



ECE 351 - SECTION 51

SYSTEM STEP RESPONSE USING CONVOLUTION

Lab 4

Submitted By:
Tristan Denning

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1 Introduction

The purpose of this lab is to practice using Convolution in Python to compute a system's step response.

*All of the code used to accomplish the goals of this lab can be accessed at my Github page: <https://github.com/Tristan-Denning>

2 Equations

Unit Step Function

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \quad (1)$$

Ramp Function

$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases} \quad (2)$$

Given Signals

$$h_1(t) = e^{-2t}(u(t) - u(t - 3)) \quad (3)$$

$$h_2(t) = u(t - 2) - u(t - 6) \quad (4)$$

$$h_3(t) = \cos(\omega_0 t)u(t) \quad (5)$$

Step Response of Given Signals (From Hand Calculation)¹

$$y_1(t) = \frac{1}{2} * ((-e^{-2t} + 1) * u(t) - (-e^{-2(t-3)} + 1) * u(t)) \quad (6)$$

$$y_2(t) = (t - 2) * u(t - 2) - (t - 6) * u(t - 6) \quad (7)$$

$$y_3(t) = \frac{1}{\omega_0} \sin(\omega_0 t)u(t) \quad (8)$$

¹Here, '*' means multiplication, not convolution.

3 Methodology

Part 1

The time shifting, scaling and linearity properties of the Unit Step (1) and Ramp (2) Functions were used to derive the user defined functions h_1 , h_2 and h_3 . Listing 1 shows how they are implemented with Python:

Listing 1 - Code for User-Defined Functions (3), (4) and (5)

```

1 def h1(t):
2     y = np.zeros(t.shape)
3     y = np.exp(-2*t)*(u(t) - u(t-3))
4     return y
5
6 def h2(t):
7     y = np.zeros(t.shape)
8     y = u(t-2) - u(t-6)
9     return y
10
11 def h3(t):
12     w = 0.25*2*np.pi
13     y = np.zeros(t.shape)
14     y = np.cos(w*t)*u(t)
15     return y

```

Part 2

Part 2 of this project utilizes the user-defined convolution function developed in Lab 3. The step response of the transfer functions is simply the result of convolving each of them with the unit step function, $u(t)$. The user-defined convolution function first appends zeroes to the end of each function so that they are in the boundaries of the resultant convolution. Then a for loop performs a summation of the product for each element in the functions' arrays. Listing 2 shows how this is implemented in Python:

Listing 2 - User-Defined Convolution from Lab 3

```

1 def conv(f1, f2):
2     Nf1 = len(f1)
3     Nf2 = len(f2)
4     f1Extended = np.append(f1, np.zeros((1, Nf2-1)))
5     f2Extended = np.append(f2, np.zeros((1, Nf1-1)))
6     result = np.zeros(f1Extended.shape)
7
8     for i in range(Nf2 + Nf1 - 2):
9         result[i] = 0
10        for j in range (Nf1):

```

```

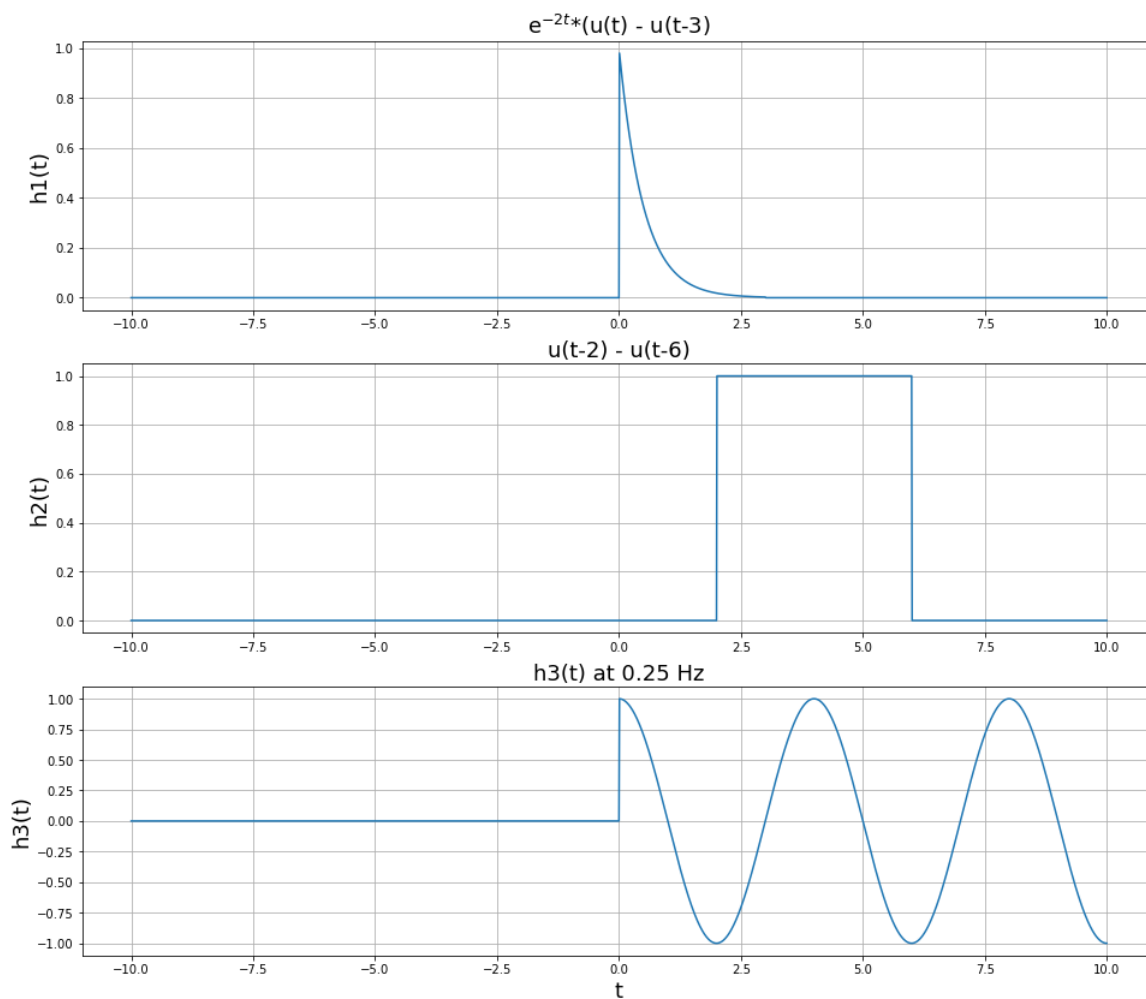
11         if (i - j + 1 > 0):
12             try:
13                 result[i] += f1Extended[j]*f2Extended[i-j+1]
14             except:
15                 print(i,j)
16     return result

```

4 Results

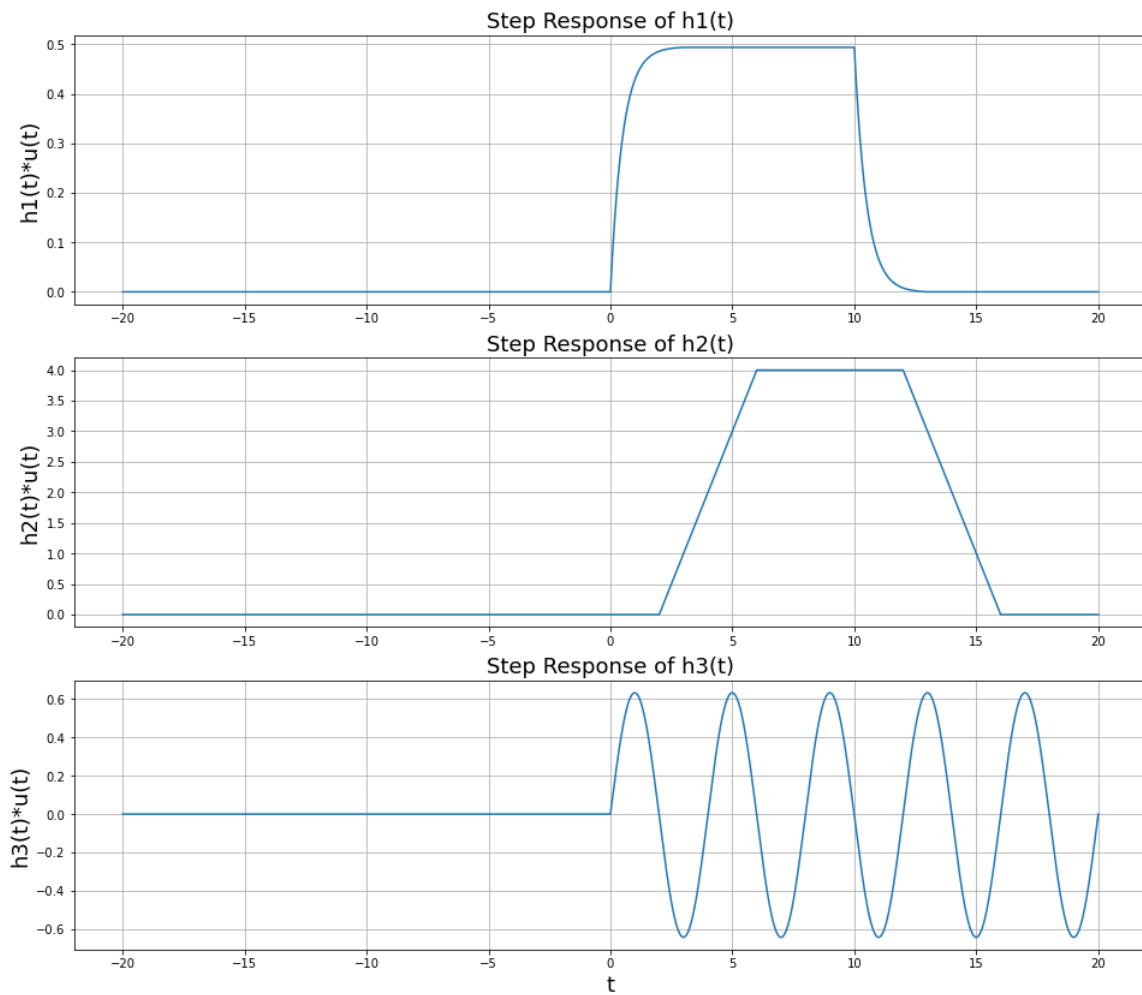
4.1 Part 1 Results

Task 1 - plots of h_1 , h_2 and h_3 .

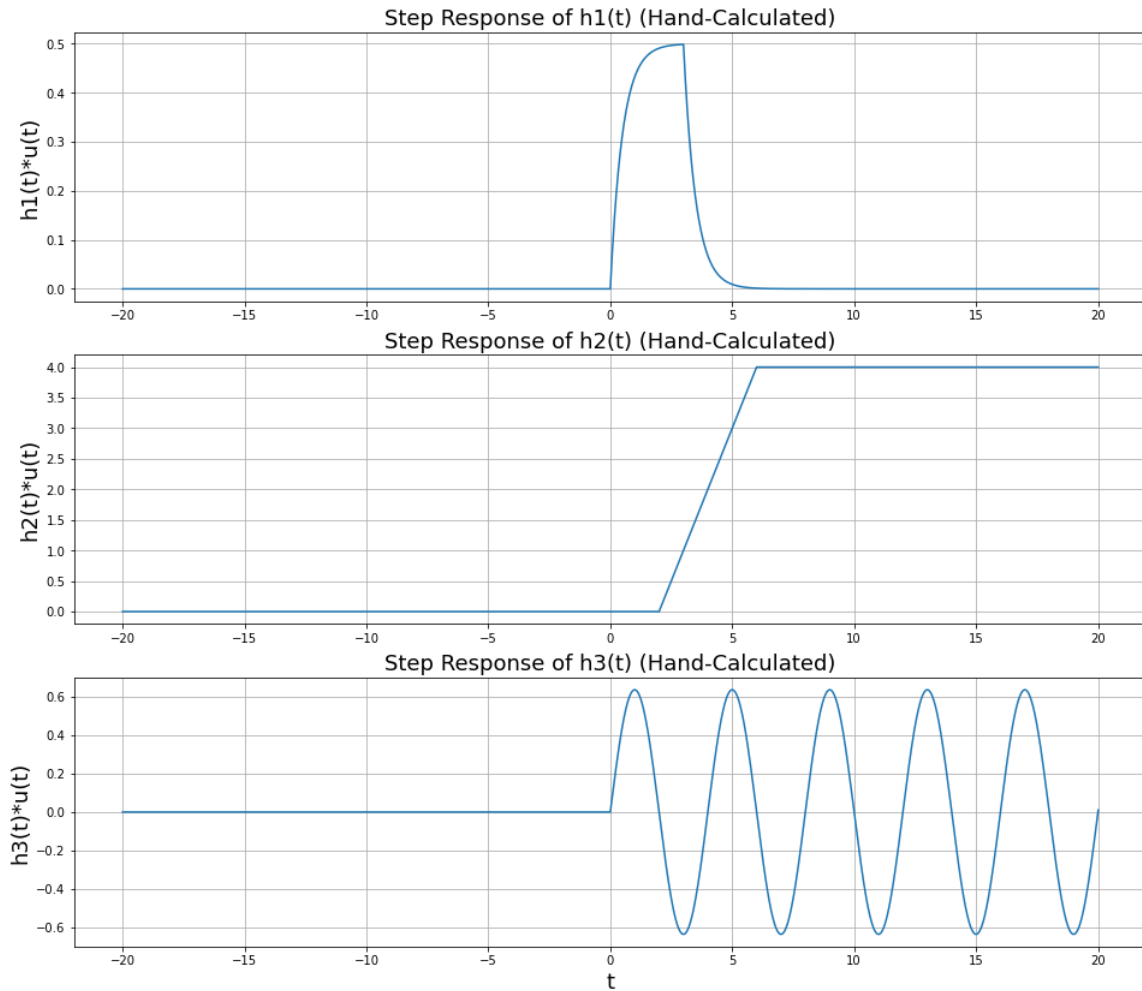


4.2 Part 2 Results

Task 1 - Step Responses of h_1 , h_2 and h_3 .



Task 2 - Step Response Plots from Hand-Calculated Equations (6), (7) and (8).



For some reason the hand calculated and computer generated convolutions of the functions do not exactly line up for h_1 and h_2 .

5 Error

Error was experienced in this lab while plotting the hand-calculated step responses for h_1 , h_2 , and h_3 . Python calculated and plotted the step responses correctly, while the hand-calculated equations did not yield the correct result. This is evident by the ranges of the results. For example, h_1 has a range of $[0, 3]$ and was convolved with the unit-step function on the interval $[0, 10]$, so the result should have a range of $[0, 13]$.

The Python-generated convolution showed a range of 13, while the hand-calculated equation had a range of about 6.5 when plotted. This error either occurred because the hand-calculations are incorrect, or because Python is unable to deal with the hand-calculated equation properly in the form they were implemented - The latter is more likely.

6 Questions

1. Leave any feedback on the clarity of lab tasks, expectations, and deliverables.
 - (a) I have no feedback regarding the clarity of lab tasks, expectations or deliverables.

7 Conclusion

The objectives for this lab were semi-fulfilled. The user-defined functions were implemented properly into Python and the expected plots were produced. In part 2, some error was experienced whilst plotting the hand-calculated step responses. The Python-generated convolution appeared to produce the expected ranges and magnitudes for each step response. Though seemingly correct, the hand-calculated step-responses did not yield correct results for step responses y_1 and y_2 . This either happened because the hand-calculations were incorrect, or because Python was unable to correctly plot the equations in their written syntax.