Show that

$$w^T X^T X w = w_0^2 \left(\sum_{n=1}^{N} x_{n1}^2 \right) + 2 w_0 w_1 \left(\sum_{n=1}^{N} x_{n1} x_{n2} \right) + w_1^2 \left(\sum_{n=1}^{N} x_{n2}^2 \right),$$

where

$$w = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}, \quad X = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \\ \vdots & \vdots \\ x_{N1} & x_{N2} \end{bmatrix},$$

Hint - expand $X^T X$ first.

$$X^T X = \begin{bmatrix} x_{11} & x_{21} & x_{31} & \cdots & x_{N1} \\ x_{12} & x_{22} & x_{32} & \cdots & x_{N2} \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \\ \vdots & \vdots \\ x_{N1} & x_{N2} \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{i=0}^{N} x_{i1}^2 & \sum_{i=0}^{N} x_{i1} x_{i2} \\ \sum_{i=0}^{N} x_{i2} x_{i1} & \sum_{i=0}^{N} x_{i2}^2 \end{bmatrix}$$

$$w^T (X^T X) = \begin{bmatrix} w_0 & w_1 \end{bmatrix} \begin{bmatrix} \sum_{i=0}^{N} x_{i1}^2 & \sum_{i=0}^{N} x_{i1} x_{i2} \\ \sum_{i=0}^{N} x_{i2} x_{i1} & \sum_{i=0}^{N} x_{i2}^2 \end{bmatrix}$$

$$= \begin{bmatrix} w_0 \sum_{i=0}^{N} x_{i1}^2 + w_1 \sum_{i=0}^{N} x_{i2} x_{i1} & w_0 \sum_{i=0}^{N} x_{i1} x_{i2} + w_1 \sum_{i=0}^{N} x_{i2}^2 \end{bmatrix}$$

$$\left(W^T X^T X\right) W = \begin{bmatrix} W_0 \sum_{i=0}^{N} x_{i1}^2 + W_1 \sum_{i=0}^{N} x_{i2} x_{i1} & W_0 \sum_{i=0}^{N} x_{i1} x_{i2} + W_1 \sum_{i=0}^{N} x_{i2}^2 \end{bmatrix} \begin{bmatrix} W_0 \\ W_1 \end{bmatrix}$$

$$= W_0^2 \sum_{i=0}^{N} x_{i1}^2 + W_0 W_1 \sum_{i=0}^{N} x_{i2} x_{i1} + W_0 W_1 \sum_{i=0}^{N} x_{i1} x_{i2} + W_1^2 \sum_{i=0}^{N} x_{i2}^2$$

Collect like terms

$$= W_0^2 \sum_{i=0}^{N} x_{i1}^2 + 2 W_0 W_1 \sum_{i=0}^{N} x_{i1} x_{i2} + W_1^2 \sum_{i=0}^{N} x_{i2}^2$$