

for $\underline{x}_n = [x_{n1}, x_{n2}]^T$, $\underline{t} = [t_1, \dots, t_N]^T$

$\underline{w} = [w_0, w_1]^T$, and, $\underline{X} = [\underline{x}_1^T, \underline{x}_2^T, \dots, \underline{x}_N^T]^T$,

Show that $\sum_n \underline{x}_n t_n = \underline{X}^T \underline{t}$

$$\underline{X}^T \underline{t} = [\underline{x}_1^T, \underline{x}_2^T, \dots, \underline{x}_N^T] \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_N \end{bmatrix}$$

$$= \begin{bmatrix} x_{11} & x_{21} & \dots & x_{N1} \\ x_{12} & x_{22} & \dots & x_{N2} \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_N \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{i=1}^N x_{i1} t_i \\ \sum_{i=1}^N x_{i2} t_i \end{bmatrix} = \sum_i \underline{x}_i t_i$$

$$\sum_n \underline{x}_n \underline{x}_n^T \underline{w} = \underline{X}^T \underline{X} \underline{w}$$

$$\underline{X}^T \underline{X} \underline{w} = [\underline{x}_1^T, \underline{x}_2^T, \dots, \underline{x}_N^T] \begin{bmatrix} \underline{x}_1^T \\ \underline{x}_2^T \\ \vdots \\ \underline{x}_N^T \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}$$

$$= \begin{bmatrix} x_{11} & x_{21} & \dots & x_{N1} \\ x_{12} & x_{22} & \dots & x_{N2} \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ \vdots & \vdots \\ x_{N1} & x_{N2} \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}$$

$$= \begin{bmatrix} \sum_i x_{i1}^2 & \sum_i x_{i1} x_{i2} \\ \sum_i x_{i1} x_{i2} & \sum_i x_{i2}^2 \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}$$

Now

$$\begin{aligned} \sum_i \underline{x}_i \underline{x}_i^T \underline{w} &= \sum_i \begin{bmatrix} x_{i1} \\ x_{i2} \end{bmatrix} [x_{i1} \ x_{i2}] \begin{bmatrix} w_0 \\ w_1 \end{bmatrix} \\ &= \sum_i \begin{bmatrix} x_{i1}^2 & x_{i1} x_{i2} \\ x_{i1} x_{i2} & x_{i2}^2 \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \end{bmatrix} \end{aligned}$$

$$\therefore LHS = RHS.$$