$$E_{p(r)}(r) = \frac{\alpha}{\alpha+\beta}$$

$$E_{p(r)}(r^2) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 r^2 \times r^{\alpha-1} (1-r)^{\beta-1} \, dr.$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 r^{\alpha+1} (1-r)^{\beta-1} \, dr.$$

$$\alpha = \alpha' - 2$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 r^{\alpha'-1} (1-r)^{\beta-1} \, dr.$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \times \frac{\Gamma(\alpha')\Gamma(\beta)}{\Gamma(\alpha'+\beta)}$$

$$= \frac{\Gamma(\alpha+\beta)\,\Gamma(\alpha+2)}{\Gamma(\alpha)\,\Gamma(\alpha+2+\beta)}$$

$$= \frac{(\alpha+1)\alpha \times \Gamma(\alpha+\beta)\,\Gamma(\alpha)}{(\alpha+\beta+1)(\alpha+\beta)\,\Gamma(\alpha)\Gamma(\alpha+\beta)}$$

$$= \frac{\alpha(\alpha+1)}{(\alpha+\beta+1)(\alpha+\beta)}$$

$$Var(r) = E_{p(r)}(r^2) - \left[E_{p(r)}(r)\right]^2$$

$$= \frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)} - \left(\frac{\alpha}{\alpha+\beta}\right)^2$$

$$= \frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)} - \frac{\alpha^2}{(\alpha+\beta)^2}$$

$$= \frac{\alpha(\alpha+1)(\alpha+\beta) - \alpha^2(\alpha+\beta+1)}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$= \frac{\cancel{\alpha^3} + \cancel{\alpha^2} + \cancel{\alpha^2\beta} + \alpha\beta - \cancel{\alpha^3} - \cancel{\alpha^2\beta} - \cancel{\alpha^2}}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$= \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$