

$$\begin{aligned}\hat{\sigma}^2 &= \frac{1}{N} \sum_{i=1}^N (t_i - \underline{x}_i \underline{\hat{w}})^2 \\ &= \frac{1}{N} \left(\sum_{i=1}^N t_i^2 - 2 \sum_{i=1}^N t_i \underline{x}_i^T \underline{\hat{w}} + \sum_{i=1}^N (\underline{x}_i \underline{\hat{w}})^2 \right)\end{aligned}$$

Concentrating on the last term.

$$\begin{aligned}\sum_{i=1}^N (\underline{x}_i^T \underline{\hat{w}})^2 &= \sum_{i=1}^N \underline{x}_i^T \underline{\hat{w}} \underline{\hat{w}}^T \underline{x}_i \\ &= \text{Tr}(\underline{X} \underline{\hat{w}} \underline{\hat{w}}^T \underline{X}^T)\end{aligned}$$

$$\begin{aligned}&\vdots \\ &= \sum_{i=1}^N t_i \underline{x}_i^T \underline{\hat{w}}\end{aligned}$$

$$\text{Therefore, } \hat{\sigma}^2 = \frac{1}{N} \left[\sum_{i=1}^N t_i^2 - \sum_{i=1}^N t_i \underline{x}_i^T \underline{\hat{w}} \right]$$

Now $\sum_{i=1}^N t_i^2 = \underline{t}^T \underline{t}$, and we know that

$$\sum_{i=1}^N t_i \underline{x}_i \underline{\hat{w}} = \underline{t}^T \underline{X} \underline{\hat{w}}$$

$$\therefore \hat{\sigma}^2 = \frac{1}{N} [\underline{t}^T \underline{t} - \underline{t}^T \underline{X} \underline{\hat{w}}]$$