

ECE 310 – Microelectronics I

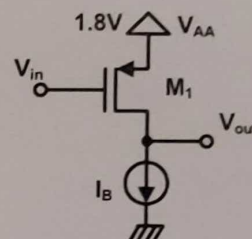
Homework #6

Fall 2021

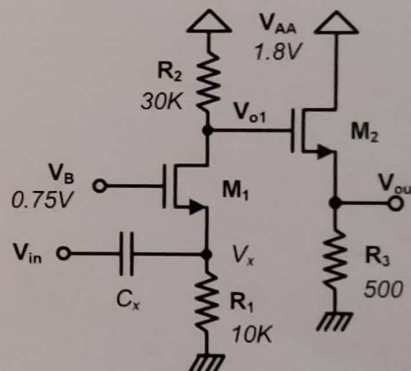
(Due Date: 11/05/2021, 8.30am)

For the problems 1 & 2, use: $KP_n=100 \mu A/V^2$, $KP_p=50 \mu A/V^2$, $V_{THn}=|V_{THp}|=0.5V$, $\lambda_n=0.05$, $\lambda_p=0.1$

1. (20pts) A common source (CS) amplifier will be designed. Assume M_1 is biased 100mV in ON region, $V_{DSAT1}=0.8V$, and $I_B=20\mu A$.
 - a. (15pts) Find $V_{in(DC)}$, $V_{out(DC)}$, and $(W/L)_1$ values.
 - b. (10pts) Find small signal voltage gain (A_v) expression and calculate its value.

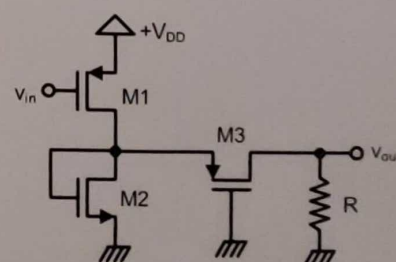


2. (55pts) Following is a multi stage amplifier. Assume; $I_{ds2}=1mA$ and $V_{X,DC}=0.2V$ by design.
 - a. (18pts) Find, I_{ds1} , V_{od1} , V_{dsat1} , $V_{o1(DC)}$, $(W/L)_1$, $(W/L)_2$, and V_{od2} , V_{dsat2} , $V_{out(DC)}$, values.
 - b. (12pts) Draw the small signal equivalent circuit (SSEC).
 - c. (15pts) Find small signal voltage gains $A_{v1}=v_{o1}/v_{in}$, $A_{v2}=v_{out}/v_{o1}$, and overall voltage gain ($A_v=v_{out}/v_{in}$) of the amplifier.
 - d. (10pts) What would be the C_x capacitor value chosen so that this circuit can accept any signal that has frequency higher than 100Hz.



3. (35 pts) Analyze following amplifier. Use; $g_{m1}=10 mS$, $g_{m2}=1mS$, $g_{m3}=4mS$, $R=1Kohm$

- a. (10pts) Draw small signal equivalent circuit for $\lambda=0$
- b. (10pts) Find small signal output resistance expression and its value, $R_{OUT}=?$
- c. (10pts) Find small signal transconductance expression and its value, $G_m=?$
- d. (5pts) Find small signal voltage gain expression, and calculate its value, $A_v=?$

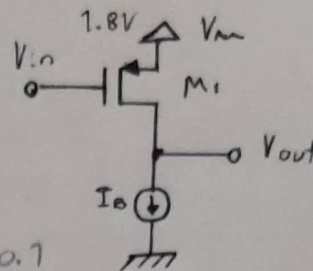


1(a)

Situation: Design a Common Source (CS) Amplifier. Assume M_1 is biased $100\mu V$ in pN Region, $V_{DSAT} = 0.8V$, $I_B = 20\mu A$. a.) Find $V_{in(DC)}$, $V_{out(DC)}$, and $(W/L)_1$ Values.

Given: $KP_n = 100 \mu A/V^2$, $KP_p = 50 \mu A/V^2$

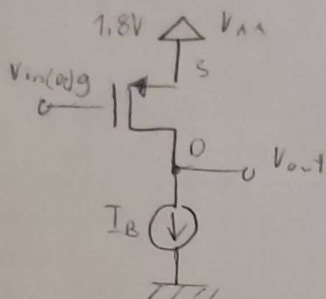
$V_{THn} = |V_{THp}| = 0.5V$, $\lambda_n = 0.05$, $\lambda_p = 0.1$



Goal: To Find / calculate Values For $V_{in(DC)}$, $V_{out(DC)}$, and $(W/L)_1$.

Plan: To use known / given Transistor equations to determine $V_{in(DC)}$, $V_{out(DC)}$ and $(W/L)_1$ For the given PMOS CS Amplifier

Solution: Redraw:



Observe:

$$V_g = V_{in}$$

$$V_s = V_{DD} = 1.8V$$

$$V_o = V_{out}$$

$$I_B = I_{SD} = 20 \times 10^{-6} A$$

Overdrive Voltage: $V_{OD} = V_{SG} - |V_{TH}|$

$$0.1 = V_{SG} - 0.5$$

$$\therefore V_{OD} = 0.1, \text{ *Given}$$

$$\therefore V_{SG} = 0.6V$$

KVL

$$V_{DD} - V_{SG} - V_{in(DC)} = 0$$

$$1.8 - 0.6 = V_{in(DC)}$$

$$\therefore V_{in(DC)} = 1.2V$$

SATURATION Voltage:

$$V_{D,SAT} = V_{SD} - V_{OD}$$

$$0.8 = V_{SD} - 0.1$$

$$\therefore V_{SD} = 0.9V$$

$$V_{SD} = V_s - V_o$$

$$0.9 = 1.8 - V_o$$

$$V_o = V_{out(DC)}$$

$$\therefore V_{out(DC)} = 0.9V$$

1(a)

 (Continued)
SAT PMOS I_{SD}

$$I_{SD} = \frac{1}{2} \mu_p \left(\frac{W}{L} \right)_1 (V_{DD})^2 (1 + 2V_{SD})$$

$$20 \times 10^{-6} = 0.5 (50 \times 10^{-6}) \left(\frac{W}{L} \right)_1 (0.1)^2 (1 + 0.1(0.9))$$

$$\therefore \left(\frac{W}{L} \right)_1 = \frac{2(20 \times 10^{-6})}{(50 \times 10^{-6})(0.1)^2(1 + 0.1(0.9))}$$

$\left(\frac{W}{L} \right)_1 = 73.39$

5 Sanity CheckCheck KVL's

$$V_{AA} = V_{out(DC)} + V_{GD} + V_{SG}$$

$$1.8 = 0.9 + (1.2 - 0.9) + 0.6$$

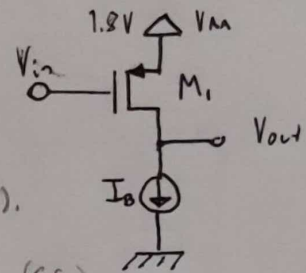
$$1.8 = 1.8 \quad \checkmark$$

$$V_{AA} - V_{SD} - V_{out(DC)} = 0$$

$$1.8 - 0.9 - 0.9 = 0 \quad \checkmark$$

1(b)

Situation: Design a (CS) Amplifier.
 Assume M_1 is biased 700mV ON, $V_{DSAT} = 0.8V$,
 $I_D = 20\mu A$, b.) Find small signal gain
 (A_v) expression and calculate its Value. Use
 Same Given Characteristic Parameters as in 1(a).

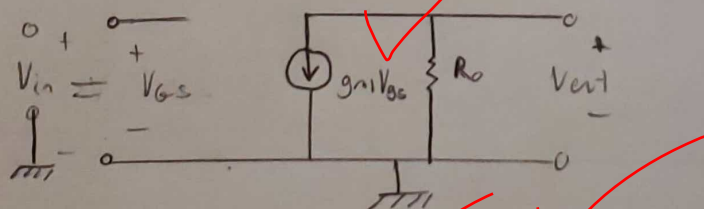
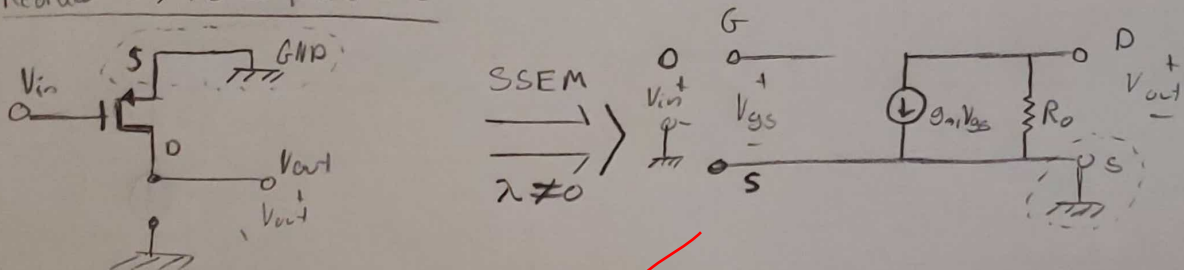


2 Goal: To Derive an expression for the given (CS) Amplifier's Gain (A_v), then Calculate its Value.

3 Plan: To find the small signal equivalent model for the CS Amp, then follow the procedure for determining A_v . Then Plug in known/calculated values to evaluate A_v .

4 Solution

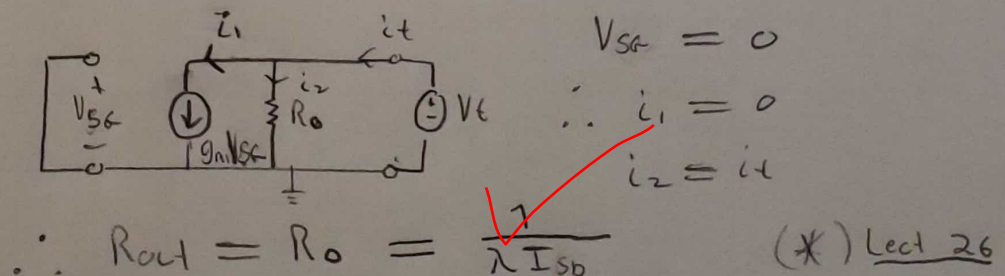
Redraw \Rightarrow DC components $= 0$



Voltage Gain: $= Z_{out} \cdot G_m = R_{out} \cdot G_m$

Find R_{out} : Solve $R_{out} = V_t / I_t |_{R_{in} = 0}$, V_t on output

Redraw

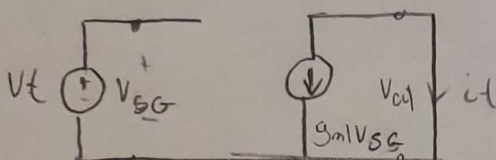
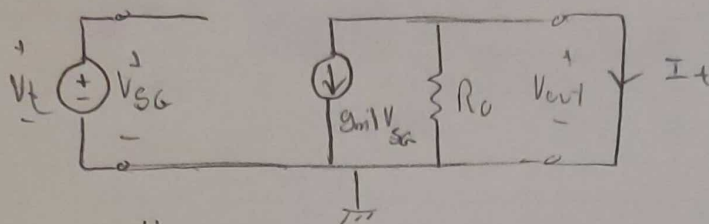


1(b) (continued)

Find G_m , Place V_t between input ports, Short output

Solve $G_m = i_t / V_t \mid R_L = 0$

Redraw:



$$i_t = -g_m V_{SG}$$

$$V_t = V_{SG}$$

$$\frac{i_t}{V_t} = -g_m$$

$$\therefore G_m = -g_m$$

$$G_m = \frac{-2 I_{SD}}{(V_{SG} - V_{TH})}$$

* Lect 27

Find A_v

$$A_v = R_{out} \cdot G_m$$

$$= \left(\frac{1}{\lambda I_{SD}} \right) \frac{2 I_{SD}}{(V_{SG} - V_{TH})}$$

$$A_v = \frac{-2}{\lambda (V_{SG} - V_{TH})}$$

Evaluate:

$$A_v = \frac{-2}{0.1(0.6 - 0.5)}$$

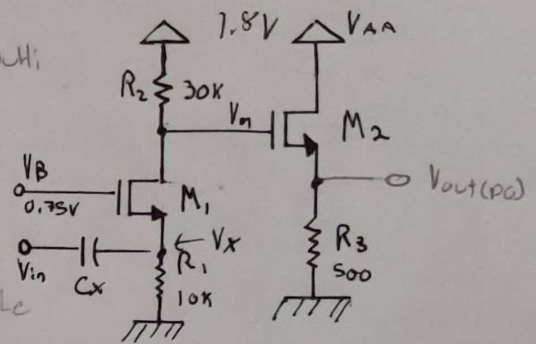
$$A_v = -200$$

Sanity Check:

$$\begin{aligned} g_m &= K_P \left(\frac{W}{L} \right) (V_{SG} - V_{TH}) (1 + \lambda V_{SD}) = \frac{2 I_{SD}}{(V_{SG} - V_{TH})} \\ &= 50 \times 10^{-6} (73.39) (0.1) (1 + 0.1(0.9)) = \frac{2 (20 \times 10^{-6})}{0.1} \\ &= 0.0004 = 0.0004 \quad \checkmark \end{aligned}$$

2(a) Situation: The following is a multi-stage amplifier. Assume $I_{D2} = 1\text{mA}$ and $V_{x,DC} = 0.2\text{V}$. Use same given transistor characteristics as 1(a).

a.) Find I_{D1} , V_{D1} , V_{DS1} , V_{D1} , $(W/L)_1$, $(W/L)_2$, V_{D2} , V_{DS2} , $V_{out(AC)}$, $V_{out(PS)}$.



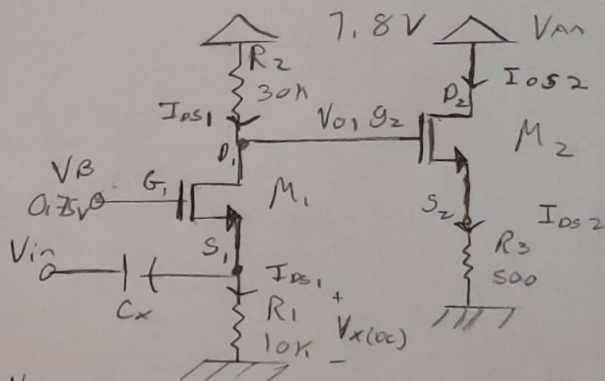
3Goal: To determine the values for the quantities listed above.

3Plan: To use known transistor voltage and current expressions along with the fundamental theorems of circuit analysis to find / calculate values for the above stated quantities.

4Solution:

Given: $I_{D2} = 0.001\text{A}$
 $V_x = 0.2\text{V}$

Observe: $V_{D1} = V_{G2} = V_{D1}$
 $V_{S1} = V_x = 0.2\text{V}$
 $V_{D1} = V_{G2}$



$$\underline{I_{D1}} \quad I_{R1} = I_{D1} = \frac{V_{x(AC)}}{R_1} \quad \therefore \text{Ohm's Law } R_1$$

$$= \frac{0.2}{10000}$$

$$\boxed{I_{D1} = 2 \times 10^{-5} \text{ A} = 20 \mu\text{A}}$$

$$\therefore V_{R2} = I_{D1}(R_2)$$

$$= 20 \times 10^{-6} (30000)$$

$$V_{R2} = 0.6\text{V}$$

$$\therefore V_{DS1} = V_{DD} - V_{x(AC)} - V_{R2} \quad \text{KVL} \rightarrow M_1$$

$$= 1.8 - 0.2 - 0.6$$

$$V_{DS1} = 1\text{V}$$

$$\therefore V_{D1(AC)} = V_{x(AC)} + V_{DS1}$$

$$= 0.2 + 1$$

$$\boxed{\therefore V_{D1(AC)} = 1.2\text{V}}$$

2(a) (Continued)

 V_{DD1}

$$V_{G1} = V_B = 0.75$$

$$V_{S1} = V_{X(OC)} = 0.2 \text{ V}$$

$$\therefore V_{GS1} = 0.55 \text{ V}$$

$$V_{DD1} = V_{GS1} - V_{TH,n}$$

$$= 0.55 - 0.5$$

$$\boxed{V_{DD1} = 0.05 \text{ V}}$$

* Lect 23, S2

 $V_{DSAT,1}$

$$V_{DSAT,1} = V_{DS1} - V_{DD1}$$

$$= 1 - 0.05$$

$$\therefore \boxed{V_{DSAT,1} = 0.95 \text{ V}}$$

* Lect 23, S2

 $(W/L)_1$

$$I_{DS1} = \frac{1}{2} k_P n \left(\frac{W}{L}\right)_1 (V_{DD1})^2 (1 + \lambda V_{DS})$$

$$\left(\frac{W}{L}\right)_1 = \frac{2 I_{DS1}}{(k_P n) (V_{DD1})^2 (1 + \lambda V_{DS})}$$

$$= \frac{(20 \times 10^{-6}) \cdot 2}{(100 \times 10^{-6}) (0.05)^2 (1 + 0.05 \cdot 1)}$$

$$\boxed{\left(\frac{W}{L}\right)_1 = 152.38}$$

 $V_{out(OC)}$

$$V_{out(OC)} = V_{R3} = I_{DS2} (R_3)$$

$$= 0.001 (500)$$

$$\therefore \boxed{V_{out(OC)} = 0.5 \text{ V}}$$

Ohm's law R_3

$$= V_{S2}$$

$$V_{G2} = V_{O1C} = 1.2 \text{ V}$$

* KVL

$$V_{DD2} = V_{GS2} - V_{TH,n}$$

$$= (1.2 - 0.5) - 0.5$$

$$\boxed{V_{DD2} = 0.2 \text{ V}}$$

2(a) (Continued)

V_{DSAT2}

$$V_{DSAT2} = V_{DS2} - V_{DS1}$$

$$\begin{aligned} V_{DS2} &= V_{D2} - V_{S2} \\ &= V_{AA} - V_{out(DC)} \\ &= 1.8 - 0.5V \end{aligned}$$

$$V_{DS2} = 1.3V$$

$$\therefore V_{DSAT2} = 1.3 - 0.2$$

$$\boxed{V_{DSAT2} = 1.1V}$$

$(\frac{W}{L})_2$

$$I_{DS2} = \frac{1}{2} K_{PN} \left(\frac{W}{L}\right)_2 (V_{DS2})^2 (1 + \lambda V_{DS2})$$

$$\begin{aligned} \left(\frac{W}{L}\right)_2 &= \frac{2I_{DS2}}{K_{PN} \cdot (V_{DS2})^2 (1 + \lambda V_{DS2})} \\ &= \frac{2(0.001)}{100 \times 10^{-6} (0.2)^2 (1 + 0.05 \cdot 1.3)} \end{aligned}$$

$$\boxed{\therefore \left(\frac{W}{L}\right)_2 = 469.484}$$

5 Sanity Check

Check KVL's

$$\begin{aligned} V_{S2} + V_{GS2} - V_{DS1} - V_{S1} &= 0 \\ 0.5 + 0.7 - 1 - 0.2 &= 0 \\ 1.2 - 1.2 &= 0 \\ 0 &= 0 \end{aligned}$$

✓

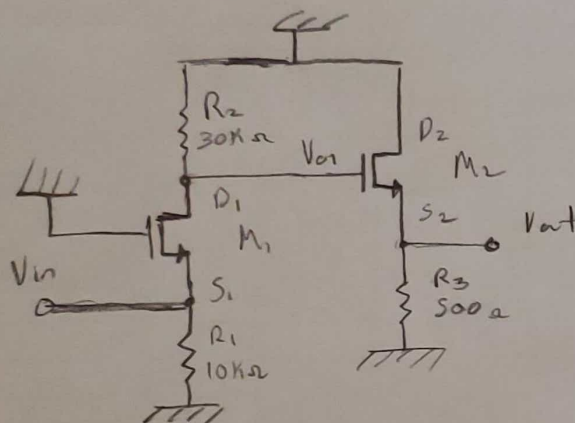
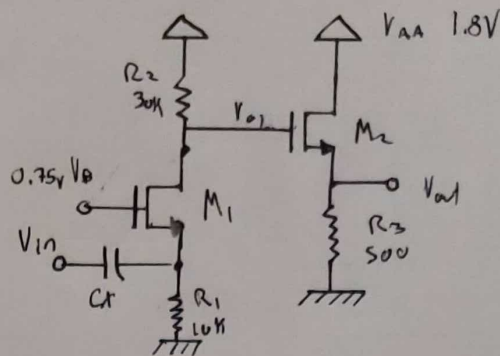
(*) Answers are reasonable and dependent on each other, and KVL is fulfilled

2(b) Situation: For the multistage amplifier described in 2(a.):
b.) Draw the small signal equivalent circuit SSEC.

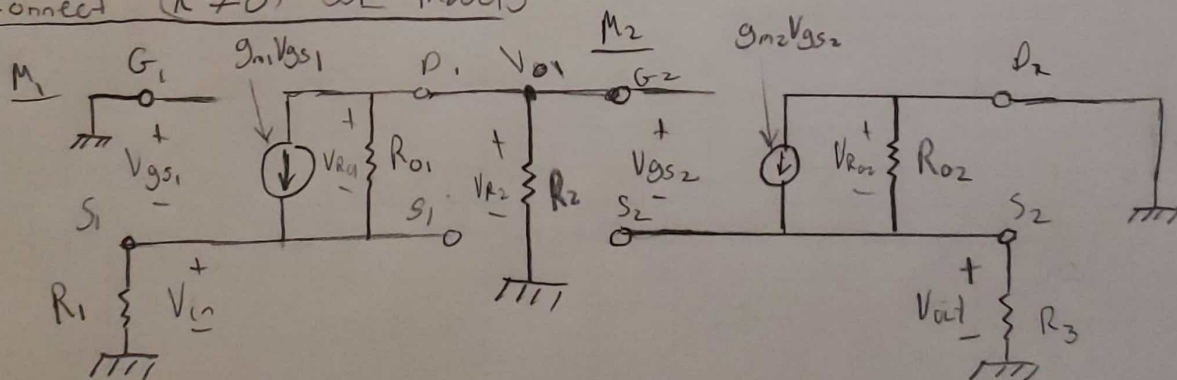
2Goal: To create a small signal equivalent model for the given multistage amplifier.

3Plan: To mesh the generic SSEM ($\lambda \neq 0$) models for each of the transistors with the architecture of the amplifier to create an overall model to perform small signal analysis.

4Solution: Redraw - Set all DC components to 0.



Connect ($\lambda \neq 0$) SSE models



Useful KVL's

$$V_{in} = V_{R1} = -V_{GS1}$$

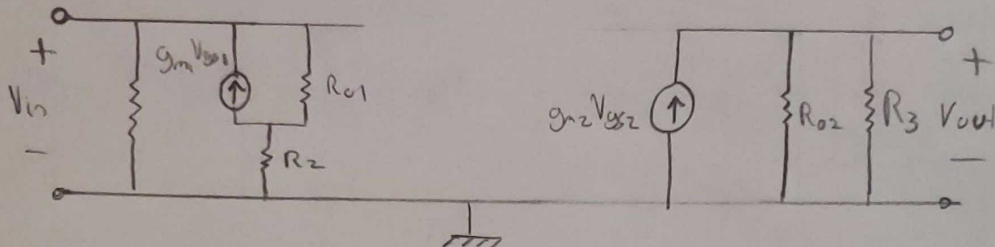
$$V_{in} + V_{R_O1} = V_{R2}$$

$$V_{R2} = V_{O1} - V_{out} = V_{GS2}$$

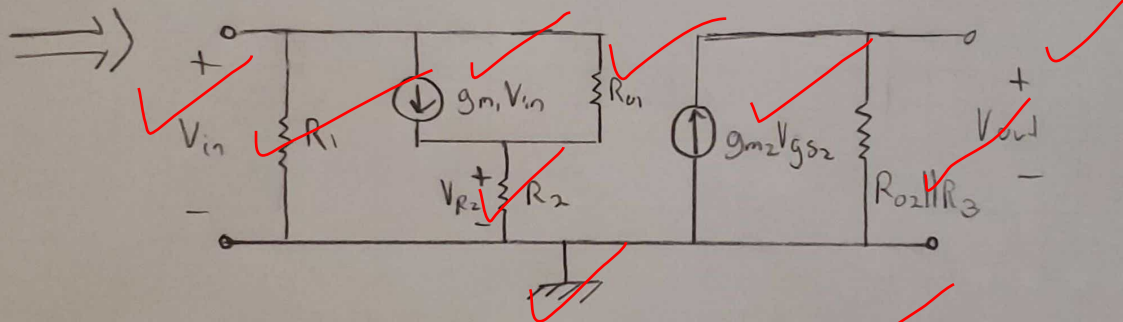
$$V_{O1} = V_{R2}$$

2(b) (Continued)

Simplify / Clean up



Apply $V_{gs1} = -V_{in}$ and $V_{gs2} = V_{o1} - V_{out}$
Combine $R_{o2} || R_3$



Apply

$$R_1 = 10k, R_2 = 30k, R_3 = 500k$$

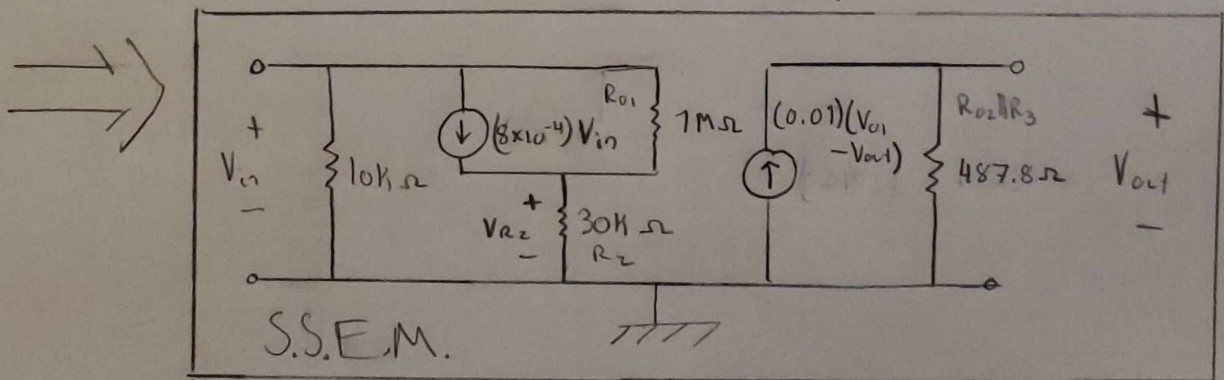
$$R_{o1} = \frac{1}{\lambda_n I_{o1}} = 1000000 \Omega = 1M\Omega$$

$$R_{o2} = \frac{1}{\lambda_n I_{o2}} = 20k\Omega$$

$$\therefore R_{o2} || R_3 = 487.8 \Omega$$

$$g_{m1} = \frac{2I_{o1}}{V_{o1}} = 8 \times 10^{-4}$$

$$g_{m2} = \frac{2I_{o2}}{V_{o2}} = 0.01$$



2(b)

 (Continued)5 Sanity Check

USEFUL KVL'S ARE FULFILLED in all iterations of the circuit.

$$V_{out} = V_{R2} = V_{R3} = V_{gm2} V_{R2} \quad \checkmark$$

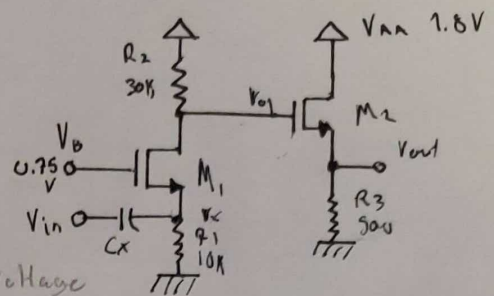
$$V_{in} + V_{gm1} V_{in} - V_{R2} = 0 \quad \checkmark$$

$$V_{in} = V_{R01} + V_{R2} \quad \checkmark$$

$$V_{gs2} = V_{R01} - V_{out} \quad \checkmark$$

2(c)

Situation For the SSEM determined in 2(b), C.) Find the Small Signal Voltage Gains $A_{v1} = V_{o1}/V_{in}$, $A_{v2} = V_{out}/V_{o1}$, and overall Voltage gain, A_v , of the amplifier

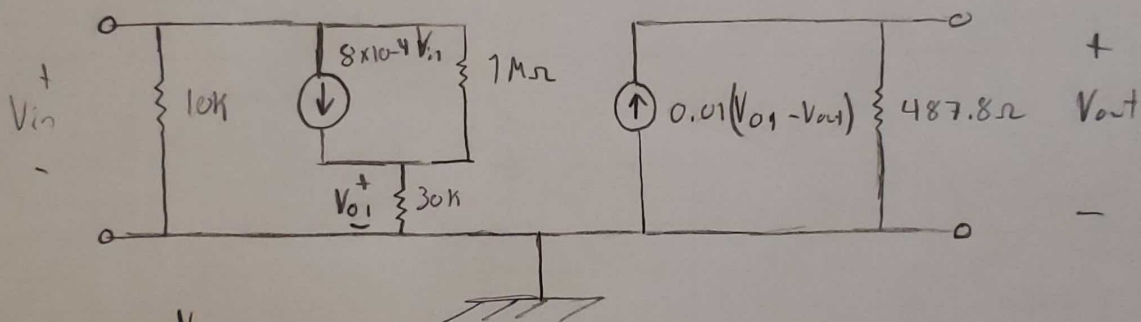


Goal: To determine the Small Signal Voltage gains as defined above.

Plan: Use the $V_{o1} = V_{R2}$ determined in 2(b) along with Fundamental circuit analysis techniques to Determine expressions for A_{v1} , A_{v2} and $A_{v1} \times A_{v2} = A_v$.

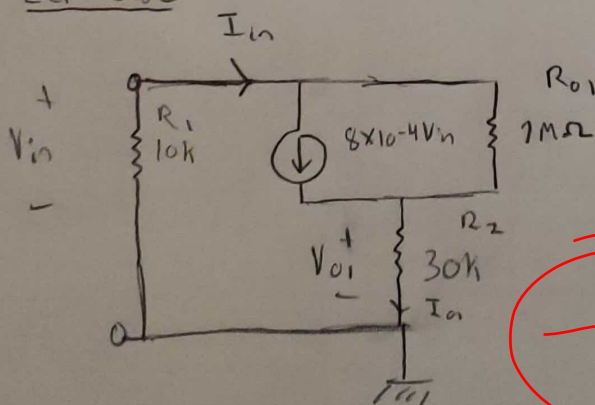
Solution: Redraw SSEM

$$V_{R2} = V_{o1} \quad (\text{From 2(b)})$$



$$A_{v1} = \frac{V_{o1}}{V_{in}}$$

Left Side



$$\text{KCL: } I_{in} = I_{o1}$$

$$\frac{-V_{in}}{R_1} = \frac{V_{o1}}{R_2}$$

$$\frac{V_{o1}}{V_{in}} = \frac{-R_2}{R_1}$$

$$= \frac{-30000}{10000}$$

-5

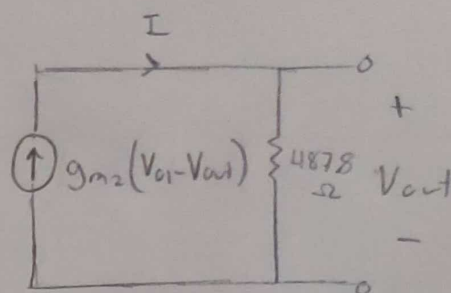
$$\therefore A_v = -5$$

2(c) (Continued)

$$A_{v2} = \frac{V_{out}}{V_{o1}}$$

Ohm's law, $V_{out} = I(487.8)$

Right Side:



$$V_{out} = g_{m2}(V_{o1} - V_{out})(487.8)$$

$$V_{out} = 487.8 g_{m2} V_{o1} - 487.8 g_{m2} V_{out}$$

$$V_{out}(1 + 487.8 g_{m2}) = V_{o1}(487.8 g_{m2})$$

$$\frac{V_{out}}{V_{o1}} = \frac{(487.8 g_{m2})}{(1 + 487.8 g_{m2})}$$

$$= \frac{4.878}{5.878}$$

$$\therefore g_{m2} = 0.01$$

$$A_{v2} = 0.8298$$

$$A_v = \frac{V_{out}}{V_{in}}$$

$$\frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_{o1}} \cdot \frac{V_{o1}}{V_{in}} = A_{v1} \cdot A_{v2}$$

$$\therefore A_v = A_{v1} \cdot A_{v2} = -3 \cdot 0.8298$$

$$A_v = -2.49$$

-1

Sanity Check:

A_v Unit Analysis

$$A_v = A_{v1} \times A_{v2} \quad [-]$$

$$= \frac{R_2}{R_1} \times \frac{487.8 g_{m2}}{1 + 487.8 g_{m2}}$$

$$= \frac{[\Omega]}{[\Omega]} \times \frac{[\frac{1}{\Omega}]}{[\frac{1}{\Omega}]}$$

$$= [-]$$

12/20

2(d)

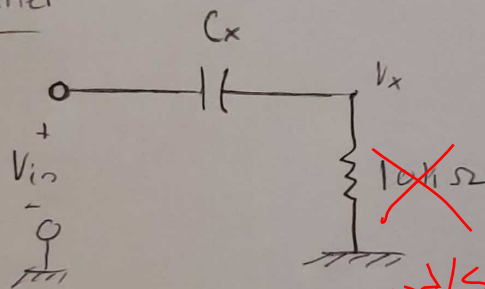
1 Situation: For the given multistage amplifier, d.) What would be the C_x value for the circuit to accept any signal with a frequency over 100 Hz?

2 Goal: Determine the capacitance of C_x to allow the amplifier to take a signal with $f \geq 100$ Hz.

3 Plan: To use the high-pass filter equation (From Lecture 28 slide 9) to solve for C_x , given frequency and drain resistance.

4 Solution:

High-Pass-Filter



Given $f = 100$ Hz $\therefore \omega = 200\pi$ Hz

Eqn $\omega_{BW} = \frac{1}{R_{in} C_x}$

$\therefore C_x = \frac{1}{R_{in} \omega_{BW}} = \frac{1}{(200\pi)(10000)}$

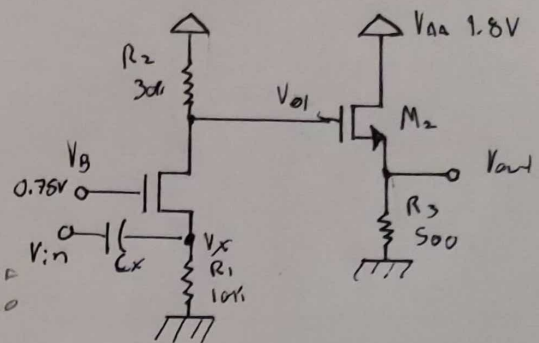
$C_x = 159.2$ nF

5 Sanity Check;

$100 = \frac{1}{(10000)(159.2 \times 10^{-9})} 2\pi$

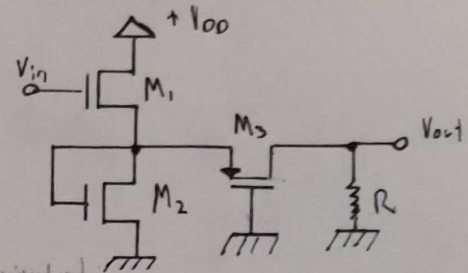
$100 \approx 99.97$ ✓

C_x is rounded, but yields $\sim f = 100$ Hz



3(a)

Situation: Analyze the following amplifier. Use $g_{m1} = 10\text{mS}$, $g_{m2} = 1\text{mS}$, $g_{m3} = 4\text{mS}$, $R = 1\text{k}\Omega$. a.) Draw the Small Signal equivalent circuit for $\lambda = 0$.

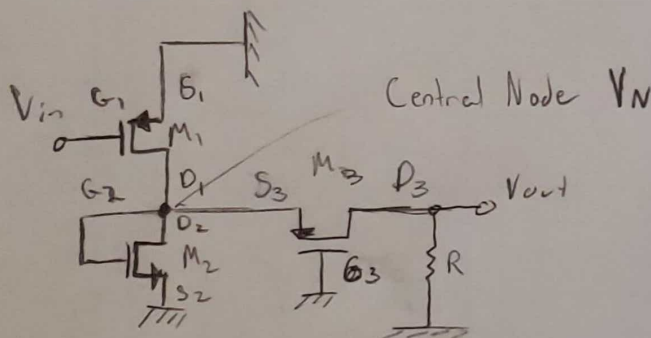


2 Goal: To Develop the Small Signal equivalent circuit for the given amplifier with the given g_m and R values.

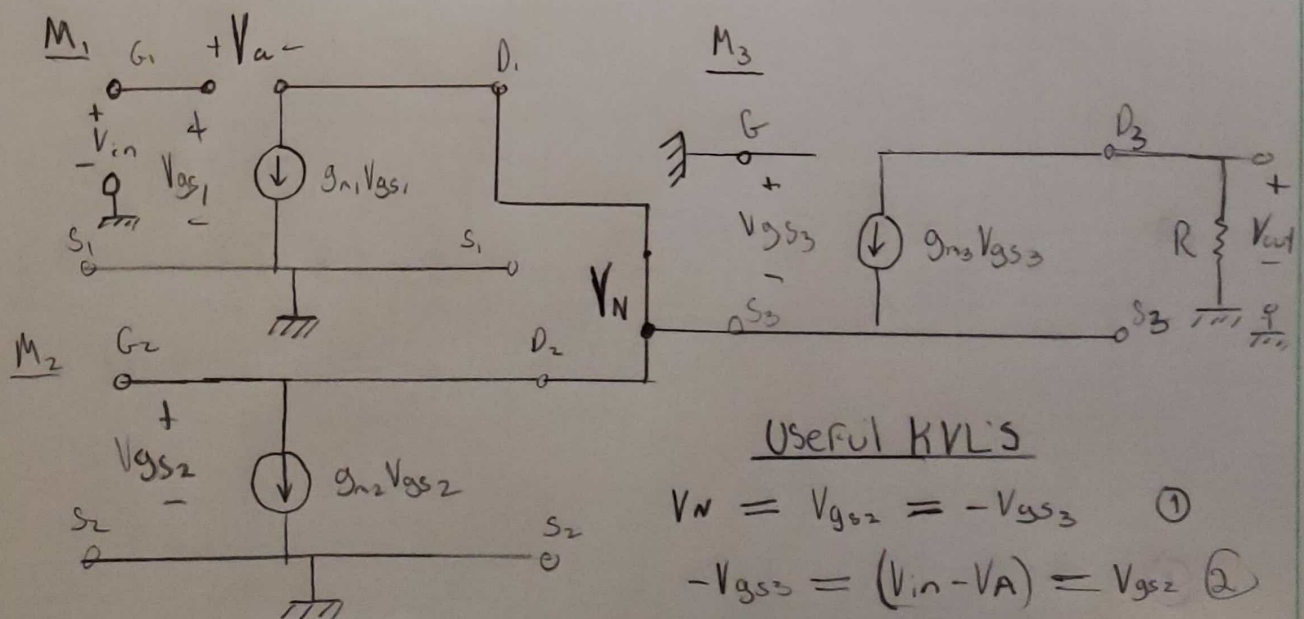
3 Plan: To make all DC components = 0, then use the generic ($\lambda = 0$) model for M_1 , M_2 , M_3 . Then connect each model in accordance with the given architecture and simplify.

4 Solution:

Force all DC's to zero



Convert to $\lambda = 0$ model



Useful KVL's

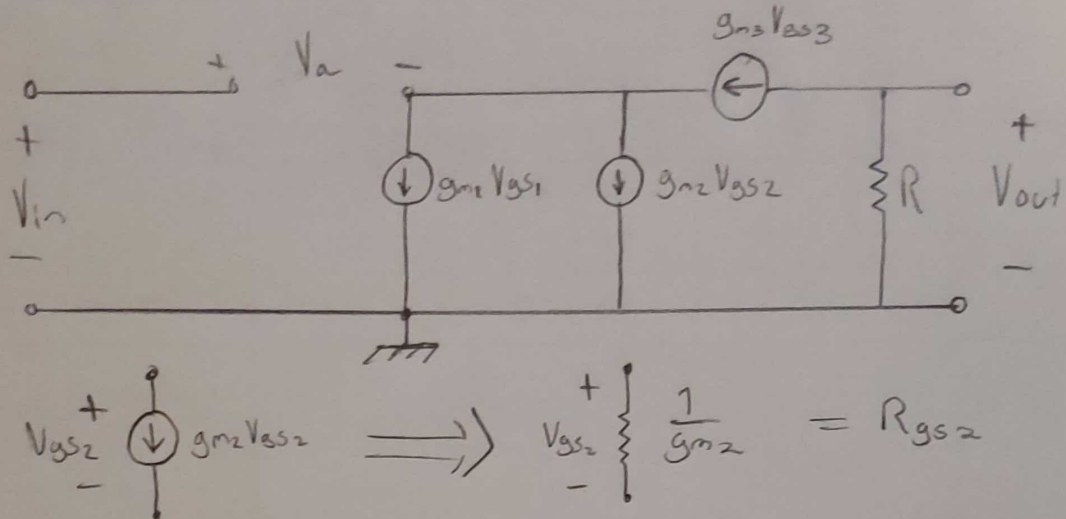
$$V_N = V_{gs2} = -V_{gs3} \quad (1)$$

$$-V_{gs3} = (V_{in} - V_A) = V_{gs2} \quad (2)$$

$$V_{gs1} = V_{in}$$

14/20

3(a) (continued)

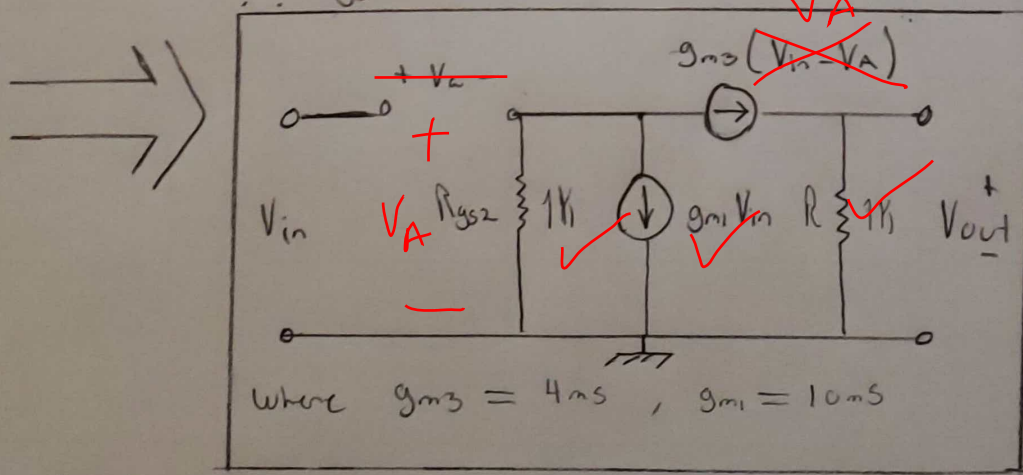
Simplifying / Draw Properly

Apply: $R = 1\text{ k}\Omega$, $g_{m3} = 4\text{ mS}$,
 $g_{m2} = 1\text{ mS}$, $g_{m1} = 10\text{ mS}$

$$V_{gs1} = V_{in}$$

$$V_{gs3} = (V_{in} - V_A)$$

$$\therefore R_{gs2} = \frac{1}{0.001} = 1\text{ k}\Omega$$

Sanity Check:

KVL's remained consistent throughout the full procedure ✓

3(b)

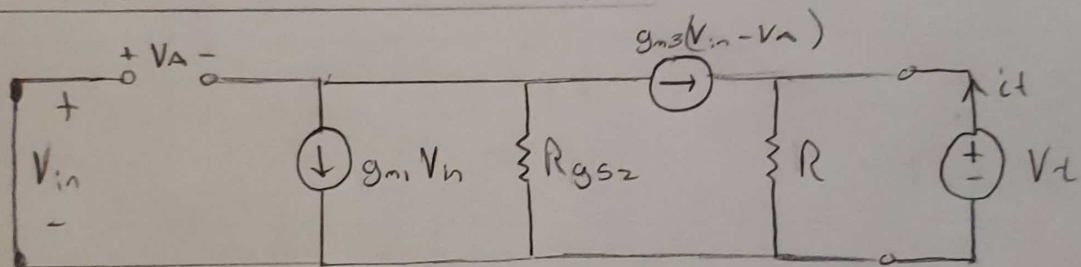
1 Situation: For the given amplifier, Use the same given values as 3(a)
b.) Find the small signal output resistance expression and its value

2 Goal: To determine an expression for the given amplifier's R_{out} .

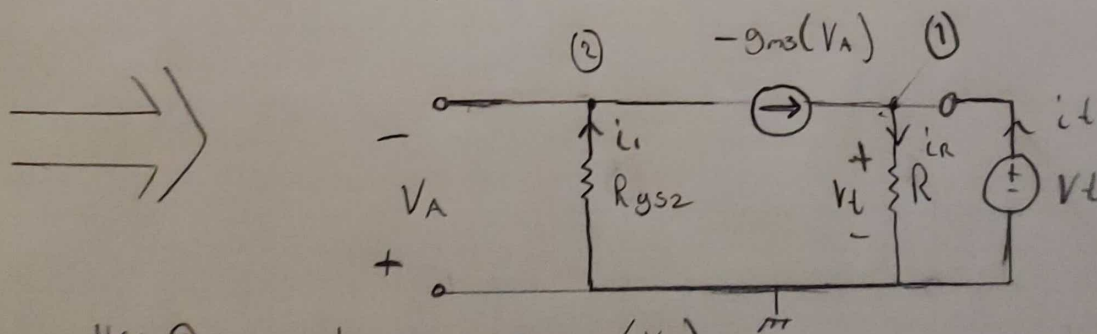
3 Plan: To perform small signal analysis, following the procedure to solve for R_{out} , then evaluate. Use SSEC found in 3a

4 Solution: Apply V_t to output, Short input, then solve for $R_{out} = V_t / i_t \mid Z_{in} = 0$

Redraw SSEC with V_t and $V_{in} \rightarrow$ Short



$\therefore V_{in} = 0$, $g_{m1} V_{in} \rightarrow$ Short



KCL @ 2: $i_1 = -g_{m3} (V_A)$

$$\frac{V_A}{R_{gsz}} = -g_{m3} (V_A)$$

$$\therefore V_A \left(\frac{1}{R_{gsz}} + g_{m3} \right) = 0$$

$$\therefore V_A = 0$$

3(b)

 (continued)

$$\text{KCL @ } \textcircled{1}: -g_m v_A + i_L = i_R$$

$$i_L = i_R$$

$$\therefore V_L / i_L = R$$

$\therefore R_{out} = R = 1000 \Omega$

Sanity Check:

I'm uncertain that this is correct, but I found no other way to solve R_{out} .

v_A is not shorted, so I'm not sure how it could be zero.

3(c)

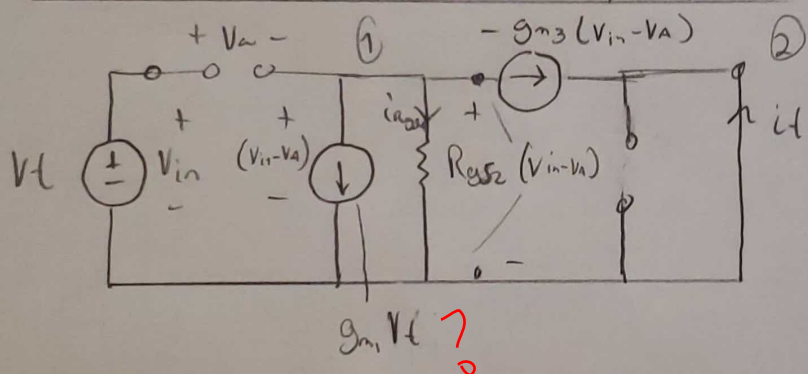
Situation: For the given amplifier, determine the small signal transconductance expression and value G_m .

Goal: To Derive an expression for G_m , then evaluate

Plan: To perform SS analysis on the SSEC found in 3(a), by shorting the output, and placing V_{test} between input nodes.

Solution: Solve $G_m = \left. \frac{i_t}{V_t} \right|_{Z_L = 0}$

Redraw with $V_{out} \rightarrow$ Short, $V_{in} = V_t$



$R \Rightarrow$ open circuit

KCL @ 1: $-g_{m3}(V_{in}-V_A) = g_{m1}V_t + g_{m2}(V_{in}-V_A)$

$$-(V_{in}-V_A)(g_{m3}+g_{m2}) = g_{m1}V_t$$

$$(V_{in}-V_A) = \frac{-g_{m1}V_t}{(g_{m3}+g_{m2})} \quad (1)$$

KCL @ 2

$$I_t = g_{m3}(V_{in}-V_A) \quad (2)$$

$(2) \rightarrow (1)$

$$I_t = \frac{-g_{m3}g_{m1}V_t}{(g_{m3}+g_{m2})}$$

$$\therefore \frac{I_t}{V_t} = \boxed{\frac{-g_{m3}g_{m1}}{g_{m3}+g_{m2}} = G_m}$$

3(c)

 (Continued)

Evaluate G_m for $g_{m1} = 10 \text{ mS}$, $g_{m2} = 1 \text{ mS}$,
 $g_{m3} = 4 \text{ mS}$

$$\therefore G_m = \frac{-4 \cdot 10}{1 + 4}$$

$G_m = -8 \text{ mS}$

Sanity Check:

Although V_A is not grounded, so $V_A = 0$ may not be the correct assumption, the unit analysis on G_m checks out.

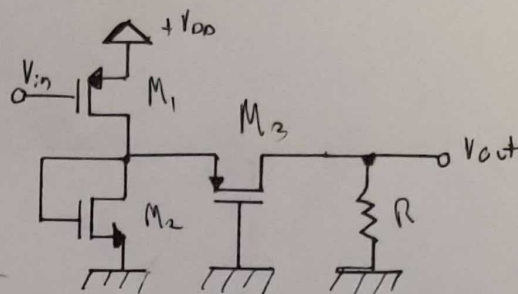
$$G_m [S] = \frac{g_{m3}[S] \cdot g_{m1}[S]}{g_{m3}[S] + g_{m2}[S]}$$

$$= \frac{[S^2]}{S}$$

$$= S \quad \checkmark$$

3(d)

1 Situation: For the given amplifier, d.) Find the small signal Voltage gain expression A_v , and evaluate.



2 Goal: To determine an expression for A_v , then evaluate.

3 Plan: To use the $A_v = R_{out} \cdot G_m$ equation to determine A_v .

4 Solution:

$$\text{Egri: } A_v = R_{out} \cdot G_m$$

$$R_{out} = R$$

$$G_m = \frac{-g_{m3} g_{m1}}{g_{m3} + g_{m2}}$$

$$\therefore A_v = \frac{-g_{m3} g_{m1} R}{g_{m3} + g_{m2}}$$

evaluate:

$$A_v = 1000 \cdot (-8 \times 10^{-3})$$

$$A_v = -8$$

5 Sanity Check:

Though my assumption that V_A was 0 may not be correct, the unit analysis is consistent for A_v .

$$\begin{aligned} A_v [-] &= \frac{-\left[\frac{1}{\Omega}\right] \left[\frac{1}{\Omega}\right] [\Omega]}{\left[\frac{1}{\Omega}\right] + \left[\frac{1}{\Omega}\right]} \\ &= \frac{\left[\frac{1}{\Omega}\right]}{\left[\frac{1}{\Omega}\right]} \\ &= [-] \text{ Unitless } \checkmark \end{aligned}$$

$$\boxed{20/20}$$