

# SETS

## Related Definitions and Formulae

### Some Important Sets of Numbers:

- (1) Set of Natural Numbers  $N = \{1, 2, 3, \dots\}$
- (2) Set of Whole Numbers  $W = \{0, 1, 2, \dots\}$
- (3) Set of Integers  $Z = \{0, \pm 1, \pm 2, \pm 3, \dots\}$
- (4) Set of Positive Prime Numbers  $P = \{2, 3, 5, 7, 11, \dots\}$
- (5) Set of Odd Numbers  $O = \{\pm 1, \pm 3, \pm 5, \dots\}$
- (6) Set of Even Numbers  $E = \{0, \pm 2, \pm 4, \pm 6, \dots\}$
- (7) Set of Rational Numbers  $Q = \{x/x = \frac{p}{q}; p, q \in Z, q \neq 0\}$
- (8) Set of Irrational Numbers  $Q' = \{x/x \neq \frac{p}{q}; p, q \in Z, q \neq 0\}$
- (9) Set of Real Numbers  $R = Q \cup Q'$

**SET:** Set is a Collection of WELL defined and DISTINCT objects".

Well defined means that a rule can be stated which determines either an object is a member of a set or not. Distinct means that each object of a set is different from all other objects of the set. We can not repeat a member in a set. For example  $A = \{x \mid x \in Z^+; x < 5\}$  it says that set "A" has member such that they are positive integers and they are less than 5 i-e set A has members  $A = \{1, 2, 3, 4\}$  So in Set A the portion  $[x \in Z^+; x < 5]$  is a rule which determines its members and each member is unique.

**ELEMENT:** Any thing belong to a Set is called an element (or member) of the Set.

### **Forms of Sets:**

**(1) Descriptive Form:** A Set which described with the help of a statement is called descriptive form.

e.g. #  $N$  = The Set of natural numbers.

**(2) Tabular Form:** In this form we list the element of a set with in curly Bracket.

$$S = \{a, b, c, d\} ; T = \{1, 2, 3\}$$

**(3) Set-Builder Form:** By enclosing within Curley bracket a rule that determines its elements is said to be set builder form.

e.g. #  $A = \{ x \mid x \in N ; \wedge ; 100 \leq x \leq 150 \}$

$$A = \{ 100, 101, \dots, 150 \}$$

**(4) Venn Diagram:** Sets can also be represent graphically using venn diagrams. In venn diagrams the Universal Set  $U$ , which contains all the elements of the Subsets under consideration, is usually represented by a rectangle. Inside this rectangle, Circles are used to represent sets.

### **SOME IMPORTANT DEFINITIONS:**

**Singleton Set:** A Set having only one element is called singleton set.

**Null/Empty Set:** A set having does not contain any element is called Null/empty set it is denoted by  $\{ \}$  or  $\phi$

e.g #  $A = \{ x \mid x \text{ is } 100 \text{ years old man living in Pakistan} \}$

$$B = \{ x \mid x \text{ is a man with } 200 \text{ feet height} \}$$

**Finite Set:** A set in which the process of counting it's elements terminates is called a finite set.

e.g #  $A = \{ 1, 2, 3, \dots, 60 \}$

**Infinite Set:** If the process of counting the elements of a set does not terminate then set is called an infinite set.

e.g #  $N$  = Set of Natural numbers.

**Equal Sets:** Two sets are said to be equal if and only if they have the same elements.

e.g #  $A = \{a, b, c, d\}$  and

$B = \{b, c, a, d\}$  are equal Sets

i – e  $A = B$  Thus  $A = B$  iff  $A \subseteq B$  and  $B \subseteq A$ .

**Equivalent Sets:** Two sets A and B are said to be equivalent, denoted by  $A \sim B$  if they have same number of elements.

e.g #  $A = \{1, 2, 3\}$  and  $B = \{a, b, c\}$  then  $A \sim B$

**Note:** If A, B and C are three sets then  $A \sim B$  and  $B \sim C \Rightarrow A \sim C$ . This is called the transitive property of equivalence of sets.

**Subset:** A set A is a subset of a set B denoted by  $A \subseteq B$ , if every element of A is also an element of B. symbolically  $A \subseteq B$ .

**Note:** (i) Null set,  $\phi$  is a subset of every set.

(ii) Every set is a subset of itself.

**Power Set:** The set of all possible subsets of a set A is called power set of A and is denoted by  $p(A)$ . The total numbers of subsets is find out by  $2^n$ . Where  $n$  = number of elements in a set.

**Proper Subset:** A set A is a proper Subset of a Set B denoted by  $A \subset B$  if A is a subset of B and if there exists atleast one element in B that is not in A.

e.g # If  $A = \{1, 2, 3\}$  and  $B = \{1, 2, 3, 4\}$

then A is proper subset of B i – e  $A \subset B$ .

**Note:** If A is a subset of B and B is a subset of C, then A is a subset of C i – e  $A \subseteq B$  and  $B \subseteq C \Rightarrow A \subseteq C$  This is called the transitive property of subsets.

**Improper Subset:** A set is called improper subset. If both sets are one by one equivalent to each other.

e.g #  $A = \{a, b, c, d\}$  ;  $B = \{b, c, a, d\}$

then set A is an improper subset of set B.

**Note:** Each Set is an improper subset of itself. Infact, the only improper subset of a set is the set itself.



### **OPERATIONS ON TWO SETS:**

**Union of Two Sets:** The Union of the Sets A and B, denoted by  $A \cup B$  is the Set that Contains those elements which are contained in A or B or both.

$$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$$

**Intersection of Two Sets:** The intersection of sets A and B, denoted by  $A \cap B$ , is the set that contains those elements which are contained in both A and B.

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$$

**Difference of Two Sets:** The difference of Sets A and B, denoted by  $A - B$  is the set containing those elements that are in A but not in B.

$$A - B = \{ x \mid x \in A \text{ and } x \notin B \}$$

**Symmetric Difference of Two Sets:** The symmetric difference of sets A and B, denoted by  $A \Delta B$ , is the set containing those elements which are either in A or in B but not in both A and B.

$$A \Delta B = (A \cup B) - (A \cap B)$$

**Universal Set:** A Set which contains all the sets under consideration is called a universal set. Usually it is denoted by U.

**Complement of a Set:** Let U be a Universal set and let  $A \subset U$ . The Complement of Set A, denoted by  $A^c$  or  $A'$ , is the set containing those elements of U, which are not in A.

**Disjoint Sets:** If the intersection of two sets is the empty set then the sets are said to be disjoint sets.

$$\text{i.e. } A \cap B = \emptyset$$

### **Fundamental Properties of Union and Intersection:**

- (1)  $A \cup B = B \cup A$   
(Commutative property of union)
- (2)  $A \cap B = B \cap A$   
(Commutative property of Intersection)
- (3)  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$   
(Associative property of Union)

**Domain and Range of a Binary Relation:** The Set of the first elements of all ordered pairs in a relation  $R$  from a Set  $A$  to a Set  $B$  is called the domain of the relation  $R$ . The Set of the second elements of all the ordered pairs in the relation is called the range of the relation  $R$ .

Domain of  $R$  is denoted by  $\text{Dom } R$  and range of  $R$  is denoted by  $\text{Range } R$ .

**Function:** Let  $A$  and  $B$  be any two sets and  $R$  be a binary relation from  $A$  to  $B$ . Then  $R$  is called a function from  $A$  to  $B$  if

- (i)  $\text{Dom } R = A$
- (ii) Every element of  $A$  is associated with exactly one element of  $B$  under  $R$  i.e.  $(a,b) \in R, (a,b') \in R$  imply that  $b = b'$ .

If a relation is a function then it is usually denoted by  $f, g$  etc.

If  $f$  is a function from  $A$  to  $B$ , it is written as  $f: A \longrightarrow B$  it is read as  $f$  is a function from  $A$  to  $B$ .

**Note:** If  $f: A \longrightarrow B$  is a function from  $A$  to  $B$  and pair  $(a,b)$ ,  $a \in A$  and  $b \in B$  is in  $f$ . Then  $b$  is called the image of  $a$  under  $f$ . It is denoted by  $f(a) = b$ .

### **Types of Function:**

**(1) Onto Function:** A function  $f$  from  $A$  to  $B$  is called an onto function if  $\text{Range } f = B$ .

**(2) One-One function:** A function  $f$  from  $A$  to  $B$  is said to be one-one if distinct elements of set  $A$  are associated with distinct elements of Set  $B$ .

**(3) One-One and onto function:** A function  $f$  from Set  $A$  to set  $B$  is called a one – one and onto function it is both one-one and onto.

- (4)  $A \cap (B \cap C) = (A \cap B) \cap C$   
(Associative property of Intersection)
- (5)  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$   
(Distributive property of Union over Intersection)
- (6)  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$   
(Distributive property of intersection over union)

**De Morgan's Laws:** If we have two sets A and B, be the subsets of (Universal Set) then

$$(i) \quad (A \cup B)' = A' \cap B' \quad (ii) \quad (A \cap B)' = A' \cup B'$$

**Ordered Pair:** An Ordered set of two elements is called an ordered pair.

e.g #  $(a, b)$ , the first component is "a" and the second component is "b"

**The Cartesian Product of Two Sets:** The Cartesian product of any set A with any other set B is the set of all ordered pairs  $(a, b)$  where  $a \in A$  and  $b \in B$  it is denoted by  $A \times B$  and is read as "A Cross B" Symbolically.

$$A \times B = \{(a, b) | a \in A; b \in B\}$$

**Note:**(1) The ordered pairs  $(a, b)$  and  $(b, a)$  are not the same unless  $a = b$

(2) Two ordered pairs  $(a, b)$  and  $(c, d)$  are equal if and only if  $a = c, b = d$ .

(3) if either A or B is empty then  $A \times B = \phi$

(4)  $A \times B \neq B \times A$  Unless  $A = B$

(5) If the number of elements in sets A and B are m and n, respectively, then the number of elements in  $A \times B$  is mn.

**Binary Relation:** A binary relation from a set A to a set B is just a subset of  $A \times B$ .

Thus, every subset of  $A \times B$  is a binary relation from A to B.

In particular, a subset of  $A \times A$  is called a binary relation in A.



## EXERCISE 1.1

**Q1.** Use both tabular and set builder forms to specify the following.

- (a) The set of positive integers greater than 2 and less than 6.

**Solution:** Tabular form  $A = \{3, 4, 5\}$

Set builder form  $A = \{x | x \in \mathbb{Z}^+ \wedge 2 < x < 6\}$

- (b) The Set of positive integers less than 20 that are divisible by 5.

**Solution:** Tabular form  $A = \{5, 10, 15\}$

Set builder from  $A = \{x | x \in \mathbb{Z}^+ \wedge x \text{ multiple of } 5 \text{ less than } 20\}$

- (c) The set of natural numbers between 4 and 12.

**Solution:** Tabular form  $A = \{5, 6, 7, 8, 9, 10, 11\}$

Set builder from  $A = \{x | x \in \mathbb{N} \wedge 4 < x < 12\}$

- (d) A set of first six positive prime numbers.

**Solution:** Tabular form  $A = \{2, 3, 5, 7, 11, 13\}$

Set builder from  $A = \{x | x \in \mathbb{P} ; 2 \leq x \leq 13\}$

**Q2.** Which of the following sets are the Null sets?

$A = \{x | x \text{ is a letter before "a" in the English alphabet}\}$

Ans. Null set

$B = \{x | x + 5 = 5\}$

Ans.  $B \{0\}$  Not Null set

$C = \{x | x \text{ is less than } 7 \text{ and greater than } 8\}$

Ans. Null set

$D = \{x | x \text{ is past President of } \bullet \bullet \bullet \text{ who was a woman}\}$

Ans. Null set

**Q3.** Which of these sets are finite and which of these are infinite?

- (a) The months of a year.

Ans. Finite

(b) The days in a year.

Ans. Finite

(c) The students of your class.

Ans. Finite

(d)  $\{2,4,6,8,10,\dots\}$

Ans. Infinite

(e) The set of lines passing through a point.

Ans. Infinite

(f) The set of lines passing through two given points.

Ans. Finite

Q4. Given  $S = \{x|x \text{ is a positive integer}\}$ , find proper subsets of  $S$  that are also subsets of  $A$ , where

$A = \{x|x \text{ is an integer less than } 3\}$

Solution:  $S = \{1,2,3,\dots\}$

Proper subsets of  $S$  are

$= \{ \}, \{1\}, \{2\}, \{3\}, \dots$

given that  $A = \{1,2\}$

subsets of  $A$  are

$= \{ \}, \{1\}, \{2\}, \{1,2\}$

According to given statement required proper subsets are  $\{ \}, \{1\}, \{2\}, \{1,2\}$ . Ans.

Q5. If  $A = \{a, b, c, d\}$  find:

(a) Proper subsets of  $A$

Solution:  $\{ \}, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, \{a,c,d\}$  Ans.

(b) An improper subset of  $A$ .

Solution:  $\{a,b,c,d\}$  Ans.

(c) Two sets  $B$  and  $C$  that are subsets of  $A$  such that  $B \subset C$

Solution: Let  $B = \{a,b\}$  and  $C = \{a,b,c\}$  this follows  $B \subset C$  Ans.

(d) Two sets  $B$  and  $C$  that are subsets of  $A$  such that  $B \subseteq C$

Solution: let  $B = \{a,b\}$

$C = \{a,b,c\}$

this follows  $B \subseteq C$  Ans.



**Q6. Find all the subsets of  $A = \{a,b,c,d\}$  Hence, or otherwise, find  $|P(A)|$**

**Solution:**  $P(A) = \{ \{ \}, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\} \}$  Ans.

**Q7. Is there any set which has no proper subset? If yes, specify the set.**

**Ans.** Yes, a Null set has no proper subset i-e  $\{ \}$

**Q8. Find a set which has only one proper subset.**

**Solution:** A set which has only one element show the only one proper subset.

Let  $A = \{1\}$

Subset of A is  $\{ \}$  Ans.

**Q9. If  $n(A) = 10$  then  $n[P(A)] = \underline{\hspace{2cm}}$ .**

**Ans.**  $= 2^n = 2^{10} = 1024$

**Q10. Use the set builder form to give an example of a Null set.**

**Solution:** If B is a Null set then in set builder notation it can be written as:

$B = \{x \in p \mid x = 4\}$  Ans.

**Q11. Consider the set  $A = \{1,2,3,4\}$ . Find a proper subset B of A then find a proper subset C of B, then find a proper subset D of C.**

**Solution:** Let proper subset is  $B = \{1,2,3\}$   $B \subset A$ . proper subset of B is  $C \Rightarrow C = \{1,2\}$  i-e  $C \subset B$ . Again proper subset of C is  $D \Rightarrow D = \{1\}$  i-e  $D \subset C$ . Ans.

**Q12. Which of the following are equivalent sets?**

(a)  $A = \{a,b,c\}, B = \{1,2,3\}$

**Solution:**  $\therefore n(A) = n(B) = 3$  So  $A \sim B$

(b)  $A = \{1,2,3,4\}, B = \{a,b,c\}$

**Solution:**  $\therefore n(A) \neq n(B)$  so  $A \not\sim B$

(c)  $A = \{x | x \text{ is a positive integers less than } 6\}$

$B = \{a, e, i, o, u\}$

Solution:  $A = \{1, 2, 3, 4, 5\}$

$B = \{a, e, i, o, u\}$

$\therefore n(A) = n(B) = 5$

So  $A \sim B$

## EXERCISE 1.2

Given that the sets  $A = \{f,a,c,e\}$  and  $B = \{e,g,d,f\}$  are subsets of the universal set  $U = \{a,b,c,d,e,f,g\}$  list the elements of.

(1)  $A'$

Solution:

$$U - A = \{a,b,c,d,e,f,g\} - \{f,a,c,e\}$$

$$A' = \{b,d,g\} \quad \text{Ans.}$$

(2)  $B'$

Solution:

$$U - B = \{a,b,c,d,e,f,g\} - \{e,g,d,f\}$$

$$B' = \{a,b,c\} \quad \text{Ans.}$$

(3)  $A \cap B$

Solution:

$$A \cap B = \{f,a,c,e\} \cap \{e,g,d,f\}$$

$$A \cap B = \{e,f\} \quad \text{Ans.}$$

(4)  $(A \cup B)'$

Solution:

$$A \cup B = \{f,a,c,e\} \cup \{e,g,d,f\}$$

$$A \cup B = \{a,c,d,e,f,g\}$$

$$U - (A \cup B) = \{a,b,c,d,e,f,g\} - \{a,c,d,e,f,g\}$$

$$(A \cup B)' = \{b\} \quad \text{Ans.}$$

(5)  $A \cap B'$

Solution:

$$U - B = \{a,b,c,d,e,f,g\} - \{e,g,d,f\}$$



$$B' = \{a, b, c\}$$

$$A \cap B' = \{f, a, c, e\} \cap \{a, b, c\}$$

$$A \cap B' = \{a, c\} \quad \text{Ans.}$$

$$(6) \quad A' \cap B'$$

**Solution:**

$$U - A = \{a, b, c, d, e, f, g\} - \{f, a, c, e\}$$

$$A' = \{b, d, g\}$$

$$U - B = \{a, b, c, d, e, f, g\} - \{e, g, d, f\}$$

$$B' = \{a, b, c\}$$

$$A' \cap B' = \{b, d, g\} \cap \{a, b, c\}$$

$$A' \cap B' = \{b, d, g\} \cap \{a, b, c\}$$

$$A' \cap B' = \{b\} \quad \text{Ans.}$$

**A**

$$(7) \quad U \cup \phi$$

**Solution:**

$$U \cup \phi = \{a, b, c, d, e, f, g\} \cup \{ \}$$

$$= \{a, b, c, d, e, f, g\}$$

$$U \cup \phi = U \quad \text{Ans.}$$

$$(8) \quad U \cap \phi$$

**Solution:**

$$U \cap \phi = \{a, b, c, d, e, f, g\} \cap \{ \}$$

$$U \cap \phi = \{ \} \quad \text{Ans.}$$

**Given the sets  $A = \{x | x \text{ is positive even integer less than } 10\}$  and  $B = \{x | x \text{ is positive odd integer less than } 10\}$  are subsets of the universal set.**

**$U = \{x | x \text{ is a positive integer less than } 10\}$  list the elements of**

$$(9) \quad A \cup B'$$

**Solution:**

**In tabular form**

$$A = \{2, 4, 6, 8\}; B = \{1, 3, 5, 7, 9\}$$

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$U - B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 3, 5, 7, 9\}$$

$$\begin{aligned}
 B' &= \{2,4,6,8\} \\
 A \cup B' &= \{2,4,6,8\} \cup \{2,4,6,8\} \\
 A \cup B' &= \{2,4,6,8\} \text{ Ans.}
 \end{aligned}$$

$$(10) \quad A' \cap B$$

**Solution:**

$$\begin{aligned}
 U - A &= \{1,2,3,4,5,6,7,8,9\} - \{2,4,6,8\} \\
 A' &= \{1,3,5,7,9\} \\
 A' \cap B &= \{1,3,5,7,9\} \cap \{1,3,5,7,9\}
 \end{aligned}$$

$$(11) \quad A' \cap B'$$

**Solution:**

$$\begin{aligned}
 U - A &= \{1,2,3,4,5,6,7,8,9\} - \{2,4,6,8\} \\
 A' &= \{1,3,5,7,9\} \\
 U - B &= \{1,2,3,4,5,6,7,8,9\} - \{1,3,5,7,9\} \\
 B' &= \{2,4,6,8\} \\
 A' \cap B' &= \{1,3,5,7,9\} \cap \{2,4,6,8\} \\
 A' \cap B' &= \{ \}
 \end{aligned}$$

$$(12) \quad A \Delta B$$

**Solution:**

$$\begin{aligned}
 A \Delta B &= \{2,4,6,8\} \Delta \{1,3,5,7,9\} \\
 A \Delta B &= \{1,2,3,4,5,6,7,8,9\}
 \end{aligned}$$

$$(13) \quad A - B'$$

**Solution:**

$$\begin{aligned}
 U - B &= \{1,2,3,4,5,6,7,8,9\} - \{1,3,5,7,9\} \\
 B' &= \{2,4,6,8\} \\
 A - B' &= \{2,4,6,8\} - \{2,4,6,8\} \\
 A - B' &= \{ \}
 \end{aligned}$$

$$(14) \quad A' \Delta B$$

**Solution:**

$$\begin{aligned}
 U - A &= \{1,2,3,4,5,6,7,8,9\} - \{2,4,6,8\} \\
 A' &= \{1,3,5,7,9\} \\
 A' \Delta B &= \{1,3,5,7,9\} \Delta \{1,3,5,7,9\} \\
 A' \Delta B &= \{ \}
 \end{aligned}$$

(15)  $(A' \cap B)'$

**Solution:**

$$U - A = \{1,2,3,4,5,6,7,8\} - \{2,4,6,8\}$$

$$A' = \{1,3,5,7,9\}$$

$$A' \cap B = \{1,3,5,7,9\} \cap \{1,3,5,7,9\}$$

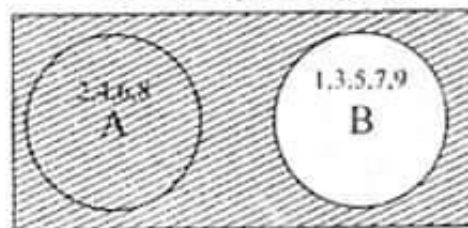
$$A' \cap B = \{1,3,5,7,9\}$$

$$U - (A' \cap B) = \{1,2,3,4,5,6,7,8,9\} - \{1,3,5,7,9\}$$

$$(A' \cap B)' = \{2,4,6,8\}$$

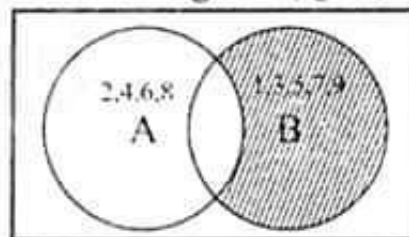
**Q16. Draw the venn diagrams for the sets in Questions # 9,10,11,12,13,14,15**

**Venn diagram (Q.9)**



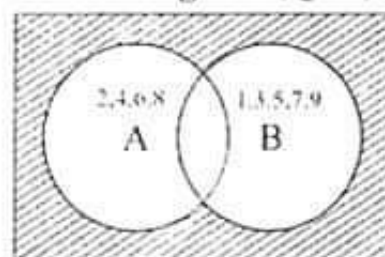
**$A \cup B'$  is shaded**

**Venn diagram (Q.10)**



**$A' \cap B$  is shaded**

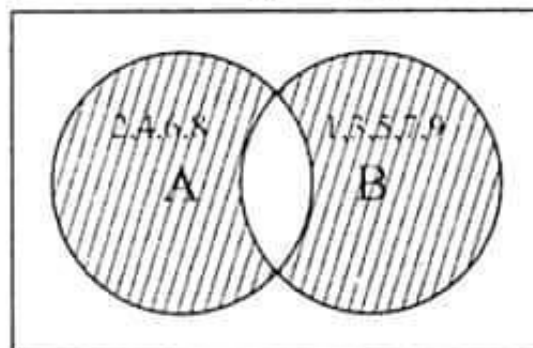
**Venn diagram (Q.11)**



**$A' \cap B'$  is shaded**

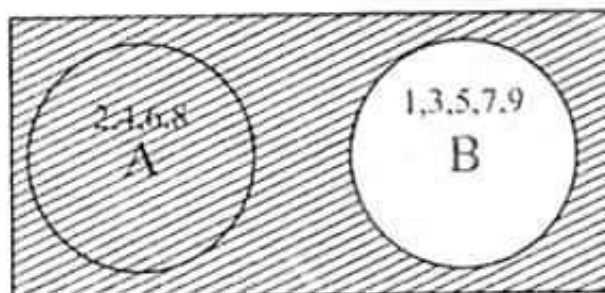


Venn diagram (Q.12)



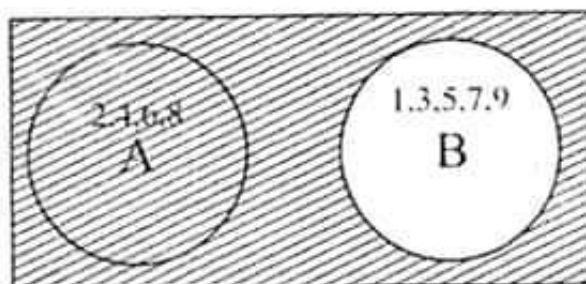
$A \Delta B$  is shaded

Venn diagram (Q.13)



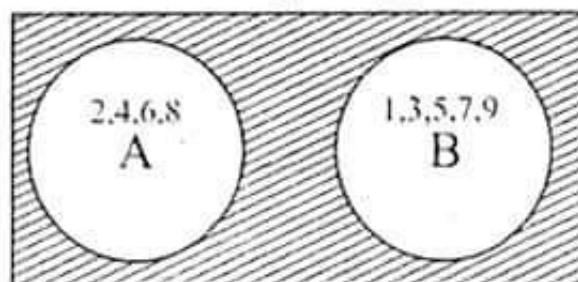
$A - B'$  is shaded

Venn diagram (Q.14)



$A' \Delta B$  is shaded

Venn diagram (Q.15)



$(A' \cap B)'$  is shaded

If  $A = \{1,2,3,4\}$  and  $B = \{2,4,6,8\}$  show that:

$$(17) \quad A \Delta B = (A \cup B) - (A \cap B)$$

Solution: Taking L.H.S

$$\begin{aligned} \text{L.H.S} &= A \Delta B \\ &= \{1,2,3,4\} \Delta \{2,4,6,8\} \\ &= \{1,3,6,8\} \text{---(1)} \\ A \cup B &= \{1,2,3,4\} \cup \{2,4,6,8\} \\ A \cup B &= \{1,2,3,4,6,8\} \\ A \cap B &= \{1,2,3,4\} \cap \{2,4,6,8\} \\ A \cap B &= \{2,4\} \\ \text{R.H.S} &= (A \cup B) - (A \cap B) \\ &= \{1,2,3,4,6,8\} - \{2,4\} \\ &= \{1,3,6,8\} \text{---(2)} \end{aligned}$$

By (1) and (2)

$$A \Delta B = (A \cup B) - (A \cap B) \quad \text{Proved.}$$

$$(18) \quad A \Delta B = (A - B) \cup (B - A)$$

Solution: Taking L.H.S

$$\begin{aligned} \text{L.H.S} &= A \Delta B \\ &= \{1,2,3,4\} \Delta \{2,4,6,8\} \\ &= \{1,3,6,8\} \text{---(1)} \\ A - B &= \{1,2,3,4\} - \{2,4,6,8\} \\ A - B &= \{1,3\} \\ B - A &= \{2,4,6,8\} - \{1,2,3,4\} \\ B - A &= \{6,8\} \\ \text{R.H.S} &= (A - B) \cup (B - A) \\ &= \{1,3\} \cup \{6,8\} \\ &= \{1,3,6,8\} \text{---(2)} \end{aligned}$$

By (1) and (2)

$$A \Delta B = (A \cup B) - (A \cap B) \quad \text{Proved.}$$

$$(19) \quad A - B = A - (A \cap B)$$

**Solution:** Taking L.H.S

$$\begin{aligned} \text{L.H.S} &= A - B \\ &= \{1,2,3,4\} - \{2,4,6,8\} \\ &= \{1,3\} \text{---(1)} \end{aligned}$$

$$A \cap B = \{1,2,3,4\} \cap \{2,4,6,8\}$$

$$A \cap B = \{2,4\}$$

$$\begin{aligned} \text{R.H.S} &= A - (A \cap B) \\ &= \{1,2,3,4\} - \{2,4\} \\ &= \{1,3\} \text{---(2)} \end{aligned}$$

By (1) and (2)

$$A - B = A - (A \cap B) \quad \text{Proved.}$$

**Q20.** If  $U = \{1,2,3,\dots, 20\}$ ,  $A = \{1,2,4,8,10,16,20\}$  and  $B = \{2,6,8,10,14,18\}$  verify De Morgan's laws.

**Solution:** (1)  $(A \cup B)' = A' \cap B'$

For L.H.S

$$A \cup B = \{1,2,4,8,10,16,20\} \cup \{2,6,8,10,14,18\}$$

$$A \cup B = \{1,2,4,8,10,14,16,18,20\}$$

$$U - (A \cup B) = \{1,2,3,\dots, 20\} - \{1,2,4,8,10,14,16,18,20\}$$

$$(A \cup B)' = \{5,6,7,9,11,12,13,14,15,17,19\} \text{---(1)}$$

For R.H.S

$$U - A = \{1,2,3,4,5,\dots,20\} - \{1,2,4,8,10,16,20\}$$

$$A' = \{3,5,6,7,9,11,12,13,14,15,17,18,19\}$$

$$U - B = \{1,2,3,4,5,\dots,20\} - \{2,4,6,8,10,14,18\}$$

$$B' = \{1,3,5,7,9,11,12,13,15,16,17,19\}$$

$$\begin{aligned} A' \cap B' &= \{3,5,6,7,9,11,12,13,14,15,17,18,19\} \cap \\ &\quad \{1,3,5,7,9,11,12,13,15,16,17,19\} \end{aligned}$$

$$A' \cap B' = \{5,6,7,9,11,12,13,14,15,17,19\} \text{---(2)}$$

By (1) and (2)

$$(A \cup B)' = A' \cap B'$$



$$(2) \quad (A \cap B)' = A' \cup B'$$

**Solution:** For L.H.S.

$$A \cap B = \{1, 2, 4, 8, 10, 16, 20\} \cap \{2, 6, 8, 10, 14, 18\}$$

$$A \cap B = \{2, 8, 10\}$$

$$U - (A \cap B) = \{1, 2, 3, \dots, 20\} - \{2, 8, 10\}$$

$$(A \cap B)' = \{1, 3, 4, 5, 6, 7, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\} \text{ ----- (1)}$$

**FOR R.H.S**

$$U - A = \{1, 2, 3, 4, 5, \dots, 20\} - \{1, 2, 4, 8, 10, 16, 20\}$$

$$A' = \{3, 5, 6, 7, 9, 11, 12, 13, 14, 15, 17, 18, 19\}$$

$$U - B = \{1, 2, 3, 4, 5, \dots, 20\} - \{2, 4, 6, 8, 10, 14, 18\}$$

$$B' = \{1, 3, 5, 7, 9, 11, 12, 13, 15, 16, 17, 19\}$$

$$A' \cup B' = \{3, 5, 6, 7, 9, 11, 12, 13, 14, 15, 17, 18, 19\}$$

$$\cup \{1, 3, 5, 7, 9, 11, 12, 13, 15, 16, 17, 19\}$$

$$A' \cup B' = \{1, 3, 5, 6, 7, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\} \text{ ----- (2)}$$

by (1) & (2)

$$(A \cap B)' = A' \cup B' \quad \text{Proved}$$

**Q21. Verify the commutative property of union and intersection for the following sets.**

(a)  $A = \{1, 2, 3, 4, 5\}, B = \{3, 5, 7, 9\}$

**Solution:** Commutative property of Union is  $A \cup B = B \cup A$

**Taking L.H.S**

$$A \cup B = \{1, 2, 3, 4, 5\} \cup \{3, 5, 7, 9\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 7, 9\} \text{ ----- (1)}$$

**Taking R.H.S**

$$B \cup A = \{3, 5, 7, 9\} \cup \{1, 2, 3, 4, 5\}$$

$$B \cup A = \{1, 2, 3, 4, 5, 7, 9\} \text{ ----- (2)}$$

By (1) and (2)  $A \cup B = B \cup A$

Commutative property for intersection  $A \cap B = B \cap A$

**Taking L.H.S**

$$\begin{aligned}\text{L.H.S} &= A \cap B \\ &= \{1,2,3,4,5\} \cap \{3,5,7,9\} \\ &= \{3,5\} \text{---(1)}\end{aligned}$$

**Taking R.H.S**

$$\begin{aligned}\text{R.H.S} &= B \cap A \\ &= \{3,5,7,9\} \cap \{1,2,3,4,5\} \\ &= \{3,5\} \text{---(2)}\end{aligned}$$

By (1) and (2)

$$A \cap B = B \cap A \quad \text{Proved.}$$

$$(b) \quad A = \{x | x \in \mathbb{Z}^+ \text{ and } x \leq 5\}$$

$$B = \{x | x \in \mathbb{Z} \text{ and } 1 \leq x \leq 4\}$$

**Solution:**  $A = \{1,2,3,4,5\}$

$$B = \{1,2,3,4\}$$

Commutative property of union.  $A \cup B = B \cup A$

**Taking L.H.S**

$$\begin{aligned}\text{L.H.S} &= A \cup B \\ &= \{1,2,3,4,5\} \cup \{1,2,3,4\} \\ &= \{1,2,3,4,5\} \text{---(1)}\end{aligned}$$

**Taking R.H.S**

$$\begin{aligned}\text{R.H.S} &= B \cup A \\ &= \{1,2,3,4\} \cup \{1,2,3,4,5\} \\ &= \{1,2,3,4,5\} \text{---(2)}\end{aligned}$$

By (1) and (2)

$$A \cup B = B \cup A$$

Commutative property of intersection  $A \cap B = B \cap A$

**Taking L.H.S**

$$\begin{aligned}\text{L.H.S} &= A \cap B \\ &= \{1,2,3,4,5\} \cap \{1,2,3,4\} \\ &= \{1,2,3,4\} \text{---(1)}\end{aligned}$$

**Taking R.H.S**

$$\begin{aligned}\text{R.H.S} &= B \cap A \\ &= \{1,2,3,4\} \cap \{1,2,3,4,5\} \\ &= \{1,2,3,4\} \text{--- (2)}\end{aligned}$$

By (1) and (2)

$$A \cap B = B \cap A$$

**Q22. Verify the following properties for the sets given below.**

- (i) Associative property of union and of intersection.
- (ii) Distributive properties of union over intersection.
- (iii) Distributive property of intersection over union.
- (a)  $A = \{1,2,3,4,5\}$ ,  $B = \{2,4,6,8\}$ ;  $C = \{4,8,10,12\}$

**Solution:**

- (i) Associative property of union and of intersection.

$$A \cup (B \cap C) = (A \cup B) \cap C$$

**Taking L.H.S**

$$\begin{aligned}B \cap C &= \{2,4,6,8\} \cap \{4,8,10,12\} \\ B \cap C &= \{2,4,6,8,10,12\} \\ A \cup (B \cap C) &= \{1,2,3,4,5\} \cup \{2,4,6,8,10,12\} \\ &= \{1,2,3,4,5,6,8,10,12\} \text{--- (1)}\end{aligned}$$

**Taking R.H.S**

$$\begin{aligned}A \cup B &= \{1,2,3,4,5\} \cup \{2,4,6,8\} \\ &= \{1,2,3,4,5,6,8\} \\ (A \cup B) \cap C &= \{1,2,3,4,5,6,8\} \cap \{4,8,10,12\} \\ &= \{1,2,3,4,5,6,8,10,12\} \text{--- (2)}\end{aligned}$$

By (1) and (2)

$$A \cup (B \cap C) = (A \cup B) \cap C$$

- (ii) Distributive property of union over intersection.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

**Taking L.H.S**

$$\begin{aligned}B \cap C &= \{2,4,6,8\} \cap \{4,8,10,12\} \\ &= \{4,8\}\end{aligned}$$

$$\begin{aligned} A \cup (B \cap C) &= \{1,2,3,4,5\} \cup \{4,8\} \\ &= \{1,2,3,4,5,8\} \text{---(1)} \end{aligned}$$

**Taking R.H.S**

$$\begin{aligned} A \cup B &= \{1,2,3,4,5\} \cup \{2,4,6,8\} \\ A \cup B &= \{1,2,3,4,5,6,8\} \\ A \cup C &= \{1,2,3,4,5\} \cup \{4,8,10,12\} \\ A \cup C &= \{1,2,3,4,5,8,10,12\} \\ (A \cup B) \cap (A \cup C) &= \{1,2,3,4,5,6,8\} \cap \{1,2,3,4,5,8,10,12\} \\ &= \{1,2,3,4,5,8\} \text{---(2)} \end{aligned}$$

By (1) and (2)

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(iii) **Distributive property of intersection over union.**

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

**Taking L.H.S**

$$\begin{aligned} B \cup C &= \{2,4,6,8\} \cup \{4,8,10,12\} \\ &= \{2,4,6,8,10,12\} \\ A \cap (B \cup C) &= \{1,2,3,4,5\} \cap \{2,4,6,8,10,12\} \\ &= \{2,4\} \text{---(1)} \end{aligned}$$

**Taking R.H.S**

$$\begin{aligned} A \cap B &= \{1,2,3,4,5\} \cap \{2,4,6,8\} \\ A \cap B &= \{2,4\} \\ A \cap C &= \{1,2,3,4,5\} \cap \{4,8,10,12\} \\ &= \{4\} \end{aligned}$$

$$\begin{aligned} (A \cap B) \cup (A \cap C) &= \{2,4\} \cup \{4\} \\ &= \{2,4\} \text{---(2)} \end{aligned}$$

By (1) and (2)

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \quad \text{Proved.}$$

$$\begin{aligned} \text{(ii)} \quad A &= \{x | x \in \mathbb{Z}^+ \text{ and } x \leq 4\} \\ B &= \{x | x \in \mathbb{Z} \text{ and } 0 < x < 5\} \\ C &= \{1,2,3\} \end{aligned}$$

**Solution:**  $A = \{1,2,3,4\}$   
 $B = \{1,2,3,4\}; \quad C = \{1,2,3\}$



(i) **Associative property of union and of intersection.**

$$A \cup (B \cap C) = (A \cup B) \cap C$$

**Taking L.H.S**

$$B \cap C = \{1,2,3,4\} \cap \{1,2,3\}$$

$$B \cap C = \{1,2,3\}$$

$$A \cup (B \cap C) = \{1,2,3,4\} \cup \{1,2,3\}$$

$$= \{1,2,3,4\} \text{---(1)}$$

**Taking R.H.S**

$$A \cup B = \{1,2,3,4\} \cup \{1,2,3,4\}$$

$$= \{1,2,3,4\}$$

$$(A \cup B) \cap C = \{1,2,3,4\} \cap \{1,2,3\}$$

$$= \{1,2,3,4\} \text{---(2)}$$

By (1) and (2)

$$A \cup (B \cap C) = (A \cup B) \cap C$$

(ii) **Distributive property of union over intersection.**

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

**Taking L.H.S**

$$B \cap C = \{1,2,3,4\} \cap \{1,2,3\}$$

$$= \{1,2,3\}$$

$$A \cup (B \cap C) = \{1,2,3,4\} \cup \{1,2,3\}$$

$$= \{1,2,3,4\} \text{---(1)}$$

**Taking R.H.S**

$$A \cup B = \{1,2,3,4\} \cup \{1,2,3,4\}$$

$$A \cup B = \{1,2,3,4\}$$

$$A \cup C = \{1,2,3,4\} \cup \{1,2,3\}$$

$$A \cup C = \{1,2,3,4\}$$

$$(A \cup B) \cap (A \cup C) = \{1,2,3,4\} \cap \{1,2,3,4\}$$

$$= \{1,2,3,4\} \text{---(2)}$$

By (1) and (2)

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(iii) **Distributive property of intersection over union.**

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

**Taking L.H.S**

$$\begin{aligned}B \cup C &= \{1,2,3,4\} \cup \{1,2,3\} \\&= \{1,2,3,4\} \\A \cap (B \cup C) &= \{1,2,3,4\} \cap \{1,2,3,4\} \\&= \{1,2,3,4\} \text{---(1)}\end{aligned}$$

**Taking R.H.S**

$$\begin{aligned}A \cap B &= \{1,2,3,4\} \cap \{1,2,3,4\} \\&= \{1,2,3,4\} \\A \cap C &= \{1,2,3,4\} \cap \{1,2,3\} \\&= \{1,2,3\} \\(A \cap B) \cup (A \cap C) &= \{1,2,3,4\} \cup \{1,2,3\} \\&= \{1,2,3,4\} \text{---(2)}\end{aligned}$$

By (1) and (2)

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \quad \textbf{Proved.}$$

### EXERCISE 1.3

Q1. If  $A = \{a, b, c, d\}$  and  $B = \{y, z\}$ .

Find (i)  $A \times B$  (ii)  $B \times A$  (iii)  $A \times A$  (iv)  $B \times B$  Show that  $A \times B \neq B \times A$  in general.

Solution:

$$\begin{aligned} \text{(i)} \quad A \times B &= \{a, b, c, d\} \times \{y, z\} \\ &= \{ (a, y), (a, z), (b, y), (b, z), (c, y), (c, z), (d, y), (d, z) \} \end{aligned}$$

Ans.

$$\begin{aligned} \text{(ii)} \quad B \times A &= \{y, z\} \times \{a, b, c, d\} \\ &= \{ (y, a), (y, b), (y, c), (y, d), (z, a), (z, b), (z, c), (z, d) \} \end{aligned}$$

Ans.

$$\begin{aligned} \text{(iii)} \quad A \times A &= \{a, b, c, d\} \times \{a, b, c, d\} \\ &= \{ (a, a), (a, b), (a, c), (a, d), (b, a), (b, b), (b, c), (b, d), \\ &\quad (c, a), (c, b), (c, c), (c, d), (d, a), (d, b), (d, c), (d, d) \} \end{aligned}$$

Ans.

$$\begin{aligned} \text{(iv)} \quad B \times B &= \{y, z\} \times \{y, z\} \\ &= \{ (y, y), (y, z), (z, y), (z, z) \} \end{aligned}$$

Ans.

Show that  $A \times B \neq B \times A$  in general.

$$\begin{aligned} A \times B &= \{a, b, c, d\} \times \{y, z\} \\ &= \{(a, y), (a, z), (b, y), (b, z), (c, y), (c, z), (d, y), (d, z)\} \text{---(1)} \end{aligned}$$

$$\begin{aligned} B \times A &= \{y, z\} \times \{a, b, c, d\} \\ &= \{(y, a), (y, b), (y, c), (y, d), (z, a), (z, b), (z, c), (z, d)\} \text{---(2)} \end{aligned}$$

By (1) and (2)

$$A \times B \neq B \times A \quad \text{Proved.}$$

Q2. Suppose that the ordered pairs  $(x+y, 2)$  and  $(4, x-y)$  are equal. Find  $x$  and  $y$ .

Solution: given that

$$(x+y, 2) = (4, x-y)$$

on equating

$$\text{on Adding} \quad x + y = 4 \text{ --- (1)}$$

$$x - y = 2 \text{ --- (2)}$$

$$2x = 6$$

$$x = \frac{6}{2}$$

$$\boxed{x = 3} \quad \text{Put the value of } x \text{ in (1)}$$

$$(1) \Rightarrow 3 + y = 4$$

$$y = 4 - 3$$

$$\boxed{y = 1} \quad \text{Ans.}$$

Q3. Let  $A = \{a, b\}$ ,  $B = \{2, 3\}$  and  $C = \{3, 4\}$  find,

(i)  $A \times (B \cup C)$

Solution:

$$B \cup C = \{2, 3\} \cup \{3, 4\}$$

$$B \cup C = \{2, 3, 4\}$$

$$A \times (B \cup C) = \{a, b\} \times \{2, 3, 4\}$$

$$= \{(a, 2), (a, 3), (a, 4), (b, 2), (b, 3), (b, 4)\} \quad \text{Ans.}$$



$$(ii) \quad (A \times B) \cup (A \times C)$$

**Solution:**

$$\begin{aligned} A \times B &= \{a, b\} \times \{2, 3\} \\ &= \{ (a, 2), (a, 3), (b, 2), (b, 3) \} \end{aligned}$$

$$\begin{aligned} A \times C &= \{a, b\} \times \{3, 4\} \\ &= \{ (a, 3), (a, 4), (b, 3), (b, 4) \} \end{aligned}$$

$$\begin{aligned} (A \times B) \cup (A \times C) &= \{ (a, 2), (a, 3), (b, 2), (b, 3) \} \cup \{ (a, 3), (a, 4), (b, 3), (b, 4) \} \\ &= \{ (a, 2), (a, 3), (a, 4), (b, 2), (b, 3), (b, 4) \} \quad \text{Ans.} \end{aligned}$$

$$(iii) \quad A \times (B \cap C)$$

**Solution:**

$$B \cap C = \{2, 3\} \cap \{3, 4\}$$

$$B \cap C = \{3\}$$

$$\begin{aligned} A \times (B \cap C) &= \{a, b\} \times \{3\} \\ &= \{ (a, 3), (b, 3) \} \quad \text{Ans.} \end{aligned}$$

$$(iv) \quad (A \times B) \cap (A \times C)$$

**Solution:**

$$\begin{aligned} A \times B &= \{a, b\} \times \{2, 3\} \\ &= \{ (a, 2), (a, 3), (b, 2), (b, 3) \} \end{aligned}$$

$$\begin{aligned} A \times C &= \{a, b\} \times \{3, 4\} \\ &= \{ (a, 3), (a, 4), (b, 3), (b, 4) \} \end{aligned}$$

$$(A \times B) \cap (A \times C) = \{ (a, 2), (a, 3), (b, 2), (b, 3) \} \cap \{ (a, 3), (a, 4), (b, 3), (b, 4) \}$$

$$(A \times B) \cap (A \times C) = \{ (a, 3), (b, 3) \} \quad \text{Ans.}$$

**Q4. For the sets given in Question # 3 find,**

$$(i) \quad A \times (B - C)$$

**Solution:**

$$B - C = \{2, 3\} - \{3, 4\}$$

$$B - C = \{2\}$$

$$A \times (B - C) = \{a, b\} \times \{2\}$$

$$A \times (B - C) = \{ (a, 2), (b, 2) \} \quad \text{Ans.}$$

$$(ii) \quad A \times (C - B)$$

**Solution:**

$$C - B = \{3,4\} - \{2,3\}$$

$$C - B = \{4\}$$

$$A \times (C - B) = \{a,b\} \times \{4\}$$

$$A \times (C - B) = \{(a,4), (b,4)\} \quad \text{Ans.}$$

$$(iii) \quad A \times (B \cap C)$$

**Solution:**

$$B \cap C = \{2,3\} \cap \{3,4\}$$

$$B \cap C = \{3\}$$

$$A \times (B \cap C) = \{a,b\} \times \{3\}$$

$$= \{(a,3), (b,3)\} \quad \text{Ans.}$$

$$(iv) \quad (A \times B) \cap (A \times C)$$

**Solution:**

$$A \times B = \{a,b\} \times \{2,3\}$$

$$= \{(a,2), (a,3), (b,2), (b,3)\}$$

$$A \times C = \{a,b\} \times \{3,4\}$$

$$= \{(a,3), (a,4), (b,3), (b,4)\}$$

$$(A \times B) \cap (A \times C) = \{(a,2), (a,3), (b,2), (b,3)\} \cap \{(a,3), (a,4), (b,3), (b,4)\}$$

$$(A \times B) \cap (A \times C) = \{(a,3), (b,3)\} \quad \text{Ans.}$$

**Q5.** Let  $A = \{1,2,3,4\}$ ,  $B = \{2,4,5,6\}$  and  $C = \{2,3,6,8\}$ . Find:

$$(i) \quad (A - B) \times (B - C)$$

**Solution:**

$$A - B = \{1,2,3,4\} - \{2,4,5,6\}$$

$$A - B = \{1,3\}$$

$$B - C = \{2,4,5,6\} - \{2,3,6,8\}$$

$$B - C = \{4,5\}$$

$$(A - B) \times (B - C) = \{1,3\} \times \{4,5\}$$

$$= \{(1,4), (1,5), (3,4), (3,5)\} \quad \text{Ans.}$$

$$(ii) \quad (A \cap B) \times (B \cap C)$$

**Solution:**

$$A \cap B = \{1,2,3,4\} \cap \{2,4,5,6\}$$

$$A \cap B = \{2,4\}$$

$$B \cap C = \{2,4,5,6\} \cap \{2,3,6,8\}$$

$$B \cap C = \{2,6\}$$

$$(A \cap B) \times (B \cap C) = \{2,4\} \times \{2,6\}$$

$$= \{(2,2), (2,6), (4,2), (4,6)\} \quad \text{Ans.}$$

$$(iii) \quad (A \times B) \cap (B \times C)$$

**Solution:**

$$A \times B = \{1,2,3,4\} \times \{2,4,5,6\}$$

$$= \{(1,2), (1,4), (1,5), (1,6), (2,2), (2,4), (2,5), (2,6), (3,2), (3,4), (3,5), (3,6), (4,2), (4,4), (4,5), (4,6)\}$$

$$B \times C = \{2,4,5,6\} \times \{2,3,6,8\}$$

$$= \{(2,2), (2,3), (2,6), (2,8), (4,2), (4,3), (4,6), (4,8), (5,2), (5,3), (5,6), (5,8), (6,2), (6,3), (6,6), (6,8)\}$$

$$(A \times B) \cap (B \times C) = \{(1,2), (1,4), (1,5), (1,6), (2,2), (2,4), (2,5), (2,6), (3,2), (3,4), (3,5), (3,6), (4,2), (4,4), (4,5), (4,6)\} \\ \cap \{(2,2), (2,3), (2,6), (2,8), (4,2), (4,3), (4,6), (4,8), (5,2), (5,3), (5,6), (5,8), (6,2), (6,3), (6,6), (6,8)\}$$

$$(A \times B) \cap (B \times C) = \{(2,2), (2,6), (4,2), (4,6)\} \quad \text{Ans.}$$

$$(iv) \quad (A \times B) - (B \times C)$$

**Solution:**

$$(A \times B) = \{1,2,3,4\} \times \{2,4,5,6\}$$

$$= \{(1,2), (1,4), (1,5), (1,6), (2,2), (2,4), (2,5), (2,6), (3,2), (3,4), (3,5), (3,6), (4,2), (4,4), (4,5), (4,6)\}$$

$$B \times C = \{2,4,5,6\} \times \{2,3,6,8\}$$

$$= \{(2,2), (2,3), (2,6), (2,8), (4,2), (4,3), (4,6), (4,8), (5,2), (5,3), (5,6), (5,8), (6,2), (6,3), (6,6), (6,8)\}$$

$$(A \times B) - (B \times C) = \{(1,2), (1,4), (1,5), (1,6), (2,2), (2,4), (2,5), (2,6), (3,2), (3,4), (3,5), (3,6), (4,2), (4,4), (4,5), (4,6)\} - \{(2,2), (2,3), (2,6), (2,8), (4,2), (4,3), (4,6), (4,8), (5,2), (5,3), (5,6), (5,8), (6,2), (6,3), (6,6), (6,8)\}$$

(4,3), (4,6), (4,8), (5,2), (5,3), (5,6), (5,8),  
(6,2), (6,3), (6,6), (6,8)}

$$(A \times B) - (B \times C) = \{(1,2), (1,4), (1,5), (1,6), (2,4), (2,5), \\ (3,2), (3,4), (3,5), (3,6), (4,4), (4,5)\}$$

Ans.

(v)  $(A \Delta B) \times (B \cap C)$

Solution:

$$A \Delta B = \{1,2,3,4\} \Delta \{2,4,5,6\}$$

$$A \Delta B = \{1,3,5,6\}$$

$$B \cap C = \{2,4,5,6\} \cap \{2,3,6,8\}$$

$$B \cap C = \{2,6\}$$

$$(A \Delta B) \times (B \cap C) = \{1,3,5,6\} \times \{2,6\} \\ = \{(1,2), (1,6), (3,2), (3,6), (5,2), (5,6), (6,2), (6,6)\}$$

Ans.

(vi)  $(B \times C) \Delta (C \times A)$

Solution:

$$B \times C = \{2,4,5,6\} \times \{2,3,6,8\} \\ = \{(2,2), (2,3), (2,6), (2,8), (4,2), (4,3), (4,6), (4,8), \\ (5,2), (5,3), (5,6), (5,8), (6,2), (6,3), (6,6), (6,8)\}$$

$$C \times A = \{2,3,6,8\} \times \{1,2,3,4\} \\ = \{(2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), \\ (6,1), (6,2), (6,3), (6,4), (8,1), (8,2), (8,3), (8,4)\}$$

$$(B \times C) \Delta (C \times A) = \{(2,2), (2,3), (2,6), (2,8), (4,2), (4,3), (4,6), \\ (4,8), (5,2), (5,3), (5,6), (5,8), (6,2), (6,3), \\ (6,6), (6,8)\} \Delta \{(2,1), (2,2), (2,3), (2,4), \\ (3,1), (3,2), (3,3), (3,4), (6,1), (6,2), (6,3), (6,4), \\ (8,1), (8,2), (8,3), (8,4)\}$$

$$(B \times A) \Delta (C \times A) = \{(2,6), (2,8), (4,2), (4,3), (4,6), (4,8), (5,2), \\ (5,3), (5,6), (5,8), (6,6), (6,8), (2,1), (2,4), \\ (3,1), (3,2), (3,3), (3,4), (6,1), (6,4), (8,1), (8,2), \\ (8,3)\}$$



**Q. 6: Let  $A = \{a,b,c\}$  and  $B = \{x,y\}$ . Write:**

**(i) Two relations in  $A \times B$ .**

**Solution:**

$$A \times B = \{a,b,c\} \times \{x,y\}$$

$$A \times B = \{(a,x), (a,y), (b,x), (b,y), (c,x), (c,y)\}$$

Two relations in  $A \times B$

$$R_1 = \{ \} ; R_2 = \{(a,x)\} \quad \text{Ans.}$$

**(ii) Two relations in  $B \times A$**

**Solution:**

$$B \times A = \{x,y\} \times \{a,b,c\}$$

$$= \{(x,a), (x,b), (x,c), (y,a), (y,b), (y,c)\}$$

Two relations in  $B \times A$

$$R_1 = \{ \} ; R_2 = \{(x,a)\} \quad \text{Ans.}$$

**(iii) Three relations in  $A$ .**

**Solution:**

$$A \times A = \{a,b,c\} \times \{a,b,c\}$$

$$= \{(a,a), (a,b), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c)\}$$

Three relations in  $A$ .

$$R_1 = \{ \} ; R_2 = \{(a,a)\}; R_3 = \{(a,b)\} \quad \text{Ans.}$$

**(iv) All relations in  $B$ .**

**Solution:**

$$B \times B = \{x,y\} \times \{x,y\}$$

$$= \{(x,x), (x,y), (y,x), (y,y)\}$$

All relations in  $B$   $2^4 = 16$

$$R_1 = \{ \} \quad R_9 = \{(x,y), (y,x)\}$$

$$R_2 = \{(x,x)\} \quad R_{10} = \{(x,y), (y,y)\}$$

$$R_3 = \{(x,y)\} \quad R_{11} = \{(y,x), (y,y)\}$$

$$R_4 = \{(y,x)\} \quad R_{12} = \{(x,x), (x,y), (y,x)\}$$

$$R_5 = \{(y,y)\} \quad R_{13} = \{(x,x), (x,y), (y,y)\}$$

$$R_6 = \{(x,x), (x,y)\} \quad R_{14} = \{(x,x), (x,y), (y,y)\}$$

$$R_7 = \{(x,x), (y,x)\} \quad R_{15} = \{(x,y), (y,x), (y,y)\}$$

$$R_8 = \{(x,x), (y,y)\} \quad R_{16} = \{(x,x), (x,y), (y,x), (y,y)\} \quad \text{Ans.}$$

**Q7. How many relations can  $A \times B$  have, if set A has four elements and set B has three elements?**

**Solution:**

$$n(A) = 4 ; \quad n(B) = 3$$

$$\begin{aligned} n(A \times B) &= n(A) \times n(B) \\ &= 4 \times 3 \\ &= 12 \end{aligned}$$

Number of relations in  $A \times B = 12^{12} = 4096$  Ans.

**Q8. List the ordered pairs in the relation R from  $A = \{1,2,3,4\}$  to  $B = \{0,1,2,3\}$  where  $(a,b) \in R$  if and only if:**

**Solution:**

$$\begin{aligned} A \times B &= \{1,2,3,4\} \times \{0,1,2,3\} \\ &= \{(1,0), (1,1), (1,2), (1,3), (2,0), (2,1), (2,2), (2,3), \\ &\quad (3,0), (3,1), (3,2), (3,3), (4,0), (4,1), (4,2), (4,3)\} \end{aligned}$$

(i)  **$a = b$**

(1,1), (2,2), and (3,3) are be the required ordered pairs.

(ii)  **$a + b = 4$**

(1,3), (2,2), (3,1) and (4,0) are be the required ordered pairs.

(iii)  **$a > b$**

(1,0), (2,0), (2,1), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2) and (4,3) are be the required ordered pairs.

(iv)  **$a$  divides  $b$**

(1,1), (2,2), (3,3), (1,0), (2,0), (3,0), (4,0), (1,2), (1,3) and (4,2) are the required ordered pairs.

**Q.9: If  $a, b$  represent elements of  $Z^+$ , the set of positive integers, find the domain and range of the relations in  $Z$  given by:**

$$R_1 = \{(a,b) \mid 2a + b = 10\}$$

$$R_2 = \{(a,b) \mid a+b = 8\}$$

$$R_3 = \{(a,b) \mid a-b = 8\}$$

**Solution:**

$$\text{As } Z = \{1,2,3,4,5,\dots\}$$

$$\text{As } 2a + b = 10$$

**For  $R_1$ :**

$$a = 1 \quad 2(1) + b = 10 \quad \text{then } b = 8$$

$$a = 2 \quad 2(2) + b = 10 \quad \text{then } b = 6$$

$$a = 3 \quad 2(3) + b = 10 \quad \text{then } b = 4$$

$$a = 4 \quad 2(a) + b = 10 \quad b = 2$$

Domain of  $R_1 = \{1, 2, 3, 4\}$  and

Range of  $R_1 = \{8, 6, 4, 2\}$

**For  $R_2$ :  $a + b = 8$**

$$a = 1 \quad 1 + b = 8 \quad b = 7$$

$$a = 2 \quad 2 + b = 8 \quad b = 6$$

$$a = 3 \quad 3 + b = 8 \quad b = 5$$

$$a = 4 \quad 4 + b = 8 \quad b = 4$$

Domain of  $R_2 = \{1, 2, 3, 4\}$

Range of  $R_2 = \{7, 6, 5, 4\}$

**For  $R_3$ :  $a - b = 8$**

$$a = 9 \quad 9 - b = 8 \quad b = 1$$

$$a = 10 \quad 10 - b = 8 \quad b = 2$$

$$a = 11 \quad 11 - b = 8 \quad b = 3$$

$$a = 12 \quad 12 - b = 8 \quad b = 4$$

Dom of  $R_3 = \{a > 8\}$  (or)  $\{x | x \in \mathbb{N} \wedge x \geq 9\}$

Range of  $R_3 = \mathbb{N}$  Ans.

**Q10.** The relations  $R = \{(a, b) \mid b = 2a\}$  in  $\mathbb{Z}$ , the set of integers, has the domain  $\{-1, 0, 1, 2\}$ . Find its range.

**Solution:** Domain =  $\{-1, 0, 1, 2\} = a$

As  $b = 2a$

$$b = 2(-1) = -2$$

$$b = 2(0) = 0$$

$$b = 2(1) = 2$$

$$b = 2(2) = 4$$

Range of  $R = \{-2, 0, 2, 4\}$  Ans.

**Q11.** The relation  $\{x,y) \mid y = x^2\}$  in the set  $Z$  has the domain  $Z^+$ . Find its range.

**Solution:**

Dom of  $Z^+ = \{1,2,3,4,5,\dots\} = x$

As  $y = x^2$

$$x = 1 \quad y = (1)^2 = 1$$

$$x = 2 \quad y = (2)^2 = 4$$

$$x = 3 \quad y = (3)^2 = 9$$

$$x = 4 \quad y = (4)^2 = 16$$

Range =  $\{1,4,9,16,\dots\}$       **Ans.**

**Q12.** The set  $A = \{1,2,3,4\}$  has the following relations in it. Find whether these are function or not. If they are functions, find their types.

(1)       $R = \{(1,2), (2,3), (3,4), (4,1)\}$

**Solution:**  $R_1$  is a function from  $A$  onto  $A$  because:

(a)      the entire set  $A$  is the domain of  $R_1$ .

(b)       $R_1$  produces only one image in  $A$  for each member of the domain.

$R_1$  is also a one-one and onto function because distinct elements of 1st set  $A$  are associated with distinct elements of 2nd set  $A$ .

(2)       $R_2 = \{ (1,2), (3,4), (4,1) \}$

**Solution:**  $R_2$  is not a function because the domain  $R_2 \neq A$ . The element 3 of set  $A$  is missing.

(3)       $R_3 = \{ (1,1), (1,2), (1,3), (1,4) \}$

**Solution:**  $R_3$  is not a function. A relation in  $A$  is a function of  $A$  into  $A$  if and only if each element of  $A$  appears as the first element in one and only one ordered pair in the relation. Here 1 is repeated.

(4)       $R_4 = \{ (2,1), (4,4), (3,1), (2,3) \}$

**Solution:**  $R_4$  is not a function as 2 is repeated as the first element.

Dom of  $R_4 \neq A$



**Q13.** List the 16 different relations on the set  $\{0,1\}$ . How many of the 16 different relations contains the pair  $(0,1)$ ?

**Solution:** If  $A = \{0,1\}$  then

$$\begin{aligned} A \times A &= \{0,1\} \times \{0,1\} \\ &= \{(0,0), (0,1), (1,0), (1,1)\} \end{aligned}$$

$$\begin{aligned} R_1 &= \{ \} & R_9 &= \{(0,1), (1,0)\} \\ R_2 &= \{(0,0)\} & R_{10} &= \{(0,1), (1,1)\} \\ R_3 &= \{(0,1)\} & R_{11} &= \{(0,0), (0,1), (1,0)\} \\ R_4 &= \{(1,0)\} & R_{12} &= \{(0,0), (0,1), (1,1)\} \\ R_5 &= \{(1,1)\} & R_{13} &= \{(0,1), (1,0), (1,1)\} \\ R_6 &= \{(0,0), (0,1)\} & R_{14} &= \{(0,0), (1,0), (1,1)\} \\ R_7 &= \{(0,0), (1,0)\} & R_{15} &= \{(1,0), (1,1)\} \\ R_8 &= \{(0,0), (1,1)\} & R_{16} &= \{(0,0), (0,1), (1,0), (1,1)\} \end{aligned}$$

There are 8 relations contains the point  $(0,1)$       **Ans.**

**Q14.** Let  $A = \{1,2,3\}$  and  $B = \{1,2,3,4\}$ . The relations  $R_1 = \{(1,1), (2,2), (3,3)\}$  and  $R_2 = \{(1,1), (1,2), (1,3), (1,4)\}$  are in  $A \times B$ . Find:

(a)  $R_1 \cup R_2$

**Solution:**

$$\begin{aligned} R_1 \cup R_2 &= \{(1,1), (2,2), (3,3)\} \cup \{(1,1), (1,2), (1,3), (1,4)\} \\ R_1 \cup R_2 &= \{(1,1), (1,2), (1,3), (1,4), (2,2), (3,3)\} \quad \text{Ans.} \end{aligned}$$

(b)  $R_1 \cap R_2$

**Solution:**

$$\begin{aligned} R_1 \cap R_2 &= \{(1,1), (2,2), (3,3)\} \cap \{(1,1), (1,2), (1,3), (1,4)\} \\ R_1 \cap R_2 &= \{(1,1)\} \quad \text{Ans.} \end{aligned}$$

(c)  $R_1 - R_2$

**Solution:**

$$\begin{aligned} R_1 - R_2 &= \{(1,1), (2,2), (3,3)\} - \{(1,1), (1,2), (1,3), (1,4)\} \\ R_1 - R_2 &= \{(2,2), (3,3)\} \quad \text{Ans.} \end{aligned}$$

(d)  $R_2 - R_1$

Solution:

$$R_2 - R_1 = \{(1,1), (1,2), (1,3), (1,4)\} - \{(1,1), (2,2), (3,3)\}$$

$$R_2 - R_1 = \{(1,2), (1,3), (1,4)\} \quad \text{Ans.}$$

(e)  $R_1 \Delta R_2$

Solution:

$$R_1 \Delta R_2 = \{(1,1), (2,2), (3,3)\} \Delta \{(1,1), (1,2), (1,3), (1,4)\}$$

$$R_1 \Delta R_2 = \{(1,2), (1,3), (1,4), (2,2), (3,3)\} \quad \text{Ans.}$$

**Q15.** Let  $f = \{(a,3), (b,2), (c,1), (d,3)\}$  be a function from  $\{a,b,c,d\}$  to  $\{1,2,3\}$ . Is  $f$  an onto function? Is  $f$  a one-one function?

Solution:

(1) Yes,  $f$  is an onto function because all the elements of Dom exist in  $f$

(2) No,  $f$  is not a one-one function  $n(\text{Dom}) \neq n(\text{Range})$

**Q16.** Let  $f = \{(a,3), (b,2), (c,1), (d,3)\}$  be a function from  $\{a,b,c,d\}$  to  $\{1,2,3,4\}$ . Is  $f$  one-one and onto?

Solution:

(1) No,  $f$  is not an onto function because  $\text{Dom of } f \neq \{1,2,3,4\}$

(2) No,  $f$  is not a one-one function because every elements of  $f$  does not hold distinct image.

**Q.17:** If  $A = \{1,2,3\}$ , then find,

(i) A function  $f$  from  $A$  to  $A$  which is one-one.

Solution:

$$A \times A = \{1,2,3\} \times \{1,2,3\}$$

$$= \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$$

then function is

$$f = \{(1,1), (2,2), (3,3)\} \quad \text{Ans.}$$

(ii) A function  $g$  from  $A$  to  $A$  which is onto.

Solution:

$$A \times A = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$$

then onto function  $g$  is

$$g = \{(1,2), (2,1), (3,3)\} \quad \text{Ans.}$$

(iii) A function  $h$  from  $A$  to  $A$  which is one-one and onto.

$$A \times A = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$$

then one-one and onto function  $h$  is.

$$h = \{(1,3), (2,1), (3,2)\} \quad \text{Ans.}$$

(iv) A function  $k$  from  $A$  to  $A$  which is neither one-one nor onto.

**Solution:**

$$A \times A = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$$

then a function  $k$  which is neither one-one nor onto is

$$k = \{(1,1), (2,1), (3,2)\} \quad \text{Ans.}$$

### EXERCISE 1.4

**Q1.** Write the quadrants in which the following points are located.

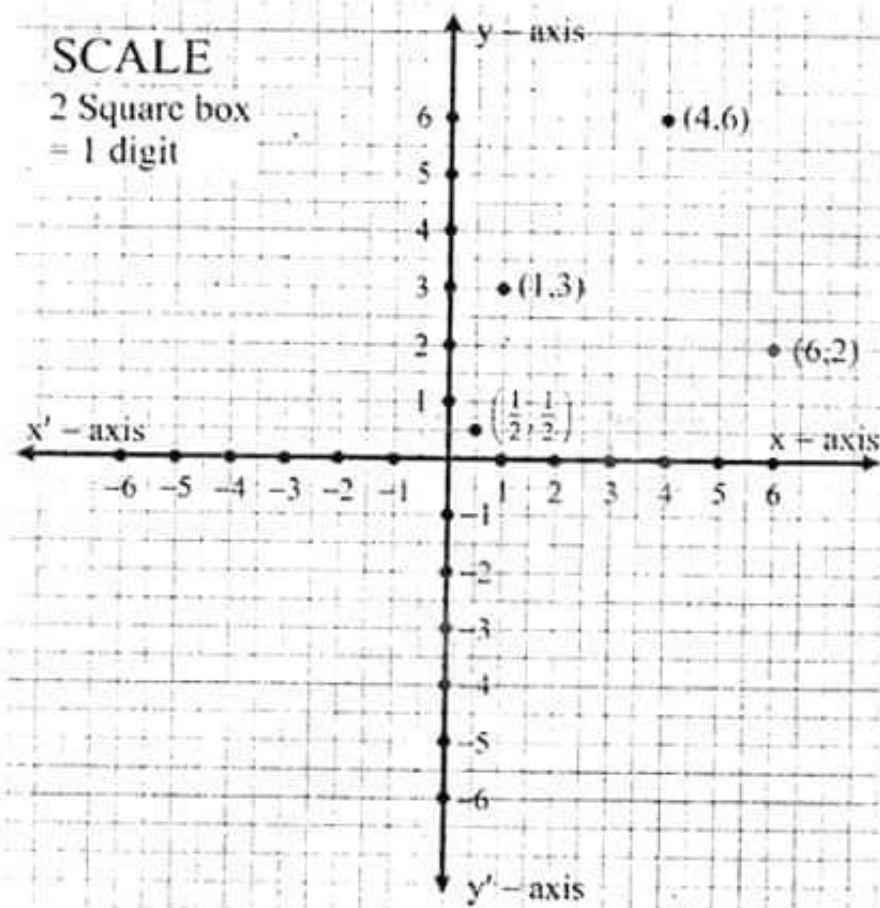
**Solution:**

Points	Quadrant	Points	Quadrant
$(1, 6)$	1st	$(3, 57)$	1st
$(\frac{1}{7}, \frac{4}{9})$	1st	$(\sqrt{5}, -6.54)$	4th
$(-1.7, 3)$	2nd	$(27, -72)$	4 <sup>th</sup>
$(\sqrt{3}, -4)$	4th	$(1.7, -2.7)$	4th
$(\sqrt{2}, -\sqrt{3})$	4th	$(-1, -11)$	3rd
$(-7, \frac{-3}{2})$	3rd	$(\sqrt{3}, -1.3)$	4th
$(\frac{-\sqrt{3}}{2}, -1.7)$	3rd	$(\frac{7}{2}, \frac{1}{2})$	1st

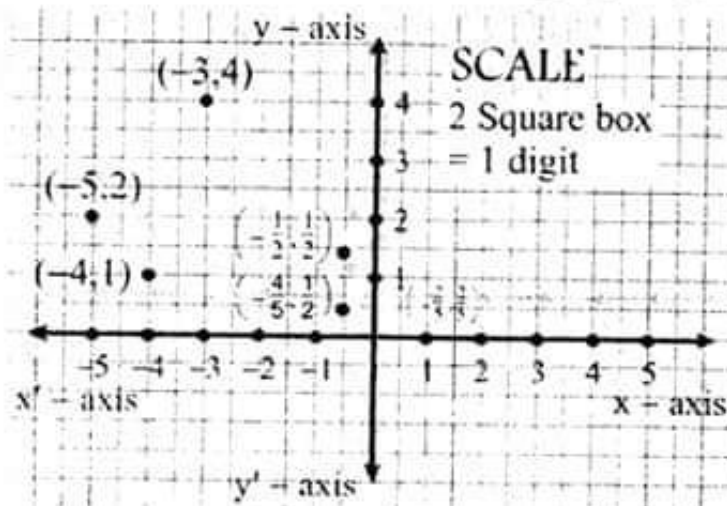


**Q2. Taking proper units, show the following points on a graph paper.**

- (i)  $(4,6), (6,2), (\frac{1}{2}, \frac{1}{2}), (1,3)$



- (ii)  $(-3,4), (-5,2), (-4,1), (-\frac{4}{5}, \frac{1}{2}), (-\frac{2}{5}, \frac{2}{5}), (-\frac{1}{2}, \frac{1}{2})$



Q3. If  $A = \{x | x \in \mathbb{N} \text{ and } 2 \leq x \leq 5\}$  and

$B = \{x | x \in \mathbb{Z} \text{ and } -6 < x < -3\}$

Find  $A \times B$ ,  $B \times A$  and  $A \times A$ .

Plot these sets on a graph paper.

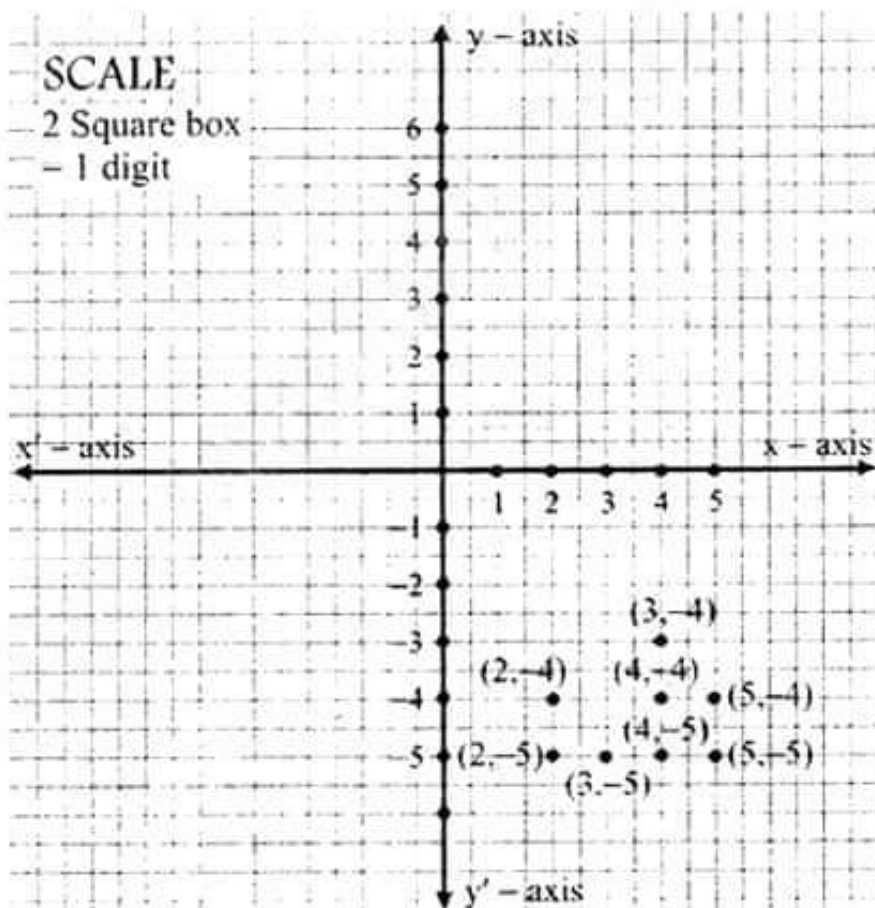
**Solution:**

$$A = \{2, 3, 4, 5\}$$

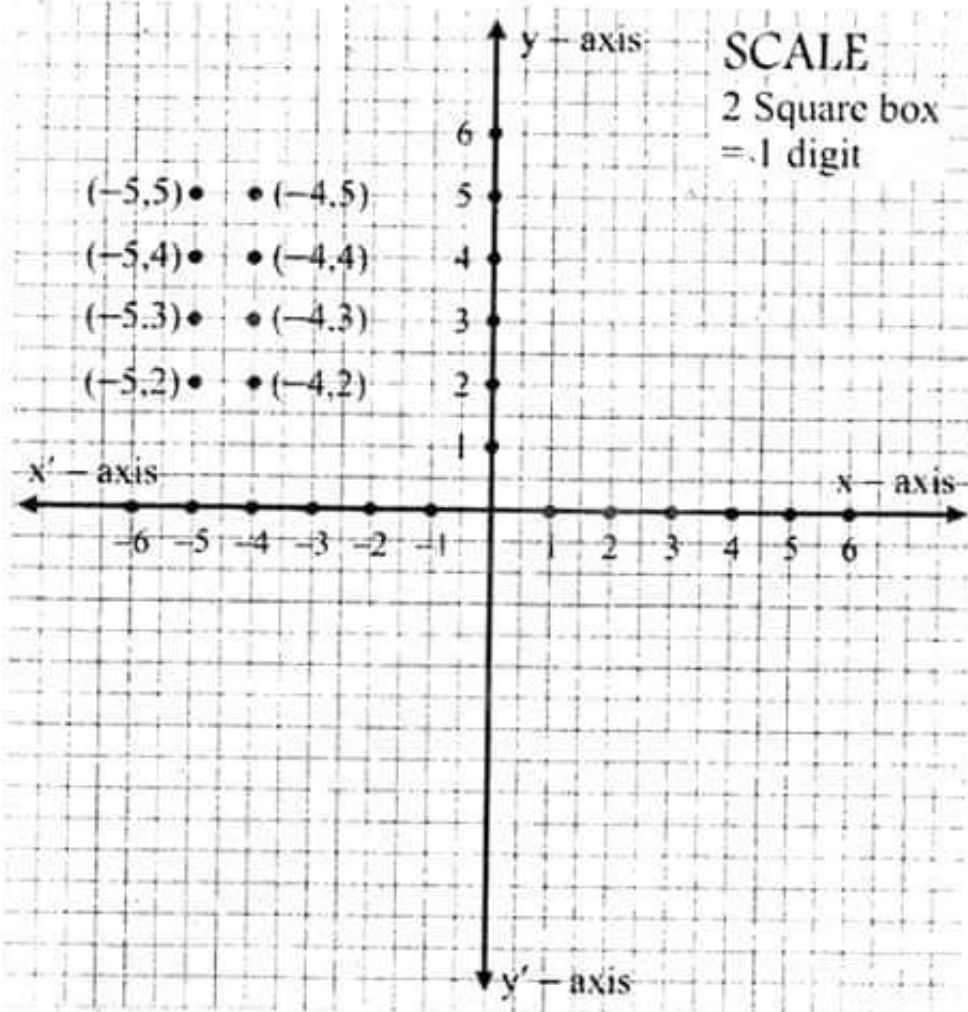
$$B = \{-5, -4\}$$

(i)  $A \times B = \{2, 3, 4, 5\} \times \{-4, -5\}$

$$= \{(2, -5), (2, -4), (3, -5), (3, -4), (4, -5), (4, -4), (5, -5), (5, -4)\}$$

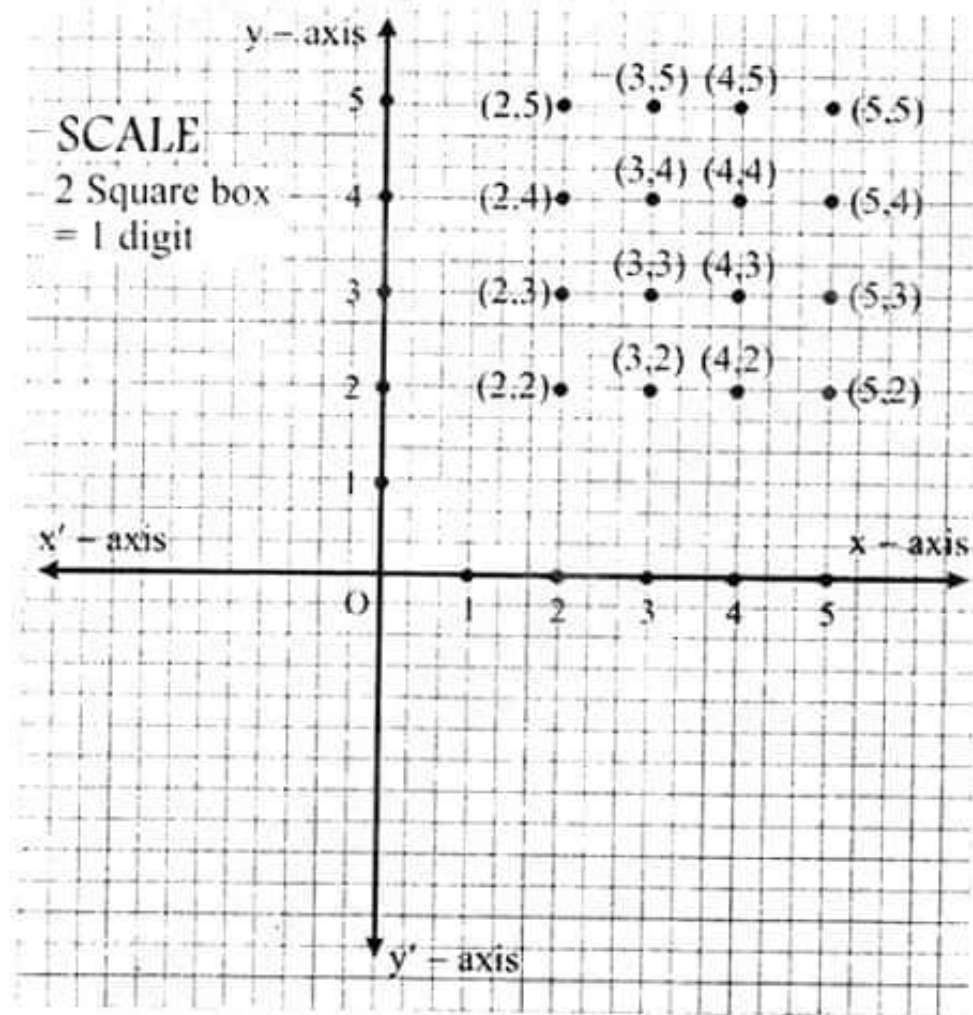


(ii)  $B \times A = \{-5, -4\} \times \{2, 3, 4, 5\}$   
 $= \{(-5, 2), (-5, 3), (-5, 4), (-5, 5), (-4, 2), (-4, 3),$   
 $(-4, 4), (-4, 5)\}$



### Unit # 1 Sets

(iii)  $A \times A = \{2,3,4,5\} \times \{2,3,4,5\}$   
 $= \{(2,2), (2,3), (2,4), (2,5), (3,2), (3,3), (3,4), (3,5),$   
 $(4,2), (4,3), (4,4), (4,5), (5,2), (5,3), (5,4), (5,5)\}$





**Q4. Find the coordinates of the points A, B, C, D, E, F, G, H, plotted on the graph paper as shown below. What are the coordinates of the origin O?**

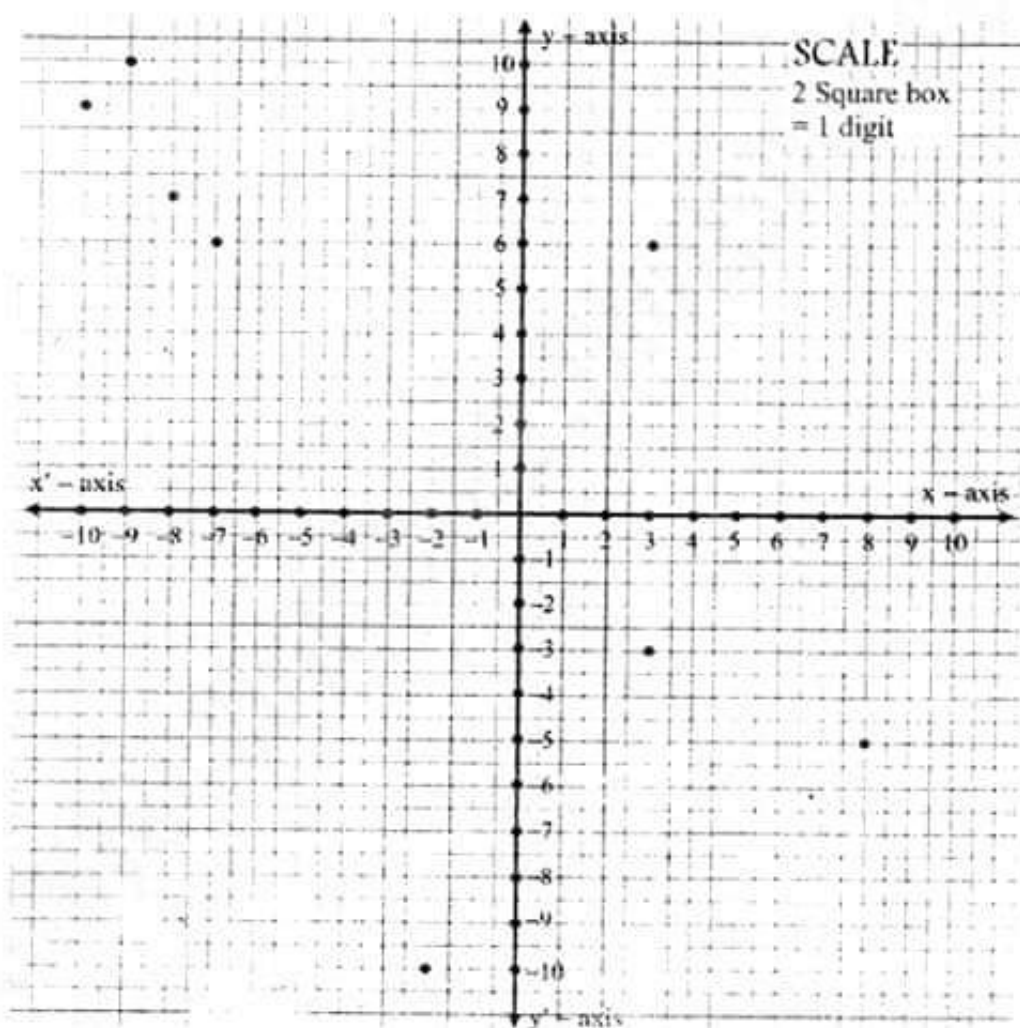
**Solution:**

The coordinates of the points A to H and O are taken as under:

A = (-9, 10)   B = (-7, 6)   C = (-8, 7)   D = (3, -3)

E = (10, 9)   F = (-2, -10)   G = (3, 6)   H = (8, -5)

O = (0, 0)





## MISCELLANEOUS EXERCISE I

**Q1. Write the following sets in tabular forms**

- (a)  $\{x \mid x \text{ is a rational number such that } x^2 = 1\}$

**Solution:**

$$A = \{-1, 1\} \quad \text{Ans.}$$

because  $x^2 = 1$

$$x = \pm\sqrt{1} = \pm 1$$

- (b)  $\{x \mid x \text{ is a positive integer less than 12}\}$

**Solution:**

$$B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\} \quad \text{Ans.}$$

**Q2. Suppose that  $A = \{2, 4, 6\}$ ,  $B = \{2, 6\}$ ,  $C = \{4, 6\}$  and  $D = \{4, 6, 8\}$ . Determine which of these sets are the subsets of the other sets.**

**Solution:**

$$\text{As } A = \{2, 4, 6\}, \quad B = \{2, 6\}$$

$$C = \{4, 6\} \text{ and } D = \{4, 6, 8\}$$

$$\Rightarrow B \subseteq A, C \subseteq A, C \subseteq D \quad \text{Ans.}$$

**Q3. Determine whether each of the following sets is the power set of a set.**

- (a)  $\phi$  (b)  $\{\phi, \{a\}\}$  (c)  $\{\phi, \{a\}, \{\phi, a\}\}$

- (d)  $\{\phi, \{a\}, \{b\}, \{a, b\}\}$

**Solution:**

- (a) No (b) Yes (c) No (d) Yes

**Q4. Let  $A = \{a, b, c, d\}$  and  $B = \{y, z\}$ . Find**

- (a)  $A \times B$

**Solution:**

$$A \times B = \{a, b, c, d\} \times \{y, z\}$$

$$= \{(a, y), (a, z), (b, y), (b, z), (c, y), (c, z), (d, y), (d, z)\} \quad \text{Ans.}$$

- (b)  $B \times A$

**Solution:**

$$B \times A = \{y, z\} \times \{a, b, c, d\}$$

$$= \{(y, a), (y, b), (y, c), (y, d), (z, a), (z, b), (z, c), (z, d)\} \quad \text{Ans.}$$

(c)  $A \times A$

Solution:

$$\begin{aligned} A \times A &= \{a,b,c,d\} \times \{a,b,c,d\} \\ &= \{(a,a), (a,b), (a,c), (a,d), (b,a), (b,b), (b,c), (b,d), (c,a), \\ &\quad (c,b), (c,c), (c,d), (d,a), (d,b), (d,c), (d,d)\} \quad \text{Ans.} \end{aligned}$$

(d)  $B \times B$

Solution:

$$\begin{aligned} B \times B &= \{y,z\} \times \{y,z\} \\ &= \{(y,y), (y,z), (z,y), (z,z)\} \quad \text{Ans.} \end{aligned}$$

Q5. In which quadrant will a point lie on the coordinate plane whose coordinates are:

(i) both positive?

Solution: 1st quadrant

(ii) both negative

Solution: 3rd quadrant

Q6. (a) What will be the y-coordinate of a point which lies on x-axis?

Solution:  $y=0$  Ans.

(b) What will be the x-coordinate of a point which lies on y-axis?

Solution:  $x=0$  Ans.

Q7. If  $A = \{-1, 1\}$  and  $B = \{\frac{1}{2}, \frac{1}{3}\}$  then write:

(a) Two binary relations from A to B.

Solution:

$$A \times B = \{-1, 1\} \times \{\frac{1}{2}, \frac{1}{3}\}$$

$$A \times B = \{(-1, \frac{1}{2}), (-1, \frac{1}{3}), (1, \frac{1}{2}), (1, \frac{1}{3})\}$$

Two binary relations from A to B

$$R_1 = \{ \}, \quad R_2 = \{(-1, \frac{1}{2})\} \quad \text{Ans.}$$

### ***Unit # 1 Sets***

**(b) Three binary relations from B to A**

**Solution:**

$$\begin{aligned} B \times A &= \left\{ \frac{1}{2}, \frac{1}{3} \right\} \times \{-1, 1\} \\ &= \left\{ \left( \frac{1}{2}, -1 \right), \left( \frac{1}{2}, 1 \right), \left( \frac{1}{3}, -1 \right), \left( \frac{1}{3}, 1 \right) \right\} \end{aligned}$$

Two binary relations from A to B

$$R_1 = \{ \}, \quad R_2 = \left\{ \left( \frac{1}{2}, -1 \right) \right\}, \quad R_3 = \left\{ \left( \frac{1}{2}, 1 \right) \right\} \quad \text{Ans.}$$

**(c) All the binary relations from A to A.**

**Solution:**

$$\begin{aligned} A \times A &= \{-1, 1\} \times \{-1, 1\} \\ &= \{ (-1, -1), (-1, 1), (1, -1), (1, 1) \} \end{aligned}$$

$$\begin{aligned} \text{Total number of binary relations} &= 2^n \\ &= 2^4 = 16 \end{aligned}$$

$$\begin{aligned} R_1 &= \{ \} \\ R_2 &= \{ (-1, -1) \} \\ R_3 &= \{ (-1, 1) \} \\ R_4 &= \{ (1, 1) \} \\ R_5 &= \{ (1, -1) \} \\ R_6 &= \{ (-1, -1), (-1, 1) \} \\ R_7 &= \{ (-1, -1), (1, -1) \} \\ R_8 &= \{ (-1, -1), (1, 1) \} \\ R_9 &= \{ (-1, 1), (1, -1) \} \\ R_{10} &= \{ (-1, 1), (1, 1) \} \\ R_{11} &= \{ (1, -1), (1, 1) \} \\ R_{12} &= \{ (-1, -1), (-1, 1), (1, -1) \} \\ R_{13} &= \{ (-1, -1), (-1, 1), (1, 1) \} \\ R_{14} &= \{ (-1, -1), (1, -1), (1, 1) \} \\ R_{15} &= \{ (-1, 1), (1, -1), (1, 1) \} \\ R_{16} &= \{ (-1, -1), (-1, 1), (1, 1), (1, -1) \} \end{aligned}$$

(d) Four binary relations from B to B.

Solution:

$$\begin{aligned} B \times B &= \left\{ \frac{1}{2}, \frac{1}{3} \right\} \times \left\{ \frac{1}{2}, \frac{1}{3} \right\} \\ &= \left\{ \left( \frac{1}{2}, \frac{1}{2} \right), \left( \frac{1}{2}, \frac{1}{3} \right), \left( \frac{1}{3}, \frac{1}{2} \right), \left( \frac{1}{3}, \frac{1}{3} \right) \right\} \end{aligned}$$

$$R_1 = \{ \}, R_2 = \left\{ \left( \frac{1}{2}, \frac{1}{2} \right) \right\}, R_3 = \left\{ \left( \frac{1}{2}, \frac{1}{3} \right) \right\}, R_4 = \left\{ \left( \frac{1}{3}, \frac{1}{2} \right) \right\} \text{ Ans.}$$

Q8. If  $S = \{1, 2, 3, 4\}$  and  $T = \left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8} \right\}$  then write:

(a) A one-one function from S to T.

Solution:

A one-one function from S to T is as under:

$$f = \left\{ \left( 1, \frac{1}{2} \right), \left( 2, \frac{1}{6} \right), \left( 3, \frac{1}{4} \right), \left( 4, \frac{1}{8} \right) \right\} \text{ Ans.}$$

(b) A function from S onto T

Solution:

A function from S onto T

$$f = \left\{ \left( 1, \frac{1}{8} \right), \left( 2, \frac{1}{4} \right), \left( 3, \frac{1}{2} \right), \left( 4, \frac{1}{6} \right) \right\}$$

(c) A function from S to T which is one-one and onto.

Solution:

A function from S to T which is one-one and onto

$$f = \left\{ \left( 1, \frac{1}{4} \right), \left( 2, \frac{1}{8} \right), \left( 3, \frac{1}{2} \right), \left( 4, \frac{1}{6} \right) \right\} \text{ Ans.}$$

(d) A function from S to T which is neither one-one nor onto.

Solution:

A function from S to T which is neither one-one nor onto.

$$f = \left\{ \left( 1, \frac{1}{2} \right), \left( 2, \frac{1}{2} \right), \left( 3, \frac{1}{4} \right), \left( 4, \frac{1}{4} \right) \right\} \text{ Ans.}$$

## Unit # 1 Sets

**Q.9** If  $U = \{x \mid x \text{ is a letter in the English alphabet.}\}$

$A = \{a,b,c,d,e,f\}$ ,  $B = \{a,e,i,o,u\}$  and  $C = \{u,v,w,x,y,z\}$ ,  
list the elements of:

- (a)  $A \cap (B \cap C)$  (b)  $A \cap (B \cup C)$  (c)  $(A \cap B) \cup (A \cap C)$   
(d)  $A \cup (B \cup C)'$  (e)  $A \cap B \cap C'$  (f)  $(A \cup B)' \cup C$

**Illustrate each set by means of a Venn diagram.**

**Solution:** (a)  $A \cap (B \cap C)$

$$B \cap C = \{a,e,i,o,u\} \cap \{u,v,w,x,y,z\}$$

$$B \cap C = \{u\}$$

$$A \cap (B \cap C) = \{a,b,c,d,e,f\} \cap \{u\}$$

$$A \cap (B \cap C) = \{\} \quad \text{Ans.}$$

(b)  $A \cap (B \cup C)$

**Solution:**

$$B \cup C = \{a,e,i,o,u\} \cup \{u,v,w,x,y,z\}$$

$$B \cup C = \{a,e,i,o,u,v,w,x,y,z\}$$

$$A \cap (B \cup C) = \{a,b,c,d,e,f\} \cap \{a,e,i,o,u,v,w,x,y,z\}$$

$$A \cap (B \cup C) = \{a,e\} \quad \text{Ans.}$$

(c)  $(A \cap B) \cup (A \cap C)$

**Solution:**

$$A \cap B = \{a,b,c,d,e,f\} \cap \{a,e,i,o,u\}$$

$$A \cap B = \{a,e\}$$

$$A \cap C = \{a,b,c,d,e,f\} \cap \{u,v,w,x,y,z\}$$

$$A \cap C = \{\}$$

$$(A \cap B) \cup (A \cap C) = \{a,e\} \cup \{\}$$

$$(A \cap B) \cup (A \cap C) = \{a,e\} \quad \text{Ans.}$$

(d)  $A \cap (B \cup C)'$

**Solution:**

$$B \cup C = \{a,e,i,o,u\} \cup \{u,v,w,x,y,z\}$$

$$B \cup C = \{a,e,i,o,u,v,w,x,y,z\}$$

$$U - B \cup C = \{a,b,c,d,\dots,z\} - \{a,e,i,o,u,v,w,x,y,z\}$$

$$(B \cup C)' = \{b,c,d,f,g,h,i,j,k,\ell,m,n,p,q,r,s,t\}$$

$$A \cap (B \cup C)' = \{a,b,c,d,e,f\} \cap \{b,c,d,f,g,h,i,j,k,\ell,m,n,p,q,r,s,t\}$$



$$A \cap (B \cup C)' = \{b, c, e, f\} \quad \text{Ans.}$$

$$(e) \quad A \cap B \cap C'$$

**Solution:**

$$U - C = \{a, b, c, d, \dots, z\} - \{u, v, w, x, y, z\}$$

$$C' = \{a, b, c, d, e, f, g, h, \dots, r, s, t\}$$

$$A \cap B = \{a, b, c, d, e, f\} \cap \{a, e, i, o, u\}$$

$$A \cap B = \{a, e\}$$

$$A \cap B \cap C' = \{a, e\} \cap \{a, b, c, d, \dots, r, s, t\}$$

$$A \cap B \cap C' = \{a, e\} \quad \text{Ans.}$$

$$(f) \quad (A \cup B)' \cap C$$

**Solution:**

$$A \cup B = \{a, b, c, d, e, f\} \cup \{a, e, i, o, u\}$$

$$A \cup B = \{a, b, c, d, e, f, i, o, u\}$$

$$U - (A \cup B) = \{a, b, c, d, \dots, z\} - \{a, b, c, d, e, f, i, o, u\}$$

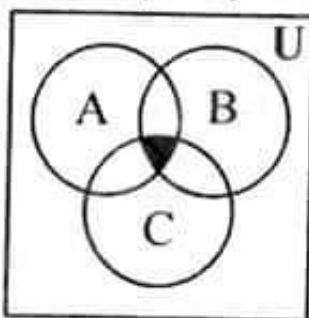
$$(A \cup B)' = \{g, h, j, k, \ell, m, n, p, q, r, s, t, v, w, x, y, z\}$$

$$(A \cup B)' \cap C = \{g, h, j, k, \ell, m, n, p, q, r, s, t, v, w, x, y, z\} \cap \{u, v, w, x, y, z\}$$

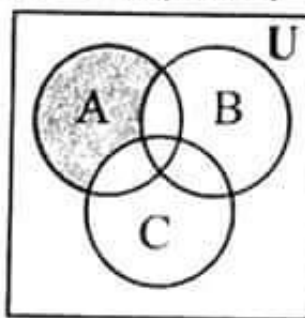
$$(A \cup B)' \cap C = \{g, h, j, k, \ell, m, n, p, q, r, s, t, u, v, w, x, y, z\} \quad \text{Ans.}$$

**Illustration by Venn Diagrams:**

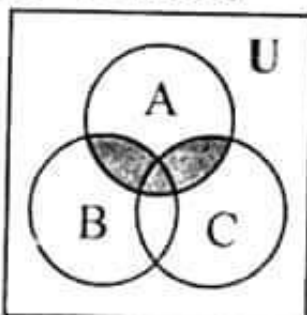
Q9(a).  $A \cap (B \cap C)$



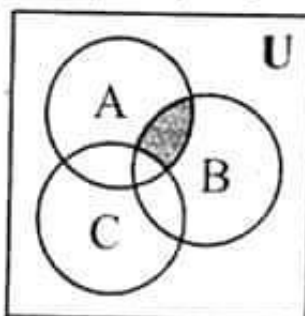
Q9(d).  $A \cap (B \cap C)'$



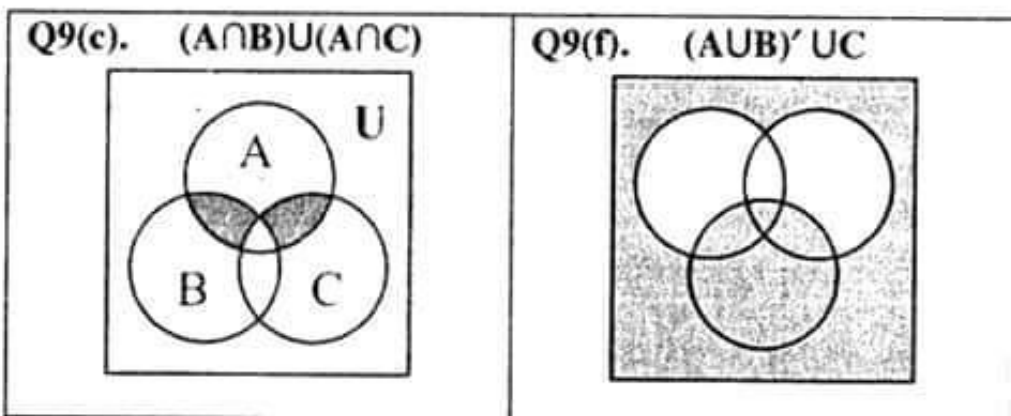
Q9(b).  $A \cap (B \cup C)$



Q9(e).  $(A \cap B) \cap C'$



## Unit #1 Sets



**Q10.** Which of the following statements are true and which of them are false?

(i) If  $A = \{x, y\}$  and  $B = \{y, z, t\}$ , then  $A \subseteq B$

**Solution:** False      **Ans.**

(ii) If  $A = \{1, 2, 3\}$  and  $U = N$ , then  $A \cup A = N$

**Solution:** False      **Ans.**

(iii) The set  $A = \{1, \frac{1}{10}, \frac{1}{100}, \dots, \frac{1}{10^{20}}\}$  is an infinite set.

**Solution:** False      **Ans.**

(iv) The intersection of two disjoint sets is empty.

**Solution:** True      **Ans.**

(v) The set of natural numbers between 40 and 42 is the empty set.

**Solution:** False      **Ans.**

(vi)  $A \cup B = AB$

**Solution:** False      **Ans.**

(vii)  $A \times B = B \times A$

**Solution:** False      **Ans.**

(viii)  $(2, -3) = (-3, 2)$

**Solution:** False      **Ans.**

(ix)  $\emptyset = \{\emptyset\}$

**Solution:** False      **Ans.**

(x) If  $A$  contains  $m$  elements and  $B$  contains  $n$  elements, then  $A \times B$  contains  $mn$  ordered pairs.

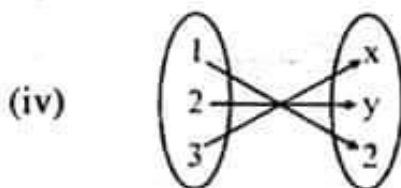
**Solution:** True      **Ans.**

**Q11. Complete the sentences.**

- (i)  $A \cap (B \cup C) = \underline{(A \cap B) \cup (A \cap C)}$
- (ii)  $A \Delta B = \underline{\{x | x \in A \text{ or } x \in B \text{ but } x \notin A \cap B\}}$ .
- (iii)  $(a,b) \neq (b,a)$
- (iv)  $(A \cup B)' = \underline{A' \cap B'}$
- (v) If  $(x + 2, 3y - 6) = (2x, y)$  then  $x = \underline{2} = y = \underline{3}$
- (vi) If  $f$  is one-one onto function from  $A$  to  $B$ , then  $n(A) \underline{=} n(B)$ .
- (vii)  $(-3, -2)$  is in Third quadrant.
- (viii)  $A = \{2, 4, 8\}$  and  $B = \{2^1, 2^2, 2^3\}$  are equal sets.
- (ix) Corresponding to any point in a plane there is an ordered pair of real numbers.
- (x) If  $R = \{(1, 2), (2, 3), (3, 4)\}$  then  $\text{Dom } R = \underline{\{1, 2, 3\}}$  and  $\text{Range } R = \underline{\{2, 3, 4\}}$ .

**Q12. Select the correct answer and write it in the blank space.**

- (i) The Cartesian product of sets  $A$  and  $B$  is written as  $A \times B$ .
- (a)  $A \cdot B$  (b)  $A \times B$  (c)  $A \Delta B$  (d)  $B \times A$
- (ii)  $\{2, 4, 6, 8, \dots, 50\}$  written in set builder form  $\{x | x \in \mathbb{R}, 2 \leq x \leq 50\}$
- (a)  $\{x | x \in \mathbb{N}, x \leq 50\}$  (b)  $\{x | x \in \mathbb{E}, x \leq 50\}$
- (c)  $\{x | x \in \mathbb{E}, x \leq x \leq 50\}$  (d)  $\{x | x \in \mathbb{Q}, x \leq 50\}$
- (iii)  $\{0, 1, 2, 3, \dots\}$  is the set of whole numbers.
- (a) prime numbers (b) integers
- (c) whole numbers (d) even numbers



Represent one to one  
and onto function.