We have N values $x_1, \dots, x_N$. Assuming these come from a Gaussian, we want to find the maximum liklihood estimate of σ and want to find the maximum liklihood estimates of the mean and variance of the Gaussian. The Gaussian PDF is

$$N(\mu, \sigma) \sim \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)$$

Assuming that the points are IID, the liklihood of all N points is given by a product over the N objects.

$$L = \prod_{i=1}^{N} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma^2}(x_i-\mu)^2\right)$$

We can then take the Natural logarithm of this.

$$\log(L) = \sum_{i=1}^{N} \left(-\frac{1}{2}\left(\log(2\pi\sigma^2\right) - \frac{1}{2\sigma^2}(x_i-\mu)^2\right)$$

To find the maximum liklihood estimate for μ, differentiate wrt μ and equate to 0.

$$\frac{\partial}{\partial\mu}\left(\log(L)\right) = \sum_{i=1}^{N} \frac{1}{\sigma^2}(x_i-\mu) = 0$$

$$\therefore \sum_{i=1}^{N} (x_i-\mu) = 0$$

$$\therefore \sum_{i=1}^{N} x_i = \sum_{i=1}^{N} \mu = N\mu \Rightarrow \mu = \frac{1}{N}\sum_{i=1}^{N} x_i.$$

and similarly for σ^2:

$$\frac{\partial}{\partial \sigma^2}\left(\log(L)\right) = \sum_{i=1}^{N}\left(-\frac{1}{2\sigma^2} + \frac{1}{2\sigma^4}(x_n - \mu)^2\right) = 0.$$

$$= -\frac{N}{2\sigma^2} + \sum_{i=1}^{N} \frac{1}{2\sigma^4}(x_n - \mu)^2$$

$$\Rightarrow N\sigma^2 = \sum_{i=1}^{N}(x_n - \mu)^2.$$

$$\therefore \sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_n - \mu)^2$$