ECE 351 - Section 51

Fourier Series Approximation of a Square Wave

Lab 8

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Contents

1 Introduction

The objective of this lab is to use Python to create a Fourier Series approximation of the square wave below:

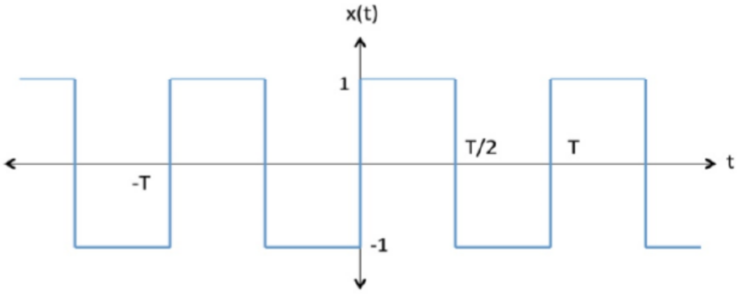


Figure 1: Square Wave Function

*All of the code used to accomplish the goals of this lab can be accessed at my Github page: `https://github.com/Tristan-Denning`

2 Equations

$$a_k = 0 \tag{1}$$

$$b_k = \frac{2}{k\pi}[1 - \cos(k\pi)] = \frac{2}{k\pi}[1 - (-1)^k] \tag{2}$$

Note that $\cos(k\pi)$ can be simplified to $(-1)^k$ because for whole values of k, the expression will evaluate to only 1 or -1. Equations (1) and (2) were implemented in Python with the following code:

```
def ak(k):
    y = 0
    return y

def bk(k):
    y = (2/(k*np.pi))*(1-(-1)**(k))
    # y = 2/((k)*np.pi)*(1-np.cos((k)*np.pi))
    return y

```

3 Methodology

This lab demonstrates how a Fourier Series Expansion can approximate a square wave signal with a variable period T. The working theory behind Fourier Series Expansion is that a periodic signal $f(t)$ can be approximated by an infinite sum of sinusoidal signals. The general Fourier series is given by the following equation:

$$x(t) = \frac{1}{2}a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t) \tag{3}$$

Expansion using equation (3) can be made simpler by noticing the orthogonality relationships of sine and cosine. In general:

For odd functions (like the one given):

$$a_0 = 0$$
$$a_k = 0$$

For even functions:

$$b_k = 0$$

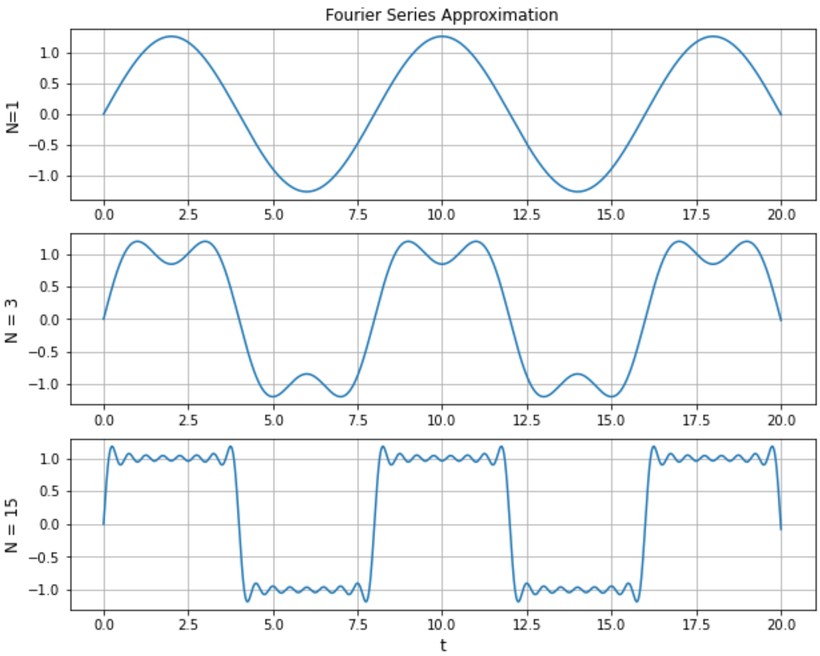
Using equations (1) and (2), equation (3) can be implemented in Python with the following for loop:

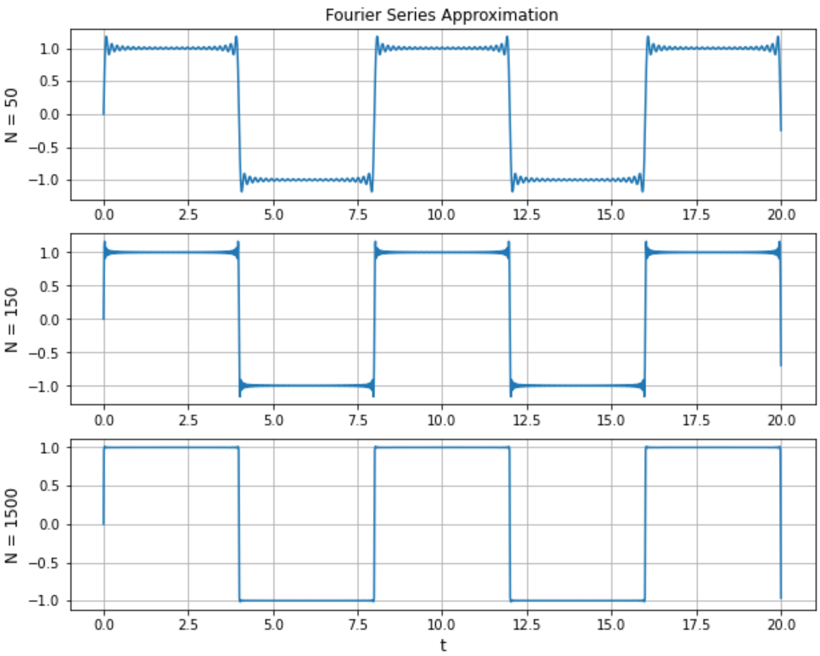
```
def fourier(N, T):
    w = 2*np.pi/T

    result = 0
    for i in np.arange(1, N+1):
        intermediate = bk(i)*np.sin(i*w*t)
        result += intermediate
    return result
```

The function takes a period value T, as well as a total number of sums to be completed N. The function returns the Fourier Series Approximation.

4 Results

Task 2 - Fourier Series Expansions for N = {1, 3, 15, 50, 150, 1500}





5 Error

No error was experienced while completing the required tasks for Lab 7.

6 Questions

1. Is $x(t)$ an even or an odd function? Explain why.
   (a) $x(t)$ is an odd function because it has symmetry about the origin. In other words:
   $$f(x) = -f(-x) \text{ for all } x$$
2. Based on your results from Task 1, what do you expect the values of $a_2, a_3, ..., a_n$ to be? Why?
   (a) The values of $a2, a3, ..., a_n$ will always be zero - as reflected in equation (1). This is a result of the function's odd symmetry.
3. How does the approximation of the square wave change as the value of N increases? In what way does the Fourier series struggle to approximate the square wave?
   (a) As N increases, the Fourier series approximation of $x(t)$ becomes more accurate. The Fourier series has difficulty approximating sharp corners (cusps).
4. What is occurring mathematically in the Fourier series summation as the value of N increases?
   (a) As N increases, the frequency of the approximating sinusoidal functions increases. For each value of N, an additional sinusoidal signal with a higher frequency is added to the total.
5. Leave any feedback on the clarity of lab tasks, expectations, and deliverables.
   (a) I have no feedback regarding the clarity of lab tasks, expectations, and deliverables.

7 Conclusion

The objective of this lab was fulfilled. Python was used to implement a Fourier series expansion for a square wave. This lab demonstrates the even and odd properties of

Fourier series, as well as how to accuracy of the approximation increases with N.

8 Appendix A

Task 1 - Output for $a_0, a_1, b_1, b_2,$ and b_3

```
a0 =  0
a1 =  0

b1 =  1.2732395447351628
b2 =  0.0
b3 =  0.4244131815783876
```