$$p(r) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} r^{\alpha-1} (1-r)^{\beta-1}$$

What is $E_{p(r)}(r)$? $\Gamma(n+1) = n\Gamma(n)$

and $\int_{r=0}^{r=1} r^{a-1}(1-r)^{b-1} dr = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$.

$$E_{p(r)}(r) = \int_{-\infty}^{\infty} r\, p(r)\, dr.$$

$$= \int_0^1 r \times \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} r^{\alpha-1}(1-r)^{\beta-1} dr$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 r^{\alpha}(1-r)^{\beta-1} dr$$

Substitute $\alpha = \alpha' - 1$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 r^{\alpha'-1}(1-r)^{\beta-1} dr$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \times \frac{\Gamma(\alpha')\Gamma(\beta)}{\Gamma(\alpha'+\beta)}$$

$$= \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+1)}{\Gamma(\alpha)\Gamma(\alpha+1+\beta)} = \frac{\alpha\Gamma(\alpha+\beta)}{(\alpha+\beta)\Gamma(\alpha+\beta)}$$

$$= \frac{\alpha}{\alpha+\beta}$$