$N = 20$, $y_N = 9$.

Assuming that the probability of heads is given by r, we observe y heads in N tosses, and r has a Beta prior with parameters α and β, the posterior density is a Beta prior with parameters $\delta = \alpha + y$ and $\gamma = \beta + N - y$. The marginal liklihood is given by

$$p(y_N \mid \alpha, \beta) = \binom{N}{y_N} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(\alpha + y_N)\Gamma(\beta + N - y_N)}{\Gamma(\alpha+\beta+N)}$$

The probability of winning is given by:

$$E_{p(r \mid y_N)}\left\{ P(Y_{new} = y_{new} \mid r) \right\}$$

$$= \binom{N_{new}}{y_{new}} \frac{\Gamma(\delta+\gamma)}{\Gamma(\delta)\Gamma(\gamma)} \frac{\Gamma(\delta + y_{new})\Gamma(\gamma + N_{new} - y_{new})}{\Gamma(\delta+\gamma+N_{new})}$$

Scenario 1: $\alpha = 1$, $\beta = 1$, $\Rightarrow$ $\delta = \alpha + y_N = 10$, $\gamma = 12$.

$$E_{p(r \mid y_N)}\left\{ P(Y_{new} = 9 \mid r) \right\} = \frac{\Gamma(22)}{\Gamma(10)\Gamma(12)} \times \frac{\Gamma(19)\Gamma(23)}{\Gamma(42)}$$