Tutorial letter 201/0/2021

Theoretical Computer Science 1

COS1501

Year module

School of Computing

This tutorial letter contains

a discussion of assignment 01, and

examination information.

Dear Student,

In this tutorial letter the solutions to the first assignment questions are discussed and examination information is provided.

Regards,
COS1501 Team

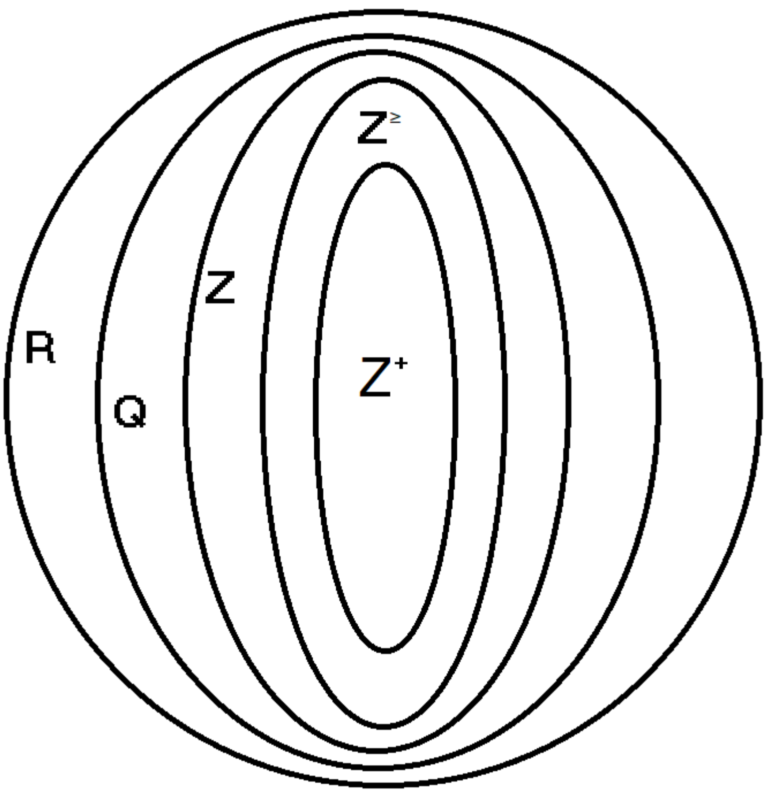
SOLUTIONS ASSIGNMENT 01

Question 1 **Alternative 3**

The different number sets are described in Chapter 1 and 2. Also see the Glossary of symbols in the Introduction chapter of the study guide. In this question we have to determine which one of the alternatives is false.

1. $\mathbb{Z}^{\geq} \subseteq \mathbb{Z}$
2. $\mathbb{Z}^{+} \subseteq \mathbb{Z}^{\geq}$
3. $\mathbb{R} \subseteq \mathbb{Q}$
4. $\mathbb{Z}^{+} \subseteq \mathbb{R}$

$\mathbb{Z} = \{... -4, -3, -2, -1, 0, 1, 2, 3, 4, ...\}$
$\mathbb{C} = \{0, 1, 2, 3, 4, ...\}$
$\mathbb{Z}^{+} = \{1, 2, 3, 4, ...\}$
From the diagram below it is clear that all elements in $\mathbb{Z}^{+}$ is also in the set $\mathbb{Z}^{\geq}$, and all elements in $\mathbb{Z}^{\geq}$ is also in the set $\mathbb{Z}$ etc.



We can therefore say that $\mathbb{Z}^{+} \subseteq \mathbb{Z}^{\geq} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

From here we can see that alternative 3 is false, because Q is in R.

Question 2 **Alternative 4**

Please refer to the correct definition in the study guide Unit 3 p. 43.

Question 3 **Alternative 1**

The set of all non-negative integers **x** less than 16 such that x^2 is an even integer should be described.

Now we can ask the question: For which non-negative integers **x** do we have that x^2 is an even number?

Remember, the required set must include as elements all non-negative integers **x** such that all the requirements for the set are met.

Alternative 1:

The set $\{\mathbf{x} \mid x \in \mathbb{Z}^{\geq}, x < 16, x^2 = 2k \text{ for some } k \in \mathbb{Z}\}$ is the set of all non-negative integers **x** less than 16 such that x^2 is an even number.

All the requirements $x \in \mathbb{Z}^{\geq}$, $x < 16$, $x^2 = 2k$ for some $k \in \mathbb{Z}$ must hold for an integer x to qualify to belong to this set.

Case 1: If $x = 0$ then $x^2 = 0$ and $0 \in \mathbb{Z}^{\geq}$, $0 < 16$, $0^2 = 2(0)$ for some $k = 0 \in \mathbb{Z}$.

Case 2: If $x = 2$ then $x^2 = 4$ and $2 \in \mathbb{Z}^{\geq}$, $2 < 16$, $2^2 = 2(2)$ for some $k = 2 \in \mathbb{Z}$.

And so we can go on to see that for all non-negative integers x = 0, 2, 4, 6, 8, 10, 12, 14 it will be the case that $x^2 = 2k$, $k \in \mathbb{Z}$, and also note that $x < 16$.

If x = 1, 3, 5, 7, 9, 11, 13, 15 then x does not qualify to belong to the required set since x is an odd number and thus x^2 is also an odd number.

Alternative 2: $x = 0$ is not an element of $\{\mathbf{x} \mid x \in \mathbb{Z}^{\geq}, x < 16, x^2 = 2k \text{ for some } k \in \mathbb{Z}^+\}$ since $k \in \mathbb{Z}^+$ thus $k = 0 \notin \mathbb{Z}^+$. This excludes the case where $x^2 = 0^2 = 0 = 2(0)$. It is required that **all** non-negative integers **x** less than 16, such that x^2 is an even integer, should live in the set described in the question statement, but in this case,

$x = 0 \notin \{\mathbf{x} \mid x \in \mathbb{Z}^{\geq}, x < 16, x^2 = 2k \text{ for some } k \in \mathbb{Z}^+\}$.

Alternatives 3 and 4 do not provide the required values for **x** as described in the question statement.

Refer to study guide, pp 3, 7, 11, 35 – 39.

Question 4 **Alternative 4**

It is required to determine the set $\{x \mid x \in \mathbb{Z}, 0 \leq x < 8\} \cap \{x \mid x \in \mathbb{R}, 4 \leq x < 16\}$.

The integers 0, 1, 2, 3, 4, 5, 6 and 7 live in the set $\{x \mid x \in \mathbb{Z}, 0 \leq x < 8\}$.

All real numbers less than 16 and greater than or equal to 4 live in the set $\{x \mid x \in \mathbb{R}, 4 \leq x < 16\}$ which means that integers less than 16 and greater than or equal to 4 also live in this set, ie the integers 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 and 15 live in this set.

The elements common to both $\{x \mid x \in \mathbb{Z}, 0 \leq x < 8\}$ and $\{x \mid x \in \mathbb{R}, 4 \leq x < 16\}$ are 4, 5, 6, and 7. Thus $\{x \mid x \in \mathbb{Z}, 0 \leq x < 8\} \cap \{x \mid x \in \mathbb{R}, 4 \leq x < 16\} = \{4, 5, 6, 7\}$. It should be clear to you that none of the sets provided in alternatives 1, 2 and 3 are equal to the set {4, 5, 6, 7}.

Set intersection: The elements 4, 5, 6 and 7 belong to both $\{x \mid x \in \mathbb{Z}, 0 \leq x < 8\}$ and $\{x \mid x \in \mathbb{R}, 4 \leq x < 16\}$.

Refer to study guide, pp 11, 25, 42.

Consider the following sets, where U represents a universal set:
U = {1, 2, {1}, {2}, {1, 2}}   A = {1, 2, {1}}   B = {{1}, {1, 2}}   C = {2, {1}, {2}}.
Questions 3 to 10 are based on the sets defined above.
NOTE: The Venn diagrams in study unit 4 will help you to understand the definitions in study unit 3.

Question 5 **Alternative 1**

$A \cup B$
$= \{1, 2, \{1\}\} \cup \{\{1\}, \{1, 2\}\}$
$= \{1, 2, \{1\}, \{1, 2\}\}$

Set union: The elements 1, 2, {1} and {1, 2} belong to A or B.

Refer to study guide, p 41.

Question 6 **Alternative 4**

$A \cap C$
$= \{1, 2, \{1\}\} \cap \{2, \{1\}, \{2\}\}$
$= \{2, \{1\}\}$.

Set intersection: The elements 2 and {1} belong to A and C.

Refer to study guide, p 42.

Question 7 **Alternative 1**

A – B is the set:

= $\{1, 2, \{1\}\} - \{\{1\}, \{1, 2\}\}$

= $\{1, 2\}$.

Set difference: The elements 1 and 2 belong to A but not to B.

Refer to study guide, pp 42, 43.

Question 8 **Alternative 3**

B + C

= $\{\{1\}, \{1, 2\}\} + \{2, \{1\}, \{2\}\}$

= $\{2, \{2\}, \{1, 2\}\}$

Set symmetric difference: The elements 2, {2} and {1, 2} belong to B or to C, but not both.

It is also the case that B + C = $(B \cup C) - (B \cap C)$, so

B + C= $(B \cup C) - (B \cap C)$ *Include elements belonging to $B \cup C$ but not to $B \cap C$.*

= $(\{\{1\}, \{1, 2\}\} \cup \{2, \{1\}, \{2\}\}) - (\{\{1\}, \{1, 2\}\} \cap \{2, \{1\}, \{2\}\})$

= $\{2, \{1\}, \{2\}, \{1, 2\}\} - \{\{1\}\}$

= $\{2, \{2\}, \{1, 2\}\}$

Refer to study guide, pp 43, 44.

Question 9 **Alternative 2**

The elements of $\mathcal{P}$(A) are all the subsets of A, so we have to determine the subsets of A.

Let's look at the definition of a subset:
For all sets F and G, F is a subset of G if and only if every element of F is also an element of G. Subsets of G can be formed by ***keeping the outside brackets*** *of G and then throwing away* ***none***, ***one*** *or* ***more*** *elements of G.*

The elements of A = $\{1, 2, \{1\}\}$ are 1, 2 and {1}.
We form the subsets of A:

Throw away no element of set A, then $\{1, 2, \{1\}\} \subseteq A$;
throw away the element 1 of set A, then $\{2, \{1\}\} \subseteq A$;
throw away the element 2 of set A, then $\{1, \{1\}\} \subseteq A$; and
throw away the elements 1 and 2 of set A, then $\{\{1\}\} \subseteq A$, and so we can go on to form all the subsets of A.

If A = $\{1, 2, \{1\}\}$ then the subsets of A, namely $\emptyset$, {1}, {2}, {{1}}, {1, 2}, {1, {1}}, {2, {1}} and {1, 2, {1}} are all the elements of $\mathcal{P}$(A), thus

$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{\{1\}\}, \{1, 2\}, \{1, \{1\}\}, \{2, \{1\}\}, \{1, 2, \{1\}\}\}$.

The set provided in alternative 2 is the set $\mathcal{P}(A)$.

The sets provided in alternatives 1 and 4 are **subsets** of $\mathcal{P}(A)$ but **not equal** to $\mathcal{P}(A)$.

The set in alternative 3 is not a subset of $\mathcal{P}$(A) since

$\{\{1, 2\}\} \in \{\emptyset, \{1\}, \{2\}, \{\{1, 2\}\}, \{1, \{1\}\}, \{2, \{1\}\}, \{1, 2, \{1, 2\}\}\}$ but $\{\{1, 2\}\} \notin \mathcal{P}(A)$.

Refer to study guide, pp 40, 45.

Question 10 **Alternative 3**

$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{\{1\}\}, \{1, 2\}, \{1, \{1\}\}, \{2, \{1\}\}, \{1, 2, \{1\}\}\}$. We consider the sets in the different alternatives:

Alternative 1: {1} is not a subset of $\mathcal{P}(A)$. We provide a counterexample:

$1 \in \{1\}$ but $1 \notin \mathcal{P}(A)$, therefore $\{1\} \nsubseteq \mathcal{P}(A)$.

Alternative 2: $1, 2 \in \{1, 2, \{1, 2\}\}$ but $1, 2 \notin \mathcal{P}(A)$, therefore $\{1, 2, \{1, 2\}\} \nsubseteq \mathcal{P}(A)$.

Alternative 3: If we throw away the elements $\emptyset$, {1}, {2}, {1, 2}, {1, {1}}, {2, {1}} and {1, 2, {1}} of $\mathcal{P}$(A), we are left with the element {{1}} that belong to the subset $\{\{\{1\}\}\}$ of $\mathcal{P}$(A),

thus $\{\{\{1\}\}\} \subseteq \mathcal{P}(A)$.

Note that $\{\{1\}\} \in \{\{\{1\}\}\}$ and $\{\{1\}\} \in \mathcal{P}(A)$ so we can say that each element of $\{\{\{1\}\}\}$ is also an element of $\mathcal{P}$(A) thus $\{\{\{1\}\}\} \subseteq \mathcal{P}(A)$. ({{1}} is the only element of $\{\{\{1\}\}\}$).

Alternative 4: {{2}} is the only element of $\{\{\{2\}\}\}$ but $\{\{2\}\} \notin \mathcal{P}(A)$, therefore $\{\{\{2\}\}\} \nsubseteq \mathcal{P}(A)$.

From the arguments provided we can deduce that alternative 3 should be selected.

Refer to study guide, pp 40, 45.

Question 11 **Alternative 2**

We determine $\mathcal{P}(A) \cap \mathcal{P}(B)$ and $\mathcal{P}(A \cap B)$:

$B = \{\{1\}, \{1, 2\}\}$

Thus $\mathcal{P}(B) = \{\emptyset, \{\{1\}\}, \{\{1, 2\}\}, \{\{1\}, \{1, 2\}\}\}$ and we have already determined that

$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{\{1\}\}, \{1, 2\}, \{1, \{1\}\}, \{2, \{1\}\}, \{1, 2, \{1\}\}\}$.

Only the elements $\emptyset$ and $\{\{1\}\}$ belong to both sets $\mathcal{P}(A)$ and $\mathcal{P}(B)$ thus

$\mathcal{P}(A) \cap \mathcal{P}(B) = \{\emptyset, \{\{1\}\}\}$.

$A \cap B$
$= \{1, 2, \{1\}\} \cap \{\{1\}, \{1, 2\}\}$
$= \{\{1\}\}$

Thus $\mathcal{P}(A \cap B) = \{\emptyset, \{\{1\}\}\} = \mathcal{P}(A) \cap \mathcal{P}(B)$.

From the arguments provided we can deduce that alternative 2 should be selected.

Refer to study guide, pp 40, 42, 45.

Question 12 **Alternative 1**

We have to determine which statement is valid if $x \notin B \cup C$?

If $x \notin B \cup C$ then
$x \in (B \cup C)'$
ie $x \in U - (B \cup C)$.

We determine $U - (B \cup C)$:
$U - (B \cup C)$
$= \{1, 2, \{1\}, \{2\}, \{1, 2\}\} - (\{\{1\}, \{1, 2\}\} \cup \{2, \{1\}, \{2\}\})$
$= \{1, 2, \{1\}, \{2\}, \{1, 2\}\} - \{2, \{1\}, \{2\}, \{1, 2\}\}$
$= \{1\}$

We can now say that if $x \notin B \cup C$ then $x \in \{1\}$.

From the arguments provided we can deduce that alternative 1 should be selected.

Refer to study guide, pp 41 - 43.

Question 13 **Alternative 4**

Given the sets **U = {a, b, c, d, e}, A = {d, e}** and **B = {a, b, c}** we have to determine which one of the alternatives is TRUE. We look at each alternative separately:

Alternative 1: | $\mathcal{P}$ (U) | **= |** $\mathcal{P}$ (A) | + | $\mathcal{P}$ (B)|

We determine **|** $\mathcal{P}$ (U) |, | $\mathcal{P}$ (A) **|** and **|** $\mathcal{P}$ (B) **|** separately. Remember that the symbol **| |** is interpreted as the cardinality of the set that it includes. You can see the definition of set cardinality on p. 44 in study unit 3 of the study guide.

We know that if a set has n elements, then the power set of the set, has 2^n has elements, that is, the cardinality of the power set is 2^n.

The set U has 5 elements, therefore $\mathcal{P}$(U) has 2^5 elements, and therefore we say | $\mathcal{P}$(U) | = 32.

The set A has 2 elements, therefore $\mathcal{P}$(A) has 2^2 elements, and therefore we say | $\mathcal{P}$(A) | = 4.

The set B has 3 elements, therefore $\mathcal{P}$(U) has 2^3 elements, and therefore we say | $\mathcal{P}$(B) | = 8.

It is clear that 32 $\neq$ 4 + 8, therefore alternative 1 is false.

Alternative 2: $\emptyset \subset \mathcal{P}$(A) and $\emptyset \subset \mathcal{P}$(B)

We know that $\emptyset$ is an element of any power set, therefore $\emptyset \in \mathcal{P}$(A) and $\emptyset \in \mathcal{P}$(B). It is however not true that $\emptyset \subset \mathcal{P}$(A) and $\emptyset \subset \mathcal{P}$(B), but $\{\emptyset\} \subset \mathcal{P}$(A) and $\{\emptyset\} \subset \mathcal{P}$(B) is true, i.e. if we want to form a subset containing the element $\emptyset$, we need to put it in curly braces as all subsets formed from a set, are sets containing 0 or more of the elements of the original set. This alternative is therefore false.

Alternative 3: U – A = A – U
U – A = {a, b, c, d, e} – {d, e} = {a, b, c}
A – U = {d, e} – {a, b, c, d, e} = { }
Clearly, U – A $\neq$ A – U

Alternative 3: A + B = U
A + B = {d, e} + {a, b, c} = {a, b, c, d, e} = U.
This alternative is therefore true and should have been selected.

Question 14 **Alternative 1**

Given the sets **U = {a, b, c, d, e}, A = {d, e}** and **B = {a, b, c}** we have to determine which one of the alternatives is FALSE. We look at each alternative separately:

Alternative 1: A′ – U = {d, e}

A′ – U = {a, b, c} – {a, b, c, d, e} = { } $\neq$ {d, e}. This alternative is FALSE and should therefore have been selected.

Alternative 2: (A $\cap$ B) + U = {a, b, c, d, e}

(A $\cap$ B) + U = ({d, e} $\cap$ {a, b, c}) + {a, b, c, d, e} = { } + {a, b, c, d, e} = {a, b, c, d, e} = U.

This statement is therefore true.

Alternative 3: $(U \cap B') = B'$

$(U \cap B') = (\{a, b, c, d, e\} \cap \{a, b, c\}') = (\{a, b, c, d, e\} \cap \{d, e\}) = \{d, e\} = B'$.

This statement is therefore true.

Alternative 4: $(U - A) + B = U - (A + B)$

LHS: $(U - A) + B = (\{a, b, c, d, e\} - \{d, e\}) + \{a, b, c\} = \{a, b, c) + \{a, b, c\} = \{ \}$.
RHS: $U - (A + B) = \{a, b, c, d, e\} - (\{d, e\} + \{a, b, c\}) = \{a, b, c, d, e\} - \{a, b, c, d, e\} = \{ \}$.
Therefore LHS = RHS, and the statement is true.

Question 15
This question did not count as it contained an error.