Bernoulli distribution : $p(X_n = x|r) = r^x(1-r)^{1-x}$
$x \in \{0,1\}$

For N samples the product is to be computed.

$$L = \prod_{i=1}^{N} r^{x_i}(1-r)^{1-x_i}$$

$$\log(L) = \sum_{i=1}^{N} x_i \log(r) + (1-x_i)\log(1-r) .$$

Differentiating wrt r, gives us.

$$\frac{\partial}{\partial r}\left(\log(L)\right) = \sum_{i=1}^{N}\left(\frac{x_i}{r} - \frac{1-x_i}{1-r}\right) = 0$$

$$\therefore \sum_{i=1}^{N} \frac{x_i}{r} = \sum_{i=1}^{N} \frac{1-x_i}{1-r}$$

$$\therefore \sum_{i=1}^{N} x_i(1-r) = \sum_{i=1}^{N} r(1-x_i)$$

$$\therefore \sum_{i=1}^{N} x_i - \cancel{r\sum_{i=1}^{N} x_i} = Nr - \cancel{r\sum_{i=1}^{N} x_i}$$

$$\therefore r = \frac{1}{N}\sum_{i=1}^{N} x_i$$