$$E_{p(r)}(r) = \frac{\alpha}{\alpha + \beta}$$

$$E_{p(r)}(r^2) = \frac{\Gamma(\alpha + \beta)}{\alpha + \beta}$$

$$E_{p(r)}(r^{2}) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_{0}^{1} r^{2} \times r^{\alpha-1}(1-r)^{\beta-1} dr.$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_{0}^{1} r^{\alpha+1}(1-r)^{\beta-1} dr.$$

$$= \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_{0}^{1} r^{\alpha'-1} (1-r)^{\beta-1} dr.$$

$$= \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \times \frac{\Gamma(\alpha')\Gamma(\beta)}{\Gamma(\alpha')\Gamma(\beta)}$$

$$\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha'+\beta)}$$

$$=\frac{\Gamma(\alpha+\beta)\Gamma(\alpha+2)}{\Gamma(\alpha+2)}$$

$$= \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+2)}{\Gamma(\alpha)\Gamma(\alpha+2+\beta)}$$

$$= \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+2)}{\Gamma(\alpha)\Gamma(\alpha+2+\beta)}$$

$$= \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+2)}{\Gamma(\alpha)\Gamma(\alpha+2+\beta)}$$

$$= \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+2+\beta)}{(\alpha+\beta)\Gamma(\alpha)}$$

$$= \frac{1}{\Gamma(\alpha) \Gamma(\alpha+2+\beta)}$$

$$= \frac{(\alpha+1)\alpha \times \Gamma(\alpha+\beta) \Gamma(\alpha)}{\Gamma(\alpha+\beta) \Gamma(\alpha+\beta) \Gamma(\alpha+\beta)}$$

$$=\frac{(\alpha+1)\alpha\times\left(\alpha+\beta\right)\left(\alpha\right)}{(\alpha+\beta)\left(\alpha+\beta\right)\left(\alpha+\beta\right)}$$

 $\frac{(\alpha+1)\alpha\times \lceil (\alpha+\beta)\rceil (\alpha)}{(\alpha+\beta+1)(\alpha+\beta)\lceil (\alpha)\rceil (\alpha+\beta)}$

$$Var(r) = E_{p(r)}(r^{2}) - \left[E_{p(r)}(r)\right]^{2}$$

$$= \frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)} - \left(\frac{\alpha}{\alpha+\beta}\right)^{2}$$

$$= \frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)} - \left(\frac{\alpha}{\alpha+\beta}\right)^{2}$$

$$= \frac{\alpha(\alpha+\beta)(\alpha+\beta+1)}{(\alpha+\beta+1)} - \left(\frac{\alpha}{\alpha+\beta}\right)^{2}$$

$$= \frac{\alpha(\alpha+\beta)(\alpha+\beta+1)}{(\alpha+\beta)} (\alpha+\beta+1)$$

$$= \frac{\alpha(\alpha+\beta)^{2}(\alpha+\beta+1)}{(\alpha+\beta)^{2}(\alpha+\beta+1)}$$

$$=\frac{x^{3}+x^{2}+x^{2}\beta+\alpha\beta-x^{3}-x^{2}\beta-x^{2}}{(\alpha+\beta)^{2}(\alpha+\beta+1)}$$

$$\frac{(\alpha+\beta)^{2}(\alpha+\beta+1)}{\alpha\beta}$$

$$\frac{\alpha\beta}{(\alpha+\beta+1)}$$

$$= (x+\beta)(x+\beta+1)$$