Tutorial letter 202/1/2019

Theoretical Computer Science 1

COS1501

Semester 1

School of Computing

Discussion of Assignment 02

Dear Student,

The solutions to the second assignment MCQ questions are discussed in this tutorial letter. A discussion of the self-assessment questions is provided in tutorial letter 102. In the exam it will be expected of you to write out the answers to all the questions and provide proofs where required, thus it is very important that you also do all the self-assessment questions. Take note of the hints provided in tutorial letter 101 since these hints will help you to avoid making common errors in the exam.

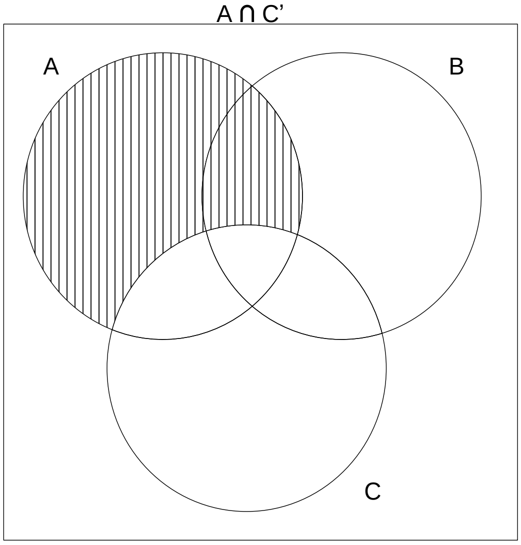
Regards,
COS1501 Team

Discussion of assignment 02, semester 1

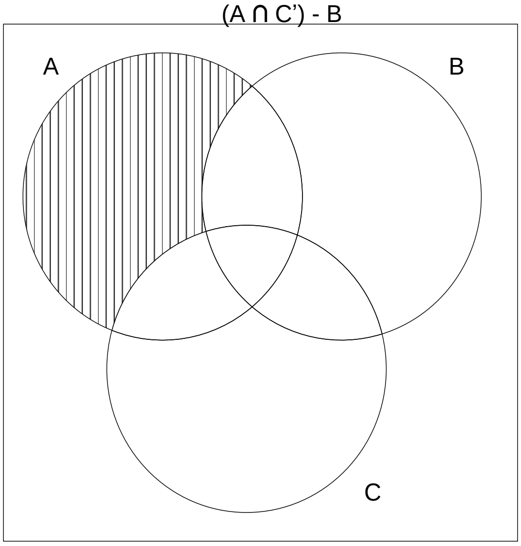
Question 1 **Alternative 3**

We determine the Venn diagram for the set $[(A \cap C') - B] \cup [(A + C) \cap B)]$ step by step:

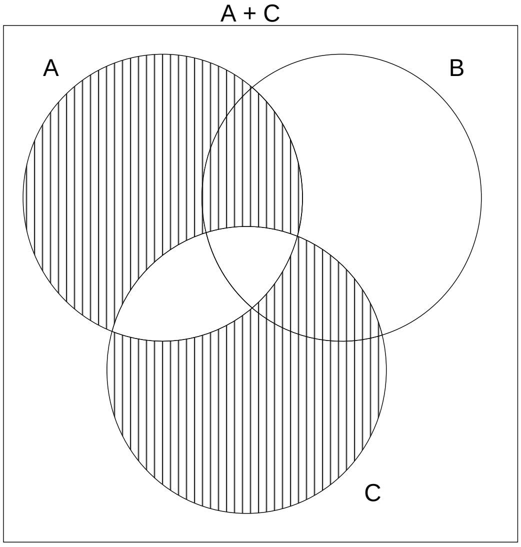
A ∩ C'



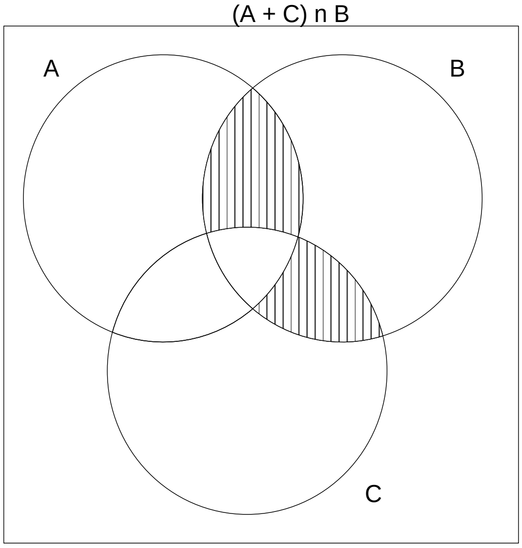
(A ∩ C') - B



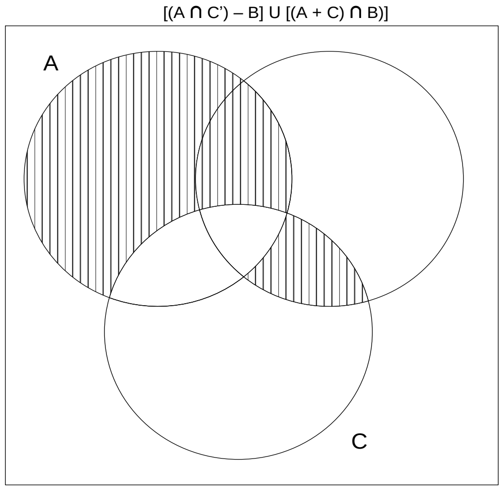
A + C



(A + C) n B



Answer:



Refer to study guide, pp 50, 51.

Question 2 **Alternative 1**

Let A, B and C be subsets of a universal set U = {a, b, c, e}.

The statement (A – B) ∪ C′ = (C′ – B) + A is NOT an identity. Which of the following sets A, B and C can be used in a *counterexample* to prove that the given statement is not an identity?

1. A = {a}, B = {b} & C = {c}
2. A = {a}, B = {a} & C = {b}
3. A = {a, b}, B = {a, b} & C = {c}
4. A = {c}, B = {c, e} & C = {e}

Discussion

Given: A, B, C ⊆ U with U = {a, b, c, e}, and (A – B) ∪ C′ = (C′ – B) + A is not an identity.

*We do **not** start our counterexample solution with (A – B) ∪ C′ ≠ (C′ – B) + A.*

***First** we **determine (A – B) ∪ C′**, **then** we **determine (C′ – B) + A** by using the sets provided in the different alternatives, then we **compare the answers** and come to a conclusion.*

We consider the different alternatives:

1. We use the sets A = {a}, B = {b} & C = {c} to determine (A – B) ∪ C′ and (C′ – B) + A then we compare the answers. Note that we use **curly brackets** for **sets**.

(A – B) ∪ C′ = [{a} – {b}] ∪ {c}′
= {a} ∪ {a, b, e}
= {a, b, e}

(C′ – B) + A = [{c}′ – {b}] + {a}
= [{a, b, e} – {b}] + {a}

$= \{a, e\} + \{a\}$
$= \{e\}$

Clearly $\{a, b, e\} \neq \{e\}$ thus $(A - B) \cup C' \neq (C' - B) + A$.

2. We use the sets $A = \{a\}$, $B = \{a\}$ & $C = \{b\}$ to determine $(A - B) \cup C'$ and $(C' - B) + A$ then we compare the answers.

$$\begin{aligned}
(A - B) \cup C' &= [\{a\} - \{a\}] \cup \{b\}' \\
&= \{\ \} \cup \{a, c, e\} \\
&= \{a, c, e\} \\
(C' - B) + A &= [\{b\}' - \{a\}] + \{a\} \\
&= [\{a, c, e\} - \{a\}] + \{a\} \\
&= \{c, e\} + \{a\} \\
&= \{a, c, e\}
\end{aligned}$$

Thus $(A \cap B) - C' = (C' - B) + A$.

3. We use the sets $A = \{a, b\}$, $B = \{a, b\}$ & $C = \{c\}$ to determine $(A - B) \cup C'$ and $(C' - B) + A$ then we compare the answers.

$$\begin{aligned}
(A - B) \cup C' &= [\{a, b\} - \{a, b\}] \cup \{c\}' \\
&= \{\ \} \cup \{a, b, e\} \\
&= \{a, b, e\} \\
(C' - B) + A &= [\{c\}' - \{a, b\}] + \{a, b\} \\
&= [\{a, b, e\} - \{a, b\}] + \{a, b\} \\
&= \{e\} + \{a, b\} \\
&= \{a, b, e\}
\end{aligned}$$

Thus $(A - B) \cup C' = (C' - B) + A$.

4. We use the sets $A = \{c\}$, $B = \{c, e\}$ & $C = \{e\}$ to determine $(A - B) \cup C'$ and $(C' - B) + A$ then we compare the answers.

$$\begin{aligned}
(A - B) \cup C' &= [\{c\} - \{c, e\}] \cup \{e\}' \\
&= \{\ \} \cup \{a, b, c\} \\
&= \{a, b, c\} \\
(C' - B) + A &= [\{e\}' - \{c, e\}] + \{c\} \\
&= [\{a, b, c\} - \{c, e\}] + \{c\} \\
&= \{a, b\} + \{c\} \\
&= \{a, b, c\}
\end{aligned}$$

Thus $(A - B) \cup C' = (C' - B) + A$.

Alternatives 2, 3 and 4 do not provide counterexamples, but a counterexample is provided in alternative 1, thus this alternative should be selected.

Refer to study guide, pp 60, 61.

Question 3 **Alternative 2**

We want to prove that for all A, B, C $\subseteq$ U,

$$A \cup [(A \cap B) - C] = A \cap (A \cup B) \cap (A \cup C') \text{ is an identity.}$$

Discussion:

In the proof we apply the definitions for union, intersection, difference and complement of sets. The notation should be correct and all the necessary steps should appear in the proof.

We complete the proof by including the correct steps 3 & 5 from alternative 2:

$z \in A \cup [(A \cap B) - C]$

iff $z \in A$ or $[(z \in (A \cap B)$ and $z \notin C]$

iff $\boldsymbol{z \in A}$ **or** $\boldsymbol{[(z \in (A \cap B)}$ **and** $\boldsymbol{z \in C']}$

iff $z \in A$ or $[z \in A$ and $z \in B$ and $z \in C']$

iff $\boldsymbol{(z \in A}$ **or** $\boldsymbol{z \in A)}$ **and** $\boldsymbol{(z \in A}$ **or** $\boldsymbol{z \in B)}$ **and** $\boldsymbol{(z \in A}$ **or** $\boldsymbol{z \in C')}$

iff $(z \in A)$ and $(z \in A$ or $z \in B)$ and $(z \in A$ or $z \in C')$

iff $(z \in A)$ and $z \in (A \cup B)$ and $z \in (A \cup C')$

iff $z \in A \cap (A \cup B) \cap (A \cup C')$

Refer to study guide, pp 41-43, 55-57.

Question 4 **Alternative 4**

52 pupils have to make a flag containing one or more of the colours red, white and blue.
Of these pupils,

> 20 use red in their flags,
> 31 use white in their flags, and
> 20 use blue in their flags.
> (pupils do not necessarily use only use one colour)
> Furthermore,
> 6 use red and white,
> 10 use red and blue,
> 5 use white and blue.
> (pupils do not necessarily use only use two colours)
>
> How many pupils use all three colours in their flags?

Solution:

$|U| = 52$, $|R| = 20$, $|B| = 20$, $|W| = 31$, (U = universal set; R = red; B = blue;)

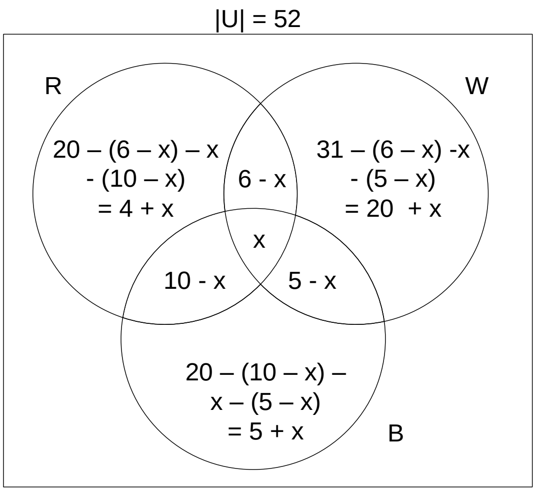
$|R \cap B| = 10$,

$|R \cap W| = 6$,

$|B \cap W| = 5$,

$|R \cap W \cap B| = x$, ie let ***x*** students use all three colours.

Now we can fill in the various regions in the following Venn diagram.
We initially fill in x for $|R \cap W \cap B|$.



$|U| = 52 = 4 + x + 6 - x + x + 10 - x + 5 - x + 20 + x + 5 + x$

ie $52 = 50 + x$

ie $x = 2$, ie 2 pupils use all three colours in their flags.

From the argument provided we can deduce that alternative 4 should be selected.

Refer to study guide, pp 63 – 66.

Let T be a relation from A = {0, 1, 2, 3} to B = {0, 1, 2, 3, 4} such that

(a, b) $\in$T iff $2b^2 - a^2$ is an odd number. (A, B $\subseteq$ U = $\mathbb{Z}$.)

(Hint: Write down all the elements of T, for example, if 4 $\in$ B and 1 $\in$ A then $(2)4^2 - 1^2 = 32 - 1 = 31$ which is an odd number, thus (1, 4) $\in$ T.)

Answer questions 5 and 6 by using the defined relation T.

Question 5 **Alternative 3**

Which one of the following alternatives provides only elements belonging to T?

1. (3, 1), (4, 1), (3, 2)
2. (0, 1), (2, 4), (2, 3)
3. (3, 0), (1, 2), (3, 4)
4. (1, 0), (1, 2), (0, 1)

We consider ordered pairs provided in the different alternatives:

1. Is (4, 1) $\in$ T? No, 4 $\notin$ A, so (4, 1) cannot be an ordered pair in T.
Since (4, 1) $\notin$ T, this alternative does not provide only elements belonging to T.

2. Is (2, 4) $\in$ T? No, $2(4)^2 - 2^2 = 28$ which is an even number thus (2, 4) $\notin$ T.
Since (2, 4) $\notin$ T, this alternative does not provide only elements belonging to T.

3. Is (3, 0) $\in$T? Yes, $2(0)^2 - 3^2 = –9$ which is an odd number thus (3, 0) $\in$ T.
Is (1, 2) $\in$T? Yes, $2(2)^2 - 1^2 = 7$ which is an odd number thus (1, 2) $\in$ T.
Is (3, 4) $\in$T? Yes, $2(4)^2 - 3^2 = 23$ which is an odd number thus (3, 4) $\in$ T.
Thus all the given ordered pairs in this alternative are elements of T.

4. Is (0, 1) $\in$ T? No, $2(1)^2 - 0^2 = 2$ which is an even number thus (0, 1) $\notin$ T.
Since (0, 1) $\notin$ T, this alternative does not provide only elements belonging to T.

In a similar way you can determine that
T = {(1, 0), (1, 1), (1, 2), (1, 3), (1, 4), (3, 0), (3, 1), (3, 2), (3, 3), (3, 4)}

From the arguments provided we can deduce that alternative 3 should be selected.

Refer to study guide, p 73.

Question 6 **Alternative 1**

Which one of the following statements regarding the relation T is true?

1. T is transitive.
2. T is symmetric.
3. T is antisymmetric.
4. T is irreflexive.

Discussion

We first provide definitions using some relation R on A:

Irreflexive:

We ask the question: Is it true that for all $x \in A$ that we have $(x, x) \notin R$?

(For ***no*** *element $x \in A$ we have that $(x, x) \in R$.)*

Symmetric: We ask the question: Is it true that for all $x, y \in A$, if $(x, y) \in R$ then $(y, x) \in R$?

Antisymmetric: We ask the question: Is it true that for all $x, y \in A$,
if $x \neq y$ and $(x, y) \in R$ then $(y, x) \notin R$?

Transitive: R is transitive iff it has the property that for all $x, y, z \in A$,
whenever $(x, y) \in R$ and $(y, z) \in R$, then $(x, z) \in R$.
We ask the question: Is it true that for all $x, y, z \in A$, if $(x, y) \in R$ and $(y, z) \in R$, then $(x, z) \in R$?

T is a relation from $A = \{0, 1, 2, 3\}$ to $B = \{0, 1, 2, 3, 4\}$ such that
$(a, b) \in T$ iff $2b^2 - a^2$ is an odd number. We provide the set T:
$T = \{(1, 0), (3, 0), (1, 1), (3, 1), (1, 2), (3, 2), (1, 3), (3, 3), (1, 4), (3, 4)\}$

We consider the different alternatives:

1. If you test all the ordered pairs for transitivity, you will see that T is indeed transitive. We leave this as an exercise for you to do. Alternative 1 should therefore be selected.

2. We provide a counterexample to prove that T is *not symmetric*:
   $(1, 2) \in T$ but $(2, 1) \notin T$.

3. We provide a counterexample to prove that T is *not antisymmetric*:
   $(1, 3) \in T$ and $(3, 1) \in T$.

4. We provide a counterexample to prove that T is *not irreflexive*:
   $(1, 1) \in T$ and $(3, 3) \in T$.

From the arguments provided it is clear that alternative 1 should be selected.

Refer to study guide, pp 75-78.

Consider the following relation on the set B = {1, b, {1}, {b}, {1, b}}:

P = {(1, b), (b, {1, b}), ({1, b}, 1), ({b}, 1), (1, {1})}.

Answer questions 7 to 10 by using the given relation P and the set B.

Question 7 **Alternative 1**

Which one of the following alternatives represents the range of P (ran(P))?

1. {1, b, {1}, {1, b}}
2. {1, b, {1}, {b}, {1, b}}
3. {1, b, {b}, {1, b}}
4. {1, b, {1, b}}

Let's first look at the definition for the range of a function:
Given a function T from X to Y, the range of T is defined by:

ran(T) = {y **|** *for some x ∈ X, (x, y) ∈ T}, ie the set of second coordinates.*

We see that only the elements 1, b, {1}, {1, b} appear as second co-ordinates, thus alternative 1 provides the range of P, ie ran(P) = {1, b, {1}, {1, b}}.

Question 8 **Alternative 3**

Which one of the following relations represents the composition relation P ○ P (ie P; P)?

1. {(1, {1, b}), (b, 1), ({1, b}, 1), ({b}, {1}), (1, {1})}
2. {(1, {1, b}), (b, 1), ({1, b}, 1), ({b}, {1})}
3. {(1, {1, b}), (b, 1), ({1, b}, {1}), ({1, b}, b), ({b}, b), ({b}, {1})}
4. {(1, {1, b}), (b, 1), ({1, b}, {1}), ({1, b}, b), ({b}, {1})}

Discussion
We first look at the definition of a composition relation:
Given relation **P** *from B to B and P from B to B, the composition of* **P** *followed by P*

(P ○ **P** *or* **P***; P) is the relation from B to B defined by*

P ○ **P** = **P**; *P = {(m, o)* **|** *there is some n ∈ B such that (m, n) ∈* **P** *and (n, o) ∈ P}.*

(P and **P** *is exactly the same relation, but for the purpose of our explanations we make the subtle distinction.)*

P = {(1, b), (b, {1, b}), ({1, b}, 1), ({b}, 1), (1, {1})} is defined on B = {1, b, {1}, {b}, {1, b}}.

To determine **P**; *P* we start with the pair (1, **b**) of **P**, and then we look for a pair in *P* that has as first co-ordinate an **b**, and then see where it takes us.

Link (1, **b**) of **P** with (**b**, {1, b}) of *P*, then (1, {1, b}) ∈ **P**; *P*.

Link (b, {**1, b**}) of **P** with ({**1, b**}, 1) of *P*, then (b, 1) ∈ **P**; *P*.

Link ({1, b}, 1) of **P** with (**1,** {1}) of *P*, then ({1, b}, {1}) ∈ **P**; *P*.

Link ({1, b}, 1) of **P** with (**1,** b) of *P*, then ({1, b}, b) ∈ **P**; *P*.

Link ({b}, 1) of **P** with (**1,** b) of *P*, then ({b}, b) ∈ **P**; *P*.

Link ({b}, 1) of **P** with (**1,** {1}) of *P*, then ({b}, {1}) $\in$ **P**; *P*.
No other pairs can be linked, so
P ○ P = {(1, {1, b}), (b, 1), ({1, b}, {1}), ({1, b}, b), ({b}, b), ({b}, {1})}

Refer to study guide, pp 79, 108, 109.

Question 9 **Alternative 3**

The relation P does not satisfy trichotomy. Which ordered pairs can be included in P so that an extended relation P_1 (say) would satisfy trichotomy?

1. (b, {1}), (b, {b}), (b, 1), ({1}, {1, b}) & ({1, b}, {b})
2. ({1}, b), (b, {b}), ({b}, {1, b}) & ({1, b}, {1})
3. (b, {1}), ({b}, b), ({1}, {b}), ({b}, {1, b}) & ({1, b}, {1})
4. (b, {1}), (b, {b}), ({1}, {b}) & ({1, b}, {1, b})

We ask the question: Which ordered pairs should be included in P_1 so that it will be true that for ***all*** *x, y $\in$ B with* ***x ≠ y****, we have* ***(x, y) $\in$ P_1 or (y, x) $\in$ P_1****?*

P = {(1, b), (b, {1, b}), ({1, b}, 1), ({b}, 1), (1, {1})} is defined on B = {1, b, {1}, {b}, {1, b}}.

We compare the elements of B: b ≠ {1}, {b} ≠ b, {1} ≠ {b}, {b} ≠ {1, b} & {1, b} ≠ {1} but these elements are not grouped in ordered pairs of P.
We only have (1, b), (b, {1, b}), ({1, b}, 1), ({b}, 1), (1, {1}) $\in$ P. We can include the ordered pairs (b, {1}), ({b}, b), ({1}, {b}), ({b}, {1, b}) & ({1, b}, {1}) in P_1 which will then satisfy trichotomy.

P_1 = {(1, b), (b, {1, b}), ({1, b}, 1), ({b}, 1), (1, {1}), (b, {1}), ({b}, b), ({1}, {b}), ({b}, {1, b}), ({1, b}, {1})} satisfies trichotomy.

From the arguments provided we can deduce that alternative 3 should be selected.

Refer to study guide, p 78.

Question 10 **Alternative 1**

Which one of the following sets is a partition of B = {1, b, {1}, {b}, {1, b}}?

1. {**{**1, b, {1}, {b}**}**, **{**{1, b}**}**}
2. {{1}, {b}, {1, b}}
3. {**{**1, b, {1}**}**, **{**{1}, {b}, {1, b}**}**}
4. {1, b, {1}, {b}, {1, b}}

Discussion
Refer to the definition of a partition provided in the question. We test the sets provided in each alternative against this definition:

1. Let P = $\{$**{**1, b, {1}, {b}**}**, **{**{1, b}**}**$\}$ (say).

   We test whether P is a partition of B = {1, b, {1}, {b}, {1, b}}:

   a. **{**1, b, {1}, {b}**}** and **{**{1, b}**}** are two **non-empty** subsets of B,
   b. **{**1, b, {1}, {b}**}** ∩ **{**{1, b}**}** = ∅, and
   c. **{**1, b, {1}, {b}**}** ∪ **{**{1, b}**}** = B.

   Since P has all the necessary properties, it is a partition of B.

2. Let T = $\{${1}, {b}, {1, b}$\}$ (say).
   We test whether T is a partition of B = {1, b, {1}, {b}, {1, b}}:

   a. {1}, {b} and {1, b} are **non-empty** subsets of B,
   b.{1} ∩ {b} ∩ {1, b} = ∅, but
   c.{1} ∪ {b} ∪ {1, b} = {1, b} ≠ B.

   Since T does not have all the necessary properties, it is not partition of B.

3. Let M = $\{$**{**1, b, {1}**}**, **{**{1}, {b}, {1, b}**}**$\}$ (say).
   We test whether M is a partition of B = {1, b, {1}, {b}, {1, b}}:

   a.**{**1, b, {1}**}** and **{**{1}, {b}, {1, b}**}** are two **non-empty** subsets of U, but
   b.**{**1, b, {1}**}** ∩ **{**{1}, {b}, {1, b}**}** = **{**{1}**}** ≠ ∅.
   c. We do have that **{**1, b, {1}**}** ∪ **{**{1}, {b}, {1, b}**}** = B = {1, b, {1}, {b}, {1, b}} but
   since M does not have all the necessary properties, it is not partition of B.

4. Let N = $\{$1, b, {1}, {b}, {1, b}$\}$ (say).
   We test whether N is a partition of B = {1, b, {1}, {b}, {1, b}}:

   a. The elements 1, b, {1}, {b} and {1, b} of N are not **subsets** of B, but rather **elements** of B.
   Since M does not have all the necessary properties, it is not partition of B.

From the arguments provided we can deduce that alternative 1 should be selected.

Refer to study guide, pp 94, 95.

Suppose U = {1, 2, 3, a} is a universal set with the subset A = {2, 3, a}.
Let R = { (2, 2), (2, a), (3, 3), (2, 3), (a, a) } be a relation on A.
Answer questions 11 & 12 by using the given sets A, U and the relation R.

Question 11 **Alternative 2**

Which one of the following statements regarding relation **R** is true?

1. R is a strict partial order.
2. R is a weak partial order.
3. R is a weak total order.
4. R is an equivalence relation.

The relations mentioned in alternatives 1-3 should have different properties which includes *antisymmetry*:
A weak total order is reflexive, *antisymmetric* and transitive, and satisfies trichotomy;
A strict partial order is irreflexive, *antisymmetric* and transitive; and
A weak partial order is reflexive *antisymmetric* and transitive.

R is **reflexive** because it contains the ordered pairs (2, 2), (3, 3) and (a, a). Looking only at this fact, R cannot be a strict partial order because it is reflexive. Therefore alternative 1 cannot be chosen.

R cannot be a weak total order, because R does not satisfy trichotomy. To satisfy trichotomy, each element in A must pair up with each other element in A in an ordered pair. Elements a and 3 do not pair up in any way, because neither (a, 3) nor (3, a) is in A. Therefore alternative 3 cannot be chosen.

Although R does not satisfy trichotomy, it is indeed reflexive, antisymmetric and transitive. It is therefore a weak partal order. Alternative 2 should therefore be chosen.

Furthermore, R is **not** an equivalence relation since it is not symmetric. We provide a counterexample:
(2, a) $\in$ R but (a, 2) $\notin$ R.

From the above discussions it is clear that alternative 2 should be selected.

Refer to study guide, pp 75 – 78, 88.

Question 12 **Alternative 4**

By adding the ordered pair(s) in only one of the alternatives below to R, R will become a relation that satisfies trichotomy. Which alternative should be chosen?

1. (3, 2)
2. (a, 2)
3. (3, 2 and (a, 2)
4. (a, 3)

Discussion

We have already shown in Question 11, that R does not satisfy trichotomy, because neither ordered pair (a, 3) not (3, a) is in R. Only one of these pairs needs to be added to satisfy trichotomy – it does not matter which one. Alternative 4 should therefore be chosen.

Refer to study guide, pp 75 – 78, 88.

Let R be the relation on $\mathbb{Z}^{\geq}$ (the set of integers) defined by

$(x, y) \in R$ iff $x^2 + y^2 = 2k$ for some integers $k \geq 0$.

Answer questions 13 to 15 by using the given relation R.

Question 13 **Alternative 3**

Which one of the following is NOT an ordered pair in R?

1. (0, 0)
2. (2, 4)
3. (3, 2)
4. (5, 7)

Relation R on $\mathbb{Z}^{\geq}$ is defined by $(x, y) \in R$ iff $x^2 + y^2 = 2k$ for some integers $k \geq 0$.
We consider the ordered pairs provided in the different alternatives:

1. Let x = 0 and y = 0 then
   $0^2 + 0^2 = 0$ which is a multiple of 2 (ie 0 x 2 = 0).
   thus $(0, 0) \in R$.

2. Let x = 2 and y = 4 then
   $2^2 + 4^2 = 20$ which is a multiple of 2.
   thus $(5, 7) \in R$.

3. Let x = 3 and y = 2 then
   $3^2 + 2^2 = 13$ which is **not** a multiple of 2.
   thus $(3, 2) \notin R$.

4. Let x = 5 and y = 7 then
   $5^2 + 7^2 = 74$ which is a multiple of 2.
   thus $(5, 7) \in R$.

From the arguments provided we can deduce that alternative 3 should be selected.

Refer to study guide, pp 71-73.

Question 14 **Alternative 2**

R is symmetric. Which one of the following is a valid proof showing that R is symmetric?

1. Let x, y $\in \mathbb{Z}^{\geq}$ be given.
   Suppose (x, y) $\in$ R
   then $x^2 + y^2 = 2k$ for some $k \geq 0$.
   ie $y^2 + x^2 = 2k$ for some $k \geq 0$.
   thus (x, y) $\in$ R.

2. Let x, y $\in \mathbb{Z}^{\geq}$ be given.
   Suppose (x, y) $\in$ R
   then $x^2 + y^2 = 2k$ for some $k \geq 0$.
   ie $y^2 + x^2 = 2k$ for some $k \geq 0$.
   thus (y, x) $\in$ R.

3. Let x, y $\in \mathbb{Z}^{\geq}$ be given.
   Suppose (x, y) $\in$ R
   then $x^2 + y^2 = 2k$ for some $k \geq 0$.
   thus (y, x) $\in$ R.

4. Let x, y $\in \mathbb{Z}^{\geq}$ be given.
   Suppose (x, x) $\in$ R
   then $x^2 + y^2 = 2k$ for some $k \geq 0$.
   ie $y^2 + x^2 = 2k$ for some $k \geq 0$.
   thus (y, y) $\in$ R.

Discussion

Refer to the definition of symmetry provided previously.

Alternative 1: The conclusion "(x, y) $\in$ R" states the same as the supposition: (x, y) $\in$ R". This is not a valid proof.

Alternative 2 provides a valid proof:

Let x, y $\in \mathbb{Z}^{\geq}$ be given.
Suppose (x, y) $\in$ R
then $x^2 + y^2 = 2k$ for some $k \geq 0$.
ie $y^2 + x^2 = 2k$ for some $k \geq 0$.
thus (y, x) $\in$ R.

Alternative 3: In a proof each step must follow logically from the previous step. In this proof the step "$y^2 + x^2 = 2k$" is omitted.

Alternative 4: The first step "(x, x) $\in$ R" does not relate to the definition of symmetry.

Refer to study guide, p 76.

Question 15 **Alternative 4**

R is not antisymmetric. Which of the following ordered pairs can be used together in a counterexample to prove that R is **not** antisymmetric? (Remember that R is defined on $\mathbb{Z}^{\geq}$.)

1. (-3, 1) & (1, –3)
2. (5, 3) & (3, 15)
3. (4, 7) & (7, 4)
4. (3, 1) & (1, 3)

Discussion

*The definition for antisymmetry requires that the mirror images of ordered pairs (x, y) with $x \neq y$ may not appear in a relation. In a counterexample we will show that $x \neq y$ and $(x, y) \in R$ but then it is also the case that $(y, x) \in R$, ie R is **not** antisymmetric.*

We discuss the different alternatives:

1. Firstly we must ask the question: Are (-3, 1) and (1, -3) elements of R? No, since R is defined on $\mathbb{Z}^{\geq}$. Thus these ordered pairs cannot be used in a counterexample to prove that R is not antisymmetric.

2. The ordered pairs (5, 3) & (3, 15) are both elements of R (we have that the sums $5^2 + 3^2$ and $3^2 + 15^2$ are multiples of 2), but these ordered pairs cannot be used in a counterexample to prove that R is not antisymmetric since these pairs are not mirror images of each other.

3. The ordered pairs (4, 7) & (7, 4) are not elements of R (we have $4^2 + 7^2 = 7^2 + 4^2 = 65$ which is not a multiple of 2), thus these ordered pairs cannot be used in a counterexample to prove that R is not antisymmetric.

4. In ordered pairs (3, 1) & (1, 3) are elements of R (we have $3^2 + 1^2 = 1^2 + 3^2 = 4$ which is a multiple of 2). It is the case that $1 \neq 3$ and $(3, 1) \in R$ but the mirror image (1, 3) is also an element of R, thus R is not antisymmetric.

From the arguments provided alternative 4 should be selected as the correct one.

Refer to study guide, pp 76, 77.