ST7: Modélisation Des Risques Financiers

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Exercice 1. *Data for exercice*
We set ρ as the correlation matrix, σ the vector of standard deviation, Σ the covariance matrix and μ as the vector of expected returns.

Equity USA, Equity EMU, Bond EUR Sovereign, Bond EUR IG Corp. & Cash

$\mu = [5.7\%, 6.6\%, 3.3\%, 1.6\%, 0.2\%]$

$\sigma = [14.3\%, 16.4\%, 8.3\%, 4.0\%, 0.5\%]$

$$\rho = \begin{pmatrix} 1.0 & 0.821 & -0.05 & 0.25 & 0 \\ & 1.0 & -0.14 & 0.10 & 0 \\ & & 1.0 & 0.67 & 0 \\ & & & 1.0 & 0 \\ & & & & 1 \end{pmatrix}$$

$\Sigma = diag(\sigma) \quad \rho \quad diag(\sigma)$

Exercice 2. *Mean Variance Optimization (MVO)*
Checking your code & the utility function :

$$w_{MVO} = argmax(\hat{\mu}^T w) \quad u.c. \quad w^T \hat{\Sigma} w \leq 10\%^2 \tag{1}$$

with $\hat{\mu}$ the vector of returns and $\hat{\Sigma}$ covariance matrix. Using the previous $\hat{\mu}$ and $\hat{\Sigma}$:

- ***a.*** Install *cvxpy*
- ***b.*** What is the value of vector w_{MVO} using cvxpy ?
- ***c.*** Check this outputs using an analytic solution i.e. this solution could be solved without a solver
- ***d.*** What is the point of these two steps ?
- ***e.*** What is the value of λ to achieve the same result with the following utility function : $w_{MVO} = argmax(\hat{\mu^T} w - \lambda w^T \hat{Sigma} w)$

Exercice 3. *Efficient frontier*
Using the same data as exercice 1 :

- ***a.*** Draw the efficient frontier for a risk (volatility) ranging from 1% to 15% (likewise, these figures are annualized volatility values).
- ***b.*** Comment the obtained results.

Exercice 4. *Constraints*
Using the same data as exercice 1 :

a. Add a positivity constraint to the utility function. What is the solution of exercise 3 ?

b. Add a positivity constraint and full investment constraint (i.e. $\sum_{i=1}^{n} w_i = 1$ to the utility function). What is the solution of exercise 3 ?

c. Plot these different efficient frontiers

d. How the risk is moving ?