Tutorial letter 201/1/2018

Theoretical Computer Science 1

COS1501

Semester 1

School of Computing

This tutorial letter contains

a discussion of assignment 01, and

examination information.

Define tomorrow.

UNISA university of south africa

Dear Student,

In this tutorial letter the solutions to the first assignment questions are discussed and examination information is provided.

Regards,
COS1501 Team

SOLUTIONS ASSIGNMENT 01 (SEMESTER 1)

Question 1 **Alternative 1**

The set of all non-negative integers **x** less than 16 such that x^2 is an even integer should be described.

Now we can ask the question: For which non-negative integers **x** do we have that x^2 is an even number?
Remember, the required set must include as elements all non-negative integers **x** such that all the requirements for the set are met.

Alternative 1:
The set $\{\mathbf{x} \mid x \in \mathbb{Z}^{\geq}, x < 16, x^2 = 2k \text{ for some } k \in \mathbb{Z}\}$ is the set of all non-negative integers **x** less than 16 such that x^2 is an even number.
All the requirements $x \in \mathbb{Z}^{\geq}$, $x < 16$, $x^2 = 2k$ for some $k \in \mathbb{Z}$ must hold for an integer x to qualify to belong to this set.
Case 1: If $x = 0$ then $x^2 = 0$ and $0 \in \mathbb{Z}^{\geq}$, $0 < 16$, $0^2 = 2(0)$ for some $k = 0 \in \mathbb{Z}$.
Case 2: If $x = 2$ then $x^2 = 4$ and $2 \in \mathbb{Z}^{\geq}$, $2 < 16$, $2^2 = 2(2)$ for some $k = 2 \in \mathbb{Z}$.
And so we can go on to see that for all non-negative integers x = 0, 2, 4, 6, 8, 10, 12, 14 it will be the case that $x^2 = 2k$, $k \in \mathbb{Z}$, and also note that $x < 16$.

If x = 1, 3, 5, 7, 9, 11, 13, 15 then x does not qualify to belong to the required set since x is an odd number and thus x^2 is also an odd number.

Alternative 2: $x = 0$ is not an element of $\{\mathbf{x} \mid x \in \mathbb{Z}^{\geq}, x < 16, x^2 = 2k \text{ for some } k \in \mathbb{Z}^{+}\}$
since $k \in \mathbb{Z}^{+}$ thus $k = 0 \notin \mathbb{Z}^{+}$. This excludes the case where $x^2 = 0^2 = 0 = 2(0)$. It is required that **all** non-negative integers **x** less than 16, such that x^2 is an even integer, should live in the set described in the question statement, but in this case,
$x = 0 \notin \{\mathbf{x} \mid x \in \mathbb{Z}^{\geq}, x < 16, x^2 = 2k \text{ for some } k \in \mathbb{Z}^{+}\}$.
Alternatives 3 and 4 do not provide the required values for **x** as described in the question statement.

Refer to study guide, pp 3, 7, 11, 35 – 39.

Question 2 **Alternative 4**

It is required to determine the set $\{x \mid x \in \mathbb{Z}, 0 \leq x < 8\} \cap \{x \mid x \in \mathbb{R}, 4 \leq x < 16\}$.

The integers 0, 1, 2, 3, 4, 5, 6 and 7 live in the set $\{x \mid x \in \mathbb{Z}, 0 \leq x < 8\}$.

All real numbers less than 16 and greater than or equal to 4 live in the set $\{x \mid x \in \mathbb{R}, 4 \leq x < 16\}$ which means that integers less than 16 and greater than or equal to 4 also live in this set, ie the integers 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 and 15 live in this set.

The elements common to both $\{x \mid x \in \mathbb{Z}, 0 \leq x < 8\}$ and $\{x \mid x \in \mathbb{R}, 4 \leq x < 16\}$ are 4, 5, 6, and 7. Thus $\{x \mid x \in \mathbb{Z}, 0 \leq x < 8\} \cap \{x \mid x \in \mathbb{R}, 4 \leq x < 16\} = \{4, 5, 6, 7\}$. It should be clear to you that none of the sets provided in alternatives 1, 2 and 3 are equal to the set {4, 5, 6, 7}.

Set intersection: The elements 4, 5, 6 and 7 belong to both
$\{x / x \in \mathbb{Z}, 0 \leq x < 8\}$ and $\{x / x \in \mathbb{R}, 4 \leq x < 16\}$.

Refer to study guide, pp 11, 25, 42.

Consider the following sets, where U represents a universal set:
U = {1, 2, {1}, {2}, {1, 2}} A = {1, 2, {1}} B = {{1}, {1, 2}} C = {2, {1}, {2}}.
Questions 3 to 10 are based on the sets defined above.
NOTE: The Venn diagrams in study unit 4 will help you to understand the definitions in study unit 3.

Question 3 **Alternative 1**

$A \cup B$
$= \{1, 2, \{1\}\} \cup \{\{1\}, \{1, 2\}\}$
$= \{1, 2, \{1\}, \{1, 2\}\}$

Set union: The elements 1, 2, {1} and {1, 2} belong to A or B.

Refer to study guide, p 41.

Question 4 **Alternative 4**

$A \cap C$
$= \{1, 2, \{1\}\} \cap \{2, \{1\}, \{2\}\}$
$= \{2, \{1\}\}$.

Set intersection: The elements 2 and {1} belong to A and C.

Refer to study guide, p 42.

Question 5 **Alternative 1**

A – B is the set:

= $\{1, 2, \{1\}\} - \{\{1\}, \{1, 2\}\}$

= $\{1, 2\}$.

Set difference: The elements 1 and 2 belong to A but not to B.

Refer to study guide, pp 42, 43.

Question 6 **Alternative 3**

B + C

= $\{\{1\}, \{1, 2\}\} + \{2, \{1\}, \{2\}\}$

= $\{2, \{2\}, \{1, 2\}\}$

Set symmetric difference: The elements 2, {2} and {1, 2} belong to B or to C, but not both.

It is also the case that B + C = (B $\cup$ C) – (B $\cap$ C), so

B + C= (B $\cup$ C) – (B $\cap$ C) *Include elements belonging to B $\cup$ C but not to B $\cap$ C.*

= $(\{\{1\}, \{1, 2\}\} \cup \{2, \{1\}, \{2\}\}) - (\{\{1\}, \{1, 2\}\} \cap \{2, \{1\}, \{2\}\})$

= $\{2, \{1\}, \{2\}, \{1, 2\}\} - \{\{1\}\}$

= $\{2, \{2\}, \{1, 2\}\}$

Refer to study guide, pp 43, 44.

Question 7 **Alternative 2**

PLEASE NOTE: This question did not count for the assignment mark, as there were typing errors in alternative 2 in Tutorial letter 101.

Wrong alternative 2: $\{\emptyset, \{1\}, \{2\}, \{\{1\}\}, \{1, 2\} \{1, \{1\}\}, \{2, \{1\}\}, \{1, 2, \{1\}\}$

Correct alternative 2: $\{\emptyset, \{1\}, \{2\}, \{\{1\}\}, \{1, 2\}, \{1, \{1\}\}, \{2, \{1\}\}, \{1, 2, \{1\}\}\}$

The elements of $\mathcal{P}$(A) are all the subsets of A, so we have to determine the subsets of A.

Let's look at the definition of a subset:
For all sets F and G, F is a subset of G if and only if every element of F is also an element of G.
Subsets of G can be formed by ***keeping the outside brackets*** *of G and then throwing away* ***none***, ***one*** *or* ***more*** *elements of G.*

The elements of A = $\{1, 2, \{1\}\}$ are 1, 2 and {1}.
We form the subsets of A:

Throw away no element of set A, then $\{1, 2, \{1\}\} \subseteq A$;
throw away the element 1 of set A, then $\{2, \{1\}\} \subseteq A$;
throw away the element 2 of set A, then $\{1, \{1\}\} \subseteq A$; *and*
throw away the elements 1 and 2 of set A, then $\{\{1\}\} \subseteq A$, *and so we can go on to form all the subsets of A.*

If $A = \{1, 2, \{1\}\}$ then the subsets of A, namely $\emptyset$, $\{1\}$, $\{2\}$, $\{\{1\}\}$, $\{1, 2\}$, $\{1, \{1\}\}$, $\{2, \{1\}\}$ and $\{1, 2, \{1\}\}$ are all the elements of $\mathcal{P}(A)$, thus

$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{\{1\}\}, \{1, 2\}, \{1, \{1\}\}, \{2, \{1\}\}, \{1, 2, \{1\}\}\}$.

The set provided in alternative 2 is the set $\mathcal{P}(A)$.

The sets provided in alternatives 1 and 4 are **subsets** of $\mathcal{P}(A)$ but **not equal** to $\mathcal{P}(A)$.

The set in alternative 3 is not a subset of $\mathcal{P}(A)$ since

$\{\{1, 2\}\} \in \{\emptyset, \{1\}, \{2\}, \{\{1, 2\}\}, \{1, \{1\}\}, \{2, \{1\}\}, \{1, 2, \{1, 2\}\}\}$ but $\{\{1, 2\}\} \notin \mathcal{P}(A)$.

Refer to study guide, pp 40, 45.

Question 8 **Alternative 3**

$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{\{1\}\}, \{1, 2\}, \{1, \{1\}\}, \{2, \{1\}\}, \{1, 2, \{1\}\}\}$. We consider the sets in the different alternatives:

Alternative 1: $\{1\}$ is not a subset of $\mathcal{P}(A)$. We provide a counterexample:

$1 \in \{1\}$ but $1 \notin \mathcal{P}(A)$, therefore $\{1\} \nsubseteq \mathcal{P}(A)$.

Alternative 2: $1, 2 \in \{1, 2, \{1, 2\}\}$ but $1, 2 \notin \mathcal{P}(A)$, therefore $\{1, 2, \{1, 2\}\} \nsubseteq \mathcal{P}(A)$.

Alternative 3: If we throw away the elements $\emptyset$, $\{1\}$, $\{2\}$, $\{1, 2\}$, $\{1, \{1\}\}$, $\{2, \{1\}\}$ and $\{1, 2, \{1\}\}$ of $\mathcal{P}(A)$, we are left with the element $\{\{1\}\}$ that belong to the subset $\{\{\{1\}\}\}$ of $\mathcal{P}(A)$,

thus $\{\{\{1\}\}\} \subseteq \mathcal{P}(A)$.

Note that $\{\{1\}\} \in \{\{\{1\}\}\}$ and $\{\{1\}\} \in \mathcal{P}(A)$ so we can say that each element of $\{\{\{1\}\}\}$ is also an element of $\mathcal{P}(A)$ thus $\{\{\{1\}\}\} \subseteq \mathcal{P}(A)$. ($\{\{1\}\}$ is the only element of $\{\{\{1\}\}\}$).

Alternative 4: $\{\{2\}\}$ is the only element of $\{\{\{2\}\}\}$ but $\{\{2\}\} \notin \mathcal{P}(A)$, therefore $\{\{\{2\}\}\} \nsubseteq \mathcal{P}(A)$.

From the arguments provided we can deduce that alternative 3 should be selected.

Refer to study guide, pp 40, 45.

Question 9 **Alternative 2**

We determine $\mathcal{P}(A) \cap \mathcal{P}(B)$ and $\mathcal{P}(A \cap B)$:

$B = \{\{1\}, \{1, 2\}\}$

Thus $\mathcal{P}(B) = \{\emptyset, \{\{1\}\}, \{\{1, 2\}\}, \{\{1\}, \{1, 2\}\}\}$ and we have already determined that

$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{\{1\}\}, \{1, 2\}, \{1, \{1\}\}, \{2, \{1\}\}, \{1, 2, \{1\}\}\}$.

Only the elements $\emptyset$ and $\{\{1\}\}$ belong to both sets $\mathcal{P}(A)$ and $\mathcal{P}(B)$ thus

$\mathcal{P}(A) \cap \mathcal{P}(B) = \{\emptyset, \{\{1\}\}\}$.

$A \cap B$
$= \{1, 2, \{1\}\} \cap \{\{1\}, \{1, 2\}\}$
$= \{\{1\}\}$

Thus $\mathcal{P}(A \cap B) = \{\emptyset, \{\{1\}\}\} = \mathcal{P}(A) \cap \mathcal{P}(B)$.

From the arguments provided we can deduce that alternative 2 should be selected.

Refer to study guide, pp 40, 42, 45.

Question 10 **Alternative 1**

We have to determine which statement is valid if $x \notin B \cup C$?

If $x \notin B \cup C$ then
$x \in (B \cup C)'$
ie $x \in U - (B \cup C)$.

We determine $U - (B \cup C)$:
$U - (B \cup C)$
$= \{1, 2, \{1\}, \{2\}, \{1, 2\}\} - (\{\{1\}, \{1, 2\}\} \cup \{2, \{1\}, \{2\}\})$
$= \{1, 2, \{1\}, \{2\}, \{1, 2\}\} - \{2, \{1\}, \{2\}, \{1, 2\}\}$
$= \{1\}$

We can now say that if $x \notin B \cup C$ then $x \in \{1\}$.

From the arguments provided we can deduce that alternative 1 should be selected.

Refer to study guide, pp 41 - 43.

EXAMINATION INFORMATION

The examination paper will test you on **study units 3 – 10** of the study guide, and the material in **all** the tutorial matter. **Study units 1 – 2** of the study guide provide background material.

Bear in mind that the assignment questions do not cover all the work that we test in the examination paper. You have to prepare **all the work** prescribed above.

It will be expected of you to write down the answers to all the examination questions and provide proofs where required.

PLEASE NOTE: The examination paper will be a fill-in paper as from 2017 onwards. You will receive a notification when a practice fill-in paper with solutions is posted on myUnisa. Please work through this paper, especially if you have never written a fill-in exam before.

How to prepare:
Cover all the study units in the study guide thoroughly and **test yourself** by doing the activities before you look at the solutions which are supplied as part of the learning units as well as under *Additional Resources* on myUnisa.

Work through the 2009 examination paper along with the solutions provided in the MO001 tutorial letter. It is very important that you also do all the self-assessment questions provided in assignments 02 and 03, and take note of the hints provided since these hints will help you to avoid making common errors in the examination. Make sure that you understand the model solutions to **all assignment questions** provided in tutorial letters.

Take note of the **structure and notation of solutions** provided in all tutorial matter. For example, when a proof is required and connectives such as "**iff**" or "**if…then**" are left out, the proof is not convincing. Also, **symbols** (e.g. "$\cap$") should be used as **connectives** for **sets** (e.g. $Y \cap W$), and **words** (e.g. "and") should be used as **connectives** in **sentences** (e.g. $x \in Y$ **and** $x \in W$). Some proofs should start with the word "**assume**" and then logic reasoning should follow. These and other notation issues are mentioned in the hints provided in the assignments.

The examination, and the supplementary examination that follows in the examination period of the following semester, will have a structure and format similar to the practice exam paper, and similar type of questions as in the previous years' exam papers available on myUnisa and the 2009 exam paper for which the solution is given as part of the learning units. The order of the sections will be in line with the order in which the concepts appear in the study guide. We do not repeat questions from previous exam papers, so please make sure that you understand the content. We do NOT provide solutions to previous exam papers. Your e-tutor is available to assist with a discussion of any old exam paper if you post your answer for a specific question that you have problems with, on the discussion forum.

Additional advice:

Venn diagrams: Draw your diagrams in stages as described on page 51 of the study guide. Also remember to draw the sets within the context of a universal set, name the sets and provide subscripts for the diagrams.

Relations and functions: You need to **apply** the definitions, not merely give them. For example, when you want to prove that a relation is functional and you only write the statement "for every *x* there is only one *y*", you will receive no marks since it is neither a properly formulated definition nor a proof. **Relate each answer to the actual definition of the specific relation or function** given in the question.

When solving problems, don't forget the useful **shortcut notations** that help you to express an English sentence in precise mathematical notation. For example, an even number can be expressed as $2k$, an odd number as $2k + 1$, a multiple of three as $3k$ and so on, and two consecutive numbers could be k and $k + 1$, with $k \in \mathbb{Z}$.

Truth tables: Use the notations T and F for *true* and *false*, and when three declarative statements (e.g. *p*, *q* and *r*) are involved, use the same order for the combinations of T and F that is used in the table on page 144 of the study guide.

Mathematical proofs and counterexamples: If you are required to provide a mathematical proof and you give an example instead of a general proof, you will receive no marks for that answer. In other words, **never** attempt to prove that something is true by using an example. However, you **should use a counterexample** to prove that something is not true. If you are required to give a **counterexample** and you attempt to give some mathematical proof instead of a **counterexample**, you will receive no marks for that answer.

Note: Read all the hints provided in tutorial letter 101. These hints will help you to avoid making general mistakes in the examination.