

# Parallelism

also know as *Parallel // programming*

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web site: <https://lms.univ-cotedazur.fr/course/view.php?id=269001>

Jan 2024

Chapter 1 : General Introduction, PRAM  
models

# Organization (flexible!)

- 12h C in total
- Approx. 30h TD or labs in total
- One individual project in C/OpenMP
- One Lab on GPGPU programming
- Perhaps One lab in C/MPI
- Individual exam (50%), around Feb 21, 2024
  - Personal notes on an A4 sheet of paper
- Bibliography on this (old) topic is huge: check slides links, LMS, and many more !

# Plan

## 1. Motivation

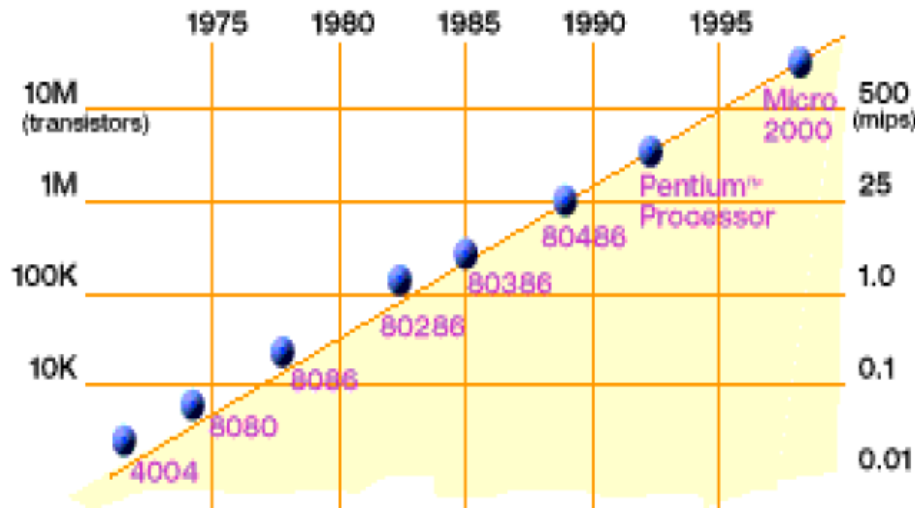
Which includes A very brief glimpse of parallel machines

## 2. Introduction to the theoretical PRAM model and typical algorithms

# Why parallel computing ?

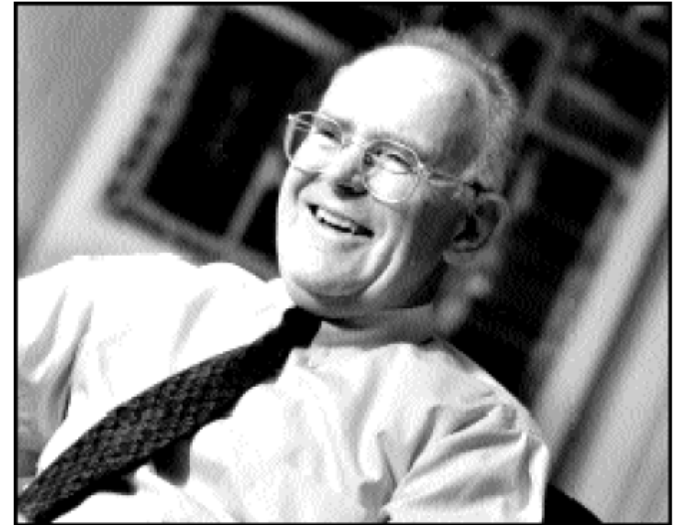
- Solve a problem faster
  - Rely on machines that are more powerful
    - Increase chip performances (Moore law)
    - Or Number of processing elements  $\gg 1$
- Solve problems whose size is too high to be handled by one single machine
  - Cut the problem into sub-problems
    - Solve sub-problems in parallel on several machines

# Chip performance: Moore Law ...



2X transistors/Chip Every 1.5 years  
Called “Moore’s Law”

**Microprocessors have become smaller, denser, and more powerful.**

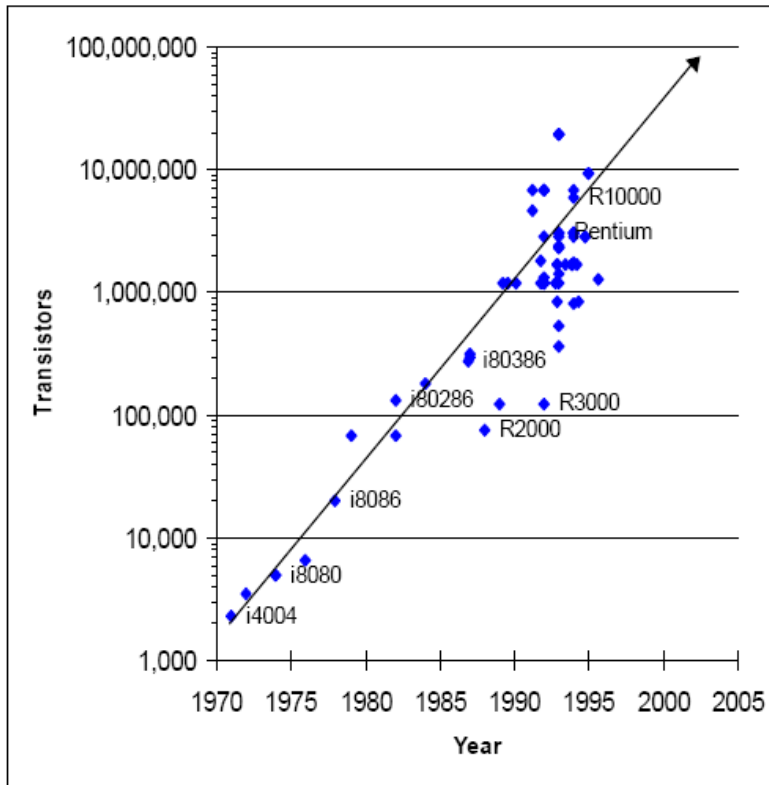


Gordon Moore (co-founder of Intel) predicted in 1965 that the transistor density of semiconductor chips would double roughly every 18 months.

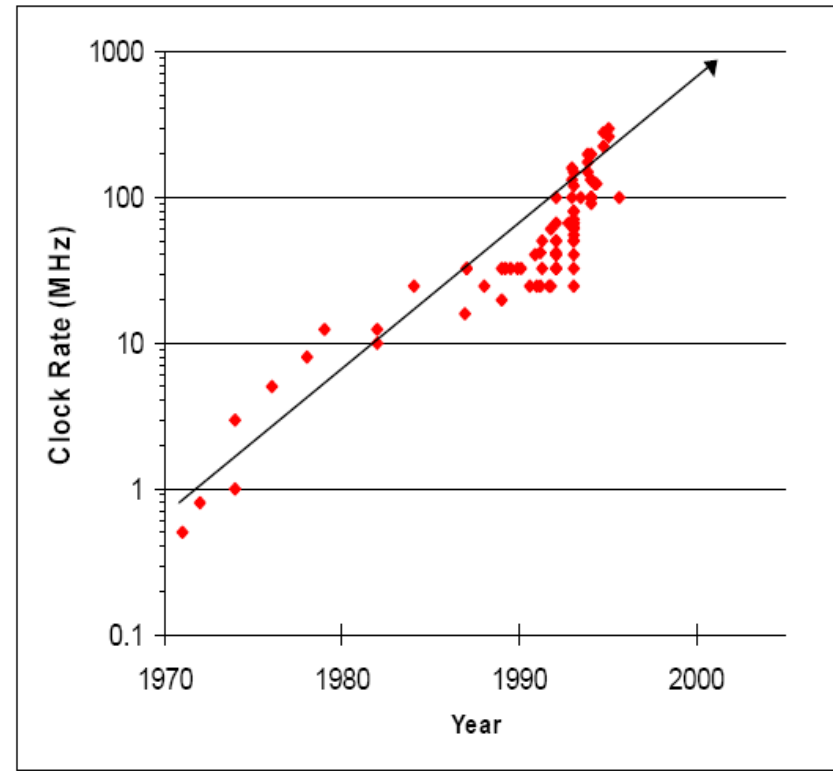
# To compute faster :

## Moore law (around the 2000's)

**Growth in transistors per chip**

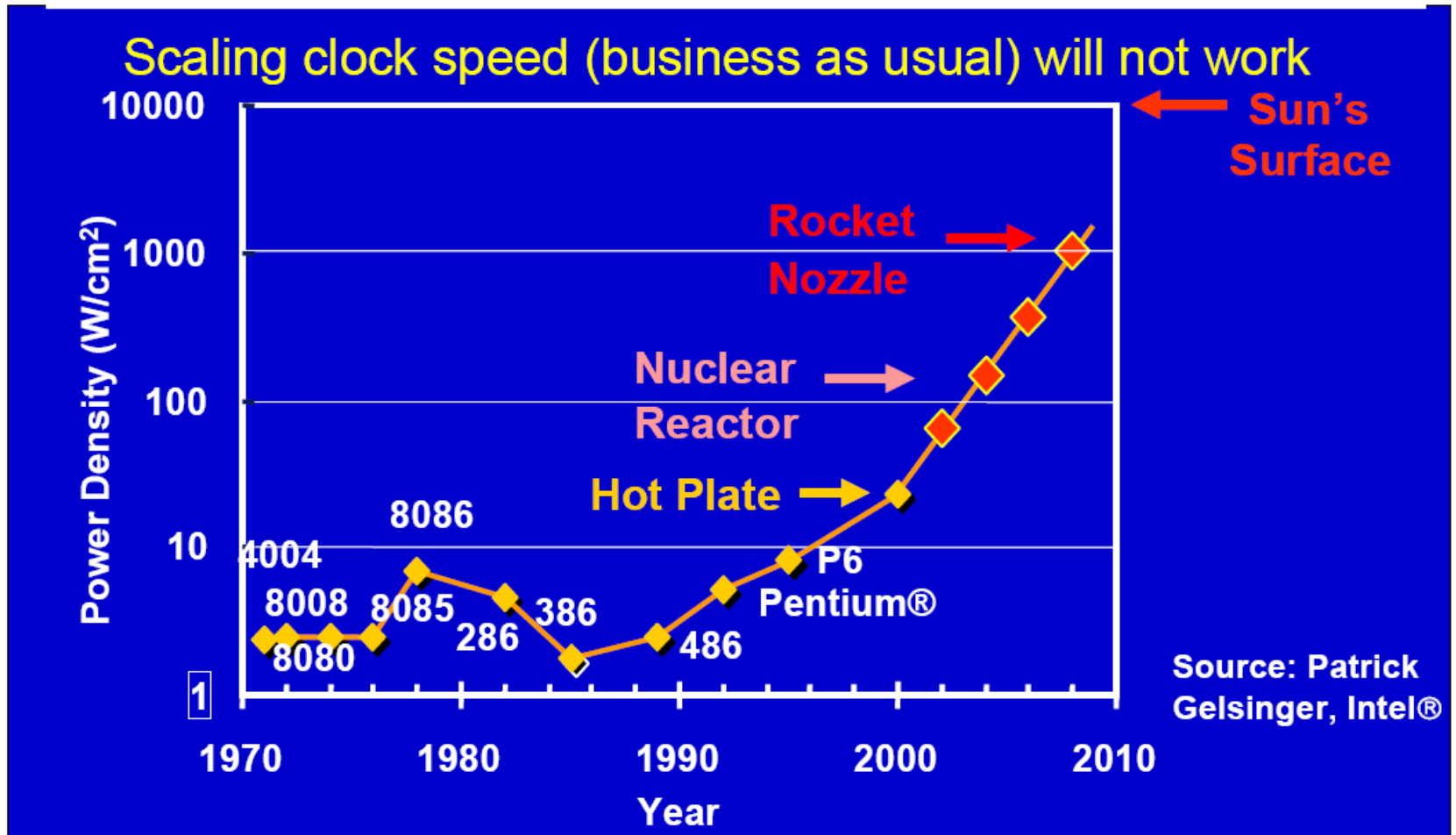


**Increase in clock rate**

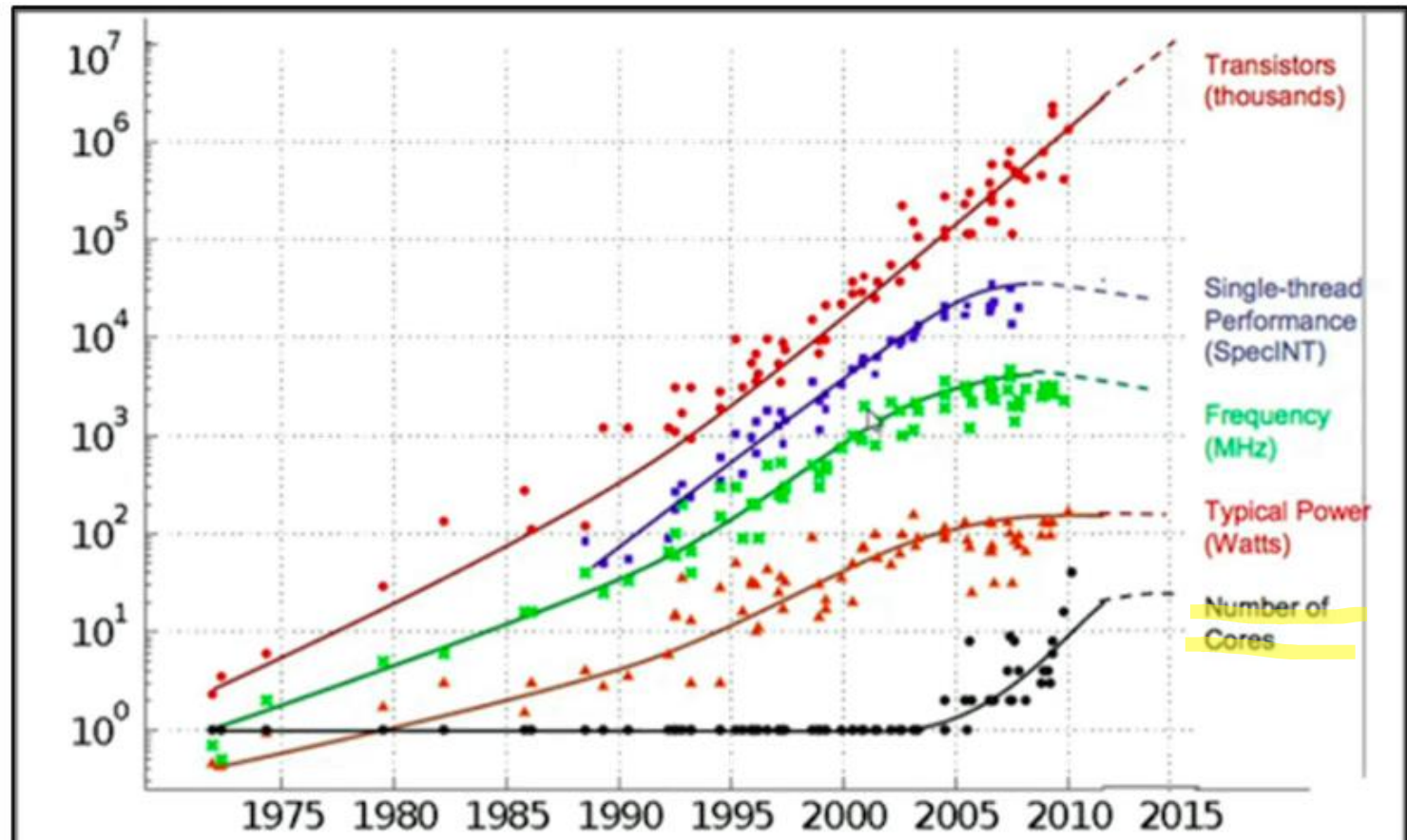


**Why bother with parallel programming? Just wait a year or two...**

# Moore's law limit



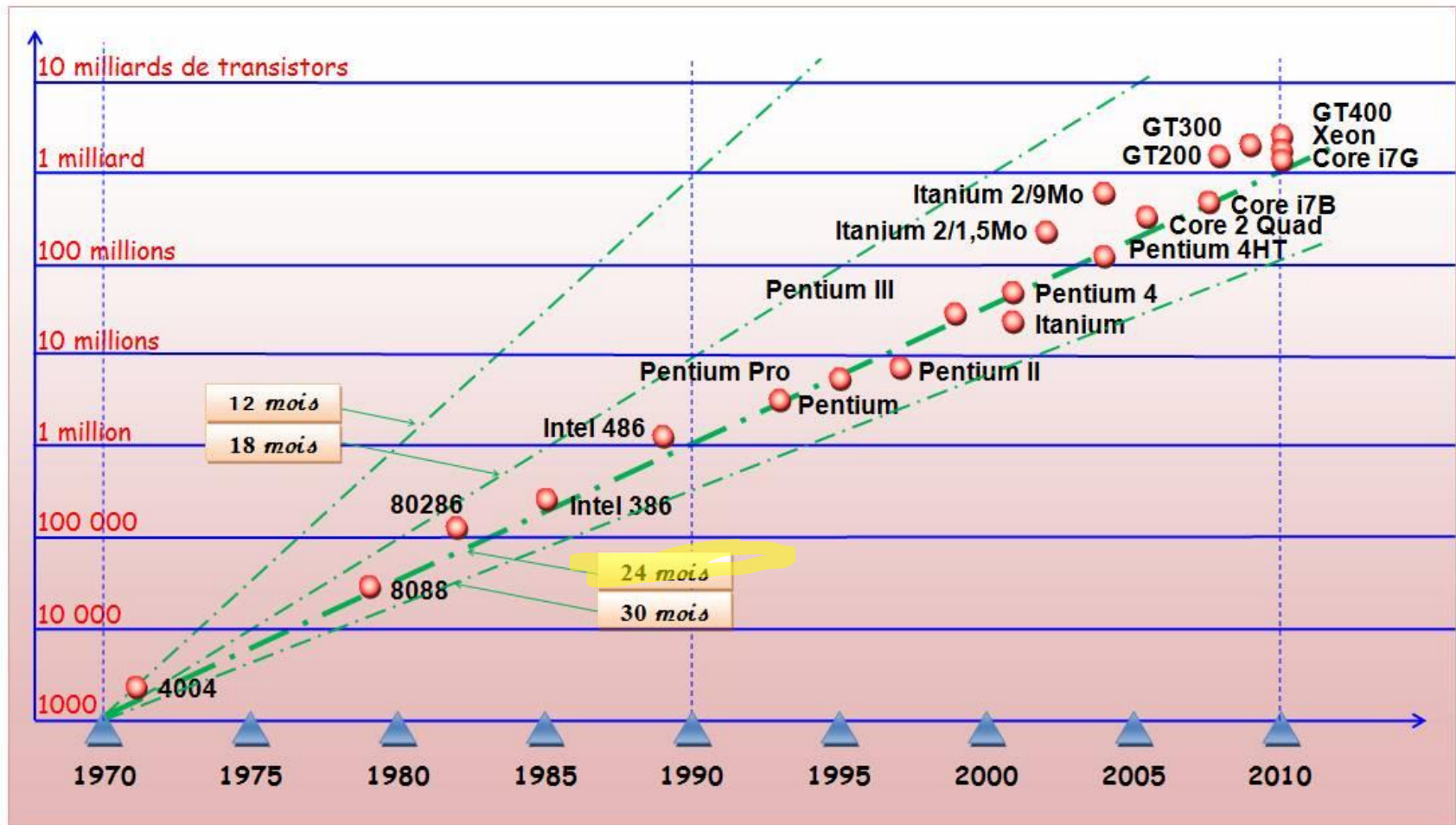
# To compute faster given the Moore's law limit: multi core



The Free Lunch Is Over  
A Fundamental Turn Toward Concurrency in Software  
By Herb Sutter, March 2005



# Moore law



# Modern chips: Tendencies to mitigate the end of Moore' law

- Even more specialized units (as FPU's are):
  - GPU's, Computational storage devices, Tensor processing units,.. => offloading workloads from CPU's
- 3D transistors
- Non silicon & nano transistors : Molecular, DNA, ...
- Quantum computers ? A survey of quantum algos  
[2310.03011.pdf \(arxiv.org\)](#)
- And... going back to more code optimization !
  - Also to decrease energy consumption, target better performance per watt, instead of pure performance

# (Explicit) Multicore programming is needed: basic concepts

- Programmers must think on how to benefit from the multiple cores, even on a single chip
  - Can be helped by general purpose/high level patterns
    - ForkJoin, bag of tasks, map&reduce functions, //workflows ...
- Each core runs its own thread(s) of computation
  - Exhibit threads in the computation
    - Could be through compilation mechanisms, dedicated APIs or language constructs
  - Hopefully, runtime+O.S. will schedule threads on the various cores to run in parallel
  - Collaboration between threads through the shared memory of the chip
    - Needs (classic) concurrency control mechanisms

# Why parallel computing ?

- Solve a problem faster
  - Rely on machines that are more powerful
    - Increase chip performances (Moore law)
    - Or Number of processing elements  $\gg 1$
- Solve problems whose size is too high to be handled by one single machine
  - 90% of data volume created in the past 2years alone!
  - Cut the problem into sub-problems
    - Solve sub-problems in parallel on **several** machines
    - Interactions between sub-computations ?
      - Message passing on a (high tech) network (eg infiniband)
      - And/or Through a physical or virtual shared memory
        - NUMA: Non Uniform Memory Access
        - typical of GPU numerous memory / cache levels

# Massively parallel computing / HPC

## Supercomputers

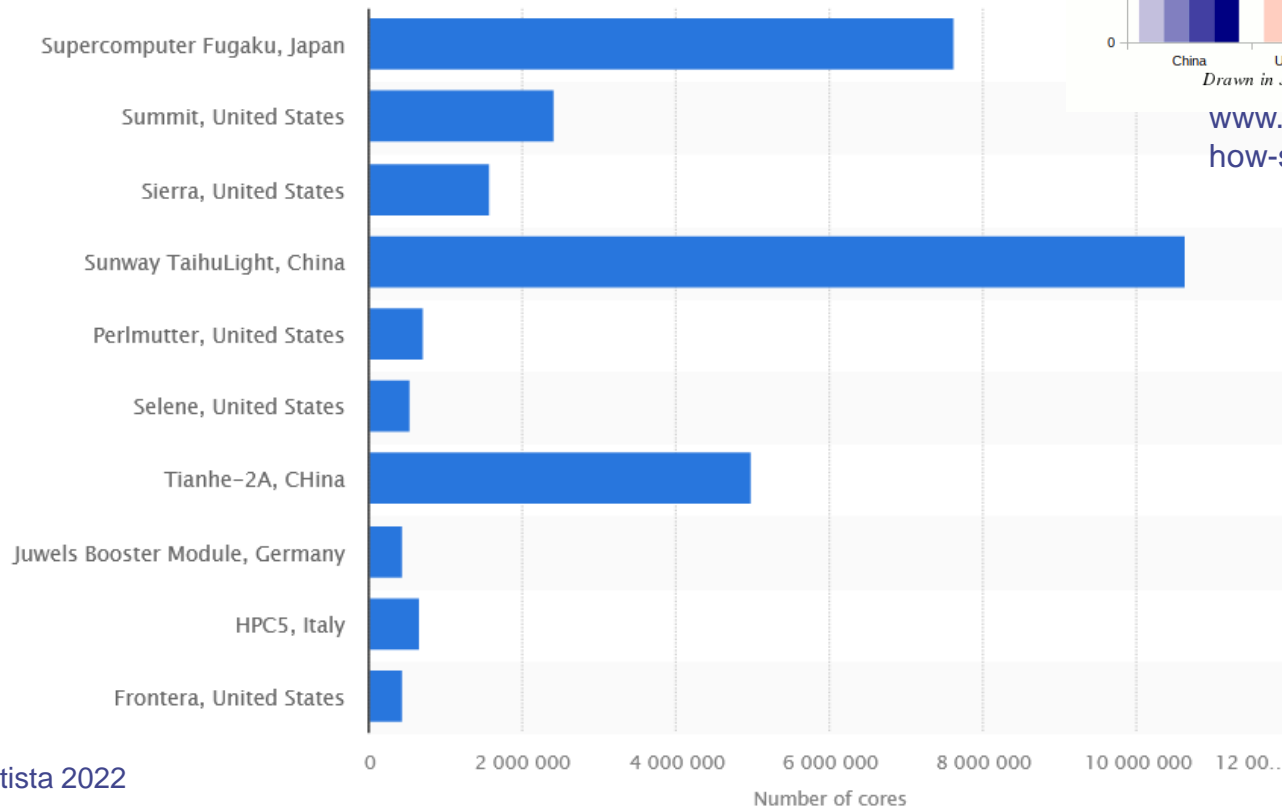
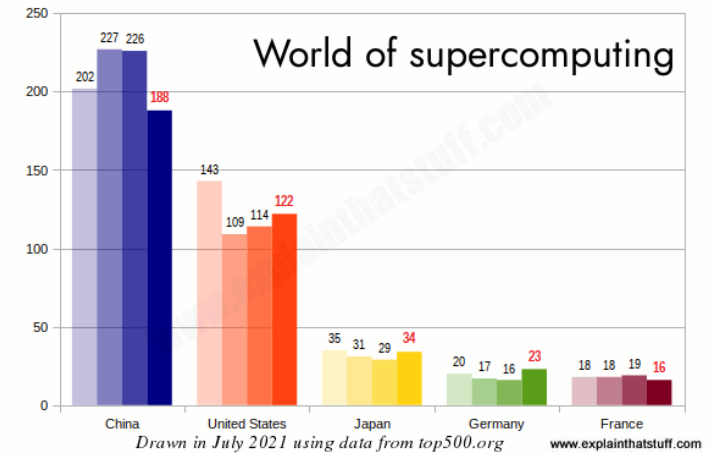
- Many-core to super computing

- Many chips, interconnected

- Target Exascale:  $10^{18}$  FLOPS

[en.wikipedia.org/wiki/Exascale\\_computing](https://en.wikipedia.org/wiki/Exascale_computing)

Who has the most supercomputers? 2017-2020



[www.explainthatstuff.com/  
how-supercomputers-work.html](https://www.explainthatstuff.com/how-supercomputers-work.html)

# Top'500

- International ranking of supercomputers
  - <https://top500.org/lists/top500/2022/11/> latest list
  - Based on their performance in FLOPS
    - Floating Point Operations Per Second (eg 10FLOPS=pocket calc)
  - on specific benchmarks (eg Linpack : solving the system  $Ax=b$ )
  - From <https://www.exascaleproject.org/what-is-exascale/>  
*At a quintillion ( $10^{18}$ ) calculations each second, exascale supercomputers will more realistically simulate the processes involved in precision medicine, regional climate, additive manufacturing, the conversion of plants to biofuels, the relationship between energy and water use, the unseen physics in materials discovery and design, the fundamental forces of the universe, and much more.*

# HPL Benchmark

HPL is a High-Performance Linpack benchmark implementation. The code solves a uniformly random system of linear equations and reports time and floating-point execution rate using a standard formula for operation count.

HPL is written in a portable ANSI C and requires an MPI implementation as well as either BLAS or VSIPPL library. Such choice of software dependencies gives HPL both portability and performance.

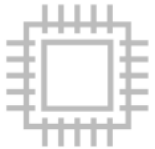
HPL is often one of the first programs run on large computer installations to produce a result that can be submitted to [TOP500](#).

## SUPERCOMPUTERS

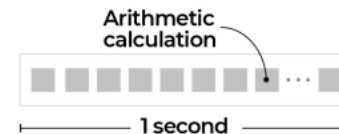
### How is computing performance measured?

The main measuring unit of supercomputer performance

**FLOPs** Floating-point operations per second



The number of arithmetic calculations the computer can perform in one second



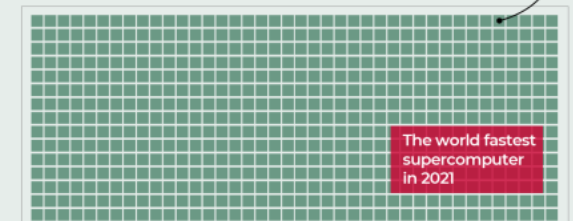
<b>kFLOPs</b>	kiloFLOPs	= $10^3$	FLOPs
<b>MFLOPs</b>	megaFLOPs	= $10^6$	FLOPs
<b>GFLOPs</b>	gigaFLOPs	= $10^9$	FLOPs
<b>TFLOPs</b>	teraFLOPs	= $10^{12}$	FLOPs
<b>PFLOPs</b>	petaFLOPs	= $10^{15}$	FLOPs

To understand how powerful the world's fastest computer is in terms of FLOPs



Supercomputer Fugaku, Japan  
442,010 TFLOPs { It is equivalent to about 600,000 iPhone 11's }

1,000 iPhone 11

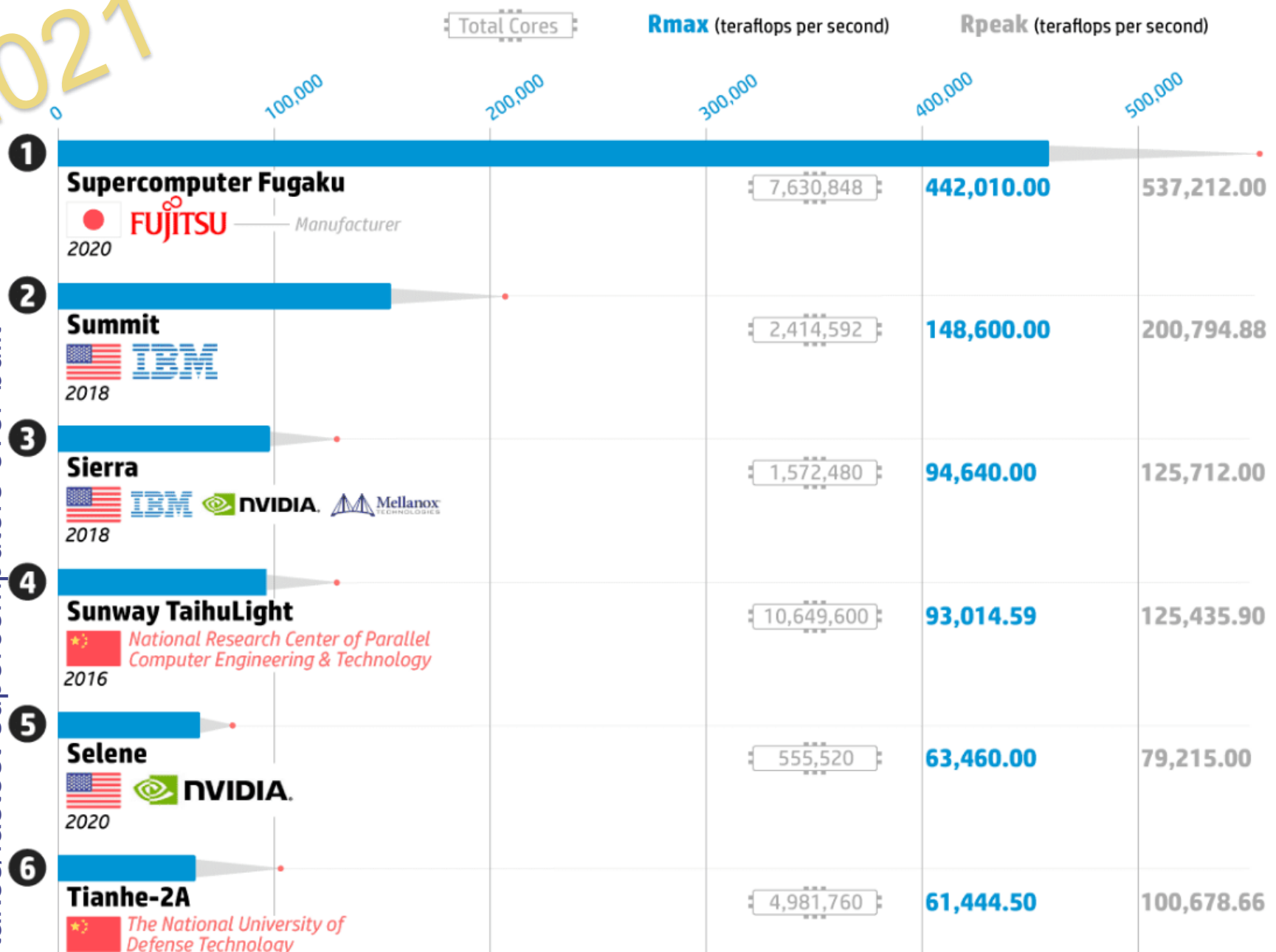


Source: Top500 | November, 2021

In supercomputing, **Rmax** and **Rpeak** are scores used to rank supercomputers on their performance. A computer's **Rmax** score ranks its maximum achieved performance, and the **Rpeak** score ranks its theoretical peak performance. They are typically measured in **flops** (floating-point operations per second).

2021

<https://www.hp.com/us-en/shop/tech-takes/fastest-supercomputers-ever-built>





# Top'500 nov 2022

[https://eurohpc-ju.europa.eu/lumi-leonardo-among-worlds-five-most-powerful-supercomputers-2022-11-14\\_en](https://eurohpc-ju.europa.eu/lumi-leonardo-among-worlds-five-most-powerful-supercomputers-2022-11-14_en)

1 **Frontier** - HPE Cray EX235a, AMD Optimized 3rd Generation EPYC 64C 2GHz, AMD Instinct MI250X, Slingshot-11, HPE

2 **Supercomputer Fugaku** - Supercomputer Fugaku, A64FX 48C 2.2GHz, Tofu interconnect D, Fujitsu

3 **LUMI** - HPE Cray EX235a, AMD Optimized 3rd Generation EPYC 64C 2GHz, AMD Instinct MI250X, Slingshot-11, HPE

4 **Leonardo** - BullSequana XH2000, Xeon Platinum 8358 32C 2.6GHz, NVIDIA A100 SXM4 64 GB, Quad-rail NVIDIA HDR100 Infiniband, Atos

5 **Summit** - IBM Power System AC922, IBM POWER9 22C 3.07GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband, IBM

•The Frontier system at the Oak Ridge National Laboratory, Tennessee, USA remains the No. 1 system on the TOP500 and is still the only system reported with an HPL performance exceeding one Exaflop/s. Frontier brought the pole position back to the USA on the June listing with an HPL score of 1.102 Exaflop/s.

•The LUMI system at EuroHPC/CSC in Finland entered the list last June at No. 3. It is again listed as No. 3 but only thanks to an upgrade of the system, which doubled its size. With its increased HPL score of 309 Pflop/s it remains the largest system in Europe.

•The only new machine to grace the top of the list was the No. 4 Leonardo system at EuroHPC/CINECA in Bologna, Italy. The machine achieved an HPL score of .174 EFlop/s with 1,463,616 cores.

# Top'500 nov 2023

1	<b>Frontier</b> - HPE Cray EX235a, AMD Optimized 3rd Generation EPYC 64C 2GHz, AMD Instinct MI250X, Slingshot-11, HPE DOE/SC/Oak Ridge National Laboratory United States
2	<b>Aurora</b> - HPE Cray EX - Intel Exascale Compute Blade, Xeon CPU Max 9470 52C 2.4GHz, Intel Data Center GPU Max, Slingshot-11, Intel DOE/SC/Argonne National Laboratory United States
3	<b>Eagle</b> - Microsoft NDv5, Xeon Platinum 8480C 48C 2GHz, NVIDIA H100, NVIDIA Infiniband NDR, Microsoft Microsoft Azure United States
4	<b>Supercomputer Fugaku</b> - Supercomputer Fugaku, A64FX 48C 2.2GHz, Tofu interconnect D, Fujitsu RIKEN Center for Computational Science Japan
5	<b>LUMI</b> - HPE Cray EX235a, AMD Optimized 3rd Generation EPYC 64C 2GHz, AMD Instinct MI250X, Slingshot-11, HPE EuroHPC/CSC Finland

- Another new system named Eagle, installed in the Microsoft Azure Cloud in the USA, has taken the No. 3 spot. This is the highest rank a cloud system has ever achieved on the TOP500. In fact, it was only 2 years ago that a previous Azure system was the first cloud system ever to enter the TOP10 at spot No. 10. This Microsoft NDv5 system has an HPL score of 561.2 PFlop/s and is based on Intel Xeon Platinum 8480C processors and NVIDIA H100 accelerators.
- Fugaku has moved to its current ranking of No. 4 after achieving No. 2 in the June 2023 list and holding the No. 1 spot from June 2020 until November 2021. This system is based in Kobe, Japan, and has an HPL score of 442.01 PFlop/s. It remains the highest ranked system outside the USA.
- The LUMI system based at Euro HPC/CSC in Kajaani, Finland, achieved the No. 5 spot with an HPL score of 379.70 PFlop/s. This system is the largest in Europe and has seen multiple upgrades that keep it near the top of the list, this time improving from an HPL score of 309.10 PFlop/s. on the last list.

# Top'500 sublist for green supercomputers, 2021

## Green500 Data






Rank	TOP500 Rank	System	Cores	Rmax (TFlop/s)	Power (kW)	Power Efficiency (GFlops/watts)
1	301	<b>MN-3</b> - MN-Core Server, Xeon Platinum 8260M 24C 2.4GHz, Preferred Networks MN-Core, MN-Core DirectConnect, Preferred Networks Preferred Networks Japan	1,664	2,181.2	55	39.379
2	291	<b>SSC-21 Scalable Module</b> - Apollo 6500 Gen10 plus, AMD EPYC 7543 32C 2.8GHz, NVIDIA A100 80GB, Infiniband HDR200, HPE Samsung Electronics South Korea	16,704	2,274.1	103	33.983
3	295	<b>Tethys</b> - NVIDIA DGX A100 Liquid Cooled Prototype, AMD EPYC 7742 64C 2.25GHz, NVIDIA A100 80GB, Infiniband HDR, Nvidia NVIDIA Corporation United States	19,840	2,255.0	72	31.538

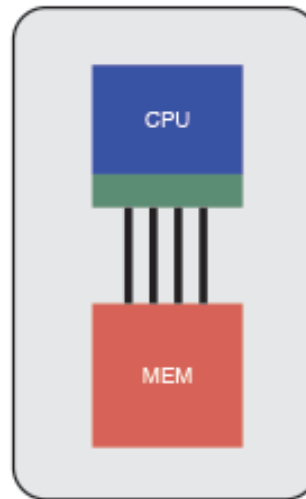
# Top'500 sublist for green supercomputers, 2023

<https://www.top500.org/lists/green500/2023/11/>

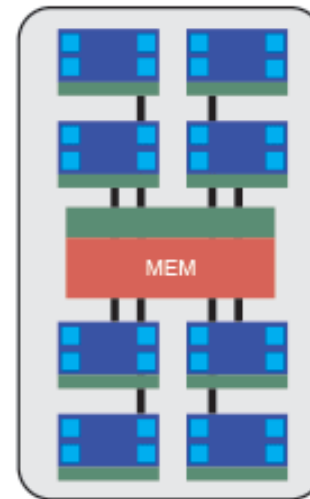
Rank	TOP500 Rank	System	Cores	Rmax (PFlop/s)	Power (kW)	Energy Efficiency (GFlops/watts)
1	293	<b>Henri</b> - ThinkSystem SR670 V2, Intel Xeon Platinum 8362 32C 2.8GHz, NVIDIA H100 80GB PCIe, Infiniband HDR, Lenovo Flatiron Institute United States	8,288	2.88	44	65.396
2	44	<b>Frontier TDS</b> - HPE Cray EX235a, AMD Optimized 3rd Generation EPYC 64C 2GHz, AMD Instinct MI250X, Slingshot-11, HPE DOE/SC/Oak Ridge National Laboratory United States	120,832	19.20	309	62.684
3	17	<b>Adastra</b> - HPE Cray EX235a, AMD Optimized 3rd Generation EPYC 64C 2GHz, AMD Instinct MI250X, Slingshot-11, HPE Grand Equipement National de Calcul Intensif - Centre Informatique National de l'Enseignement Suprieur (GENCI-CINES) France	319,072	46.10	921	58.021

# Evolution of memory archis

-  Central processing unit (CPU)
-  Multicore CPU
-  Memory (MEM)
-  Cache
-  Graphic processing unit (GPU)

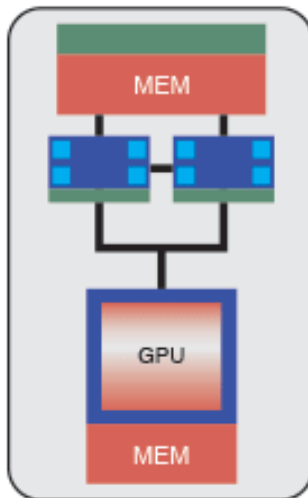


1995  
Single CPU per node  
with main memory

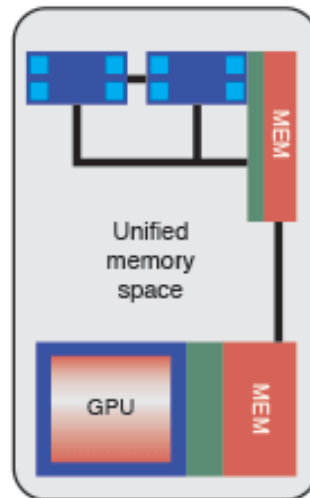


2000–2010  
Multiple CPUs per node  
sharing main memory

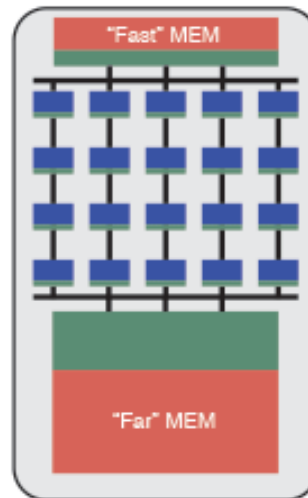
## New programming models



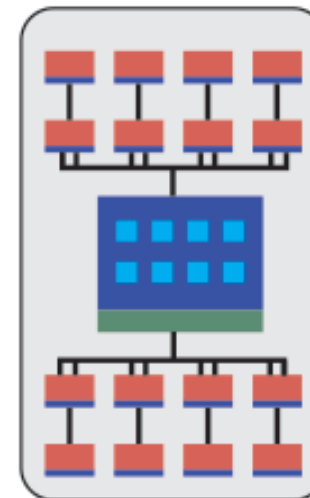
2000–2010  
Accelerators usher in  
era of heterogeneity



2014  
Accelerators share common  
view of memory with CPU



2015  
Simple low-power cores and  
non-uniform memory access



2017–2019  
Processor in memory

# Plan

## 1. Motivation

Which includes A very brief glimpse of parallel machines

## 2. Introduction to the theoretical PRAM model and typical parallel algorithms

# Theoretical model of parallelism

- On the Parallel Random Access Machine: PRAM
- Why studying problems in this PRAM model ?
  - Establish the complexity of a problem to be solved in parallel
  - Provides a complexity of a problem dependant of the number of processors to be involved to solve it
  - A lower bound: even with an unlimited number of processors, a problem cannot be solved in less than time  $T$ , in parallel
  - Optimality of the way to solve a problem with  $P$  processors compared to the time it would take to solve it optimally and sequentially (on 1 processor)

# PRAM

- A PRAM (Parallel Random Access Machine) is an abstract machine (as a RAM is)
  - An unbounded number of parallel processors
  - A shared (flat) global memory with direct (Random) access
  - A sequence of instructions to run
  - A **single** instruction pointer that all procs. follow
- => the execution is synchronous: one single (and same) instruction at a time, on each proc. (SIMD style)
- => no need of synchronisation barriers to cross before going to the next PRAM instruction!!



# PRAM variants regarding access mode to the global shared memory

- As all processors may access at the same time the memory, at same memory cell/address
- ... need to establish some rules
  - EREW (exclusive read, exclusive write) :
    - It is forbidden to read or write to a same @ in parallel
  - CREW (concurrent read exclusive write)
    - Reading the same @ by several procs allowed
  - CRCW (concurrent read, concurrent write)
    - Arbitrary mode: the write op. of only one arbitrarily chosen process to that same @ succeeds
    - Consistant mode: all write operations succeed only if the value to write at a same @ is the same
    - Associative mode: an associative operation is run on all values to be written at a same @, before the result is written at that @

# Simulation between PRAM variants

- Simulate (or emulate) one variant of a PRAM onto another variant
  - The cost in //time and proc. needed is well known
  - $\Rightarrow$  the complexity of one algorithm for a given PRAM variant becomes easily transposable onto another PRAM variant
  - Ex: a computation that takes Parallel time=  $O(1)$  on a CRCW PRAM takes  $O(\log p)$  on a p-CREW PRAM (using  $p$  processors)



# Elements of the algorithmic language

- A simple Pseudo-language
  - seq and // loops, data structures (eg arrays, lists, ...) with random access operations to any element (index j), conditional tests/branches
- All variables are shared -by default
- //Loop instruction example
  - Pour chaque (proc number) i en parallèle  
faire
    - $x[i] = y[i]$
  - FinPour
    - Here, read operations of the  $y[i]$  are executed in //, then, in // the write operations into the  $x[i]$  are run.
  - Notice that For each i in parallel => proc number i is active

# Example: compute the maximum (v1)

- We look for  $\max(T[i])$  where  $T$  has  $n$  entries
  - Each  $T[i]$  stores a number
- We use a PRAM having  $n^2$  procs.
  - Each PRAM proc will access  $T[i]$  and  $T[j]$
- We use an auxiliary array of  $n$  booleans:  $m(i)$

```
pourchaque  $1 < i < n$  en parallele
     $m[i] = \text{TRUE}$ 
pourchaque  $1 < i, j < n$  en parallele
    if (  $T[i] < T[j]$  )
         $m[i] = \text{FALSE}$ 
pourchaque  $1 < i < n$  en parallele
    if (  $m[i] = \text{TRUE}$  )
         $\text{max} = T[i]$ 
```

## ■ Complexity :

- //time=  $O(1)$  on an arbitrary CRCW (is enough!)
- $O(\log n)$  // time on a EREW or a CREW

Exo: intrinsic seq  
complexity

Exo: simulation  
complexity

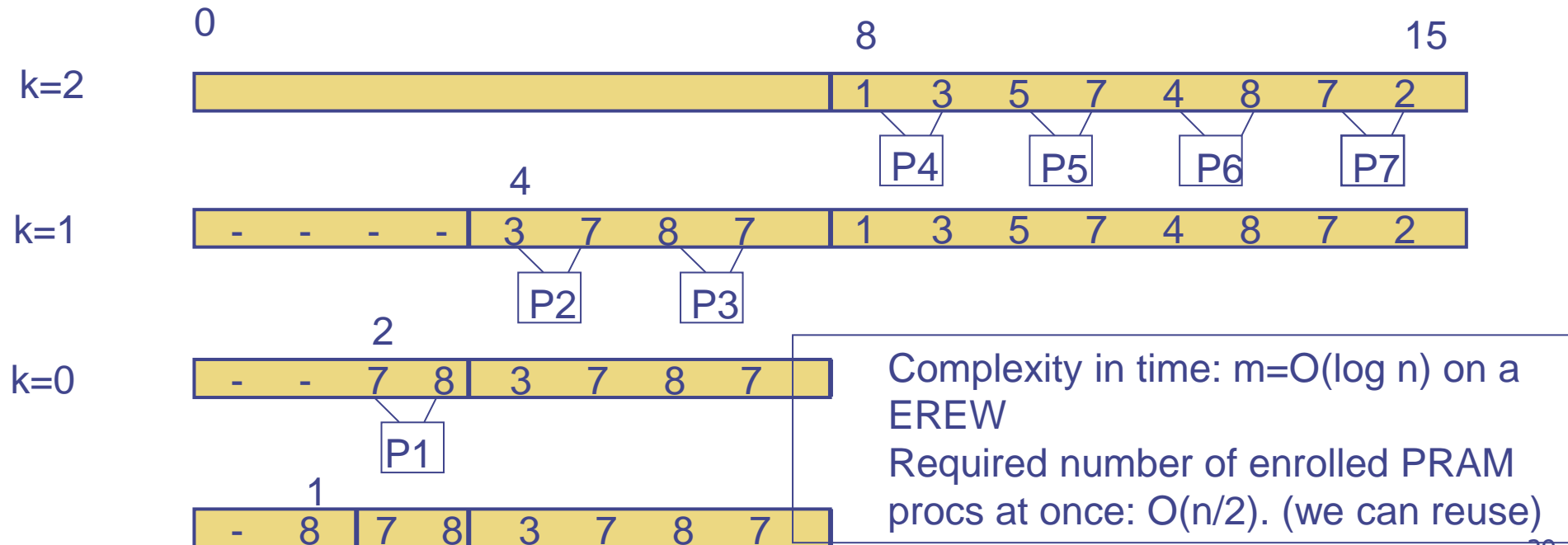
# Example: compute the maximum (v2)

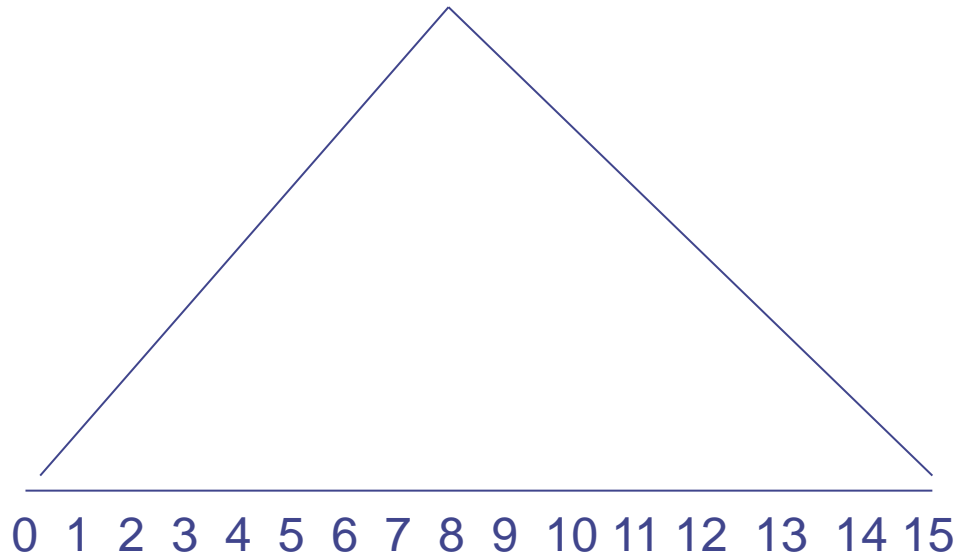
if  $n=2^m$ , if array  $A$  has size  $2n$ , and if one aims to compute the maximum of the  $n$  values of  $A$  stored at position  $A[n], A[n+1], \dots, A[2n-1]$ , the algorithm outputs the result in  $A[1]$  as follows:

Pour ( $k=m-1; k \geq 0; k--$ )

Pour chaque  $j$  from  $2^k$  to  $2^{(k+1)}-1$  en parallele

$A[j] = \max(A[2j], A[2j+1]);$





$$\text{WORK} = \text{//Time} * \text{Procs} = O(\log n) * O(n)$$

procs

$O(n)$

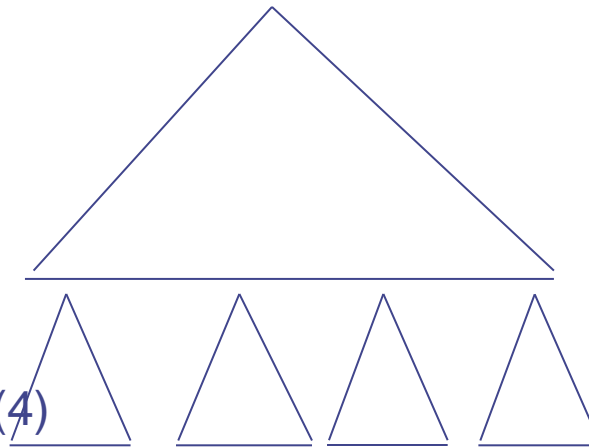
$2^4$

$$\log(2^4) = 4$$

$$O(\log(n))$$

Time //

Time for treatment of subtree in sequential =  $O(4)$



Time //  $O(4)$

Time //

$$\log(2^4 / 4) = 2$$

$$\log(n / \log n)$$

procs

$n / \log n$

$$2^4 / 4 = 4$$

$$\text{WORK} = O(\log(n)) * O(n / \log n) = O(n)$$

So, it is optimal compared to the seq. work =  $O(n)$

# Generalisation

- Traversal of a complete binary tree of logarithmic depth
  - From bottom (leaves) to the top (root), and sub problems results are merged in pair
    - Still, it is not a pure 'divide and conquer' approach
    - As the sub problems standing at the leaves level exist naturally
    - As new sub problems pop up when the algorithm goes one level up
- How to get an **optimal PRAM WORK complexity** ?
  - Reduce the needed total number of processors (resources)...
  - ...while not increasing the time complexity
  - By reducing the problem size to be solved by the PRAM algo
    - Instead of  $n$  procs  $\Rightarrow n/\log n$  procs in order to solve problem of size  $n/\log n$
    - Instead of tree traversal  $= \log n$  //time  $\Rightarrow \log (n/\log n) \Rightarrow O(\log n)$  //time
  - Add to it the //time needed by all processes to solve each « big » initial sub-problem using the seq. approach:
    - $O(\log n)$  //time using the  $n/\log n$  procs. , each proc handle  $\log n$  input data
    - No interaction needed during sub problems solving  $\Rightarrow$  Embarrassingly //

Exo: write  
PRAM algo