Denning

# ECE 310 – Microelectronics I

*Prof. Suat Ay*

## Homework #1

***(Due Date: 09/10/2021, 8.30am)***

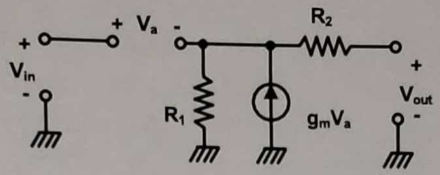
*Fall 2021*

| | |
|---|---|
| 1 | 10 |
| 2 | 30 |
| 3 | 40 |
| 4 | 20 |
| T | 100 |

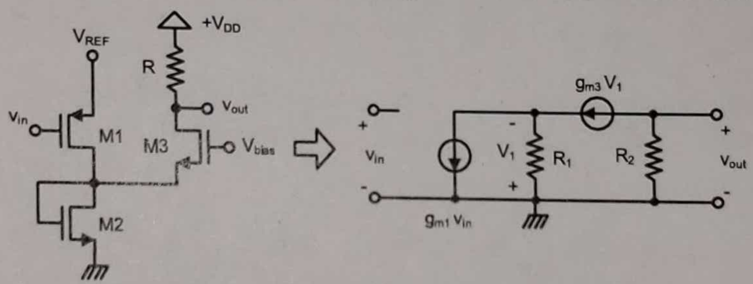
1. (10 pts) Explain what small signal is and AC analysis means to a non-technical person using real life analogies.

2. (30 pts) Find the small signal;
   a. (10pts) Output impedance ($R_{out}$),
   b. (10pts) Transconductance ($G_m$)
   c. (10pts) Voltage gain ($A_v$)

   expressions of the following circuit.

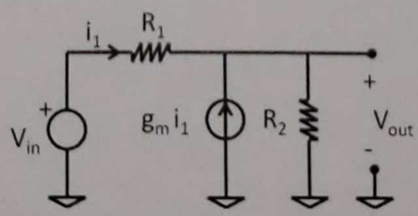



3. (40 pts) Analyze following amplifier (small signal equivalent model is shown on the right).




   a. (10pts) Find small signal input resistance, $R_{IN}$=?
   b. (10pts) Find small signal output resistance, $R_{OUT}$=?
   c. (10pts) Find small signal transconductance, $G_m$=?
   d. (10pts) Find small signal voltage and current gain expression of the amplifier, $A_V$=? $A_I$=?

4. (20 pts) Find the small signal (a) input impedance ($R_{in}=V_{in}/i_1$) <u>expression</u> and (b) its <u>value</u> for the following circuit. ($g_m$ is a unitless constant)




$R_1$=20K, $R_2$=1K, $g_m$=50 ~~mS~~

**1.**

1 **Situation:** Explain what Small Signal is and AC analysis means to a non-technical person using real life analogies.

2 **Goal** To explain the concepts of AC Analysis and Small Signal in Layman's terms.

3 **Plan** N/A

4 **Solution**

In the field of microelectronics, engineers typically deal with very small voltages. For these voltages to be useful, they usually must be amplified by some means. In most cases, any DC voltage or offset, will have some noise, or unpredictable variation deviating from the offset value. This is called a small signal. We are concerned with small signals because amplifiers will carry them through, resulting in greater distortion of the original signal.

These small signals are created and affected by sound, temperature, light, and other nondeterministic variables.

Small signals are somewhat analogous to standing water. A serene lake that has flat calm water on top would be some DC offset. If someone threw a pebble into the lake, there would be some "small signal" created on the surface. This does not change the depth of the lake - ie the DC offset, but it is a noticable fluctuation.

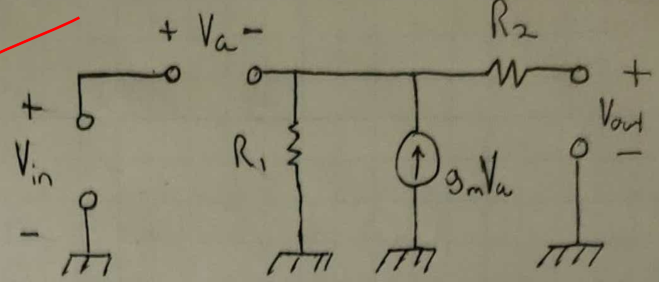
AC Analysis aims to measure or quantify how these small signals will affect an output. It can give us models for transconductance and transresistance — or how drastically the small signal will be amplified.

In terms of the lake, amplification factor would tell us how big of waves or ripples would be created by throwing a pebble in.

Tristan Denning | ECE 310 - Homework 1, Problem 2(a), 1 | HW#01

# 2 (a)

Given Circuit:




1. **Situation**

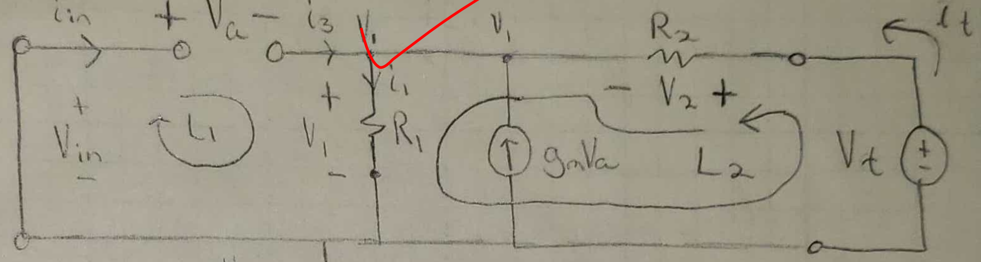
To derive an expression for the OUTPUT impedance of the given circuit.

2. **Goal** To find a general expression for Rout

3. **Plan** Under the assumption that the given circuit is already the small signal equivalent for some amplifier, apply a test voltage $V_t$ between OUTPUT ports and drive $V_t/i_t$ relation, making input source to be zero.

4. **Solution**

Redraw:




Observe: $R_{out} = V_t/i_t \big|_{V_{in}=0}$

KCL @ $V_1$: $i_t + g_m V_a + i_3 - i_1 = 0$
$i_t + g_m V_a = i_1$ (1) $\quad$ * $i_3 = 0$, No current through $V_a$

KVL on $L_1$: $V_{in} - V_a - V_1 = 0$
$V_1 = -V_a$ (2) $\quad$ * $V_{in} = 0$

Ohm's law $R_1$: $V_1 = R_1 i_1$
$i_1 = V_1/R_1$ (3)

(3) → (1) : $i_t + g_m V_a = V_1/R_1$
$i_t = V_1/R_1 - g_m V_a$ (4)

(2) → (4) : $i_t = V_1/R_1 - g_m(-V_1)$
$i_t = V_1/R_1 + V_1 g_m$ (5)

KVL on $L_2$ : $V_1 = V_t - V_2$ (6)

Ohm's law $R_2$ : $V_2 = i_t R_2$ (7)

(7) → (6) : $V_1 = V_t - V_2$ (8)

Rearrange ⑤ : $i_t = V_1\left(\frac{1}{R_1} + g_m\right)$

⑦ → ⑧ : $V_1 = V_t - i_t R_2$ ⑨

⑨ → ⑤ : $i_t = (V_t - i_t R_2)\left(\frac{1}{R_1} + g_m\right)$

$$i_t = \frac{V_t}{R_1} - \frac{i_t R_2}{R_1} + V_t g_m - i_t R_2 g_m$$

$$i_t + \frac{i_t R_2}{R_1} + i_t R_2 g_m = \frac{V_t}{R_1} + V_t g_m$$

$$i_t\left(\frac{R_2}{R_1} + R_2 g_m + 1\right) = V_t\left(\frac{1}{R_1} + g_m\right)$$

$$\frac{V_t}{i_t} = \frac{\left(\frac{R_2}{R_1} + R_2 g_m + 1\right)}{\left(\frac{1}{R_1} + g_m\right)}$$

$$\therefore \frac{V_t}{i_t} = \boxed{R_{out} = \frac{\left(\frac{R_2}{R_1} + R_2 g_m + 1\right)}{\left(\frac{1}{R_1} + g_m\right)}}$$

5 Sanity Check

$$R_{out}[\Omega] = \frac{\left(\left[\frac{\Omega}{\Omega}\right] + [\Omega]\left[\frac{1}{\Omega}\right] + 1\right)}{\left(\left[\frac{1}{\Omega}\right] + \left[\frac{1}{\Omega}\right]\right)}$$

$$= \frac{1}{\left[\frac{1}{\Omega}\right]}$$

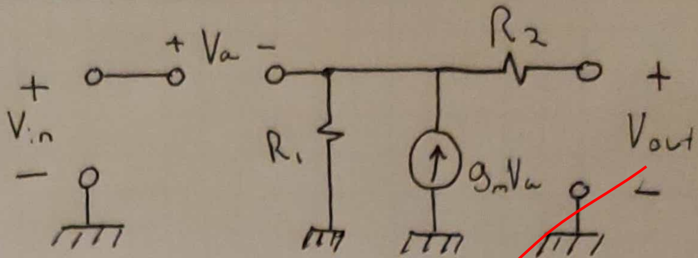
$$= [\Omega]$$

∴ My answer is not insane

# 2(b)

## 1 Situation

To derive an expression for the TRANSCONDUCTANCE of the given circuit.




## 2 Goal

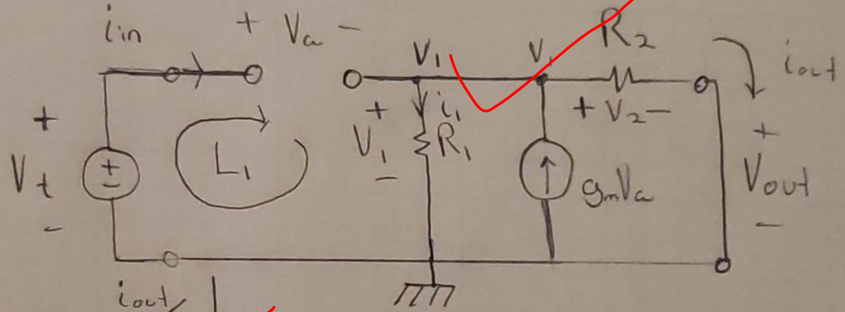
To find a general expression for the circuits transconductance in terms of $R_1$, $R_2$ and $g_m$.

## 3 Plan

To Redraw the circuit Applying a test Voltage $V_t$ between input ports and drive the $i_{out}/V_t$ relationship, where $i_{out}$ is the short circuit output current exiting toward the load. Then solve $G_m = i_{out}/V_t$.

## 4 Solution

Redraw:




Known: $G_m = i_{out}/V_t \big|_{V_{out}} = 0$ (Shorted)

Observe: $V_1 = V_2$ — * $V_{out}$ is shorted, so $V_1 \parallel V_2$

$i_{in} = 0$ — * No current across open Voltage $V_a$

KCL @ $V_1$: $g_m V_a = i_1 + i_{out}$

$i_{out} = g_m V_a - i_1$ (1)

KVL on $L_1$: $V_t - V_a - V_1 = 0$ (2)

Ohm's law $R_2$: $V_2 = V_1 = i_{out} R_2$ (3)

Ohm's law $R_1$: $V_1 = i_1 R_1$ (4)

(3) = (4) : $i_1 R_1 = i_{out} R_2$

$i_1 = \frac{i_{out} R_2}{R_1}$ (5)

(5) → (1) $i_{out} = g_m V_a - \frac{(i_{out} R_2)}{R_1}$ (6)

$\boxed{2(b)}$ (continued)

Rearrange (2) : $V_a = V_t - V_1$ (7)

(3) $\longrightarrow$ (7) : $V_a = V_t - i_{out} R_2$ (8)

(8) $\longrightarrow$ (6) : $i_{out} = g_m(V_t - i_{out}R_2) - \left(\frac{i_{out} R_2}{R_1}\right)$ (9)

* Eqn with desired quantities that can be rearranged to $i_{out}/V_t = G_m$

Algebra on (9):

$$i_{out} = g_m V_t - g_m i_{out} R_2 - \frac{(i_{out} R_2)}{R_1}$$

$$g_m V_t = i_{out}\left(1 + g_m R_2 + \frac{R_2}{R_1}\right)$$

$$\frac{i_{out}}{V_t} = \frac{g_m}{\left(1 + g_m R_2 + \frac{R_2}{R_1}\right)}$$

* desired form $G_m = i_{out}/V_t$

$$\therefore \boxed{G_m = \frac{g_m}{\left(1 + g_m R_2 + \frac{R_2}{R_1}\right)}}$$

s Sanity Check

$$G_m = \frac{i_{out}}{V_t} \quad \left[\frac{1}{\Omega}\right]$$

$$= \frac{g_m}{1 + g_m R_2 - \frac{R_2}{R_1}}$$

$$= \frac{\left[\frac{1}{\Omega}\right]}{\left(\left[\frac{\Omega}{\Omega}\right] + \left[\frac{1}{\Omega}\right]\left[\Omega\right] - \left[\frac{\Omega}{\Omega}\right]\right)}$$

$$= \frac{\left[\frac{1}{\Omega}\right]}{1} = \left[\frac{1}{\Omega}\right] \quad \checkmark$$

* desired Units for transconductance

$\therefore$ The derived expression for $G_m$ yields the correct unit $[1/\Omega]$

| Tristan Denning | Problem 2(c) | HW # 01 |
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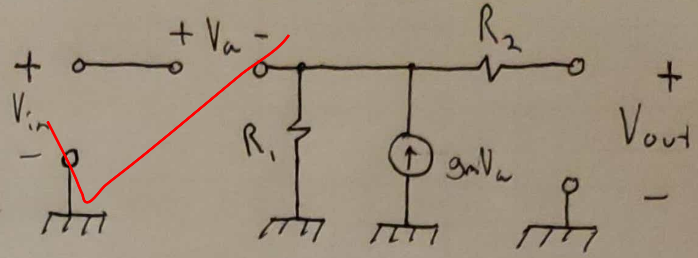
## $\boxed{2(c)}$

### 1 Situation




To derive an expression for the Voltage Gain, $A_v$, of the given circuit

### 2 Goal

To find a general expression for $A_v$ in terms of $R_1$, $R_2$, and $g_m$.

### 3 Plan

To Utilize known relationships of $Z_{in}$, $Z_{out}$ and $G_m$ along with the expressions derived in parts a.) and b.) to create an expression for $A_v$.

### 4 Solution

Known:

$$A_v = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{i_{out}} \times \frac{i_{out}}{V_{in}} = Z_{out} \times G_m$$

$$Z_{out} = R_{out} = \frac{\left(\frac{R_2}{R_1} + R_2 g_m + 1\right)}{\left(\frac{1}{R_1} + g_m\right)}$$

$$G_m = \frac{g_m}{\left(1 + g_m R_2 + \frac{R_2}{R_1}\right)}$$

Substitute:

$$A_v = \frac{g_m}{\left(1 + g_m R_2 + \frac{R_2}{R_1}\right)} \times \frac{\left(1 + g_m R_2 + \frac{R_2}{R_1}\right)}{\left(\frac{1}{R_1} + g_m\right)}$$

$$= \frac{g_m}{\left(\frac{1}{R_1} + g_m\right)}$$

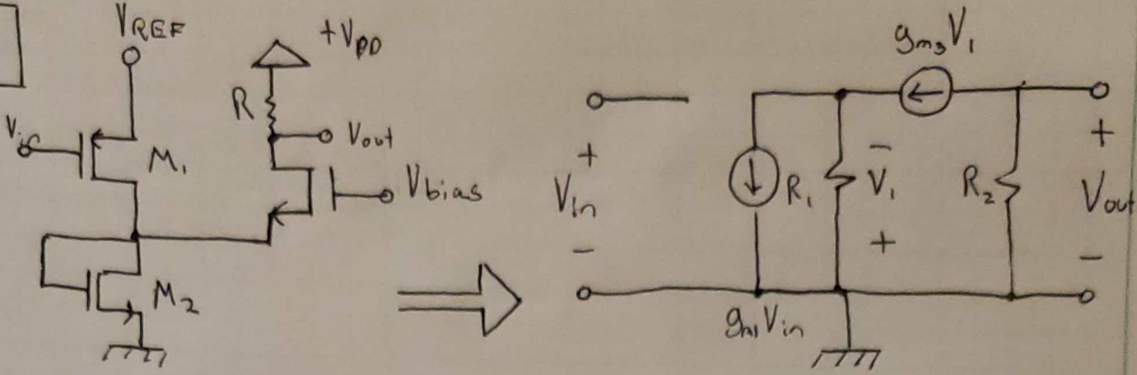
$$\therefore \boxed{A_v = \frac{g_m}{\left(\frac{1}{R_1} + g_m\right)}}$$

### 5 Sanity Check

$$A_v[1] = \frac{\left[\frac{1}{\Omega}\right]}{\left[\frac{1}{\Omega}\right] + \left[\frac{1}{\Omega}\right]} = [1] \checkmark$$

$\therefore$ Result is unitless which is desired for gain expression

## 3(a)



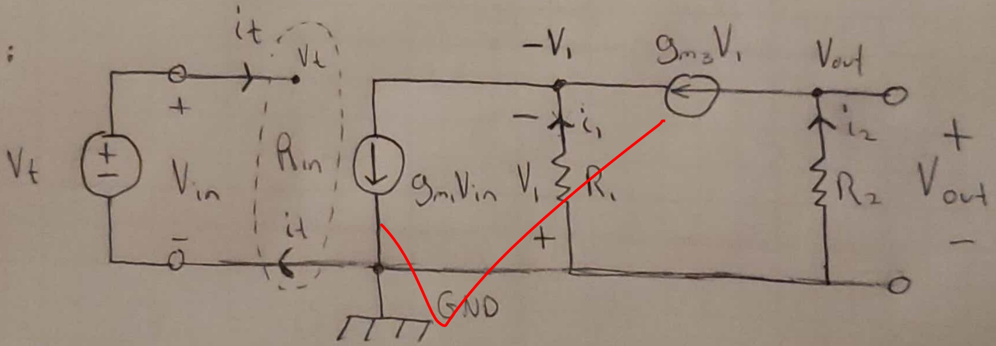

**1 Situation** To derive an expression for the INPUT resistance, $R_{in}$, of the small signal equivalent of the given Amplifier.

**2 Goal** To find a general expression for $R_{in}$ in terms of $R_1$, $R_2$, $g_{m1}$ and $g_{m3}$.

**3 Plan** To redraw the circuit applying a test Voltage $V_t$ between input ports and drive $V_t/i_t$ relation while leaving the output ports open, (Load disconnected). Then use KVL, KCL, and Ohms law to derive an expression relating $V_t$, $i_t$.

**4 Solution** Solve $R_{in} = V_t/i_t \Big|_{R_L=\infty}$ * Load disconnected

Redraw:




Observations: $V_t = V_{in}$

$i_t = 0$

$\therefore \quad R_{in} = V_t/0$

$= \infty$

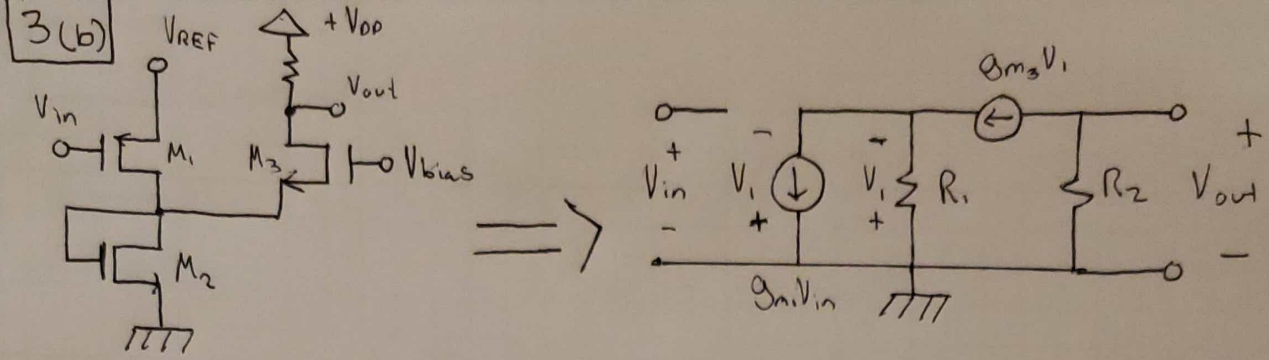
$$\boxed{R_{in} = \infty}$$

**5 Sanity Check**

$$R_{in}[\Omega] = \frac{V_{in}\,[V]}{0\,[A]} = [\Omega] \checkmark$$

Tristan Denning | Problem 3(b) part 1 | HW # 01

$\boxed{3(b)}$



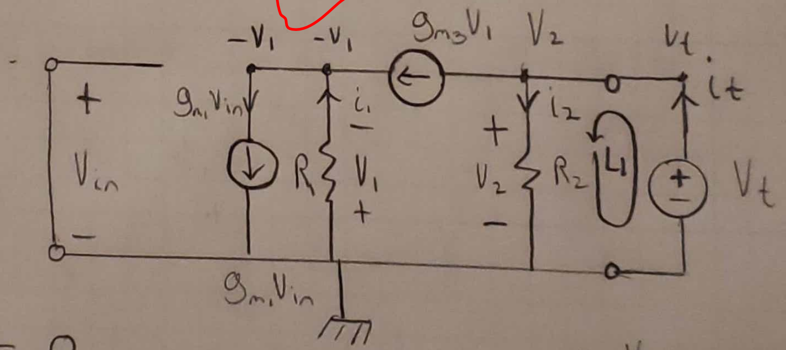

1 **Situation** To derive an expression for the output resistance, $R_{out}$ for the given circuit.

2 **Goal** To find a general expression for $R_{out}$ in terms of $g_{m1}$, $g_{m3}$, $R_1$, and $R_2$.

3 **Plan** To Redraw the circuit applying a test voltage between output ports and shorting the input terminals, then use fundamental circuit analysis techniques to find an equation relating $V_t$ and $i_t$. Solve $R_{out} = V_t/i_t$

4 **Solution** Solve $R_{out} = V_t/i_t \big|_{R_{in}=0}$

Redraw:




Observe: $V_{in} = 0$ * $V_{in}$ is shorted

KVL on $L_1$: $V_t = V_2$ (1)

Ohm's law $R_2$: $V_2 = i_2 R_2$ (2)

KCL @ $V_2$ : $i_t = i_2 + g_{m3}V_1$ (3)

KCL @ $-V_1$ : $g_{m3}V_1 + i_1 = g_{m1}V_{in}$

$-g_{m3}V_1 = i_1$ (4) * $V_{in} = 0$

Ohm's law $R_1$: $V_1 = i_1 R_1$ (5)

(1) → (2) : $V_t = i_2 R_2$

$i_2 = V_t/R_2$ (6)

$\boxed{8/14}$

$\boxed{3(b)}$ (continued)

⑥ → ③ $\quad i_t = V_t/R_2 + g_{m3}V_1$ ⑩

KCL @ GND : $\quad i_2 = i_1 + i_t + g_{m1}V_{in}$

$i_2 = i_1 + i_t \qquad * \; V_1 = 0$

$i_1 = i_2 - i_t$ ⑦

⑥ → ⑦ $\quad i_1 = V_t/R_2 - i_t$ ⑧

⑧ → ⑤ $\quad V_1 = \left(\frac{V_t}{R_2} - i_t\right) R_1$ ⑨

⑨ → ⑩ $\quad i_t = \frac{V_t}{R_2} + g_{m3}R_1\left(\frac{V_t}{R_2} - i_t\right)$ ⑪

* Rearrange ⑪ to $V_t/i_t$ form to obtain eqn for $R_{out}$

$$i_t = V_t/R_2 + \frac{g_{m3}R_1V_t}{R_2} - g_{m3}R_1 i_t$$

$$i_t + g_{m3}R_1 i_t = V_t/R_2 + \frac{g_{m3}R_1V_t}{R_2}$$

$$i_t(1+g_{m3}R_1) = V_t\left(\frac{1}{R_2} + \frac{g_{m3}R_1}{R_2}\right)$$

$$V_t/i_t = \frac{(1 + g_{m3}R_1)}{\left(\frac{1}{R_2} + \frac{g_{m3}R_1}{R_2}\right)}$$

$$V_t/i_t = \frac{(1+g_{m3}R_1)}{\frac{1}{R_2}(1+g_{m3}R_1)}$$

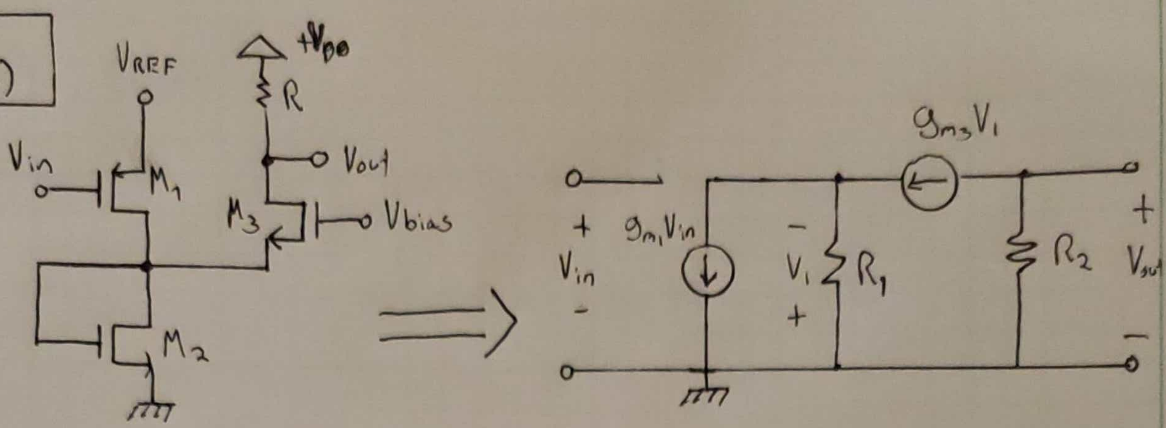
$$V_t/i_t = R_2$$

$$\therefore \boxed{R_{out} = R_2}$$

<u>Sanity Check</u>

$R_{out}\,[\Omega] = R_2\,[\Omega]$ ✓

**3(c)**



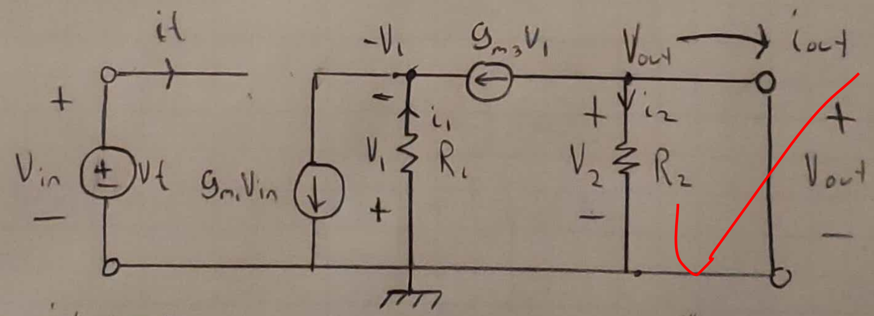

**1 Situation** To derive an expression for the transconductance, $G_m$, of the small signal equivalent given.

**2 Goal** To find a general expression for $G_m$ in terms of $R_1$, $R_2$, $g_{m1}$ and $g_{m3}$.

**3 Plan** To redraw the circuit applying a test voltage $V_t$ between input ports and short the load. Then use Fund. theorems of circuit analysis to find an equation relating the OUTPUT current and test voltage. Solve $G_m = i_{out}/V_t$

**4 Solution** Solve $G_m = \left. i_{out}/V_t \right|_{Z_L=0}$

Redraw:




Observe: $i_t = 0$, $V_2 = V_{out} = 0$, $i_2 = 0$ $\}$ * Open, * $V_{out}$ Shorted

KCL $V_{out}$: $i_{out} = -g_{m3}V_1$ (1)

KCL $-V_1$: $i_1 + g_{m3}V_1 = g_{m1}V_{in}$ (2)

Ohm's law $R_1$: $V_1 = i_1 R_1$ (3)

KCL @ GND: $g_{m1}V_{in} + i_{out} = i_1$ * $i_t = i_2 = 0$

$g_{m1}V_t + i_{out} = i_1$ (4) * $V_{in} = V_t$

$\boxed{3(c)}$ (Continued)

④ → ③ $\quad V_1 = (g_{m1} V_t + i_{out}) R_1 \quad$ ⑤

⑤ → ① $\quad i_{out} = -g_{m3}(g_{m1} V_t + i_{out}) R_1 \quad$ ⑥

* Rearrange eqn ⑥ to $i_{out}/V_t$ form

$$i_{out} = -g_{m3} g_{m1} V_t R_1 - g_{m3} i_{out} R_1$$

$$i_{out} + g_{m3} i_{out} R_1 = -g_{m3} g_{m1} R_1 V_t$$

$$i_{out}(1 + g_{m3} R_1) = V_t(-g_{m3} g_{m1} R_1)$$

$$i_{out}/V_t = \frac{(-g_{m3} g_{m1} R_1)}{(1 + g_{m3} R_1)}$$

$$\therefore \boxed{G_m = \frac{(-g_{m3} g_{m1} R_1)}{(1 + g_{m3} R_1)}}$$

<u>Sanity Check</u>

$$G_m[\mho] = \frac{\left(\left[\frac{1}{\Omega}\right]\left[\frac{1}{\Omega}\right][\Omega]\right)}{\left([-] + \left[\frac{1}{\Omega}\right][\Omega]\right)}$$

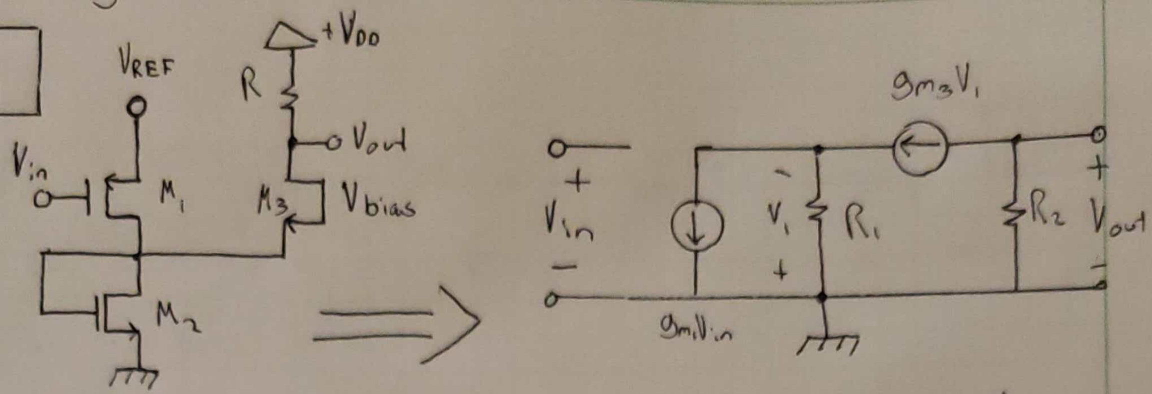
$$= \frac{\left[\frac{1}{\Omega}\right][\Omega]}{\left[\frac{\Omega}{\Omega}\right]}$$

$$= \left[\frac{1}{\Omega}\right]$$

$$= [\mho] \quad ✓$$

Tristan Denning | Problem 3(d) | HW #01

## 3(d)




1 **Situation** To derive an expression for the Voltage gain $A_v$, and current gain $A_i$ for the given small signal equivalent.

2 **Goal** To find a general expression for $A_v$ and $A_i$ in terms of $R_1, R_2, g_{m1}$ and $g_{m3}$.

3 **Plan** To utilize known equations for $A_v$ and $A_i$ in terms of $R_{in}$, $R_{out}$ and $G_m$ calculated in parts a, b, and c of this problem. Plug in derivations from previous parts to known relationships.

4 **Solution**

**Known:**

$$A_v = Z_{out} \times G_m = R_{out} \times G_m \quad (1)$$

$$A_i = Z_{in} \times G_m = R_{in} \times G_m \quad (2)$$

**Derived:**

$$R_{out} = R_2 \quad * \text{ Part (b)} \quad (3)$$

$$R_{in} = \infty \quad * \text{ Part (a)} \quad (4)$$

$$G_m = \frac{(-g_{m3}g_{m1}R_1)}{(1+g_{m3}R_1)} \quad * \text{ Part (c)} \quad (5)$$

**$A_v$:**

3 5 → 1:

$$\boxed{A_v = R_2\left(\frac{-g_{m3}g_{m1}R_1}{1+g_{m3}R_1}\right)}$$

Sanity Check $A_v[-] = [\Omega]\left[\frac{1}{\Omega}\right] = [-]$ ✓

**$A_i$:**

4, 5 → 2:

$$A_i = \infty\left(\frac{-g_{m3}g_{m1}R_1}{1+g_{m3}R_1}\right)$$

$$\boxed{A_i = \infty}$$

Sanity Check: $A_i[-] = [\Omega]\left[\frac{1}{\Omega}\right] = [-]$ ✓

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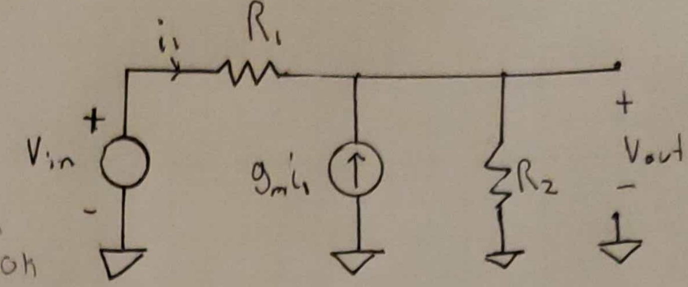
## 4 (a)



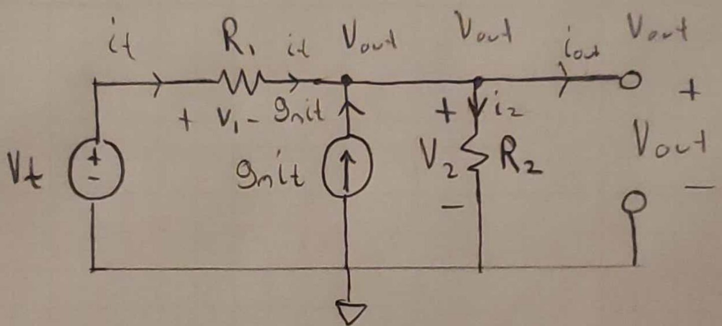

1 Situation: To derive an expression for the INPUT resistance $R_{in}$ of the given circuit. (a) then use $R_1 = 20k$, $R_2 = 1k$, $g_m = 50$ to calculate the value of $R_{out}$

2 Goal: To find a general expression for $R_{in}$ and its value given the specified components above.

3 Plan: To redraw applying a test voltage between input ports and disconnecting the load (open), then use fundamental circuit analysis techniques to find an equation relating the test voltage and test current. Solve $R_{in} = V_t / i_t$ then plug in.

4 Solution

Redraw:




$R_1 = 20k, \quad R_2 = 1k, \quad g_m = 50$

Observe: $V_2 = V_{out}$ (1)

$i_{out} = 0$

KCL @ $V_{out}$: $i_2 = i_t + g_m i_t$ (2)

Ohm's law $R_2$: $V_2 = i_2 R_2$ (3)

(1) → (3) $V_{out} = i_2 R_2$

$i_2 = V_{out}/R_2$ (4)

(4) → (2) $V_{out}/R_2 = i_t + g_m i_t$ (5)

KVL Outside Loop: $V_t - V_1 - V_{out} = 0$

$V_t - i_t R_1 - V_{out} = 0$ * $V_1 = i_t R_1$

$V_{out} = V_t - i_t R_1$ (6)

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**4(a)** (continued)

(6) → (5)

$$\frac{(V_t - i_t R_1)}{R_2} = i_t + g_m i_t \quad (7)$$

Manipulate (7) to $V_t/i_t$ Form

$$\frac{V_t}{R_2} - \frac{i_t R_1}{R_2} = i_t + g_m i_t$$

$$V_t\left(\frac{1}{R_2}\right) = i_t\left(\frac{R_1}{R_2} + g_m + 1\right)$$

$$V_t/i_t = R_1 + g_m R_2 + R_2$$

$$\therefore \boxed{R_{out} = R_1 + g_m R_2 + R_2}$$

5 Sanity Check

$$R_{out}\,[\Omega] = [\Omega] + [-][\Omega] + [\Omega]$$
$$= [\Omega] \checkmark$$

**4(b)**

Plug in given Values to expression derived in (a.)

$R_2 = 1k$ , $R_1 = 20k$ , $g_m = 50$

$$R_{out} = 20000 + 50\times10^{-3}(1000) + 1000$$

$$\boxed{R_{out} = \cancel{21050}\ \Omega}$$

71000 Ω

5 Sanity Check

Answer is in Ohms which was the expected result, and is consistant with previous Sanity Check.