

for the diagonal Covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_D^2 \end{bmatrix}, \text{ the determinant is the}$$

product of the leading diagonal $\therefore \prod_{d=1}^D \sigma_d^2$

$$p(w) = \frac{1}{(2\pi)^{\frac{D}{2}} \left(\prod_{d=1}^D \sigma_d^2 \right)^{\frac{1}{2}}} \exp \left(-\frac{1}{2\sigma^2} (w_d - \mu_d)^2 \right)$$

$$= \frac{1}{(2\pi)^{\frac{D}{2}} \left(\prod_{d=1}^D \sigma_d^2 \right)^{\frac{1}{2}}} \prod_{d=1}^D \exp \left(-\frac{1}{2\sigma^2} (w_d - \mu_d)^2 \right)$$

$$= \prod_{d=1}^D \frac{1}{(2\pi)^{\frac{D}{2}} \sigma_d} \exp \left(-\frac{1}{2\sigma^2} (w_d - \mu_d)^2 \right)$$

This is the product of D independent Univariate Gaussians