ECE 351 - Section 51

Step and Impulse Response of an RLC Bandpass Filter

Lab 5 Prelab

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1 Solution

Task 1 Solution:

$$H(S) = \frac{S}{RCS^2 + S + \frac{R}{L}} \tag{1}$$

Task 2 Solution

$$h(t) = 1.03556e^{-5000t}\sin(18584.1t + 105.06^o) * u(t) \tag{2}$$

The appended work shows the steps and process for obtaining equations (1) and (2).

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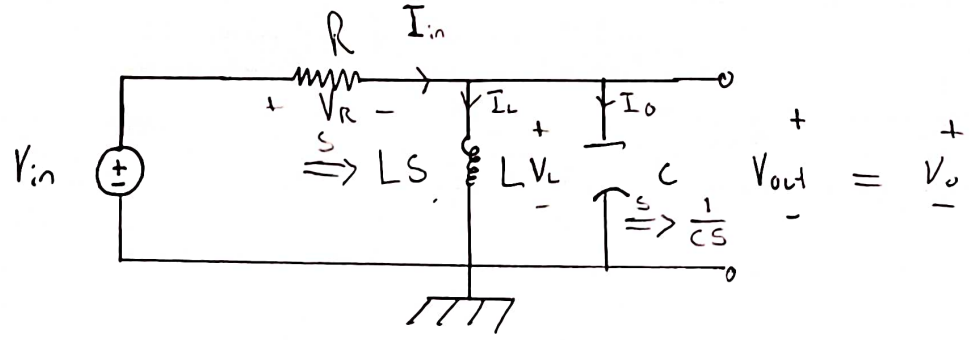
1.) Find the transfer function $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$ symbolically in terms R, L, and C.



Nodal @ V_{out} node

$$I_{in} = I_L + I_o$$

$$\frac{(V_{in} - V_o)}{R} = \frac{V_o}{LS} + V_o SC$$

$$\frac{V_{in}}{R} = V_o\left(\frac{1}{R} + \frac{1}{LS} + CS\right)$$

$$V_{in} = V_o\left(1 + \frac{R}{LS} + RCS\right)$$

$$\frac{V_o}{V_{in}} = \frac{1}{\left(1 + \frac{R}{LS} + RCS\right)}$$

Multiply by S/S

$$\Rightarrow \frac{V_o}{V_{in}} = \frac{LS}{\left(LS + \frac{R}{L} + LRCS^2\right)}$$

$$\frac{V_o}{V_{in}} = \frac{LS}{\frac{R}{L} + S + RCS^2}$$

$$\therefore H(s) = \frac{S}{\frac{R}{L} + S + RCS^2}$$

2.) Find the impulse response $h(t)$

$$H(s)\Big|_{R=1k\Omega,\ L=27mH,\ C=100nF} = \frac{s}{\frac{1000}{27\times10^{-3}} + s + 100\times10^{-6}s^2}$$

Roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1 - 4(100\times10^{-6})\left(\frac{1000}{27\times10^{-3}}\right)}}{2(100\times10^{-6})}$$

$$= \frac{-1 \pm \sqrt{1 - \frac{400}{27}}}{200\times10^{-6}}$$

$$= \frac{-1 \pm \sqrt{-13.8148}}{200\times10^{-6}}$$

$$= \frac{-1 \pm j\sqrt{13.8148}}{200\times10^{-6}}$$

$$\Rightarrow H(s) = \frac{s}{\left(s - \left(\frac{-1 + j\sqrt{13.8148}}{200\times10^{-6}}\right)\right)\cdot\left(s - \left(\frac{-1 - j\sqrt{13.8148}}{200\times10^{-6}}\right)\right)}$$

Sine Method

$$h_s(t) = \frac{|G|}{\omega} e^{\alpha t} \sin(\omega t + \angle G)\, u(t) \quad (1)$$

Where

$$p = \frac{-1 + j\sqrt{13.8146}}{200\times10^{-6}} = \alpha + j\omega$$

$$H_o(s) = s$$

2.) (continued)

$$\therefore \quad g = H_o(s)\Big|_{s=p}$$

$$= \frac{-1 + j\sqrt{13.8148}}{200 \times 10^{-6}}$$

$$\therefore \quad = -5000 + j18584.1$$

$$= (19245 \angle 105.06^\circ)$$

$$\therefore \quad \left.\begin{aligned} \alpha &= -5000 \\ \omega &= 18584.1 \\ |g| &= 19245 \\ \angle g &= 105.06^\circ \end{aligned}\right\} \text{ Plug into (1)}$$

$$h_s(t) = \frac{19245}{18584.1} \cdot e^{-5000t} \cdot \sin(1854.1t + 105.06^\circ)$$

$$\therefore \quad h(t) = h_s(t)$$

$$\boxed{h(t) = 1.03556\, e^{-5000t} \sin(1854.1t + 105.06^\circ)\, u(t)}$$