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Dynamic Risk Aversion and the Business Cycle: A Cause or a Symptom?

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Abstract

The provision of a comprehensive overview of the academic literature relating to the dynamic aspect of investors' appetite towards risk and its relation to the business cycle. In this context the most influential research studies, both theoretical and empirical, are discussed. Lastly, the research question my dissertation addresses is posed and a model consisting of heterogeneous agents is created, with the aim of observing cyclicality when the RRA coefficient is allowed to fluctuate dynamically.

1 Introduction

Risk aversion is defined by D. J. Meyer and J. Meyer (2006) as *"a formal way to measure ones propensity to choose the certain outcome over an unknown in a gamble with a certain payoff"*. It is important to understand the concept in order to govern risk taking and often destructive behaviours. Risk aversion can be parameterised by the risk aversion coefficient, a risk averse investor will accept a lower payoff for greater certainty, with the intensity determined by the curvature of their utility function. In 1964 and 1965 *John Pratt* and *Kenneth Arrow* showed that risk aversion can be quantified as follows:

$$\text{Absolute Risk Aversion:} \quad -\frac{U''(W)}{U'(W)} \equiv R_{ar}, \tag{1.1a}$$

$$\text{Relative Risk Aversion:} \quad -\frac{U''(W)W}{U'(W)} \equiv R_{ra}, \tag{1.1b}$$

where $U(W)$ denotes the utility function characterising the agent, and W her wealth.

- Absolute Risk Aversion [Eq. (1.1a)]: This captures the investors preferences and also the propensity to take a risk. It is assumed that an investor wishes to maximise 'Expected Utility' (U) when outcomes are uncertain with the coefficient showing the absolute monetary value investors will pay to avoid a gamble of a specific size.

- Relative Risk Aversion [Eq. (1.1b)]: This considers the proportion of total wealth W an investor will pay to avoid a gamble sized relative to her wealth. It is assumed that an individual prefers more wealth to less; $U'(W) > 0$ Captured by the first order derivative of the utility function.

The sign of the coefficient defined by Eqs. (1.1a), (1.1b) categorises the agent as risk averse, or risk loving. In finance it is assumed that agents are averse to risk and the formula captures the rate of diminishing preference when faced with decreasing probabilities. $U''(W) < 0$ reflects the diminishing marginal returns in expected utility as the probability of the outcome decreases, causing the function to be concave.

Most literature surrounding the topic revolves around measuring risk tolerance, a difficult task due to the individual differences across persons. It can be intuitively concluded that a person's risk tolerance is influenced by an abundance of factors including but not limited to; age, health, wealth, experiences, emotion, personality and culture (see Sahm (2012) We will conduct this review with a focus on the effect of experience in the market.

The majority of the academic literature in finance favour two categories of concave utility functions corresponding to

1. The absolute risk aversion is a constant (Constant Absolute Risk Aversion-CARA). This condition is met by the exponential utility function over the wealth.

2. The relative risk aversion is a constant (Constant Relative Risk Aversion-CRRA). The power utility (also known as iso-elastic) satisfies this condition [see Eq. (1.2)].

The concept of CARA is limited in relevance, in view of the empirical evidence regarding risk aversion, see J. Y. Campbell and Viceira (2002b).

$$U(W) = \frac{W^{1-a}}{1-a} \tag{1.2}$$

Under the power utility assumption ARA decreases as wealth increases whilst RRA is a constant.

In the following,the empirical studies linking time-varying risk aversion with the business cycle, have been considered. Furthermore the theoretical literature on risk aversion in the dynamic sense has been reviewed.

2 Empirical Studies

There are a handful of experimental studies which showing risk preferences to change across time; the most relevant being the research of Guiso, Sapienza, and Zingales (2018) who studied qualitative and quantitative measures of 1,541 Italian household's risk preferences before and after the crisis. This was done using survey methods, and by monitoring the composition of individual portfolios, in 2007 and in 2009. This research is extremely holistic given that both a lab experiment and a data study were conducted.

The lab experiment was done by playing an extract from a horror film to the 'treated' sample, they then gave a hypothetical lottery to both samples where individuals were given several choices between a prospect which paid €10,000.00 or €0.00 with equal probability. They had an alternative option to take a certain amount of money which was steadily increasing from €100.00 to €9,000.00, meaning the more risk averse individuals will take the certain outcome at a lower sum.

It could be argued that the method of using a film to condition fear, along with the hypothetical version of the lottery is far detached from real life experience, which diminishes the value of this experiment; however the purpose was only to give a preliminary understanding of the influence of fear. The results were statistically significant at the 1% level and showed that the treated sample had a risk premium 27% higher than the control sample.

The more important part of Guiso, Sapienza, and Zingales (2018) study is that involving the study of the Italian investors across time, the same hypothetical lottery and other qualitative questions were given to 1,541 individuals of whom had authorised access to the administrative records of their holdings. This allowed the observation of the proportion of risky assets in their portfolio on both sides of the crisis and the consideration of changes in wealth.
The proportions of investors claiming not to wish to take any financial risk increase from 16% to 43% and aversion is correlated with wealth levels across both years. There was also a decrease in the holdings of risky assets post crisis in 2009. Note that the study is only representative of Italian investors and it is difficult to isolate the effects of fear from wealth fluctuation.

These findings are similar to that of Menkhoff and Sakha (2016) who conducted a panel data study on 2000 households across rural Thailand. Also using a certainty equivalent lottery with but improving on the issues of hypothetically involved in Guiso, Sapienza, and Zingales (2018) with the expected value of the lottery being around one days unskilled wage (150 Baht). The study was able to capture change over five years and collected detailed characteristics including expenditures, savings, health, employment, socio-economic characteristics and experience of micro-level shocks. The results showed that risk aversion decreases from 2008 to 2010 and normalises in 2013. This time period incorporates the crash and recovery stages of the financial crisis; furthermore, at the 5% level of significance, economic shocks have a greater effect on risk tolerance for households in the bottom 20% of income distribution. The risk preference variability can be attributed to micro-level shocks, income and total asset shocks.

Sahm (2012), also uses a hypothetical gamble response taken from the Health and Retirement Study 1992-2002. This measure of risk aversion could then be compared with changes in income, age and macroeconomic conditions. Risk aversion moderately increased with age and decreased as macroeconomic conditions improved. However, unlike the results of Guiso, Sapienza, and Zingales (2018) and Menkhoff and Sakha (2016), there was no significant impact of a change in income. There was a modest relationship in terms of a 10% higher level of average income being associated with a 0.9% higher relative risk tolerance; there are causality implications for this finding since those individuals who are naturally more risk tolerant may gravitate toward high risk/return occupations. On the other hand, the higher income level could be the causal factor in increasing tolerance.

There are studies such as that of Andersen et al. (2008) who did not find significant change in risk aversion over time, using the same certainty equivalent lottery method as above on 253 Danish adults; although it should be highlighted that this was done over a 17 month period limiting the ability for preferences to vary measurably.

Parada-Contzen (2017) did a longitudinal study between 2002 and 2009 into the various characteristics impacting individual risk aversion in a dynamic life-cycle. The study compares the estimated marginal effects of variables impacting investment decisions, when risk aversion is not modelled, endogenously modelled, exogenously and when its evolution is modelled as a function of its previous realizations. The research is thorough in relation to time-varying measures of aversion, with the model depending on the curvature of the current period utility function along with the curvature of discounted future utility.

The study models both observed measures of risk preference and unobserved heterogeneity, an important feature given the heterogeneous nature of individual risk tolerance. It was found that there are correlations between observed measures and behaviours such as occupation selection, investment decisions and health status; which confirms their usefulness as proxies in other literature. There were however no significant effects of wealth levels and previous investment decisions on individual risk aversion; although this is when it is modelled as an endogenous determinant.

A different approach to examining counter-cyclical risk aversion is that of Cohn et al. (2015). They conduct a controlled experiment on 162 financial professionals, endowed with 200 CHF and told to decide the proportions to which they will allocate between a risky asset and a risk- free account with zero interest. Subjects were primed with either a boom or bust scenario by exposing them to images depicting either boom or bust price charts. Other literature such as Guiso, Sapienza, and Zingales (2018) shows the relationship between fear and risk aversion. In order to help define causality in this association, the subjects were informed they would receive either painful, mild or painless electric shocks during the investment task.

The results showed that those primed with a bust scenario invest an average of 22% less into the risky asset than the boom primed subjects. Furthermore, rather than the receiving an electric shock it was the expectation of doing so which diminished risk taking by 10%. The outcome of Cohn et al. (2015) provides strength to the intuitive reasoning for counter-cyclical risk aversion of fear arising from losses which in turn heightens aversion and leads to a snowballing feedback loop of asset sales. This is also consistent with a neurostudy by Kandasamy et al. (2014), who found risk aversion to increase with administration of cortisol. It should be noted that there are likely issues with selection bias since Cohn et al. (2015) only observe individuals who participate in the financial markets and likely have a higher risk tolerance.
In regard to wealth variability as a determinant of risk aversion, Brunnermeier and Nagel (2008) create a model of portfolio choice in order to examine 20 years of household data from the 'Panel Study of Income Dynamics'. They found that fluctuation in liquid wealth is correlated with stock market entry and exit; however, it does not significantly explain change in asset allocation. This provides little evidence for the difference-habit theory of risk aversion the authors base their model on (difference-habits will be discussed further in the theoretical section of this review). It could be argued that the absence of impact on allocation is down to inertia. Portfolios are slow to be rebalanced and to react to information, contradicting the assumption of an efficient market, but nevertheless true at the individual level.

One empirical study which highly strengthens our hypothesis is that of Malmendier and Nagel (2009) who investigate the impact of macroeconomic experiences on risk attitude, particularly in relation to stock and bond market behaviour. It is a thorough longitudinal study between 1964 and 2004 using 'Survey of Consumer Finances' data, and controls for wealth, income and age effects. Both at the micro-level and on aggregate, they find that historical asset returns experienced across investors lives impact the risk tolerance in further participation, both in bond and stock markets. Moreover, it is found that recent returns are comparatively more salient than longer lagged return memories, although impacts still fade slowly with time.

Although this is important research, it should be said that that the causality cannot be distinguished between variation in risk tolerance in the endogenous respect, and the future belief impacts which can depend on historical experience and other exogenous information sources.

On the topic of the persistence of effects, Cameron and Shah (2013) study the effect of natural disasters in Indonesia on risk aversion and find recent experiences to have a stronger negative effect on risk aversion than older ones. Although not a directly macroeconomic shock, natural disasters increase background risk and risk vulnerability. A large sample of 1550 people from 120 villages were given a lottery game, (similar to that of Sahm 2012; Guiso, Sapienza, and Zingales 2018; Menkhoff and Sakha 2016) and results compared to earthquake and flood data.

In a very similar panel study of risk aversion before and after the 2011 Japan earthquake by Hanaoka, Shigeoka, and Watanabe (2018) it was found that the greater the intensity of the disaster effects the more risk tolerant males become, however with no significant change exhibited by females. This unexpected result could possibly be due to the severity of the event; the radiation exposure and death toll from the Fukushima accident could impact subject perception of life expectancy, accompanied with the reported post-disaster increase in drinking and gambling behaviours.

There are also two studies supporting time varying and counter-cyclical risk aversion using reduced form models and market level data. Smith and Whitelaw (2009) compute excess returns on data starting in 1952 and Antell and Vaihekoski (2016) use a reverse estimation on a Mertonian model to measure the equity premium from 1928 onwards. These long time periods aid in producing robust conclusions, both studies assume the Merton (1969) model to be in full accordance with stylized facts.

3 Theoretical Literature

In terms of theory there seems to be support for dynamic risk aversion albeit coming from many angles and foundations. Gollier and Pratt (1996) proposed the theory of risk-vulnerability. This is underpinned by the idea of an agents risk aversion being susceptible to the exogenous introduction of new 'unfair risks' to capital, explaining fluctuations in the equity premium.

Bommier and Rochet (2006) consider the assumptions of 'additive separability' in the von Neumann-Morgenstern utility function and find that when these are disregarded and providing consumption levels are specific substitutes across time, then risk aversion decreases with respect to horizon length. On the subject of risk aversion and consumption influence, Campbell and Cochrane (1999) formulated the 'habit-preferences' theory, mentioned earlier in the empirical section. This is based on a subsistence habit level which increases with consumption, but as consumption converges towards the habit level, risk aversion increases; as it diverges away, aversion falls. This could explain depressed prices in recessions, despite the high expected return, consumption is dampened and individuals are extremely averse to falling below a developed level of habit.

Although based on different foundations, habit preference shares similarity with 'cumulative prospect theory' formulated by Kahneman and Tversky (1979), the theory proposed a concave value function for gains and convex for losses; however, with a steeper loss function gradient. Both theories share the concept of high loss aversion which creates a disposition effect. The disposition effect means investors hold onto losing assets longer than they hold onto winners since they dislike losses more than they like gains. In terms of business cycles this could be hypothesised to increase the inertia of sales once investors reach a threshold of loss on the convex function, therefore creating more visible free-falls in a bust cycle.

Weber and Camerer (1998) ran portfolio experiments which proved the disposition effect but cannot eliminate the possibility of investors expecting mean reversion of both their falling and rising assets.
Bringing together prospect theory and time-varying risk aversion, Barberis, Huang, and Santos (1999) propose an asset pricing framework which accounts for both, where utility is derived from levels and changes in value, whilst still considering the disposition effect. This theory is very important as it ties in with empirical studies showing that utility depends on prior investment outcomes. (see Guiso, Sapienza, and Zingales (2018), Cohn et al. (2015), and Brunnermeier and Nagel (2008)) The loss subsequent aversion rise cannot however be derived from prospect theory as this is yet to be understood dynamically.

Constantinides and Ghosh (2017) also develop an asset pricing model using the counter-cyclical left skewness of household consumption risk. It is the first study to find equilibrium using a heterogeneous agent model with households exhibiting recursive utility. Epstein and Zin (1989) developed a two-component recursive utility function which results in a time additive von Neumann-Morgenstern utility, removing the iso-elasticity and using 'first-order risk aversion' parameters (where risk aversion is independent of history). Recursive utility is important in Constantinides and Ghosh (2017) since it assumes intertemporal substitution (the inverse of the Arrow-Pratt approximation, 1.2) to be counter-cyclical. They find the left skewness of the consumption risk to be significantly related to business cycles.

Li (2007) study time-varying risk aversion and asset prices and find that counter-cyclical aversion results in a pro-cyclical premium, this is against the intuition of Campbell and Cochrane (1999) habit preferences but does not disprove it: higher risk premia are demanded in recession, but the effect of low volatility when risk aversion is high, in turn lowering the premium. The second effect dominates the first.

4 Methodology

The aim of this research project is to fill the gap in the theoretical literature in regard to the relation between dynamic risk aversion and the business cycle. The forthcoming study will be investigating this through an agent-based dynamic computational model; similar to Iori (2002) who simulates market activity to explain some stylized facts of returns. The model will simulate a market consisting of two assets, i.e. one risk-free (cash) and one risky (stock) and two types of agents:

1. A representative noise-trader creating demand side shocks on the risky asset.
2. A number of heterogeneous boundedly rational utility-maximising agents characterised by a CRRA utility function [see equation (1.1b)] over their wealth. The innovation consists in allowing the relative risk aversion coefficient to depend (dynamically) on the agents' recent experience (making losses, or profits) in accordance to an exponentially moving average. More specifically the current measure of relative risk aversion[1] follows a simple (deterministic) AR(1) process, such that the current appetite to risk varies according to an exponentially moving average with respect to the most recent experience of making profit, or losses in the previous period. Profits

[1]In general in the vast majority of dynamic models in finance assume that the agents' measure of relative risk aversion is an (endogenously) set parameter versus an endogenously determined (dynamical) variable.

will be assumed to be leading to an increase of agents' risk appetite and, conversely, losses will be assumed to be making them more risk averse.

In the context of this model I will investigate the interaction of a representative noise trader (who in the absence of the utility-maximising agents would cause the price of the risky asset to fluctuate such that the logarithmic returns on the asset are normally distributed, according to the efficient market hypothesis (see Thurner, Farmer, and Geanakoplos (2012) pg. 7). A differing result would suggest that the memory of positive or negative market experience is a sufficient condition for the creation of booms and busts, i.e. the "business cycle", for example returns may be unexpectedly distributed or replicate stylized facts such as fat tailed returns.

Much of the current literature contains models in which a CARA utility function is used, (see Levy (1991), Brunnermeier and Nagel (2008)) and many of the empirical studies estimate an ARA coefficient (Cohn et al. (2015), Babcock, Choi, and Feinerman (1993), Cameron and Shah (2013)). In order to observe the effects of dynamical risk aversion, a relative risk aversion coefficient must be used and allowed to fluctuate. Campbell and Cochrane (1999), Maio (2007), and Azar and Karaguezian-Haddad (2014) show that the RRA coefficient should vary with time and has a negative relationship with the business cycle.

The RRA coefficients are usually estimated using the intertemporal CAPM through the use of equity return data. So as to represent the heterogeneity between investors, I will be using a range of RRA coefficients through in the model. There is limited research into the effect of gains and losses on the relative risk aversion coefficient, however Paravisini, Rappoport, and Ravina (2010) use loan portfolio data to compute an average RRA coefficient of 2.82; they use a fixed-effects study and find that a shock to wealth corresponds to change in the RRA coefficient with an elasticity of -1.18. This is particularly useful as these results can be used to calibrate the risk-taking behaviours of the sophisticated investors in the model.

Paravisini, Rappoport, and Ravina (2010) provide a single elasticity and therefore assume symmetry between gains and losses, however the works of Kahneman and Tversky (1979) should be considered. Prospect theory implies that agents are more sensitive to losses than gains, this is often referred to as the reflection effect. A number of studies including Abdellaoui, Bleichrodt, and L'Haridor (2008), Camerer (1998) both find evidence for loss aversion and the reflection effect at both the individual and aggregate level. (Sousa 2007) conduct a very thorough empirical study into the effects of wealth shocks on RRA; they find little evidence for the inertia and delayed portfolio re-balancing effects discussed earlier in this paper and in (Brunnermeier and Nagel 2008). Sousa (2007) use US quarterly

data ranging from 1953-2004 to observe changes in portfolio composition. They find that there is significant cyclicality in relative risk aversion and that it is related to wealth shocks. This is the case for financial, and both directly and indirectly held stock market wealth. In terms of housing wealth, this is generally held as a hedge, and so a larger real-estate allocation is seen upon a negative wealth shock.
Sousa (2007) find that a 1% shock to wealth corresponds to a change in risky assets held of 2.682%. These results are significant at the 1% level of confidence.

5 Heterogeneous Agent Model

The economy consists of two types of agents, a representative uninformed noise-trader and M sophisticated (utility-maximising) agents trading two assets, a risk-free (cash) and N items of a risky one (stock). The sophisticated agents are characterised by a CRRA utility function. The agents are assumed to have heterogeneous appetites towards risk. Moreover, the risk-appetite is assumed to explicitly depend on the profit or loss making experienced at the previous period.

In the following, the model is presented in finer detail.

5.1 Noise Traders

The noise-traders demand at the future period $t+1$, $t \in \mathbb{N}$ (in terms of dollar value) is assumed to follow a mean-reverting AR(1) process (Thurner, Farmer, and Geanakoplos 2012; Poledna et al. 2014). Let ξ_t denote the noise traders' demand; the demand for the risky asset at the next period is

$$\log \xi_{t+1} = \rho \log \xi_t + (1-\rho) \log (VN) + \sigma \chi_{t+1}, \tag{5.1}$$

where ρ determines the rate of reversion to the mean, N is the total number of risky assets supplied at each period, V determines the long-run expected logarithmic demand $\mathbb{E}[\log \xi_{t+1}]$, and $\chi_{t+1} \sim N(0,1)$ is the random shock on the demand side. Since the stochastic process governing the noise-traders' demand for the risky asset is a mean-reverting AR(1) process, in the long run there exists a steady state distribution with respect to $\log \xi_{t+1}$. Therefore, in the absence of the utility-maximising agents, a steady state distribution of the future logarithmic equilibrium price $\log p_{t+1}$ and the logarithmic returns on the risky asset $r_{t+1} = \log (p_{t+1}/p_t)$ also exists. As shown in Appendix A both stochastic variables, $\log p_{t+1}$, r_{t+1} are normally distributed. Specifically,

$$\begin{aligned} \mathbb{E}[\log p_{t+1}] &= \log V \\ \mathrm{Var}[\log p_{t+1}] &= \frac{\sigma^2}{1-\rho^2}. \end{aligned} \tag{5.2a}$$

and

$$\begin{aligned} \mathbb{E}[r_{t+1}] &= 0, \\ \mathrm{Var}[r_t] &= \frac{2\sigma^2}{1+\rho}, \end{aligned} \tag{5.3}$$

5.2 Sophisticated Investors

The risky asset is also traded by utility-maximising investors assumed to have an iso-elastic utility function with respect to their wealth [see Eq. (1.2)], referred to herein as the "sophisticated investors" (SIs). The jth utility investor seeks to maximise her expected utility as a function of her wealth at the next period $\mathbb{E}[U^j(W^j_{t+1}]$ given her wealth at period t.[2] The optimal demand for the sophisticated investors is bounded between -1 and 1, this is because they are allowed to take both long and short positions however they must be unleveraged, due to the possible cycles which could arise through borrowing. SIs are assumed to be aware that consecutive returns on the risky asset are correlated because of the trading behaviour of the NTs and use this information in order to estimate the expected return one period ahead [see details in Appendix B]. Assuming further that returns are log-normally distributed and that the risk-free rate $r_f = 0$ the optimal demand of the jth SI is

$$D_t = \max\left\{-1, \min\left[\frac{\mathbb{E}_t(r_{t+1}|r_t) + \frac{1}{2}\mathrm{Var}(r_{t+1})}{a^j_t \ \mathrm{Var}\ (r_{t+1})}, 1\right]\right\} W^j_t/p_t, \tag{5.4}$$

where $\mathbb{E}_t(r_{t+1}|r_t)$ denotes the conditional expected future return given the information set $I_t = \{r_1, r_2, \ldots r_t\}$ at time t, and W^j_t, a^j_t her wealth and relative risk aversion (RRA) coefficient, respectively. The RRA coefficient depends on the most recent rate of return $\Delta W_t/W_t$ of the agent. Specifically, the time-evolution of a^j_t is

$$a^j_{t+1} = a_t + \epsilon_a a^j_{t-1}\frac{\Delta W^j_t}{W^j_t}, \tag{5.5}$$

where ϵ_a is the wealth elasticity of the RRA coefficient; a figure of -1.18 has been used, taken from the study by (Paravisini, Rappoport, and Ravina 2010).

5.3 Price formation

Given that the risk-free rate of return is assumed 0, the wealth of the jth SI evolves as

$$W_{t+1} = (p_{t+1} - p_t)D_t + W_t. \tag{5.6}$$

The market clears when

$$\sum_j D^j_t + \xi_t/p_t = N. \tag{5.7}$$

Therefore, the equilibrium price is determined by solving Eq. (5.7) using Eqs. (5.1), (5.4) and (5.6).[3]

[2]Hence, the SIs are myopic.

[3]The conditional expected value and variance of the next-period return, given the most recent observation, is estimated as the running mean and variance utilising the algorithm found in (Knuth 2014, p 282).

5.4 Parameter values

In all numerical calculations the number of SIs is $K = 10$. The stochastic process determining NTs behaviour cannot be directly calibrated using empirical data. Therefore, we source the relevant parameter values from (Thurner, Farmer, and Geanakoplos 2012). Namely, we set $\sigma = 0.035$, $V = 1$ and $N = 1000$. We deviate from the calibration in Thurner, Farmer, and Geanakoplos (2012) with respect to the value of ρ which in their study was set close to 1. However, given the aim of my study such a choice could lead to the emergence of "spurious" cycles in the price of the risky asset because of the NTs rather than the dynamic dependence of the RRA coefficient. However, notice that too small a value of ρ would make the last observed return more "informative", as the smaller ρ becomes, the stronger the dependence of the conditional expected value of the next return on the most recent observation [see Fig. 3], which will increase SI demand. This would lead to a rapid increase of the market share of the SIs and therefore strong non-stationarity in all observables of interest. Bearing these points in mind, after searching the parameter space with respect to ρ, the optimal choice is $\rho = 0.85$ The endowment of the SIs is the same across all agents $W_0 = 10$. For this choice of an initial value of the wealth of the SIs the market power of the NTs dominates by an order of magnitude over the (collective) one of the SIs. This ensures that any deviation from the statistical properties of the market when only the NTs are trading do not stem from the choice of the initial conditions and it can only emerge dynamically. Lastly, the RRA coefficient is uniformly distributed in $[0.37, 15.97]$ according to the values reported by Bliss and Panigirtzoglou (2004).

6 Results

6.1 Wealth dynamics: booms and busts

We start by first investigating the evolution of the net wealth of the SIs. A natural question that arises is whether the SIs, given their strategy, can exploit the irrational behaviour of the NTs to (on average) increase their wealth indefinitely. The unbounded increase in their wealth is a possibility given that there are no transaction or operational costs taken into account in the model.

In Fig. 1 the time series of net wealth for 5 of the SIs is shown [panel (c)], as well as the corresponding rate of return [panel (d)]. In order to have a measure of comparison, the same observables are shown assuming that the RRA coefficient is fixed [panels (a), and (b)]. As observed in Fig. 1 (a) where the wealth of 5 of the agents is plotted as a function of time for 10^5 periods, fixing the RRA coefficient, all agents, who differ only in their appetite for risk, demonstrate an upward trend. Therefore, when the RRA coefficient is constant, the SIs systematically profit at the expense of the NTs. Moreover, the loss experienced in

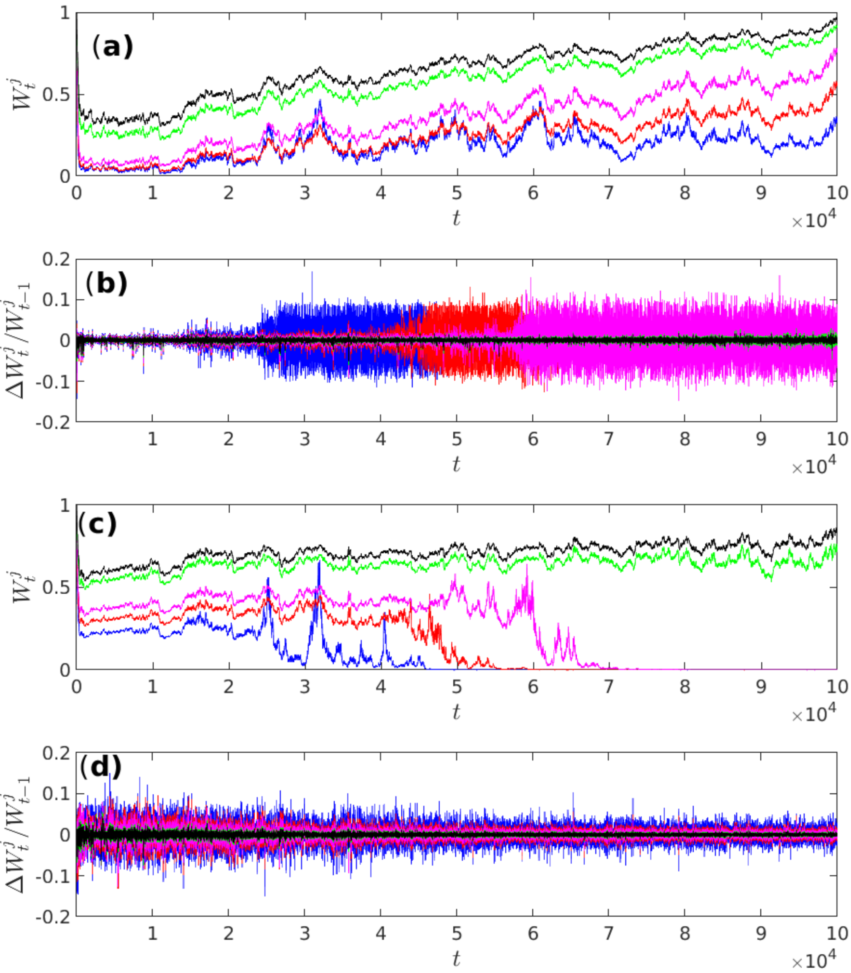

Figure 1: The net wealth [panels (a), and (c)] and the rate of return [(b) and (d)] $\Delta W_t^j / W_t^j$ for 5 of the SIs trading in the market as a function of time, for $t \in [1, 10^5]$. Panels (a) and (b) correspond to a constant RRA coefficient, while in (c) and (d) the RRA is varying according to Eq. (5.5). The blue, red, magenta, green and black lines correspond to SIs in ascending order of the (initial) value of RRA.

the short-run, i.e. when the number of observations is small and their beliefs about the parameters of the distribution of returns are inaccurate, they lose a different proportion of their endowment. In fact, the loss experienced by the SIs for small t is inversely proportional to their RRA. This can be interpreted as follows: in the absence of sufficient information about the statistics of returns, the more risk averse an agent is, the lesser the loss a SI experiences.

As t increases and the beliefs of the SIs become more accurate, their wealth consistently increases, i.e. shows a clear upward trend. In contrast, when the RRA coefficient is allowed to vary [Fig. 1 (c)] the three most risky agents at $t = 0$ [blue, red, and magenta lines] experience a consistent increase of their wealth for a period of time. This, however, results in the decrease of their aversion to risk, according to Eq. (5.5), resulting in a substantial loss, from which they never recover (the wealth approaches 0). In Fig. 2 we run the model for an additional 3×10^4 number of periods, until a substantial rapid decrease of the wealth of all agents is detected to ensure that the "booms" and "busts" are independent of the initial value of the RRA coefficient. It can be seen that the SIs with the highest initial value of the RRA coefficient (most risk averse) continue to increase their wealth, after more risky investors have been "eliminated" from the market. However, as the initially less risky SIs continue to increase their wealth (on average), they become increasingly risk tolerant. As this occurs they also become more susceptible to demand shocks in the risky asset caused by the NTs and virtually all of their accumulated wealth is wiped out. The fluctuations in the rate of returns however, follow an opposite trend when the RRA is a constant [panel (b)] and dynamic [panel (c)]. When RRA is a constant, the volatility of the rate of return increases for $t \lesssim 2.6 \times 10^4$ after which point in time it attains a constant value. Therefore, as agents adopt their beliefs in response to their information set, their demand increases which in turn leads to an increase in the fluctuations of their wealth. Yet, when the RRA coefficient is allowed to vary, the change in their appetite towards risk is dominating over the convergence of their beliefs to their asymptotic values.

In conclusion, the dynamic dependence of the RRA coefficient on wealth shocks does indeed lead to periods of increasing wealth, followed by an abrupt and sharp decrease, reminiscent of financial cycles. However, given that SIs are assumed to be characterised by a power utility and under the assumption of log-Normal returns, they never go bankrupt ($W_t^j < 0$), nor do they ever recover. This can be understood by considering that the SI demand is given in terms of the fraction of their wealth invested in the risky asset. Hence, if their wealth is close to 0, their demand in cash terms will also be close to 0.

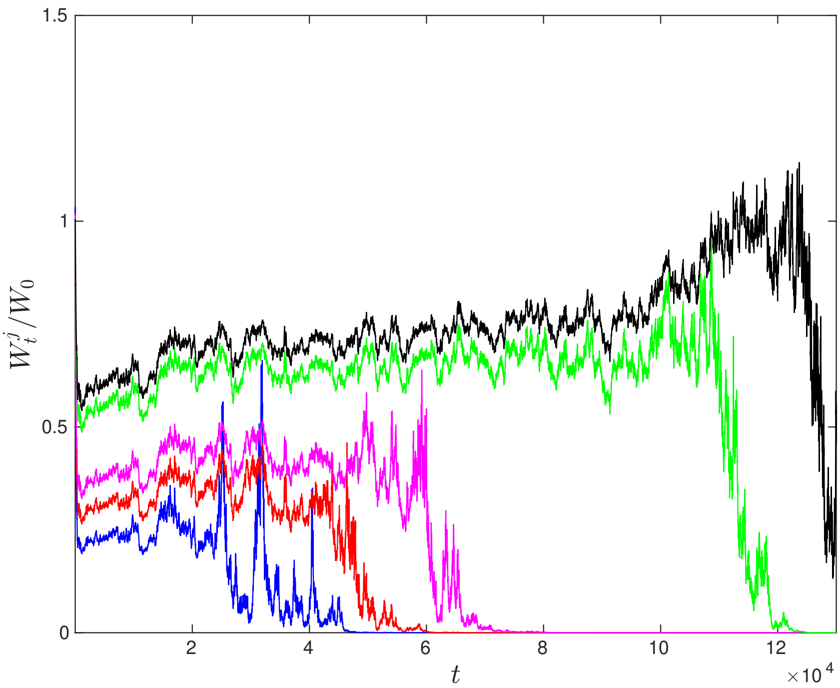

Figure 2: The net wealth as a fraction of the agents' endowments $W_0 = 10$ The blue, red, magenta green and black lines correspond to SIs with a descending initial value of RRA.

7 Conclusion

The model has some limitations, for example the assumption that the SIs do not pay transaction costs is unrealistic. This may be a reason for the dominance in the overall trend, with very low variance across smaller time intervals; there is a very steady increase in wealth when the RRA is fixed. The model could be improved with the introduction of trading costs. Another refinement could involve the incorporation of asymmetric loss and gain aversion, to observe the effects of prospect theory. The results show evidence of a cycle when the RRA coefficient is not held constant, namely in the form of an increase followed by a rapid fall in wealth, as shown in figure (2). However the evidence is limited in the sense that cycles develop slowly and the design of the model results in the eventual bankruptcy of the SI's as their wealth converges to 0, and therefore shows a singular cycle. Further research may involve the reintroduction of a new SI upon bankruptcy, thus allowing the observation of repeated cycles. Furthermore, the cycles have been graphically presented in terms of wealth, it may be interesting to observe these effects in terms of the logarithmic price of the risky asset, and to scrutinize the distribution of returns occurring when the sophisticated investors interact with the noise traders.

References

Abdellaoui, Mohammed, Han Bleichrodt, and Olivier L'Haridor (2008). "A tractable method to measure utility and loss aversion under prospect theory". In: *Journal of Risk and Uncertainty* 36.3, pp. 245–266. ISSN: 08955646, 15730476. URL: http://www.jstor.org/stable/41761343.

Ait-Sahalia, Yacine and Andrew Lo (2000). "Nonparametric risk management and implied risk aversion". In: *Journal of Econometrics* 94.1-2, pp. 9–51. URL: https://EconPapers.repec.org/RePEc:eee:econom:v:94:y:2000:i:1-2:p:9-51.

Andersen, Steffen et al. (2008). "Lost In State Space: Are Preferences Stable?" In: *International Economic Review* 49.3, pp. 1091–1112. DOI: 10.1111/j.1468-2354.2008.00507.x. URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1468-2354.2008.00507.x.

Antell, Jan and Mika. Vaihekoski (2016). "Countercyclical and Time-Varying Risk Aversion and Equity Premium". In: 2139. URL: http://www.fmaconferences.org/Boston/AV15_FMA_Boston.pdf.

Arthur, W Brian (1991). "Designing Economic Agents that Act Like Human Agents: A Behavioral Approach to Bounded Rationality". In: *American Economic Review* 81.2, pp. 353–59. URL: https://EconPapers.repec.org/RePEc:aea:aecrev:v:81:y:1991:i:2:p:353-59.

Azar, Samih Antoine (2010). "Bounds To The Coefficient Of Relative Risk Aversion". In: *Banking and Finance Letters* 2.4, pp. 391–398. URL: https://www.academia.edu/4438011/Bounds_To_The_Coefficient_Of_Relative_Risk_Aversion.

Azar, Samih Antoine and Vera Karaguezian-Haddad (2014). "Simulating the market coefficient of relative risk aversion". In: *Cogent Economics & Finance* 2.1. Ed. by David McMillan, p. 990742. DOI: 10.1080/23322039.2014.990742. URL: https://doi.org/10.1080/23322039.2014.990742.

Babcock, Bruce A., Eun Kwan Choi, and Eli Feinerman (1993). "Risk and Probability Premiums for CARA Utility Functions". In:

Barberis, Nicholas, Ming Huang, and Tano Santos (July 1999). "Prospect Theory and Asset Prices". In: *National Bureau of Economic Research*. Working Paper Series 7220. DOI: 10.3386/w7220. URL: http://www.nber.org/papers/w7220.

Berlinger, Edina and Kata Váradi (Jan. 2015). "Risk Appetite". In: *Public Finance Quarterly* 60.1, pp. 49–62.

Bliss, Robert R and Nikolaos Panigirtzoglou (2004). "Option-implied risk aversion estimates". In: *The journal of finance* 59.1, pp. 407–446.

Bommier, Antoine (2005). *Risk Aversion, Intertemporal Elasticity of Substitution and Correlation Aversion*. en. Tech. rep. Zürich. DOI: 10.3929/ethz-a-010574890.

Bommier, Antoine and Jean-Charles Rochet (2006). "Risk Aversion and Planning Horizons". In: *Journal of the European Economic Association* 4.4, pp. 708–734. ISSN: 15424766, 15424774. URL: http://www.jstor.org/stable/40004815.

Brunnermeier, Markus K. and Stefan Nagel (2008). "Do Wealth Fluctuations Generate Time-Varying Risk Aversion? Micro-Evidence on Individuals' Asset Allocation". In: *The American Economic Review* 98, pp. 713–736.

Camerer, Colin (1998). "Prospect Theory in the Wild: Evidence From the Field". In: *California Institute of Technology, Division of the Humanities and Social Sciences* 1037. URL: https://EconPapers.repec.org/RePEc:clt:sswopa:1037.

Cameron, Lisa and Manisha Shah (Oct. 2013). "Risk-Taking Behavior in the Wake of Natural Disasters". In: *National Bureau of Economic Research*. Working Paper Series 19534. DOI: 10.3386/w19534. URL: http://www.nber.org/papers/w19534.

Campbell, John Y. and Luis M. Viceira (2002a). *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors*. OUP Catalogue 9780198296942. Oxford University Press. ISBN: ARRAY(0x40564038).

— (2002b). *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors*. OUP Catalogue 9780198296942. Oxford University Press. ISBN: ARRAY(0x3fd15bb0). URL: https://ideas.repec.org/b/oxp/obooks/9780198296942.html.

Campbell and Cochrane (1999). "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior". In: *Journal of Political Economy* 107.2, pp. 205–251. ISSN: 00223808, 1537534X. URL: http://www.jstor.org/stable/10.1086/250059.

Chou, Ray, Robert F Engle, and Alex Kane (Mar. 1991). *Measuring Risk Aversion From Excess Returns on a Stock Index*. Working Paper 3643. National Bureau of Economic Research. DOI: 10.3386/w3643. URL: http://www.nber.org/papers/w3643.

Chuang, Yating and Laura Schechter (2015). "Stability of experimental and survey measures of risk, time, and social preferences: A review and some new results". In: *Journal of Development Economics* 117.C, pp. 151–170. URL: https://EconPapers.repec.org/RePEc:eee:deveco:v:117:y:2015:i:c:p:151-170.

Cochrane, John H (Aug. 2016). *Macro-Finance*. Working Paper 22485. National Bureau of Economic Research. DOI: 10.3386/w22485. URL: http://www.nber.org/papers/w22485.

Cohn, Alain et al. (Feb. 2015). "Evidence for Countercyclical Risk Aversion: An Experiment with Financial Professionals". In: *American Economic Review* 105.2, pp. 860–85. DOI: 10.1257/aer.20131314. URL: http://www.aeaweb.org/articles?id=10.1257/aer.20131314.

Conine, Thomas E., Michael B. McDonald, and Maurry Tamarkin (2017). "Estimation of relative risk aversion across time". In: *Applied Economics* 49.21, pp. 2117–2124. DOI:

10.1080/00036846.2016.1231910. URL: https://doi.org/10.1080/00036846.2016.1231910.

Constantinides, George M. and Anisha Ghosh (2017). "Asset Pricing with Countercyclical Household Consumption Risk". In: *The Journal of Finance* 72.1, pp. 415–460. URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/jofi.12471.

Davis, Richard A and Peter J Brockwell (1987). *Time series: theory and methods*. Springer-Verlag.

Díaz, Antonio and Carlos Esparcia (July 2019). "Assessing Risk Aversion From the Investor's Point of View." In: *Frontiers in psychology*. DOI: doi:10.3389/fpsyg.2019.01490. URL: %22https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6614341/%22,.

Eeckhoudt, Louis, Christian Gollier, and Harris Schlesinger (2005). *Economic and Financial Decisions under Risk*. Princeton University Press. ISBN: 9780691122151. URL: http://www.jstor.org/stable/j.ctvcm4j15.

Eisenhauer, Joseph G. (2010). "Relative Risk Aversion as an Arc Elasticity". In: *Risk Management and Insurance Review* 13.1, pp. 161–172. DOI: 10.1111/j.1540-6296.2010.01180.x. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1540-6296.2010.01180.x. URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6296.2010.01180.x.

Epstein, Larry G. and Stanley E. Zin (1989). "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework". In: *Econometrica* 57.4, pp. 937–969. ISSN: 00129682, 14680262. URL: http://www.jstor.org/stable/1913778.

Gollier, Christian and John W. Pratt (1996). "Risk Vulnerability and the Tempering Effect of Background Risk". In: *Econometrica* 64.5, pp. 1109–1123. ISSN: 00129682, 14680262. URL: http://www.jstor.org/stable/2171958.

Guiso, Luigi and Monica Paiella (2008). "Risk Aversion, Wealth, and Background Risk". In: *Journal of the European Economic Association* 6.6, pp. 1109–1150. ISSN: 15424766, 15424774. URL: http://www.jstor.org/stable/40282700.

Guiso, Luigi, Paola Sapienza, and Luigi Zingales (2018). "Time varying risk aversion". In: *Journal of Financial Economics* Volume 128.Issue 3, pp. 403–421. URL: https://www.nber.org/papers/w19284.pdf.

Hagerman, Robert L. (1978). "More Evidence on the Distribution of Security Returns". In: *The Journal of Finance* 33.4, pp. 1213–1221. ISSN: 00221082, 15406261. URL: http://www.jstor.org/stable/2326950.

Hamill, Lynne and G. Nigel Gilbert (2016). *Agent-based Modelling in Economics.*

Hanaoka, Chie, Hitoshi Shigeoka, and Yasutora Watanabe (Apr. 2018). "Do Risk Preferences Change? Evidence from the Great East Japan Earthquake". In: *American Eco-*

nomic Journal: Applied Economics 10.2, pp. 298–330. DOI: 10.1257/app.20170048. URL: http://www.aeaweb.org/articles?id=10.1257/app.20170048.

Hens, Thorsten and Marc Rieger (May 2016). "Financial Economics". In: *A Concise Introduction to Classical and Behavioural Finance* 2, pp. 90–140.

Hung, Mao-Wei and Jr-Yan Wang (2005). "Asset Prices Under Prospect Theory and Habit Formation". In: *Review of Pacific Basin Financial Markets and Policies (RPBFMP)* 8.01, pp. 1–29. DOI: 10.1142/S0219091505000324. URL: https://ideas.repec.org/a/wsi/rpbfmp/v08y2005i01ns0219091505000324.html.

Iori, G. (May 2002). "A microsimulation of traders activity in the stock market: the role of heterogeneity, agents' interactions and trade frictions". In: *Journal of Economic Behavior & Organization* 49.2, pp. 269–285. DOI: 10.1016/S0167-2681(01)00164-0. URL: http://openaccess.city.ac.uk/id/eprint/14604/.

Kahneman, Daniel and Amos Tversky (1979). "Prospect Theory: An Analysis of Decision under Risk". In: *Econometrica* 47.2, pp. 263–91. URL: https://EconPapers.repec.org/RePEc:ecm:emetrp:v:47:y:1979:i:2:p:263-91.

Kandasamy, Narayanan et al. (2014). "Cortisol shifts financial risk preferences." In: *Proceedings of the National Academy of Sciences of the United States of America* 111 9, pp. 3608–13.

Knuth, Donald E (2014). *Art of computer programming, volume 2: Seminumerical algorithms*. Addison-Wesley Professional.

Levin, Jonathan (2006). "Choice under Uncertainty". In: *Stanford Lecture Notes*. URL: https://web.stanford.edu/~jdlevin/Econ20202/Uncertainty.pdf.

Levy, Haim (1991). "Absolute and Relative Risk Aversion: An Experimental Study". In: *Journal of Risk and Uncertainty* 8.3, pp. 289–307. ISSN: 08955646, 15730476. URL: http://www.jstor.org/stable/41760729.

Li, George (Jan. 2007). "Time-varying risk aversion and asset prices". In: *Journal of Banking & Finance* 31.1, pp. 243–257. URL: http://www.sciencedirect.com/science/article/B6VCY-4KGG5WY-B/1/32340f26ec53b59f58372d29f88b7d6d.

Maio, Paulo (Feb. 2007). *ICAPM with time-varying risk aversion*. Money Macro and Finance (MMF) Research Group Conference 2006 111. Money Macro and Finance Research Group. URL: https://ideas.repec.org/p/mmf/mmfc06/111.html.

Malmendier, Ulrike and Stefan Nagel (Mar. 2009). *Depression Babies: Do Macroeconomic Experiences Affect Risk-Taking?* Working Paper 14813. National Bureau of Economic Research. DOI: 10.3386/w14813.

Menkhoff, Lukas and Sahra Sakha (Jan. 2016). "Determinants of Risk Aversion Over Time: Experimental Evidence from Rural Thailand". In: *SSRN Electronic Journal*. DOI: 10.2139/ssrn.2785467. URL: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2785467.

Merton, Robert C. (1969). "Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case". In: *The Review of Economics and Statistics* 51.3, pp. 247–257. ISSN: 00346535, 15309142. URL: http://www.jstor.org/stable/1926560.

Meyer, Donald J. and Jack Meyer (2006). "Measuring Risk Aversion". In: *Foundations and Trends(R) in Microeconomics* 2.2, pp. 107–203.

Natalia, Khorunzhina and Gayle Wayne Roy (July 2011). *Heterogenous intertemporal elasticity of substitution and relative risk aversion: estimation of optimal consumption choice with habit formation and measurement errors*. MPRA Paper 34329. University Library of Munich, Germany. URL: https://ideas.repec.org/p/pra/mprapa/34329.html.

Parada-Contzen, Marcela V (2017). "Individual Risk Aversion through the Life Cycle : Incorporation of Observed Measures of Individual Risk Aversion in the Estimation of Dynamic Life Cycle Decision Models ". In:

Paravisini, Daniel, Veronica Rappoport, and Enrichetta Ravina (June 2010). *Risk Aversion and Wealth: Evidence from Person-to-Person Lending Portfolios*. Working Paper 16063. National Bureau of Economic Research. DOI: 10.3386/w16063. URL: http://www.nber.org/papers/w16063.

Poledna, Sebastian et al. (2014). "Leverage-induced systemic risk under Basle II and other credit risk policies". In: *Journal of Banking & Finance* 42, pp. 199–212.

Pozzi, Lorenzo (2003). "The coefficient of relative risk aversion: a Monte Carlo study investigating small sample estimator problems". In: *Economic Modelling* 20.5, pp. 923–940. ISSN: 0264-9993. DOI: https://doi.org/10.1016/S0264-9993(02)00049-4. URL: http://www.sciencedirect.com/science/article/pii/S0264999302000494.

Pratt, John W. (1964). "Risk Aversion in the Small and in the Large". In: *Econometrica* 32.1/2, pp. 122–136. ISSN: 00129682, 14680262. URL: http://www.jstor.org/stable/1913738.

Sahm, Claudia R. (2012). "How Much Does Risk Tolerance Change?" In: *Quarterly Journal of Finance* 02.04, p. 1250020. DOI: 10.1142/S2010139212500206. URL: https://doi.org/10.1142/S2010139212500206.

Smith, Douglas R. and Robert F. Whitelaw (2009). "Time-Varying Risk Aversion and the Risk-Return Relation". In: *SSRN Electronic Journal*.

Sousa, Ricardo (2007). *Wealth Shocks and Risk Aversion*. NIPE Working Papers 28/2007. NIPE - Universidade do Minho. URL: https://EconPapers.repec.org/RePEc:nip:nipewp:28/2007.

Thurner, Stefan, J Doyne Farmer, and John Geanakoplos (2012). "Leverage causes fat tails and clustered volatility". In: *Quantitative Finance* 12.5, pp. 695–707. URL: https://vermontcomplexsystems.org/share/papershredder/thurner-volatility-finance.pdf.

Weber, Martin and Colin Camerer (1998). “The disposition effect in securities trading: an experimental analysis”. In: *Journal of Economic Behavior & Organization* 33 (2), pp. 167–184.

Wilensky, Uri and William Rand (2015). *An Introduction to Agent-Based Modeling: Modeling Natural, Social, and Engineered Complex Systems with NetLogo*. Mit Press. ISBN: 9780262731898. URL: http://www.jstor.org/stable/j.ctt17kk851.

A Appendix: Noise Traders

A.1 Moments

In the following I determine the properties of the AR(1) process dictating the trading behaviour of the noise-traders and, moreover, the stochastic properties of the equilibrium price and the logarithmic returns in the absence of the utility-maximisers.

Equation (5.1) can be rewritten

$$\log(\xi_{t+1}/N) = \rho \log(\xi_t/N) + (1-\rho)\log V + \sigma\chi_{t+1} \tag{A.1}$$

The demand for the risky asset at a given price in the absence of the SIs is

$$N = \frac{\xi_t}{p_t} \tag{A.2}$$

Hence, from equation (A.1) it follows

$$\log p_{t+1} = \rho \log p_t + (1-\rho)\log V + \sigma\chi_{t+1} \tag{A.3}$$

Acting with the expectation operator $\mathbb{E}[\ ]$ on equation (A.3) we obtain

$$\mathbb{E}[\log p_{t+1}] = \rho\mathbb{E}[\log p_t] + (1-\rho)\mathbb{E}[\log V] + \sigma\mathbb{E}[\chi_{t+1}] \tag{A.4}$$

Given that $\chi_{t+1} \sim N(0,1)$, $\sigma\mathbb{E}[\chi_{t+1}] = 0$, thus

$$\mathbb{E}[\log p_{t+1}] = \rho \cdot \mathbb{E}[\log p_t] + (1-\rho)\cdot\mathbb{E}[\log V]. \tag{A.5}$$

At the steady state the expected value and the variance are constant across all time-periods. Therefore, $\mathbb{E}[\log p_t] = \mathbb{E}[\log p_{t+1}]$ and $\mathrm{Var}[\log p_{t+1}] = \mathrm{Var}[\log p_t]$. Therefore,

$$\mathbb{E}[\log p_{t+1}] = \log V, \tag{A.6}$$

and

$$\begin{aligned} \mathrm{Var}[\log p_{t+1}] &= \rho^2 \mathrm{Var}[\log p_t] + \sigma^2 \Leftrightarrow \\ (1-\rho^2)\mathrm{Var}[\log p_{t+1}] &= \sigma^2 \Leftrightarrow \\ \mathrm{Var}[\log p_{t+1}] &= \frac{\sigma^2}{1-\rho^2}. \end{aligned} \tag{A.7}$$

Let us now turn our attention to the statistical properties of the log-returns r_t.

$$\begin{aligned} N = \frac{\xi_{t+1}}{p_{t+1}} \Leftrightarrow \log(N) = \log \xi_{t+1} - \log p_{t+1} \xRightarrow{\text{Eq. (5.1)}} \\ \log N = \rho \cdot \log \xi_t + \sigma \cdot \chi_{t+1} + (1-\rho) \log (V \cdot N) - \log p_{t+1} \end{aligned} \tag{A.8}$$

Hence,

$$\begin{aligned} \log N =& \rho \log \xi_t + \sigma \chi_{t+1} + (1-\rho) \log N + (1-\rho) \log V - \log p_{t+1} \Leftrightarrow \\ \log p_{t+1} =& \rho \log \left(\frac{\xi_t}{N} \right) + \sigma \chi_{t+1} + (1-\rho) \log V \Leftrightarrow \\ \log p_{t+1} =& \rho \log p_t + (1-\rho) \log V + \sigma \chi_{t+1}. \end{aligned} \tag{A.9}$$

For the equation (A.9) it follows that the log-returns follow the ARMA(1,1) process

$$r_{t+1} = \rho r_t + \sigma(\chi_{t+1} - \chi_t). \tag{A.10}$$

Therefore, the unconditional expected value of the future log-returns is

$$\mathbb{E}(r_{t+1}) = 0, \tag{A.11}$$

whereas the unconditional variance is

$$\begin{aligned} \mathrm{Var}[r_{t+1}] =& \rho^2 \mathrm{Var}(r_t) + \sigma^2 \cancelto{1}{\mathrm{Var}(\chi_{t+1})} + \sigma^2 \cancelto{1}{\mathrm{Var}(\chi_t)} + 2\rho\sigma \cancelto{0}{\mathrm{Cov}(\chi_{t+1}, \chi_t)} + \\ & + 2\rho\sigma \cancelto{0}{\mathrm{Cov}(r_t, \chi_{t+1})} - 2\rho\sigma \cancelto{\sigma}{\mathrm{Cov}(r_t, \chi_t)} \xRightarrow{\mathrm{Var}(r_{t+1}) = \mathrm{Var}(r_t)} \\ (1-\rho^2)\mathrm{Var}[r_{t+1}] =& 2\sigma^2 - 2\rho\sigma \mathrm{Cov}(r_t, \chi_t) = 2\sigma^2 - 2\rho\sigma^2 \Leftrightarrow \\ \mathrm{Var}(r_{t+1}) =& \frac{2\sigma^2}{1+\rho}. \end{aligned} \tag{A.12}$$

Let us now turn our attention to the expected value of the next-period return conditional on the last observation. To gain an insight, I computed the (sample) conditional expected value and variance numerically iterating equations (5.1) and (A.2) for 10^6 periods, for three different values of the parameter $\rho \in \{0.99, 0.5, 0.01\}$. The results for the conditional sample mean and variance is illustrated in Figure 3 (blue and red circles, respectively). The results show that the larger the value of ρ, the stronger the dependence of the expected future return given the last one observed. Moreover, the conditional variance is a constant, i.e. independent of the last observation, increasing as ρ decreases.

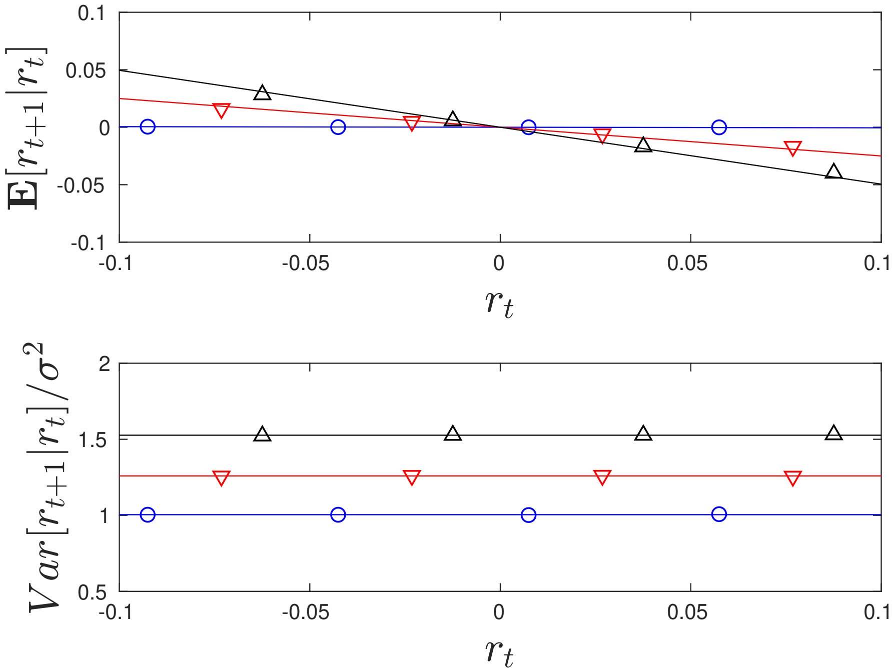

Figure 3: The sample conditional expected value and variance of r_{t+1} as a function of the last observed log-return r_t when only the NTs are trading, for three different values of ρ [$\rho = 0.99$ (blue circles), $\rho = 0.5$ (red downward triangles), and $\rho = 0.01$ (black upward triangles)] Notice that $\text{Var}[r_{t+1}|r_t]$ is normalised by σ^2.

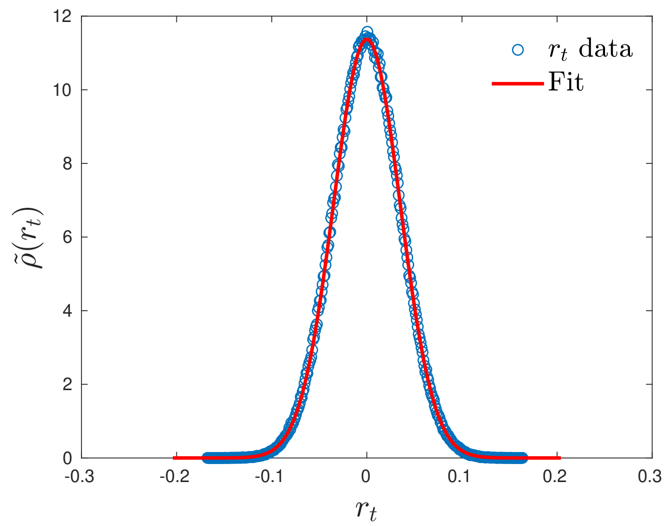

Figure 4: The unconditional PDF of log-returns when only the NTs are present in the market when $\rho = 0.99$. The fit with a Normal PDF yields $\mathbb{E}[r_t] = 0 \pm 4 \times 10^{-5}$, $\sqrt{\mathrm{Var}[r_t]} = \sqrt{\frac{2\sigma^2}{1+\rho}} \approx 0.0351$. in agreement with equation (A.12)

.

A.2 Probability density functions

Given that the log-returns r_{t+1} is a linear combination of normally distributed random variables [see Eq. (A.10) in the absence of the SIs both the unconditional and conditional probability density function (PDF) of the log-return on the risky asset are Normal distributions. Let $\tilde{\rho}(r_t)$ denote the PDF of r_t. Given equations (A.11) and (A.12) it follows that

$$\tilde{\rho}(r_{t+1}) \sim N\left(0, \frac{2}{1+\rho}\sigma^2\right) \tag{A.13}$$

The numerically estimated PDF is shown in Fig. 4[4] on the basis of a sample of log-returns generated by iterating equations (A.2) and (5.1) for 10^6 periods. The numerically estimated PDF is fitted by a Normal distribution. The estimated parameters of the best fit agree with the analytical results given by equations (A.11) and (A.12).

Lastly, the conditional distribution of consecutive returns will be also a Normal distribution since: (a) The expected next-period return depends on time only through r_t, and (b) The conditional variance is a constant stemming from the fact that $\mathrm{Cov}(r_{t+1}, r_t) = const.$ given that the covariance of an ARMA(p,q) process depends only on the time-lag, which in this case is fixed to 1 (Davis and Brockwell 1987) and therefore is a constant.

[4]The Normality of the PDF is independent of the value of the parameter ρ. The mean and the variance of the PDF calculated numerically are in agreement with the analytically derived values for all values of ρ.

B Appendix: Sophisticated Investors

We seek to determine the optimal demand for each of the sophisticated investors, given their information set I_t at time t.

$$\underset{D_t^j \in [-1,1]}{\operatorname{argmax}} \left\{ \mathbb{E}_t \left[U(W_{t+1}^j) \right] \right\}, \tag{B.1}$$

where $U\left(W_{t+1}^j\right) = {W_{t+1}^j}^{1-a_t^j}/(1-a_t^j) \sim {W_{t+1}^j}^{1-a_t^j}$, and W_{t+1}^j is the wealth of the jth SI at the next period.

It is assumed that the SIs are aware of the fact that consecutive log-returns are (directly) correlated, i.e. the AR component of the data generating process is of order 1. Therefore, it is assumed that only the most recent observed return can be used to forecast the next period return. Hence, $\mathbb{E}[r_{t+1}|I_t] = \mathbb{E}_{\approx}[r_{t+1}|r_t]$, where the subscript t denotes the estimate of the conditional moment given their information set up to time t.

To simplify the notation we drop the subscript t and the superscript j.

Equation (B.1) is equivalent to the maximisation of the logarithm of the expected utility. Furthermore, given that returns are log-normally distributed (J. Y. Campbell and Viceira 2002a)[pp. 17-21] it follows that

$$\log \mathbb{E}\left[{W_{t+1}}^{1-a_t^j}\right] = \mathbb{E}\left[\log {W_{t+1}}^{1-a_t^j}\right] + \frac{\operatorname{Var}\left[\log {W_{t+1}}^{1-a_t^j}\right]}{2} \tag{B.2}$$

Consequently, the problem becomes

$$\underset{D_t \in [-1,1]}{\operatorname{argmax}} \left\{ \left(1-a_t^j\right) \mathbb{E}\left[\log W_{t+1}\right] + (1-a_t^j)^2 \frac{\operatorname{Var}\left[\log W_{t+1}\right]}{2} \right\}. \tag{B.3}$$

The wealth of the jth SI at the next period is

$$W_{t+1} = (1 + x_t R_{t+1}) W_t, \tag{B.4}$$

where x is the fraction of its wealth invested into the risky asset, and R the (arithmetic) return of the portfolio. Re-expressing Eq. (B.4) in terms of the logarithmic returns r we get

$$\log\left(W_{t+1}\right) = \log W_t + \log\left[1 + x_t\left(\exp(r_{t+1}) - 1\right)\right], \tag{B.5}$$

albeit a transcendental equation with respect to r. An approximate solution can be obtained by performing a Taylor expansion of Eq (B.5) with respect to r to obtain

$$\log(W_{t+1}) = \log(W_t) + x_t r_{t+1}\left(1 + \frac{r_{t+1}}{2}\right) - \frac{{x_t}^2}{2} r_{t+1}^2 + \mathcal{O}\left(r^3\right). \tag{B.6}$$

Substituting Eq. (B.6) into Eq. (B.3), and furthermore approximating $\mathbb{E}(r_{t+1}^2)$ with $\mathrm{Var}(r_{t+1})$ we obtain

$$\underset{D_t\in[0,\lambda_{\max}]}{\operatorname{argmax}} \left\{ \log W_t + x_t \mathbb{E}(r_{t+1}) + \frac{x_t}{2}(1-x_t)\mathrm{Var}(r_{t+1} + (1-a_t^j)x_t^2 \mathrm{Var}(r_{t+1}) \right\}. \tag{B.7}$$

Finally the first-order condition yields

$$x_t = \max\left\{-1, \min\left[\frac{\mathbb{E}(r_{t+1}) + \frac{1}{2}\mathrm{Var}(r_{t+1})}{a_t^j \; \mathrm{Var}\; (r_{t+1})}, 1\right]\right\}. \tag{B.8}$$

Therefore, in terms of number of risky assets d_t^j traded by the jth SI

$$d_t = \max\left\{-1, \min\left[\frac{\mathbb{E}(r_{t+1}) + \frac{1}{2}\mathrm{Var}(r_{t+1})}{a_t^j \; \mathrm{Var}\; (r_{t+1})}, 1\right]\right\} W_t^j / p_t. \tag{B.9}$$

C Appendix: Matlab Code

lstlisting breaklines=true

```
    function DRA(simTime,V,riskAversionMin,riskAversionMax,...
    DeltaT,corrLogReturnFlag,randRiskAversion,...
    tdAlpha,writeTimeSeries,runNu,seed) %#codegen
%Write time-series:
% Do not write any time-series,
% 1. Log-returns and price,
% 2. SIs' demand for the risky asset,
% 3. SIs' Wealth,
% 4. NTs' demand for the risky asset.


%% Initialize
coder.extrinsic('forecast');
coder.extrinsic('exist');
coder.extrinsic('mkdir');
clear global

% Ndl = arima('AR',AR,'MA',MA,'Variance',V,'Constant',0);
% MdlData=load('PreEstimatedMdls/OptARMAMdl.mat');
% muSigmaMdl = MdlData.OptArmaMdl;
% varMdl = MdlData.OptGarch;
% Mdl = arima('ARLags',1:4,'MALags',1,'Constant',0);

% RW = int32(1e3);

randomizeExt=int32(1);
hists = int32(0);
if (randomizeExt==1)
    rng(seed,'twister');
end
NTsOnly = int32(0);
randomizeInTime = int32(0);
% rng('shuffle','twister');
simTime=simTime+int32(1);
```

```
%has been removed from the market
if (tdAlpha==1)
    savepath = ['./Results/WithSIs/TDAlpha/run',num2str(runNu),'/'];
else
    savepath = ['./Results/WithSIs/TIAlpha/run',num2str(runNu),'/'];
end
if (~exist(savepath,'dir'))
    mkdir(savepath);
end
N = double(1000);
H = int32(10); %number of sophisticated traders
global rho
rho = double(0.8); %reversion to the mean parameter
sigma = double(0.035); %volatility induced by the noise traders
Wo = double(10);
Wmin = 0; %Wo/double(100);
reintroductionTime = double(100); %reintroduction time after an SI has gone
    bankrupt
TreMin = 10;
Tre = int32(rand(H,1)*(reintroductionTime-TreMin)+TreMin);
% TreDeltaT = zeros(H,DeltaT,'int32');
% TreDeltaT(:,1) = Tre;
tt = int32(0);
maxLagR=1e3;




fprintf('\n%s\n','Parameter values:');
fprintf('%s%d\n', ' T = ',simTime);
fprintf('%s%g\n', ' V = ',V);
fprintf('%s%g\n', ' min[a_j] = ',riskAversionMin);
fprintf('%s%g\n', ' max[a_j] = ',riskAversionMax);
fprintf('%s%d\n', ' Buffer length = ',DeltaT);

fprintf('%s\n','Parameter values:');
fprintf('%s%g%s%g%s%g\n%s%g%s%g%s%g\n','sigma = ',sigma,', rho = ',rho,...
    ', V = ',V,', N = ', N,', Wo = ',Wo,', Wmin = ',Wmin);
```

```
if ( corrLogReturnFlag == 1)
    fprintf('%s\n','The autocovariance of Log returns will be computed');
end

if (randRiskAversion == 1)
    fprintf('%s\n', 'Random risk aversion parameter');
end
fprintf('%s%d\n','Maximum lag: ',maxLagR);

fprintf('%s\n',' ');

alpha = ones(H,simTime);

if (randRiskAversion==1)
    alpha(:,1) = (riskAversionMax-riskAversionMin)...
        .*rand(H,1)+riskAversionMin;
    alpha(:,1) = sort(alpha(:,1),1);
    alpha = alpha(:,1).*ones(H,simTime);
else
    if (riskAversionMin~=riskAversionMax)
        alpha(:,1) = linspace(riskAversionMin,riskAversionMax,H)';
        alpha = alpha(:,1).*ones(H,simTime);
    else
        alpha = riskAversionMin.*ones(H,simTime);
    end
end

p_t = horzcat(V, zeros(1,DeltaT,'double'));
logReturn = zeros(1,simTime,'double');
W_t = horzcat(Wo.*ones(H,1), zeros(H,DeltaT,'double'));
D_t = zeros(H,DeltaT+1,'double');
xi_t = horzcat(N, zeros(1,DeltaT,'double'));
C_t = zeros(H,DeltaT+1,'double');
NAV = zeros(H,DeltaT+1,'double');
Tre_t = zeros(H,1,'int32');
W_tm1 = Wo.*ones(H,1,'double');
% W_tm2 = zeros(H,1,'double');
% deltaW = zeros(H,1,'double');
% percentileDeltaW = zeros(H,1,'double');
```

```
% coder.extrinsic('tic','toc','xcov','sprintf','disp');
rnd = randn(1,DeltaT+1);


ctr = double(0);
meanPrice=double(0);
counterPrice = double(0);
M2Price = double(0);
meanLogPrice = double(0.);
counterLogPrice = double(0.);
M2LogPrice = double(0.);
meanST = zeros(H,1,'double');
M2ST = zeros(H,1,'double');
% counterST = zeros(H,1,'double');



lastLogReturnEdges = linspace(-0.5,0.5,51)';
dr_tm1 = lastLogReturnEdges(2)-lastLogReturnEdges(1);
condLogReturnBinsNu = int32(length(lastLogReturnEdges)-1);
condLogReturnBins = lastLogReturnEdges(1:end-1)+dr_tm1*0.5;

condMeanLogReturn = zeros(condLogReturnBinsNu,1,'double');
counterCondLogReturn = zeros(condLogReturnBinsNu,1,'double');
M2CondLogReturn = zeros(length(lastLogReturnEdges),1,'double');
global condMuLR condM2LR condVarLR nCondLRObs LRedges LRBinsNu LRBins
condMuLR = condMeanLogReturn;
condM2LR = M2CondLogReturn;
condVarLR = condM2LR;
nCondLRObs = counterCondLogReturn;
LRedges = lastLogReturnEdges;
LRBinsNu = condLogReturnBinsNu;
% LRBins = condLogReturnBins;

global muLR M2LR varLR nLRobs
meanLogReturn = double(0);
counterLogReturn = double(0);
M2LogReturn = double(0);
muLR = 0.;
```

```
varLR = 0.;
nLRobs = 0.;
M2LR = 0.;

M2Wt = zeros(H,1,'double');
meanWt = zeros(H,1,'double');
counterWt = zeros(H,1,'double');
meanXi = double(0);
counterXi = double(0);
M2Xi = double(0);
performance = zeros(H,1,'double');
meanPerformance = zeros(H,1,'double');
counterPerformance = zeros(H,1,'double');
M2Performance = zeros(H,1,'double');



nBinsWt = 400;
DeltaWBinsPos = 2.^linspace(-16,log2(10*Wo),nBinsWt);
DeltaWBinsNeg = -DeltaWBinsPos(end:-1:1);
DeltaWHistNeg = zeros(H,nBinsWt,'double');
DeltaWHistPos = zeros(H,nBinsWt,'double');


NReturnBins=2e2;
logReturnHist = zeros(1,NReturnBins);
binsLogReturn = linspace(-0.5,0.5,NReturnBins);
% [binsCon,binsLogR] = meshgrid(binsLogReturn,binsLogReturn);
% binsReturn = linspace(9.5,2,NReturnBins);
% returnHist = zeros(1,NReturnBins);
NbinsPrice = 2e2;
% binsPrice = linspace(0,1.5,NbinsPrice);
% priceHist = zeros(1,NbinsPrice);
binsLogP = linspace(log(1e-3),log(1.5),NbinsPrice);
logPHist = zeros(1,NbinsPrice);


numberOfSIsDefaulted = zeros(H,1,'double');
% timesDefaultedInDeltaT = zeros(H,1,'double');
ttOfBBankruptcy = zeros(H,DeltaT,'int32');
```

```
timeActive = zeros(H,1,'double'); %time each fund is active
timesSpawned = zeros(H,1,'double'); %times a fund is introduced
totalTimeActive=zeros(H,1,'double');
survivalTime = zeros(H,1,'double');
MaxST = double(0);
% coder.varsize('ttOfBBankruptcy', [H ceil(DeltaT/1e2)]);
timesBankrupt = zeros(H,1,'int32');

% tstart=tic;
t0 = int32(0);
dt = int32(simTime/10);




%% Time loop
for i=2:simTime

    %    if (any(timeActive==0)||i==3135)
    %        fprintf('stop');
    %    end
    t0 = t0 + int32(1);

    t1 = t0 + int32(1);

    tt = int32(1) + tt;

    AST = find(W_t(:,t0)>=Wmin);

    IST = find(W_t(:,t0)<Wmin);

    if (~isempty(AST))
        timeActive(AST) = timeActive(AST) + ones(length(AST),1);
        totalTimeActive(AST) = totalTimeActive(AST) + ones(length(AST),1);
    end

    if ( mod(i,dt)==0 )
        fprintf('%s%d\n\n',' Time step ',i);
%         fprintf('%s%g\n\n','Max ST: ',MaxST);
    end
```

```
    xi_t(t1) = xi(rho,sigma,V,N,xi_t(t0),rnd(1,t0));

    %        if (i<10)
    %               fprintf('%s%g\n','xi = ',xi_t(t1));
    %        end


%     W_tm2 = W_tm1;

    W_tm1 = W_t(:,t0);



    if (NTsOnly==0)


%         if (mod(i-10,RW)==0&i~=10)
%             onlineEstMdl = estimate(Mdl,logReturn(51:i)');%,'Display','off');
%         end
%         if (i>50)
%             [mu,~,sigma_sq] = forecast(muSigmaMdl,1,'Y0',logReturn(i-50:i-1)');
%             sigma_sq = forecast(varMdl,1,'Y0',logReturn(i-50:i-1)');
%         else
%             mu = 0.;
%             sigma_sq = sigma^2;
%         end

%     if (i==61)
%         fprintf('%s\n','stop');
%     end

        [p_t(t1),W_t(:,t1),D_t(:,t1)] = p(p_t(t0),logReturn(i-1),W_t(:,t0),...
            D_t(:,t0),xi_t(t1),N,V,Wmin,alpha(:,i-1));
    else
        p_t(t1) = xi_t(t1)/N;
    end
    %     assert(p_t(t1)>1e-2,'Unexpected price value %g\n',p_t(t1))

    if (tdAlpha==1)
```

```
    deltaW = W_t(:,t1)-W_tm1;
    alpha(:,i) = ((-1.82)*alpha(:,i-1)).*deltaW./W_tm1 + alpha(:,i-1);
end

Tre_t(IST) = Tre_t(IST) + 1;

W_t(Tre_t == Tre,t1) = Wo;

NAV(:,t1) = D_t(:,t1).*p_t(t1);

ctr = 1. + ctr;

%Calculation fo the first two moments for
%1.Wealth
%2 NTs' demand (in cash)
%3 Price of the risky asset
%4 Log-price
%5 SIs' rate of return

[M2Wt,meanWt,counterWt] = onlineMoments(W_t(:,t1),...
    meanWt,M2Wt,counterWt);

[M2Xi,meanXi,counterXi] = onlineMoments(xi_t(t1)...
    ,meanXi,M2Xi,counterXi);

[M2Price,meanPrice,counterPrice] = onlineMoments(p_t(t1),...
    meanPrice,M2Price,counterPrice);
if (i>2)
    logReturn(i) = log(p_t(t1)/p_t(t0));

    [M2LogReturn,meanLogReturn,counterLogReturn] = onlineMoments(logReturn(i)
        ,...
        meanLogReturn,M2LogReturn,counterLogReturn);

    muLR = meanLogReturn;
    nLRobs = counterLogReturn;
    if (counterLogReturn>2)
        varLR = M2LogReturn/(counterLogReturn - 1.);
        M2LR = M2LogReturn;
    else
```

```
        M2LR = M2LogReturn;
    end
end

if (i>3)
%       jC = ceil( (logReturn(i)-lastLogReturnEdges(1))/dr_tm1 );
%       if (jC<=0)
%           jC=1;
%       elseif (jC>condLogReturnBinsNu)
%           jC=condLogReturnBinsNu;
%       end

    jC=find(lastLogReturnEdges>=logReturn(i-1),1,'first');
    jC = jC - 1;
%       if (~isempty(jC)&&logReturn(i)<0)
%           fprintf('%s%g\n','r_{t-1} = ',logReturn(i));
%           fprintf('%g%s%g\n',lastLogReturnEdges(jC),'< r_{t-1} <',
lastLogReturnEdges(jC+1));
%           fprintf('\n');
%       end
    if (isempty(jC)||jC<1)
        fprintf('%s%e%s%e\n','r = ',logReturn(i-1),' > ',lastLogReturnEdges(
            end));
    end
    if (~isempty(jC)&&jC<condLogReturnBinsNu)
        [M2CondLogReturn(jC),condMeanLogReturn(jC),counterCondLogReturn(jC)]
            = ...
            onlineMoments(logReturn(i),condMeanLogReturn(jC),M2CondLogReturn(
                jC),...
            counterCondLogReturn(jC));
%           if (counterCondLogReturn(jC)>2)
            condMuLR(jC) = condMeanLogReturn(jC);
            condM2LR(jC) = M2CondLogReturn(jC);
            condVarLR(jC) = M2CondLogReturn(jC)/(counterCondLogReturn(jC) -1.)
                ;
            nCondLRObs(jC) = counterCondLogReturn(jC);
%           end
    end
end
```

```
logP = log(p_t(t1));

[M2LogPrice,meanLogPrice,counterLogPrice] = onlineMoments(logP,...
    meanLogPrice,M2LogPrice,counterLogPrice);

for j=1:H
    if (W_t(j,t0)~=0)
        performance(j) = ((W_t(j,t1)-W_t(j,t0))/W_t(j,t0));
        [M2Performance(j),meanPerformance(j),counterPerformance(j)] = ...
            onlineMoments(performance(j),meanPerformance(j),...
            M2Performance(j),counterPerformance(j));
    end
end

for j=1:H
    if Tre_t(j)==1
        survivalTime(j) = timeActive(j);
        numberOfSIsDefaulted(j)= 1. + numberOfSIsDefaulted(j);
        timesBankrupt(j) = int32(1) + timesBankrupt(j);
        ttOfBBankruptcy(j,tt) = t0;
        if (MaxST<survivalTime)
            MaxST = survivalTime(j);
        end
    end
end

for j=1:H
    if Tre(j)==Tre_t(j)
        Tre(j) = int32(rand(1,1)*(reintroductionTime-TreMin)+TreMin);
        Tre_t(j) = 0;
        if (randomizeInTime==1)
            if (randRiskAversion==1)
                alpha(j,i) = (riskAversionMax-riskAversionMin)...
                    *rand(1,1)+riskAversionMin;
            end
        end
    end
end

if (mod(i-1,DeltaT)==0)
```

```
if (hists==1)
    deltaW = W_t(:,2:(end-1))-W_t(:,1:(end-2));
    for j=1:H
        DeltaWJTemp = deltaW(j,:);
        DeltaWJPosTemp = DeltaWJTemp(DeltaWJTemp>0);
        DeltaWJPos = DeltaWJPosTemp(DeltaWJPosTemp~=Wo);
        DeltaWJNeg = DeltaWJTemp(DeltaWJTemp<0);
        if (~isempty(DeltaWJPos))
            fDeltaWJTempPos = histc(DeltaWJPos,DeltaWBinsPos);
            DeltaWHistPos(j,:) = fDeltaWJTempPos + DeltaWHistPos(j,:);
        end
        if (~isempty(DeltaWJNeg))
            fDeltaWJTempNeg = histc(DeltaWJNeg,DeltaWBinsNeg);
            DeltaWHistNeg(j,:) = fDeltaWJTempNeg + DeltaWHistNeg(j,:);
        end
    end

    if (i==(DeltaT+1))
        logReturnHist=hist(logReturn(i),binsLogReturn);
    else
        logReturnTemp = histc(logReturn(logReturn~=0.),binsLogReturn);
        logReturnHist = logReturnTemp + logReturnHist;
    end
end
%-----------------------------------------------------------------------------

%                       Record time-series
%-----------------------------------------------------------------------------

if (writeTimeSeries~=0)
    if (writeTimeSeries>0)
%                 fileIDReturn = fopen([savepath,'logReturn.txt'],'a');
%
%                 for j=2:DeltaT+1
%                     fprintf(fileIDReturn,'%g\n',logReturn(j));
%                 end

%                 fclose(fileIDReturn);

        fileIDPrice = fopen([savepath,'price.txt'],'a');
```

```
    for j=2:DeltaT+1
        fprintf(fileIDPrice,'%g\n',p_t(j));
    end
    fclose(fileIDPrice);
end

if (writeTimeSeries>1)
    filenamesDemand=cell(H,1);
    for j=1:H
        filenamesDemand{j}=['D_',...
            Double2String(double(j)),'.txt'];
    end
    fileIDs = zeros(H,1);
    for k=1:H
        fileIDs(k) = fopen([savepath,filenamesDemand{k}],'a');
        for j=2:size(D_t,2)
            fprintf(fileIDs(k),'%g\n',D_t(k,j));
        end
        fclose(fileIDs(k));
    end
end
if (writeTimeSeries>2)
    fileNames=cell(H,1);
    for j=1:H
        fileNames{j}=['Wt_',...
            Double2String(double(j)),'.txt'];
    end
    fileIDs = zeros(H,1);
    for k=1:H
        fileIDs(k) = fopen([savepath,fileNames{k}],'a');
        for j=2:size(W_t,2)
            fprintf(fileIDs(k),'%g\n',W_t(k,j));
        end
        fclose(fileIDs(k));
    end
end
if (writeTimeSeries>3)
    filenameXi=fopen([savepath,'Xi_t.txt'],'a');
    for j=2:length(xi_t)
        fprintf(filenameXi,'%g\n',xi_t(j));
```

```
                end
                fclose(filenameXi);
            end
        end
        %
            xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

        for j=1:H
            C_t(j,2:end) = W_t(j,2:end) - D_t(j,2:end).* p_t(2:end);
        end
        for j=1:H
            W_t(j,1)=W_t(j,end);
            D_t(j,1)=D_t(j,end);
            C_t(j,1)=C_t(j,end);
        end
        p_t(1) = p_t(end);
        xi_t(1) = xi_t(end);
        rnd = randn(1,DeltaT+1);
%        logReturn(1) = logReturn(end);
        tt = int32(0);
        t0 = int32(0);
    end

    if ~isempty(IST)
        if any(timeActive(IST)~=0)
            timeActive(IST) = 0;
        end
    end
end


%% Post-processing results
fprintf('%s\n','Time loop finished. Writing output files, please wait');
fprintf('%s\n','****************************************************');
fprintf('%s%g%s%g\n','E[log(p)] = ',meanLogPrice,', Var[log(p)] = ',M2LogPrice
    ./(counterLogPrice-1.))
fprintf('%s%g%s%g\n','E[r] = ',meanLogReturn,', Var[r] = ',M2LogReturn./(
    counterLogReturn-1.))
% fprintf('%s\n','Writing times HF defaulted');
fileID = fopen([savepath,'timesBankrupt.txt'],'w');
```

```
for i=1:H
    fprintf(fileID,'%d\n',timesBankrupt(i));
end

fclose(fileID);

fileIDReturn = fopen([savepath,'logReturn.txt'],'w');

for j=10:simTime
    fprintf(fileIDReturn,'%g\n',logReturn(j));
end

fclose(fileIDReturn);

fileID = fopen([savepath,'parameters.txt'],'w');
fprintf(fileID,'%s%g\n%s%g\n%s%g\n','V = ',V,'rho = ',rho,' Sigma = ',sigma);
fclose(fileID);

fileID = fopen([savepath,'meanST.txt'],'w');
for i=1:H
    if (timesBankrupt(i)~=0)
        fprintf(fileID,'%g\n',meanST(i));
    else
        fprintf(fileID,'%g\n',double(simTime));
    end
end
fclose(fileID);

fileID = fopen([savepath,'stdST.txt'],'w');
stdST = zeros(H,1,'double');
for i=1:H
    if (timesBankrupt(i)>1)
        stdST(i) = sqrt(M2ST(i)/(double(timesBankrupt(i))-1.));
        fprintf(fileID,'%g\n',stdST(i));
    else
        fprintf(fileID,'%g\n',0);
    end
end
fclose(fileID);
```

```
% fprintf('Writing times each SI was spawned\n');
fileID = fopen([savepath,'timesSpawned.txt'],'w');
for i=1:H
    fprintf(fileID,'%g\n',timesSpawned(i));
end
fclose(fileID);

% fprintf('Writing performance\n');
fileID = fopen([savepath,'meanPerformance.txt'],'w');
for i=1:H
    fprintf(fileID,'%g\n',meanPerformance(i));
end
fclose(fileID);

% fprintf('Writing performance\n');
fileID = fopen([savepath,'stdPerformance.txt'],'w');
for i=1:H
    fprintf(fileID,'%g\n',sqrt(M2Performance(i))/(counterPerformance(i)-1));
end
fclose(fileID);

% fprintf('Writing risk aversion coefs. used\n');
fileID = fopen([savepath,'riskAversion.txt'],'w');
if (tdAlpha==1)
    for i=1:H
        for j=2:simTime
            fprintf(fileID,'%g%s',alpha(i,j),' ');
        end
        fprintf(fileID,'\n');
    end
    fclose(fileID);
else
    for i=1:H
        fprintf(fileID,'%g\n',alpha(i,1));
    end
end

% fprintf('Writing sim. time\n');
fileID = fopen([savepath,'simTime.txt'],'w');
fprintf(fileID,'%d\n',simTime);
```

```
fclose(fileID);


fileID = fopen([savepath,'logReturnHist.txt'],'w');
for i=1:int32(size(logReturnHist,2))
    fprintf(fileID,'%g%s%g\n',binsLogReturn(i),' ',logReturnHist(i));
end
fclose(fileID);


fileID = fopen([savepath,'logPriceHist.txt'],'w');
for i=1:int32(NbinsPrice)
    fprintf(fileID,'%g%s%g\n',binsLogP(i),' ',logPHist(i));
end
fclose(fileID);

stdPt = sqrt(M2Price/(counterPrice-1));
fileID = fopen([savepath,'MeanStdPt.txt'],'w');
fprintf(fileID,'%g\n%g\n',meanPrice,stdPt);
fclose(fileID);

stdLogPt = sqrt(M2LogPrice/(counterLogPrice-1));
fileID = fopen([savepath,'MeanStdLogPt.txt'],'w');
fprintf(fileID,'%g\n%g\n',meanLogPrice,stdLogPt);
fclose(fileID);

fileID = fopen([savepath,'MeanStdLogReturn.txt'],'w');
fprintf(fileID,'%g\n%g\n',meanLogReturn,sqrt(M2LogReturn/(counterLogReturn-1.)))
    ;
fclose(fileID);

for i=1:H
    fileID = fopen([savepath,'MeanStdWt.txt'],'w');
    fprintf(fileID,'%s\n%s\n',meanWt(i,1),sqrt(M2Wt(i,1)/(counterWt-1.)));
end
fclose(fileID);

fileID = fopen([savepath,'totalTimeActive.txt'],'w');
for i=1:H
    fprintf(fileID,'%g\n',totalTimeActive(i));
```

```
end
fclose(fileID);

fileID = fopen([savepath,'condMeanLogR.txt'],'w');

for i=1:condLogReturnBinsNu
    if (counterCondLogReturn(i)>10)
        fprintf(fileID,'%g%s%e%s%d\n',condLogReturnBins(i),' ',condMeanLogReturn(
            i),' ',counterCondLogReturn(i));
%         fprintf('%s%g%s%e\n','r_{t-1} =',condLogReturnBins(i),', E[r_t|r_{t-1}]
     = ',condMeanLogReturn(i));
    end
end
fclose(fileID);

fileID = fopen([savepath,'condVarLogR.txt'],'w');
for i=1:condLogReturnBinsNu
    if (counterCondLogReturn(i)>10)
    fprintf(fileID,'%g%s%e%s%e\n',condLogReturnBins(i),' ',...
        M2CondLogReturn(i)/(counterCondLogReturn(i)-1.),' ',...
        counterCondLogReturn(i));
%     fprintf('%s%g%s%e%s%g\n','r_{t-1} = ',condLogReturnBins(i),', Var[r_t|r_{t
    -1}] =',...
%         M2CondLogReturn(i)/(counterCondLogReturn(i)-1.),' #Obs ',
    counterCondLogReturn(i));
%     fprintf('%e\n',1./counterCondLogReturn(i));
    end
end
fclose(fileID);

% fprintf('\n\n%s\n','Error');
% for i=1:condLogReturnBinsNu
%     if (counterCondLogReturn(i)>10)
%         fprintf('%e\n',sqrt(1./counterCondLogReturn(i)));
%     end
% end
fprintf('\n');
% fprintf('%s\n','The pre-estimated model was:')
% summarize(EstMdl);
% fprintf('%s\n','The re-estimated model is:')
```

```
% onlineEstMdl = estimate(Mdl,logReturn(51:end)');
% save([savepath,'reEstimatedMdl.mat'],'onlineEstMdl');
fprintf('%s\n\n','====================================');
fprintf('%s\n\n','Finished');
fprintf('%s\n\n','====================================');

% for i=1:H
%       fprintf('%s%d%s%g\n\n','Total time SI j active ',...
%               i,' was active ',totalTimeActive(i));
% end

% if (corrLogReturnFlag==1)
%    fprintf('\n%s%d\n','length(Abs(Log(R)))',ctrR);
%    fprintf('%s%d\n','length(Abs(CondLog(R)))',ctrCondR);
%    yy=ReturnsV(ReturnsV~=0);
%    y=abs(yy)-mean(abs(yy),2);
%    fprintf('\n%s\n','calculating absolute log-returns autocorrelation');
%    fprintf('\n%s%g%s%g\n','length y = ',length(y),' maxLag = ',maxLagR)
%    corrReturns = xcorr(y,maxLagR,'unbiased');
%    corrReturns = corrReturns(1,(maxLagR+1):end);
%    SigmaR = var(abs(yy),1);
%    if (SigmaR~=0)
%        corrReturns = corrReturns/SigmaR;
%    end
%
%    fprintf('\n%s\n','calculating conditional absolute log-returns
   autocorrelation');
%
%    fileID = fopen([savepath,'absReturnsCorr.txt'],'w');
%    if (SigmaR~=0)
%        for j=1:int32(maxLagR+1)
%            fprintf(fileID,'%g\n',corrReturns(j));
%        end
%    end
%    fclose(fileID);
% end

% fprintf('Time elapsed %g',telapsed)

% % fprintf('Writing WTBD histogram\n');
```

```
% fileID = fopen([savepath,'WTBDHist.txt','w');
% for i=1:H
%     for j=1:int32(size(fWTBDvsBeta,2))
%         if j==int32(size(fWTBDvsBeta,2))
%             fprintf(fileID,'%g\n',fWTBDvsBeta(i,j));
%         else
%             fprintf(fileID,'%g%s',fWTBDvsBeta(i,j),' ');
%         end
%     end
% end
% fclose(fileID);

% fileID = fopen([savepath,'DeltaWHistPos.txt','w');
% for i=1:H
%     for j=1:int32(size(DeltaWHistPos,2))
%         if j==int32(size(DeltaWHistPos,2))
%             fprintf(fileID,'%g\n',DeltaWHistPos(i,j));
%         else
%             fprintf(fileID,'%g%s',DeltaWHistPos(i,j),' ');
%         end
%     end
% end
% fclose(fileID);

% fileID = fopen([savepath,'DeltaWHistNeg.txt','w');
% for i=1:H
%     for j=1:int32(size(DeltaWHistNeg,2))
%         if j==int32(size(DeltaWHistNeg,2))
%             fprintf(fileID,'%g\n',DeltaWHistNeg(i,j));
%         else
%             fprintf(fileID,'%g%s',DeltaWHistNeg(i,j),' ');
%         end
%     end
% end
% fclose(fileID);
%
%
% fileID = fopen([savepath,'totalDeltaWHistPos.txt','w');
% totalDeltaWHistPos = sum(DeltaWHistPos,2);
% for i=1:H
```

```
%     fprintf(fileID,'%g\n',totalDeltaWHistPos(i));
% end
% fclose(fileID);

% % fprintf('Writing the mean of WTBD\n');
% fileID = fopen([savepath,'meanWTBD.txt'],'w');
% for i=1:length(meanWTBD)
%     fprintf(fileID,'%g\n',meanWTBD(i));
% end
% fclose(fileID);
%
% fileID = fopen([savepath,'stdWTBD.txt'],'w');

% stdWTBD = zeros(nBins,1,'double');
% for i=1:nBins
%     if (ctrMeanWTBD(i)>1)
%         stdWTBD(i) = sqrt(M2WTBD(i)/(ctrMeanWTBD(i)-1));
%     end
%     fprintf(fileID,'%g\n',stdWTBD(i));
% end
% fclose(fileID);

% % fprintf('Writing WTBD histogram\n');
% fileID = fopen([savepath,'fWTBD.txt','w');
% for i=1:int32(length(fWTBD))
%     fprintf(fileID,'%g%s%g\n',binsWTBD(i),' ',fWTBD(i));
% end
% fclose(fileID);

% % fprintf('Writing condLogReturns\n\n');
% fileID = fopen([savepath,'condLogReturnHist.txt'],'w');
% for i=1:int32(size(condLogReturnHist,2))
%         fprintf(fileID,'%g%s%g\n',binsCondLogReturns(i),...
%                 ' ',condLogReturnsHist(i));
% end
% fclose(fileID);

% fileID = fopen([savepath,'condReturnHist.txt'],'w');
% for i=1:int32(size(condReturnHist,2))
%         fprintf(fileID,'%g%s%g\n',binsReturns(i),' ',condReturnsHist(i));
```

```
% end
% fclose(fileID);

% fileID=fopen([savepath,'condLogPriceHist.txt'],'w');
% for i=1:int32(size(condLogPricesHist,2))
%        fprintf(fileID,'%g%s%g\n',binsLogPrices(i),' ',condLogPricesHist(i));
% end
% fclose(fileID);
%
% fileID=fopen([savepath,'returnsHist.txt'],'w');
% for i=1:int32(size(returnsHist,2))
%        fprintf(fileID,'%g%s%g\n',binsReturns(i),' ',returnsHist(i));
% end
% fclose(fileID);
%
% fileID = fopen([savepath,'priceHist.txt'],'w');
% for i=1:int32(NbinsPrice)
%        fprintf(fileID,'%g%s%g\n',binsPrice(i),' ',priceHist(i));
% end
% fclose(fileID);
%
% fileID = fopen([savepath,'MeanStdCondLogReturn.txt'],'w');
% fprintf(fileID,'%g\n%g\n',meanCondLogReturns,...
%        sqrt(M2CondLogReturns/(ctrCondLogReturns-1.)));
% fclose(fileID);

% fileID = fopen([savepath,'MeanStdCondLogPt.txt'],'w');
% fprintf(fileID,'%g\n%g\n',meanCondLogPt,...
%        sqrt(M2CondLogPt/(ctrCondLogPt-1.)));
% fclose(fileID);
% fileID = fopen([savepath,'MeanStdArithmeticReturns.txt'],'w');
% fprintf(fileID,'%g\n%g\n',meanDeltaP,sqrt(M2DeltaP/(ctr-1.)));
% fclose(fileID);
end

function d_t = Dtilde(p,p_tm1,r_tm1,w_t,alpha) %#codegen

% global lastLogReturnEdges mu sigma_sq condLogReturnBinsNu
global muLR M2LR nLRobs
global condMuLR nCondLRObs condM2LR LRedges LRBinsNu LRBins rho
```

```
LRbar=0.;m2LR=0.;varLR=0.;
condMuLRNew = condMuLR;
condM2LRNew = condM2LR;
nCondLRObsNew = nCondLRObs;
d_t = zeros(size(w_t,1),1,'double');

%   CRRA Iso-elastic with log-normal returns

logR = log(p/p_tm1);

%forward lookers. SIs use the MLE of the expected future
%return

% [m2LR,LRbar,~]=onlineMoments(logR,muLR,M2LR,nLRobs);
%
% if (nLRobs>=2)
%     varLR = m2LR/nLRobs;
% end

%forward lookers. SIs use the MLE of the expected next
%return given the previous and the information set at time
% t

j=find(LRedges>=r_tm1,1,'first');

j = j- 1;


if (isempty(j)||j == 1)
    fprintf('%s%e%s%e%s%e\n','r = ',logR,' , min(r) =',...
        lastLogReturnEdges(1),' max(r) = ,',lastLogReturnEdges(end));
end
if (~isempty(j) && j <= LRBinsNu && j >=1)
    [condM2LRNew(j),condMuLRNew(j),nCondLRObsNew(j)] = ...
        onlineMoments(logR,condMuLR(j),condM2LR(j),...
        nCondLRObs(j));

    k=find(LRedges>=logR,1,'first');
    k = k - 1;
```

```
% interpolate the running mean at point logR for more precision

%     if (~isempty(k) && k>=1 && k<= LRBinsNu && nCondLRObsNew(k)>1)
%         LRbar = condMuLRNew(k);
%         varLR = condM2LRNew(k)/(nCondLRObsNew(k)-1);
%     end
% end

    if (k==1)
        if (nCondLRObsNew(1)>=2&&nCondLRObsNew(2)>=2)
            LRbar = interp1(LRedges(1:2),condMuLRNew(1:2),logR);
            varLR = interp1(LRedges(1:2),...
                condM2LRNew(1:2)./(nCondLRObsNew(1:2)-1),logR);
        end
    elseif (k==LRBinsNu)
        if (nCondLRObsNew(end-1)>=2&&nCondLRObsNew(end)>=2)
            LRbar = interp1(LRedges(end-1:end),...
                condMuLRMew(end-1:end),logR);
            varLR = interp1(LRedges(end-1:end),...
                condM2LRNew(end-1:end)./...
                (nCondLRObsNew(end-1:end)-1),logR);
        end
    elseif (nCondLRObsNew(k)>=2&&nCondLRObsNew(k+1)>=2)
        LRbar = interp1(LRedges(k:k+1),condMuLRNew(k:k+1),logR);
        varLR = interp1(LRedges(k:k+1),condM2LRNew(k:k+1)./...
            (nCondLRObsNew(k:k+1)-1),logR);
    end
end

% Backward lookers
% j=find(LRedges>=logR,1,'first');
%
% j = j - 1;

% if (~isempty(j)&&j <= LRBinsNu&&j > 1)
%     if (j==1)
%         if (nCondLRObs(1)>1&&nCondLRObs(2)>1)
%             LRbar = interp1(LRedges(1:2),condMuLR(1:2),logR);
%             varLR = interp1(LRedges(1:2),condM2LrunR(1:2)./(nCondLRObs(1:2)-1.)
    ,logR);
```

```
%         end
%     elseif (j==LRBinsNu)
%         if (nCondLRObs(end-1)>1&&nCondLRObs(end)>1)
%             LRbar = interp1(LRedges(end-1:end),condMuLR(end-1:end),logR);
% varLR = interp1(LRedges(end-1:end),condM2LR(end-1:end)./(nCondLRObs(end-1:end)
    -1.),logR);
%         end
%     else
%         if (nCondLRObs(j)>1&&nCondLRObs(j+1)>1)
%             LRbar = interp1(LRedges(j:j+1),condMuLR(j:j+1),logR);
%             varLR = interp1(LRedges(j:j+1),condM2LR(j:j+1)./(nCondLRObs(j:j+1)
    -1.),logR);
%         end
%     end
% end

% LRbar = 0.5*(rho-1)*logR;
% varLR = 0.035^2.*(0.5*(1-rho)+1);


if (varLR~=0)
    d_t = max(-1, min( (LRbar+varLR*0.5)./(alpha.*varLR),1) ).*w_t;
else
    d_t = min( 1./(2.*alpha), 1) .*w_t;
end

d_t(w_t <= 0) = 0;

end

function [p_t,W_t,D_t] = p(p_last,r_last,W_last,D_last,xi_t,N,...
    V,Wmin,alpha)%#codegen




% persistent a b
%
% if (isempty(a)|isempty(b))
%     a = double(2);
%     b = 0.035^2;
```

```
% end

p_t = fzero(@pfunc,p_last,...
    [],W_last,p_last,r_last,D_last,Wmin,V,xi_t,N,...
    alpha);

W_t = W(W_last,p_t,p_last,D_last,Wmin);

r = log(p_t/p_last);

D_t = Dtilde(p_t,p_last,r_last,W_t,alpha)./p_t;

% % if (p_t>2.25)
%      figure(1)
%      axis;
%     hold on
%     x=linspace(0,3,1e3);
%     fplot(@(x)pfrununc(x,W_last,p_last,r_last,D_last,Wmin,V,xi_t,N,...
%     alpha),[0 3])
%     plot([0 3],[0 0],'-b');
%     hold off
%     drawnow
%     disp(p_t)
%     keyboard
% % end

% [sigma_sq,ap,bp] = bayesianUpdate(r,a,b);

% a = ap; b = bp;

end

function W_t = W(W_last,p_t,p_last,D_last,Wmin) %#codegen

W_t = W_last + (p_t - p_last) .* D_last;

% W_t = (1-2.5e-5).*W_t;

W_t(W_last < Wmin) = 0.;
```

```
end

function y = pfunc(p_t,W_last,p_last,r_last,...
    D_last,Wmin,V,...
    xi_t,N,alpha)%#codegen

W_t = W(W_last,p_t,p_last,D_last,Wmin);

d_t = Dtilde(p_t,p_last,r_last,W_t,alpha);

y = xi_t + sum(d_t,1) - N*p_t;

end

function xi_t = xi(rho,sigma,V,N,xi_last,chi)%#codegen

xi_t = exp(rho*log(xi_last) + sigma * chi + (1-rho)*log(V*N));

% xi_t = rho*xi_last + sigma * chi + (1-rho)*V*N;

end
```