ECE 351 - Section 51

Fourier Series Approximation of a Square Wave

Lab 8 Prelab

Submitted By:
Tristan Denning

1 Solution

Task 1 - Derivations for a$_k, b_k, x(t)$

a$_k$.)

$$a_k = \frac{2}{T}\int_0^T x(t)\cos(k\omega_0 t)dt$$

$$\therefore b_k = \frac{2}{T}\left[\int_0^{\frac{T}{2}} \cos(kw_0 t)dt - \int_{\frac{T}{2}}^{T} \cos(k\omega_0 t)dt\right]$$

$$= \frac{2}{Tkw_0}\left[\left(\sin\left(kw_0\frac{T}{2}\right) - \sin(kw_0 * 0)\right) - \left(\sin(kw_0 T) - \sin\left(kw_0\frac{T}{2}\right)\right)\right]$$

$$= \frac{2}{2\pi kw_0}\left[\left(\sin\left(kw_0\frac{2\pi}{2w_0}\right) + 0\right) - \left(\sin\left(kw_0\frac{2\pi}{w_0}\right) - \sin\left(kw_0\frac{2\pi}{2w_0}\right)\right)\right]$$

$$= 0$$

$$a_k = 0 \tag{1}$$

This result is consistent with the symmetry expectations of a Fourier series.

For odd functions (like the one given):

$$a_0 = 0 \tag{2}$$
$$a_k = 0$$

$\mathbf{b}_k$.)

$$b_k = \frac{2}{T}\int_0^T x(t)\sin(k\omega_0 t)dt$$

$$x(t) = \begin{cases} 1, 0 < t < \frac{T}{2} \\ -1, \frac{T}{2} < t < T \end{cases}$$

$$\therefore b_k = \frac{2}{T}\left[\int_0^{\frac{T}{2}} \sin(kw_0 t)dt - \int_{\frac{T}{2}}^{T} \sin(k\omega_0 t)dt\right]$$

$$= \frac{2}{Tkw_0}\left[\left(-\cos\left(kw_0\frac{T}{2}\right) + \cos(kw_0 * 0)\right) - \left(-\cos(kw_0 T) + \cos\left(kw_0\frac{T}{2}\right)\right)\right]$$

$$= \frac{2}{2\pi kw_0}\left[\left(-\cos\left(kw_0\frac{2\pi}{2w_0}\right) + 1\right) - \left(-\cos\left(kw_0\frac{2\pi}{w_0}\right) + \cos\left(kw_0\frac{2\pi}{2w_0}\right)\right)\right]$$

$$= \frac{1}{k\pi}[-2\cos(k\pi) + 1 + \cos(2k\pi)]$$

$$= \frac{2}{k\pi}[-\cos(k\pi) + 1] \because \cos(2k\pi) = 1$$

$$= \frac{2}{k\pi}[1 - (-1)^n]$$

$$\therefore b_k = \frac{2}{k\pi}[1 - (-1)^n] \tag{3}$$

Now plug (1) and (2) into x(t)

$$x(t) = \sum_{n=1}^{\infty}\left[\frac{2}{k\pi}(1 - (-1)^n)\sin(kw_0 t)\right] \tag{4}$$