110
100

Tristan Denning

ECE 310 – Microelectronics I

Prof. Suat Ay **Homework #2** *Fall 2021*

(Due Date: 09/22/2021, 8.30am or before)

For the problem below, use: k=1.38E-23 (J/K), q=1.602E-19 (J/eV), ε_0=8.854E-14 (F/cm)

30

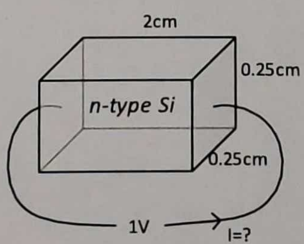
1. (30 pts) A Gallium-Arsenide (GaAs) PN junction has doping concentrations of $7\times10^{+9}$ cm^{-3} on the n-side and $2\times10^{+10}$ cm^{-3} on the p-side of the junction.
   a. (20pts) Determine the majority and minority carrier concentrations on both sides of the junction at T=300K, and 400K.
   b. (10pts) Calculate the built-in potential at T=300K, and 400K.

20

2. (20 pts) For this problem, assume D_n=35 cm^2/s and μ_n=1356 cm^2/V-s for electrons and D_p=12 cm^2/s and μ_p=465 cm^2/V-s for holes.
   a. (10pts) Consider majority carrier concentration in a p-type semiconductor varies linearly from $4\times10^{+13}$ (on right) to 10^{+11} (cm^{-3}) (on left) in a distance of 10μm at time t=0 while minority carrier concentration is uniform and does not change throughout the volume. Calculate the current density (J in A/cm^2) at time $t=0^+$.
   b. (10pts) Which direction does the current flows? Explain why?

30

3. (30 pts) A uniformly doped n-type piece of Silicon with a length of 2cm and a cross sectional area of 0.25cm x 0.25cm sustains a voltage difference of 1.0V. If the doping level of the substrate is 3.18×10^{13} cm^{-3}, calculate;
   a. (10pts) the total current flowing through the piece at room temperature (27C°)
   b. (5pts) its resistance
   c. (10pts) Calculate its resistance when temperature is increased to 100 C° assuming mobility of electrons and hole do not change.
   d. (5pts) Looking at part (b) and (c) do you consider this structure (under given assumptions) to be a good resistor? Explain why or why not?



30

4. (30 pts) Junction capacitance of a Germanium diode with a cross-sectional area of 0.1 cm^2 was measured using a capacitance meter that applies 1V during measurement. On one direction instrument reads 0pF while the opposite direction 10pF. Datasheet of the diode mentions that the build-in potential of it is 250mV at room temperature.
   a. (15 pts) Find the unit area junction capacitance, and possible doping concentrations (N_A, and N_D) of this diode.
   b. (15pts) If temperature doubles in C°, what would you expect the measured capacitance to be?

1(a)

1 **Situation:** A Gallium-Arsenide PN junction has doping concentrations of $7\times10^{9}\ cm^{-3}$ on the n-side and $2\times10^{10}\ cm^{-3}$ on the P-side. Determine the majority and minority carrier concentrations on both sides at $T = 300K$ and $T = 400K$.
Given: $K = 1.38\times10^{-23}\ J/K$, $q = 1.602\times10^{-19}\ J/eV$, $\varepsilon_0 = 8.854\times10^{-14}\ F/cm$

2 **Goal** To calculate the majority and minority concentrations on the P and N side at 300K and 400K.

3 **Plan** To use known values of E_g and B along with the given quantities and known equation for n_i to determine the intrinsic carrier concentration at 300K and 400K. Then use majority and minority carrier concentration equations for the P side and N side at 300 and 400 Kelvin.

4 **Solution:**

Visual:

n	p
As	Ga
$7\times10^{9}/cm^3$	$2\times10^{10}/cm^3$
Group V Majority e^- Minority h^+	Group III Majority h^+ Minority e^-

Given Values

$K = 1.38\times10^{-23}$ J/K
$q = 1.602\times10^{-19}$ J/eV
$\varepsilon_0 = 8.854\times10^{-14}$ F/cm

Known Eqn's

Intrinsic CC: $$n_i = B\cdot T^{3/2}\exp\left(\frac{-E_g\cdot q}{2\cdot K\cdot T}\right)$$

P-Type Maj CC: $p \cong N_A$ ①

Min. CC: $n = n_i^2/N_A$ ②

N-Type Maj CC: $n = N_D$ ③

Min CC: $p = n_i^2/N_D$ ④

Known Values / Constants:

* From Lecture 6

$E_g = 1.42\ eV$

$B = 3.56\times10^{14}\ cm^{-3}(K^{-3/2})$

1(a) (continued)

Calculate n_i at $T = 300K$, $T = 400K$

T = 300K

$$n_{i300} = (3.56\times10^{14})(300)^{3/2} \exp\left(-\frac{1.42 \cdot 1.602\times10^{-19}}{2 \cdot 300 \cdot 1.38\times10^{-23}}\right)$$

$$= 2.165\times10^{6} \quad [\#/cm^3]$$

T = 400K

$$n_{i400} = 3.56\times10^{14}(400)^{3/2} \exp\left(\frac{-1.42 \cdot 1.602\times10^{-19}}{2 \cdot 400 \cdot 1.38\times10^{-23}}\right)$$

$$= 3.204\times10^{9} \quad [\#/cm^3]$$

Plug n_{i300}, n_{i400}, 7×10^9, 2×10^9 into respective eqn's 1 2 3 4

P-Type Side, eqn's 1, 2

At 300K

Majority: $p \cong \boxed{N_A = 2\times10^{10}\ h^+/cm^3}$

Minority: $n = (n_{i300})^2 / N_A$

$$= \frac{(2.165\times10^{6})^2}{(2\times10^{10})}$$

$$= \boxed{234.361\ e^-/cm^3}$$

At 400K

Majority: $p \cong \boxed{N_A = 2\times10^{10}\ h^+/cm^3}$

Minority: $n = (n_{i400})^2 / N_A$

$$= \frac{(3.204\times10^{9})^2}{2\times10^{10}}$$

$$= \boxed{5.133\times10^{8}\ e^-/cm^3}$$

$\boxed{1(a)}$ (continued)

n-Side

At 300K

Majority: $n \cong N_D$

$$\boxed{n \cong 7 \times 10^9 \ e^-/cm^3}$$

Minority: $p = (n_{i300})^2 / N_D$

$$= \frac{(2.165 \times 10^6)^2}{(7 \times 10^9)}$$

$$\boxed{p = 669.6 \ h^+/cm^3}$$

At 400K

Majority: $n \cong N_D$

$$\boxed{n \cong 7 \times 10^9 \ e^-/cm^3}$$

Minority: $p = (n_{i400})^2 / N_D$

$$= \frac{(3.204 \times 10^9)^2}{7 \times 10^9}$$

$$\boxed{p = 1.467 \times 10^9 \ h^+/cm^3}$$

$\boxed{1(a)}$ (continued)

5 Sanity Check

Unit analysis of assumed equations:

Intrinsic CC:

$$n_i = B \cdot T^{1.5} \exp\left(\frac{-E_g \cdot q}{2kT}\right)$$

$$= \left[\frac{1}{cm^3 K^{3/2}}\right]\left[K^{3/2}\right] e^{\left[\frac{[eV][C]}{[K][J/K]}\right]}$$

$$= \left[\frac{1}{cm^3}\right] \checkmark$$

Minority CC:

$$p = n = \frac{n_i^2}{N}$$

$$= \frac{[cm^{-3}]^2}{[cm^{-3}]}$$

$$= [cm^{-3}] \checkmark$$

$\therefore$ Unit Analysis yields the appropriate dimensions for density

Tristan Denning | Problem 1(b) | HW #2

1(b)

1 **Situation:** A Gallium Arsenide (GaAs) PN junction has doping concentrations of $7\times10^{9}\ cm^{-3}$ on the n-Side, and $2\times10^{10}\ cm^{-3}$ on the p-Side. Calculate the built-in Potential at 300 and 400 Kelvin. Use Values Given in 1(a).

2 **goal:** To determine the built-in potential barrier for free carriers to move between two regions freely.

3 **Plan:** To utilize the known equation for the built-in potential, V_o, along with Values determined in 1(a) to calculate the built in potential for the PN-junction.

4 **Solution:**

Built in Potential:

$$V_o = \frac{KT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Given Values:

$$K = 1.38\times10^{-23}\ J/K$$

$$q = 1.602\times10^{-19}\ J/eV$$

$$N_A = 2\times10^{10}\ cm^{-3}$$

$$N_D = 7\times10^{9}\ cm^{-3}$$

Calculated in 1(a):

$$n_{i300} = 2.165\times10^{6}\ cm^{-3}$$

$$n_{i400} = 3.204\times10^{9}\ cm^{-3}$$

Plug in Values:

At 300K

$$V_{o300} = \frac{(1.38\times10^{-23})(300)}{1.602\times10^{-19}} \ln\left(\frac{(2\times10^{10})(7\times10^{9})}{(2.165\times10^{6})^2}\right)$$

$$\boxed{V_{o300} = 0.445\ V}$$

At 400K

$$V_{o400} = \frac{(1.38\times10^{-23})(400)}{1.602\times10^{-19}} \ln\left(\frac{(2\times10^{10})(7\times10^{9})}{3.204\times10^{9}}\right)$$

$$\boxed{V_{o400} = 0.090\ V}$$

5/24

$\boxed{1(b)}$ (continued)

5 Sanity Check

The answers seem reasonable. At room temperature, the diode has a slight voltage drop across it of about 0.445 V. At high temperature, the voltage drop is much lower and the diode has essentially failed.

Unit Analysis

$$V_o = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$= \frac{[J/K][K]}{J/eV} \ln\left(\frac{[cm^{-3}][cm^{-3}]}{[cm^{-3}]^2}\right)$$

$$= \frac{[J]}{[J/eV]}$$

$$= \frac{[J]}{[C]}$$

$$= [V] \checkmark$$

$\therefore$ Appropriate Unit For Potential

2(a)

1 **Situation** Consider majority carrier concentration in a P-type Semiconductor varies linearly from 4×10^{13} on the right to 10^{11} (cm^{-3}) (on left) in a distance of 10 µm at time $t=0$ while minority carrier concentration is uniform and does not change throughout the volume. Calculate the current density (J in A/cm^2) at time $t=0^+$. Use the following given values: $D_n = 35\ cm^2/s$, $\mu_n = 1356\ cm^2/V{\cdot}s$ for e^-'s and $D_p = 12\ cm^2/s$, $\mu_p = 465\ cm^2/Vs$ for holes.

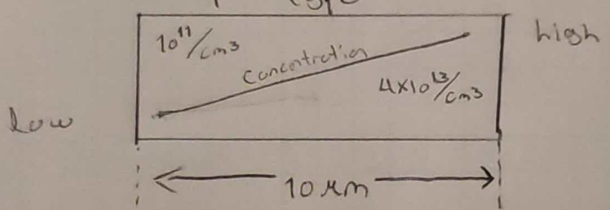
2 **Goal** : To calculate the current density at $t=0^+$ for the majority carrier concentration diffusion current in a P-type Semi-Conductor.

3 **Plan** : To use the known diffusion current equation for P-type Semi-Conductors along with the given quantities to determine the current density.

4 **Solution:**

Visual:

P — type



* For P-type, majority is holes (h^+)

Diffusion Current Eqn:

$$J_p = -q D_p \, dp/dx$$

Where:

$q = 1.602\times10^{-19}\ C$ * Known

$D_p = 12\ \frac{cm^2}{s}$ * given

$dp = (4\times10^{13} - 10^{11})$

$dx = 10\times10^{-4}\ cm$ * given

Plug in Values:

$$J_p = -1.602\times10^{-19}(12)\left(\frac{4\times10^{13}-10^{11}}{10\times10^{-4}}\right)$$

$$\boxed{J_p = -0.0767\ A/cm^2}$$

* At instant $t=0^+$

$\boxed{7/24}$

$\boxed{2(a)}$ (Continued)

5 Sanity Check

Unit analysis of J_p

$$J_p = -q D_p \frac{dp}{dx}$$

$$= [C] \frac{[cm]^2}{[S]} \frac{[cm^{-3}]}{[cm]}$$

$$= \left[\frac{C}{S}\right] [cm^{-2}]$$

$$= \frac{[A]}{[cm^2]} \checkmark$$

$\therefore$ dimensional Analysis yields appropriate units

2(b)

1 **Situation** For the Same Situation as given in 2(a), which direction does the Current Flow?

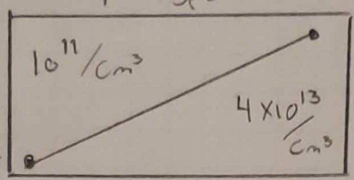
2 **Goal:** To determine the direction of current flow using results from 2(a) and given information

3 **Plan:** To Examine the type (h^+) or (e^-) Concentration at hand as well as the Sign on the result for 2(a) to make a determination of current flow direction.

4 **Solution:**

Visual

P-type



High h^+ concentration

low h^+ conc. →

* For P-type Semiconductors, h^+ are the majority carrier.

At $t = 0^+$, the holes (h^+) repel eachother and begin moving from high Concentration (right) to low Concentration (left). The resulting current occurs in the same direction of h^+ movement.

∴ The Current flow is from <u>right to left</u>

9/24

Tristan Denning | Problem 3(a) | HW # 2

$\boxed{3(a)}$

1 **Situation:** A uniformly doped n-type piece of Silicon with a length of 2cm and a cross section area of $(0.25 \times 0.25)\ cm^2$ sustains a voltage difference of 1.0V. If the doping level of the substrate is $3.18 \times 10^{13}\ cm^{-3}$ Calculate: The total current flowing through the piece at $(27°C)$.

2 **Goal:** To calculate the total current flowing through the substrate.

3 **Plan:** To plug in the given values to known equations for μ_n, $\vec{E}$, n. Then plug those quantities into the drift current equation.

4 **Solution:**

Drift Current Eqn's:

$$I_{drift,n} = -\mu_n \vec{E} A n q$$

$$I_{drift,p} = \mu_p \vec{E} A p q$$

* drift current is the only current through the substrate under the given conditions.

$I_{drift,n}$ Components:

μ_n:
$$\mu_n(N_d) = 65 + \frac{1265}{1 + \left(\frac{N_D}{8.5 \times 10^{16}}\right)^{0.72}}$$

$$= 65 + \frac{1265}{\left(1 + \left(\frac{3.18 \times 10^{13}}{8.5 \times 10^{16}}\right)^{0.72}\right)}$$

$$= 1325.7\ cm^2/V\cdot s \quad *$$

$\vec{E}$:
$$\vec{E} = -d\vec{V}/dx$$

$$= \frac{-1.0\ [V]}{0.02\ [m]}$$

$$= -50\ V/m = -0.5\ V/cm \quad *$$

A:
$$A = (0.25 \cdot 0.25)\ cm$$

$$= 0.0625\ cm^2 \quad *$$

$\boxed{3(a)}$ (continued)

Idrift,n Components Continued

n: $\quad n \cong N_D$ * For n-type majority carrier concentration *

q: $\quad q = 1.602\times10^{-19}$ [C] *

Idrift,p Components:

μ_p: $\quad \mu_p(N_D) = 130 + \dfrac{370}{\left(1+\left(\frac{N_d}{8\times10^{17}}\right)^{1.25}\right.}$

$= 500.0 \ \ cm^2/V\cdot s$

$\vec{E}$: $\quad \vec{E} = -0.5 \ V/cm$

A: $\quad A = 0.0625 \ cm^2$

p: $\quad p = n_i^2/N_D$

n_i $\quad n_i = B_{Si} T^{1.5} \exp\left(\dfrac{-E_{gSi}\cdot q}{2\cdot k\cdot T}\right)$

$= 5.2\times10^{15}(273+27)^{1.5}\exp\left(\dfrac{(-1.12)\,1.602\times10^{-19}}{2(300)\,1.38\times10^{-23}}\right)$

$n_i = 1.049\times10^{10} \ h^+/cm^3$ *

q: $\quad q = 1.602\times10^{-19}$ C

$\boxed{3(a)}$ (continued)

Plug in Component Values to $I_{drift,n}$, $I_{drift,p}$

$$I_{drift,n} = -\mu_n E A n q$$

$$= -1325.7(-0.5)(0.0625)(3.18\times10^{13})(1.602\times10^{-19})$$

$$\because n \cong N_D = 3.18\times10^{13}$$

$$I_{drift,n} = 2.11\times10^{-4} \text{ A}$$ *

$$I_{drift,p} = \mu_p E A p q$$

$$= \mu_p E A\left(\frac{n_i^2}{N_D}\right)q$$

$$= 500(-0.5)(0.0625)\left(\frac{(1.049\times10^{10})^2}{3.18\times10^{13}}\right)1.602\times10^{-19}$$

$$I_{drift,p} = -8.662\times10^{-12} \text{ A}$$ *

$$I_{drift} = I_{drift,n} + I_{drift,p}$$

$$= 2.11\times10^{-4} + 8.662\times10^{-12}$$

$$\boxed{I_{drift} \approx 2.11\times10^{-4} \text{ A}}$$

$\boxed{3(a)}$ (Continued)

5. Sanity Check

Unit analysis for $I_{drift,n}$:

$$I_{drift} = -\mu_n E A n q$$

$$[A] = \frac{\cancel{[cm^2]}}{\cancel{[V]}[s]} \frac{\cancel{[V]}}{\cancel{[cm]}} \cancel{[cm^2]} \cancel{[cm^{-3}]} [C]$$

$$= \frac{[C]}{[s]}$$

$$= [A] \checkmark$$

$\therefore$ Amperes is the expected unit for current.

3(b) 1 Situation: For the Same situation as 3(a) calculate the substrate's resistance. Assuming at room temperature (27°C)

2 Goal: To utilize Kown and calculated Values to determine the resistance of the Substrate at 27°C.

3 Plan: To use Ohm's law relationship $R = V/I$ with the given Value for V, and Calculated Value of I.

4 Solution:

Known: $V = 1.0\ V$

Calculated in 3(a): $I_{drift} = 2.11\times10^{-4}\ A$

Ohm's law: $R = V/I_{drift}$

$$R = \frac{1}{(2.11\times10^{-4})} \quad \frac{[V]}{[A]}$$

$$\boxed{R = 4738.22\ \Omega}$$

$\boxed{3(c)}$ 1. **Situation**: For the situation listed in 3(a), calculate the resistance when the temperature is raised from 27°C to 100°C. Assumption: the mobility of e^- and h^+ do not change.

2. **Goal**: To calculate the resistance of the substrate under the affect of 100°C temperature

3. **Plan**: To use equations and values found in 3(b), recalculating the temperature dependent intrinsic carrier constant n_i at 100°C (373 K), then calculate the new drift current. Then use Ohm's law relationship $R = V/I$ to solve for the new resistance.

4. **Solution**:

* From 3(b), only $I_{drift,p}$ contained a temperature-dependent component, n_i.

n_i @ 373 K

$$n_{i373} = B(373)^{1.5} \exp\left(\frac{-E_{g_{Si}}\, q}{2k(373)}\right)$$

$$= 5.2\times10^{15}(373)^{1.5} \exp\left(\frac{-1.12 \cdot 1.602\times10^{-19}}{2(1.38\times10^{-23})\,373}\right)$$

$$n_{i373} = 1.01 \times 10^{12}$$

Plug into $I_{drift,p}$ eqn

$$I_{drift,p} = \mu_p \vec{E} A \left(\frac{(n_{i373})^2}{N_D}\right) q$$

$$= 500 \cdot (-0.5)(0.0625)\left(\frac{(1.01\times10^{12})^2}{3.18\times10^{13}}\right) 1.602\times10^{-19}$$

$$I_{drift,p} = -8.033 \times 10^{-8}$$

3(c) (continued)

Calculate I_{total}

$$I_{drift,n} = 2.11 \times 10^{-4} \text{ A}$$

* does not have a temp-dependent component

$$I_{drift} = I_{drift,n} + I_{drift,p}$$

$$= (2.11 \times 10^{-4}) + 8.033 \times 10^{-8}$$

$$I_{drift} = 2.1097 \times 10^{-4} \text{ A}$$

Plug into Ohm's law

$$R_{373} = V / I_{drift}$$

$$R_{373} = \frac{1}{(2.1097 \times 10^{-4})}$$

$$\boxed{R_{373} = 4740.02 \ \Omega}$$

5 Sanity Check:

Answer makes sense because the intrinsic carrier concentration n_i is barely affected by the temperature change. Furthermore, $I_{drift,p}$ changed pretty significantly from -8.662×10^{-12} + -8.033×10^{-8}. However, this is still very small in comparison to $I_{drift,n}$ which was 2.11×10^{-4}. This resulted in minimal change to the result, I_{drift}, and subsequent R_{373}.

I am omitting dimensional analysis since the only new value was 373 K instead of 300 K. The unit analysis would still result in [A] for I drift since I only substituted [K] for [K].

3(d)

1 **Situation:** For the Situation described in 3(a), and Analyzing results for 3(b) and 3(c), is the defined Substrate a good resistor. Explain.

2 **Goal:** Make a Supported Claim on the effectiveness of the Substrate as a resistor.

4 **Plan:** Analyze the effect of temperature change under Constant Voltage on the Substrates' resistance. Then make a Claim based on the analysis.

5 **Solution:**

Resistance @ room temperature: $R_{300} = 4738.22\ \Omega$ * From 3(b)

Resistance @ 100°C (373 K) $R_{373} = 4740.02\ \Omega$

Determine ΔR:

$$\Delta R = R_{373} - R_{300}$$
$$= 4740.02 - 4738.22$$
$$= 1.80\ \Omega$$

$$\Delta R \%\ R_{300} = \left(\frac{1.80}{4738.22}\right) 100$$
$$= 0.038\ \%$$ *

Determine ΔT:

$$\Delta T = 373 - 300$$
$$= 73\ K$$

$$\Delta T \%\ T_{300} = \left(\frac{73}{300}\right) \cdot 100$$
$$= 24.3\bar{3}\ \%$$ *

> **Claim:** The Structure is a good resistor under the given assumptions

$\boxed{3(d)}$ (continued)

5 Sanity Check / Explanation

The Substrate is a good resistor because a large % Change in temperature yielded a very low % Change in resistance.

For a 24.33% increase in surrounding Average molecular kinetic energy around the Substrate, the resistance only changed by 0.038% of its Value at room temperature

Tristan Denning | Problem 4(a) | HW # 2

4(a) 1**Situation**: Junction capacitance of a Germanium diode with a cross sectional Area of $0.1\ cm^2$ was measured using a capacitance meter that applies 1V during measurement. On one direction, instrument reads 0pF, on the other, 10pF. Datasheet of the diode mentions that the diode has built-in potential of 250mV at Room temperature. Assume: Room temperature is $27°C = 300K$, P4(a): Find the Unit Area junction Capacitance, and possible doping concentrations N_A, N_D for the diode.

2**Goal**: To find the Unit Area junction Capacitance, and possibilities for N_A, and N_D.

3**Plan**: To utilize the known equations and constants associated with Total junction Capacitance, (C_J) and Unit area junction capacitance, (C_{J0}) as well as equation for V_0 to set up and solve 2 equations with 2 unknowns, N_A, N_D, then solve for N_A, N_D using Ti-nspire CX II CAS, Finally, plug the calculated values into the (C_{J0}) equation to calculate the Unit area Junction Capacitance.

4**Solution**

Known Equations;

Intrinsic CC: $n_i = B \cdot T^{1.5} \exp\left(\frac{-E_g \cdot q}{2kT}\right)$

Built-in Potential: $V_0 = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$ (1)

Junction Capacitance: $C_J = \frac{C_{J0} \times A}{\sqrt{1 + V_R/V_0}}$

Unit Area Junction Capacitance: $C_{J0} = \sqrt{\frac{\varepsilon_{dep} \cdot q \cdot N_A N_D}{2(N_A + N_D) \cdot V_0}}$

(2)

Given Constants/Values

$C_J = 10\,pF = 10 \times 10^{-12}\ F$

$V_0 = 250\,mV = 0.25\ (V)$

$V_R = 1\ (V)$

19/24

$\boxed{4(a)}$ (continued)

Given Constants/Values (continued):

$$B_{Ge} = 1.66\times10^{15} \quad (cm^{-3} K^{-3/2})$$

$$E_{gGe} = 0.66 \quad (eV)$$

$$q = 1.602\times10^{-19} \quad (C)$$

$$K = 1.38\times10^{-23} \quad (J/K)$$

$$A = 0.1 \quad (cm^2)$$

$$\varepsilon_{dep,Ge} = 16\varepsilon_0 = 16(8.854\times10^{-14}) \quad (F/cm)$$

Compute n_i^2 @ 300K

$$n_i^2 = \left(BT^{1.5}\exp\left(\frac{-E_g\cdot q}{2KT}\right)\right)^2$$

$$= \left(1.66\times10^{15}(300)^{1.5}\exp\left(\frac{(-0.66\cdot1.602\times10^{-19})}{2\cdot1.38\times10^{-23}\cdot300}\right)\right)^2$$

$$n_i^2 = 6.0267\times10^{26} \quad (\#/cm^3)^2$$

* Needed for eqn (1)

$(n_i^2)\rightarrow$ (1) and Simplify:

$$V_0 = \frac{KT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$= \frac{(1.38\times10^{-23})(300)}{1.602\times10^{-19}}\ln\left(\frac{N_A N_D}{6.0267\times10^{26}}\right)$$

20/24

$\boxed{4(a)}$ (continued)

Simplify eqn ① (continued)

$$V_0 = 0.025843 \ln\left(\frac{N_A N_D}{6.0267 \times 10^{26}}\right) = 0.25 \quad ①$$

Build eqn ② from C_J, C_{J0}, Solve C_{J0}

$$C_J = \frac{C_{J0} A}{\sqrt{1 + V_R/V_0}}$$

$$C_{J0}^{-1} = \frac{A}{C_J \sqrt{1 + \frac{V_R}{V_0}}}$$

$$C_{J0}^{-1} = \left(\frac{0.1}{10 \times 10^{-12} \sqrt{1 + \left(\frac{1}{0.250}\right)}}\right)$$

$$C_{J0}^{-1} = 4.47214 \times 10^{9}$$

$$\boxed{C_{J0} = 2.2361 \times 10^{-10} \text{ F}}$$

* Unit Area Junction Capacitance

recall C_{J0} eqn:

$$C_{J0} = \sqrt{\frac{\varepsilon_{dep} \cdot q \cdot N_A N_D}{2(N_A + N_D) \cdot .25}}$$

$$2.2361 \times 10^{-10} = \sqrt{\frac{16(8.854 \times 10^{-14}) \cdot 1.602 \times 10^{-19} (N_A N_D)}{2(N_A + N_D)}}$$

$$2.2361 \times 10^{-10} = \sqrt{\frac{2.269 \times 10^{-31} (N_A N_D)}{2(N_A + N_D) \cdot 0.25}} \quad ②$$

* Equations ①, ② with 2 unknowns, N_A, N_D

Continued on next page

$\boxed{4(a)}$ (Continued)

Solve eqn's ① & ② For N_A, N_D

Plug in Calculator:

Solve:
$$\begin{cases} 0.025843 \ln\left(\dfrac{N_A N_D}{6.0267\times10^{26}}\right) = 0.250 & ① \\ \sqrt{\dfrac{2.269\times10^{-31}(N_A N_D)}{2(N_A+N_D)0.26}} = 2.2361\times10^{-10} & ② \end{cases}$$

Output:

$$*1 \begin{cases} N_A = 1.1016\times10^{11}\ cm^{-3} \\ N_D = 8.697\times10^{19}\ cm^{-3} \end{cases}$$

or

$$*2 \begin{cases} N_A = 8.697\times10^{19}\ cm^{-3} \\ N_D = 1.1016\times10^{11}\ cm^{-3} \end{cases}$$

Sanity Check:

Plug Values of N_A, N_D into V_o eqn to yield given 0.250 V

*1
$$V_o = 0.025843 \ln\left(\frac{N_A N_D}{6.0267\times10^{26}}\right)$$
$$= 0.025843 \ln\left(\frac{1.1016\times10^{11}\times8.697\times10^{19}}{6.0267\times10^{26}}\right)$$
$$= 0.250003 \approx 250\ mV \checkmark$$

*2
$$V_o = 0.025843 \ln\left(\frac{8.697\times10^{19}\times1.1016\times10^{11}}{6.0267\times10^{26}}\right)$$
$$= 0.250003 \approx 250\ mV \checkmark$$

∴ The two combinations of N_A, N_D yield the given build in potential, So they are feasible answers

$\boxed{22/24}$

4(b)

1 **Situation:** For the same situation described in 4(a), If temperature doubles in °C, what would the measured capacitance change to

2 **Goal:** To determine / Predict what the capacitance of the diode if surrounding temperature increased from 27°C to 54°C. (327 K)

3 **Plan:** To recalculate temperature-dependent quantities using equations and constants listed in 4(a). Substitute 327 for T in appropriate eqn's, then recalculate the junction capacitance C_J.

4 **Solution:**

Temp-Dependent Eqn's:

$$n_i = BT^{1.5} \exp\left(\frac{-E_g \cdot q}{2kT}\right)$$

$$V_0 = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

recalculate @ T = 54°C = 327 K

n_i

$$n_i = 1.66\times10^{15}(327)^{1.5} \exp\left(\frac{-0.66\times1.602\times10^{-19}}{2\times1.38\times10^{-23}\times327}\right)$$

$$= 8.01833\times10^{13} \ \#/cm^3$$

$$\therefore n_i^2 = 6.4294\times10^{27} \ (\#/cm^3)^2$$

V_0

$$V_0 = \frac{(1.38\times10^{-23})(327)}{1.602\times10^{-19}} \ln\left(\frac{1.1016\times10^{11}\times8.697\times10^{19}}{6.4294\times10^{27}}\right)$$

$$\therefore V_0 = 0.2066 \ V$$

$\boxed{4(b)}$ (continued)

C_{J_0}

$$C_{J_0} = \sqrt{\frac{2.269\times10^{-31}(N_A N_D)}{2(N_A+N_D)\,\underbrace{V_0}}}$$

* Changed with T

$$= \sqrt{\frac{2.269\times10^{-31}(1.1016\times10^{11})(8.8967\times10^{11})}{2(1.106\times10^{11}+8.8967\times10^{11})\cdot 0.2066}}$$

$$C_{J_0} = 2.45905\times10^{-10}\ \text{F}$$

Plug into C_J eqn From 4(a)

$$C_J = \frac{C_{J_0} A}{\sqrt{1+V_R/V_0}}$$

$$= \frac{(2.45905\times10^{-10})(0.1)}{\sqrt{1+\left(\frac{1}{0.2066}\right)}}$$

$$\therefore \boxed{C_J = 1.0175\times10^{-11}\ \text{F}} \ ^{*}= 10.175\ \text{pF}$$

5 Sanity Check

Conceptually, this answer makes sense. As temperature increases, more free carriers enter the depletion region. As the depletion region increases, the ability of the reverse biased diode to store charge increases. This charge is stored in the depletion region.

The results from 4(a) and (b) support this

Capacitance @ 27°C : 10 pF

Capacitance @ 54° C : 10.175 pF

10.175 > 10

$\boxed{24/24}$