

Using w and X as defined in the previous exercise, show that $(Xw)^T = w^T X^T$ by multiplying out both sides.

$$X = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ \vdots & \vdots \\ x_{n1} & x_{n2} \end{bmatrix} \quad w = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}$$

$$Xw = \begin{bmatrix} w_0 x_{11} + w_1 x_{12} \\ w_0 x_{21} + w_1 x_{22} \\ \vdots \\ w_0 x_{n1} + w_1 x_{n2} \end{bmatrix} \quad \therefore (Xw)^T = \begin{bmatrix} w_0 x_{11} + w_1 x_{12} & w_0 x_{21} + w_1 x_{22} \\ \dots & w_0 x_{n1} + w_1 x_{n2} \end{bmatrix}$$

$$w^T X^T = \begin{bmatrix} w_0 & w_1 \end{bmatrix} \begin{bmatrix} x_{11} & x_{21} & \dots & x_{n1} \\ x_{12} & x_{22} & \dots & x_{n2} \end{bmatrix}$$

$$= \begin{bmatrix} w_0 x_{11} + w_1 x_{12} & w_0 x_{21} + w_1 x_{22} & \dots & w_0 x_{n1} + w_1 x_{n2} \end{bmatrix}$$

$$\therefore (Xw)^T = w^T X^T$$