Discussion Class: COS1501

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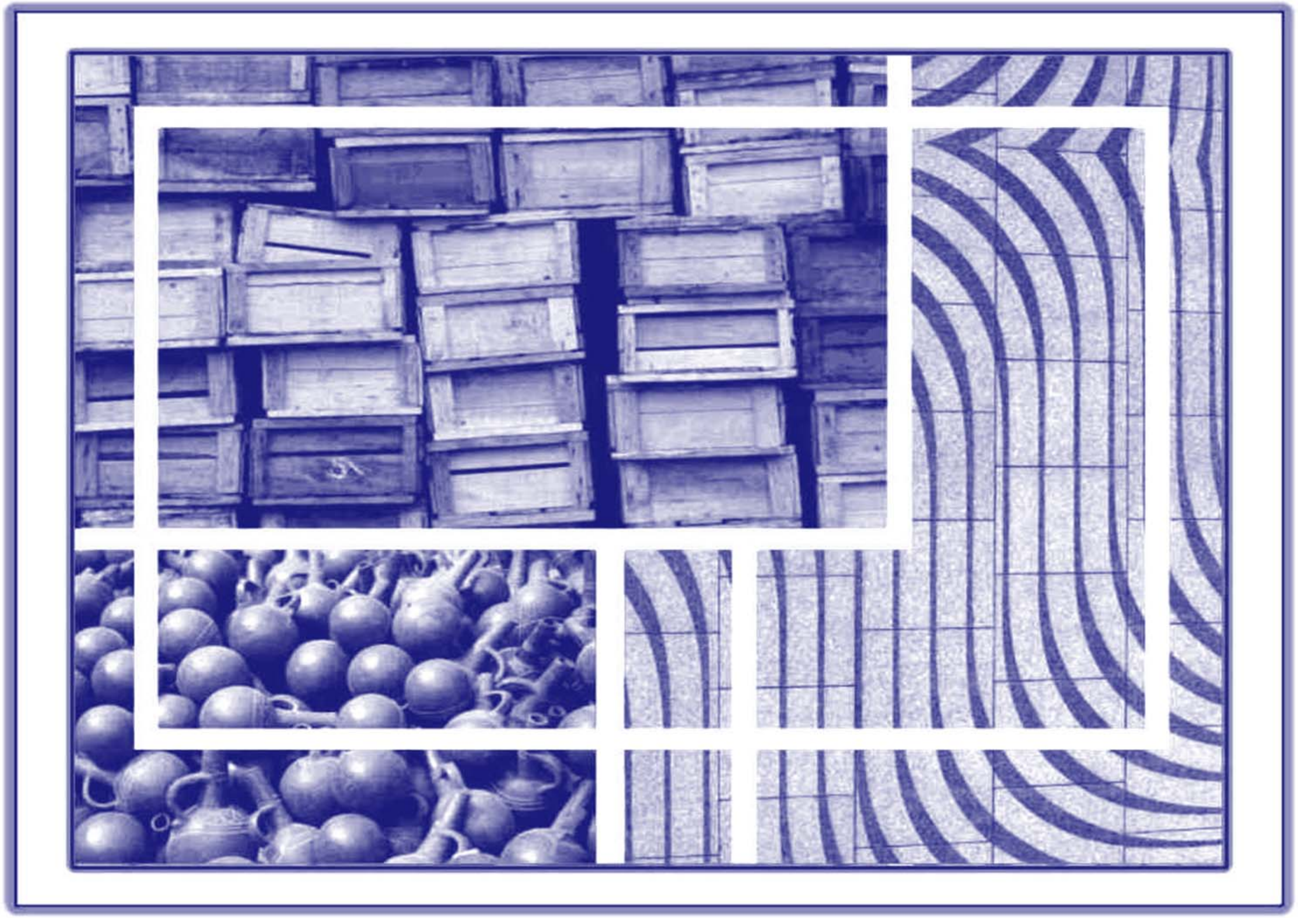
COS1501

THEORETICAL COMPUTER SCIENCE 1



SCHOOL OF COMPUTING

CONTENTS

- **Sets: Question 1; study units 3, 4**
- ***Relations: Question 2; study units 5, 6***
- ***Functions: Question 3; study units 6.5, 7***
- **Operations: Question 4; study unit 8**
- ***Logic: Question 5; study units 9, 10***
- **Mixed concepts: Question 6**

SETS *(Study Guide, pp. 40 - 43)*

Subset: $A \subseteq B$ every element of A also element of B

Form subsets: throw away some element(s) from B

Union: $A \cup B = \{x \mid x \in A$ or $x \in B\}$

Intersection: $A \cap B = \{x \mid x \in A$ and $x \in B\}$

Difference: $A - B = \{x \mid x \in A$ and $x \notin B\}$

Complement: $A' = \{x \mid x \in U$ and $x \notin A\}$

Symm. diff.: $A + B = \{x \mid x \in A$ or $x \in B$, but not both$\}$

$$A + B = (A \cup B) - (A \cap B)$$

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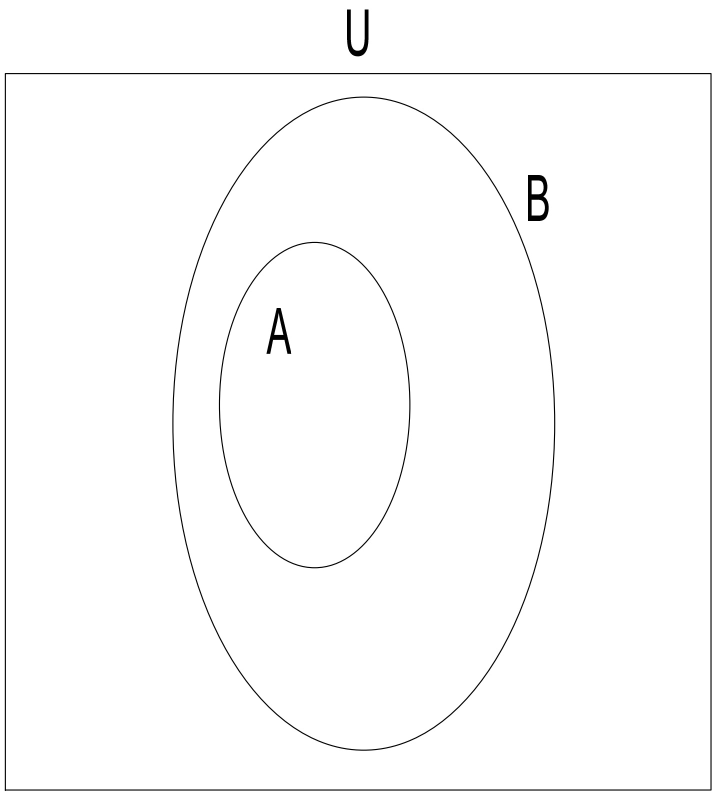
SUBSETS *(Study Guide, pp. 40, 44)*

Subset: $A \subseteq B$
Proper subset: $A \subset B$



Example:

Let $C = \{\varnothing, \{a\}\}$
2 elements in C namely $\varnothing$ and $\{a\}$
Cardinality: $|C| = 2$

Form subsets of C:

Throw away:

- **both elements to form subset $\{\ \}$;**
- **element $\varnothing$ to form subset $\{\{a\}\}$;**
- **element $\{a\}$ to form subset $\{\varnothing\}$;**
- **no element then $C \subseteq C$.**

Subsets of C: $\{\ \}$, $\{\{a\}\}$, $\{\varnothing\}$ and C.

Powerset:
$\boldsymbol{P}(C) = \{\{\ \}, \{\{a\}\}, \{\varnothing\}, C\}$

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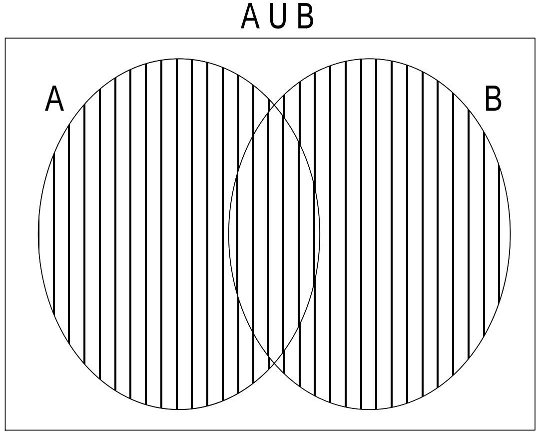
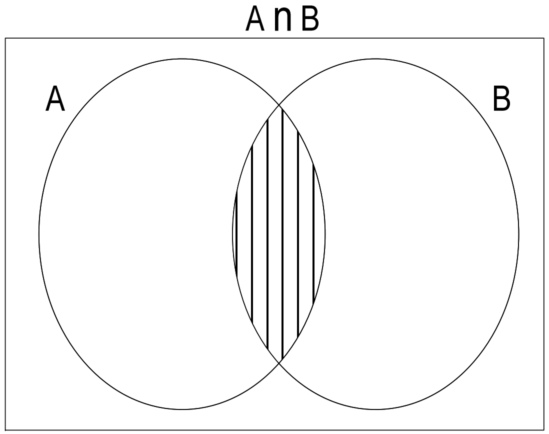
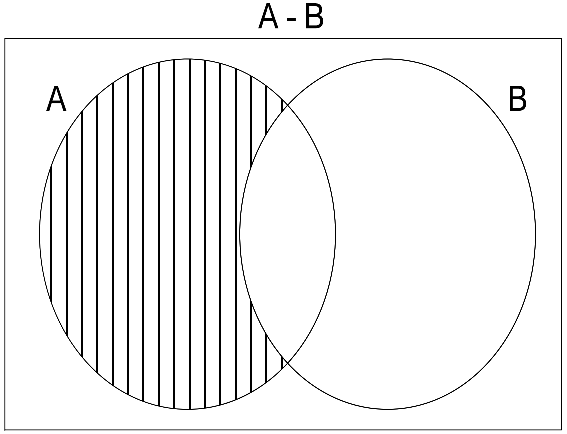
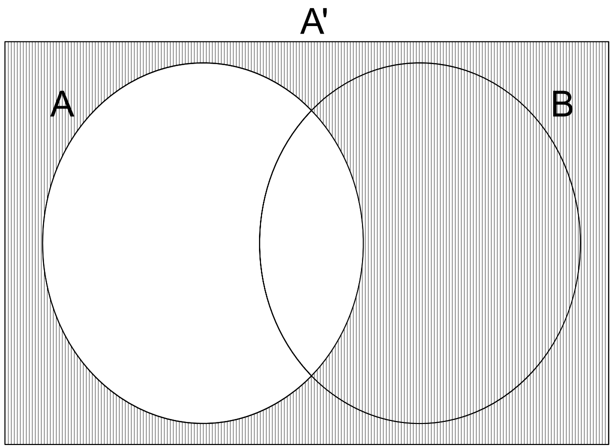
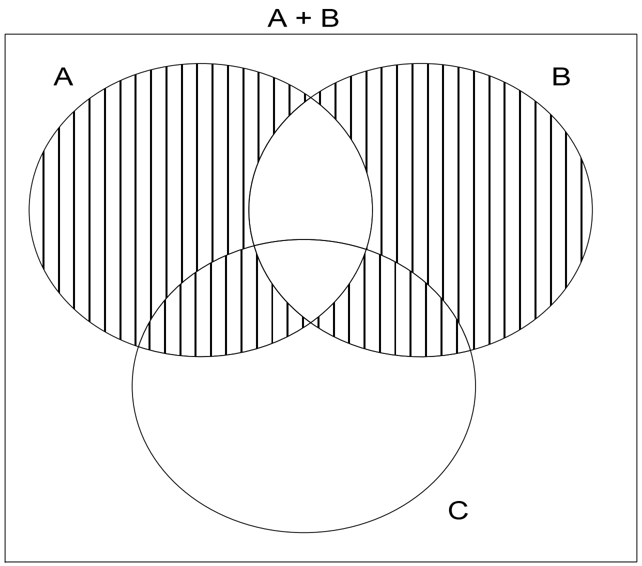
VENN DIAGRAMS *(Study Guide, pp. 48 - 51)*











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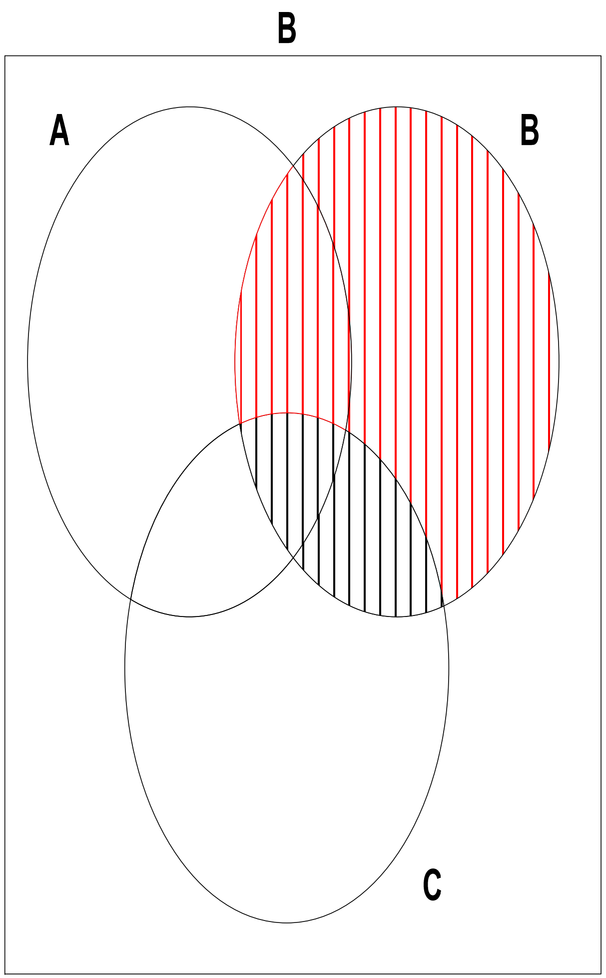
Question 1

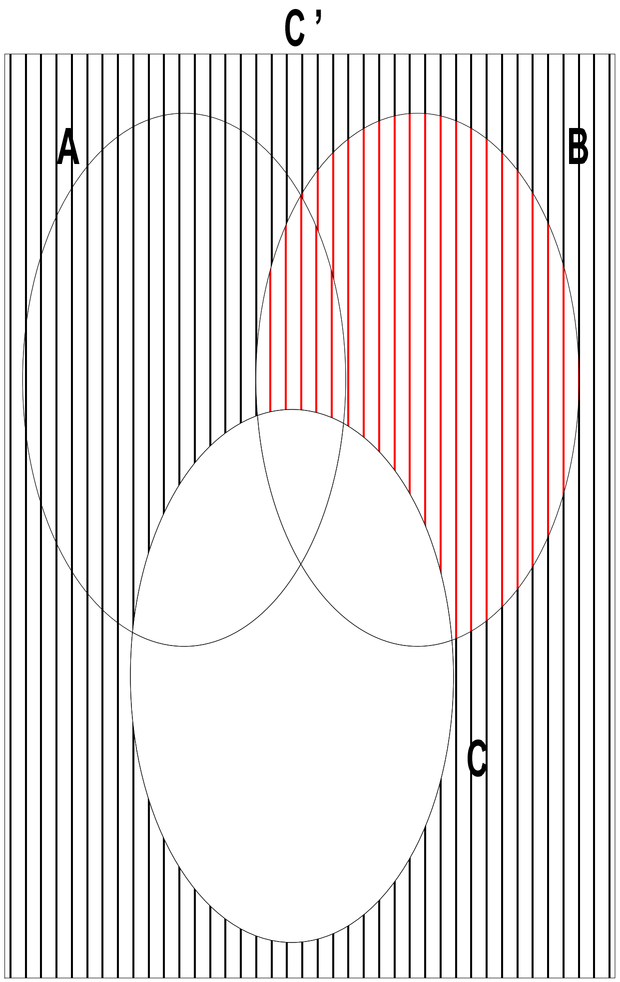
Use Venn diagrams to investigate whether, for all $A,B,C \subseteq U$,

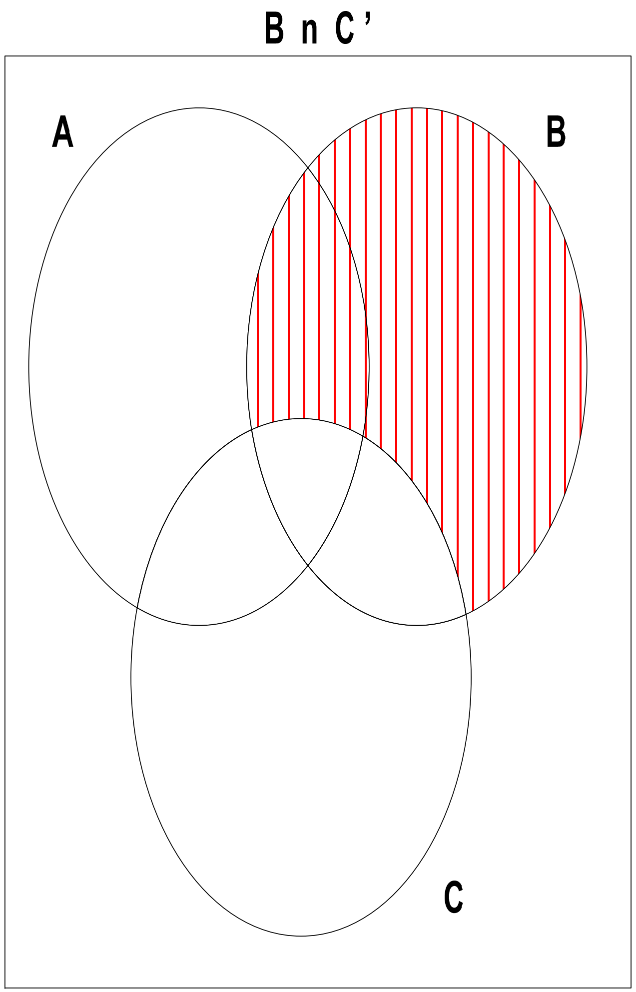
$$A \cup (B \cap C') = (A + B) - C.$$

If an identity, give proof; if not, give a counterexample.

Solution: Left-hand side:



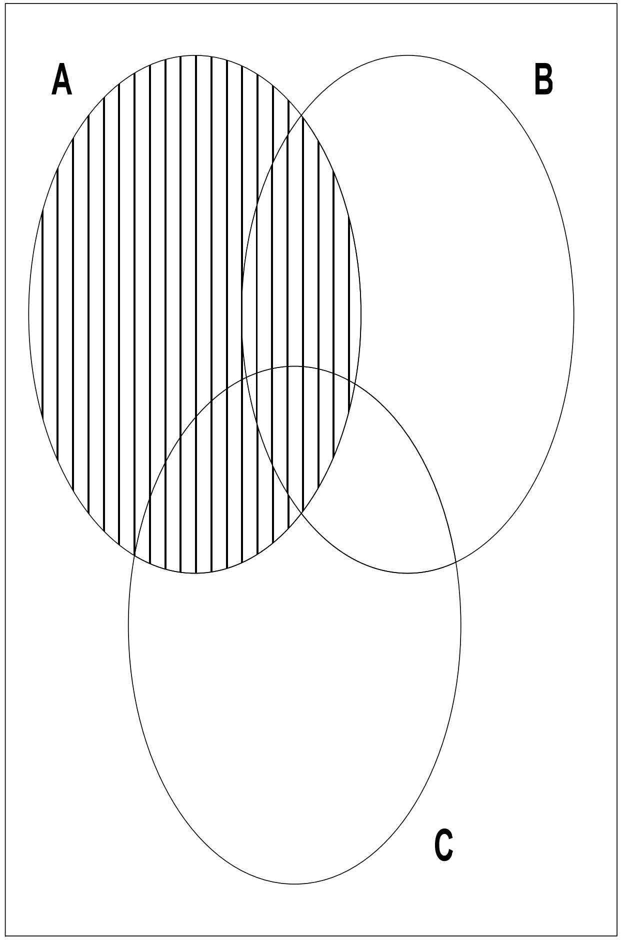




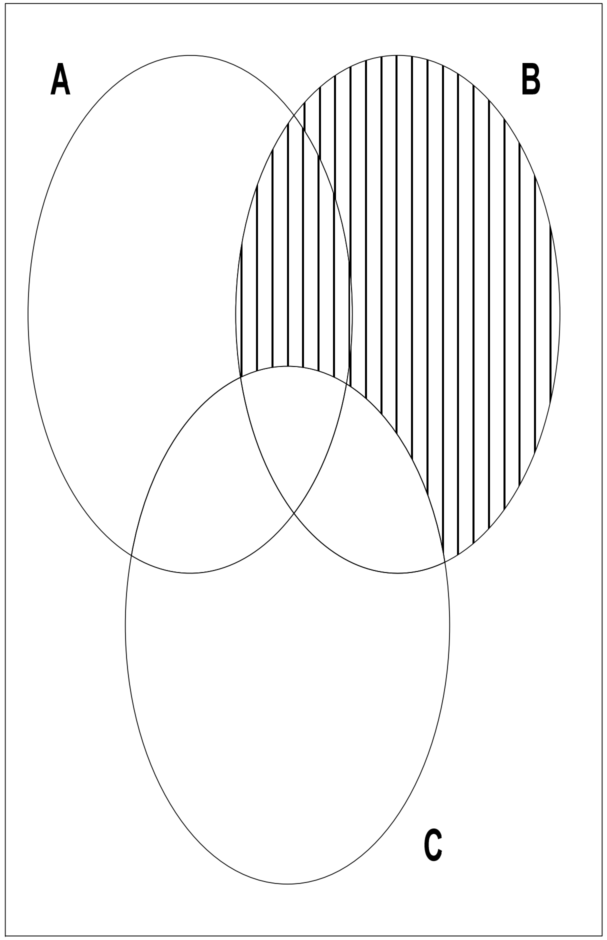
Is A $\cup$ (B $\cap$ C′) = (A + B) − C an identity?

Left-hand side:

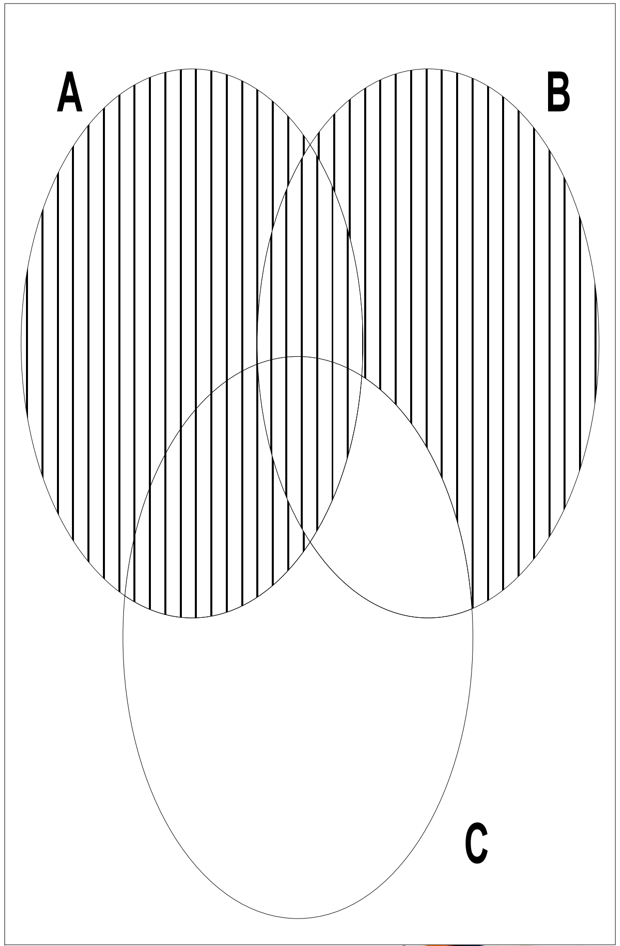
A



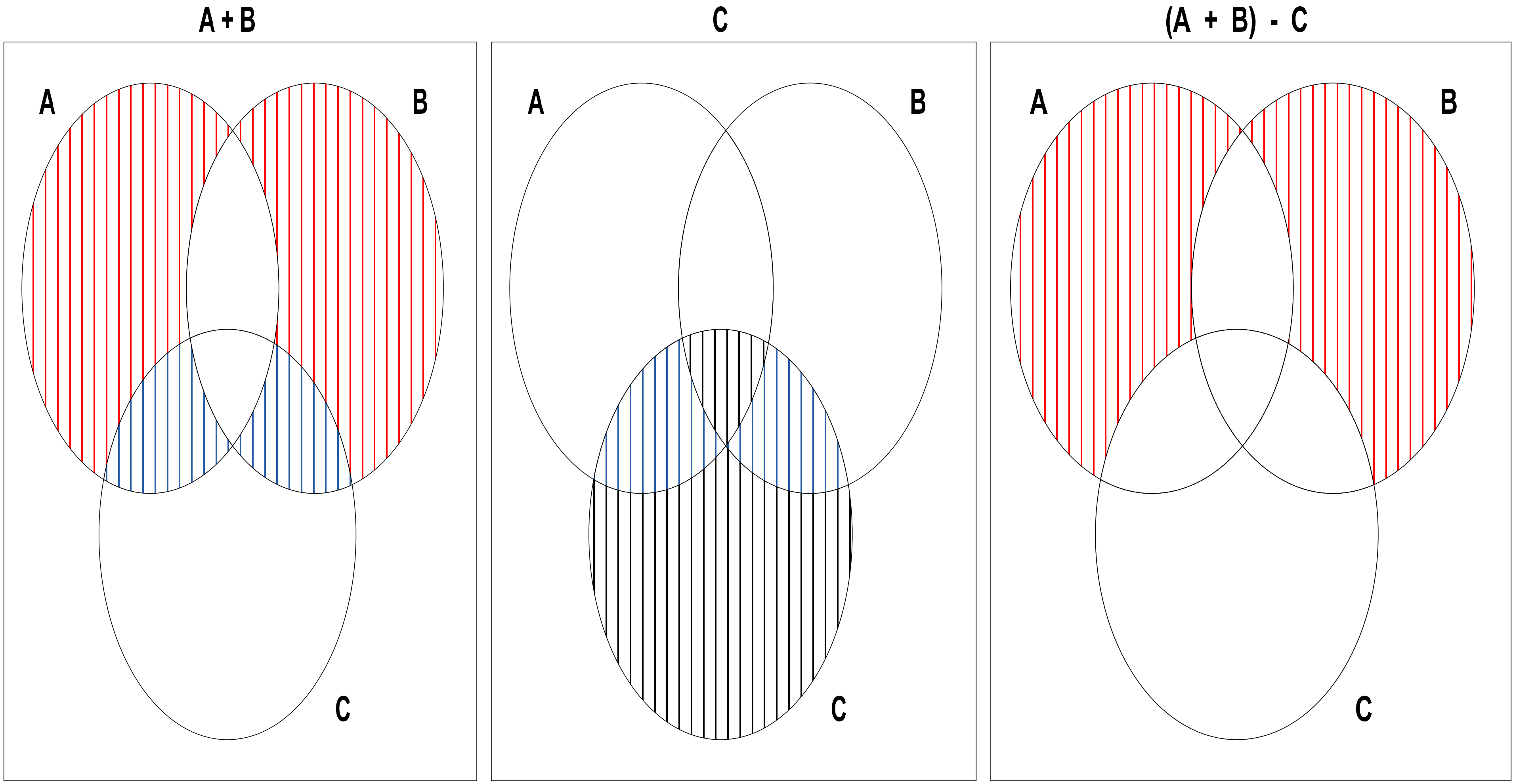
B n C ’



A U (B n C ’)



Right-hand side:

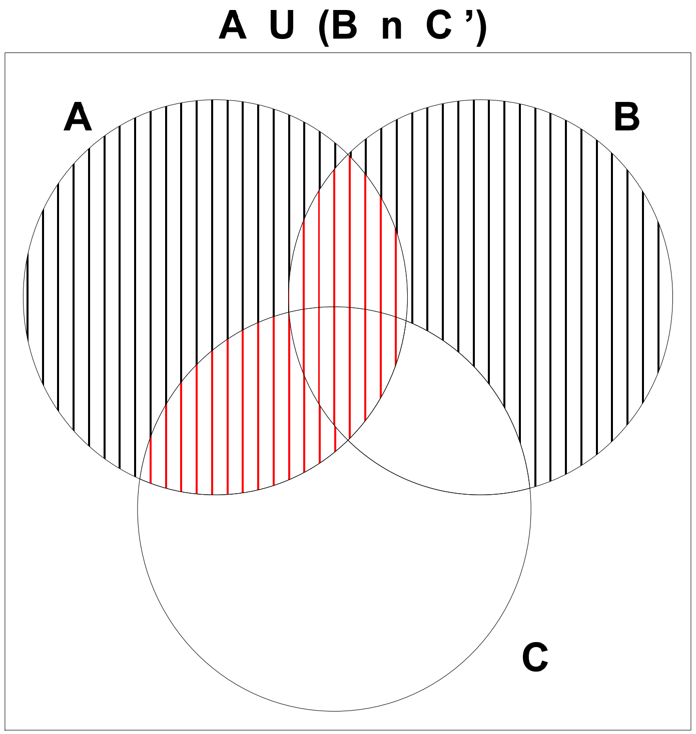


LHS ≠ RHS:

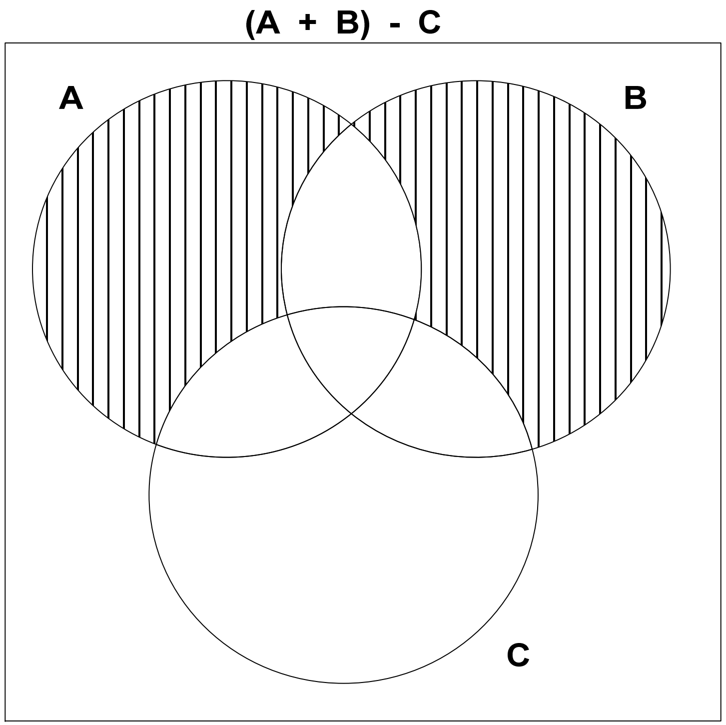
$A \cup (B \cap C') = (A + B) - C$ is not an identity.

Provide counterexample.

Left-hand side:



Right-hand side:



C*ounterexample*: *Choose a ∈ A, B and C.*

Let U = {a, b, c}, A = {a}, B = {a, b} and C = {a}.

L-H: $A \cup (B \cap C')$

$= \{a\} \cup (\{a,b\} \cap \{b,c\})$

$= \{a\} \cup \{b\}$

$= \{a, b\}$

R-H: $(A + B) - C$

$= \{b\} - \{a\}$

$= \{b\}$

L-H≠R-H

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(Study Guide, pp. 55, 72)

Set equality:

For any sets A, B $\subseteq$ U,

A = B iff A $\subseteq$ B and B $\subseteq$ A.

To prove A = B: prove that x $\in$ A iff x $\in$ B

Cartesian product:

$A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$

Example:

Suppose A = {2, 3, 4} and B = {5, 6}, then

$A \times B = \{ (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6) \}$

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Question 1b)

Determine whether, for all X, Y, W $\subseteq$ U,

$$(X - Y) \times W \subseteq (X \times W) - (Y \times W).$$

Solution:

Suppose $(p, q) \in (X - Y) \times W$,

then $p \in (X - Y)$ **and** $q \in W$

i.e. $(p \in X$ **and** $p \notin Y)$ **and** $q \in W$

i.e. $(p \in X$ **and** $q \in W)$ **and** $(p \notin Y$ **and** $q \in W)$

i.e. $(p, q) \in (X \times W)$ **and** $(p, q) \notin (Y \times W)$

i.e. $(p, q) \in (X \times W) - (Y \times W)$.

Thus $(X - Y) \times W \subseteq (X \times W) - (Y \times W)$.

Handy notations:

an even number can be expressed as $2n$, an odd number as $2m + 1$, and a multiple of three as $3t$, for some n, m, $t \in \mathbb{Z}$.

…

Two consecutive numbers: k and $k + 1$ for some $k \in \mathbb{Z}$.

RELATIONS *(Study Guide, pp.74 - 78)*

R $\subseteq$ A $\times$ A: R a binary relation *from A to A* (or *on A*).

R *reflexive* on A: $\forall x \in A$, $(x, x) \in R$.
R *irreflexive*: $\forall x \in A$, $(x, x) \notin R$.

R *symmetric*: $\forall x, y \in A$, if $(x, y) \in R$, then $(y, x) \in R$.

R *antisymmetric*: $\forall x, y \in A$, if $x \neq y$ and $(x, y) \in R$, then $(y, x) \notin R$.

if $(x, y) \in R$ and $(y, x) \in R$, then $x = y$.

R on A satisfies *trichotomy*: $\forall x, y \in A$, such that $x \neq y$ we have $(x, y) \in R$ or $(y, x) \in R$.

E.g. R on $\mathbb{Z}$: $(v, w) \in R$ iff $w - v = 7k$, $k \in \mathbb{Z}$.

Note order of variables: $(x, y) \in R$: $y - x = 7k$

$(y, z) \in R$: $z - y = 7m$

$(x, z) \in R$: $z - x = 7t$

Def: R *transitive*: $\forall\ x, y, z \in \mathbb{Z}$,

if $(x, y) \in R$ and $(y, z) \in R$, then $(x, z) \in R$.

Proof: Assume $(x, y) \in R$ and $(y, z) \in R$,

then $y - x = 7k$ ① and $z - y = 7m$ ②

① + ②: $z - x = 7(k + m)$,

thus $(x, z) \in R$.

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(Study Guide, pp. 84 - 88)

***Kinds of Relations*:**

R on A

- ***weak partial order*:**

  reflexive on A, antisymmetric, and transitive.

- ***strict partial order*:**

  irreflexive, antisymmetric, and transitive.

Weak or strict *total (or linear)* order also satisfies *trichotomy*.

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(Study Guide, pp. 90 – 92, 94)

A relation R on A is an equivalence relation

iff R is reflexive on A, symmetric, & transitive.

***Equivalence classes*:**

$[x] = \{y \mid y \in A \text{ and } x\,R\,y\}$

Say $[x_1]$ **&** $[x_2]$ **eq. classes of R:** $[x_1], [x_2] \subseteq A$,

then

$P = \{[x_1], [x_2]\}$ **is a *partition* of *A*:**

- $[x_1] \neq \varnothing$, $[x_2] \neq \varnothing$,
- $[x_1] \cap [x_2] = \varnothing$, and
- $[x_1] \cup [x_2] = A$

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Question 2a)

R on $\mathbb{Z}$: $(x, y) \in R$ iff $y - x$ is even.

Prove: R an equivalence relation. Show equivalence classes.

Solution:

Eq. rel: Reflexive on $\mathbb{Z}$, symmetric & transitive

$y - x$ is even, i.e. multiple of two, so

$y - x = 2k$ for some $k \in \mathbb{Z}$.

Reflexivity: (Is $(x, x) \in R$ for all $x \in \mathbb{Z}$?

i.e. is $x - x = 2k$ for all $x \in \mathbb{Z}$?)

Proof:

For all $x \in \mathbb{Z}$, $x - x = 0 = 2(0)$ with $0 \in \mathbb{Z}$.

Hence $(x, x) \in R$.

Thus R is reflexive on $\mathbb{Z}$.

Symmetry: (If $(x, y) \in R$, is $(y, x) \in R$?)

Suppose $(x, y) \in R$,

then $y - x = 2k$ for some $k \in \mathbb{Z}$ ①

$-$ ①: $-(y - x) = -(2k)$

i.e. $x - y = 2(-k)$, $-k \in \mathbb{Z}$

Thus $(y, x) \in R$, hence R is **symmetric**.

Transitivity: (If $(x, y) \in R$ & $(y, z) \in R$. Is $(x, z) \in R$?)

Suppose $(x, y) \in R$ and $(y, z) \in R$ then

$y - x = 2k$ ① and $z - y = 2m$ ②

① + ②: $(y - x) + (z - y) = 2k + 2m$

i.e. $z - x = 2(k + m)$, $(k+m) \in \mathbb{Z}$

Thus $(x, z) \in R$, hence R is **transitive**.

Continue

Equivalence classes of R:

$[x] = \{ y \mid (x, y) \in R \} = \{ y \mid y - x = 2k \text{ for some } k \in \mathbb{Z} \}$

$= \{ y \mid y = 2k + x \text{ for some } k \in \mathbb{Z} \}$

Let $x = 0$:

$[0] = \{ y \mid y = 2k + 0, \text{ for some } k \in \mathbb{Z} \}$

$= \{\ldots, -8, -6, -4, -2, 0, 2, 4, 6, 8, \ldots\}$ (even integers)

Let $x = 1$:

$[1] = \{ y \mid y = 2k + 1, \text{ for some } k \in \mathbb{Z} \}$

$= \{ \ldots, -7, -5, -3, -1, 1, 3, 5, 7, \ldots \}$ (odd integers)

Try x = ..-4,-2, 2, 4,.. then ...= [-4] = [-2] = [0] = [2] = [4] =...

Try x = ..-3,-1,3,5,.. then … = [-3] = [-1] = [1] = [3] = [5] =…

Two eq classes: $[0] \cup [1] = \mathbb{Z}$.

($S = \{ [0], [1] \}$ partition of $\mathbb{Z}$.)

Question 2

R on $\mathbb{Z}$:

(x, y) $\in$ R iff mx = y for some m $\in \mathbb{Z}^+$.

(y is a multiple of x)

bi) Give element in R &

element not in R.

Solution:

(3, 12) $\in$ R **($3 \times 4 = 12$);**

(3, 4) $\notin$ R **(4 is not a multiple of 3)**

R is a weak partial order on $\mathbb{Z}$ because R is reflexive on $\mathbb{Z}$, antisymmetric, and transitive.

R does not satisfy trichotomy:
Counterexample: (3, 4) $\notin$ R and (4, 3) $\notin$ R

FUNCTIONS *(Study Guide, pp. 98 – 114)*

$R \subseteq X \times Y$:

domain of R: $dom(R) \subseteq X$,

range of R: $ran(R) \subseteq Y$ (codomain = Y)

- **$dom(R) = \{x \mid \text{for some } y \in Y, (x, y) \in R\}$**
- **$ran(R) = \{y \mid \text{for some } x \in X, (x, y) \in R\}$**

R is function iff R is *functional* and *dom(R)=X*.

(functional: each $x \in A$ appears exactly once as first co-ordinate)

Function denoted by $R: X \to Y$.

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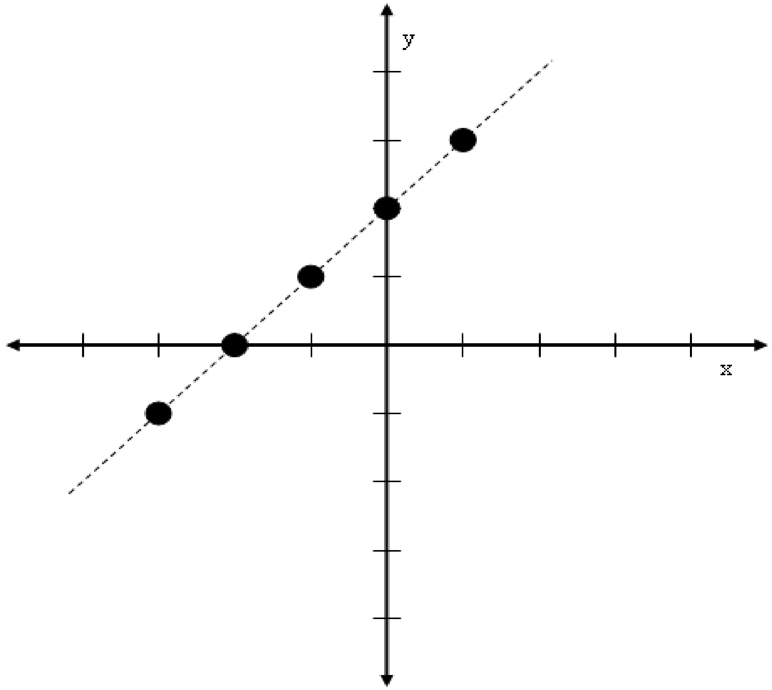
Ex. f: $\mathbb{Z} \to \mathbb{Z}$ is def. by
$f(x) = x + 2$

Graph for f: 

Consider function h: $A \to B$:

h *injective*: if $h(a_1) = h(a_2)$ then $a_1 = a_2$

h *surjective*: $\mathrm{ran}(h) = B$

h *bijective* iff h injective & surjective

Images *(Study Guide, pp. 110 – 111)*

Ex: function f: $\mathbb{Z} \to \mathbb{Z}$ is def by $f(x) = x^2 - 3x$ and

function g: $\mathbb{Z} \to \mathbb{Z}$ is def by $g(x) = 5x + 4$

The image of x under f: $f(x) = x^2 - 3x$

[Examples: $f(u) = u^2 – 3u$ or even

$f(m + 1) = (m + 1)^2 - 3(m + 1)$ etc.]

Now

$f \circ g(x) = f(g(x)) = (g(x))^2 - 3g(x)$ (image of x under $f \circ g$)

i.e. $f(5x + 4) = (5x +4)^2 - 3(5x +4)$

…

Question 3 f on $\mathbb{Z}$: $(x, y) \in f$ iff $y = 3x - 1$
g on $\mathbb{Z}$: $(x, y) \in g$ iff $y = 3 - x$
(a)Prove f a function: $f: \mathbb{Z} \to \mathbb{Z}$ (i.e. f functional & dom $(f) = \mathbb{Z}$)

Solution:

Is f functional? Yes.

Suppose $(x, y) \in f$ and $(x, z) \in f$,
then $y = 3x - 1$ and $z = 3x - 1$
i.e. $y = 3x - 1 = z$
i.e. $y = z$.

Is dom (f) = $\mathbb{Z}$? Yes.

$$\begin{aligned} \mathrm{dom}(f) &= \{ x \mid \text{for some } y \in \mathbb{Z}, (x, y) \in f\} \\ &= \{ x \mid \text{for some } y \in \mathbb{Z},\ y = 3x - 1\} \\ &= \{ x \mid 3x - 1 \text{ is an integer}\} \\ &= \mathbb{Z} \end{aligned}$$

Question 3b)

f ($y = 3x - 1$) and g ($y = 3 - x$) functions on $\mathbb{Z}$.
Which function not bijective? Do two tests on it.

Solution:

f not bijective: injective (one-to-one); but
not surjective (not onto).

f injective:

Suppose $f(u) = f(v)$ for some $u, v \in \mathbb{Z}$,
then $3u - 1 = 3v - 1$ i.e. $u = v$.

f not surjective: Counterexample:

Choose $y = 3$.
There is no $x \in \mathbb{Z}$ such that $f(x) = y$,
i.e. such that $3x - 1 = 3$. ($x = 4/3 \notin \mathbb{Z}$.)

Hence $3 \notin \mathrm{ran}(f)$ and thus $\mathrm{ran}(f) \neq \mathbb{Z}$.

Question 3 c) Give inverse of bijective function g.

Solution: g ($y = 3 - x$) is bijective.

$$
\begin{aligned}
(y, x) \in g^{-1} \quad & \text{iff} \quad (x, y) \in g \\
& \text{iff} \quad y = 3 - x \\
& \text{iff} \quad x = 3 - y
\end{aligned}
$$

Hence $g^{-1}: \mathbb{Z} \times \mathbb{Z}$ def. by $g^{-1}(y) = 3 - y$

Question 3 d) Determine $f \circ g$. ($f(x) = 3x - 1$; $g(x) = 3 - x$)

Solution:

$$
\begin{aligned}
f \circ g(x) &= f(g(x)) \\
&= f(\underline{3 - x}) \qquad \text{(replace } g(x) \text{ by } 3 - x\text{)} \\
&= 3(\underline{3 - x}) - 1 \qquad (f(\underline{x}) = 3\underline{x} - 1) \\
&= 8 - 3x
\end{aligned}
$$

Thus $f \circ g$: $\mathbb{Z} \times \mathbb{Z}$ is def. by $f \circ g(x) = 8 - 3x$.

OPERATIONS *(Study Guide, pp. 119 – 122)*

Ex. Binary operation $\lozenge$: $X \times X \rightarrow X$

- ***commutative*:**

  $x \lozenge y = y \lozenge x$ for all $x, y \in X$.

- ***associative*:**

  $(x \lozenge y) \lozenge z = x \lozenge (y \lozenge z)$ for all $x, y, z \in X$.

- ***an identity element*:**

  $e \lozenge x = x \lozenge e = x$ for all $x \in X$.

Question 4

Let X = { b, c, d }

ai) Give example of binary operation * on X in a table:

* is commutative & has identity element.

Solution:

*	b	c	d
b	b	c	d
c	c	c	c
d	d	c	d

*	b	c	d
b	b	c	d
c	c	c	c
d	d	c	d

Commutative: Symmetry around diagonal from top left to bottom right corner. b is the identity element.

Question 4 Let X = { b, c, d }

aii) Show that your operation has both these properties, and name the identity element. **Solution:**

Commutative

b * b = b = b * b; b * c = c = c * b
b * d = d = d * b; c * c = c = c * c
c * d = c = d * c; d * d = d = d * d

Check commutativity:
Diagonal top left to bottom right.

*	b	c	d
b	b	c	d
c	c	c	c
d	d	c	d

Identity element: b, because

b * b = b = b * b,
c * b = c = b * c, and
d * b = d = b * d.

Question 4 Let X = { b, c, d }

aiii) Using your table, test for associativity (one ex.).

Decide from example: Does your binary operation has the property of associativity? Justify your answer.

*	b	c	d
b	b	c	d
c	c	c	c
d	d	c	d

Solution:

$(b * c) * d = c * d = c$ and

$b * (c * d) = b * c = c$

Example shows associativity, but <u>one</u> example does not show that it is true for all possible combinations.

Question 4

bi) Let Y = {$\varnothing$, {$\varnothing$}}. Complete the following table for the **binary operation** $\cap$ (intersection) **on Y**:

$\cap$	$\varnothing$	{$\varnothing$}
$\varnothing$		
{$\varnothing$}		

Solution:

$\cap$	$\varnothing$	{$\varnothing$}
$\varnothing$	$\varnothing$	$\varnothing$
{$\varnothing$}	$\varnothing$	{$\varnothing$}

Question 4

Let $Y = \{\varnothing, \{\varnothing\}\}$

bii) Write $\cap$ in list notation.

$\cap$	$\varnothing$	$\{\varnothing\}$
$\varnothing$	$\varnothing$	$\varnothing$
$\{\varnothing\}$	$\varnothing$	$\{\varnothing\}$

Solution:

$\varnothing \cap \varnothing = \varnothing$, so $((\varnothing,\varnothing), \varnothing)$ is an element in the set;
$\varnothing \cap \{\varnothing\} = \varnothing$, so $((\varnothing,\{\varnothing\}), \varnothing)$ is an element in set; etc.

Note:

$\varnothing = \{ \}$ ($\varnothing$ has ***no elements***) and

$\{\varnothing\}$ has ***one element*** namely $\varnothing$, so $\varnothing$ and $\{\varnothing\}$ has no common elements, thus $\varnothing \cap \{\varnothing\} = \varnothing$.

$\cap$ in list notation:

$$\{ ((\varnothing,\varnothing), \varnothing), ((\varnothing,\{\varnothing\}), \varnothing), ((\{\varnothing\},\varnothing), \varnothing), ((\{\varnothing\},\{\varnothing\}), \{\varnothing\}) \}$$

LOGIC *(Study Guide, study units 9 & 10)*

Connectives for declarative statements p & q: $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$

p	q	conjunction $p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	disjunction $p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	conditional $p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	biconditional $p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

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Declarative statement True or False

Compound statement always true: *tautology*

Compound statement always false: *negation*

Statements p and q *logically equivalent*:

$p \equiv q$ iff $p \leftrightarrow q$ a tautology

De Morgan's laws:

- $\neg (p \vee q) \equiv \neg p \wedge \neg q$
- $\neg (p \wedge q) \equiv \neg p \vee \neg q$

NOTE: $p \rightarrow q \equiv \neg p \vee q$

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Universal quantifiers e.g.:

- **"For all x $\in \mathbb{Z}$ …"   i.e. "$\forall$ x $\in \mathbb{Z}$ …"**

Existential quantifiers e.g.:

- **"There exists an x $\in \mathbb{Z}$ …"   i.e."$\exists$ x $\in \mathbb{Z}$ …"**

***Predicate* P(x):**

P(x) is true for any variable x $\in$ A that satisfies the property, and P(x) is false otherwise.

e.g. if P(x) is the pred. "x is an even integer": True for all even integers and false for all odd integers.

Negation:

- **$\neg$ ($\forall$ x $\in$ A, P(x))   i.e. $\exists$ x $\in$ A, $\neg$ P(x)**
- **$\neg$ ($\exists$ x $\in$ A, P(x))   i.e. $\forall$ x $\in$ A, $\neg$ P(x)**

Question 5a)

Write the English sentence

‘If the wind is blowing then it will bring wind or rain’

in symbolic logic notation.

Use

the letter t for ‘the wind is blowing’,

the letter d for ‘it will bring wind’ and

the letter h for ‘it will bring rain’.

Solution:

Symbolic logic notation:

$$t \rightarrow (d \vee h)$$

Question 5b)

Use the double negation property and De Morgan's laws to rewrite the following expression as an equivalent statement that does not have the not symbol (¬) outside parentheses.

$\neg(q \wedge (\neg q \vee p))$

Solution:

$\neg(q \wedge (\neg q \vee p))$

$\equiv \neg q \vee \neg(\neg q \vee p)$ De Morgan's law

$\equiv \neg q \vee (\neg\neg q \wedge \neg p)$ De Morgan's law

$\equiv \neg q \vee (q \wedge \neg p)$ Double negation

Question 5c)

Use a **truth table** to determine whether the compound statement

$$(\neg p \lor q) \land [\neg(p \to q)]$$

is a **tautology**, a **contradiction** or **neither**.

Solution:

p	q	$\neg p$	$\neg p \lor q$	$p \to q$	$\neg(p \to q)$	$(\neg p \lor q) \land [\neg(p \to q)]$
T	T	F	T	T	F	F
T	F	F	F	F	T	F
F	T	T	T	T	F	F
F	F	T	T	T	F	F

All values false, the statement is a contradiction.

Question 5d)

Let D = {1, 2, 4}. Provide negation of following statement:

$\forall\, x \in D,\ 4x + 1 \leq 16$

Which is true, the original statement or the negation?

Solution:

Negation: $\exists\, x \in D,\ 4x + 1 > 16$

x	4x + 1
1	5
2	9
4	17

$4x + 1 > 16$ true for $x = 4$.

So there exists an $x \in D$ such that $4x + 1 > 16$.

Thus negation is true.

Question 5e)

Prove "for any n $\in \mathbb{Z}$, if $5n^2$ is odd then n is odd" ($p \to q$) by using contrapositive of given statement.

Solution:

(To prove: $(\neg q) \to (\neg p)$,

i.e. if n is not odd, then $5n^2$ is not odd,

i.e. if n is even then $5n^2$ is even.)

Suppose n is even, then n=2k, for some k $\in \mathbb{Z}$.

Then $5n^2 = 5(2k)^2 = 5(4k^2) = 2(10k^2)$

i.e. $5n^2$ **is even.**

Question 5f)

Prove by **contradiction** (reduction ad absurdum) that for any integer n, **if $n^2 + 2n$ is even then n is even**.

Solution:

Suppose $n^2 + 2n$ is **even**.

Two possibilities: **either n is odd or n is even.**

Suppose n is odd, i.e. $n = 2k + 1$ for some $k \in \mathbb{Z}$.

$$
\begin{aligned}
\text{Then } n^2 + 2n &= (2k + 1)^2 + 2(2k + 1) \\
&= 4k^2 + 8k + 3 \\
&= (4k^2 + 8k + 2) + 1 \\
&= 2(2k^2 + 4k + 1) + 1, \quad \text{i.e. } n^2 + 2n \text{ is odd}
\end{aligned}
$$

This contradicts initial supposition,
so questionable supposition wrong,
thus n is even if n^2+2n is even.

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Power set of A: **set that has as members all the subsets of A**

Example:

A = {1, {1}}: 2 elements: 1 and {1} (n = 2)

$\mathcal{P}$ (A) = { ∅, {1}, {{1}}, {1, {1}} } (nr of elements: 2 to power n)

Ex. of factorisation:

$x^2 - 4x + 3 < 0$

i.e. $(x - 3)(x - 1) < 0$

then $(x - 3) > 0$ and $(x - 1) < 0$ $(+ \times - = -)$

i.e. $x > 3$ and $x < 1$

OR

$(x - 3) < 0$ and $(x - 1) > 0$

i.e. $x < 3$ and $x > 1$,

i.e. $1 < x < 3$ thus $x > 0$.

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Question 6 Let $A = \{1, 2, 3\}$, $B = \{0, 1\}$ and $C = \{\varnothing\}$.

a) Give A + B and an equivalence relation on A + B (not identity relation).

Solution: $A+B=\{0, 2, 3\}$ Equivalence relation on A+B:

$$\{(0, 0), (2, 2), (3, 3), (0, 2), (2, 0)\}$$

b) Determine values of sets:

$\mathcal{P}(B)\cap\mathcal{P}(C)$; $\mathcal{P}(B\cap C)$; $\mathcal{P}(B)-\mathcal{P}(C)$; $\mathcal{P}(B)+\mathcal{P}(C)$

Solution:

$\mathcal{P}(B)=\{\varnothing, \{0\}, \{1\}, \{0, 1\}\}$; $\mathcal{P}(C)=\{\varnothing, \{\varnothing\}\}$;

hence $\mathcal{P}(B)\cap\mathcal{P}(C) = \{\varnothing\}$.

$B \cap C = \varnothing$; hence $\mathcal{P}(B\cap C) = \{\varnothing\}$.

$\mathcal{P}(B) - \mathcal{P}(C) = \{\{0\}, \{1\}, \{0, 1\}\}$

Question 6 Let $A = \{ 1, 2, 3 \}$, $B = \{ 0, 1 \}$ and $C = \{ \varnothing \}$.

c) Give injective function on $B \times B$.

Solution:

$B \times B = \{ (0, 0) , (0, 1) , (1, 0) , (1, 1) \}$;

Injective function on $B \times B$:

$\{((0,0), (0,0)), ((0, 1), (0, 1)), ((1, 0), (1, 0)), ((1, 1), (1, 1))\}$

d) Give example of surjective function from $B \times B$ to $B \cup C$.

Solution: Surjective: $Ran(f) = B \cup C$. Function: Each member of $B \times B$ must appear only once as first co-ordinate.

$B \times B = \{ (0,0), (0,1), (1,0), (1,1) \}$ and
$B \cup C = \{ 0, 1, \varnothing \}$

Surjective function: $\{ ((0,0), 0), ((0,1), 1),$
$((1,0), 1), ((1,1), \varnothing) \}$

Question 6 Let A = { 1, 2, 3 }, B = { 0, 1 } and C = { $\varnothing$ }.

e) Give simplest equivalence relation on $\mathcal{P}$ (C).

Solution:

Relation: Reflexive on $\mathcal{P}$ (C), symmetric and transitive.

$\mathcal{P}$ (C) = { $\varnothing$, { $\varnothing$ } }

Simplest equivalent relation on $\mathcal{P}$ (C):

Identity relation: { ($\varnothing$, $\varnothing$) , ({ $\varnothing$ }, { $\varnothing$ }) }

f) Give a partition of B.

Solution:

{{0,1}} or {{0}, {1}}