If and Only If Proofs

The purpose of an iff proof is to shorten a proof where you need to show that it works both forwards and backwards. Remember the symbol for iff is $\leftrightarrow$.

To do this, you convert the statement into words.

Example

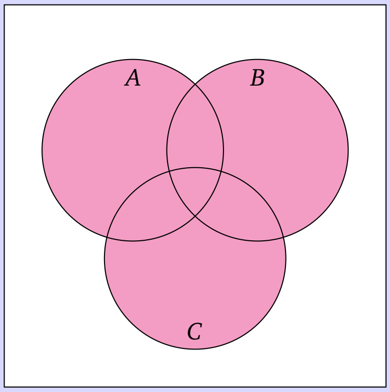
1. **Prove that $A \cup (B \cup C) = (A \cup B) \cup C$ for all sets $A, B, C \subseteq U$.**

   To start off, assume that x is an element of the statement on the left:

   Let $x \in A \cup (B \cup C)$.

   Then start the proof. Convert all the $\cup$ and $\cap$ symbols to words.

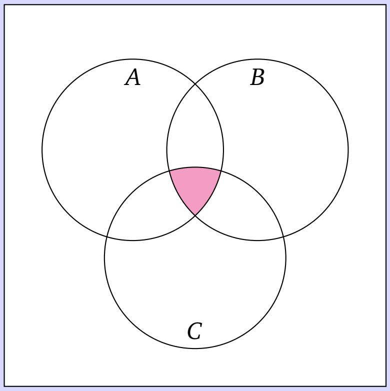
	$x \in A \cup (B \cup C)$
iff	$x \in A$ or $x \in (B \cup C)$
iff	$x \in A$ or $x \in B$ or $x \in C$
iff	$(x \in A$ or $x \in B)$ or $x \in C$
iff	$(x \in (A \cup B))$ or $x \in C$
iff	$x \in (A \cup B) \cup C$
$\therefore$	$A \cup (B \cup C) = (A \cup B) \cup C$

2. **Prove that $A \cap (B \cap C) = (A \cap B) \cap C$ for all sets $A, B, C \subseteq U$.**

   Let $x \in A \cap (B \cap C)$

	$x \in A \cap (B \cap C)$
iff	$x \in A$ and $x \in (B \cap C)$
iff	$x \in A$ and $x \in B$ and $x \in C$
iff	$(x \in A$ and $x \in B)$ and $x \in C$
iff	$(x \in (A \cap B))$ and $x \in C$
iff	$x \in (A \cap B) \cap C$
$\therefore$	$A \cap (B \cap C) = (A \cap B) \cap C$

Using Nots

A basic application of the not symbol swaps $\in$ to $\notin$. If the statement has a $-$ in, this is the equivalent of and $\notin$. For example,

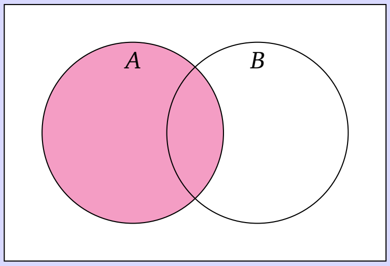
$$x \in A - B = x \in A \text{ and } x \notin B$$

Example

1. **Prove that $(A')' = A$ for all sets $A \subseteq U$.**

   Let $x \in (A')'$

	$x \in (A')'$
iff	$x \notin A'$
iff	x is not $\notin A$
iff	$x \in A$
$\therefore$	$(A')' = A$



The words swap!

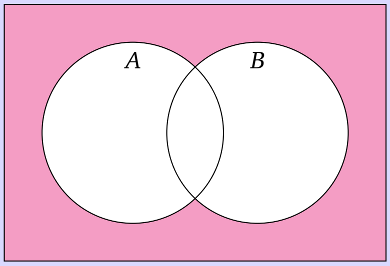
When you apply a $\notin$ sign in words, then $\cup$ means *and* instead of or, and $\cap$ means *or* instead of and.

Example

1. **Prove that $(A \cup B)' = A' \cap B'$. (De Morgan's Theorem)**

   Let $x \in (A \cup B)'$.

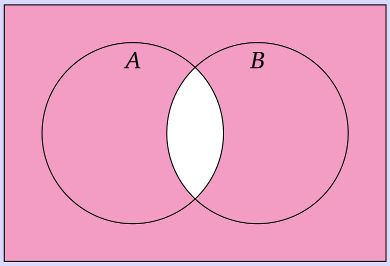
	$x \in (A \cup B)'$	
iff	$x \notin (A \cup B)$	
iff	$x \notin A$ *and* $x \notin B$	(Note the use of and instead of or for $\cup$)
iff	$x \in A'$ and $x \in B'$	
iff	$x \in A' \cap B'$	
$\therefore$	$(A \cup B)' = A' \cap B'$	



2. **Prove that $(A \cap B)' = A' \cup B'$. (De Morgan's Theorem)**

   Let $x \in (A \cap B)'$.

	$x \in (A \cap B)'$	
iff	$x \notin (A \cap B)$	
iff	$x \notin A$ *or* $x \notin B$	(Note the use of or instead of and for $\cap$)
iff	$x \in A'$ or $x \in B'$	
iff	$x \in A' \cup B'$	
$\therefore$	$(A \cap B)' = A' \cup B'$	



Self-Assessment Exercise 6

1. **Prove the following using iff statements. The answers are on the next page.**
   - **(a)** $X - (Y \cap W) = (X - Y) \cup (X - W)$
   - **(b)** $X \cap (Y \cup W) = (X \cap Y) \cup (X \cap W)$

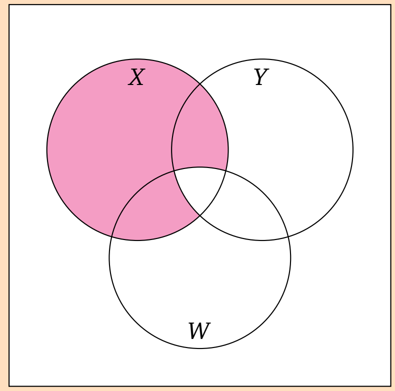
Self-Assessment Exercise 6 Answers

1. Prove the following using iff statements.

(a) $X - (Y \cap W) = (X - Y) \cup (X - W)$

Let $x \in X - (Y \cap W)$.

	$x \in X - (Y \cap W)$
iff	$x \in X$ and $x \notin (Y \cap W)$
iff	$x \in X$ and $(x \notin Y$ or $x \notin W)$
iff	$x \in X$ and $(x \in Y'$ or $x \in W')$
iff	$(x \in X$ and $x \in Y')$ or $(x \in X$ and $x \in W')$
iff	$(x \in (X \cap Y'))$ or $(x \in (X \cap W'))$
iff	$(x \in (X - Y))$ or $(x \in (X - W))$
iff	$x \in (X - Y) \cup (X - W)$
$\therefore$	$X - (Y \cap W) = (X - Y) \cup (X - W)$



(b) $X \cap (Y \cup W) = (X \cap Y) \cup (X \cap W)$

Let $x \in X \cap (Y \cup W)$.

	$x \in X \cap (Y \cup W)$
iff	$x \in X$ and $x \in (Y \cup W)$
iff	$x \in X$ and $(x \in Y$ or $x \in W)$
iff	$(x \in X$ and $x \in Y)$ or $(x \in X$ and $x \in W)$
iff	$(x \in (X \cap Y))$ or $(x \in (X \cap W))$
iff	$x \in (X \cap Y) \cup (X \cap W)$
$\therefore$	$X \cap (Y \cup W) = (X \cap Y) \cup (X \cap W)$