ECE 351 - Section 51

Step and Impulse Response of a RLC Band Pass Filter

Lab 5

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Contents

1 Introduction

The purpose of this lab is to use Laplace transforms to find the time-domain response of an RLC bandpass filter to impulse and step inputs.

*All of the code used to accomplish the goals of this lab can be accessed at my Github page: `https://github.com/Tristan-Denning`

2 Equations

Unit Step Function

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \tag{1}$$

Transfer Function for Bandpass Filter

$$H(S) = \frac{S}{RCS^2 + S + \frac{R}{L}} \tag{2}$$

Impulse Response (Inverse Laplace Transform of (2))

$$h(t) = 10355.6e^{-5000t} \sin(18584.1t + 105.06^o)u(t) \tag{3}$$

Final Value Theorem

$$\lim_{t \to \infty} h(t) \ = \lim_{s \to 0} H(s)$$

$$10355.6e^{-\infty * 5000} \sin(18584.1t + 105.06^o)u(\infty) = \frac{0^2}{0^2 * RC + 0 + \frac{R}{L}}$$

$$0 = 0$$

$$\therefore \lim_{t \to \infty} h(t) \ = 0 = \lim_{s \to 0} H(s)$$

3 Methodology

Part 1

Part one of this lab aims to compare the impulse response calculated by hand to that calculated by the scipy.signal import. Equation (2) is obtained by transforming the circuit in *Figure 1* to the s-domain, then solving for $H(s) = \frac{V_{out}}{V_{in}}$.

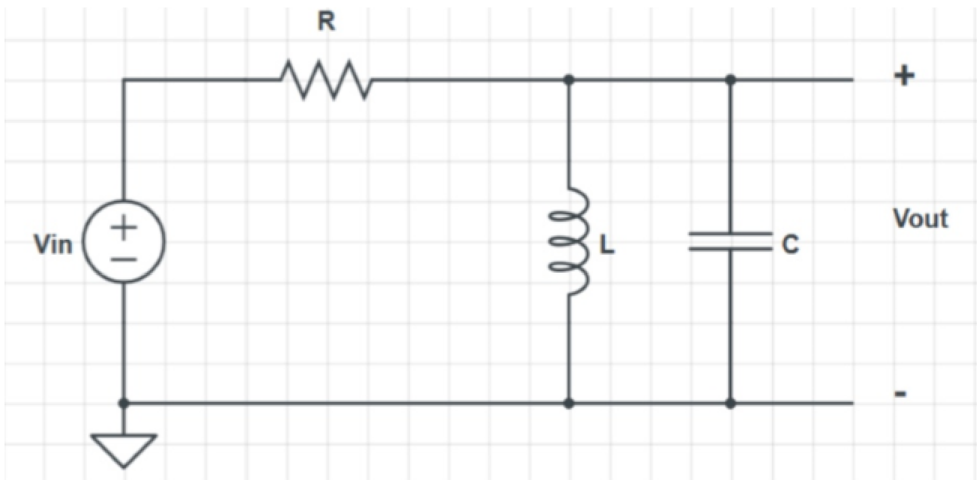


Figure 1: R = 1kΩ, L = 27 mH, C = 100 nF

Equation (3) is obtained by performing the inverse Laplace transform of equation (2) and plugging in the given R, L and C values.

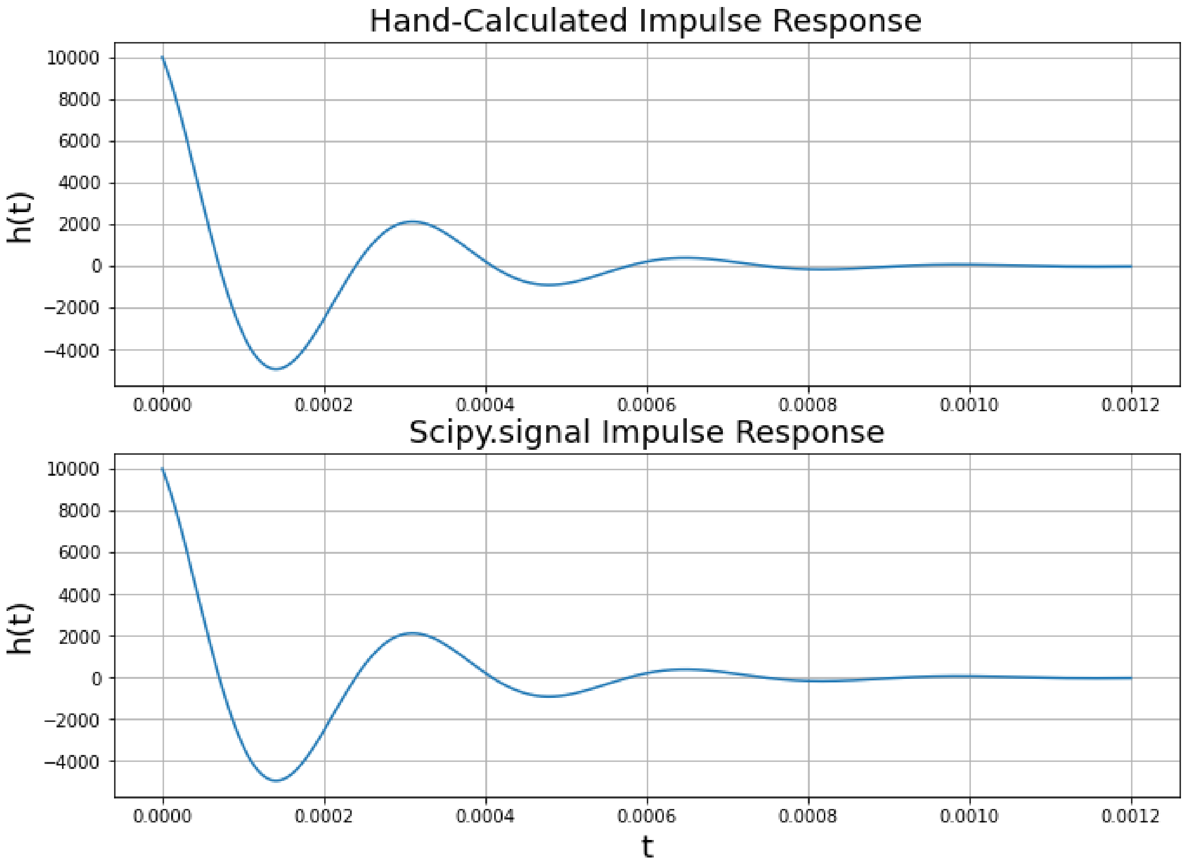
Part 2

Part 2 of this project demonstrates the Final Value Theorem for the equations (2) and (3) developed in in part 1. The Final Value Theorem for the corresponding expressions is listed in the **Equations** section.

4 Results

4.1 Part 1 Results

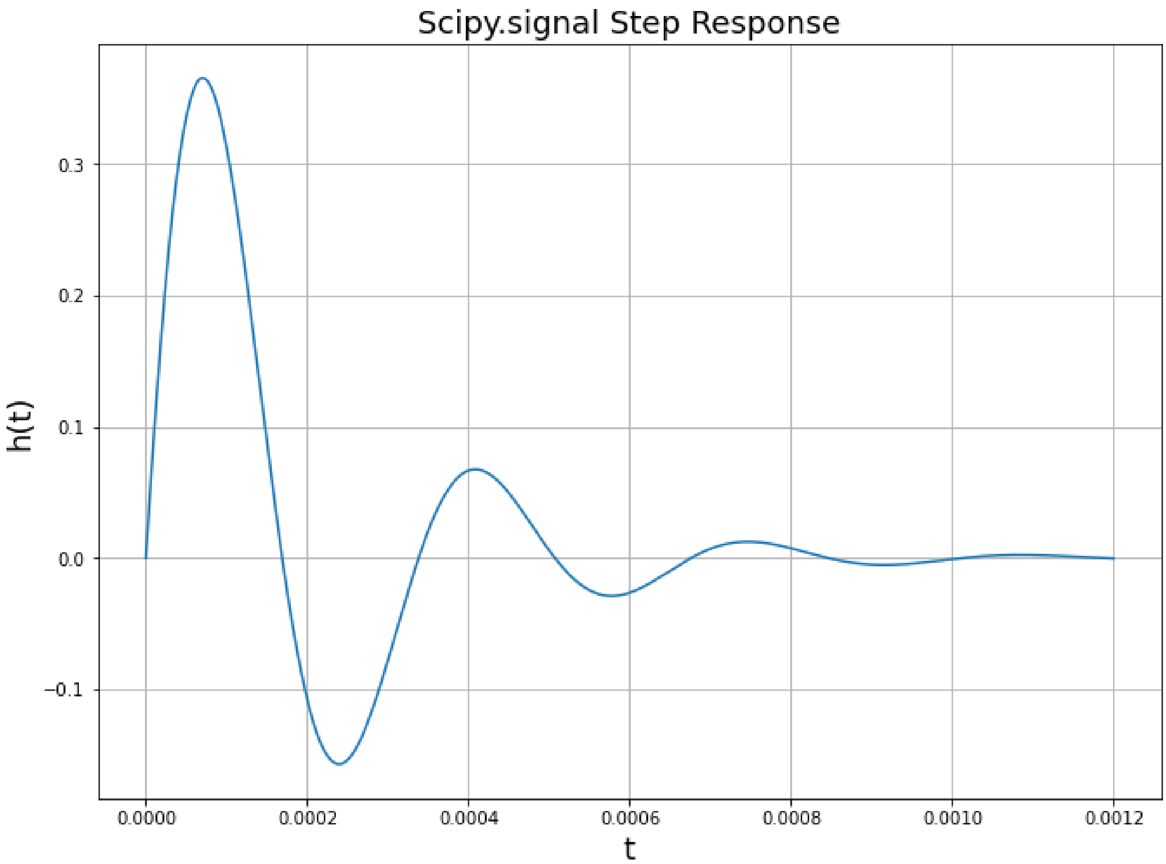
Task 2



Visually, the two graphs are identical. This verifies that the equation developed in the preliminary calculations is correct.

4.2 Part 2 Results

Task 1 - Step Response of H(s)



The graph appears to be the integral of the impulse response, where the minimums and maximums on the step response graph occur at the same time as the zeros on the impulse response. This is the expected result as the impulse response is simply the derivative of the step response:

$$\delta(t) = \frac{du(t)}{dt}$$

5 Error

Error was experienced in this lab while attempting to plot the hand calculated impulse response. Initially the hand calculated equation was entered into python with the phase shift in degrees, as in equation (3). This resulted in the hand calculated graph being slightly different from the scipy.signal graph. The error was corrected by converting the phase shift to radians. Equation (3) was changed to:

$$h(t) = 10355.6e^{-5000t} \sin(18584.1t + 1.8336)u(t) \tag{4}$$

No other significant error or difficulties were experienced during this lab.

6 Questions

1. Explain the result of the Final Value Theorem from Part 2 Task 2 in terms of the physical circuit components.
   (a) The circuit contains an inductor and capacitor in parallel with each other. When a unit step voltage is applied to the input, the capacitor and inductor will charge/discharge for a slight period of time, dampening at each iteration. As time approaches infinity, the system approaches DC Steady State - in which the capacitor acts as an open circuit and the inductor acts as a short. With the inductor as a short, no potential difference exists across it. So as time goes to infinity, the voltage measured across the inductor and capacitor, V_{out} goes to 0. This is reflected in the result of the Final Value Theorem shown in the **Equations** section.
2. Leave any feedback on the clarity of the expectations, instructions, and deliverables.
   (a) I have no feedback regarding the clarity of lab expectations, instructions or deliverables.

7 Conclusion

The objectives for this lab were fulfilled. The impulse response was accurately calculated and plotted using both the scipy.signal.impulse function and solving it by hand. The step and impulse response of this circuit both support the result of the Final Value Theorem.