

The Hessian for a general model of our form is:  $-\frac{1}{\sigma^2} \underline{X}^T \underline{X}$ .

For a linear model,  $\underline{X}$  has a Column of ones for the Constant of Set then  $x_i$  for the gradient.

$$\underline{X} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_N \end{bmatrix}^T$$

Therefore, the Hessian is-

$$-\frac{1}{\sigma^2} \begin{bmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_N \end{bmatrix} \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$$

$$= -\frac{1}{\sigma^2} \begin{bmatrix} N & \sum_{i=1}^N x_i \\ \sum_{i=1}^N x_i & \sum_{i=1}^N x_i^2 \end{bmatrix}$$

The leading diagonal terms are  $-\frac{N}{\sigma^2}$  &  $-\frac{1}{\sigma^2} \sum_{i=1}^N x_i^2$ , which only differ by a Constant.