$$\widehat{\sigma^2} = \frac{1}{N}\sum_{i=1}^{N}\left(t_i - \underline{x}_n \underline{w}\right)^2$$

$$= \frac{1}{N}\left(\sum_{i=1}^{N} t_i^2 - 2\sum_{i=1}^{N} t_i \underline{x}_i^T \underline{\widehat{w}} + \sum_{i=1}^{N}\left(\underline{x}_i^T \underline{\widehat{w}}\right)^2\right)$$

Concentrating on the last term.

$$\sum_{i=1}^{N}\left(\underline{x}^T \underline{\widehat{w}}\right)^2 = \sum_{i=1}^{N} \underline{x}_i^T \underline{\widehat{w}}\, \widehat{w}^T \underline{x}_i$$

$$= \mathrm{Tr}\left(\underline{X}\, \widehat{w}\, \widehat{w}^T \underline{X}^T\right)$$

$$\vdots$$

$$= \sum_{i=1}^{N} t_i \underline{x}_n^T \widehat{w}$$

Therefore, $\widehat{\sigma_2} = \frac{1}{N}\left[\sum_{i=1}^{N} t_i^2 - \sum_{i=1}^{N} t_i \underline{x}_i^T \widehat{w}\right]$

Now $\sum_{i=1}^{N} t_i^2 = \underline{t}^T \underline{t}$, and we know that

$$\sum_{i=1}^{N} t_i \underline{x}_i \underline{\widehat{w}} = \underline{t}^T \underline{X}\, \underline{\widehat{w}}$$

$$\therefore\ \widehat{\sigma^2} = \frac{1}{N}\left[\underline{t}^T \underline{t} - \underline{t}^T \underline{X}\, \widehat{w}\right]$$