

Bernoulli distribution : $P(x|\theta) = \theta^x (1-\theta)^{1-x}$

The Fisher information is the negative expected value of the second derivative of the log likelihood, evaluated about some point.

$$F = -E_{P(x|\theta)} \left\{ \frac{\partial^2 \log(P(x|\theta))}{\partial \theta^2} \right\}$$

$$\log(P(x|\theta)) = x \log \theta + (1-x) \log(1-\theta)$$

$$\frac{\partial}{\partial \theta} (\log(P(x|\theta))) = \frac{x}{\theta} - \frac{1-x}{1-\theta}$$

$$\frac{\partial^2}{\partial \theta^2} (\log(P(x|\theta))) = -\frac{x}{\theta^2} - \frac{1-x}{(1-\theta)^2}$$

$$\therefore F = \frac{1}{\theta^2} E_{P(x|\theta)} \{x\} + \frac{1}{(1-\theta)^2} E_{P(x|\theta)} \{1-x\}$$

$$E_{P(x|\theta)} \{x\} = \theta.$$

$$\therefore F = \frac{\theta}{\theta^2} + \frac{1-\theta}{(1-\theta)^2} = \theta^{-1} + (1-\theta)^{-1}$$