4. **Let g be a function from $\mathbb{Z}^+$ (the set of positive integers) to $\mathbb{Q}$ (the set of rational numbers) defined by**

$$(x, y) \in g \text{ iff } y = 4x - 3/7(g \subseteq \mathbb{Z}^+ \times \mathbb{Q})$$

and let f be a function on $\mathbb{Z}^+$ defined by

$$(x, y) \in f \text{ iff } y = 5x^2 + 2x - 3(f \subseteq \mathbb{Z}^+ \times \mathbb{Z}^+)$$

Consider the function f on $\mathbb{Z}^+$. For which values of x is it the case that $5x^2 + 2x - 3 > 0$?

$$\begin{aligned} & 5x^2 + 2x - 3 > 0 \\ \Rightarrow\ & (5x-3)(x+1) > 0 \end{aligned}$$

For the equation to be positive, either both terms are negative, or both terms are positive.

(i) **Both terms are negative.**

$$\begin{array}{llll} & 5x - 3 < 0 & \text{AND} & x + 1 < 0 \\ \Rightarrow & 5x \quad < 3 & \text{AND} & x < -1 \\ \Rightarrow & x \quad < \dfrac{3}{5} & \text{AND} & x < -1 \end{array}$$

As both conditions need to be true, take the one that includes both terms. For example, the number 0 satisfies the first term, but not the second. But every number that satisfies the second term also satisfies the first.

$$x < -1$$

However, x is only defined on $\mathbb{Z}^+$, so $x \nleq 0$.
Therefore, there are no x values where both terms can be negative.

(ii) **Both terms are positive.**

$$\begin{array}{llll} & 5x - 3 > 0 & \text{AND} & x + 1 > 0 \\ \Rightarrow & 5x \quad > 3 & \text{AND} & x > -1 \\ \Rightarrow & x \quad > \dfrac{3}{5} & \text{AND} & x > -1 \end{array}$$

As both conditions need to be true, take the one that includes both terms. For example, the number 0 satisfies the second term, but not the first. But every number that satisfies the first term also satisfies the second.

$$x > \frac{3}{5}$$

But we know that x is only defined on $\mathbb{Z}^+$. So x has to be equal to, or bigger than the nearest positive integer that is greater than 3/5, which would be 1. So,

$$x \geq 1 \text{ where } x \in \mathbb{Z}^+$$