100/100

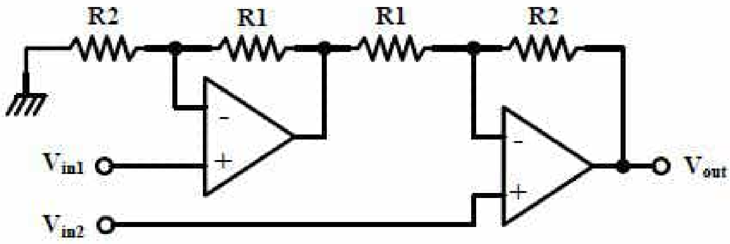
Tristan Denning

ECE 310 – Microelectronics I

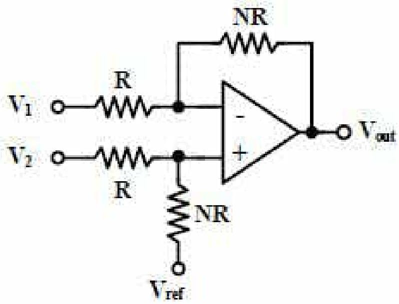
Dr. Suat Ay | **Homework #7** | *Fall 2021*

Due Date: 11/29/2021

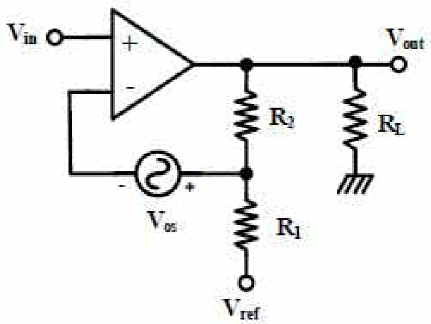
1. **(10 points)** Find the input-output relation (V_{out} versus V_{in1} and V_{in2}), using ideal Opamp model.



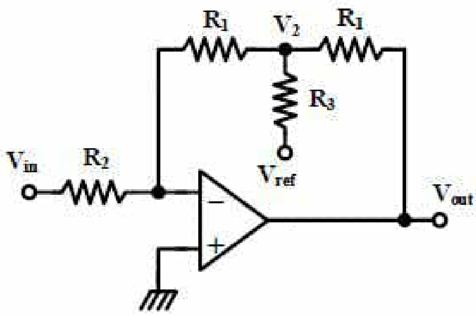
2. **(20 points)** Find the input-output relation (V_{out} versus N, V_{ref}, V_1 and V_2) using non-ideal Opamp model and verify that your derivation is correct. (NR means; N times R)



3. **(30 points)** Find the input-output relation (V_{out} versus V_{in}, V_{os} and V_{ref}) under following conditions.
   a. Find it using ideal Opamp model.
   b. Verify that your derivation is correct.



4. **(40 points)** For the circuit shown, assume R_1=100K, R_2=10K, and R_3=1K-100K (a potentiometer/trimmer).
   a. The input-output relation and voltage gain expression of the circuit using ideal OPAMP model.
   b. Verify that your derivation is correct.
   c. Calculate the voltage gain for minimum and maximum values of the R_3 (for V_{ref}= 0).
   d. Derive the voltage gain expression using non-ideal OPAMP model and verify that your derivation is correct for V_{ref}=0.



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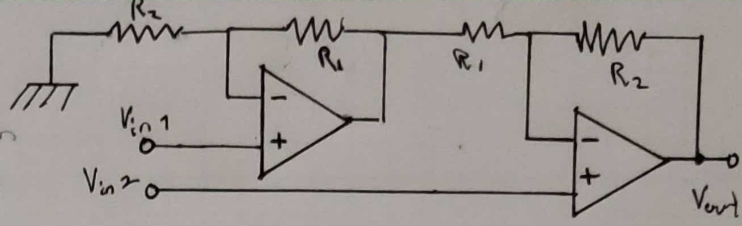
$\boxed{1.}$

1 **Situation:** Find the input-output relation (V_{out} vs V_{in1} and V_{in2}), using the ideal op-amp model.



2 **Goal:** To determine the i/o relationship between V_{out} and the two inputs, V_{in1} and V_{in2}.

3 **Plan:** To utilize the "3-zero" Rule and ideal opamp characteristics along with the fundamental circuit analysis techniques to determine a relation for the output in terms of the input.

4 **Solution:**

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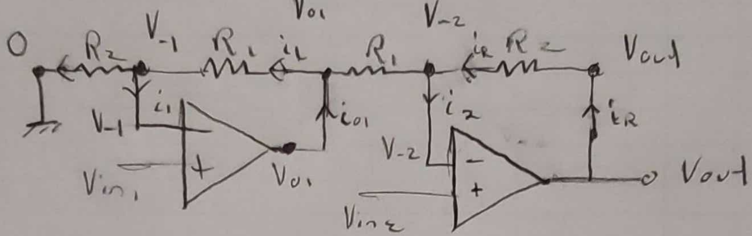


Observe:

$$i_1 = i_2 = 0$$
$$V_{in1} = V_{+1} = V_{-1}$$
$$V_{in2} = V_{+2} = V_{-2}$$

KCL V_{-1}

$$\frac{(V_{o1} - V_{-1})}{R_1} = \frac{V_{-1}}{R_2}$$

$$\therefore \quad V_{o1} = \frac{V_{-1}(R_1)}{R_2} + V_{-1}$$

$$\therefore \quad V_{o1} = V_{in1}\left(1 + \frac{R_1}{R_2}\right)$$

KCL V_{-2}

$$\frac{(V_{out} - V_{-2})}{R_2} = \frac{(V_{-2} - V_{o1})}{R_1}$$

$$(V_{out} - V_{-2})R_1 = (V_{-2} - V_{o1})R_2$$

$$V_{out}R_1 = V_{-2}R_1 + V_{-2}R_2 - V_{o1}R_2$$

$$V_{out} = V_{in2} + \frac{V_{in2}R_2}{R_1} - V_{in1}\left(1 + \frac{R_1}{R_2}\right)\frac{R_2}{R_1}$$

$$= V_{in2}\left(1 + \frac{R_2}{R_1}\right) - V_{in1}\left(1 + \frac{R_2}{R_1}\right)$$

$$\therefore \quad \boxed{V_{out} = (V_{in2} - V_{in1})\left(1 + \frac{R_2}{R_1}\right)}$$

$\boxed{1}$ (continued)

sSanity Check

Unit analysis:

$$V_{out} = (V_{in2} - V_{in1})\left(1 + \frac{R_2}{R_1}\right)$$

$$[V] = ([V] - [V])\left(1 + \frac{[\Omega]}{[\Omega]}\right)$$

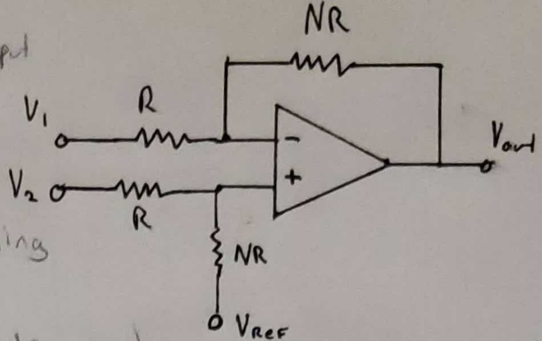
$$= [V]\,([-])$$

$$= [V] \checkmark$$

Units check out and process was straightforward.

2

1 Situation: Find the input-output relation (V_{out} vs N, V_{REF}, V_1 and V_2) using non-ideal opamp model and verify the derivation.



2 Goal: To determine an expression relating V_{out} to V_1, V_2, V_{REF}, and N.

3 Plan: To utilize the "2-zero" rule and nonideal opamp characteristics along with the fundamental circuit analysis techniques to determine a relation for the output in terms of V_1, V_2, V_{ref}, and N.

4 Solution:

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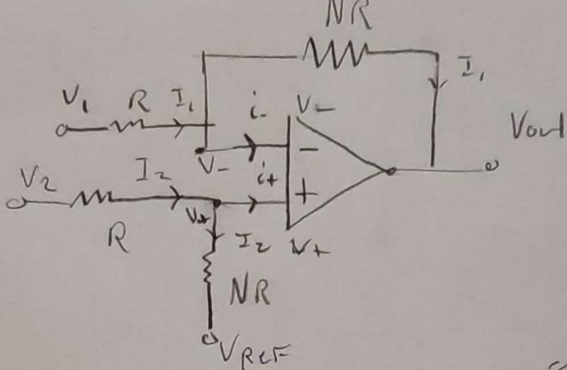


Observe:
$$I_+ = I_- = 0 \quad \because \text{"2-zero"}$$
$$V_{out} = A_o(V_+ - V_-) \qquad (1)$$

KCL V_-
$$\frac{(V_1 - V_-)}{R} = \frac{(V_- - V_{out})}{NR}$$
$$(V_1 - V_-) = \frac{(V_- - V_{out})}{N}$$
$$V_1 = V_- + \frac{V_-}{N} - \frac{V_{out}}{N}$$
$$V_1 + \frac{V_{out}}{N} = V_-\left(1 + \frac{1}{N}\right)$$
$$\therefore \quad V_- = \frac{\left(V_1 + \frac{V_{out}}{N}\right)}{\frac{N+1}{N}}$$
$$V_- = \frac{NV_1 + V_{out}}{N+1} \qquad (2)$$

$\boxed{2}$ (Continued)

KCL V_+
$$\frac{(V_2 - V_+)}{R} = \frac{V_+ - V_{Ref}}{NR}$$

$$\therefore \quad V_+ = \frac{NV_2 + V_{Ref}}{N+1} \quad (3)$$

(2), (3) $\rightarrow$ (1):
$$V_{out} = A_o \frac{1}{N+1}\left((NV_2 + V_{ref}) - (NV_1 + V_{out})\right)$$

$$V_{out} = A_o \frac{1}{N+1}(NV_2 + V_{Ref}) - \frac{A_o V_1 N}{N+1} - \frac{A_o V_{out}}{N+1}$$

$$V_{out}\left(1 + \frac{A_o}{N+1}\right) = \frac{A_o}{N+1}(NV_2 + V_{Ref} - NV_1)$$

$$\therefore \quad V_{out} = \frac{\frac{A_o}{N+1}(NV_2 + V_{Ref} - NV_1)}{\left(1 + \frac{A_o}{N+1}\right)}$$

$$= \frac{A_o(NV_2 + V_{ref} - NV_1)}{(N+1 + A_o)}$$

$$\therefore \quad \boxed{V_{out} = \frac{A_o(NV_2 + V_{REF} - NV_1)}{N+1 + A_o}}$$

Verify: Use Ideal Model, then Check $\lim_{A_o \to \infty} (V_{out})$ For Non-Ideal

$$V_+ = V_-$$

$$\therefore \quad NV_2 + V_{Ref} = NV_1 + V_{out}$$

$$\therefore \quad V_{out,ideal} = NV_2 + V_{Ref} - NV_1$$

$\boxed{2}$ (Continued)

Sanity Check:

Check $\lim_{A_0 \to \infty} (V_{out}) \stackrel{?}{=} V_{out,\ Ideal}$

$$\lim_{A_0 \to \infty} \left(\frac{A_0 (NV_2 + V_{REF} - NV_1)}{N+1 + A_0} \right)$$

$$= \frac{\infty (NV_2 + V_{REF} - NV_1)}{\infty + (N+1)}$$

$$= \frac{\infty (NV_2 + V_{REF} - NV_1)}{\infty}$$

$$= NV_2 + V_{REF} - NV_1$$

$$= V_{out,\ Ideal}$$

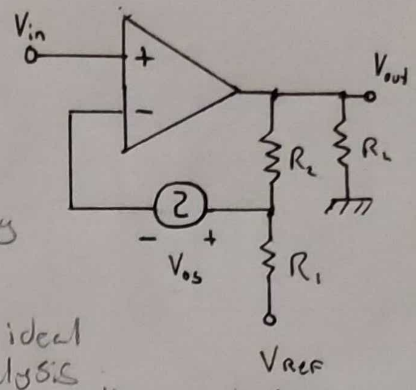
$$\therefore \lim_{A_0 \to \infty} (V_{out}) = V_{out,\ Ideal}$$

So the derivation for V_{out} using Non-Ideal Op Amp model is correct

Simplified: $$V_{out} = \frac{A_0 (N(V_2 - V_1) + V_{REF})}{A_0 + N + 1}$$

3

1. **Situation:** Find the input-output relation (V_{out} vs V_{in}, V_{os}, and V_{ref}) for the given circuit. a.) Use ideal opamp model b.) Verify the derivation is correct.



2. **Goal:** To relate V_{out} to V_{in}, V_{os}, V_{ref} using ideal opamp model, then verify.

3. **Plan:** To use the "3-zero" rule along with ideal opamp characteristics and fundamental circuit analysis techniques - including superposition - to determine the i/o relation and then verify the derivation:

4. **Solution:**

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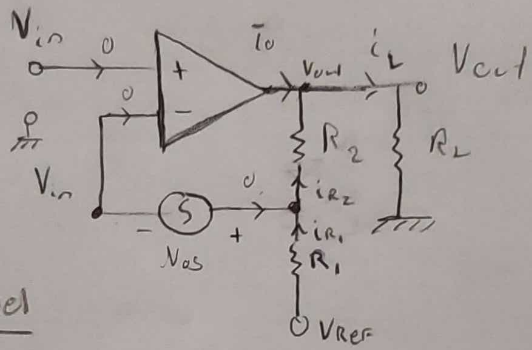


a.) Ideal Opamp Model

Observe: $i_{R_2} = i_{R_1}$

$V_{R_1} = V_{in} + V_{os}$ ∵ "3-Zero Rule"

KCL $+V_{os}$: $$\frac{(V_{in}+V_{os}) - V_{out}}{R_2} = \frac{V_{ref} - (V_{in}+V_{os})}{R_1}$$

$$\left((V_{in}+V_{os}) - V_{out}\right)R_1 = \left(V_{ref} - (V_{in}+V_{os})\right)R_2$$

$$V_{in}R_1 + V_{os}R_1 - V_{out}R_1 = V_{ref}R_2 - V_{in}R_2 - V_{os}R_2$$

$$R_1(V_{in}+V_{os}) - R_2(V_{ref} - V_{in} - V_{os}) = V_{out}R_1$$

$$V_{out} = (V_{in}+V_{os}) - \left(V_{ref} - (V_{in}+V_{os})\right)\frac{R_2}{R_1}$$

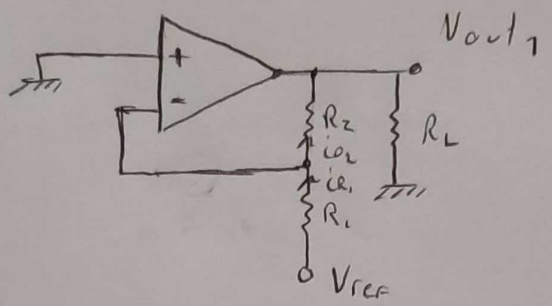
$$\boxed{V_{out} = (V_{in}+V_{os}) - \left(V_{ref} - (V_{in}+V_{os})\right)\frac{R_2}{R_1}} \quad (1)$$

$\boxed{3}$ (continued)

b.) Verify derivation from a.)

Superposition 1.) Set $V_{in} = 0$, $V_{os} = 0$, $V_{ref} \neq 0$

Redraw



Observe:

$$V_+ = V_- = -V_{os} = 0 \quad \because \text{ Grounded}$$

$$\therefore V_{os} = 0$$

then

$$i_{R_1} = i_{R_2}$$

$$\frac{V_{Ref} - 0}{R_1} = \frac{-V_{out1}}{R_2}$$

$$V_{out1} = -V_{Ref}\frac{R_2}{R_1} \qquad (*)$$

Superposition 2.) Set $V_{ref} = 0$, $V_{in} = 0$, $V_{os} \neq 0$

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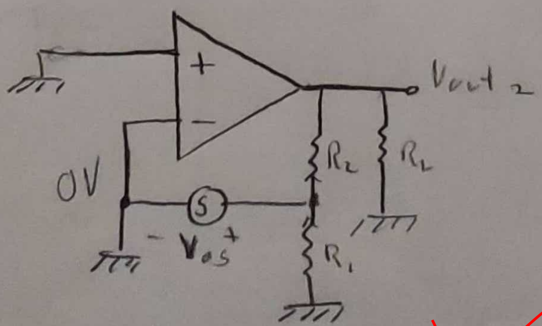


Observe:

$$V_{in} = V_+ = V_- = -V_{os}$$

$$i_{R_1} = i_{R_2}$$

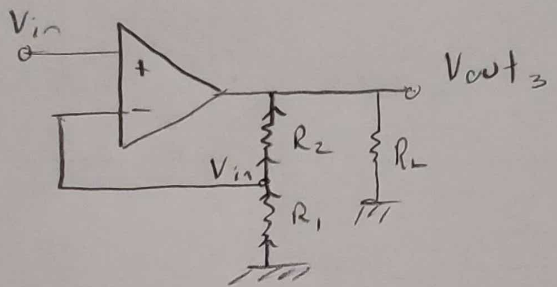
$\boxed{3}$ (continued)

$$\frac{0 - V_{os}}{R_1} = \frac{V_{os} - V_{out2}}{R_2}$$

$$-V_{os}\frac{R_2}{R_1} - V_{os} = -V_{out2}$$

$$V_{os}\left(1 + \frac{R_2}{R_1}\right) = V_{out2} \qquad (*)$$

Superposition 3: $V_{in} \neq 0$, $V_{os} = 0$, $V_{ref} = 0$



$$\frac{0 - V_{in}}{R_1} = \frac{V_{in} - V_{out3}}{R_2}$$

$$\therefore \quad V_{out3} = V_{in}\left(1 + \frac{R_2}{R_1}\right) \qquad (*)$$

Add $V_{out1} + V_{out2} + V_{out3}$, Check $= V_{out}$

$$\Longrightarrow \quad -V_{Ref}\frac{R_2}{R_1} + V_{os}\left(1 + \frac{R_2}{R_1}\right) + V_{in}\left(1 + \frac{R_2}{R_1}\right)$$

$$= -V_{Ref}\frac{R_2}{R_1} + V_{os} + V_{os}\frac{R_2}{R_1} + V_{in} + V_{in}\frac{R_2}{R_1}$$

$$= (V_{os} + V_{in}) + \frac{R_2}{R_1}\left(-V_{ref} + V_{os} + V_{in}\right)$$

$$= (V_{os} + V_{in}) - \left(V_{Ref} - (V_{os} + V_{in})\right)\frac{R_2}{R_1}$$

$$= 1$$

$\boxed{3}$ (continued)

5 Sanity Check:

Topology Check: The given looks like it is Almost a Non-inverting Amp. Set V_{os}, $V_{Ref} = 0$ to Check

$$V_{out} = (V_{in} + V_{os}) - (V_{ref} - (V_{in} + V_{os}))\frac{R_2}{R_1}$$

$$V_{out}\Big|_{V_{os}=V_{ref}=0} = V_{in} - (0 - (V_{in}))\frac{R_2}{R_1}$$

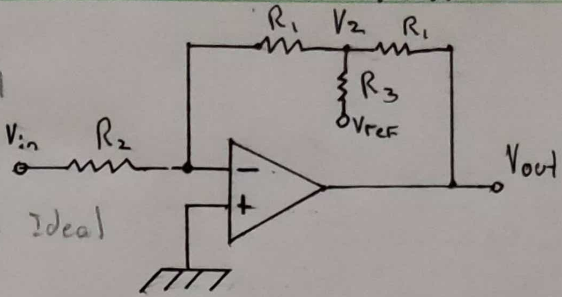
$$= V_{in} + V_{in}\frac{R_2}{R_1}$$

$$= \left(1 + R_2/R_1\right) V_{in} \quad \checkmark$$

For a non-inverting Amp, $V_{out} = V_{in}\left(1 + \frac{R_2}{R_1}\right)$

Since I verified my expression for V_{out} in part a.) with Superposition in b.), and the topology Check Worked, I am Confident in my expression

4 (a.) 1 **Situation:** For the Circuit shown, assume $R_1 = 100k$, $R_2 = 10k$, $R_3 = 1k - 100k$.
a.) Find the i/o relation and Voltage gain expression using the Ideal Opamp model.



2 **Goal:** To determine an expression relating V_{out} to V_{in}, V_{ref}, V_2 and the resistors, then find $A_V = V_{out}/V_{in}$ for the given amplifier.

3 **Plan:** To use the fundamental circuit analysis techniques along with the "3-zero" rule to derive a relationship between V_{out} and the inputs.

4 **Solution:**

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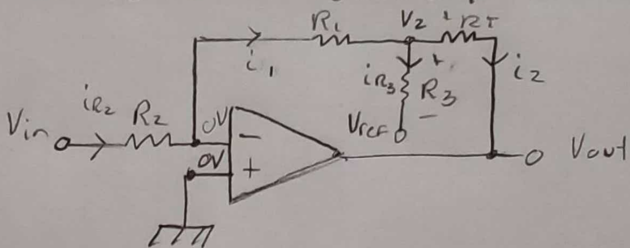

Observe:

$$V_+ = V_- = 0 \quad \because \text{"3-zero" rule}$$
$$i_{R2} = i_1$$

KCL V_-

$$\frac{V_{in} - 0}{R_2} = \frac{0 - V_2}{R_1}$$
$$\frac{V_{in}}{R_2} = \frac{-V_2}{R_1}$$
$$\therefore V_2 = -V_{in}\frac{R_1}{R_2} \quad (1)$$

KCL V_2

$$i_1 = i_{R3} + i_2$$
$$\frac{0 - V_2}{R_1} = \frac{V_2 - V_{REF}}{R_3} + \frac{V_2 - V_{out}}{R_1}$$
$$-V_2\left(\frac{1}{R_1}\right) = V_2\left(\frac{1}{R_3}\right) - V_{ref}\left(\frac{1}{R_3}\right) + V_2\left(\frac{1}{R_1}\right) - V_{out}\left(\frac{1}{R_1}\right)$$
$$V_{out}\left(\frac{1}{R_1}\right) = V_2\left(\frac{1}{R_1} + \frac{1}{R_1} + \frac{1}{R_3}\right) - V_{ref}\left(\frac{1}{R_3}\right)$$
$$V_{out} = V_2\left(2 + \frac{R_1}{R_3}\right) - V_{ref}\cdot\frac{R_1}{R_3} \quad (2)$$

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$\boxed{4(a)}$ (continued)

① → ②

$$V_{out} = -V_{in}\frac{R_1}{R_2}\left(2+\frac{R_1}{R_3}\right) - V_{ref}\cdot\frac{R_1}{R_3}$$

$$V_{out} = -V_{in}\frac{100k}{10k}\left(2+\frac{100k}{R_3}\right) - V_{ref}\left(\frac{100k}{R_3}\right)$$

$$V_{out} = -V_{in}\left(10\left(2+\frac{100k}{R_3}\right)\right) - V_{ref}\left(\frac{100k}{R_3}\right)$$

$$V_{out} = -V_{in}\left(20+\frac{1000k}{R_3}\right) - V_{ref}\left(\frac{100k}{R_3}\right)$$

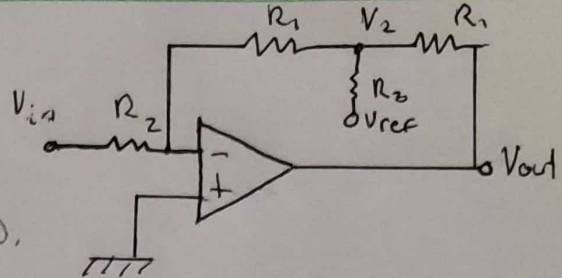
For $V_{ref} = 0$

$$\boxed{V_{out} = -V_{in}\left(20+\frac{1000k}{R_3}\right)}$$

$$\therefore \frac{V_{out}}{V_{in}} = -\left(20+\frac{1000k}{R_3}\right)$$

$$\boxed{A_v = -\left(20+\frac{1000k}{R_3}\right)}$$

5 Sanity Check: See part 4(b).



4(b)

1 Situation: For the gain expression determined in 4(a), Verify that it is correct

2 Goal: To Verify the result for 4(a).

3 Plan: To use a topology check to Verify the gain expression determined in 4(a).

4 Solution:

Sanity Check For 4(a):

Impose Inverting amplifier by setting $V_{REF} = 0$, $R_3 \to \infty$

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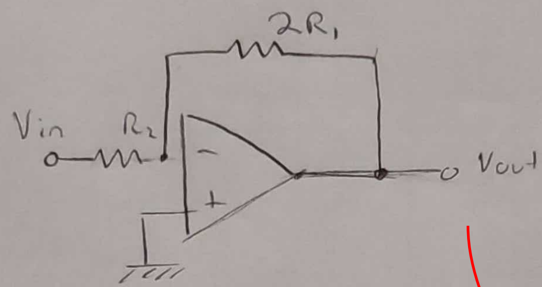


For inverting Amplifier (Ideal):

$$A_{v,inv} = -\frac{2R_1}{R_2} = \frac{-200k}{10k}$$

$$A_{v,inv} = -20$$

Impose $V_{REF} = 0$, $R_3 \to \infty$ to A_v from 4(a).

$$A_v = -\left(20 + \frac{1000k}{R_3}\right)$$

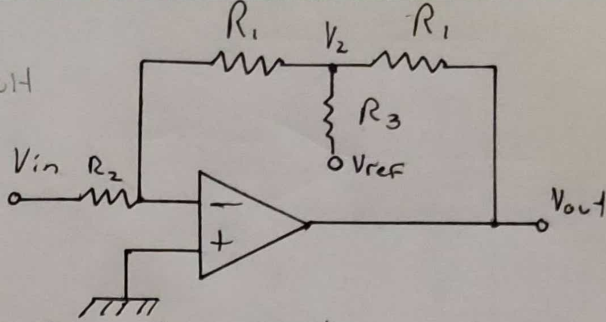
$$\lim_{R_3 \to \infty} (A_v) = -\left(20 + \frac{1000k}{\infty}\right)$$

$$= -20 + 0$$

$$= -20 \quad \checkmark$$

5 Sanity Check:

Imposing an inverting amplifier topology yields the same gain both mathematically, and by observation for known inverting Amp A_v equation



4(c)

1 **Situation:** For the Circuit shown with values given in 4(a), c.) Calculate the voltage gain for minimum and maximum values of R_3 (let $V_{ref} = 0$).

2 **Goal:** To determine the circuit's gain at $R_3 = 1k$, and for $R_3 = 100k$.

3 **Plan:** To evaluate the gain expression found in 4(a) for $A_v|_{R_3=1k}$ and $A_v|_{R_3=100k}$

4 **Solution:**

R_3 Minimum

$$A_v|_{R_3=1k} = -\left(20 + \frac{1000k}{1k}\right)$$

$$= \boxed{-1020 \quad [V/V]}$$

R_3 Maximum

$$A_v|_{R_3=100k} = -\left(20 + \frac{1000k}{100k}\right)$$

$$= \boxed{-30 \quad [V/V]}$$

∵ Assumption: $V_{ref} = 0$

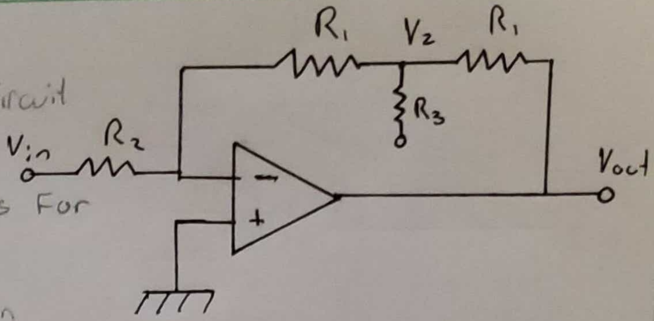
5 **Sanity Check:**

This result makes sense. R_3 acts as a controller for the inverting amplifier by increasing or decreasing the effective resistance between the V_- node and Vout.

Tristan Denning | Problem 4(d) | HW # 07

4(d)

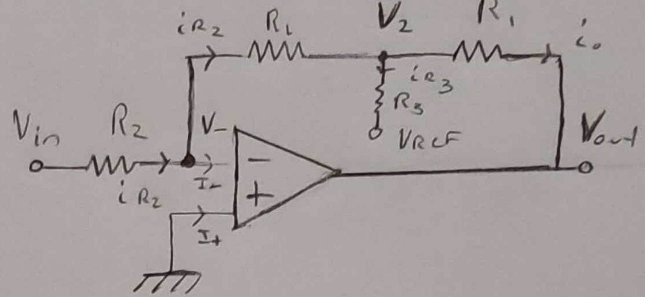
1 **Situation:** For the Circuit shown, d.) Derive the Voltage gain expression using non-ideal Opamp model and Verify correctness For $V_{REF} = 0$.



2 **Goal:** To derive an expression For the Voltage gain Using NON-ideal model.

Plan: To utilize "2-Zero" Rule along with NON-ideal Opamp characteristics to derive an expression for the gain, A_v of the circuit.

Solution: Redraw:



Observe:

$$I_+ = 0$$
$$I_- = 0$$
$$V_{out} = A_o(V_+ - V_-)$$
$$V_+ = 0 \Rightarrow V_{out} = -A_o V_- \quad (1)$$

KCL V_-

$$i_{R_2} = i_{R_2}$$
$$\frac{V_{in} - V_-}{R_2} = \frac{V_- - V_2}{R_1}$$
$$(V_{in} - V_-)R_1 = (V_- - V_2)R_2$$
$$V_{in}R_1 + V_2R_2 = V_-(R_1 + R_2)$$
$$\therefore V_- = \frac{(V_{in}R_1 + V_2R_2)}{(R_1 + R_2)} \quad (2)$$

KCL V_2

$$i_{R_2} = i_o + i_{R_3}$$
$$\frac{(V_- - V_2)}{R_1} = \frac{V_2 - V_{REF}}{R_3} + \frac{V_2 - V_{out}}{R_1}$$
$$\therefore V_2 = \frac{V_-R_3 + V_{REF}R_1 + V_{out}R_3}{R_1 + 2R_3} \quad (3)$$

$\because$ From Calculator

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$\boxed{4(d)}$ (Continued)

③ → ②

$$V_- = \frac{\left(V_{in}R_1 + \frac{(V_-R_3 + V_{REF}R_1 + V_{out}R_3)R_2}{(R_1 + 2R_3)}\right)}{R_1 + R_2} \quad (4)$$

④ → ①

$$V_{out} = -A_o\left(\frac{\left(V_{in}R_1 + \frac{(V_-R_3 + V_{REF}R_1 + V_{out}R_3)R_2}{R_1 + 2R_3}\right)}{R_1 + R_2}\right)$$

$$\frac{V_{out}}{V_{in}} = A_v = \frac{-20(R_3 + 50000)A_o}{100000A_oV_{REF} + R_3(A_o + 21) + 1100000}$$

$\because$ $R_1 = 100k$, $R_2 = 10k$, Plugged into Calculator to obtain A_v expression

Now $$A_v\Big|_{V_{REF}=0} = \frac{-20(R_3 + 50000)A_o}{R_3(A_o + 21) + 1100000}$$

$$\therefore \boxed{A_v' = \frac{-20A_o(R_3 + 50000)}{R_3(A_o + 21) + 1100000}}$$

5 Sanity Check: Verify $V_{REF} = 0$ gain expression

$$\lim_{A_o \to \infty}(A_v') = \frac{-20(\infty)(R_3 + 50000)}{R_3(\infty + 21) + 1100000}$$

$$= \frac{-20(R_3 + 50000)}{R_3} \quad \because \text{Calculator}$$

$$= -20 + \frac{1{,}000{,}000}{R_3} \quad ✓$$

This is the same result as 4(a.)

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