Poisson Process Project - Testing for homogeneity of a Poisson process

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Introduction

The Poisson process, both homogeneous and inhomogeneous, plays a central role in statistical modeling due to its ability to describe random events occurring over time or space. We note $N \sim P(\int_0^t \lambda(x)dx)$ where N is the Poisson Process and lambda is called the intensity function.

The distinction between homogeneous and inhomogeneous lies in the intensity function: for a homogeneous Poisson process, the intensity function is constant over time or space, reflecting a uniform rate of occurrences. In contrast, the intensity function of an inhomogeneous Poisson process varies, capturing changes in the rate of events. This flexibility makes Poisson processes particularly useful for a wide range of real-world applications, including modeling customer arrival times in a queue, analyzing the occurrences of earthquake, and tracking the arrival times of insurance claim.

Our project builds upon the article "Tests for an Increasing Trend in the Intensity of a Poisson Process: A Power Study"[1]. This study evaluates the hypothesis of a constant intensity function against the alternative of an increasing intensity function. Various statistical tests are introduced, and their performances are compared using power studies.

Despite its valuable contributions, the study has some limitations. For instance, the explanation of the tests is relatively underexplored. This limitation in the study motivates further investigation and refinement of the proposed methods [1].

The aim of our project is to test the hypothesis of a constant intensity function versus the alternative of an increasing intensity function, following the framework laid out in the article.

To achieve this, we construct several statistical tests, including the Laplace test and the Weibull test. We also perform numerical simulations to evaluate the effectiveness of these tests under various conditions.

The structure of this report is as follows: First, we detail the construction of the statistical tests. Next, we compare the power of these tests using numerical simulations. Finally, we apply our tests to real-world data, specifically Danish fire data, to evaluate their performance in a practical context.

I - Definition and construction of tests

In the first part of this report, we will focus on the theoretical study of the tests described in the first reference article [1].

We will define and explore both homogeneous and inhomogeneous Poisson processes mathematically, outlining their assumptions and fundamental properties. Additionally, we will demonstrate the mathematical constructions underlying these tests, particularly for the Laplace and Weibull distributions, and examine the hypotheses associated with these models.

I.1 - Definitions
Homogeneous Poisson process:

Let $N = (N_t)_t$ be a counting process representing the total number of events that have occurred up to time t. A point process N is a *homogeneous Poisson process* with rate $\lambda > 0$ if:

- $N_0 = 0$
- The number of points in disjoint time intervals are independent (independent increments).
- The distribution of the number of points in any time interval depends only on the length of the interval, not its position (stationary increments).

  We denote this process as $N \sim \mathcal{P}(\lambda t)$.

Inhomogeneous Poisson process:

In the same way, an *inhomogeneous Poisson process* with intensity function λ is defined as follows:

- $N_0 = 0$
- The increments are independent.

  We denote this process as $N \sim \mathcal{P}\left(\int_0^t \lambda(x)\,dx\right)$.

I.2 - Laplace test
We test,

$$H_0 : \lambda(.) \text{ is constant} \quad \text{versus} \quad H_1 : \lambda(.) \text{ is increasing}$$

The test statistic proposed in [1] is as follows.

$$L = \sum_{i=1}^{n} \frac{T_i}{T*} = \frac{1}{T*} \sum_{i=1}^{n} T_i \quad \text{with } T* \text{ s the last observed time,}$$
$$\text{et } T_i \text{ is the } i\text{-th observed time.}$$

Moreover, under H_0 and conditionally on $N = n$, based on Property 2.23 from [2], we have:

$$(T_1, T_2, ..., T_n)|\{n = N\} \sim (U_{(1)}, U_{(2)}, ..., U_{(n)},)$$
$$\text{where } (U_1, U_2, ..., U_n) \text{ are i.i.d. r.v} \sim U([0, T*])$$

Thus, $\frac{T_i}{T^*}$ follows the same distribution as the i-th order statistic $U_{(i)}/T^*$, which is a uniform random variable $U([0, 1])$.

$$\text{Reminder: If } X \sim U([0,1]) \text{ then } \mathbb{E}[X] = \frac{1}{2} \quad \text{and} \quad Var(X) = \frac{1}{12}$$

By the Central Limit Theorem,

$$\frac{L - \mathbb{E}[L]}{\sqrt{\mathbb{Var}(L)}} = \frac{\sum_{i=1}^{n} \frac{T_i}{T*} - \frac{n}{2}}{\sqrt{\frac{n}{12}}} \underset{n \to +\infty}{\overset{L}{\to}} \mathcal{N}(0, 1)$$

This is therefore our pivotal statistic.

We construct the rejection zone. The hypothesis H_0 is rejected when L is large. The intensity function $\lambda(.)$ is constant under H_0, so the T_i/T^* are uniformly distributed over $[0,1]$. However, under H_1, $\lambda(.)$ is increasing, meaning the T_i/T^* tend to take larger values. Therefore, L will be larger under H_1 than under H_0. So

$$R_\alpha = \{L > l\}$$

Error of the first order:

$e_1 = P_{H_0}(L > l | \{N = n\}) = P_{H_0}\left(\frac{\sum_{i=1}^n \frac{T_i}{T^*} - \frac{n}{2}}{\sqrt{\frac{n}{12}}} > \frac{l-\frac{n}{2}}{\sqrt{\frac{n}{12}}} | \{N = n\}\right) \underset{n\to+\infty}{\xrightarrow{L}} P(Z > \frac{l-\frac{n}{2}}{\sqrt{\frac{n}{12}}})$

where $Z \sim \mathcal{N}(0,1)$. And,

$$P(Z > \frac{l-\frac{n}{2}}{\sqrt{\frac{n}{12}}}) = 1 - P(Z \leq \frac{l-\frac{n}{2}}{\sqrt{\frac{n}{12}}})$$

We are conducting a level alpha test.

Then

$$1 - P(Z \leq \frac{l-\frac{n}{2}}{\sqrt{\frac{n}{12}}}) = \alpha <=> P(Z \leq \frac{l-\frac{n}{2}}{\sqrt{\frac{n}{12}}}) = 1 - \alpha$$

So, we have $\frac{l-\frac{n}{2}}{\sqrt{\frac{n}{12}}} = z_{1-\alpha}$, where z_η is the η-quantile of a normal distribution $\mathcal{N}(0,1)$. Finally,

$$l = \sqrt{\frac{n}{12}} z_{1-\alpha} + \frac{n}{2}$$

We conclude that the rejection region is given by $R_\alpha = \{L > l\} = \{L > \sqrt{\frac{n}{12}} z_{1-\alpha} + \frac{n}{2}\} = \{\frac{L-\frac{n}{2}}{\sqrt{\frac{n}{12}}} > z_{1-\alpha}\}$

We denote $X_n = \frac{L-\frac{n}{2}}{\sqrt{\frac{n}{12}}}$ our test statistic.

The p-value is defined as: $\hat{\alpha} = P_{H_0}(X_n > X_n^{obs}) = 1 - P_{H_0}(X_n < X_n^{obs}) \underset{n\to+\infty}{\xrightarrow{L}} 1 - \mathcal{F}_{\mathcal{N}(0,1)}(X_n^{obs})$

where $\mathcal{F}_{\mathcal{N}(0,1)}$ is the cumulative distribution function of a distribution $\mathcal{N}(0,1)$.

I.3 - Weibull test

In this test, we consider $\lambda(t) = (\beta/\theta)(t/\theta)^{\beta-1}$ for $\beta, \theta > 0$. We observe that if $\beta = 1$ we recover the exponential distribution with parameter $\lambda(t) = 1/\theta$ corresponding to the simple Poisson process. If $\beta > 1$, the failure rate increases over time (system wear and tear).

We test

$$H_0 : \beta = 1 \quad \text{versus} \quad H_1 : \beta > 1$$

We denote $Z = 2\sum_{i=1}^n log(T^*/T_i)$, the test statistic proposed in [1].

Conditionally on $\{N_{T^*} = n\}$, $(T_1, \ldots, T_n)$ behaves as the order statistic associated to n independent and identically distributed random variable with commun density f:

$$f : s \longmapsto \frac{\lambda(s)}{\int_0^{T^*} \lambda(x)dx} \mathbf{1}_{0<s\leq T^*} = \frac{\beta}{\theta}(\frac{s}{\theta})^{\beta-1} \frac{1}{\Lambda(T^*)} \mathbf{1}_{0<s\leq T^*} = \frac{\beta s^{\beta-1}}{\theta^\beta \Lambda(T^*)} \mathbf{1}_{0<s\leq T^*}$$

with $\Lambda(T^*) = \int_0^{T^*} \lambda(x)dx = \int_0^{T^*} \frac{\beta}{\theta}(\frac{x}{\theta})^{\beta-1}dx = \frac{\beta}{\theta^\beta}\int_0^{T^*} x^{\beta-1}dx = \frac{\beta}{\theta^\beta}\left[\frac{x^\beta}{\beta}\right]_0^{T^*} = (\frac{T^*}{\theta})^\beta$

Consequently, $f(s) = \frac{\beta s^{\beta-1}}{\theta^\beta \frac{(T^*)^\beta}{\theta^\beta}} \mathbf{1}_{0<s\leq T^*} = \frac{\beta s^{\beta-1}}{T^*} \mathbf{1}_{0<s\leq T^*}$

To determine the distribution of Z, we calculate its moment-generating function.

$$\mathcal{L}_Z(u) = \mathbb{E}[e^{uZ} | \ \{N_{T^*} = n\} \ ]$$

We recall that

$$(T_1, T_2, ..., T_n \mid \{N_T = n\}) =^{(d)} (U_{(1)}, U_{(2)}, \ \dots \ , U_{(n)})$$

where U_i are independent and identically distributed random variables and of density f defined above.

Therefore,

$$\begin{aligned}
\mathcal{L}_Z(u) &= \mathbb{E}\left[e^{u\,2\sum_{i=1}^{n} \ln\left(\frac{T^*}{T_i}\right)} \middle| \ \{N_T = n\} \right] \\
&= \mathbb{E}\left[e^{u\,2\sum_{i=1}^{n} \ln\left(\frac{T^*}{U_{(i)}}\right)} \right] \\
&= \mathbb{E}\left[e^{u\,2\sum_{i=1}^{n} \ln\left(\frac{T^*}{U_i}\right)} \right] \\
&= \mathbb{E}\left[\prod_{i=1}^{n} e^{u\,2\ln\left(\frac{T^*}{U_i}\right)} \right] \\
&= \mathbb{E}\left[\prod_{i=1}^{n} \left(\frac{T^*}{U_i}\right)^{2u} \right] \\
&= \left(\mathbb{E}\left[\left(\frac{T^*}{U_1}\right)^{2u} \right] \right)^n
\end{aligned}$$

and,

$$\mathbb{E}\left[(\frac{T*}{U_1})^{2u} \right] = \int_{\mathbb{R}} (\frac{T^*}{x})^{2u} \frac{\beta x^{\beta-1}}{T*^{\beta}} \mathbf{1}_{0<x\leq T^*} dx = T^{*2u-\beta} \beta \int_0^{T^*} \frac{x^{\beta-1}}{x^{2u}} dx = T^{*2u-\beta} \beta \left[\frac{x^{\beta-2u}}{\beta-2u} \right]_0^{T^*} = T^{*2u-\beta} \beta \frac{T^{*2u-\beta}}{\beta-2u} = \frac{\beta}{\beta-2u}$$

Then, $(\mathbb{E}\left[(\frac{T*}{U_1})^{2u} \right])^n = (\frac{\beta}{\beta-2u})^n$

Finally, under the hypothesis H_0, the parameter $\beta = 1$, therefore we have$\mathcal{L}_Z(u) = (\frac{1}{1-2u})^n = (\frac{\frac{1}{2}}{\frac{1}{2}-u})^n$

Here we recognise the generating function of the moments of a Gamma distribution $\Gamma(n, 1/2)$.

Moreover, as explain , $\Gamma(n, 1/2) =^{(d)} \chi^2(2n)$ which is the law of Z under H_0.

We construct the rejection zone. We reject H_0 when Z is significantly small, as the reasoning is reversed compared to the previous test due to the use of the reciprocal ratio. Therefore,

$$R_\alpha = \{Z < s_\alpha\}$$

Error of first order:

$e_1 = P_{H_0}(Z < s_\alpha | N = n) = P_{H_0}(2\sum_{i=1}^{n} log(T^*/T_i) < s_\alpha | N = n) = P(X < s_\alpha)$ where $X \sim \chi^2(2n)$

We are conducting a level alpha test:

Then,

$$P(X < s_\alpha) \Leftrightarrow \mathbb{P}(X < x_{2n,\alpha}) \leq \alpha$$

and $\quad x_{2n,\alpha}$ is the $\alpha -$ quantile of a $\chi^2(2n)$

We conclude that the rejection region is given by: $R_\alpha = \{Z < x_{2n,\alpha}\}$ and Z is our test statistic.

The p-value is defined as:

$$\hat{\alpha} = \mathbb{P}_{H_0}(Z < Z^{obs}) = \mathcal{F}_{\chi^2(2n)}(Z^{obs})$$

where $\mathcal{F}_{\chi^2(2n)}$ is the cumulative distribution function of a distribution $\chi^2(2n)$.

II - Numerical study

In this section, we digitally construct the previously developed Laplace and Weibull tests. We analyze the outcomes of these constructions by verifying the alpha levels of the tests and then compare their performance in terms of power.

II.1.a - Poisson processes simulations
Here, we build the functions that allow us to construct homogeneous and inhomogeneous Poisson processes. We construct inhomogeneous Poisson processes in two ways: as discussed in [2], and as outlined in [1].

II.1.b - Laplace test
We define the function that calculates our test statistic and applies the test to the arrival times of a homogeneous or inhomogeneous Poisson process.

```
## [1] "Rejection of H_0: TRUE"

## [1] "Statistical test observed: 1.88319410442914"

## [1] "p-value: 0.0298370338652485"
```

In this case, for a process with a constant intensity function, we do not reject the hypothesis H_0 at the 5% significance level because the p-value is greater than 0.05. This is indeed the expected result.

We compare two methods to simulate an inhomogeneous Poisson process: the thinning algorithm and the method described in [1].

```
## [1] "Rejection of H_0: TRUE"

## [1] "Statistical test observed: 6.29345204434486"

## [1] "p-value: 1.55241264288009e-10"

## [1] "Rejection of H_0: TRUE"

## [1] "Statistical test observed: 3.92666303507497"

## [1] "p-value: 4.30662580696017e-05"
```

In this case, we consider an inhomogeneous Poisson process with an increasing intensity function $\lambda(t) = 2t$. We observe a p-value less than 0.05, so we reject H_0 at the 5% significance level. This is also the expected result.

The Laplace tests provide consistent results in the trials we conducted.

II.1.c - Weibull tests

Here, we construct the Weibull test statistic from the arrival times as demonstrated in the first section.

Here, we construct our intensity function dependent on the parameters θ and $\beta > 0$.

```
## [1] "Rejection of H_0: FALSE"
```

```
## [1] "Statistical test observed: 23.9181613936174"
```

```
## [1] "p-value: 0.533714923618714"
```

```
## [1] "Rejection of H_0: FALSE"
```

```
## [1] "Statistical test observed: 0.354796448646747"
```

```
## [1] "p-value: 0.361371044548681"
```

In the case where $\beta = 1$, which corresponds to the null hypothesis H_0 of the Weibull test, we obtain a p-value greater than 0.05. Therefore, we do not reject H_0 at the 5% significance level. This is consistent with our intensity function, which is constant.

```
## [1] "Rejection of H_0: TRUE"
```

```
## [1] "Statistical test observed: 107.754179791905"
```

```
## [1] "p-value: 6.90830606986647e-14"
```

```
## [1] "Rejection of H_0: TRUE"
```

```
## [1] "Statistical test observed: 3.16685101536986"
```

```
## [1] "p-value: 0.000770496223111206"
```

In this second case, we consider an intensity function with $\beta = 2 > 1$, which is therefore increasing. We obtain a very low p-value and thus reject H_0 at the 5% significance level, which is consistent with our hypotheses.

II.2.a - Comparison of tests in the article

In this section, we study the power of the different tests. The function `calculate_power` takes as arguments a test function, the number of simulations to perform in order to estimate the power, and the necessary arguments for the functions performing the test (last arrival time, upper bound of the intensity function, and the intensity function itself).

We revisit the different cases presented in [1]. First, we study the power when the intensity function is a Weibull function. Next, we examine cases where the intensity function is exponential, logarithmic, and finally, a step function. To do so, as in [1], we fix both T_{max} and the number of occurrences n.

In Figure 1, we obtain the same results as in [1]. First, the larger β is, the more the function increases, and the more confidently we reject H_0. This result holds when $\beta = 1$, under H_0, the power is approximately 0.05 for both tests. When $\beta = 2$, the power is about 0.6 for $n = 10$, increasing to 0.98 for $n = 40$. For $\beta = 4$, the power is almost always equal to 1 for both tests.

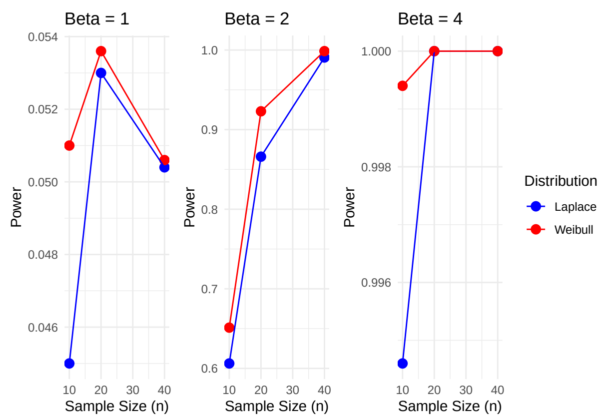

Figure 1: Power Analysis for Weibull Function

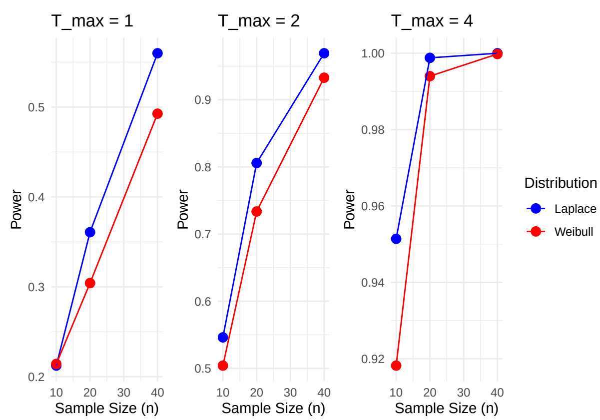

Figure 2: Power Analysis for Exponential Lambda

The larger n is, the greater the power for $\beta = 2$ or $\beta = 4$. As a result, the more powerful the test.

Finally, we observe that the red curve (Weibull) is always above the blue curve (Laplace). For this type of function, the Weibull test is more powerful and is therefore recommended.

In Figure 2, we observe that as $T_{\max}$ increases, the power also increases. For $n = 40$, the power is around 0.5 when $T_{\max} = 1$, whereas it reaches 1 when $T_{\max} = 4$.

Similarly, as n increases, the power also increases. For instance, for any $T_{\max}$, the curves are increasing.

As the intensity function follows an exponential pattern, the larger $T_{\max}$ is, the more the increase is significant. Consequently, as $T_{\max}$ increases, the power approaches 1.

Additionally, the power curve for Laplace is always above that for Weibull. The Laplace test is therefore more powerful for exponential-type functions.

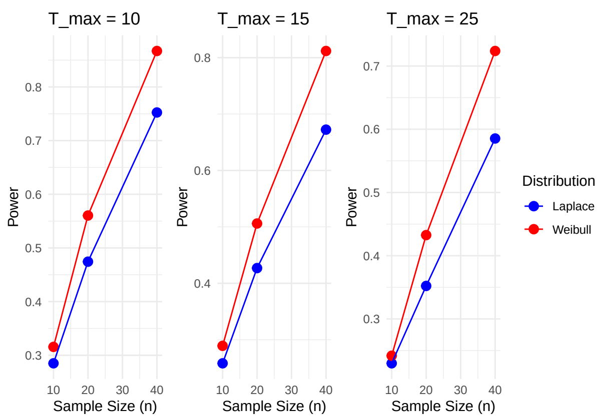

Figure 3: Power Analysis for Log

Remark: When integrating the function $\lambda(t) = \log(1 + t)$, the resulting function does not have an explicit inverse. We approximate it using the `uniroot` function in **R**.

We can see in Figure 3, as $T_{\max}$ increases, the power decreases. For example, for $n = 40$, the power for the Laplace test is 0.75 when $T_{\max} = 10$ and decreases to 0.6 when $T_{\max} = 25$.

This occurs because the function $\log(1 + t)$ exhibits decreasing growth as t becomes larger. Consequently, for a high $T_{\max}$, the intensity function may appear almost constant, as the rapid initial growth is mitigated by the stabilization of the function. Therefore, as $T_{\max}$ increases, the power of our tests decreases.

As n increases, the power also increases, for the same reason as mentioned earlier.

The power of the Weibull test is always greater than that of the Laplace test.

We can observe some intersting points in the figure Figure 4.

As n increases, the power becomes greater for the same reasons as mentioned previously.

For the Weibull test, the power tends to decrease as θ increases. Lower values are observed for $\theta = 2/3$, while similar values are obtained for $\theta = 1/3$ and $\theta = 1/2$.

In contrast, for the Laplace test, the power appears to slightly increase with θ. Lower values are observed for $\theta = 1/3$, while similar values are obtained for $\theta = 1/3$ and $\theta = 2/3$.

Thus, for $\theta = 1/3$, the Weibull test is more powerful. For the other θ value studied, the Laplace test is more powerful.

Overall, these simulations confirm the results presented in [1].

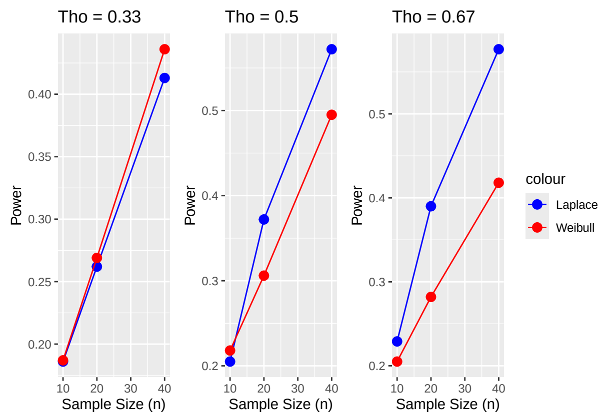

Figure 4: Power Analysis for Step Function Intensity

II.2.b - Study of the Weibull test parameters

In this section, we study the impact of the parameters β and θ of the Weibull function on the power of the tests. The number of points, n, is not fixed. We set $T_{max} = 10$ for both studies. The results are summarized in Figure 5.

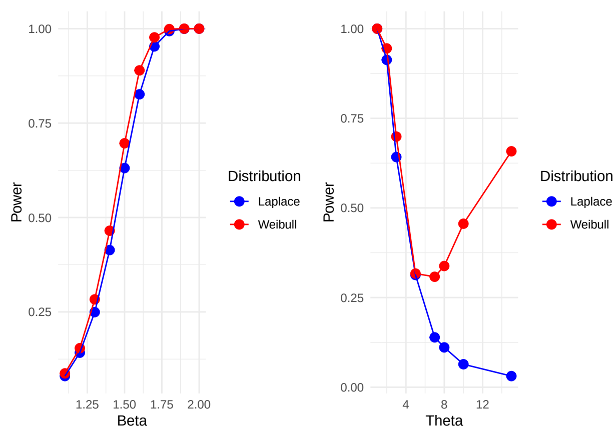

Figure 5: Power as function of beta and theta

First, we fix $\theta = 1$, and consider different values of β between 1 and 2. We observe similar results as previously: for both tests, the power increases with β. When $\beta = 1$ (under H_0), the power is 0.05, which is expected as it corresponds to the significance level α of the test. The larger β becomes, the steeper the function increases. Consequently, the power of the test grows with β.

Next, we fix $\beta = 2$ and consider values of θ between 1 and 15. As θ increases, the intensity function approaches 0 and increases more slowly. This explains why the power decreases for the Laplace test and initially for the Weibull test. However, for the Weibull test, we observe that the power stops decreasing and starts increasing when $\theta = 7$.

In both studies, the power of the Weibull test is consistently greater than that of the Laplace test. This conclusion aligns with the findings in Section II.2.B.

III - Study of Danish fires since 1980

Finally, we apply the homogeneity tests to a real-world dataset consisting of large fire insurance claims in Denmark recorded. This dataset, available in the *evir* R-package under the name "danish," includes the dates of each observation, allowing for a practical evaluation of the tests' performance [3].

These data represent large fire insurance claims in Denmark, spanning from Thursday, January 3rd, 1980, to Monday, December 31st, 1990. The claims are stored in a numeric vector, with the corresponding dates provided in a `times` attribute, an object of class `POSIXct` (refer to `DateTimeClasses` in R). The dataset was supplied by Mette Rytgaard from Copenhagen Re. It is important to note that these claims constitute an irregular time series.

Why can we consider that the fires follow a Poisson process?

We assume that at time t_0, no fires have occurred, so $N_0 = 0$. Furthermore, we hypothesize that the occurrence of one fire does not affect the likelihood of another fire occurring, ensuring that the increments of the process are independent. These assumptions align with the fundamental properties of a Poisson process, making it a suitable model for this phenomenon.

```
##
## Attachement du package : 'evir'

## L'objet suivant est masqué depuis 'package:ggplot2':
##
##     qplot
```

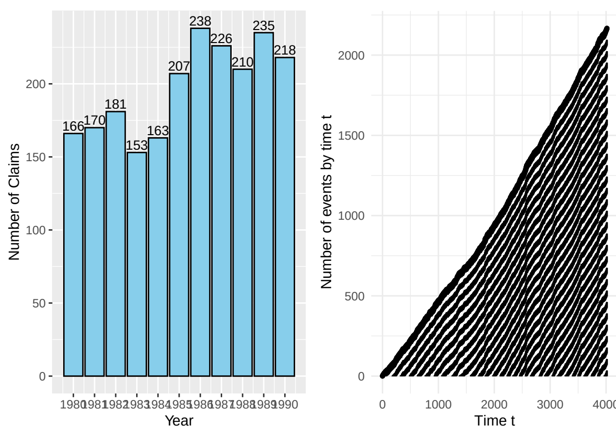

Figure 6: Analysis of Fire Insurance Claims and Point Process

The left graph of Figure 6 shows the annual number of large fire insurance claims in Denmark from 1980 to 1990, highlighting fluctuations over the decade. The number of claims was relatively stable in the early 1980s, ranging from 166 to 181, before dipping to a low of 153 in 1984. This was followed by a sharp increase, peaking at 238 claims in 1986, the highest number during the period. Claimns in the latter half of the decade remained consistently high, with minor fluctuations. Overall, the trend suggests increasing claim activity over time, possibly influenced by external factors such as changing reporting practices, economic conditions, or environmental events.

The cumulative number of events shows an increasing trend in the right graph of Figure 6. This growth appears to have slightly accelerated as time progressed.

We aim to study the function λ that describes a Poisson process. We consider the four functions previously analyzed in the power tests to determine which one best describes our process.

To do so, we begin by finding the optimal parameters for these functions using the maximum likelihood method.

Now that we have the functions well-defined with their various parameters, we calculate the AIC (Akaike Information Criterion) to determine which function best models our process.

```
## Exponential Logarithmic  Two Values     Weibull
##    6980.128    6991.789    6976.859    6988.364
```

The AIC criterion must be minimized. Therefore, the two best models for our intensity function are the exponential function and the step function. This is consistent with the visual analysis conducted earlier in Section II.2.a.

Let us study the behavior of the intensity function using the Laplace and Weibull tests constructed in the Section I. First, we extract the maximum time value from our data.

We now apply our tests to the data we have just retrieved:

```
## [1] "(Laplace) Rejection of H_0: TRUE"

## [1] "(Laplace) Statistical test observed: 5.62632759710445"

## [1] "(Laplace) p-value: 9.20432952078443e-09"
```

In the context of the Laplace test, we reject H_0 at a 5% significance level. This suggests that the intensity function is increasing.

These results provide strong evidence to support the hypothesis of an increasing intensity function.

```
## [1] "(Weibull) Rejection of H_0: TRUE"

## [1] "(Weibull) Statistical test observed: 3898.50957058286"

## [1] "(Weibull) p-value: 6.59452174463987e-07"
```

For the Weibull test, we also reject H_0 at a 5% significance level with certainty. Therefore, the intensity function can be assumed to be increasing.

In conclusion, based on the Danish fires dataset, the results of the Laplace and Weibull tests consistently indicate that the intensity function is increasing. This suggests a growing frequency of large fire insurance claims over the observed period. These findings highlight the non-homogeneous nature of the underlying process and demonstrate the practical utility of the proposed tests in real-world applications.

Conclusion

The Poisson process remains a cornerstone of statistical modeling, with its capacity to model random events over time or space. In this study, we focused on testing the hypothesis of a constant intensity function versus the alternative of an increasing intensity function. Guided by the framework established in [1] we constructed and analyzed statistical tests, only the Laplace and Weibull tests, while addressing limitations in the original study.

Through numerical simulations, we evaluated the power of these tests under a range of conditions. The results highlighted their strengths, emphasizing the importance of selecting the appropriate test based on the underlying intensity function. The Weibull test is more powerful for logarithmic and weibull intensity functions, while the Laplace test is more powerful for exponential and step intensity functions. Furthermore, the application of these tests to Danish fire data provided a practical demonstration of their effectiveness, offering insights into the behavior of real-world intensity processes.

Our findings underscore the utility of statistical tests for identifying trends in Poisson processes, contributing to their broader applicability in fields such as risk management and event prediction. Future work could explore the development and evaluation of alternative statistical tests tailored to different characteristics of Poisson processes. Additionally, conducting power studies for a broader variety of intensity functions, such as periodic or more complex functions, could provide deeper insights into the robustness and versatility of these methods.

List of Figures

References

[1] Lee J Bain, Max Engelhardt, and FT Wright. "Tests for an increasing trend in the intensity of a Poisson process: A power study". *Journal of the American Statistical Association*, 80(390):419–422, 1985.

[2] Mélissande Albert. "Poisson processes and application to reliability theory and actuarial science". *INSA Toulouse*, 2024.

[3] R Core Team. "Package evir: Extreme Value Inference in R". CRAN, 2024.