Tutorial letter 101/3/2020

Theoretical Computer Science 1

COS1501

Semesters 1 & 2

School of Computing

IMPORTANT INFORMATION:

This tutorial letter contains important information about your module.

Please register on myUnisa, activate your myLife e-mail address and make sure that you have regular access to the myUnisa site **COS1501-20-S1** (for sem 1) or **COS1501-20-S1** (for sem 2), as well as your e-tutor group site

Note: This is an online module and therefore it is available on myUnisa. However, in order to support you in your learning process, you will also receive some study material in printed format.

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CONTENTS

Dear Student,

As part of this tutorial letter, we wish to inform you that Unisa has implemented a transformation charter based on five pillars and eight dimensions. In response to this charter, we have also placed curriculum transformation high on the agenda. For your information, curriculum transformation includes the following pillars: student-centred scholarship, the pedagogical renewal of teaching and assessment practices, the scholarship of teaching and learning, and the infusion of African epistemologies and philosophies. These pillars and their principles will be integrated at both the programme and module levels, as a phased-in approach. You will notice the implementation thereof in your modules, and we encourage you to fully embrace these changes during your studies at Unisa.

1 INTRODUCTION

Welcome to COS1501! This is a semester module presented by the School of Computing. (The previous module code was COS101S.)

This module is a blended online module as from 2017. This means that you have to work through the lessons on myUnisa, and follow the instructions when you are guided to work through the respective study guide units.

This module focuses on discrete mathematics and its application in Computer Science. Concepts and skills needed for a theoretical understanding of Computer Science are introduced. If you feel a flash of fear when you hear the word "theoretical", just remember that theory is much tidier than practice: one knows precisely what the meaning of the terms is and the problems always have solutions. Rigorous proofs are expected in this module.

This module does not involve practical work on a computer, but there is a strong emphasis on practicing the skills acquired by means of doing many exercises and the assignments.

Studying a mathematical module is rather like learning to play a musical instrument – the more you practice, the better you become! If you put an honest effort into trying to do the self-assessment exercises and assignments, we believe that by the end of the semester you will be able to do a lot of things you had never heard of before, and enjoy doing them.

2 PURPOSE OF AND OUTCOMES FOR THE MODULE

2.1 Purpose

On completing this module, you will be able to critically apply the fundamental knowledge and skills of discrete mathematics. The module forms part of the theoretical foundation of a Computer Science major. This background is relevant to computing fields such as relational databases, the development of provably correct programs, and the analysis of algorithms that will contribute to the development of computing in Southern Africa, Africa, or globally. The module will support further studies and applications in the computing discipline.

2.2 Outcomes

Specific outcome 1: Manipulate logical arguments, using a variety of mathematical tools.

Specific outcome 2: Construct proofs in a clear and concise way using mathematical reasoning techniques.

Specific outcome 3: Demonstrate knowledge and understanding of the definitions, laws and operations of set theory.

Specific outcome 4: Synthesise and critically analyse relations, functions and binary sets that are represented as sets containing ordered pairs.

Specific outcome 5: Perform operations on vectors and matrices.

3 LECTURERS AND CONTACT DETAILS

3.1 Lecturers

It occasionally happens that the study material does not instantly transfer itself to one's brain. In such an event, you are welcome to make an appointment with one of the COS1501 lecturers. The names and telephone numbers of the lecturers, as well as relevant email addresses and URLs, will be supplied in a COSALL tutorial letter.

If there is something that is not clear in the tutorial matter or in the model solutions, you are welcome to make an appointment or send an email. You should make use of the module contact email address on myUnisa because a specific lecturer might not be available at the time. The response time with email is generally fast since we try to respond within 24 hours. Should you prefer, you may send a letter to the following address:

The Lecturers, COS1501
School of Computing
P O Box 392
UNISA
0003

Note that lecturers are only responsible for module content-related queries.

You can communicate with fellow COS1501 students on the myUnisa **discussion forum** for this module. You will also be assigned to an e-tutor on myUnisa that will assist you. If your questions are not answered there, you can contact the lecturers. Also read section 5.2 of this letter regarding e-tutors.

3.2 School of Computing

In 2013 the School of Computing moved to Florida. The general contact number for the school is **011 670 9200**. Should you have difficulty in contacting your lecturers regarding an urgent matter, you can phone the pilot number of the school. Your message will then be conveyed to the relevant lecturer. Remember to provide your student number and relevant module code.

3.3 University

If you need to contact the university about administrative matters, you should send your queries via e-mail to the specific department whose contact details are provided in the brochure *Study@Unisa* that you received in your study package. This brochure also contains other important information about Unisa.

4 MODULE-RELATED RESOURCES

Please note that as from 2020 onwards you will only receive the Study guide and the Tutorial letter 101 in printed form for this module. All other tutorial matter will only be available on myUnisa. The Tutorial letter 101 will also be available on myUnisa – so please do not wait for your printed copy – **everything you need to start studying is available on myUnisa**. If you do not find a tutorial letter under Official Study Material, you will find it under Additional Resources.

4.1 Study guide (no prescribed book)

No book is prescribed for this module. Instead, you are provided with a study guide.

In the introductory unit of the study guide, information about this module and aspects on “how to study” are provided.

Number sets are introduced and then we move on to set theory, studying sets based on these number sets. We investigate elements of, and operations on sets. Equality between sets is investigated by using Venn diagrams or by providing a formal proof. The concept of inclusion-exclusion is introduced.

Certain kinds of sets turn out to be particularly useful, so they get special names such as relation and function. Relations are investigated, examining their properties and testing for particular types of relation. We also study functions and their properties, as well as operations on functions. Binary operations are studied and then vectors and matrices are introduced.

An introduction to logic is provided. Concepts from the field of logic, such as truth tables, connectives, quantification and predicates are introduced. Different mathematical proof strategies are implemented. Throughout, great emphasis is placed on sound reasoning and the ability to construct mathematical proofs.

Note: Tutorial letter 103 under Additional Resources on myUnisa provides solutions to all the self-assessment exercises that are provided in the study guide. Tutorial letter 102 also under Additional Resources, provides other information including solutions to additional self-assessment questions and examples.

4.2 Lessons

A pdf version of the lessons on myUnisa is also available under Additional resources on myUnisa. As explained before, the lessons will guide you through the sections in the study guide.

4.3 Recommended books

Should you wish to know more about a particular topic, you may consult the following books. Please note that these books are not necessarily included in the Study Collection in the Unisa library. The library cannot guarantee that they will be available.

ENSLEY, D.E. AND CRAWLEY, J.W. Discrete Mathematics: Mathematical Reasoning and Proof with Puzzles, Patterns and Games. John Wiley & Sons, Inc., 2006.

GRIMALDI, R.P. Discrete and Combinatorial Mathematics: An applied Introduction, 5th edition. Pearson Education, 2004.

JOHNSONBAUGH, R. Discrete Mathematics, 7th edition. Pearson Education Inc., 2009.

LABUSCHAGNE, W.A. A User-friendly Introduction to Discrete Mathematics for Computer Science. Pretoria, UNISA, 1999.

ROSEN, K.H. Discrete Mathematics and its Application, 6th edition. McGraw-Hill, 2007.

4.4 Electronic Reserves (e-Reserves)

The computer-aided instruction (CAI) tutorial “Relations” is will not be available on CD as from 2020. You can download it from the website as explained in Section 9 (Frequently asked questions) in this Tutorial letter. The interactive CAI tutorial is a supplementary study aid. It deals with sets and the main properties of relations, such as reflexivity, irreflexivity, symmetry, antisymmetry and transitivity. It also explores the properties of different types of relations. These concepts are discussed in study units 3, 5 and 6 of the study guide. This CAI is not compulsory and you will not be examined on any specific example in the CAI, but if you do not understand sets and relations and their properties, the CAI tutorial is a good exercise to do.

4.5 Joining myUnisa

If you have access to a computer that is linked to the internet, you can quickly access resources and information at the University. The myUnisa learning management system is the University's online campus that will help you communicate with your lecturers, with other students and with the administrative departments at Unisa – all through the computer and the internet.

You can start at the main Unisa website at http://www.unisa.ac.za and then click on the myUnisa orange block. This will take you to the myUnisa website. To go to the myUnisa website directly, go to https://mymodules.unisa.ac.za. Click on the **Claim UNISA Login** on the right-hand side of the screen on the myUnisa website. You will then be prompted to give your student number in order to claim your initial myUnisa details as well as your myLife e-mail login details.

For more information on myUnisa, consult the brochure *Study@Unisa*, which you received with your study material.

4.6 Library services and resource information

The Unisa Library offers a range of information services and resources:

- for brief information go to: https://www.unisa.ac.za/library/libatglance
- for more detailed Library information, go to http://www.unisa.ac.za/sites/corporate/default/Library
- for research support and services (e.g. Personal Librarians and literature search services), go to http://www.unisa.ac.za/sites/corporate/default/Library/Library-services/Research-support

The Library has created numerous Library guides: http://libguides.unisa.ac.za

Recommended guides:

- request and find library material/download recommended material: http://libguides.unisa.ac.za/request/request
- postgraduate information services: http://libguides.unisa.ac.za/request/postgrad
- finding and using library resources and tools: http://libguides.unisa.ac.za/Research_skills
- Frequently asked questions about the Library: http://libguides.unisa.ac.za/ask
- Services to students living with disabilities: http://libguides.unisa.ac.za/disability

Important contact information:

- https://libguides.unisa.ac.za/ask - ask a Librarian
- Lib-help@unisa.ac.za - technical problems accessing library online services
- Library-enquiries@unisa.ac.za - general library related queries
- Library-fines@unisa.ac.za - for queries related to library fines and payments

5 STUDENT SUPPORT SERVICES FOR THE MODULE

5.1 *Study@Unisa* brochure

Important information appears in your *Study@Unisa* brochure. When you register, you receive an inventory letter containing information about your tutorial matter. Check the study material that you received in your study package against the inventory letter. You should have received all the items listed in the inventory, unless there is a statement such as “out of stock” or “not available”. If any item is missing, follow the instructions on the back of the inventory letter without delay. All tutorial matter is downloadable from myUnisa. The URL is https://mymodules.unisa.ac.za. (Log on with your student number and password.)

5.2 E-tutors

With effect from 2013, Unisa offers online tutorials (e-tutoring) to students registered for modules at NQF level 5, 6 and 7, this means qualifying undergraduate modules.

Once you have been registered for a qualifying module, you will be allocated to a group of students with whom you will be interacting during the tuition period as well as with an e-tutor who will be your tutorial facilitator. Thereafter you will receive an sms informing you about your group, the name of your e-tutor and instructions on how to log onto myUnisa in order to receive further information on the e-tutoring process.

Online tutorials are conducted by qualified e-tutors who are appointed by Unisa and are offered free of charge. All you need to be able to participate in e-tutoring is a computer with internet connection. If you live close to a Unisa regional center, please feel free to visit this center to access the internet. E-tutoring takes place on myUnisa where you are expected to connect with other students in your allocated group. It is the role of the e-tutor to guide you through your study material during this interaction process. To get the most out of online tutoring, you need to participate in the online discussions that the e-tutor will be facilitating. Please make use of this opportunity. The e-tutor is also available if you need any help with questions from old exam papers – post your questions on the discussion forum of your e-tutor group, and the e-tutor and your fellow students will comment and assist.

There are modules which students have been found to repeatedly fail, these modules are allocated face-to-face tutors and tutorials for these modules take place at the Unisa regional centers. These tutorials are also offered free of charge, however, it is important for you to register at your nearest Unisa regional center to secure attendance of these classes.

5.3 Free computer and internet access

Unisa has entered into partnerships with establishments (referred to as Telecentres) in various locations across South Africa to enable you (as a Unisa student) free access to computers and the Internet. This access enables you to conduct the following academic related activities: registration; online submission of assignments; engaging in e-tutoring activities and signature courses; etc. Please note that any other activity outside of these is for your own costing e.g. printing, photocopying, etc. For more information on the Telecentre nearest to you, please visit **www.unisa.ac.za/telecentres.**

6 MODULE-SPECIFIC STUDY PLAN

Refer to your *Study@Unisa* brochure for general administrative matters, time management and planning skills.

In the introductory study unit of the COS1501 study guide, guidelines are provided on study skills that you can apply. For instance, if you draw a mind map for each study unit it may save time when you study for the examinations.

We provide two study programmes, one for each semester. You have to follow the programme for the **semester for which you are registered**. It can serve as a guide for you to work through the study units of the study guide at a steady pace and enable you to submit the assignments on time.

Note: You need to complete the assignments during the week before the due dates or else you will not be able to cover the whole syllabus.

Please turn over

Study programme for semester 1 registration 2020:

Week	Dates (Mondays)	Activity	Due dates semester 1
1	10 February	Study unit 1, 2, 3	
2	17 February	Study unit 4 Start with assignment 1	
3	24 February	***Compulsory*** *Assignment 01*	**24 February** (multiple choice)
4	2 March	Study unit 5	
5	9 March	Study unit 6.1-6.3 Start with ass 2	
6	16 March	Complete *Assignment 02* Study unit 6.4, 6.5, 7	
7	23 March	Submit Assignment 2	**23 March** (multiple choice)
8	30 March	Study unit 8, 9	
9	6 April	Study unit 10 Start with assignment 3	
10	13 April	Complete Assignment 3 And submit	**17 April** (multiple choice)
11-13	20 April up to examination date. Exams begin 3 May.	Revision Work through old exam Papers and assignments	

Study programme for semester 2 registration 2020:

Week	Date (Mondays)	Activity	Due dates semester 2
1	3 August	Study unit 1, 2, 3	
2	10 August	Start with ***Compulsory*** *Assignment 01* Study unit 4	
3	17 August	Submit assignment 1 Study unit 5	**17 August** (multiple choice)
4	24 August	Study units 6.1-6.3,	
5	31 August	Start with Assignment 2	
6	7 September	Complete *Assignment 02*	
7	14 September	Submit assignment 2 Study units 6.4, 6.5, 7	**14 September** (multiple choice)
8	21 September	Study unit 8, 9	
9	28 September	Study unit 10. Start with assignment 3	
10	5 October	Complete and submit *Assignment 03*	**8 October** (multiple choice)
11-13	12 October up to examination date. Exams begin 19 Oct.	Revision	

7 MODULE PRACTICAL WORK AND WORK-INTEGRATED LEARNING

There is no practical work for this module.

8 ASSESSMENT

8.1 Assessment criteria

The assessment criteria is given per specific outcome:

Specific outcome 1:

Think in an abstract way, to manipulate logical arguments, using a variety of mathematical tools.

Assessment criteria

- Predicates and symbols are used, to represent properties or relations, all formulated as English sentences;
- A given set of logical connectives is used to combine propositions and predicate logic atoms, correctly, from given English sentences into equivalent logic sentences;
- Truth tables illustrate the result of logical connectives, with the correct relationships;
- Quantifiers generalise over predicate logic sentences, within the context;
- Classifications of compound statements include tautology, contradiction or neither;
- Arguments around propositional and a predicate logic sentences are valid.

Specific outcome 2:

Construct proofs in a clear and concise way using mathematical reasoning techniques.

Assessment criteria

- Diagrams and mathematical notation are used to represent the structure of the problem correctly;
- Rigorous, precise and convincing proofs are constructed correctly (direct proofs, proof by contraposition, proof by contradiction);
- A counterexample is provided correctly in the case where a mathematical statement is not always true.

Specific outcome 3:

Demonstrate knowledge and understanding regarding the definitions, laws and operations of set theory.

Assessment criteria

- Sets are represented correctly using various notations;
- New sets constructed from existing one using set operations are valid;
- Set equality are determined correctly;
- A counter-example in the case of set inequality is correct;
- Sets represented using Venn diagrams are valid;
- Equality of Venn diagrams are determined correctly;
- A counter-example in the case of inequality of Venn diagrams is correct;
- New Venn diagrams constructed from existing ones using set operations are valid.

Specific outcome 4:

Synthesise and critically analyse relations, functions and binary sets that are represented as sets containing ordered pairs.

Assessment criteria

- Particular properties of relations are identified correctly;
- Different kinds of relation are defined correctly;
- Synthesised relations of a given kind are correct;
- New relations constructed from existing ones are valid;
- Functions having particular properties are identified correctly;
- Inverse function of a given function are defined correctly;
- The composition of two given functions is valid;
- Synthesised functions of a given kind are valid;
- Properties of binary operations can be determined correctly;
- Synthesised binary operations satisfying given properties are valid.

Specific outcome 5:

Perform operations on vectors and matrices.

Assessment criteria

- Operations on vectors and matrices are applied in order to construct different ways of storing and listing numbered information correctly.
- The synthesised vector or matrix that fits a place-holder within an equation or that holds defined properties is correct.

8.2 Assessment plan

Why do assignments? In the first place, we need to provide proof to the Department of National Education that you are an active student, therefore it is **compulsory** to submit at least one assignment by **the date specified for assignment 1 in section 6** above for the respective semester that you are registered for, otherwise **you will not have exam admission.** Furthermore, experience has shown that most students who do not work systematically during the semester, give up and do not write the examination.

Important: Take special note of the solutions to the self-assessment exercises of the study guide that are provided in tutorial letter 103. These will **help you to eliminate some of the mistakes that often occur in students' examination solutions**.

An integrated assessment system is used for this module. This means that your final mark is based not only on your examination mark, but also on your performance during the semester. Assignments do not only provide you with an opportunity to evaluate your understanding of the prescribed material (or to give you feedback on your readiness for the examination), but also make a contribution towards your semester mark.

Your **final mark** will be calculated as follows:

Semester mark (out of 100) × 20% + Examination mark (out of 100) × 80%

In order to pass this module, a final mark of at least 50% is required.

Your **semester mark** is based on your assignment marks. Different weights are allocated to the individual assignments. If an assignment is not submitted or is submitted late (for whatever reason), no marks are awarded for such an assignment. *It is your responsibility to ensure that your assignments are submitted on time*. Multiple choice assignments are marked by a computer system at a time set out by the Assignment Section of Unisa – lecturers can therefore not give any extension for multiple choice assignments.

The following weights are allocated to the individual assignments:

Assignment 01: **20%** Assignment 02: **40%** Assignment 03: **40%**

Example: The following example shows how the assessment system works, assuming that assignments 01, 02 and 03 were all submitted.

Assignment	**Mark**	× **Weight**	**Contribution to semester mark**
01	90%	× 0.20	18%
02	90%	× 0.40	36%
03	90%	× 0.40	36%
		Semester mark	90%

The resulting semester mark is 90%.

Suppose you obtain 80% in the examination. The final mark will be calculated as follows:

(90 × 0.20)% + (80 × 0.80)% = (18.0 + 64.0)% = 82%.
Note: The semester mark will not contribute towards the results of students writing a supplementary examination.

8.3 Assignment numbers

8.3.1 General assignment numbers

The letter you are reading is very important. This is the only tutorial letter that you received in printed form. All other tutorial letters are available on myUnisa under Additional Resources. All tutorial letters contain important information, so make sure you read each one carefully.

The letters are of three kinds:

- those numbered 101 (such as this one), 102 are concerned specifically with the tutorial matter for COS1501;
- those numbered 201, 202, 203 contain solutions to assignments;
- Tutorial letter 103 contains some important formation, including a copy of the lessons on myUnisa, as well as solutions to the self-assessment exercises in the 101;
- the COSALL tutorial letter contains important information (including the names and contact numbers of lecturers) for all students in the School of Computing. The School of Computing is situated at the Florida campus.

8.3.2 Unique assignment numbers

Each assignment has its own unique assignment number which you will have to provide when you submit your assignments.

Assignments semester 1	**Unique number**
Assignment 01	**629979**
Assignment 02	**813948**
Assignment 03	**598243**
Assignments semester 2	**Unique number**
Assignment 01	**698943**
Assignment 02	**716776**
Assignment 03	**833636**

Note: If you do not use the correct unique number, your assignment will be marked against the wrong memorandum!

8.4 Assignment due dates

The table below provides a summary of the due dates for the assignments of this module.

Assignment	**Due dates semester 1**	**Due dates semester 2**	**Weight**
01	24 February	17 August	20%
02	23 March	14 September	40%
03	17 April	8 October	40%

8.5 Submission of assignments

Submit assignments **electronically** via myUnisa. For detailed information and requirements as far as assignments are concerned, refer to *Study@Unisa*, which you received with your study package. Instructions on how to register to become a myUnisa user, and how to submit your assignments electronically, are provided on the myUnisa website. You can follow the steps below when you submit your assignments via myUnisa:

- Go to myUnisa at https://mymodules.unisa.ac.za
- Log on with your student number and password.
- Choose the relevant module (COS1501) in the orange block.
- Click on assessment info in the menu on the left-hand side of the screen.
- Click on the assignment number of the assignment that you want to submit.
- Follow the instructions.

Note: Administrative enquiries about assignments should be addressed to *assign@unisa.ac.za*.

NOTE: THERE ARE SEPARATE ASSIGNMENTS FOR SEMESTER 1 AND SEMESTER 2. ONLY DO THE ASSIGNMENTS FOR THE SEMESTER THAT YOU ARE REGISTERED FOR.

PLEASE TURN THE PAGE TO FIND THE ASSIGNMENTS FOR THIS MODULE.

8.6 Assignments

FIRST SEMESTER ASSIGNMENTS

8.6.1 Semester 1: Assignment 01

Unique number:	**629979**
Due date:	**24 February 2020**
Material to be tested: **Additional material:**	**Study guide, study units 1 – 3** **Tutorial letters 102 and 103, lessons 1 – 3** ***NOTE: The Venn diagrams in study unit 4 will help you to understand the definitions in study unit 3.***
Submission procedure:	**Electronic submission via myUnisa.**
Weight towards semester mark:	**20%**

Important: *Do the self-assessment exercises in the activities in the study guide before attempting this assignment. The solutions are in Tutorial letter 103 and in the lessons on* myUnisa.

CAI tutorial: *In sections 4.3 & 11 of this tutorial letter it is mentioned that you can work through the CAI tutorial "Relations" that is downloadable from the web. In preparation for this assignment, go to the menu of the tutorial, choose "Prerequisite background" then attempt the "Sets" part. Graphical illustrations are also provided.*

Note: *Answer all the questions for this assignment. The mark that you achieve from the possible 10 marks will be converted to a percentage.*

Question 1

The set of **all** non-negative integers **x** less than 16 such that x^2 is an even integer can be described as the set:

1. $\{\mathbf{x} \mid x \in \mathbb{Z}^{\geq}, x < 16, x^2 = 2k \text{ for some } k \in \mathbb{Z}\}$
2. $\{\mathbf{x} \mid x \in \mathbb{Z}^{\geq}, x < 16, x^2 = 2k \text{ for some } k \in \mathbb{Z}^{+}\}$
3. $\{0, 2, 4, 16, 36, 64, 100, 144, 196\}$
4. $\{0, 2, 4\}$

(Note: The required set must include as elements all non-negative integers **x** such that all the requirements for the set are met.)

Question 2

$\{x \mid x \in \mathbb{Z}, 0 \leq x < 8\} \cap \{x \mid x \in \mathbb{R}, 4 \leq x < 16\}$ is the set:

1. $\{x \mid x \in \mathbb{R}, 4 \leq x < 8\}$
2. $\{x \mid x \in \mathbb{Z}, 0 \leq x \leq 8\}$
3. $\{x \mid x \in \mathbb{Z}, 0 \leq x \leq 16\}$
4. {4, 5, 6, 7}

Consider the following sets, where U represents a universal set:

U = {1, 2, {1}, {2}, {1, 2}} A = {1, 2, {1}} B = {{1}, {1, 2}} C = {2, {1}, {2}}.

(Hint: U has 5 elements namely 1, 2, {1}, {2}, and {1, 2}, A has 3 elements, B has 2 elements and C has 3 elements. List the elements of A, B and C before answering the questions.)

Questions 3 to 10 are based on the sets defined above.

NOTE: The Venn diagrams in study unit 4 will help you to understand the definitions in study unit 3.

Question 3

A ∪ B is the set:

1. {1, 2, {1}, {1, 2}}
2. {1, 2, {1, 2}}
3. {{1}, {1, 2}}
4. {{1}}

Question 4

A ∩ C is the set:

1. {2, {1, 2}}
2. {1, 2, {1}}
3. {2, {1}, {2}}
4. {2, {1}}

Question 5

A – B is the set:

1. {1, 2}
2. {1, {1, 2}}
3. { }
4. {{1, 2}}

Question 6

B + C is the set:

1. $\{\{1, 2\}\}$
2. $\{\{1\}, \{2\}\}$
3. $\{2, \{2\}, \{1, 2\}\}$
4. $\{2, \{1\}, \{2\}, \{1, 2\}\}$

Question 7

$\mathcal{P}$(A) is the set:

1. $\{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{1, 2, \{1\}\}\}$
2. $\{\emptyset, \{1\}, \{2\}, \{\{1\}\}, \{1, 2\} \{1, \{1\}\}, \{2, \{1\}\}, \{1, 2, \{1\}\}$
3. $\{\emptyset, \{1\}, \{2\}, \{\{1, 2\}\}, \{1, \{1\}\}, \{2, \{1\}\}, \{1, 2, \{1, 2\}\}\}$
4. $\{\emptyset, \{1\}, \{2\}, \{\{1\}\}, \{1, \{1\}\}, \{2, \{1\}\}, \{1, 2, \{1\}\}\}$

Question 8

Which one of the following is a *subset* of $\mathcal{P}$(A)?

1. $\{1\}$
2. $\{1, 2, \{1, 2\}\}$
3. $\{\{\{1\}\}\}$
4. $\{\{\{2\}\}\}$

Remember, subsets of some set G can be formed by ***keeping the outside brackets*** *of G and then throwing away* ***none***, ***one*** *or* ***more*** *elements of G. Refer to study guide, p 40.*

Question 9

Which one of the following sets represents both $\mathcal{P}(A) \cap \mathcal{P}(B)$ and $\mathcal{P}(A \cap B)$?

1. $\{\{1\}\}$
2. $\{\emptyset, \{\{1\}\}\}$
3. $\{\emptyset, \{1\}\}$
4. Not one of the above alternatives since $\mathcal{P}(A) \cap \mathcal{P}(B) \neq \mathcal{P}(A \cap B)$.

Question 10

Which one of the following statements is valid if $x \notin B \cup C$?
(Hint: Determine $U - (B \cup C)$.)

1. $x \in \{1\}$.
2. $x \in \emptyset$.
3. $x \in \{1, 2\}$.
4. $x \in B$ and $x \in C$.

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8.6.2 Semester 1: Assignment 02

Unique number:	**813948**
Due date:	**23 March 2020**
Material to be tested:	**Study guide, study units 4 – 6.3**
Additional material:	**Tutorial letters 102 and 103, lessons 4 – 6**
Submission procedure:	**Electronic submission via myUnisa.**
Weight towards semester mark:	**40%**

Important: *Do the self-assessment exercises in the activities in the study guide before attempting this assignment. The solutions are in the tutorial letter 102 as well as in the lessons on myUnisa.*

CAI tutorial: *In sections 4.3 & 11 of this tutorial letter it is mentioned that you can work through the CAI tutorial "Relations" that is downloadable from the web. In preparation of this assignment, navigate through all the sections in the tutorial. Graphical illustrations are also provided.*

Additional self-assessment questions: *Attempt these written questions which are provided in the tutorial letter 103 as well as in the lessons on myUnisa. You can do these questions in preparation of this assignment. Only submit the solutions to the multiple choice questions given in this assignment.*

***Examination paper**: Although the exam paper is an MCQ paper, you will need to write down proofs on rough, for example, or draw Venn diagrams on rough before you will be able to select the correct alternative. It is therefore **very important** to attempt the self-assessment questions. You will receive a practice exam paper later in the semester.*

Note: *Answer all the questions for this assignment. The mark that you achieve from the possible 15 marks will be converted to a percentage.*

Hints:

*If a relation is provided in a question, make sure that your alternative of choice (relation) is defined on the set indicated in the question. For example, if a relation is defined on **A = {a, b, c}** but a relation R = {a, b), (b, c), (c, **d**)} is provided in some alternative, whatever the question is, this alternative is immediately disqualified because (c, **d**) ∈ R but **d** ∉ A.*

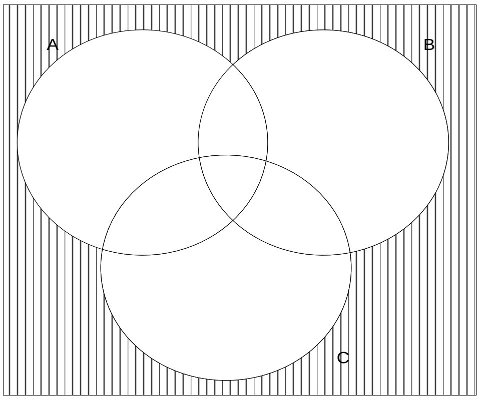
*To form a subset of some set L (say), **keep the outside brackets** of L, then throw away some elements of L to form a subset of L. Refer to the definition of a subset in the study guide.*

Question 1

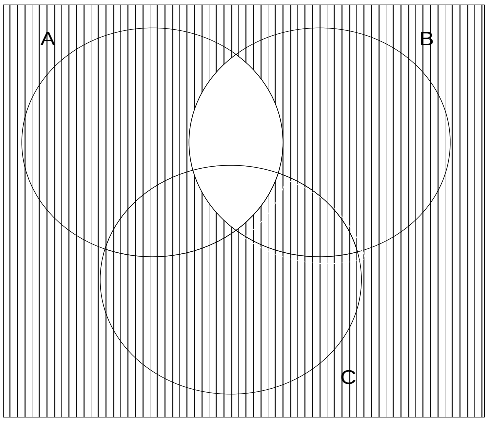
Let A, B and C be subsets of a universal set U.

Which one of the following four Venn diagrams presents the set $[(A \cap B)' - C] \cap [(A + B) - C]$?

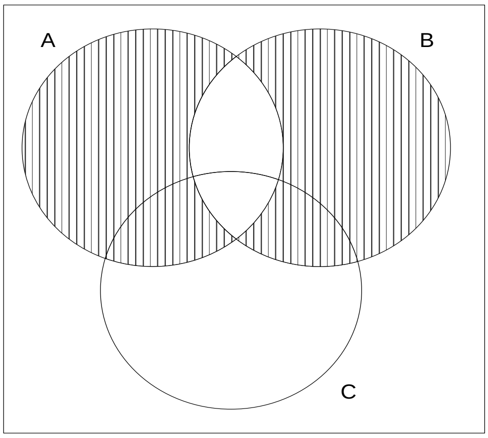
1.



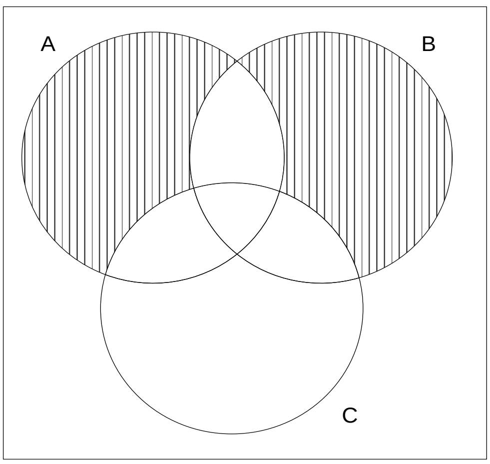
2.



3.



4.



Question 2

Let A, B and C be subsets of a universal set U = {1, 2, 3, 4}.

The statement $(A - B) \cup C' = (C' - B) + A$ is NOT an identity. Which of the following sets A, B and C can be used in a *counterexample* to prove that the given statement is not an identity?

1. A = {1}, B = {2} & C = {3}
2. A = {1}, B = {1} & C = {2}
3. A = {1, 2}, B = {1, 2} & C = {3}
4. A = {3}, B = {3, 4} & C = {4}

Question 3

We want to prove that for all $A, B, C \subseteq U$,

$(A \cap B) \cup (C - B) = (A \cup C) \cap (A \cup B') \cap (B \cup C)$ is an identity.

Consider the following proof:

$z \in (A \cap B) \cup (C - B)$

iff $(z \in A$ and $z \in B)$ or $(z \in C$ and $z \notin B)$

iff $(z \in A$ or $z \in C)$ and $(z \in A$ or $z \notin B)$ and $(z \in B$ or $z \in C)$ and $(z \in B$ or $z \notin B)$

iff Step 4

iff $z \in (A \cup C)$ and $z \in (A \cup B')$ and $z \in (B \cup C)$ and $z \in (B \cup B')$

iff Step 6

iff $z \in (A \cup C) \cap (A \cup B') \cap (B \cup C) \cap U$

iff $z \in (A \cup C) \cap (A \cup B') \cap (B \cup C)$ [For any sets U and G, $(G \cap U) = G$.]

Which one of the following alternatives provides valid steps 4 and 6 to complete the given proof?

1. Step 4: iff $(z \in A$ or $z \in C)$ and $(z \in A$ or $z \in B')$ and $(z \in B$ or $z \in C)$ and $(z \in B$ or $z \in B')$
   Step 6: iff $z \in (A$ or $C)$ and $z \in (A$ or $B')$ and $z \in (B$ or $C)$ and $z \in U$

2. Step 4: iff $(z \in A$ or $z \in C)$ and $(z \in A$ or $z \in B')$ and $(z \in B$ or $z \in C)$ and $(z \in B$ or $z \in B')$
   Step 6: iff $z \in (A \cup C)$ and $z \in (A \cup B')$ and $z \in (B \cup C)$ and $z \in U$

3. Step 4: iff $(z \in A$ or $z \in C)$ and $(z \in A$ or $z \notin B')$ and $(z \in B$ or $z \in C)$ and $(z \in B$ or $z \notin B')$
   Step 6: iff $z \in (A \cup C)$ or $z \in (A \cup B')$ or $z \in (B \cup C)$ or $z \in U$

4. Step 4: iff $(z \in A$ and $z \in C)$ or $(z \in A$ and $z \in B')$ or $(z \in$ and $z \in C)$ or $(z \in B$ and $z \in B')$
   Step 6: iff $z \in (A \cup C)$ and $z \in (A \cup B')$ and $z \in (B \cup C)$ and $z \in U$

Question 4

Fourty (40) students go to a party wearing red, white and blue.
Of these students,

20 wear red,
21 wear blue,
21 wear white,
(Students do not necessarily wear only one colour.)
Furthermore,
7 wear red and white,
9 wear red and blue,
12 wear blue and white.
(Students do not necessarily wear only two colours.)
(***Hint***: always start in die middle filling in the Venn diagram and work outwards. Then determine the value of the unknown, say *x*)

Which one of the following alternatives is TRUE?

1. 4 students wear red, white and blue.
   3 students wear read and white, but not blue.

2. 18 students wear red, white and blue.
   7 student wears read and white, but not blue.

3. 6 students wear red, white and blue.
   13 students wear read and white, but not blue.

4. 6 students wear red, white and blue.
   1 student wears read and white, but not blue.

Let T be a relation from A = {0, 1, 2, 3} to B = {0, 1, 2, 3, 4} such that

$(a, b) \in T$ iff $b^2 - a^2$ is an odd number. ($A, B \subseteq U = \mathbb{Z}$.)

(Hint: Write down all the elements of T. For example, if $4 \in B$ and $1 \in A$ then $4^2 - 1^2 = 16 - 1 = 15$ which is an odd number, thus $(1, 4) \in T$.)

Answer questions 5 and 6 by using the defined relation T.

Question 5
Which one of the following alternatives provides only elements belonging to T?

1. (3, 1), (4, 1), (3, 2)
2. (0, 1), (2, 4), (2, 3)
3. (3, 0), (1, 2), (3, 4)
4. (1, 0), (1, 2), (1, 3)

Question 6
Which one of the following statements regarding the relation T is true?

1. T is transitive.
2. T is symmetric.
3. T is antisymmetric.
4. T is irreflexive.

Consider the following relation on set B = {a, b, {a}, {b}, {a, b}}:

P = {(a, b), (b, {a, b}), ({a, b}, a), ({b}, a), (a, {a})}.

Answer questions 7 to 10 by using the given relation P and the set B.

Question 7
Which one of the following alternatives represents the range of P (ran(P))?
1. {a, b, {a}, {a, b}}
2. {a, b, {a}, {b}, {a, b}}
3. {a, b, {b}, {a, b}}
4. {a, b, {a, b}}

Question 8
Which one of the following relations represents the composition relation P ○ P (ie P; P)?
1. {(a, {a, b}), (b, a), ({a, b}, a), ({b}, {a}), (a, {a})}
2. {(a, {a, b}), (b, a), ({a, b}, a), ({b}, {a})}
3. {(a, {a, b}), (b, a), ({a, b}, {a}), ({a, b}, b), ({b}, b), ({b}, {a})}
4. {(a, {a, b}), (b, a), ({a, b}, {a}), ({a, b}, b), ({b}, {a})}

Question 9
The relation P does not satisfy trichotomy. Which ordered pairs should be included in P so that an extended relation P_1 (say) would satisfy trichotomy?

*(For trichotomy, **each** element of B must be paired with **each other different** element in B to form elements of P_1. For example, we see that b $\neq$ {a} but neither (b, {a}) nor ({a}, b) are elements of P, but at least one of these elements should be an element of P_1.*
*We include **(b, {a})** in P_1: P_1 = {(a, b), (b, {a, b}), ({a, b}, a), ({b}, a), (a, {a}), **(b, {a})**, ...}.*

Now check all the different elements of B and if two different elements of B are not already grouped in an ordered pair of P, they should be paired and be included as elements of P_1. You can ask the question: Which ordered pairs should be included in P_1 so that it will be true that for ***all*** *x, y $\in$ B with* ***x ≠ y****, we have* ***(x, y) $\in$ P_1 or (y, x) $\in$ P_1****? Refer to study guide, p 78.)*

Choose the alternative that provides all the missing ordered pairs that should be included as elements of P_1 in order for P_1 to be a relation that satisfies trichotomy.

1. (b, {a}), (b, {b}), (b, a), ({a}, {a, b}) & ({a, b}, {b})
2. ({a}, b), (b, {b}), ({b}, {a, b}) & ({a, b}, {a})
3. (b, {a}), ({b}, b), ({a}, {b}), ({b}, {a, b}) & ({a, b}, {a})
4. (b, {a}), (b, {b}), ({a}, {b}) & ({a, b}, {a, b})

Question 10

Which one of the following sets is a partition S of B = {a, b, {a}, {b}, {a, b}}?

1. {**{**a, b, {a}, {b}**}**, **{**{a, b}**}**}
2. {{a}, {b}, {a, b}}
3. {**{**a, b, {a}**}**, **{**{a}, {b}, {a, b}**}**}
4. {a, b, {a}, {b}, {a, b}}

(A partition of the given set B can be defined as a set S = {S_1, S_2, S_3, …}. The members of S are subsets of B (each set S_i is called a part of S) such that

a. for all i, $S_i \neq \emptyset$ (that is, each part is nonempty),

b. for all i and j, if $S_i \neq S_j$, then $S_i \cap S_j = \emptyset$ (that is, different parts have nothing in common), and

c. $S_1 \cup S_2 \cup S_3 \cup \ldots = B$ (that is, every element in B is in some part S_i).

It is possible to form different partitions of B depending on which subsets of B are formed to be elements of S.

Test whether the sets given in the different alternatives meet all the criteria given in the above definition. Note that the ***elements*** *of a* ***partition*** *of B must be* ***subsets*** *of* ***B****. Subsets of B are formed when you keep the outside brackets of B and then throw away all, some or no element of B. For example, keep the outside brackets of B, then throw away the element {a}, then the subset {a, b, {b}, {a, b}} of B are formed. Refer to study guide, pp 94, 95.)*

Suppose U = {1, 2, 3, a, b, c} is a universal set with the subset A = {a, c, 2, 3}.

Let R = { (a, a), (a, c), (3, c), (3, a), (2, 3), (2, a), (2, c), (c, 2) } be a relation on A.

Answer questions 11 & 12 by using the given sets A, U and the relation R.

Question 11
Which one of the following statements regarding relation **R** is true?

1. R is a weak total order.
2. R is a strict total order.
3. R is a weak partial order.
4. R is **not** an equivalence relation.

Question 12
Which ordered pairs should be removed from relation R in order for the changed relation R_1 (say) to be a strict partial order?

1. only (2, c)
2. only (a, a)
3. (a, a) & (c, 2)
4. (a, a) & (2, c)

Let R be the relation on $Z^{\geq}$ (the set of integers) defined by

$$(x, y) \in R \text{ iff } x^2 + y^2 = 2k \text{ for some integers } k \geq 0.$$

Answer questions 13 to 15 by using the given relation R.

Question 13
Which one of the following is an ordered pair in R?

1. (1, 0)
2. (2, 9)
3. (3, 8)
4. (5, 7)

Question 14

R is symmetric. Which one of the following is a valid proof showing that R is symmetric?

1. Let $x, y \in \mathbb{Z}^{\geq}$ be given.
   Suppose $(x, y) \in R$
   then $x^2 + y^2 = 2k$ for some $k \geq 0$.
   ie $y^2 + x^2 = 2k$ for some $k \geq 0$.
   thus $(x, y) \in R$.

2. Let $x, y \in \mathbb{Z}^{\geq}$ be given.
   Suppose $(x, y) \in R$
   then $x^2 + y^2 = 2k$ for some $k \geq 0$.
   ie $y^2 + x^2 = 2k$ for some $k \geq 0$.
   thus $(y, x) \in R$.

3. Let $x, y \in \mathbb{Z}^{\geq}$ be given.
   Suppose $(x, y) \in R$
   then $x^2 + y^2 = 2k$ for some $k \geq 0$.
   thus $(y, x) \in R$.

4. Let $x, y \in \mathbb{Z}^{\geq}$ be given.
   Suppose $(x, x) \in R$
   then $x^2 + y^2 = 2k$ for some $k \geq 0$.
   ie $y^2 + x^2 = 2k$ for some $k \geq 0$.
   thus $(y, y) \in R$.

Question 15

R is not antisymmetric. Which of the following ordered pairs can be used together in a counterexample to prove that R is **not** antisymmetric? (Remember that R is defined on $\mathbb{Z}^{\geq}$.)

1. $(-1, 1)$ & $(1, -1)$
2. $(5, 9)$ & $(13, 15)$
3. $(8, 7)$ & $(7, 8)$
4. $(3, 1)$ & $(1, 3)$

8.6.3 *Semester 1: Assignment 03*

Unique number:	**598243**
Due date:	**17 April 2020**
Material to be tested:	**Study guide, study units 6.4 – 10**
Additional material:	**Tutorial letters 102 and 103, lessons 7 – 10**
Submission procedure:	**Electronic submission via myUnisa.**
Weight towards semester mark:	**40%**

Important: *Do the self-assessment exercises in the activities in the study guide before attempting this assignment. The solutions are in the tutorial letter 103 as well as in the lessons on myUnisa.*

Additional self-assessment questions: *Attempt these written questions which are provided in the tutorial letter 103 as well as in the lessons on myUnisa. You can do these questions in preparation of this assignment. Only submit the solutions to the multiple choice questions given in this assignment.*

***Examination paper**: Although the exam paper is an MCQ paper, you will need to write down proofs on rough, for example, or draw the diagrams for binary operations, or truth tables on rough before you will be able to select the correct alternative. It is therefore **very important** to attempt the self-assessment questions. You will receive a practice exam paper later in the semester.*

Note: *Answer all the questions for this assignment. The mark that you achieve from the possible 15 marks will be converted to a percentage.*

Suppose U = {1, 2, 3, 4, 5, a, b, c} is a universal set with the subset A = {a, b, c, 1, 2, 3, 4}.

Answer questions 1 and 2 by using the given sets U and A.

Question 1

Which one of the following relations on A is NOT functional?

1. {(1, 3), (b, 3), (1, 4), (b, 2), (c, 2)}
2. {(a, c), (b, c), (c, b), (1, 3), (2, 3), (3, a)}
3. {(a, a), (c, c), (2, 2), (3, 3), (4, 4)}
4. {(a, c), (b, c), (1, 3), (3, 3)}

Question 2
Which one of the following alternatives represents a **surjective function** from **U** to **A**?

1. {(1, 4), (2, b), (3, 3), (4, 3), (5, a), (a, c), (b, 1), (c, b)}
2. {(a, 1), (b, 2), (c, a), (1, 4), (2, b), (3, 3), (4, c)}
3. {(1, a), (2, c), (3, b), (4, 1), (a, c), (b, 2), (c, 3)}
4. {(1, a), (2, b), (3, 4), (4, 3), (5, c), (a, a), (b, 1), (c, 2)}

Question 3
Let G and L be relations on A = {1, 2, 3, 4} with
G = {(1, 2), (2, 3), (4, 3)} and L = {(2, 2), (1, 3), (3, 4)}.
Which one of the following alternatives represents the relation $L \circ G = G; L$?

1. {(2, 3), (3, 3)}
2. {(1, 2), (2, 4), (4, 4)}
3. {(1, 2), (2, 1), (3, 3), (4, 4)}
4. {(2, 4), (4, 4)}

Let g be a function from $\mathbb{Z}^+$ (the set of positive integers) to $\mathbb{Q}$ (the set of rational numbers) defined by

(x, y) $\in$ g iff y = $4x - \frac{3}{7}$ ($g \subseteq \mathbb{Z}^+ \times \mathbb{Q}$) and

let f be a function on $\mathbb{Z}^+$ defined by

(x, y) $\in$ f iff y = $5x^2 + 2x - 3$ ($f \subseteq \mathbb{Z}^+ \times \mathbb{Z}^+$).

Answer questions 4 to 7 by using the given functions g and f.
Hint: Drawing graphs of f and g before answering the questions, may assist you. Keep in mind that $g \subseteq \mathbb{Z}^+ \times \mathbb{Q}$ and $f \subseteq \mathbb{Z}^+ \times \mathbb{Z}^+$.

Question 4
Consider the function f on $\mathbb{Z}^+$. For which values of x is it the case that $5x^2 + 2x - 3 > 0$?
Hint: Solve $5x^2 + 2x - 3 > 0$ and keep in mind that $x \in \mathbb{Z}^+$.

1. $x < 5, x \in \mathbb{Z}^+$
2. $\frac{3}{5} < x < 1, x \in \mathbb{Z}^+$
3. $x \geq 1, x \in \mathbb{Z}^+$
4. $x < 1, x \in \mathbb{Z}^+$

Question 5

Which one of the following is an ordered pair belonging to f?

1. (–1, 0)
2. (2, 21)
3. (1, 5)
4. (3, 44)

Question 6

Which one of the following alternatives represents the image of x under g ○ f (ie g ○ f(x)))?

1. $20x^2 + 8x - 12\,\frac{3}{7}$
2. $80x^2 + 4\,\frac{4}{7}x - \frac{180}{49}$
3. $20x^2 + 8x + 3\,\frac{3}{7}$
4. $80x^2 + 4\,\frac{4}{7}x - 3$

Question 7

Which one of the following statements regarding the function g is TRUE?
(Remember, $g \subseteq \mathbb{Z}^+ \times \mathbb{Q}$.)

1. g can be presented as a straight **line** graph.
2. g is injective.
3. g is surjective.
4. g is bijective.

Let A = {□, ◊, ☼, ⌂} and let # be a binary operation from A × A to A presented by the following table:

#	□	◊	☼	⌂
□	□	◊	☼	⌂
◊	◊	□	◊	□
☼	☼	◊	☼	⌂
⌂	⌂	□	⌂	⌂

Answer questions 8 and 9 by referring to the table for #.

Question 8

Which one of the following statements pertaining to the binary operation # is TRUE?

1. ☼ is the identity element for #.
2. # is symmetric.
3. # is associative.
4. [(⌂ # ◊) # ☼] = [⌂ # (◊ # ☼)]

Question 9

can be written in list notation. Which one of the following ordered pairs is an element of the list notation set representing #?

1. ((□, ◊), ⌂)
2. ((⌂, ☼), ◊)
3. ((☼, ◊), ◊)
4. ((⌂, ◊), ◊)

Question 10

Perform the following matrix multiplication operation:

$$\begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 1 \\ -1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -3 & 0 & 0 \\ -1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

Which one of the following alternatives represents the correct answer to the above operation?

1. The operation is not possible.
2. $\begin{bmatrix} -1 & 1 & 0 \\ 2 & 4 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

3. $\begin{bmatrix} -7 & 0 & -1 \\ -12 & 1 & -3 \\ 2 & -1 & -1 \end{bmatrix}$

4. $\begin{bmatrix} 0 & 4 & 3 \\ 5 & 9 & 8 \\ -5 & -1 & -2 \end{bmatrix}$

Question 11

Consider the truth table for the connective '$\leftrightarrow$' with two simple declarative statements p and q.

p	**q**	**p $\leftrightarrow$ q**
T	T	**T**
T	F	**F**
F	T	**F**
F	F	**T**

(a) Which one of the given alternatives represents '$\leftrightarrow$' as a binary operation on the set of truth values {T, F}? (b) Does the binary operation '$\leftrightarrow$' have an identity element?

1.
(a)

$\leftrightarrow$	**T**	**F**
T	F	T
F	T	F

(b) The binary operation '$\leftrightarrow$' does not have an identity element.

2.
(a)

$\leftrightarrow$	**T**	**F**
T	T	F
F	F	F

(b) The binary operation '$\leftrightarrow$' has an identity element.

3.
(a)

$\leftrightarrow$	**T**	**F**
T	T	F
F	T	F

(b) The binary operation '$\leftrightarrow$' does not have an identity element.

4.

(a)

$\leftrightarrow$	**T**	**F**
T	T	F
F	F	T

(b) The binary operation '$\leftrightarrow$' has an identity element.

Question 12

Let p, q and r be simple declarative statements. Which alternative provides the truth values for the biconditional '$\leftrightarrow$' of the compound statement provided in the given table?

Hint: Determine the truth values of $p \rightarrow r$, $q \vee r$, $(p \rightarrow r) \wedge (q \vee r)$, $q \rightarrow p$, $\neg(q \rightarrow p)$ and $\neg(q \rightarrow p) \wedge r$ in separate columns before determining the truth values of
$[(p \rightarrow r) \wedge (q \vee r)] \leftrightarrow [\neg(q \rightarrow p) \wedge r]$.

p	q	r	$[(p \rightarrow r) \wedge (q \vee r)] \leftrightarrow [\neg(q \rightarrow p) \wedge r]$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

1.

$\leftrightarrow$
F
T
F
T
T
F
F
T

2.

$\leftrightarrow$
F
T
F
T
F
T
F
T

3.

$\leftrightarrow$
T
F
F
F
T
F
F
F

4.

$\leftrightarrow$
F
F
T
F
T
F
F
T

Question 13

Consider the following quantified statement:

$$\forall x \in \mathbb{Z}\ [(x^2 \geq 0) \lor (x^2 + 2x - 8 > 0)].$$

Which one of the alternatives provides a true statement regarding the given statement or its negation?

1. The negation $\exists x \in \mathbb{Z}\ [(x^2 < 0) \lor (x^2 + 2x - 8 \leq 0)]$ is not true.
2. $x = -3$ would be a counterexample to prove that the negation is not true.
3. $x = -6$ would be a counterexample to prove that the statement is not true.
4. The negation $\exists x \in \mathbb{Z}\ [(x^2 < 0) \land (x^2 + 2x - 8 \leq 0)]$ is true.

Question 14

Consider the following proposition:

For any predicates P(x) and Q(x) over a domain D, the negation of the statement

$$\exists x \in D,\ P(x) \land Q(x)$$

is the statement

$$\forall x \in D,\ P(x) \rightarrow \neg Q(x).$$

We can use this truth to write the negation of the following statement:

"There exist integers a and d such that a and d are negative and a/d = 1 + d/a."

Which one of the alternatives provides the negation of this statement?

1. There exist integers a and d such that a and d are positive and $a/d = 1 + d/a$.
2. For all integers a and d, if a and d are positive then $a/d \neq 1 + d/a$.
3. For all integers a and d, if a and d are negative then $a/d \neq 1 + d/a$.
4. For all integers a and d, a and d are positive and $a/d \neq 1 + d/a$.

Question 15

Which one of the alternatives is a proof by contrapositive of the statement

"If $x^3 - x + 4$ is not divisible by 4 then x even."

1. Required to prove: If $x^3 - x + 4$ is not divisible by 4 then x even.
   Proof:
   Suppose x is odd. Let $x = 2k + 1$, then we have to prove that $x^3 - x + 4$ is divisible by 4.
   $x^3 - x + 4 = (2k + 1)^3 - (2k + 1) + 4$
   $= (2k + 1)(4k^2 + 4k + 1) - 2k - 1 + 4$
   $= 8k^3 + 8k^2 + 2k + 4k^2 + 4k + 1 - 2k - 1 + 4$
   $= 8k^3 + 12k^2 + 4k + 4$
   $= 4(2k^3 + 3k^2 + k + 1)$, which is divisible by 4. (4 multiplied by any integer is divisible by 4)

2. Required to prove: If $x^3 - x + 4$ is not divisible by 4 then x even.
   Proof:
   Assume that $x^3 - x + 4$ is not divisible by 4. Then x can be even or odd. We assume that x is odd.
   Let $x = 2k + 1$, then
   $x^3 - x + 4 = (2k+1)^3 - (2k + 1) + 4$
   $= (2k + 1)(4k^2 + 4k + 1) - 2k - 1 + 4$
   $= 8k^3 + 8k^2 + 2k + 4k^2 + 4k + 1 - 2k - 1 + 4$
   $= 8k^3 + 12k^2 + 4k + 4$
   $= 4(2k^3 + 3k^2 + k + 1)$, which is divisible by 4. (4 multiplied by any integer is divisible by 4)

   But this is a contradiction to our original assumption. Therefore x must be even
   if $x^3 - x + 4$ is not divisible by 4.

3. Required to prove: If $x^3 - x + 4$ is not divisible by 4 then x even.
   Proof:
   Let $x = 4$ be an even element of $\mathbb{Z}$. We can replace x with 4 in the expression $x^3 - x + 4$.
   $x^3 - x + 4$
   $= (4)^3 - (4) + 4$
   $= 64 - 4 + 4$
   $= 64$ which is divisible by 4.

4. Required to prove: If $x^3 - x + 4$ is not divisible by 4 then x even.
   Proof:
   Assume that x is even, ie $x = 4k$,
   then $x^3 - x + 4$
   $= (4k)^3 - (4k) + 4$
   $= 64k^3 - 4k + 4$
   $= 4(16k^3 - k + 1)$, which is divisible by 4.

SECOND SEMESTER ASSIGNMENTS

8.6.4 *Semester 2: Assignment 01*

Unique number:	**698943**
Due date:	**17 August 2020**
Material to be tested: **Additional material:**	**Study guide, study units 1 – 3** **Tutorial letters 102 and 103, lessons 1 – 3** ***NOTE: The Venn diagrams in study unit 4 will help you to understand the definitions in study unit 3.***
Submission procedure:	**Electronic submission via myUnisa.**
Weight towards semester mark:	**20%**

Important: *Do the self-assessment exercises in the activities in the study guide before attempting this assignment. The solutions are in the tutorial letter 103as well as in the lessons on myUnisa.*

CAI tutorial: *In sections 4.3 & 11 of this tutorial letter it is mentioned that you can work through the CAI tutorial "Relations" that is downloadable from the web. In preparation for this assignment, you can go to the menu, choose "Prerequisite background" then attempt the "Sets" part. Graphical illustrations are also provided.*

Note: *Answer all the questions for this assignment. The mark that you achieve from the possible 10 marks will be converted to a percentage.*

Consider the following sets, where U represents a universal set:

U = {1, 2, {1}, {1, 2}, b, {c}} A = {1, 2, {c}} B = {{1, 2}, b} C = {2, {1}, {c}}

(Hint: U has 6 elements namely 1, 2, {1}, {1, 2}, b and {c}, A has 3 elements, B has 2 elements and C has 3 elements. List the elements of A, B and C before answering the questions.)

Questions 1 to 10 are based on the sets defined above.

Note: The Venn diagrams in study unit 4 will help you to understand the definitions in study unit 3.

Question 1

A ∪ B is the set:

1. {1, 2, b, c}
2. {{1, 2}, b, {c}}
3. {1, 2, {1, 2}, b, {c}}
4. {2, {1}, {c}}

Question 2

A $\cap$ C is the set:

1. $\{2, \{c\}\}$
2. $\{1, 2, \{c\}\}$
3. $\{\{1, 2\}, \{c\}\}$
4. $\{1, 2, c\}$

Question 3

B - C (set difference) is the set:

1. $\{b, \{c\}\}$
2. $\emptyset$
3. $\{\{1, 2\}, b\}$
4. $\{b\}$

Question 4

B′ is the set:

1. $\{c\}$
2. $\{1, 2, \{1\}, \{c\}\}$
3. $\{1, 2, c\}$
4. $\{\{1\}, \{c\}\}$

Question 5

A + C (symmetric difference) is the set:

1. $\{1, 2, \{1\}, \{c\}\}$
2. $\{2, \{c\}\}$
3. $\{1, 2, c\}$
4. $\{1, \{1\}\}$

Question 6

What is the cardinality of the set U – C?

1. 2
2. 3
3. 5
4. 6

Question 7
Which of the following is a subset of U?

1. $\{\{1, 2\}, \{2, \{1\}\}\}$
2. $\{\{b, \{c\}\}, \{1, 2\}\}$
3. $\{1, 2, b, c\}$
4. $\{1, \{1\}, \{1, 2\}\}$

Remember, subsets of some set G can be formed by ***keeping the outside brackets*** *of G and then throwing away* ***none***, ***one*** *or* ***more*** *elements of G. Refer to study guide, p 40.*

Question 8
Which one of the following alternatives provides (an) element(s) of $\mathcal{P}$(B) (the power set of B)?

1. $\{1, 2\}$, b
2. $\{\{\{1, 2\}, b\}\}$
3. $\{\{1, 2\}\}$, $\{b\}$
4. $\{\emptyset\}$

Question 9
Which one of the following sets is a *proper subset* of $\mathcal{P}$(B)?

(Remember to distinguish between *elements* and *subsets* of $\mathcal{P}$(B).)

1. $\{\{\{1, 2\}, b\}\}$
2. $\{\{1, 2, b\}\}$
3. $\{\{\ \}, \{\{1, 2\}\}, \{b\}, \{\{1, 2\}, b\}\}$
4. $\{b\}$

Remember, proper subsets of some set G can be formed by ***keeping the outside brackets*** *of G and then throwing away* ***one*** *or* ***more*** *elements of G. Refer to study guide, p 40.*

Question 10
Which one of the following is a set D such that D + U = $\{\{1, 2\}, \{c\}\}$?

1. $\{1, 2, \{1\}, \{1, 2\}, b, \{c\}\}$
2. $\{1, \{1, 2\}, \{c\}\}$
3. $\{\{1, 2\}, \{c\}\}$
4. $\{1, 2, \{1\}, b\}$

---oooOooo---

8.6.5 *Semester 2: Assignment 02*

Unique number:	**716776**
747 number:	
Due date:	**14 September 2020**
Material to be tested:	**Study guide, study units 4 – 6.3**
Additional material:	**Tutorial letters 102 and 103, lessons 4 – 6.3**
Submission procedure:	**Electronic submission via myUnisa.**
Weight towards semester mark:	**40%**

Important: *Do the self-assessment exercises in the activities in the study guide before attempting this assignment. The solutions are in the tutorial letter 103 as well as in the lessons on myUnisa.*

CAI tutorial: *In sections 4.3 & 11 of this tutorial letter it is mentioned that you can work through the CAI tutorial "Relations" that is downloadable from the web. In preparation of this assignment, navigate through all the sections in the tutorial. Graphical illustrations are also provided.*

Additional self-assessment questions: *Attempt these written questions which are provided in the tutorial letter 103 as well as in the lessons on myUnisa. You can do these questions in preparation of this assignment. Only submit the solutions to the multiple choice questions given in this assignment.*

Examination paper*: Although the exam paper is an MCQ paper, you will need to write down proofs on rough, for example, or draw Venn diagrams on rough before you will be able to select the correct alternative. It is therefore* ***very important*** *to attempt the self-assessment questions. You will receive a practice exam paper later in the semester.*

Note: *Answer all the questions for this assignment*. *The mark that you achieve from the possible 15 marks will be converted to a percentage.*

Hints:

If a relation is provided in a question, make sure that your alternative of choice (relation) is defined on the set indicated in the question. For example, if a relation is defined on ***A = {a, b, c}*** *but a relation R = {a, b), (b, c), (c,* ***d****)} is provided in some alternative, whatever the question is, this alternative is immediately disqualified because (c,* ***d****)* $\in$ *R but* ***d*** $\notin$ *A.*

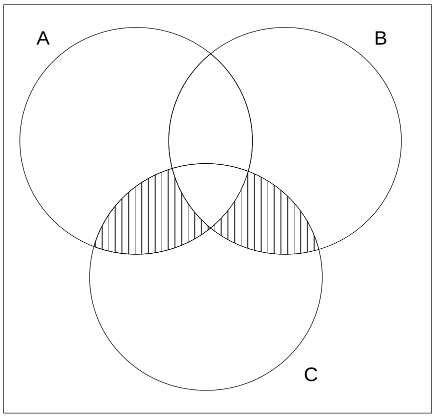
To form a subset of some set L (say), ***keep the outside brackets*** *of L, then throw away some element(s) of L to form a subset of L. Refer to the definition of a subset in the study guide.*

Question 1

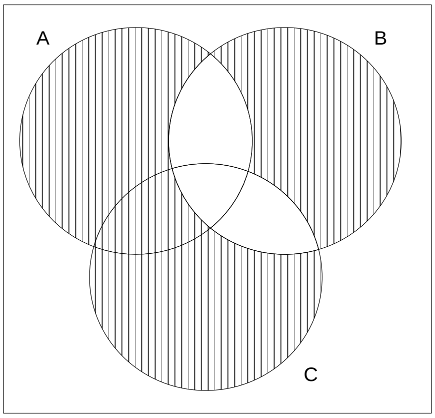
Let A, B and C be subsets of a universal set U.

Which one of the following four Venn diagrams represents the set $((A + B) \cup C) - (B \cap C)$?

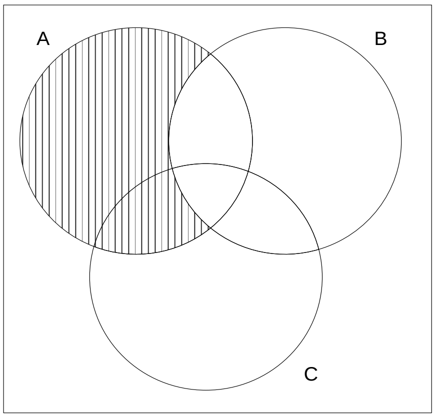
1.



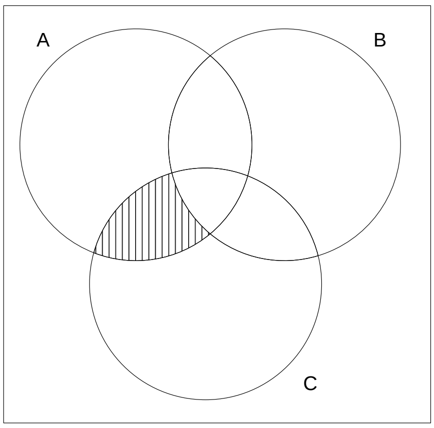
2.



3.



4.



Question 2

Let A, B and C be subsets of a universal set U = {1, 2, 3}.

The statement $A \cup (B' \cap C) = (A - B) \cup C$ is NOT an identity. Which of the following sets A, B and C can be used in a *counterexample* to prove that the given statement is not an identity?

1. A = {1}, B = {2, 3} & C = {3}
2. A = {1}, B = {2} & C = {1}
3. A = {1}, B = {1} & C = {1}
4. A = {1, 2, 3}, B = {1} & C = {1}

Question 3

We want to prove that, for all $X, Y \subseteq U$,

$$(X - Y) \times (X \cup Y) = ((X - Y) \times X) \cup ((X - Y) \times Y) \text{ is an identity.}$$

Consider the following proof:

$(u, v) \in (X - Y) \times (X \cup Y)$

iff $u \in (X - Y)$ and $v \in (X \cup Y)$

iff Step 3

iff ($u \in X$ and $u \notin Y$ and $v \in X$) or ($u \in X$ and $u \notin Y$ and $v \in Y$)

iff ($u \in (X - Y)$ and $v \in X$) or ($u \in (X - Y)$ and $v \in Y$)

iff Step 6

iff $(u, v) \in ((X - Y) \times X) \cup ((X - Y) \times Y)$

Which one of the following alternatives provides valid steps 3 and 6 to complete the given proof?

1. Step 3: iff ($u \in X$ and $u \notin Y$) or ($v \in X$ and $v \in Y$)
   Step 6: iff $(u, v) \in (X - Y) \times X$) or $(u, v) \in (X - Y) \times Y$)
2. Step 3: iff ($u \in X$ and $u \notin Y$) and ($v \in X$ or Y)
   Step 6: iff $(u, v) \in (X - Y) \times X$) or $(u, v) \in (X - Y) \times Y$)
3. Step 3: iff ($u \in X$ and $u \notin Y$) and ($v \in X$ or $v \in Y$)
   Step 6: iff $(u, v) \in (X - Y) \times X$) and $(u, v) \in (X - Y) \times Y$
4. Step 3: iff ($u \in X$ and $u \notin Y$) and ($v \in X$ or $v \in Y$)
   Step 6: iff $(u, v) \in (X - Y) \times X$) or $(u, v) \in (X - Y) \times Y$)

Question 4

A newly built old age home has 40 townhouses. Residents may only plant clivias, rose bushes and lavenders in their gardens. Of the 40 gardens

18 grow clivias,
30 grow roses, and
17 grow lavenders.

(Residents do not necessarily plant only one of the kinds of plants.)

Furthermore, some gardens grow the following:

10 grow clivias and rose bushes
5 grow clivias and lavenders,
15 grow rose bushes and lavenders.

How many gardens have clivias, rose bushes and lavenders?

1. 0
2. 5
3. 19
4. 40

Question 5

Which one of the following sets is equal to the set $\{u \in \mathbb{Z} \mid 2u^2 + 4u - 30 < 0\}$?

1. $\{u \in \mathbb{Z} \mid u > -5 \text{ or } u < 3\}$
2. $\{w \in \mathbb{Z} \mid -5 > w > 3\}$
3. $\{w \in \mathbb{Z} \mid -5 < w < 3\}$
4. $\{u \in \mathbb{Z} \mid -5 > u < 3\}$

Consider the following relations on the set B = {1, 4, a, c, f}:

P = {(1, 4), (4, 1), (a, 4), (c, 4), (f, 1)} and R = {(1, c), (4, 1), (a, 4)}.

Answer questions 6 to 8 by using the given relations P and R defined on the set B.

Question 6

Which one of the following alternatives represents the **range** of P?

1. {1, 4, a, c, f}
2. {1, a, c, f}
3. {1, 4}
4. 1, 4

Question 7

Which one of the following relations represents the composition relation P; R (i.e R ○ P)?

1. { (1, 1), (4, c), (a, 1), (c, 1), (f, c) }
2. { (1, 4), (1, c), (4, 1), (a, 4), (c, 4), (f, 1) }
3. { (1, 4), (4, 4), (a, 1) }
4. { (4, 1), (a, 4) }

Question 8

The relation R = {(1, c), (4, 1), (a, 4)} is not transitive. Which ordered pair(s) should be included in R so that the extended relation would satisfy transitivity?

1. only (4, c)
2. only (a, 1)
3. only (4, c) & (a, 1)
4. (4, c), (a, 1), & (a, c)

Let A = {1, 2, {2}, 3, {3, 4}}.

Answer questions 9 and 10 by using the given set A.

Question 9

Which one of the following sets is NOT a partition of **A**?

1. { {{3, 4}}, {1, 2}, {{2}, 3} }
2. { {1, {2}}, 2, 3, {{3, 4}} }
3. { {1, 3, {3, 4}}, {2}, {{2}} }
4. { {1, {2}, 3}, {2, {3, 4}} }

(A partition of the given set B can be defined as a set S = {S_1, S_2, S_3, …}. The members of S are subsets of B (each set S_i is called a part of S) such that

a. for all i, $S_i \neq \emptyset$ (that is, each part is nonempty),

b. for all i and j, if $S_i \neq S_j$, then $S_i \cap S_j = \emptyset$ (that is, different parts have nothing in common), and

c. $S_1 \cup S_2 \cup S_3 \cup \ldots = B$ (that is, every element in B is in some part S_i).

It is possible to form different partitions of B depending on which subsets of B are formed to be elements of S.

*Test whether the sets given in the different alternatives meet all the criteria given in the above definition. Note that the **elements** of a **partition** of B must be **subsets** of **B**. Subsets of B are formed when you keep the outside brackets of B and then throw away all, some or no element of B. For example, keep the outside brackets of B, then throw away the element {a}, then the subset {a, b, {b}, {a, b}} of B are formed. Refer to study guide, pp 94, 95.)*

Question 10
Consider the following relation on A:

$$R = \{(1, 1), (1, \{2\}), (1, \{3, 4\}), (\{2\}, 1), (\{2\}, \{2\}), (\{2\}, \{3, 4\}), (\{3, 4\}, \{2\})\}.$$

Which one of the following statements regarding the relation R is TRUE?

1. R is reflexive, transitive and anti-symmetric.
2. R is irreflexive, transitive and symmetric.
3. R is neither reflexive nor irreflexive, and is not transitive and not symmetric.
4. R is neither reflexive nor irreflexive, and is transitive and not symmetric.

Let S be some relation from A = {1, 3, 4, 6} to B = {1, 2, 4} defined by

$$(a, b) \in S \text{ iff } 3a - b \text{ is odd. } (S \subseteq A \times B, \text{ and } A, B \subseteq U = \mathbb{Z}.)$$

Answer questions 11 and 12 by using the given relation S.

Hint: First determine the relations $A \times B$ (a Cartesian product) and S.

Question 11
In which one of the following alternatives is it true that all the ordered pairs belong to S?

1. (1, 2), (3, 1), (6, 1)
2. (4, 1), (6, 1), (6, 4)
3. (1, 1), (3, 1), (4, 2)
4. (1, 4), (3, 2), (4, 1)

Question 12
Which one of the following statements is TRUE?

1. $(4, 2) \in A \times B$ but $(4, 2) \notin S$.
2. $(3, 1) \in A \times B$ and $(3, 1) \in S$.
3. $(2, 3) \notin A \times B$ and $(2, 3) \in S$.
4. $(2, 6) \in A \times B$ and $(6, 2) \in S$.

Let R be the relation on $\mathbb{Z}$ defined by

$$(x, y) \in R \text{ iff } y > 1 - x \text{ with } x, y \in \mathbb{Z}.$$

Answer questions 13 to 15 by using the given relation R.

Question 13
Which one of the following is NOT an ordered pair in R?

1. (5, 0)
2. (0, 1)
3. (0, 5)
4. (5, -3)

Question 14
R is a symmetric relation. Which of the following alternatives can be used to prove that R is a symmetric relation?

1. Let $x, y \in \mathbb{Z}$ be given such that $(x, y) \in R$ and $(y, x) \in R$
   then $y > 1 - x = x > 1 - y$
   ie $y > 1 - x = x - 1 > -y$
   ie $y > 1 - x = 1 - x < y$ (multiply by -1, which changes > to < as well)

2. Let $x, y \in \mathbb{Z}$ be given such that $(x, y) \in R$. We have to prove that $(y, x) \in R$, ie. $x > 1 - y$
   If $(x, y) \in \mathbb{Z}$ then
   $y > 1 - x$
   ie $y - 1 > -x$
   ie $1 - y < x$ (multiply by -1, which changes > to <)
   ie $x > 1 - y$
   thus $(y, x) \in R$.

3. $(5, -3) \in R$ and $(-3, 5) \in R$, therefore R is symmetric.
   We substitute the ordered pairs (5, -3) and (-3, 5) in $y > 1 - x$ and $1 - x < y$ for the proof:
   $-3 > 1 - 5$ and $5 > 1 - (-3)$
   ie $-3 > -4$, which is true ie $5 > 4$, which is also true

4. R is not a symmetric relation. We give a counterexample:
   The ordered pairs (5, -4) and (-4, 5) are not ordered pairs in R.
   We substitute the ordered pairs (5, -4) and (-4, 5) in $y > 1 - x$ and $1 - x < y$ for the proof:
   $-4 > 1 - 5$ and $5 > 1 - (-4)$
   ie $-4 > -4$, which is false ie $5 > 5$, which is also false
   We have proved that R is not symmetric.

Question 15
Which one of the following statements regarding the relation R is TRUE?

1. R satisfies trichotomy.
2. R is not transitive.
3. (1, 0), (0, 1) $\in$ R.
4. R is an equivalence relation.

*(For trichotomy, **each** element of some set S must be paired with **each other different** element in S. For example, let S = {(1, 2), (1, 3), (2, 3)} be a relation on the set T = {1, 2, 3}. We see that **each** element in T is paired with **each other different** element in T in the ordered pairs of S.)*

8.6.6 *Semester 2: Assignment 03*

Unique number:	**833636**
Due date:	**8 October 2020**
Material to be tested:	**Study guide, study units 6.4 - 10** **Tutorial letters 102 and 103, lessons 7 - 10**
Submission procedure:	**Electronic submission via myUnisa.**
Weight towards semester mark:	**40%**

Important: *Do the self-assessment exercises in the activities in the study guide before attempting this assignment. The solutions are in the tutorial letter 103 as well as in the lessons on myUnisa.*

Additional self-assessment questions: *Attempt these written questions which are provided in the tutorial letter 103 as well as in the lessons on myUnisa. You can do these questions in preparation of this assignment. Only submit the solutions to the multiple choice questions given in this assignment.*

Examination paper*: Although the exam paper is an MCQ paper, you will need to write down proofs on rough, for example, or draw Venn diagrams on rough before you will be able to select the correct alternative. It is therefore **very important** to attempt the self-assessment questions. You will receive a practice exam paper later in the semester.*

Note: *Answer all the questions for this assignment. The mark that you achieve from the possible 15 marks will be converted to a percentage.*

Suppose U = {2, 4, 6, a, b, c, {b, c}} is a universal set with the following subsets:

A = {6, b, c, {b, c}} and B = {2, 6, b, c}.

Answer questions 1 and 2 by using the given sets.

Question 1

Which one of the following relations from **A** to **B** is functional?

1. {(2, 6), (6, b), (b, c), (c, {b, c}}
2. {(6, 6), (c, c), (b, 2), ({b, c}, 6)}
3. {(6, 2), (b, 6), (c, b), (6, {b, c})}
4. {(b, 2), (b, 6), (b, b), (b, c)}

Question 2

Let F = {(6, {b, c}), (b, 2), (c, c), ({b, c}, 6)} be a function from **A** to **U**. Which one of the following alternatives is true for function **F**?

1. F is injective and surjective.
2. F is surjective, but not injective.
3. F is neither injective nor surjective.
4. F is injective, but not surjective.

Question 3

Which one of the following relations is a bijective function on the set B = {t | ($-4 < t < 4$, $t \in \mathbb{Z}$}?
Hint*: First list the elements of set B.*

1. {(-3, -3), (-1, -1), (1, 1), (3, 3)}
2. {(0, -3), (0, -2), (0, -1), (0, 0), (0, 1), (0, 2), (0, 3)}
3. {(-3, 0), (-2, -1), (-1, -1), (0, 0), (1, -3), (2, -3), (3, 3)}
4. {(-3, 0), (-2, -3), (-1, 3), (0, -1), (1, -2), (2, 2), (3, 1)}

Let f be a function on $\mathbb{Z}^+$ (the set of positive integers) defined by

$(x, y) \in f$ iff $y = 2x^2 - 7$ ($f \subseteq \mathbb{Z}^+ \times \mathbb{Z}^+$)

and g be a function from $\mathbb{Z}^+$ to $\mathbb{Q}$ (the set of rational numbers) defined by

$(x, y) \in g$ iff $y = \frac{3}{5}x + 5$ ($g \subseteq \mathbb{Z}^+ \times \mathbb{Q}$).

Answer questions 4 to 7 by using the given functions f and g.

Question 4

Which one of the following is NOT an ordered pair in g?

1. $(3, 6\frac{4}{5})$
2. $(4, 7\frac{1}{5})$
3. (5, 8)
4. $(6, 8\frac{3}{5})$

Question 5

Which one of the following alternatives represents the range of g (ie ran(g))?

1. $\{y \mid \frac{5}{3}(y - 5) \in \mathbb{Z}^+ \}$
2. $\{y \mid \frac{3}{5}x + 5 \in \mathbb{Q} \}$
3. $\{y \mid$ for some $y \in \mathbb{Q}$, $y = \frac{3}{5}x + 5 \in \mathbb{Q}\}$
4. $\mathbb{Q}$

Question 6

The function f is NOT surjective. Which of one of the following values for y provides a counterexample that can be used to prove that f is not surjective?

1. $y = 43$
2. $y = 25$
3. $y = 12$
4. $y = 1$

Question 7

Which one of the following alternatives represents the image of x under $g \circ f$ (ie $g \circ f(x)$)?

1. $\frac{6}{5}x^2 + \frac{4}{5}$
2. $\frac{18}{25}x^2 + 12x + 43$
3. $\frac{9}{25}x^2 - 8$
4. $4x^2 - 21$

Let A = {□, ◊, ☼, ⌂}. Consider the following table for the binary operation *: A × A → A:

*	□	◊	☼	⌂
□	⌂	☼	◊	⌂
◊	☼	⌂	□	◊
☼	◊	◊	⌂	☼
⌂	⌂	◊	☼	☼

Answer questions 8 and 9 by referring to the table of *.

Question 8

The binary operation * does not satisfy associativity. Which one of the following alternatives can be used in the calculations of a counterexample to prove that * does **NOT** satisfy associativity?

1. Determine (☼ * □) * ◊ and ☼ * (□ * ◊).

2. Determine (□ * ⌂) * ⌂ and □ * (⌂ * ⌂).

3. Determine (□ * ☼) * □ and □ * (☼ * □).

4. Determine (□ * ⌂) * □ and □ * (⌂ * □).

Question 9

Which of the following is true regarding an identity element for operation * ?

1. ◊ is the identity element.

2. ⌂ is the identity element.

3. * does not have an identity element

4. ☼ is the identity element.

Question 10

Perform the following matrix multiplication operation:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & -3 \end{bmatrix} \cdot \begin{bmatrix} -4 & 0 \\ -2 & -1 \\ 0 & 1 \end{bmatrix}$$

Which one of the following alternatives represents the correct answer to the above operation?

1. The operation is not possible.

2. $\begin{bmatrix} -8 & -16 \\ 1 & -3 \end{bmatrix}$

3. $\begin{bmatrix} -4 & 0 \\ -4 & 0 \\ 0 & -3 \end{bmatrix}$

4. $\begin{bmatrix} -8 & 1 \\ -16 & -3 \end{bmatrix}$

Question 11

Let p and q be simple declarative statements. Which one of the following statements are **not** a logical equivalence – in other words which one of the expressions is **false**? (*Hint:* Use truth tables to get to a conclusion)

1. $(\neg p \vee q) \equiv ((\neg(\neg p)) \rightarrow q)$

2. $(\neg q \rightarrow (p \wedge q)) \equiv ((\neg q \vee p) \wedge \neg q)$

3. $((q \wedge p) \rightarrow (\neg p \vee \neg q)) \equiv (p \rightarrow \neg q)$

4. $(p \vee (q \rightarrow p)) \equiv (p \vee \neg q)$

Question 12

Let p, q and r be simple declarative statements. Which alternative provides the truth values for the biconditional '↔' of the compound statement provided in the given table?

Hint: Determine the truth values of $p \to q$, $(p \to q) \to r$, $\neg q$, $\neg p$, $(\neg q \vee \neg p)$ and $(r \wedge (\neg q \vee \neg p))$ in separate columns before determining the truth values of $((p \to q) \to r) \leftrightarrow (r \wedge (\neg q \vee \neg p))$.

p	**q**	**r**	**$((p \to q) \to r) \leftrightarrow (r \wedge (\neg q \vee \neg p))$**
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

1.

↔
F
T
T
F
T
T
T
T

2.

↔
T
T
T
T
T
F
T
F

3.

↔
T
T
T
T
T
T
T
T

4.

$\leftrightarrow$
F
F
F
F
F
F
F
F

Question 13

Which one of the alternatives provides the negation of the statement

$$\exists x \in \mathbb{R}, [(\frac{x}{2} \geq 0) \wedge (x - 4 < 0)]?$$

1. $\forall x \in \mathbb{R}, [(\frac{x}{2} < 0) \wedge (x - 4 \geq 0)]$
2. $\exists x \in \mathbb{R}, [(\frac{x}{2} < 0) \vee (x - 4 \geq 0)]$
3. $\forall x \in \mathbb{R}, [(\frac{x}{2} < 0) \vee (x \geq 4)]$
4. $\forall x \in \mathbb{R}, [(\frac{x}{2} \leq 0) \vee (x > 4)]$

Question 14

The negation of the statement

$$\forall x \in \mathbb{Z}^+, [(x^2 > 2x) \vee (x - 1 \geq 0)]$$

can be written as:

$$\exists x \in \mathbb{Z}^+, [(x^2 - 2x \leq 0) \wedge (x - 1 < 0)].$$

Which one of the following alternatives is true regarding the statement and its negation?

1. The statement and its negation are false.
2. The statement and its negation are true.
3. The statement is false, but its negation is true.
4. The statement is true, but its negation is false.

Question 15

Which one of the alternatives is a proof by contradiction for the statement

“If $2x^2 – 3x + 7$ is odd, then x is even.”

1. Required to prove: If x is odd, then $2x^2 – 3x + 7$ is even.
   Proof: Suppose x is odd. Let $x = 2k + 1$, then we have to prove that $2x^2 – 3x + 7$ is even.
   $2x^2 – 3x + 7 = 2(2k+1)^2 – 3(2k + 1) + 7$
   $= 2(4k^2 + 4k +1) – 6k – 3 + 7$
   $= 8k^2 + 8k + 2 – 6k – 3 + 7$
   $= 8k^2 + 2k + 6$
   $= 2(4k^2 + k + 3)$, which is even (2 multiplied by any integer is even)
2. Assume that $2x^2 – 3x + 7$ is odd. Then x can be even or odd. We assume that x is odd.
   Let $x = 2k + 1$, then
   $2x^2 – 3x + 7 = 2(2k+1)^2 – 3(2k + 1) + 7$
   $= 2(4k^2 + 4k +1) – 6k – 3 + 7$
   $= 8k^2 + 8k + 2 – 6k – 3 + 7$
   $= 8k^2 + 2k + 6$
   $= 2(4k^2 + k + 3)$, which is even (2 multiplied by any integer is even)

   But this is a contradiction to our original assumption. Therefore x must be even
   if $2x^2 – 3x + 7$ is odd.

3. Let $x = 2$ be an even element of $\mathbb{Z}$. We can replace x with 2 in the expression $2x^2 – 3x + 7$.
   $2x^2 – 3x + 7$
   $= 2(2)^2 – 3(2) + 7$
   $= 8 – 6 + 7$
   $= 9$, which is odd.
   We have therefore proven that if $2x^2 – 3x + 7$ is odd, then x is even.

4. Required to prove: if $2x^2 – 3x + 7$ is odd, then x is even.
   Proof: Assume that x is even, ie $x = 2k$,
   then $2x^2 – 3x + 7$
   $= 2(2k)^2 – 3(2k) + 7$
   $= 8k^2 – 6k + 7$
   $= 2(4k^2 – 3k + 3) + 1$, which is odd. We have therefore proven that if if $2x^2 – 3x + 7$ is odd, then x is even.

8.7 Other assessment methods

Except for the activities in the study guide and the self-assessment exercises in the Tutorial letter 102, which has already been discussed, there are no other formal assessment methods.

8.8 The examination

A two hour examination will be scheduled for this module. Please refer to the brochure *Study@Unisa* for general examination guidelines and examination preparation guidelines.

Examination information will be provided to you with more specific details on the examination paper that will be an MCQ paper. You will still have to draw diagrams on rough or do proofs on rough before you will be able to select the correct alternative for a question in the exam, so it is important that you attempt the self-assesment assignments questions. A past examination paper with solutions is provided in tutorial letter 103. You will also be provided with a practice exam later in the semester.

Note that you gain **admission to the examination** of COS1501 by **submitting at least one assignment by the date specified for assignment 1**.

9 FREQUENTLY ASKED QUESTIONS

The *Study@Unisa* brochure contains an A-Z guide of the most relevant study information.

Is each COS1501 student assigned to an e-tutor?

You can communicate with your assigned e-tutor via myUnisa and e-mail. Please activate your *my*Life e-mail account to simplify the process. Students are assigned to tutors in batches of 200. It could happen that an overflowing group of less than 200 students is not assigned to a tutor. Unfortunately this is how the system functions.

Are past examination papers available?

Past papers are available on the COS1501 myUnisa web site. No solutions are provided for these. You can do the questions in the old examination papers and discuss your answers with the e-tutor and other students on your e-tutor site. A past examination paper and solutions are provided in the tutorial letter 103.

A practice examination paper will be put on myUnisa under Additional Resources about 1 month before the exam. Your e-tutor will discuss the answers to this paper with you.

Are solutions to the self-assessment exercises in the activities in the study guide available?

As mentioned in this tutorial letter, these are provided in tutorial letter *103*

Is it possible to download the CAI tutorial from the web?

Yes, the following URL is available where you can access the CAI tutorial.
http://osprey.unisa.ac.za/TechnicalReports/cos1501/cos1501.zip

To download the tutorial:

Go to the given web link.
Save cos1501.zip to your computer (Choose C drive *Documents* or wherever you want to save it) and then double click on the saved cos1501.zip.
Choose ***extract*** from top row of buttons on the opened page. Then click on ***extract*** **to the right** of the open window.
Then double click on the cos1501 folder that appears, then double click on the relations.exe icon that looks like a round ball with a red ribbon around it.
You can now navigate through the tutorial.

OR

Enter this URL on the web, then you will be prompted to save the file to your computer. A COS1501 folder is then saved - double click on this folder. Look for the Relations file in the list of files that appear. Then double click on the file “Relations” (note not RELATIONS). You will get the message that files are compressed, so click on the button "extract all", then extract. Now go back to the COS1501 folder and double click on it. The Relations file now has to the left of the word a blue round circle with a red ribbon draped around it. Double click on this file and the tutorial will open.

We have tried these steps without experiencing any problem. Depending on your browser and operating system, there might be a slight variation in these steps. Ask someone experienced with computers to help you if you struggle. Also note that you should have WINZIP already loaded on your computer. Find it free on the internet.

How do I navigate through the tutorial?

The main menu appears at the beginning of the tutorial, providing all the sections in the tutorial. You can choose to do some section and at the end of the section a submenu/menu will appear, providing a choice as to the section of work to do next. A section already completed will be indicated by a red tick. You may do sections in any sequence but we recommend that you do them in the sequence shown. You may repeat sections as often as you like. The red tick showing that a section has already been done can be removed on the main menu.

In order to progress from one screen to another in the tutorial, you should click on one of the buttons that appears at the bottom of the screen. The words on the buttons appear either in white or grey. A white word indicates that the button is active, while the buttons with the grey text are inactive for that particular screen.

Back: Return to the previous screen
Repeat: Re-do the current screen
Exit: Get directly out of the tutorial
Menu: Return to main or submenu
Forward: Continue to the next screen

Problems while running the tutorial:

If the first screen is unattractive and spotty, your screen setting may be too low quality. Change your computer’s display settings to 16-bit or higher.
<u>To change the screen colours:</u> (if not 16-bit or higher)
- Click on the **Start** menu.
- Move your mouse to **Settings**.
- Click on **Control panel**.
- Double click on the **Display** icon.
- Click on the **Settings** tab.
- Click on the **Colours** drop down menu and choose High Color(16-bit) or True Color (32-bit).
- Click on OK.

The bottom part of the screen and left or right of the screen appear to be “missing”. Your computer is probably set at the incorrect resolution. This should be changed.
<u>To change the screen resolution:</u> (if not 800x600)
- Click on the **Start** menu.
- Move your mouse to **Settings**.
- Click on **Control panel**.
- Double click on the **Display** icon.

- Click on the **Settings** tab.
- Adjust the screen area to 800X600 pixels by dragging the arrow.
- Click on OK.

The picture on your screen appears in a window in the centre of the screen with open space around it. Your resolution setting is higher than what is required. It is not essential that you reset it. Reset it if you want a bigger picture on your screen. (See procedure above.)

10 SOURCES CONSULTED

The COS1501 study guide & the CAI tutorial “Relations”.

11 IN CONCLUSION

In your first year you will acquire skills that you will apply in further studies. If you study hard and purposefully, you are on your way to success. We hope that you will enjoy your studies at Unisa. Everything of the best with your studies this year!