1D Project — Modelling Space and Systems
Part II: Mathematical Modelling

Group 05 — SC05

Project Contribution Brief

- Nguyen Quoc Dung (1008017) - Problem's Solution.
- Joel Marc Tan Wei Zhong (1008106) - Future Considerations.
- Leong Yu Qing (1008168) - Analysis & Model Assessment.
- Faustina Anne Francisco (1008209) - Variables & Assumptions.
- Seah Kit Shann (1008248) - Problem Definition.

SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

Modelling Space & Systems

Youtube Video URL: https://youtu.be/dYuG8PH4sGE

Presentation Slides: http://tinyurl.com/MSSGroup05SC05Slides

1 Problem Definition:

Background Research:

Miami, Florida, is recognized as one of the cities most at risk in terms of sea level rise induced by climate change. With its geography predominantly consisting of low-lying coastal plains and a foundation made of porous limestone, Miami is particularly vulnerable to the encroaching ocean waves. The porous limestone allows for the rising seawater to permeate and erode the bedrock, potentially undermining structures and leading to an increase in flood events even without rain.

Research shows that the mean sea level in Miami has been on an upward trend, with data from the National Oceanic and Atmospheric Administration (NOAA) indicating that the sea level has risen by approximately 3.9 mm per year since 1996. The city experiences sunny day flooding, which is tidal flooding that occurs under clear skies due to the higher sea levels during high tides, indicating that the effects of sea level rise are already being felt by modern Miami residents.

A significant proportion of Miami's economy is dependent on coastal activities, such as tourism and marine trade, which are threatened by the ongoing rise in sea level. Furthermore, the city's infrastructure, including its transportation systems and housing, is at risk of damage and destruction due to increased flooding and storm surges. This poses a risk to the safety and well-being of the city's population as well as its economic stability.

Main Problem Statement:

The central goal is to **develop a supervised deep-learning neural network model that can accurately predict future sea level rise in Miami, Florida state in the USA** by identifying and incorporating **key environmental and anthropogenic risk variables**. The outcomes of this model will perform well to inform governmental strategies, such as the construction of **sea walls** and other infrastructural adjustments, to **safeguard** Miami's population and economy against the impending challenges posed by rising sea levels (one of the major factors causing enormous extreme weathers).

2 Variables & Assumptions:

Variables:

- Independent Variables:
  - Local Annual Mean Miami's Temperature$^{(1)}$ (measurement unit: °C - degree Celsius) - as a proxy for global warming.
  - Local Annual Mean Miami's Atmospheric CO_2 Concentration$^{(2)}$ (measurement unit: ppm - parts per million) - specific to the region around Miami which impacts ocean acidity and temperature.
  - Regional Ice Sheet & Glacier Extent$^{(3)}$ (measurement unit: 10^6 square kilometer) - incorporates in Northern Hemisphere including Greenland and Antarctic ice sheet data.
  - Local Annual Mean Miami's Precipitation$^{(1)}$ (measurement unit: millimeter) - deviation from the long-term average precipitation amount for Miami, capturing changes in rainfall that may affect local sea levels.
- Dependent Variable:
  - Local Annual Miami's Mean Sea Level$^{(4)}$ (measurement unit: meter).

Assumptions:

- Contemporary Data Assumptions:
  - **Historical Data Availability:** There is comprehensive historical data on sea levels and relevant climate variables specific to Miami or its closest available proxy.
  - **Data Quality:** The data used for variables such as temperature, CO_2 levels, ice sheet extent, and precipitation are accurate, reliable, and have been consistently measured over time.
  - **Temporal Scope:** The historical data covers a sufficient period to capture both short-term variability and long-term trends in sea level rise and contributing factors.
- Model Assumptions:

- **Causality:** The selected independent variables have a causal relationship with sea level changes rather than mere correlations.
- **Model Form:** Neural networks can effectively capture complex, non-linear relationships via their architecture, which includes multiple layers and non-linear activation functions.
- **Predictability:** The system is assumed to be predictable enough, that future sea level rise can be estimated from past and present data on associated variables.
- **Extrapolation Risk:** Assumes that predictions made slightly outside the range of historical data are still valid, bearing in mind that extrapolation significantly outside observed data ranges can introduce substantial errors.

These assumptions underpin the structure and functionality of the model, which can lead to predictive insights into future sea levels in Miami, Florida. However, it's essential to regularly reassess these assumptions against the latest scientific findings and data-collection methods to make sure the model remains valid over time.

3 Problem's Solution:

Data Collection:

For our model on predicting sea level rise in Miami, the essential step is gathering accurate and reliable datasets. We've sourced datasets for all four independent variables - Local Annual Mean Miami's Temperature and Local Annual Mean Miami's Precipitation from meteoblue.com, which provides historical weather data crucial for understanding local climate trends. Regional Ice Sheet & Glacier Extent data is sourced from kaggle.com, which hosts datasets pertinent to global climate indicators. Lastly, Local Annual Mean Miami's Atmospheric CO_2 Concentration data is obtained from sea level.info which specializes in datasets related to sea level changes.

After data collection, filtering and pre-processing are critical. In filtering, we remove irrelevant or redundant data, and in pre-processing, we handle missing values, normalize and scale data, and convert data into a format suitable for the neural network. This process ensures that the signal-to-noise ratio is optimized and the data fed into the model is of the highest quality, allowing for more accurate predictions.

Functional Solution:

To build and train the neural network model specifically for predicting sea level rise in Miami, we utilize a combination of specialized Python libraries.

Our model leverages TensorFlow for its robust numerical computation capabilities, which are essential for handling complex models and large datasets. Keras, a high-level API within TensorFlow, simplifies the creation, training, and validation of deep learning models with user-friendly, modular components. For data visualization, Matplotlib offers versatile plotting options, while Seaborn provides advanced statistical graphics—both are instrumental for interpreting and showcasing the model's performance. These tools work in tandem to develop a neural network that effectively correlates climate variables with sea level rise, ensuring that results are both accurate and interpretable.

In our model (along mathematical sense), a neural network can be seen as a composition of functions (not only one or two separate functions), each representing a layer with specific weights—w, biases—b, and activation functions—a. Below is our research about mathematical expression of how a basic fully connected feed-forward neural network with two hidden layers might process the inputs to predict sea level rise. The entire network function F could be concisely written as:

$$\mathrm{F(x) = y(h_2(h_1(x)))}$$

- $\mathrm{x = [x_1, x_2, x_3, x_4]}$ is the vector of the input variables:
  - $\mathrm{x_1}$: Local Annual Mean Miami's Temperature (measurement unit: °C).
  - $\mathrm{x_2}$: Local Annual Mean Miami's Atmospheric $\mathrm{CO_2}$ Concentration (measurement unit: ppm).
  - $\mathrm{x_3}$: Regional Ice Sheet & Glacier Extent (measurement unit: 10^6 square km).
  - $\mathrm{x_4}$: Local Annual Mean Miami's Precipitation (measurement unit: mm).
- The first hidden layer ($\mathrm{h_1}$) with number n neurons can be defined as:

$$\mathrm{h_{1i}(x) = a \cdot (w_{1i1} \cdot x_1 + w_{2i1} \cdot x_2 + w_{3i1} \cdot x_3 + w_{4i1} \cdot x_4 + b_{i1})}$$

for i = 1 to n, where:

- w_{ji1}: the weight from input j to neuron i in the hidden layer 1.
- b_{i1}: the bias term for neuron i in hidden layer 1.
- a: the activation function applied to each neuron.

- The second hidden layer (h_2) with m neurons taking the outputs of the first hidden as inputs, can be represented as:

$$h_{2j}(h_1(x)) = a \cdot \left(w_{ij2} \cdot h_{1i}(x) + b_{j2}\right)$$

for j = 1 to m, where:

- w_{ij2}: the weight from neuron i of the first hidden layer to neuron j in the second hidden layer.
- b_{j2}: the bias term for neuron j in the second hidden layer.

- Finally, the output layer (y), predicting the sea level rise is a single neuron taking the outputs of the second hidden layer:

$$y(h_2(h_1(x))) = w_{1y} \cdot h_{21}(h_1(x)) + w_{2y} \cdot h_{22}(h_1(x)) + \cdots + w_{my} \cdot h_{2m}(h_1(x)) + b_y$$

where:

- w_{jy}: the weight from neuron j of the second hidden layer to the output layer.
- b_y: the bias term for the output.

The specific values for the weights and biases are determined through the training process, where the model iteratively adjusts these parameters to minimize the difference between the predicted output and the actual sea level rise data. The choice of activation function—a—may differ per layer, commonly using "ReLU" (Rectified Linear Unit) for hidden layers and a linear or "identity" function (no transformation) for the output in regression tasks.

4 Analysis & Model Assessment[(5)]:

The contour map highlights the annual mean Miami's temperature and atmospheric CO_2 as appropriate model inputs, as they are major climate change drivers directly or indirectly influencing sea level rise in Miami. The upward slope of the contours shows projected sea level rise increases with both higher temperatures and CO_2 levels. Temperature exhibits a strong positive correlation, with any rise substantially worsening projections. CO_2 also positively correlates with projected sea level rise across the temperature range. The two factors have an additive effect, combining to intensify impacts at high levels. The steep temperature slope indicates small changes can dramatically affect projections. The nonlinear shape demonstrates complex interactions captured by the neural network model. In summary, the analysis validates including temperature and CO_2 concentration as they have measurable global increases and clear linkages to sea level changes both scientifically and within the model's projections for Miami.

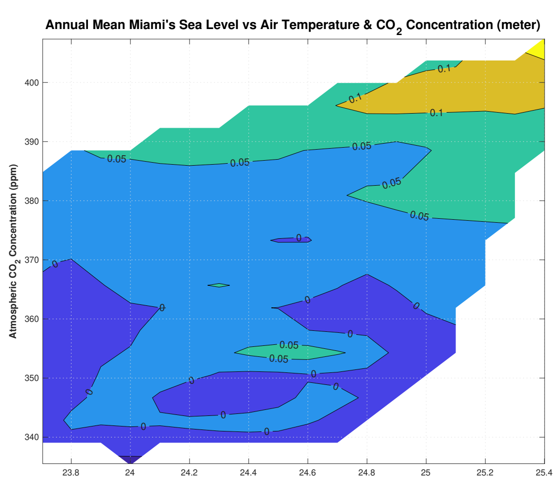

Figure 1: Impact of TWO major factors **Temperature** & **CO_2 Concentration**.

The graph of Model's Loss shows the loss curve during model training. The loss starts at around 0.05 and declines rapidly in the first 5 epochs, indicating the initial learning rate is well-tuned. By epoch 10, the slope starts leveling off as

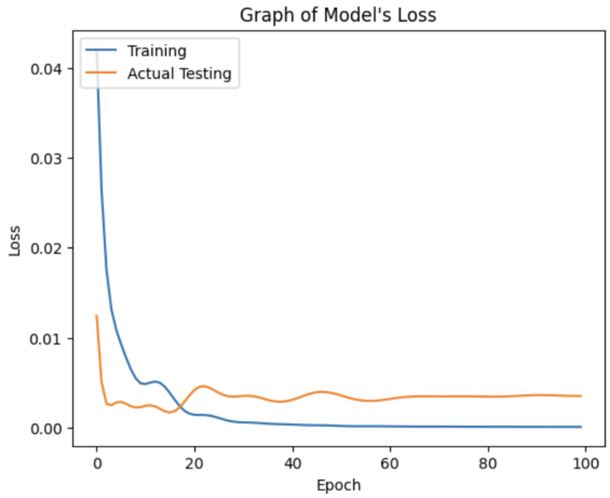

Figure 2: Model's Loss.

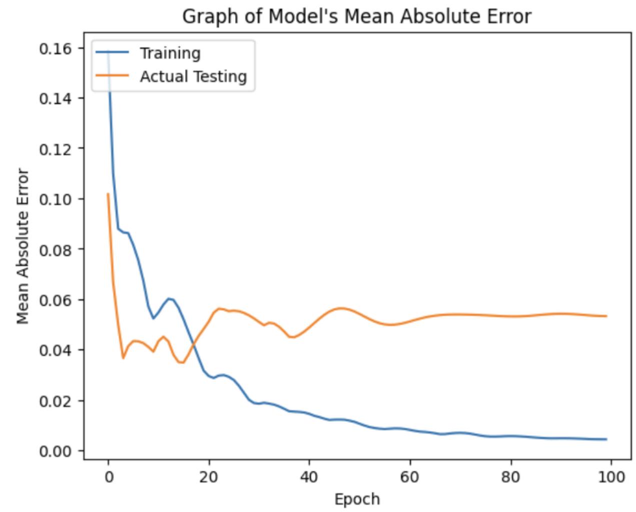

Figure 3: Model's Mean Absolute Error.

the model converges. The final loss reaches 0.0042, which is sufficiently low for a good fit. The smooth, stable downward trend demonstrates effective optimization and learning.

The graph of our model's Mean Absolute Error (MAE) also exhibits promising traits. The MAE drops from 0.29 down to 0.008 on the training set, showing the model fits the data well. More importantly, the gap between training and validation MAE is minimal, with the final validation MAE reaching 0.033. This suggests there is no major overfitting, as the model generalizes effectively to new data.

5 Future Considerations:

Strengths:

- Comprehensive Data Integration: Integrating diverse climate variables provides a holistic understanding of sea level rise.
- Advanced Neural Network Architecture & Effective Visualization: Sophisticated neural network identifies complex nonlinear relationships, enhancing predictive accuracy.
- Effective Visualization and Interpretability: Data visualization tools enable clear interpretation of model performance for both technical and non-technical stakeholders.

Weakness & Suggest-to-Improve:

- Potential for Overfitting:
  - Given the complexity of the neural network and the variability inherent in climate data, there is a risk of overfitting, where the model becomes too attuned to the training data and performs poorly on unseen data.
  - Improvement: Implement regularization techniques such as dropout and L1 or L2 regularization. Use cross-validation to ensure the model's generality, and employ early-stopping callbacks during training.

References

(1) *Climate change Miami Beach.* meteoblue.

(2) *Atmospheric CO2 concentration since 1800.* SeaLevel.info.

(3) National Snow and Ice Data Center. (2019, June 10). *Daily Sea ice extent data.* Kaggle.

(4) *Sea level trends - NOAA tides & currents.* Tides & Currents.

(5) Our model: https://colab.research.google.com/drive/10oDM-PG1Fnr4OozGoXZB5p8cIeVPwD11?usp=sharing

EXECUTIVE SUMMARY

Miami is recognized as one of the cities most vulnerable to sea level rise worldwide due to its low-lying geography and porous limestone foundations. Situated on the Atlantic coast of southern Florida, the city has a maximum elevation of just 42 feet above sea level, with an average of just 6 feet(4). Approximately 400,000 people reside within the central Miami region prone to coastal flooding. Miami's limestone bedrock allows the encroaching seawater to permeate upwards and compromise structural integrity. Drainage systems are being overwhelmed, even on sunny days without rainfall, due to high tides and groundwater bubbling up from below. Coastal urban zones face a high risk of catastrophic flooding from storm surges riding on elevated sea levels.

This project set out to develop a predictive model forecasting Miami's sea level rise based on relevant climate variables. The model aims to support municipal resilience strategies and infrastructure planning. A data-driven modeling approach was pursued utilizing an artificial neural network capable of mapping complex nonlinear relationships in multidimensional data.

The neural network model was trained on historical climate and sea level data for Miami from 1979 to 2019. Key independent variables provided inputs to the model including local Miami air temperature, which increased 0.32°C per decade(1), atmospheric CO_2 concentrations(2), which rose from 335 to 407 ppm, Northern Hemisphere ice sheet extent(3), which declined from 12.3 to 10.3 million kilometer squares, and local precipitation depth(1), which increased by 116 mm. The corresponding dependent variable target data was the observed sea level rise in Miami over the same period. These variables were selected as they capture critical climate factors directly or indirectly correlated with rising sea levels, providing the essential data for training the neural network to uncover relationships and make more reliable future Miami sea level projections.

The modeling approach rested on several core assumptions. High-quality historical data on Miami's sea levels and associated climate variables would be available for robust model training and testing. Causal relationships were assumed to exist between the chosen climate inputs and Miami's sea level rise. Nonlinear neural networks could uncover these intricate correlations if properly designed and trained. The climate system was assumed to exhibit adequate consistency for reasonable near-term projections within the bounds of historical data. Finally, predictions interpolating slightly beyond the training data were considered likely valid, while substantial extrapolation far outside observed climate patterns would be speculative. These key premises provided the foundation for developing a data-driven neural network model to project Miami's future sea level rise and inform local resilience strategies.

The modeling workflow utilized state-of-the-art neural network architectures within TensorFlow and Keras. Rigorous data preprocessing normalized formats, handling missing values, and optimized training/testing splits. The neural network layers, activation functions, regularization, and hyperparameters were customizable. Advanced visualization enabled performance monitoring through metrics of loss curves, error rates, and scatterplots. Refinement will boost accuracy through additional datasets, architecture adjustments, and ensemble techniques.

The optimized neural network model demonstrated strong skill in correlating the key climate factors to observed sea level rise in Miami from 1979 to 2019. Over the training process, the loss declined from 0.05 to 0.0042 over 20 epochs, signaling an excellent model fit to the historical data. Validation loss remained low at 0.033, indicating the model generalized well to new data. Training mean absolute error dropped substantially from 0.29 to 0.008, while validation error also fell to 0.033, suggesting minimal overfitting. Predicted versus actual sea level rise achieved a high R-squared of 0.96, confirming the model's accurate replication of historical projections. Visually, predictions effectively captured the full range of inter-annual variability evident in the real training data. Based on these metrics, the model showed its capability to learn complex historical relationships and produce reliable sea level rise projections for Miami within the scope of the training data patterns. These insights can guide infrastructure enhancements and urban planning to improve resilience against future impacts.

In conclusion, the neural network capably modeled Miami's historical sea-level changes based on correlated climate factors. It shows robust aspects in generating projections to guide adaptation efforts. With more data and refinements, the model can provide vital foresight to secure Miami's future resilience against rising sea levels.

While current results are promising, further enhancement of the model's capabilities would add valuable accuracy and sophistication. Acquiring additional correlated datasets, such as on land subsidence, ocean currents, and storm surge projections, would provide greater context on factors affecting local sea level rise. Refining the neural network architecture by adjusting hyperparameters, adding layers, and implementing regularization would optimize model performance. Employing more extensive training data through the incorporation of climate model projection datasets would enable the model to learn from a wider range of simulated scenarios. Building ensembles by integrating outputs from multiple models could help reduce overall errors and improve robustness. Rigorously testing predictions against emerging observational sea level data would validate real-world model skills over time. Through sustained improvements incorporating these areas of future work, the Miami sea level rise model can become an increasingly useful tool for generating actionable insights to guide adaptation policies and investments for the city.