COS1501 Assignment 4 Questions

1. Let set P = {a, b, {b}, {{a}, 3}}. Which one of the following does NOT represent element(s) of the set P?
   - **A.** a, b
   - **B.** b, {b}
   - **C.** {{a}, 3}
   - ✔ **D.** a, {a}, 3

   Answer Key:D

2. Suppose U = {1, {2, 3}, 3, d, {d, e}, e} is a universal set with the following subsets:

   A = {{2, 3}, 3, {d, e}}, B = {1, {2, 3}, d, e} and C = {1, 3, d, e}.

   Which one of the following sets represents C n B' ?
   - ✔ **A.** {3}
   - **B.** {1, 3, d, e}
   - **C.** {1, 3, {d, e}}
   - **D.** {1, d, e}

   Answer Key:A

3. Suppose U = {1, {2, 3}, 3, d, {d, e}, e} is a universal set with the following subsets:

   A = {{2, 3}, 3, {d, e}}, B = {1, {2, 3}, d, e} and C = {1, 3, d, e}.

   Let S = {(1, 3), (1, d), (1, e), (d, e), (3, d)} be a relation on set C.

   Which one of the following statements regarding S is true?
   - ✔ **A.** If (e, 3) is added to S, S would satisfy trichotomy.
   - **B.** S is a weak partial order.
   - **C.** S is a strict total order.
   - **D.** If (e, 3) is added to S, it would make S transitive.

   Answer Key:A

4. Let set A = {1, 2, {2}, {{1}, 3}} .

   Which one of the following is NOT a subset of the set A?

   - A. {{{1},3}}
   - B. {2, {2}}
   - ✔ C. {2, {2}, {1}, 3}
   - D. {1, 2, {2}, {{1}, 3}}

   Answer Key:C

5. Suppose U = {1, {2, 3}, 3, d, {d, e}, e} is a universal set with the following subsets:

   A = {{2, 3}, 3, {d, e}}, B = {1, {2, 3}, d, e} and C = {1, 3, d, e}.

   Which one of the following sets represents U + B?

   - ✔ A. {3, {d, e}}
   - B. {1, {2, 3}, d, e}
   - C. (U – A) – C
   - D. U

   Answer Key:A

6. Suppose U = {1, {2, 3}, 3, d, {d, e}, e} is a universal set with the following subsets:

   A = {{2, 3}, 3, {d, e}}, B = {1, {2, 3}, d, e} and C = {1, 3, d, e}.

   Which one of the following sets represents C – A?

   - A. {1, {2, 3}, d, e, {d, e}}
   - B. {3, d, e}
   - ✔ C. {1, d, e}
   - D. { }

   Answer Key:C

7. Consider the following sets, where U represents a universal set:

U = {1, 2, {1}, {1, 2}, 3, {4}} A = {1, 2, {4}} B = {{1, 2}, 3} C = {2, {1}, {4}}

A ∩ C is the set:

- **A.** {2, {4}}
- **B.** {1, 2, {4}}
- **C.** {{1, 2}, {4}}
- **D.** {1, 2, 4}

Answer Key:A

8. Suppose U = {{a, b}, a, c, d, {a, d}, e} is a universal set with the following subsets:

A = {{a, b}, a, {a, d}}, B = {a, d, e} and C = {{a, b}, a, d, {a, d}, e}.

Which one of the following sets represents A U C?

- **A.** {a, b, d, e}
- **B.** U
- **C.** {{a, b}, d, {a, d}}
- **D.** {{a, b}, a, d, {a, d}, e}

Answer Key:D

9. Let U = {1, 2, 3} and A, B and C be subsets of U.

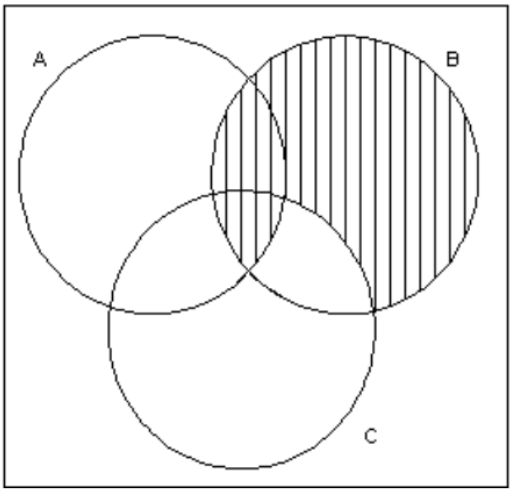
The statement (A ∩ (B – C')) = (A ∩ B) is NOT an identity.

Which of the following alternatives contains sets A, B and C that can be used as counterexample to prove that the statement is NOT an identity.

- **A.** A = {1}, B = {1, 2}, C = {3}
- **B.** A = { }, B = {1, 3}, C = {1, 2}
- **C.** A = {1}, B = {3}, C = {1, 2, 3}
- **D.** A = {1}, B = {1, 2 }, C = {1, 3}

Answer Key:A

10. Consider the Venn diagram below:



Which one of the following alternatives is represented by the Venn diagram?

- **A.** [(A U B) - A] + (B ∩ C)
- **B.** [B + (A ∩ B)] - (C ∩ B)
- ✔ **C.** [B - (B ∩ C)] U (A ∩ B)
- **D.** [B - (A ∩ B)] + B

Answer Key:C

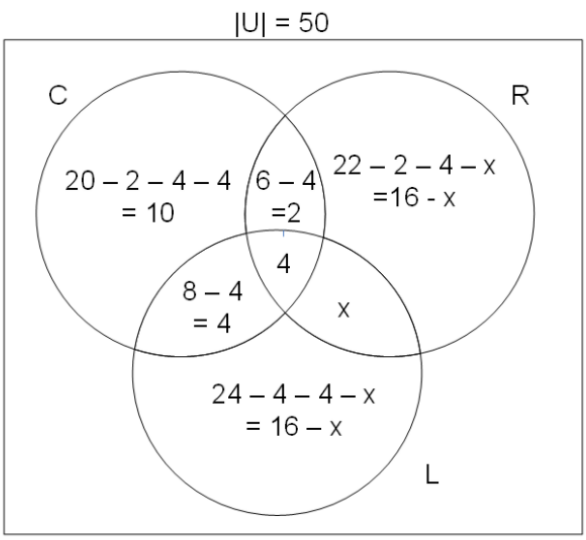
11. A newly built old age home has 50 townhouses. Residents may only plant clivias, rose bushes and lavenders in their gardens.

Of the 50 gardens 20 grow clivias, 22 grow roses, and 24 grow lavenders. (Residents do not necessarily plant only one of the kinds of plants.)

Furthermore, some gardens grow the following: 6 grow clivias and rose bushes 8 grow clivias and lavenders, (Residents do not necessarily plant only two of the kinds of plants.)

4 grow clivias, rose bushes and lavenders.

The Venn diagram below represents the above-given information:



How many gardens have rose bushes and lavenders, but no clivias?

- A. 1
- ✔ **B.** 2
- **C.** 6
- **D.** 28

Answer Key:B

12. We want to determine whether or not for all A, B, C $\subseteq$ U,

$$A \cap (B \cap C)' = (A - B) \cup (A - C) \text{ is an identity.}$$

Which one of the following alternatives is the correct way to determine this?

- ✔ **A.**

$x \in A \cap (B \cap C)'$
iff $x \in A$ and $x \in (B \cap C)'$
iff $x \in A$ and $x \notin (B \cap C)$
iff $x \in A$ and ($x \notin B$ or $x \notin C$)
iff ($x \in A$ and $x \notin B$) or ($x \in A$ and $x \notin C$)
iff $x \in (A - B)$ or $x \in (A - C)$
iff $x \in (A - B) \cup (A - C)$, which is equal to the RHS of the given equation.
We can therefore say that the given statement is an identity.

- **B.**

$x \in A \cap (B \cap C)'$
iff $x \in A$ and $x \in (B \cap C)'$
iff $x \in A$ and $x \in (B' \cap C')$
iff $x \in A$ and ($x \in B'$ and $x \in C'$)
iff ($x \in A$ and $x \in B'$) and ($x \in A$ and $x \in C'$)
iff $x \in (A - B')$ and $x \in (A - C')$
iff $x \in (A - B') \cap (A - C')$, which is NOT equal to the RHS of the given equation.
We can therefore say that the given statement is NOT an identity.

- **C.**

$x \in A \cap (B \cap C)'$
iff $x \in A$ and $x \in (B \cap C)'$
iff $x \in A$ and $x \notin (B - C)$
iff $x \in A$ and ($x \notin B$ or $x \notin C$)
iff ($x \in A$ and $x \notin B$) or ($x \in A$ and $x \notin C$)
iff $x \in (A - B)$ or $x \in (A - C)$
iff $x \in (A - B) \cup (A - C)$, which is equal to the RHS of the given equation.
We can therefore say that the given statement is an identity.

$x \in A \cap (B \cap C)'$
iff $x \in A$ and $x \in (B \cap C)'$
iff $x \in A$ and $x \notin (B \cap C)$
- **D.** iff $x \in A$ and ($x \notin B$ and $x \notin C$)
iff ($x \in A$ and $x \notin B$) and ($x \in A$ and $x \notin C$)
iff $x \in (A - B)$ and $x \in (A - C)$
iff $x \in (A - B) \cap (A - C)$, which is NOT equal to the RHS of the given equation.
We can therefore say that the given statement is NOT an identity.

Answer Key:A

13. **Let R be the relation on $\mathbb{Z}^{\geq}$ (the set of integers) defined by**

$(x, y) \in R$ iff $x^2 + y^2 = 2k$ for some integers $k \geq 0$.

R is not antisymmetric. Which of the following ordered pairs can be used together in a counterexample to prove that R is **not** antisymmetric? (Remember that R is defined on $\mathbb{Z}^{\geq}$.)

- **A.** (-3, 1) & (1, –3)
- **B.** (5, 3) & (3, 15)
- **C.** (4, 7) & (7, 4)
- ✔ **D.** (3, 1) & (1, 3)

Answer Key:D

14. Let A = {a, b, c}, B = {b, c, d} and C = {a, b, c, d}.

Let F = {(b, b), (a, c), (d, b), (c, a)} be a relation from C to A.

Which one of the following alternatives regarding F is TRUE?

- **A.** F is an injective function from C to A.
- ✔ **B.** F is a surjective function from C to A.
- **C.** F is a bijective function from C to A.
- **D.** F is neither an injective nor a surjective function from C to A.

Answer Key:B

15. Let U = {a, b, c, 1, 2}.

Let A = {1, c, 2}, B = {b, c, 1} and C = {a, c, 1, 2}.

Which one of the following relations is both a relation on C and on A?

- **A.** {(1, a), (2, c), (c, c)}
- **B.** {(a, 1), (1, 2), (a, c), (2, 2)}
- ✔ **C.** {(1, 2), (2, c), (c, 1), (c, c)}
- **D.** {(c, 2), (2, 2), (1, a), (c, c)}

Answer Key:C

16. Let U = {a, b, c, 1, 2}.

Let A = {1, c, 2}, B = {b, c, 1} and C = {a, c, 1, 2}.

Which one of the following relations from A to C is a function?

- **A.** {(c, a), (2, c), (1, 1), (c, 2)}
- **B.** {(a, a), (2, 2), (c, c)}
- ✖ **C.** {c, 1), (2, 2)}
- **D.** {(c, a), (c, c), (c, 1), (c, 2)}

For Question 16, the lecturer has made an error – the correct answer is C, not B.

Answer Key:B

17. Let C = {1, 3, d, e}.

Let R = {(1, 1), (1, 3), (1, e), (3, 3), (3, d), (e, 3) be a relation on C.

Which one of the following alternatives is needed to make R transitive and irreflexive?

- ✔ **A.** Add the ordered pairs (1, d) and (e, d), and remove ordered pairs (1, 1) and (3, 3).
- **B.** Add the ordered pair (d, 1) and remove ordered pairs (1, 1) and (3, 3).
- **C.** Add the ordered pair (1, d), (d, d) and (e, e).
- **D.** Add the ordered pair (d, d) and (e, e).

Answer Key:A

18. Consider the following relation on set B = {1, b, {1}, {b}, {1, b}}:

P = {(1, b), (b, {1, b}), ({1, b}, 1), ({b}, 1), (1, {1})}.

Which one of the following relations represents the composition relation

P ○ P (ie P; P)?

- **A.** {(1, {1, b}), (b, 1), ({1, b}, 1), ({b}, {1}), (1, {1})}
- **B.** {(1, {1, b}), (b, 1), ({1, b}, 1), ({b}, {1})}
- ✔ **C.** {(1, {1, b}), (b, 1), ({1, b}, {1}), ({1, b}, b), ({b}, b), ({b}, {1})}
- **D.** {(1, {1, b}), (b, 1), ({1, b}, {1}), ({1, b}, b), ({b}, {1})}

Answer Key:C

19. Let f be a function on Z^+ (the set of positive integers) defined by (x, y) ∈ f

iff $y = 2x^2 - 7$ ($f \subseteq Z^+ \rightarrow Z^+$) The relation f is NOT surjective.

Which of one of the following values for y provides a counterexample that can be used to prove that f is not surjective?

- **A.** y = 43
- **B.** y = 25
- ✔ **C.** y = 12
- **D.** y = 1

Answer Key:C

20. Let f and g be functions on Z defined by:

(x, y) ∈ g iff $y = -x^2 + 3$ and (x, y) ∈ f iff y = 5 - 3x

Which one of the following alternatives represents an ordered pair that does NOT belong to f?

- **A.** (-2, 11)
- **B.** (0, 5)
- **C.** (1, 2)
- ✔ **D.** (-3, 13)

Answer Key:D

21. Let f and g be functions on Z defined by:

(x, y) ∈ g iff $y = 2x^2 - 5$ and (x, y) ∈ f iff $y = -3x + 7$

Which one of the following statements regarding functions f and g is TRUE?

- **A.** Neither function f nor function g is injective.
- **B.** Function f is bijective, but function g is not bijective.
- **C.** Both functions f and g are surjective.
- ✔ **D.** Function f is injective, but function g is not injective.

Answer Key:D

22. Let f and g be functions on Z defined by:

(x, y) ∈ g iff $y = 2x^2 + 4x$ and (x, y) ∈ f iff $y = 5 - x$.

Which one of the following alternatives represents $g \circ f(x)$ (i.e. g(f(x))?

- ✔ **A.** $2x^2 - 24x + 70$
- **B.** $2x^2 - 28x + 90$
- **C.** $2x^2 - 16x + 50$
- **D.** $2x^2 - 20x + 54$

Answer Key:A

23. Let set P = {a, b, {b}, {{a}, 3}}. Which one of the following sets is a partition of the set P?

- ✔ **A.** {{{{a}, 3}}, {a, {b}}, {b}}
- **B.** {{{a} ,3}, {a, b, {b}}}
- **C.** {{b}, {a, b}, {{a}, 3}}
- **D.** {a, b, {b}, {{a}, 3}}

Answer Key:A

24. Consider the following matrices:

$$Let\ A = \begin{bmatrix} 1 & 6 \\ 2 & 5 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix} \ and \ C = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}.$$

Which one of the following alternatives regarding operations on the given matrices is TRUE?

- **A.** The result of (C · A) · B is a 2 x 2 matrix.
- **B.** The result of (A · A) is a 2 x 2 matrix.
- ✔ **C.** The result of (C · A) is a 1 x 2 matrix.
- **D.** The result of (B · A) is a 2 x 3 matrix.

25. Consider the following matrices:

$$Let\ A = \begin{bmatrix} 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 3 \\ 2 & 1 \end{bmatrix} \quad and\ C = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}.$$

Which one of the following alternatives regarding operations on the given matrices is FALSE?

- **A.** Performing the operation A · C will result in a 1 x 3 matrix.
- **B.** The result of C · B is the matrix
$$\begin{bmatrix} 13 & 11 \\ 11 & 13 \end{bmatrix}.$$
- ✘ **C.** Calculating C + B will result in a 2 x 2 matrix.
- **D.** The result of B · C is the matrix
$$\begin{bmatrix} 7 & 6 & 5 \\ 12 & 12 & 12 \\ 5 & 6 & 7 \end{bmatrix}$$

For Question 25, the lecturer has made an error – the correct answer is C, not B.

Answer Key:B

26. Let A = {a, b, c, d}. Consider the following table for the binary operation *: A x A → A:

*	a	b	c	d
a	d	c	b	d
b	c	d	a	b
c	b	b	d	c
d	d	b	c	c

Which of the following is true regarding an identity element for operation * ?

- [ ] **A. b is the identity element.**
- [x] **B. d is the identity element.**
- [ ] **C. * does not have an identity element.**
- [ ] **D. c is the identity element.**

Answer Key:C

27. Consider the binary operation * below:

*	a	b	c
a	b	c	a
b	c	b	a
c	c	a	b

Which one of the following elements is NOT a part of the list notation of the binary operation * ?

- **A.** ((b, b), b)
- **B.** ((c, c), b)
- **C.** ((c, c), a)
- **D.** ((a, b), c)

Answer Key:C

28. Consider the following incomplete truth table:

p	q	¬q	¬q → p	p → (¬q → p
T	T	F	T	T
T	F	T	T	T
F	T	F	T	T
F	F			

Which one of the following alternatives correctly reflects the incomplete row?

- **A.** | F | F | T | T | F |
- **B.** | F | F | T | F | F |
- **C.** | F | F | T | T | T |
- **D.** | F | F | T | F | T |

Answer Key:D

29. Which one of the statements in the following alternatives is NOT equivalent to $p \wedge q$? (Hint: simplify the statement in each alternative using de Morgan's rules or a truth table to find the statement that is NOT equivalent to $p \wedge q$.)

- **A.** $p \wedge (\neg(p \to \neg q))$
- ✔ **B.** $p \wedge (\neg(\neg p \to \neg q))$
- **C.** $\neg(\neg p \vee (p \to \neg q))$
- **D.** $p \wedge (\neg(\neg(p \wedge q)))$

Answer Key:B

30. Which one of the following alternatives is equivalent to $\neg p \vee q$? (*Hint*: Simplify the statement in each alternative using de Morgan's rules.)

(Hint: simplify the statement in each alternative using de Morgan's rules.)

- **A.** $(p \to \neg q) \vee (p \vee \neg q)$
- ✔ **B.** $(p \wedge \neg q) \to (\neg p \vee q)$
- **C.** $(p \wedge \neg q) \vee (\neg p \to \neg q)$
- **D.** $(p \wedge \neg q) \vee (\neg p \vee q)$

Answer Key:B

31. Consider the following statement:

$$\forall x \in Z^+, [(x + 3 > 0) \vee (3x + 1 < 0)]$$

Which one of the following statements about the given statement is TRUE?

- **A.** The negation of the statement is TRUE.
- ✔ **B.** The negation of the given statement is $\exists x \in Z^+, [(x + 3 \leq 0) \wedge (3x + 1 \geq 0)]$
- **C.** The given statement is TRUE for $x = 0$.
- **D.** The given statement is FALSE.

Answer Key:B

32. Consider the following two quantified statements:

Statement 1: $\forall x \in Z$, $[(2x - 1 > 3) \vee (x^2 + 1 \geq 0)]$.

Statement 2: $\exists x \in Z$, $[(3x + 2 \leq 0) \wedge (x + 2 > 0)]$.

Which one of the following statements is true regarding statements 1 and 2 above?

- **A.** Statement 1 is true and statement 2 is false.
- **B.** Statement 1 is false and statement 2 is true.
- ✔ **C.** Both statements 1 and 2 are true.
- **D.** Both statements 1 and 2 are false.

Answer Key:C

33. Consider the statement:

If n is a multiple of 3, then 6n + 9 is even.

Which one of the following statements provides the contrapositive of the given statement?

- **A.** If n is a multiple of 3, then 6n + 9 is even.
- **B.** If n is not a multiple of 3, then 6n + 9 is odd.
- **C.** If 6n + 9 is even, then n is a multiple of 3.
- ✔ **D.** If 6n + 9 is odd, then n is not a multiple of 3.

Answer Key:D

34. Which of the following alternatives provides a **direct** proof to show that for all $n \in \mathbb{Z}$,

If n + 1 is even, then $n^3 + 3n + 4$ is even.

- **A.** Let n be even, then n = 2k, for some $k \in \mathbb{Z}$,
  then $n^2 + 3n + 4$
  ie $(2k)^2 + 3(2k) + 4$,
  ie $4k^2 + 6k + 4$,
  ie $2(2k^2 + 3k + 2)$, which is even.

- **B.** Let n + 1 = 2, which is even, then n = 2 – 1 = 1,
  then $n^2 + 3n + 4$
  ie $(1)^2 + 3(1) + 4$,
  ie 8 which is even.

- C. Let n + 1 be even,
then $n^2 + 3n + 4$
ie $(n + 1)^2 + 3(n + 1) + 4$,
ie $n^2 + 2n + 1 + 3n + 3 + 4$,
ie $n^2 + 5n + 1 + 3 + 4$,
ie $n^2 + 5n + 2(4)$, which is even because odd + odd + even is even.

- ✔ D. Let n + 1 be even, then n + 1 = 2k, for some $k \in \mathbb{Z}$, then n = 2k – 1,
then $n^2 + 3n + 4$
ie $(2k - 1)^2 + 3(2k - 1) + 4$,
ie $4k^2 - 4k + 1 + 6k - 3 + 4$,
ie $4k^2 + 2k + 2$
ie $2(2k^2 + k + 1)$, which is even.

Answer Key:D

35. Which one of the alternatives is a proof by contradiction for the statement
"If $2x^2 - 3x + 7$ is odd, then x is even."

- A. Required to prove: If x is odd, then $2x^2 - 3x + 7$ is even.
Proof: Suppose x is odd. Let x = 2k + 1,
then we have to prove that $2x^2 - 3x + 7$ is even.
$2x^2 - 3x + 7 = 2(2k+1)^2 - 3(2k + 1) + 7$
$= 2(4k^2 + 4k + 1) - 6k - 3 + 7$
$= 8k^2 + 8k + 2 - 6k - 3 + 7$
$= 8k^2 + 2k + 6$
$= 2(4k^2 + k + 3)$, which is even (2 multiplied by any integer is even)

- ✔ B. Assume that $2x^2 - 3x + 7$ is odd. Then x can be even or odd.
We assume that x is odd.
Let x = 2k + 1, then
$2x^2 - 3x + 7 = 2(2k+1)^2 - 3(2k + 1) + 7$
$= 2(4k^2 + 4k + 1) - 6k - 3 + 7$
$= 8k^2 + 8k + 2 - 6k - 3 + 7$
$= 8k^2 + 2k + 6$
$= 2(4k^2 + k + 3)$, which is even (2 multiplied by any integer is even)
But this is a contradiction to our original assumption.
Therefore x must be even if $2x^2 - 3x + 7$ is odd.

Let $x = 2$ be an even element of $\mathbb{Z}$.
We can replace x with 2 in the expression $2x^2 - 3x + 7$.
$2x^2 - 3x + 7$

- **C.** $= 2(2)^2 - 3(2) + 7$
$= 8 - 6 + 7$
$= 9$, which is odd.
We have therefore proven that if $2x^2 - 3x + 7$ is odd, then x is even.

Required to prove: if $2x^2 - 3x + 7$ is odd, then x is even.
Proof: Assume that x is even, i.e. $x = 2k$,
then $2x^2 - 3x + 7$

- **D.** $= 2(2k)^2 - 3(2k) + 7$
$= 8k^2 - 6k + 7$
$= 2(4k^2 - 3k + 3) + 1$, which is odd.
We have therefore proven that if if $2x^2 - 3x + 7$ is odd, then x is even.

Answer Key:B