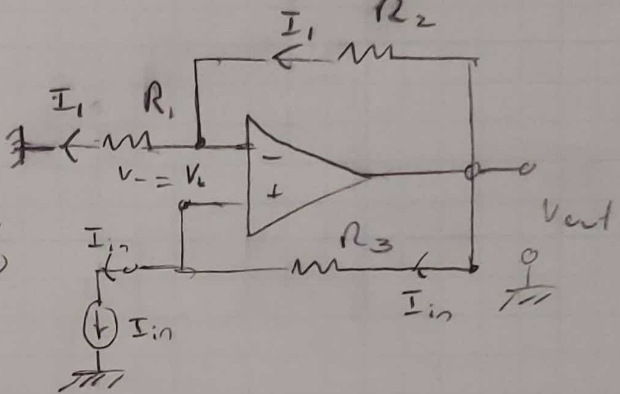
$\boxed{1(a.)}$ 1 **Situation:** An opamp based sensor readout circuit is to be analyzed. a.) Find the transresistance gain $A_r = V_{out}/I_{in}$ of the circuit using Ideal OpAmp Model



2 **Goal:** To use the "3-zero" rule along with nodal analysis to derive an expression for $A_R = V_{out}/I_{in}$.

3 **Plan:** To use the "3-Zero" rule along with Nodal Analysis to derive an expression for the transrestance gain $A_R = V_{out}/I_{in}$.

4 **Solution:** Redraw/Label;



Observe:

$I_{R_1} = I_{R_2} = I_1 \quad \because I_- = 0$
$I_{in} = I_{R_3} \quad \because I_+ = 0$
$V_+ = V_-$

Nodal Analysis:

@ V_+ ;
$$\frac{(V_{out} - V_+)}{R_3} = I_{in}$$
$$\therefore V_+ = V_{out} - I_{in}R_3 = V_- \quad (1)$$

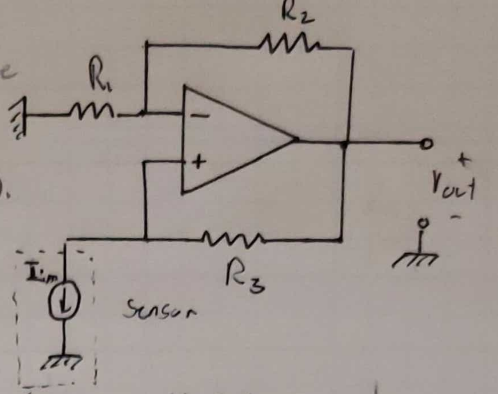
@ V_- ;
$$\frac{(V_{out} - V_+)}{R_2} = \frac{V_+}{R_1}$$
$$(V_{out} - V_+)R_1 = V_+ R_2$$
$$V_{out} R_1 = V_+ R_2 + V_+ R_1$$
$$V_{out} = V_+ \left(\frac{R_2 + R_1}{R_1}\right)$$

(1) =>
$$V_{out} = (V_{out} - I_{in}R_3)\left(1 + \frac{R_2}{R_1}\right)$$
$$\therefore V_{out} = \frac{I_{in}(R_1 + R_2)R_3}{R_2}$$
$$\therefore \boxed{A_R = \frac{(R_1 + R_2)R_3}{R_2}}$$

5 **Sanity Check:**

Problem 1(b) will serve as sanity check. Units check out.

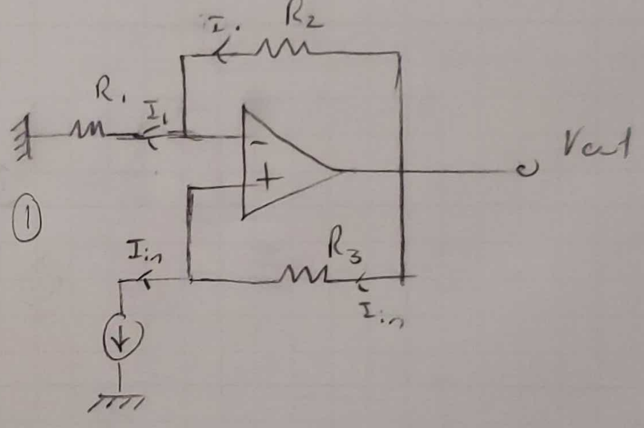
$\boxed{1(b)}$ 1 **Situation:** Find the transresistance gain ($A_r = V_{out}/I_{in}$) of the circuit using non-ideal opamp model. Verify the derivation is right from part 1(a).



2 **Goal:** Derive an expression for the transresistance gain (A_R) using the non-ideal opamp model.

3 **Plan:** To use Nodal Analysis and the "2-zero" rule and non-ideal opamp characteristics to derive an expression for A_R.

4 **Solution:** Redraw / Label



Observe: $I_+ = I_- = 0$

$$V_{out} = (V_+ - V_-)A_o$$

$$V_{out} = V_+ A_o - V_- A_o \quad (1)$$

$$V_+ = \frac{V_{out}}{A_o} + V_-$$

Nodal Analysis;

@ V_-

$$\frac{(V_{out} - V_-)}{R_2} = \frac{V_-}{R_1}$$

$$(V_{out} - V_-)R_1 = V_- R_2$$

$$\therefore V_- = \frac{V_{out} \cdot R_1}{R_1 + R_2} \quad (2)$$

Nodal @ V_+

$$\frac{(V_{out} - V_+)}{R_3} = I_{in}$$

$$\therefore V_+ = V_{out} - I_{in} R_3 \quad (3)$$

(3), (2) $\Rightarrow$ (1)

$$V_{out} = \left((V_{out} - I_{in}R_3) - \frac{V_{out} R_1}{R_1 + R_2}\right) A_o \quad (4)$$

Simplify 4:

$$V_{out} = \frac{-A_o I_{in} (R_1 + R_2) \cdot R_3}{R_1 - (A_o - 1) \cdot R_2}$$

$\boxed{1(b)}$ (continued)

$$\therefore \frac{V_{out}}{I_{in}} = \boxed{\frac{-A_o(R_1+R_2)\cdot R_3}{R_1-(A_o-1)\cdot R_2} = A_R}$$

Sanity Check (a),(b):

$$\text{Check } \lim_{A_o\to\infty}\left(A_{R,\,Nonideal}\right) = A_{R,\,Ideal}$$

$$\lim_{A_o\to\infty}\left(A_{R,\,NoNideal}\right) = \frac{-\infty\,(R_1+R_2)\cdot R_3}{R_1-\infty R_2+R_2}$$

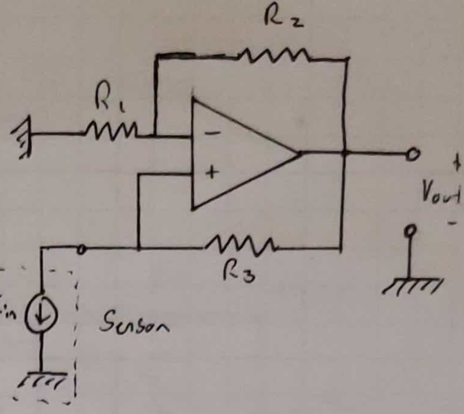
$$= \frac{-\frac{\infty}{\infty}(R_1+R_2)\cdot R_3}{\frac{R_1}{\infty}-\frac{\infty}{\infty}R_2+\frac{R_2}{\infty}}$$

$$= \frac{-1\,(R_1+R_2)\cdot R_3}{0-R_2+0}$$

$$\boxed{A_R = \frac{(R_1+R_2)\cdot R_3}{R_2}}$$

1(c)

1 **Situation:** Find the input impedance (R_{in}) seen by the Sensor using Non-ideal OPAmp Model



2 **Goal:** To determine an expression for the circuit's input impedance R_{in}.

3 **Plan:** To use the equation for $R_{in} = V_{in}/I_{in}$ and previously determined relations for V_{in} and I_{in} to derive an expression for R_{in}.

4 **Solution:**

$$R_{in} = \frac{V_+}{I_{in}}$$

$$V_+ = V_{out} - I_{in} R_3$$

$$\therefore R_{in} = \frac{V_{out}}{I_{in}} - R_3$$

$$= \frac{\left(\dfrac{-A_o I_{in}(R_1+R_2)\cdot R_3}{R_1-(A_o-1)\cdot R_2}\right)}{I_{in}} - R_3$$

$$\therefore R_{in} = \frac{-A_o(R_1+R_2)\cdot R_3}{R_1-(A_o-1)\cdot R_2} - R_3$$

$$\boxed{R_{in} = R_3\left(\frac{-A_o(R_1+R_2)}{R_1-(A_o-1)\cdot R_2} - 1\right)}$$

5 **Sanity Check:** Unit Analysis:

$$[\Omega] = [\Omega]\left(\frac{-[-]\left([\Omega]+[\Omega]\right)}{[\Omega]-([-])\cdot[\Omega]} - [-]\right)$$

$$= [\Omega]\,[-]$$

$$= [\Omega] \quad \checkmark$$

2.

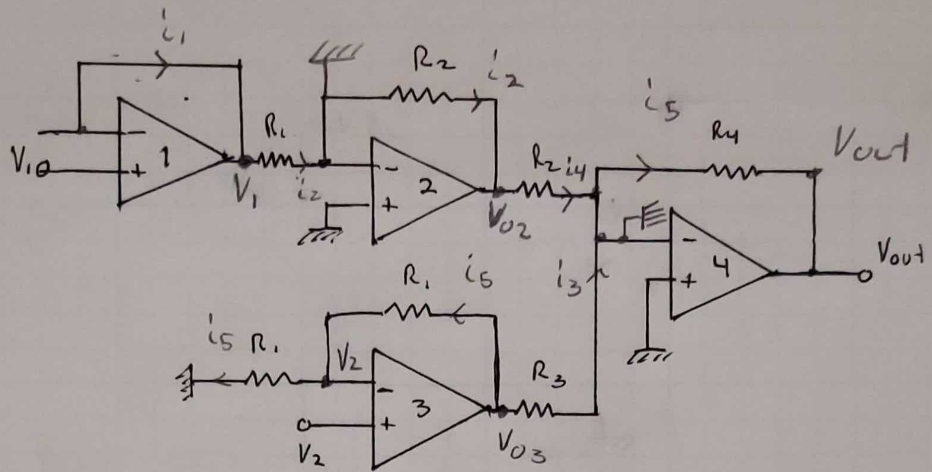
1. **Situation:** Find V_{out} vs V_1 and V_2 using the Ideal opamp model for the circuit shown below:

$R_1 = 1K$

$R_2 = 3K$

$R_3 = 2K$

$R_4 = 10K$



2. **Goal:** To derive an expression relating V_{out} to V_1 and V_2.

3. **Plan:** To use the "3-zero" rule along with Nodal circuit analysis to develop a system of equations to solve for V_{out} in terms of V_1, V_2.

4. **Solution:** Label the given diagram:

Observations Under "3-Zero" Rule:

$V_1 = V_{+1} = V_{-1} = V_{out1}$

$V_{+2} = V_{-2} = 0$

$V_2 = V_{+3} = V_{-3}$

$V_{+4} = V_{-4} = 0$

Nodal Analysis:

@ V_{-2}:

$$\frac{V_1}{R_1} = \frac{-V_{o2}}{R_2}$$

$$\therefore \; V_{o2} = \frac{-V_1 R_2}{R_1} \qquad (1)$$

@ V_{-3}:

$$\frac{(V_{o3} - V_2)}{R_1} = \frac{V_2}{R_1}$$

$$\therefore \; V_{o3} = 2V_2 \qquad (2)$$

$\boxed{2}$ (Continued)

@ V_{-4}: $i_4 + i_3 = i_5$

$$\frac{V_{o2}}{R_2} + \frac{V_{o3}}{R_3} = \frac{-V_{out}}{R_4} \qquad (3)$$

(2), (1) => (3):

$$\frac{-V_1}{R_1} + \frac{2V_2}{R_3} = \frac{-V_{out}}{R_4}$$

$$V_{out} = R_4\left(\frac{V_1}{R_1} - \frac{2V_2}{R_3}\right)$$

$R_4 = 10k$, $R_1 = 1k$, $R_3 = 2k$

$$V_{out} = 10k\left(\frac{V_1}{1k} - \frac{2V_2}{2k}\right)$$

$$\boxed{V_{out} = 20(V_1 - V_2)}$$

Sanity Check:

This result makes sense because:

(1) is a buffer with $V_{o1} = V_1$

(2) is an inverting Amp with $V_{o2} = -V_{o1}\left(\frac{R_2}{R_1}\right)$

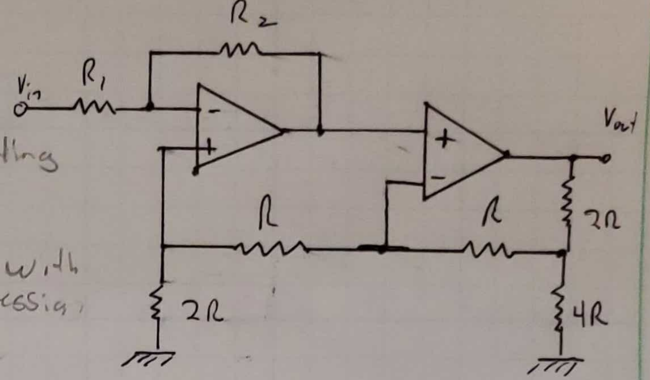
(3) is a non-inverting Amp with $V_{o3} = V_2\left(1 + \frac{R_1}{R_1}\right) = 2V_2$

then: (4) is a Summing Amplifier that adds the two, V_{o2}, V_{o3} and multiplies them by R_4/R_2 and R_4/R_3 Respectively

The individual topologies combine to get the correct relation

$\boxed{3(a)}$

1. **Situation:** Find the Vout expression Using Ideal Vs Vin model



2. **Goal:** To derive an expression relating Vout to Vin.

3. **Plan:** To use "3-zero" Rule along with Nodal analysis to derive an expression relating Vout to Vin.

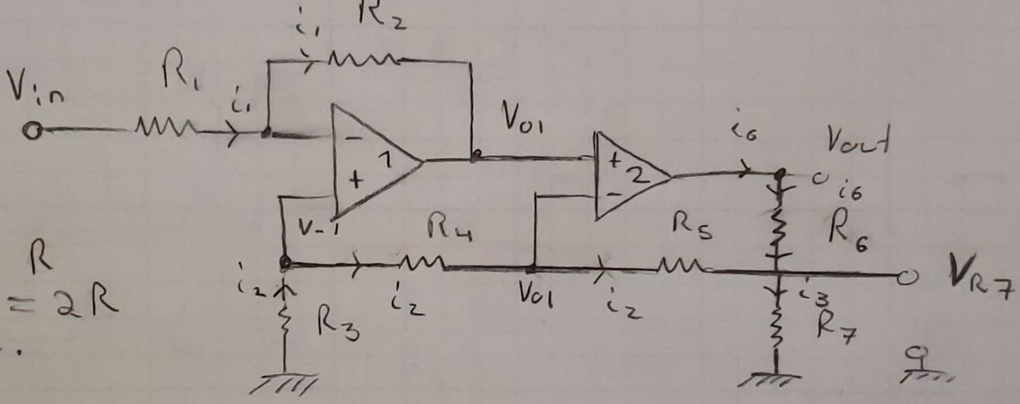
4. **Solution:** Redraw / Label.

Let:

$R_4 = R_5 = R$

$R_3 = R_6 = 2R$

$R_7 = 4R.$



Nodal @ V_{-1}

$$\frac{(V_{in} - V_{-1})}{R_1} = \frac{(V_{-1} - V_{o1})}{R_2} \quad (1)$$

@ V_{+1}

$$\frac{(-V_{-1})}{R_3} = \frac{(V_{-1} - V_{o1})}{R_4} \quad (2)$$

@ V_{-2}

$$\frac{(V_{-1} - V_{o1})}{R_4} = \frac{(V_{o1} - V_{R7})}{R_5} \quad (3)$$

@ V_{R7}

$$i_6 + i_2 = i_3$$

$$\frac{(V_{o1} - V_{R7})}{R_5} + \frac{(V_{out} - V_{R7})}{R_6} = \frac{V_{R7}}{R_7} \quad (4)$$

$\boxed{3(a)}$ (continued)

* 4 eqn's, 4 unknowns W.R.T Vout: V_{in}, V_{-1}, V_{o1}, V_{R7}.

∴ Solvable w/ Algebra

I will use my Calculator to minimize the liklihood of error.

Solve (1) for V_-:

$$\Longrightarrow \quad V_- = \frac{(V_{o1}R_1 + V_{in}R_2)}{R_1+R_2} \qquad (5)$$

(5) → (2) For V_{o1}:

$$\Longrightarrow \quad V_{o1} = \frac{-V_{in}R_2(3R)}{R_1 \cdot R - 2R_2R} \qquad (6)$$

(6), (5) → (3) for V_{R7}:

$$\Longrightarrow \quad V_{R7} = \frac{(V_{o1}-V_{in})R_2}{(R_1+R_2)} + V_{o1} \qquad (7)$$

Solve (6), (7), and (4) for Vout

$$V_{out} = 2V_{R7} \qquad (8)$$

$$= 2\,\frac{(V_{o1}-V_{in})R_2}{R_1+R_2} + V_{o1}$$

$$= 2\left(\frac{\left[\left(\frac{-V_{in}R_2(3R)}{R_1 \cdot R - 2R_2R}\right) - V_{in}\right]R_2}{R_1+R_2} + \left(\frac{-V_{in}R_2(3R)}{R_1 \cdot R - 2R_2 \cdot R}\right)\right)$$

$$\boxed{V_{out} = \frac{-8V_{in}R_2}{R_1 - 2R_2}}$$

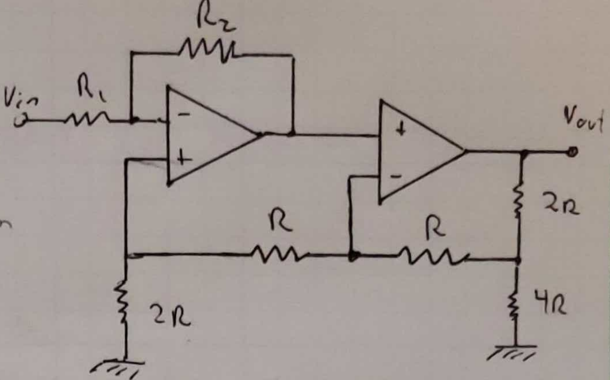
Sanity Check

Part 3(b) is Sanity Check

$\boxed{8/22}$

$\boxed{3(b)}$

1 **Situation:** Verify the derivation from 3(a), using two different methods. Unit Check is not allowed.



2 **Goal:** To verify the result from 3(a) using two methods.

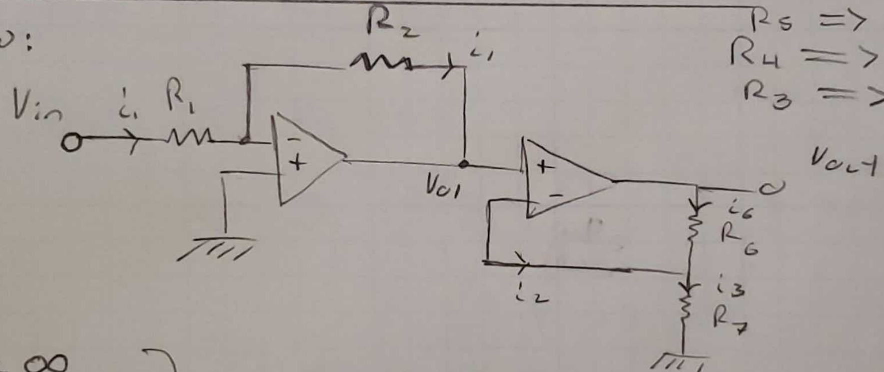
3 **Plan:** To ▷1 to an inverting Amp and Vice-Versa then solve the system's for Vout and compare to their mathematical equivalent operations to the expressions found in 3(a).

4 **Solution:**

1.) ▷1 => inverting Amp, ▷2 => Non inverting;

$R_5 \Rightarrow$ Short
$R_4 \Rightarrow$ Open
$R_3 \Rightarrow$ Short

Redraw:



$$\left.\begin{aligned} R_4 &= \infty \\ R_3 &= 0 \\ V_- &= 0 \end{aligned}\right\}$$

expected V_{o1}:

$$V_{o1} = -R_2/R_1 \, V_{in}$$

Check eqn 1 from Part 3(a):

$$\frac{(V_{in} - V_{-1})}{R_1} = \frac{(V_{-1} - V_{o1})}{R_2}$$

$$\frac{V_{in}}{R_1} = \frac{-V_{o1}}{R_2} \quad \because \quad V_{-1} = 0$$

$$\therefore \; V_{o1} = \frac{-R_2}{R_1} V_{in}$$

So eqn 1 holds true for $R_4 = \infty$, $R_3 = 0$.

$\boxed{3(b)}$ (Continued)

Check eqn (2) from 3(a) for $R_4 = \infty$, $R_3 = 0$

* $V_{-1} = 0$

$$\therefore \quad \frac{-V_{-1}}{R_3} = \frac{(V_{-1} - V_{o1})}{R_4}$$

$$0 = \frac{-V_{o1}}{\infty}$$

$$0 = 0$$

So eqn (2) holds true for the conditions

Check eqn for V_{out} (8)

$$V_{out} = 2\,V_{R7}$$

$$= 2\left(\frac{(V_{o1} - V_{in})\,R_2}{R_1 + R_2} + V_{o1}\right)$$

$$= 2\left(\frac{\left(\frac{-R_2}{R_1} V_{in} - V_{in}\right) \cdot R_2}{R_1 + R_2} - \frac{R_2}{R_1} V_{in}\right)$$

$$= \frac{-4 R_2 V_{in}}{R_1} = -4 V_{in} \frac{R_2}{R_1}$$

For noninverting amp, the output should be $V_{out} = (1 + R_6/R_7)\,V_{o1}$

$$V_{out} = \left(1 + \frac{2R}{4R}\right)\left(-\frac{R_2}{R_1} V_{in}\right)$$

$$= \ldots\ldots$$

$\boxed{3(b)}$ (continued)

5 Sanity Check:

I'm not sure how to conveniently do a topology check, because any resistors that I could remove are a multiple of R. So if I short one, $R=0$, or open one, $R=\infty$, this changes the circuit parameters completely.

Furthermore, If I did solve the new Vout expressions, I will not be able to relate them to my original expression from 3(a),

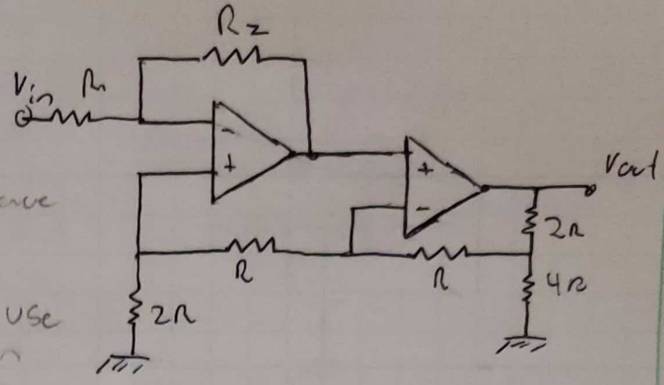
So I will continue to part c.) under the assumption my result from a.) is correct.

I did check it with multiple sets of equations.

$$\boxed{V_{out} = \frac{-8V_{in}R_2}{R_1 - 2R_2}}$$

$\boxed{3(c)}$

1 **Situation:** Calculate transresistance gain if $R_1 = 2R_2$.



2 **Goal:** To Calculate the transresistance gain for $R_1 = 2R_2$.

3 **Plan:** To Set $R_1 = 2R_2$, then use the transresistance gain équation to Solve for the transresistance gain. $A_r = V_{out}/I_{in}$

4 **Solution:**

Equation: $A_R = V_{out}/I_{in}$

$$V_{out} = \frac{-8 V_{in} R_2}{R_1 - 2R_2}$$

$$\frac{V_{out}}{V_{in}} = A_V = \frac{-8R_2}{R_1 - 2R_2}$$

$$A_R = R_{in} \times A_V$$

$R_{in} = R_1$ ∵ Lecture 37 Slide 4.

$$\therefore A_R = \frac{-8R_1R_2}{R_1 - 2R_2}$$

For $R_1 = 2R_2$

$$A_R = \frac{-8R_1R_2}{0} \longrightarrow \infty$$

$$\therefore \boxed{A_R \longrightarrow \infty}$$

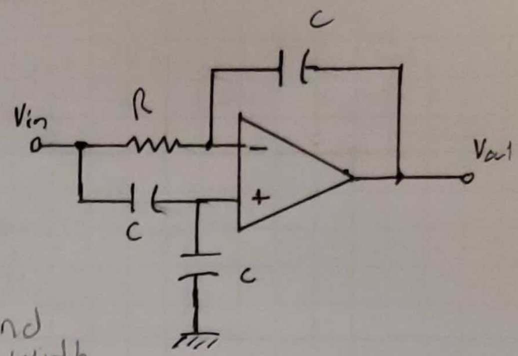
5 **Sanity Check:** This can also be Verified with:

$$A_R = V_{out}/I_{in} = \frac{\left(\frac{-8V_{in}R_2}{2R_2 - 2R_2}\right)}{I_{in}} = \frac{\infty}{I_{in}}$$

$$= \infty \quad \checkmark$$

$\boxed{4}$

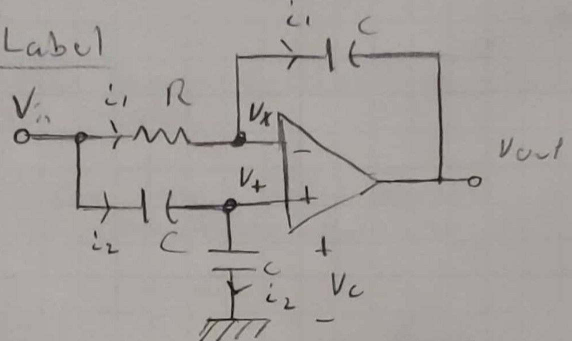
1 **Situation:** Find Vout vs Vin in the time domain for the given Op Amp. Use Ideal op amp model.



2 **Goal:** To derive the input-output relation for the given amplifire.

3 **Plan:** To use the "3-zero" rule and capacitor current equation along with Nodal analysis to derive a relation between Vout and Vin.

4 **Solution.** Redraw / Label



$V_+ = V_-$

$I_+ = I_- = 0$

$V_{in} = 2V_c$

$V_+ = V_c = \frac{V_{in}}{2}$

Nodal @ V_-:

$$\frac{(V_{in} - V_+)}{R} = C\,\frac{d(V_+ - V_{out})}{dt}$$

$$\therefore \quad \frac{(V_{in} - V_{in}/2)}{R} = C\,\frac{d(V_{in}/2 - V_{out})}{dt}$$

$$V_{in} = 2RC\left(\frac{dV_{in}}{dt}\cdot\frac{1}{2} - \frac{dV_{out}}{dt}\right)$$

$$\frac{1}{2RC}(V_{in}\,dt) = \frac{dV_{in}}{2} - dV_{out}$$

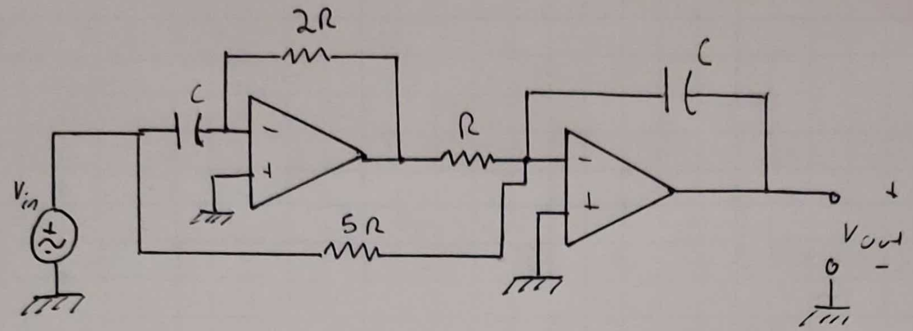
$$\frac{1}{2RC}\int_{t_1}^{t_2} V_{in}\,dt = \frac{1}{2}\int dV_{in} - \int dV_{out}$$

$$\frac{1}{2RC}\int_{t_1}^{t_2} V_{in}(t)\,dt = \frac{V_{in}}{2} - V_{out}$$

$$\therefore \quad \boxed{V_{out} = \frac{V_{in}}{2} - \frac{1}{2RC}\int_{t_1}^{t_2} V_{in}(t)\,dt}$$

5.

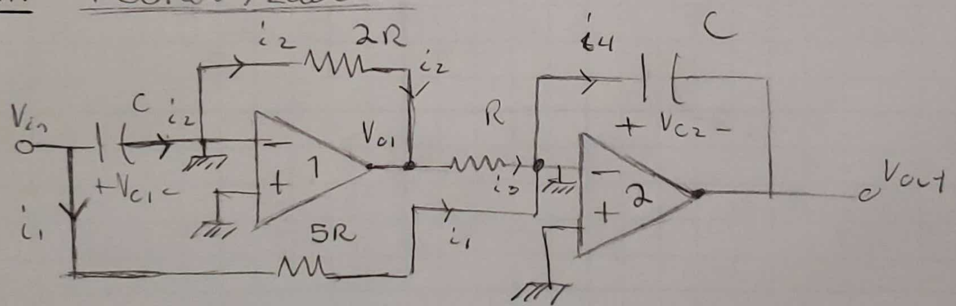
1 **Situation:** Find the input-output relation for the following circuit in the time domain, using the Ideal opamp models



2 **Goal:** To find the time domain relation of V_{out} to V_{in} under the ideal opAmp model.

3 **Plan:** To use the "3-zero" rule, Nodal analysis, as well as the time domain capacitor current equation to derive an expression relating V_{out} to V_{in}.

4 **Solution:** Redraw / Label:



Observe:

$$V_{in} = V_{c1}$$

$$V_{out} = -V_{c2}$$

$$i_1 = \frac{V_{in}}{5R} = \frac{V_{c1}}{5R}$$

Nodal @ V_{-1} Node:

$$i_2 = C\frac{dV_{c1}(t)}{dt} = C\frac{dV_{in}(t)}{dt}$$

$$\therefore \quad C\frac{dV_{in}(t)}{dt} = -\frac{V_{o1}}{2R}$$

$$\therefore \quad V_{o1} = -2RC \frac{d}{dt}\left(V_{in}(t)\right) \quad (1)$$

$\boxed{5}$ (Continued)

Nodal @ V-2 Node: $\quad i_1 + i_3 = i_4$

$$\frac{V_{o1}}{R} + \frac{V_{in}}{5R} = C\frac{dV_{c2}(t)}{dt}$$

$$\therefore \quad \frac{V_{o1}}{R} + \frac{V_{in}}{5R} = -C\frac{d}{dt}\left(V_{out}(t)\right)$$

$$5V_{o1} + V_{in} = -5RC\,\frac{d}{dt}V_{out}(t)$$

$$-\frac{V_{o1}}{RC}dt - \frac{V_{in}}{5RC}dt = dV_{out}$$

$$\frac{-1}{RC}\left[\int_{t_1}^{t_2} V_{in}\,dt + \int_{t_1}^{t_2} V_{o1}\,dt\right] = \int dV_{out} \qquad (2)$$

(1) → (2)

$$\therefore \quad V_{out} = \frac{-1}{RC}\left[\int_{t_1}^{t_2} V_{in}dt - \int_{t_1}^{t_2} 2RC\frac{d}{dt}(V_{in}(t))dt\right]$$

$$V_{out} = \frac{-1}{RC}\int_{t_1}^{t_2} V_{in}(t)\,dt + 2V_{in}(t)$$

$$\boxed{V_{out}(t) = 2V_{in}(t) - \frac{1}{RC}\int_{t_1}^{t_2} V_{in}(t)\,dt}$$

5 Sanity Check:

- The output of the derivative opamp was a derivative quantity of $V_{in}(t)$. (1)
- The output of the integration OpAmp was an integral quantity of $V_{in}(t)$, shifted by the integral of the derivative quantity from (1). This makes sense because the second op amp has input schematic similar to an adder, but it also integrates.

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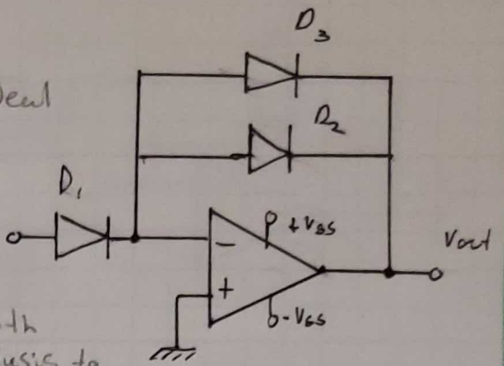
6.

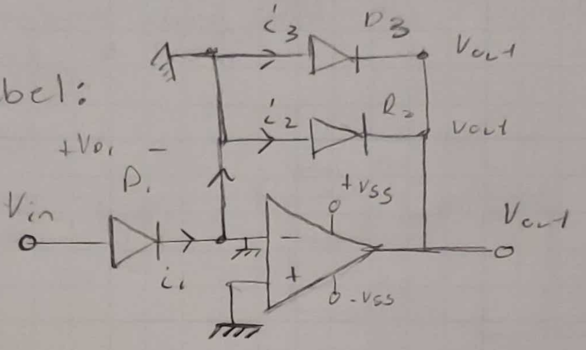
1 Situation: Find the input output relation for the following circuit. Use ideal opamp model. All diodes are identical.



2 Goal: To derive an expression relating Vout to Vin using the ideal opamp model.

3 Plan: To use the "3-zero" rule along with the diode current equation and Nodal analysis to relate Vout to Vin.

4 Solution: Redraw/Label:



Observe:

$V_{D1} = V_{in}$

$V_{D2} = V_{D3} = -V_{out}$

Diode Current eqn: $I_D = I_S \exp(V_D / nV_T)$

$V_T = 26 \times 10^{-3}$ V

$\therefore \; i_1 = I_S \exp\left(\frac{V_{in}}{nV_T}\right)$

Nodal @ V_- Node

$$i_1 = i_2 + i_3$$

$$I_S \exp\left(V_{in}/nV_T\right) = 2 I_S \exp\left(\frac{-V_{out}}{nV_T}\right)$$

$$\frac{V_{in}}{nV_T} = -\frac{2V_{out}}{nV_T}$$

$$V_{in} = -2V_{out}$$

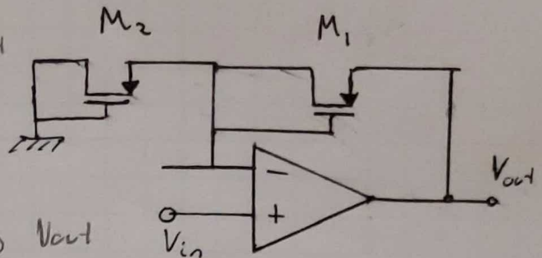
$$\therefore \boxed{V_{out} = -V_{in}/2}$$

5 Sanity Check:

Here, the diodes can be thought of like resistors, assuming forward bias in saturation. Then the effective resistances would be $R_1 = R$, $R_2 = R \| R = R/2$, and the inverting amplifier would have a gain: $V_{out} = \frac{-R_2}{R_1} V_{in}$

$= -\frac{1}{2} V_{in}$

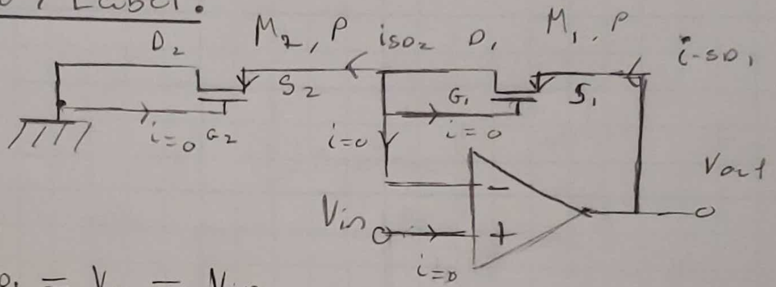
$\boxed{7}$ 1 **Situation:** Find the input-output relation Vout vs Vin. Use Ideal Opamp Model and $\lambda = 0$ for Mosfet in Circuit Shown.



2 **Goal:** To derive an expression relating Vout to Vin using $\lambda = 0$ and Ideal Opamp model.

3 **Plan:** To use nodal analysis along with the Mosfet current equation to relate Vout to Vin.

4 **Solution:** Redraw / Label:



Observe:

$V_{G2} = V_{D2}$

$V_{S2} = V_{G1} = V_{D1} = V_- = V_{in}$

$V_{S1} = V_{out}$

MOSFET Current: $I_{SD} = KP_p (W/L)_q (V_{OD})^2$

where $V_{OD} = (V_{GS} - V_{TH})$

$V_{gs_1} = V_{in} - V_{out}$ $\qquad$ $\because$ $V_G = V_- = V_+ = V_{in}$, $V_S = V_{out}$

$V_{gs2} = V_{G_2} - V_{S2}$

$= 0 - V_{in}$

$\therefore$ $V_{gs2} = -V_{in}$

$\therefore$ $I_{DS2} = I_{DS1}$

$KP_p (W/L)_p (V_{OD_2})^2 = KP_p (W/L)_p (V_{OD_1})^2$

$V_{OD2}^2 = V_{OD1}^2$

$V_{OD2} = V_{OD1}$

$(-V_{in} - V_{TH}) = (V_{in} - V_{out} - V_{TH})$

$\boxed{7}$ (continued)

$$\therefore \quad -V_{in} = V_{in} - V_{out}$$

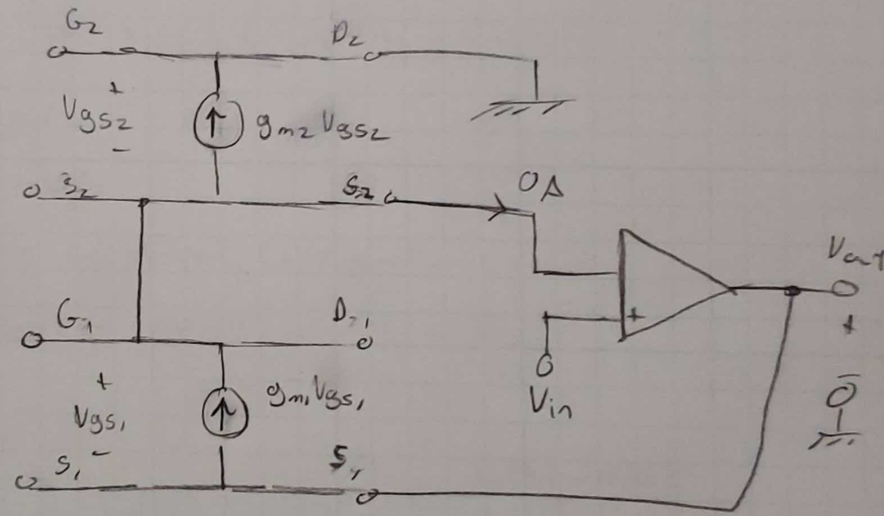
$$\boxed{V_{out} = 2V_{in}}$$

s Sanity Check:

Solve using Small Signal instead: $\lambda = 0$

$M_2 \Rightarrow$

$M_1 \Rightarrow$



$$V_{gs2} = -V_{in}$$
$$V_{gs1} = V_{in} - V_{out}$$

KVL; Nodal at S_2:

$$g_{m2}V_{gs2} = g_{m1}V_{gs1}$$
$$-V_{in} = V_{in} - V_{out}$$
$$\boxed{V_{out} = 2V_{in}} \quad ✓$$

$\boxed{8}$

1 **Situation:** Design an OPAMP based circuit that divides two inputs. Derive the circuits input-output relation, i.e.) $V_{out} = A(V_{in1}/V_{in2} + B)$

2 **Goal:** To create an OPAMP circuit to divide two inputs.

3 **Plan:** To use the equation: $V_1/V_2 = \exp(\ln(V_1) - \ln(V_2))$ as a basis to create a divider op amp circuit using diodes and their associated current equation (exponential).

4 **Solution:**

Equation:
$$V_1/V_2 = \exp(\ln(V_1) - \ln(V_2))$$
$$= \exp(\ln(V_1) + (-\ln(V_2)))$$

Needed Topologies:

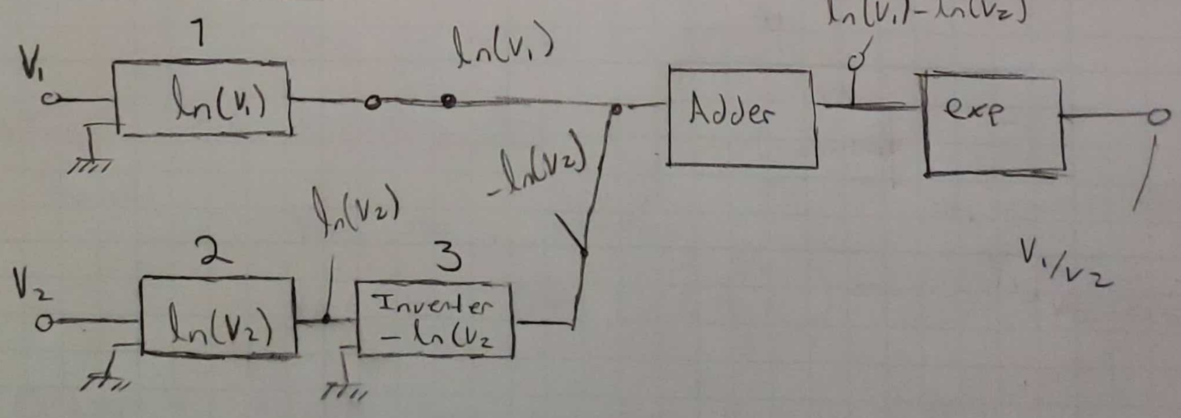
(2) log opamps

(1) exp OPAMP in Adder Configuration

(1) Inverting Buffer OpAmp $\therefore R_1 = R_2$

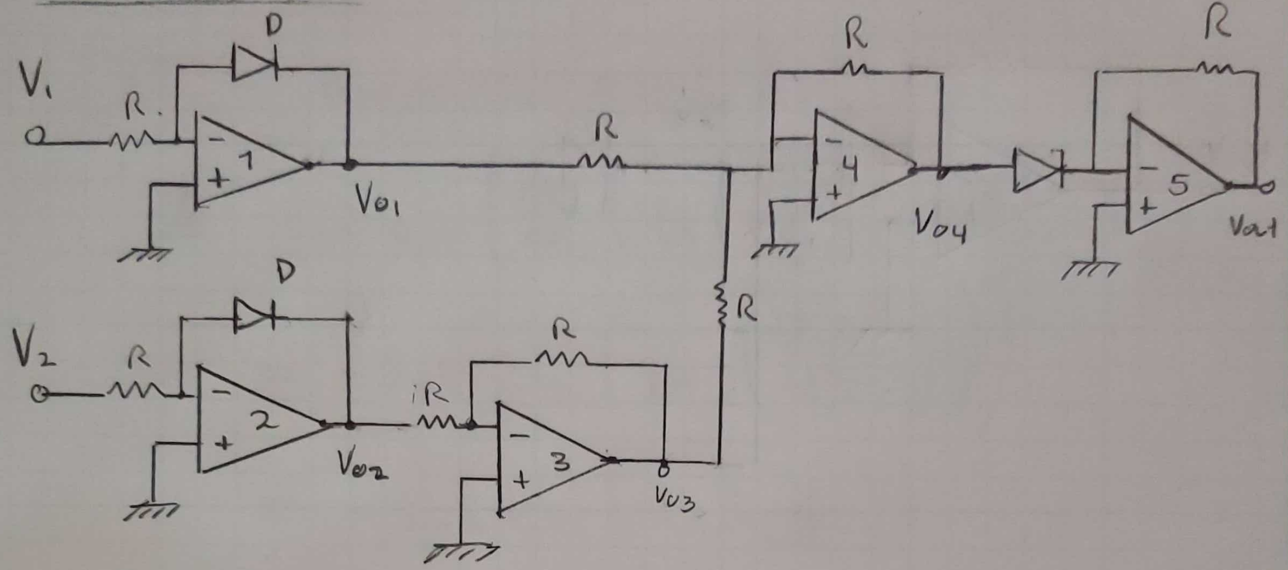
(1) Adder

Block Diagram



8 (Continued)

Draw Schematic



OPAMPS 1, 2: $\ln(v_{in})$

Known:

$$V_{o1} = -nV_T \ln\left(\frac{V_1}{I_S R}\right)$$

$$V_{o2} = -nV_T \ln\left(\frac{V_2}{I_S R}\right)$$

$\therefore$ lect 38

OPAmp 3: Inverting Amp

Known:

$$V_{o3} = -\frac{R}{R} V_{o2}$$

$$V_{o3} = nV_T \ln\left(\frac{V_2}{I_S R}\right)$$

$\therefore$ Known I/O Relation

OPAMP 4: Adder Amp: (with $R_1 = R_2 = R$)

Known:

$$V_{o4} = -(V_{o1} + V_{o3})$$

$$= -\left(-nV_T \ln\left(\frac{V_1}{I_S R}\right) + nV_T \ln\left(\frac{V_2}{I_S R}\right)\right)$$

$$\therefore V_{o4} = nV_T\left(\ln\left(\frac{V_1}{I_S R}\right) - \ln\left(\frac{V_2}{I_S R}\right)\right)$$

$\boxed{8}$ (continued)

OpAmp 5 : exp

Known:

$$V_{out} = -I_S R \exp\left(\frac{V_{o4}}{n V_T}\right)$$

$$= -I_S R \exp\left(\frac{nV_T}{nV_T}\left(\ln\left(\frac{V_1}{I_S R}\right) - \ln\left(\frac{V_2}{I_S R}\right)\right)\right)$$

$$= -I_S R \exp\left(\ln\left(\frac{\frac{V_1}{I_S R}}{\frac{V_2}{I_S R}}\right)\right)$$

$$= -I_S R \exp\left(\ln\left(V_1/V_2\right)\right)$$

$$= -I_S R \cdot V_1/V_2$$

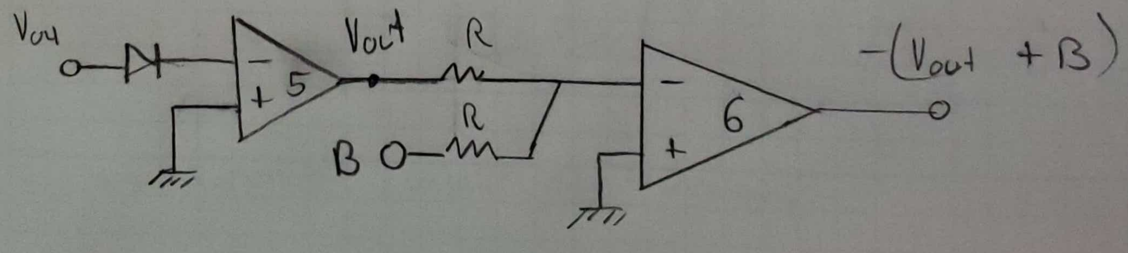
$$\therefore \boxed{V_{out} = -I_S R \, V_1/V_2}$$

This is in the form $V_{out} = A(V_1/V_2) + B$

Where $A = -I_S R$, $B = 0$

If a Non-Zero B term was desired, I could put another Adder after OpAmp 5 with B as another input:

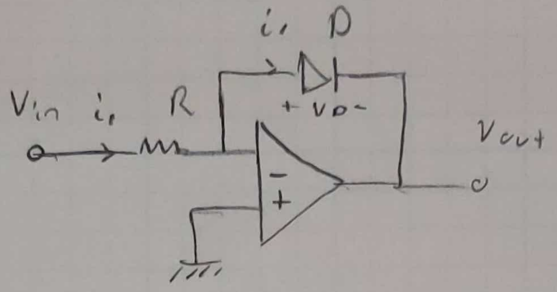
For B



$\boxed{8}$ (continued)

5 Sanity Check: Check assumptions

OpAmps 1, 2



$V_D = -V_{out}$

$i_1 = i_1$

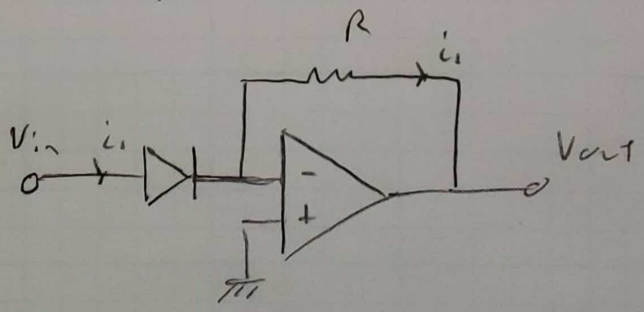
$$\frac{V_{in}}{R} = I_s \exp\left(\frac{-V_{out}}{nV_T}\right)$$

$$\ln\left(\frac{V_{in}}{I_s R}\right) = \ln\left(\exp\left(\frac{-V_{out}}{nV_T}\right)\right)$$

$$\boxed{-nV_T \ln\left(\frac{V_{in}}{I_s R}\right) = V_{out}}$$ ✓

OpAmps 3, 4 have been verified in class multiple times.

OpAmp 5



$V_D = V_{in}$

$i_1 = i_1$

$$I_s \exp\left(\frac{V_{in}}{nV_T}\right) = -V_{out}/R$$

$$\boxed{V_{out} = R I_s \exp\left(V_{in}/nV_T\right)}$$ ✓

I am confident with my design.

$\boxed{22/22}$