Assignment 2

Uniform plane waves at oblique incidence in non-magnetic lossless dielectric media

ELG 3106 - Electromagnetic Engineering

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Executive Summary

This report delves into the study of reflectivity and transmissivity of a plane wave striking at an angle, encompassing both perpendicular (TE) and parallel (TM) polarizations, into a substance with a relative permittivity of ε_r =8.

The methodology involves:

- Utilization of specific equations to determine reflection and transmission for both TE and TM polarizations.
- Calculation tools that determine impedances, wave numbers, and Brewster's angle.
- Implementation of a Python code, structured based on a defined algorithm, to facilitate calculations.

Notable findings include:

- The visualization of the tabulated results in Figure 1.0 via the matplotlib Python library.
- The Brewster angle, a critical parameter in the study, was identified at approximately 70.5287°, aligning the tabulated results with theoretical predictions.
- Further validation of the results was achieved using the Module 8.3 app (F. Ulaby, 2020).

In a more detailed examination, a specific incident angle of 60° was analyzed. The parameters considered an incident wave traversing through air at a frequency of 1 GHz with H_{io} =1. Advanced calculations were performed to determine TM reflectivity and transmissivity, leading to the assessment of magnitudes of the reflected and transmitted waves. These results were also verified using the aforementioned Module 8.3 app.

In conclusion, the findings were effectively organized and exported to an Excel file using the DataFrame function from the pandas Python library. The report's execution was smooth, with a minor exception related to the Python code. Based on the efficiency experienced with Python, there is an inclination towards exploring MATLAB for future assignments.

Introduction

This report delves into the study of plane waves striking a boundary, focusing on their reflectivity and transmissivity as they interact with a substance having a relative permittivity of ε_r of 8. Both perpendicular (TE) and parallel (TM) polarizations will be considered.

An overview of the relevant theory will elucidate the equations for reflection and transmission. We'll address crucial concepts like impedance, wave number, and Brewster's angle. Using phasors, we will describe the properties of the incoming, reflected, and transmitted waves for a given incidence angle of 60°.

Utilizing Python libraries, namely numpy, pandas, and matplotlib, the report will compute and visualize these phenomena. A clear methodology will be presented, with a highlight on the Brewster's Angle, where a specific polarization sees no reflection.

The report's results will compare our data against known theories, using the Module 8.3 app by F. Ulaby (2020) as a reference point.

Concluding, we'll discuss the findings and their implications, aiming to bridge theory with practical observations. The goal is to deepen our understanding of wave interactions at a boundary layer, powered by computational techniques.

Theory

Our reflection and transmission equations for TE (perpendicular) and TM (parallel) polarizations are:

$$\Gamma_{TE} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \text{ and } \tau_{TE} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t},$$

$$\Gamma_{TM} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} \text{ and } \tau_{TM} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}.$$

The impedance of a material can be calculated by:

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

The wave number of a material can be calculated by:

$$k = \omega\sqrt{\mu\varepsilon}$$

Brewster's angle is the angle of incidence at which the transmitted wave will transmit through a dielectric medium without any reflection which can be calculated by:

$$\theta_B = arctan(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}})$$

The general phasor forms for the incident wave are:

$\tilde{E}_i = \widehat{a}_{ei} H_{i0} e^{-jk_i \cdot R}$ $\tilde{H}_i = \widehat{a}_{hi} H_{i0} e^{-jk_i \cdot R}$

The general phasor forms for the reflected wave are:

$\tilde{E}_r = \widehat{a}_{er} H_{r0} e^{-jk_r \cdot R}$ $\tilde{H}_r = \widehat{a}_{hr} H_{r0} e^{-jk_r \cdot R}$

The general phasor forms for the transmitted wave are:

$\tilde{E}_t = \widehat{a}_{et} H_{t0} e^{-jk_t \cdot R}$ $\tilde{H}_t = \widehat{a}_{ht} H_{t0} e^{-jk_t \cdot R}$

Snell's law states that the relationship between the incident angle, transmitted angle, wavenumber and indexes of refraction can be calculated by:

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{k_1}{k_2} = \frac{n_1}{n_2}$$

Where n can be calculated by:

$$n = \frac{c}{u_p} = \sqrt{\mu_r \varepsilon_r}$$

We know that the relationship between the coefficients are:

$$\tau_{TE} = \Gamma_{TE} + 1 \quad \tau_{TM} = (\Gamma_{TM} + 1)\frac{\cos\theta_i}{\cos\theta_t}$$

Where reflectivity and transmissivity can be calculated by:

$$R_{TE} = \left|\Gamma_{TE}\right|^2 \quad T_{TE} = \left|\tau_{TM}\right|^2 \frac{\eta_1 \cos\theta_t}{\eta_2 \cos\theta_i}$$

$$R_{TM} = \left|\Gamma_{TM}\right|^2 \quad T_{TM} = \left|\tau_{TM}\right|^2 \frac{\eta_1 \cos\theta_t}{\eta_2 \cos\theta_i}$$

Where reflectivity and transmissivity can be related by:

$$T_{RE} = 1 - R_{RE} \quad T_{TM} = 1 - R_{TM}$$

*The entirety of this section was created using the works of Dr. H. Schrimer. (Schriemer, 2023)

Derivation

$$n_{air} = 1$$

$$n_{dielectric} = \sqrt{\varepsilon_r} = \sqrt{8}$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_{air}}{n_{dielectric}}$$

$$\theta_t = \arcsin\left(\sin\theta_i \frac{1}{\sqrt{8}}\right)$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\ \Omega$$

$$\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r}} = \frac{120\pi}{\sqrt{8}}\ \Omega$$

$$\Gamma_{TE} = \frac{(\eta_2)\cos\theta_i - \eta_1\cos\left(\arcsin\left(\sin\theta_i\frac{1}{\sqrt{8}}\right)\right)}{(\eta_2)\cos\theta_i + \eta_1\cos\left(\arcsin\left(\sin\theta_i\frac{1}{\sqrt{8}}\right)\right)} \qquad \tau_{TE} = \frac{2(\eta_2)\cos\theta_i}{(\eta_2)\cos\theta_i + \eta_1\cos\left(\arcsin\left(\sin\theta_i\frac{1}{\sqrt{8}}\right)\right)}$$

$$\Gamma_{TM} = \frac{(\eta_2)\cos\left(\arcsin\left(\sin\theta_i\frac{1}{\sqrt{8}}\right)\right) - \eta_1\cos\theta_i}{(\eta_2)\cos\left(\arcsin\left(\sin\theta_i\frac{1}{\sqrt{8}}\right)\right) + \eta_1\cos\theta_i} \qquad \tau_{TM} = \frac{2(\eta_2)\cos\theta_i}{(\eta_2)\cos\left(\arcsin\left(\sin\theta_i\frac{1}{\sqrt{8}}\right)\right) + \eta_1\cos\theta_i}$$

Flow Chart

Start

Initialize Libraries:

- Import numpy as np
- Import pandas as pd
- Import matplotlib.pyplot as plt

Define compute_data Function:

- Input: n2_value, incident_angle_deg
- Constants:
    - $n_1 = 120 \times \pi$
- Calculate:
    - incident_angle_rad
    - transmission_angle_rad
    - transmission_angle_deg
    - Γ_TE
    - τ_TE
    - Γ_TM
    - τ_TM
    - T_TE_squared
    - R_TE
    - T_TE
    - R_TM
    - T_TM
- Return a dictionary of the calculated values.

Define plot_reflectivity_transmissivity Function:

- Input: n2_value, incident_angles
- For each incident_angle in incident_angles:
- Compute data using compute_data function
- Create a DataFrame df from the computed data list
- Plot values from df:
    - $R(TE)$
    - $T(TE)$
    - $R(TM)$
    - $T(TM)$
- Display the plot

Main Execution:

- Compute n2_value
- Define incident_angles from 0 to 90 in steps of 0.5 degrees
- Call plot_reflectivity_transmissivity function
- Create a DataFrame df from the computed data list for all incident_angles
- Extract a subset of columns from df to create condensed_data
- Save df to "computed_data.xlsx"
- Save condensed_data to "condensed_data.xlsx"

End

Tabulated Results

Incident Angle	Transmission Angle	R (TE)	T (TE)	R (TM)	T (TM)
0	0	0.228094	0.771906	0.228094	0.771906
0.5	0.176775	0.228107	0.771893	0.228082	0.771918
1	0.353538	0.228143	0.771857	0.228045	0.771955
1.5	0.530277	0.228205	0.771795	0.227984	0.772016
2	0.706981	0.228291	0.771709	0.227898	0.772102
2.5	0.883638	0.228402	0.771598	0.227787	0.772213
3	1.060236	0.228537	0.771463	0.227652	0.772348
3.5	1.236763	0.228697	0.771303	0.227492	0.772508
4	1.413208	0.228882	0.771118	0.227308	0.772692
4.5	1.589559	0.229091	0.770909	0.227098	0.772902
5	1.765804	0.229325	0.770675	0.226865	0.773135
5.5	1.94193	0.229585	0.770415	0.226606	0.773394
6	2.117928	0.229869	0.770131	0.226323	0.773677
6.5	2.293783	0.230178	0.769822	0.226014	0.773986
7	2.469486	0.230513	0.769487	0.225681	0.774319
7.5	2.645024	0.230872	0.769128	0.225323	0.774677
8	2.820385	0.231257	0.768743	0.22494	0.77506
8.5	2.995557	0.231668	0.768332	0.224532	0.775468
9	3.170529	0.232104	0.767896	0.224099	0.775901
9.5	3.345289	0.232565	0.767435	0.223641	0.776359
10	3.519825	0.233053	0.766947	0.223158	0.776842
10.5	3.694125	0.233566	0.766434	0.222649	0.777351
11	3.868178	0.234106	0.765894	0.222115	0.777885

11.5	4.041971	0.234671	0.765329	0.221556	0.778444
12	4.215494	0.235263	0.764737	0.220971	0.779029
12.5	4.388733	0.235882	0.764118	0.220361	0.779639
13	4.561678	0.236527	0.763473	0.219725	0.780275
13.5	4.734316	0.237199	0.762801	0.219063	0.780937
14	4.906635	0.237899	0.762101	0.218375	0.781625
14.5	5.078625	0.238625	0.761375	0.217662	0.782338
15	5.250272	0.239379	0.760621	0.216922	0.783078
15.5	5.421566	0.240161	0.759839	0.216157	0.783843
16	5.592494	0.240971	0.759029	0.215365	0.784635
16.5	5.763045	0.241808	0.758192	0.214547	0.785453
17	5.933206	0.242674	0.757326	0.213703	0.786297
17.5	6.102967	0.243569	0.756431	0.212832	0.787168
18	6.272314	0.244493	0.755507	0.211934	0.788066
18.5	6.441237	0.245445	0.754555	0.21101	0.78899
19	6.609723	0.246427	0.753573	0.210058	0.789942
19.5	6.777761	0.247439	0.752561	0.20908	0.79092
20	6.945339	0.24848	0.75152	0.208075	0.791925
20.5	7.112445	0.249552	0.750448	0.207042	0.792958
21	7.279067	0.250654	0.749346	0.205983	0.794017
21.5	7.445193	0.251787	0.748213	0.204895	0.795105
22	7.610812	0.252952	0.747048	0.20378	0.79622
22.5	7.775911	0.254147	0.745853	0.202638	0.797362
23	7.94048	0.255375	0.744625	0.201468	0.798532
23.5	8.104506	0.256634	0.743366	0.200269	0.799731
24	8.267977	0.257926	0.742074	0.199043	0.800957

24.5	8.430881	0.259251	0.740749	0.197788	0.802212
25	8.593207	0.260609	0.739391	0.196505	0.803495
25.5	8.754943	0.262	0.738	0.195194	0.804806
26	8.916077	0.263426	0.736574	0.193854	0.806146
26.5	9.076598	0.264886	0.735114	0.192485	0.807515
27	9.236493	0.26638	0.73362	0.191087	0.808913
27.5	9.39575	0.26791	0.73209	0.189661	0.810339
28	9.554359	0.269475	0.730525	0.188205	0.811795
28.5	9.712307	0.271077	0.728923	0.18672	0.81328
29	9.869582	0.272715	0.727285	0.185206	0.814794
29.5	10.02617	0.274389	0.725611	0.183662	0.816338
30	10.18207	0.276101	0.723899	0.182088	0.817912
30.5	10.33725	0.277851	0.722149	0.180485	0.819515
31	10.49172	0.27964	0.72036	0.178852	0.821148
31.5	10.64546	0.281467	0.718533	0.177189	0.822811
32	10.79845	0.283333	0.716667	0.175496	0.824504
32.5	10.95069	0.28524	0.71476	0.173773	0.826227
33	11.10216	0.287187	0.712813	0.17202	0.82798
33.5	11.25285	0.289174	0.710826	0.170236	0.829764
34	11.40276	0.291204	0.708796	0.168422	0.831578
34.5	11.55186	0.293275	0.706725	0.166578	0.833422
35	11.70015	0.295389	0.704611	0.164703	0.835297
35.5	11.84762	0.297546	0.702454	0.162797	0.837203
36	11.99425	0.299746	0.700254	0.160861	0.839139
36.5	12.14003	0.301992	0.698008	0.158894	0.841106
37	12.28495	0.304282	0.695718	0.156897	0.843103

37.5	12.429	0.306618	0.693382	0.154869	0.845131
38	12.57217	0.309	0.691	0.15281	0.84719
38.5	12.71445	0.311429	0.688571	0.150721	0.849279
39	12.85582	0.313906	0.686094	0.148601	0.851399
39.5	12.99627	0.316431	0.683569	0.146451	0.853549
40	13.13579	0.319004	0.680996	0.14427	0.85573
40.5	13.27438	0.321628	0.678372	0.142059	0.857941
41	13.41202	0.324302	0.675698	0.139818	0.860182
41.5	13.54869	0.327027	0.672973	0.137546	0.862454
42	13.68439	0.329804	0.670196	0.135245	0.864755
42.5	13.8191	0.332633	0.667367	0.132914	0.867086
43	13.95282	0.335516	0.664484	0.130554	0.869446
43.5	14.08553	0.338453	0.661547	0.128164	0.871836
44	14.21723	0.341445	0.658555	0.125746	0.874254
44.5	14.34789	0.344493	0.655507	0.123299	0.876701
45	14.47751	0.347597	0.652403	0.120824	0.879176
45.5	14.60608	0.350759	0.649241	0.118321	0.881679
46	14.73359	0.353979	0.646021	0.115791	0.884209
46.5	14.86003	0.357258	0.642742	0.113234	0.886766
47	14.98538	0.360597	0.639403	0.110651	0.889349
47.5	15.10964	0.363997	0.636003	0.108042	0.891958
48	15.23279	0.367458	0.632542	0.105409	0.894591
48.5	15.35482	0.370983	0.629017	0.102751	0.897249
49	15.47573	0.374571	0.625429	0.10007	0.89993
49.5	15.5955	0.378223	0.621777	0.097367	0.902633
50	15.71412	0.381941	0.618059	0.094642	0.905358

50.5	15.83158	0.385726	0.614274	0.091897	0.908103
51	15.94787	0.389578	0.610422	0.089132	0.910868
51.5	16.06298	0.393498	0.606502	0.08635	0.91365
52	16.1769	0.397488	0.602512	0.083551	0.916449
52.5	16.28962	0.401548	0.598452	0.080737	0.919263
53	16.40112	0.40568	0.59432	0.077909	0.922091
53.5	16.51141	0.409885	0.590115	0.075069	0.924931
54	16.62047	0.414163	0.585837	0.07222	0.92778
54.5	16.72828	0.418516	0.581484	0.069362	0.930638
55	16.83485	0.422945	0.577055	0.066499	0.933501
55.5	16.94015	0.427451	0.572549	0.063632	0.936368
56	17.04418	0.432035	0.567965	0.060764	0.939236
56.5	17.14693	0.436698	0.563302	0.057898	0.942102
57	17.2484	0.441442	0.558558	0.055037	0.944963
57.5	17.34856	0.446267	0.553733	0.052184	0.947816
58	17.44741	0.451175	0.548825	0.049341	0.950659
58.5	17.54494	0.456167	0.543833	0.046514	0.953486
59	17.64115	0.461244	0.538756	0.043706	0.956294
59.5	17.73602	0.466407	0.533593	0.04092	0.95908
60	17.82954	0.471658	0.528342	0.038163	0.961837
60.5	17.92171	0.476998	0.523002	0.035438	0.964562
61	18.01252	0.482429	0.517571	0.032751	0.967249
61.5	18.10195	0.48795	0.51205	0.030107	0.969893
62	18.19	0.493565	0.506435	0.027514	0.972486
62.5	18.27666	0.499273	0.500727	0.024976	0.975024
63	18.36192	0.505077	0.494923	0.022503	0.977497

63.5	18.44578	0.510977	0.489023	0.0201	0.9799
64	18.52822	0.516976	0.483024	0.017777	0.982223
64.5	18.60924	0.523074	0.476926	0.015542	0.984458
65	18.68882	0.529272	0.470728	0.013405	0.986595
65.5	18.76697	0.535573	0.464427	0.011376	0.988624
66	18.84367	0.541977	0.458023	0.009467	0.990533
66.5	18.91891	0.548486	0.451514	0.00769	0.99231
67	18.9927	0.555102	0.444898	0.006057	0.993943
67.5	19.06501	0.561825	0.438175	0.004582	0.995418
68	19.13584	0.568658	0.431342	0.003281	0.996719
68.5	19.2052	0.575601	0.424399	0.00217	0.99783
69	19.27306	0.582656	0.417344	0.001266	0.998734
69.5	19.33942	0.589825	0.410175	0.00059	0.99941
70	19.40428	0.597109	0.402891	0.00016	0.99984
70.5	19.46762	0.604509	0.395491	4.88E-07	1
71	19.52945	0.612027	0.387973	0.000135	0.999865
71.5	19.58975	0.619665	0.380335	0.000589	0.999411
72	19.64852	0.627423	0.372577	0.001392	0.998608
72.5	19.70576	0.635304	0.364696	0.002574	0.997426
73	19.76145	0.643309	0.356691	0.004169	0.995831
73.5	19.81559	0.65144	0.34856	0.006213	0.993787
74	19.86818	0.659697	0.340303	0.008745	0.991255
74.5	19.91921	0.668083	0.331917	0.011809	0.988191
75	19.96868	0.676599	0.323401	0.015452	0.984548
75.5	20.01657	0.685247	0.314753	0.019723	0.980277
76	20.06289	0.694027	0.305973	0.024679	0.975321

Table 1.0 - Reflectivity and Transmitivity for TE and TM Polarization

76.5	20.10763	0.702943	0.297057	0.030381	0.969619
77	20.15078	0.711994	0.288006	0.036894	0.963106
77.5	20.19234	0.721183	0.278817	0.044291	0.955709
78	20.23231	0.730511	0.269489	0.052651	0.947349
78.5	20.27068	0.73998	0.26002	0.062061	0.937939
79	20.30745	0.749592	0.250408	0.072615	0.927385
79.5	20.34261	0.759347	0.240653	0.08442	0.91558
80	20.37617	0.769247	0.230753	0.097588	0.902412
80.5	20.4081	0.779294	0.220706	0.112248	0.887752
81	20.43843	0.789489	0.210511	0.128538	0.871462
81.5	20.46713	0.799834	0.200166	0.146611	0.853389
82	20.4942	0.810331	0.189669	0.166638	0.833362
82.5	20.51966	0.82098	0.17902	0.188808	0.811192
83	20.54348	0.831783	0.168217	0.213327	0.786673
83.5	20.56567	0.842742	0.157258	0.240427	0.759573
84	20.58622	0.853857	0.146143	0.270366	0.729634
84.5	20.60514	0.865132	0.134868	0.303429	0.696571
85	20.62243	0.876566	0.123434	0.339937	0.660063
85.5	20.63807	0.888162	0.111838	0.380245	0.619755
86	20.65207	0.89992	0.10008	0.424754	0.575246
86.5	20.66442	0.911842	0.088158	0.473912	0.526088
87	20.67514	0.92393	0.07607	0.528224	0.471776
87.5	20.6842	0.936184	0.063816	0.588258	0.411742
88	20.69162	0.948606	0.051394	0.654656	0.345344
88.5	20.69739	0.961198	0.038802	0.728145	0.271855
89	20.70151	0.97396	0.02604	0.80955	0.19045

Graphical Results

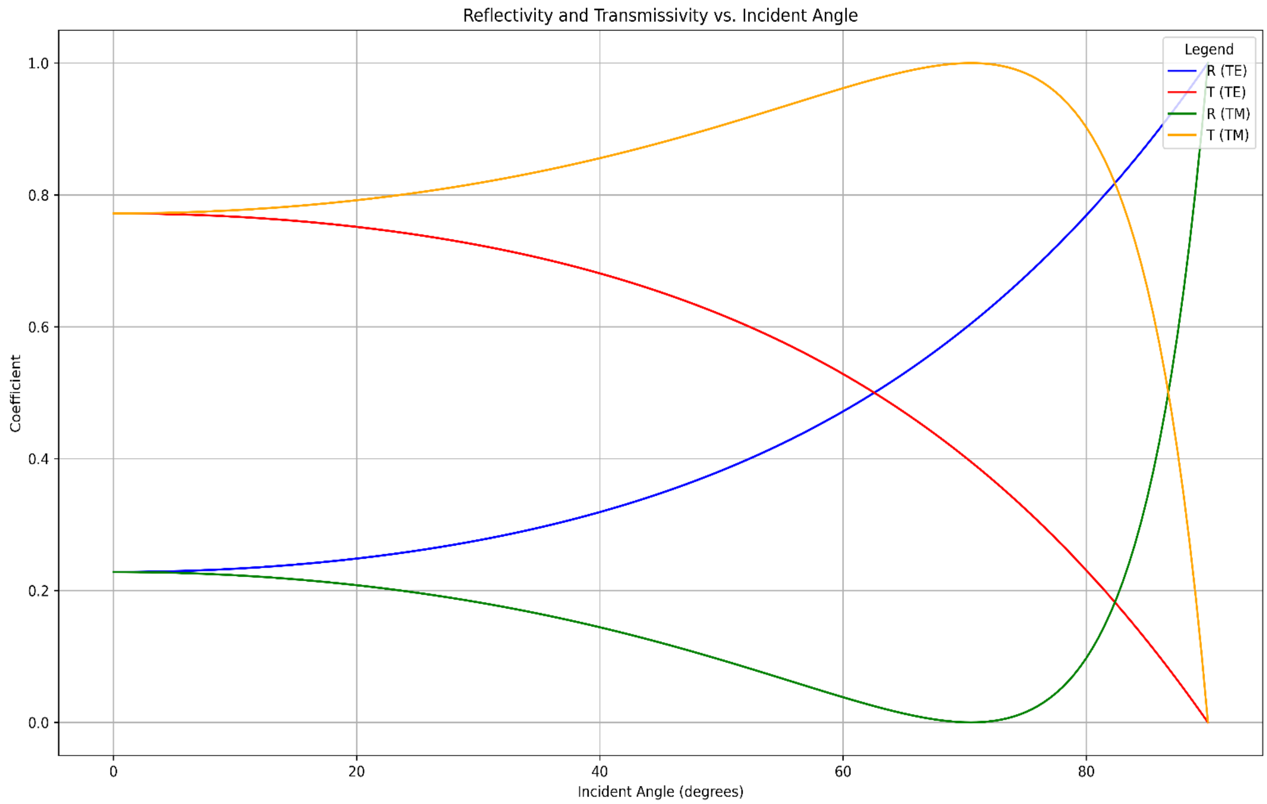

Figure 1.0 - Reflectivity and Transmissivity vs. Incident Angle

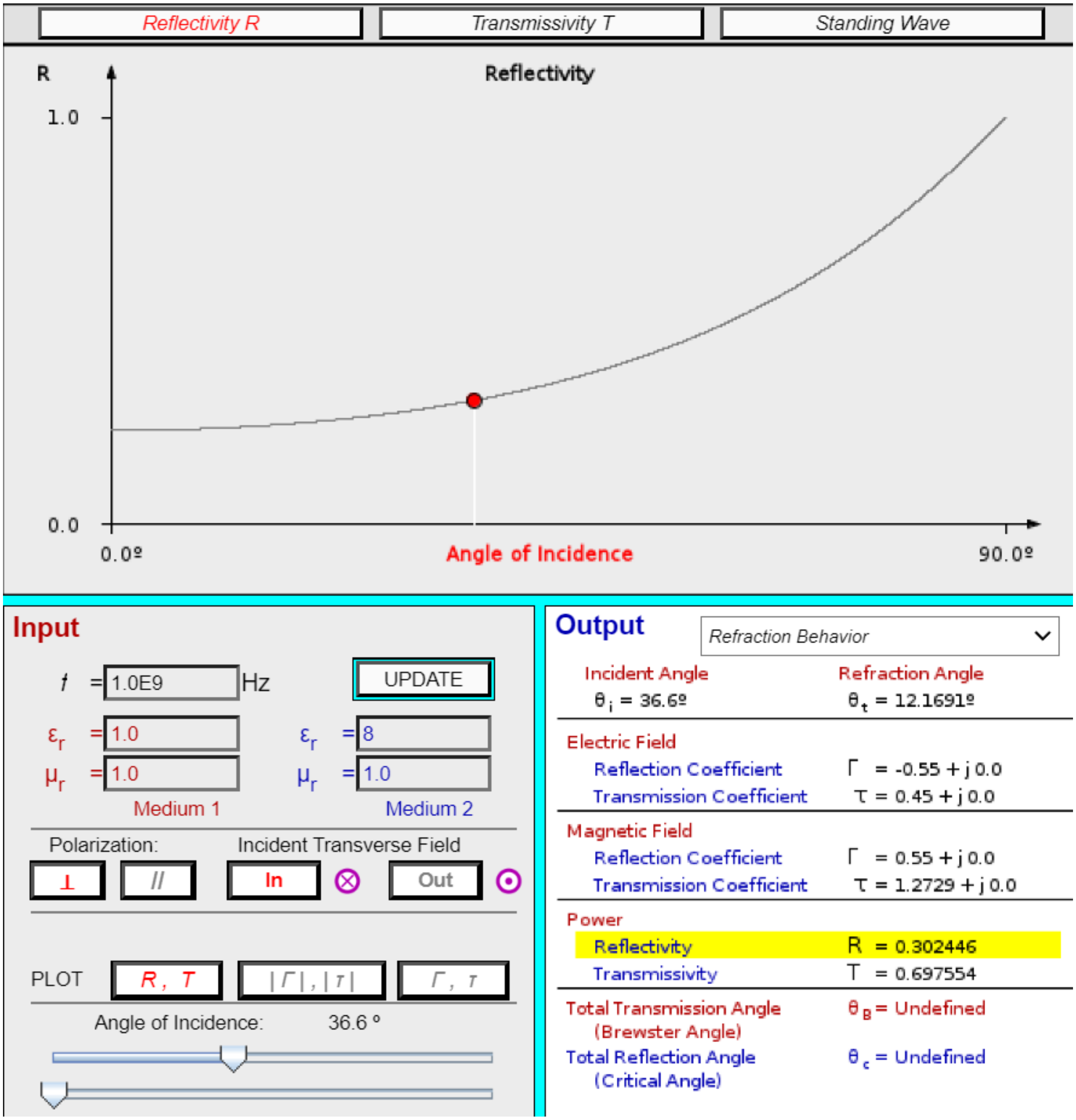

Figure 1.1 - TE Reflectivity vs. Incident Angle

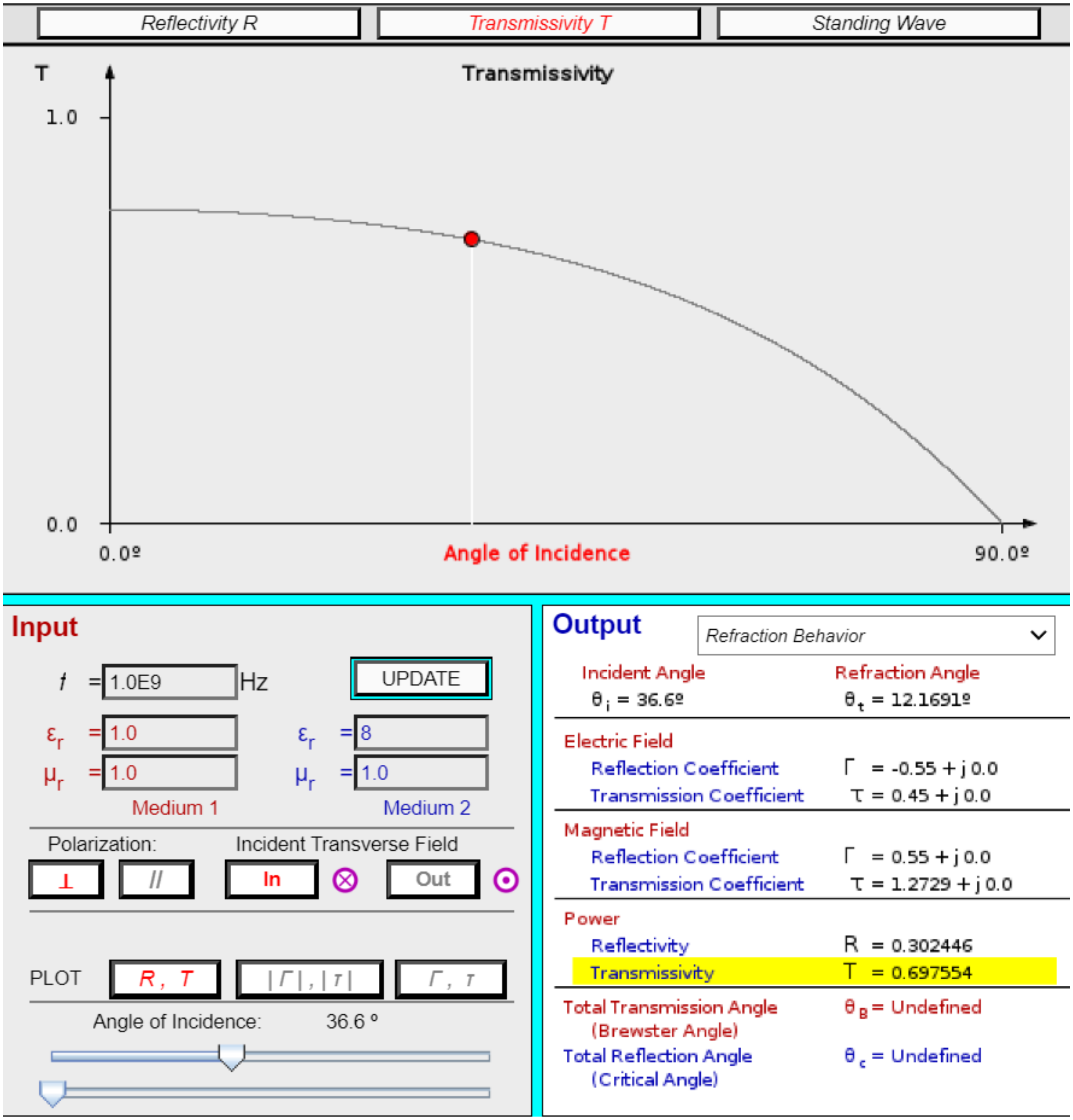

Figure 1.2 - TE Transmissivity vs. Incident Angle

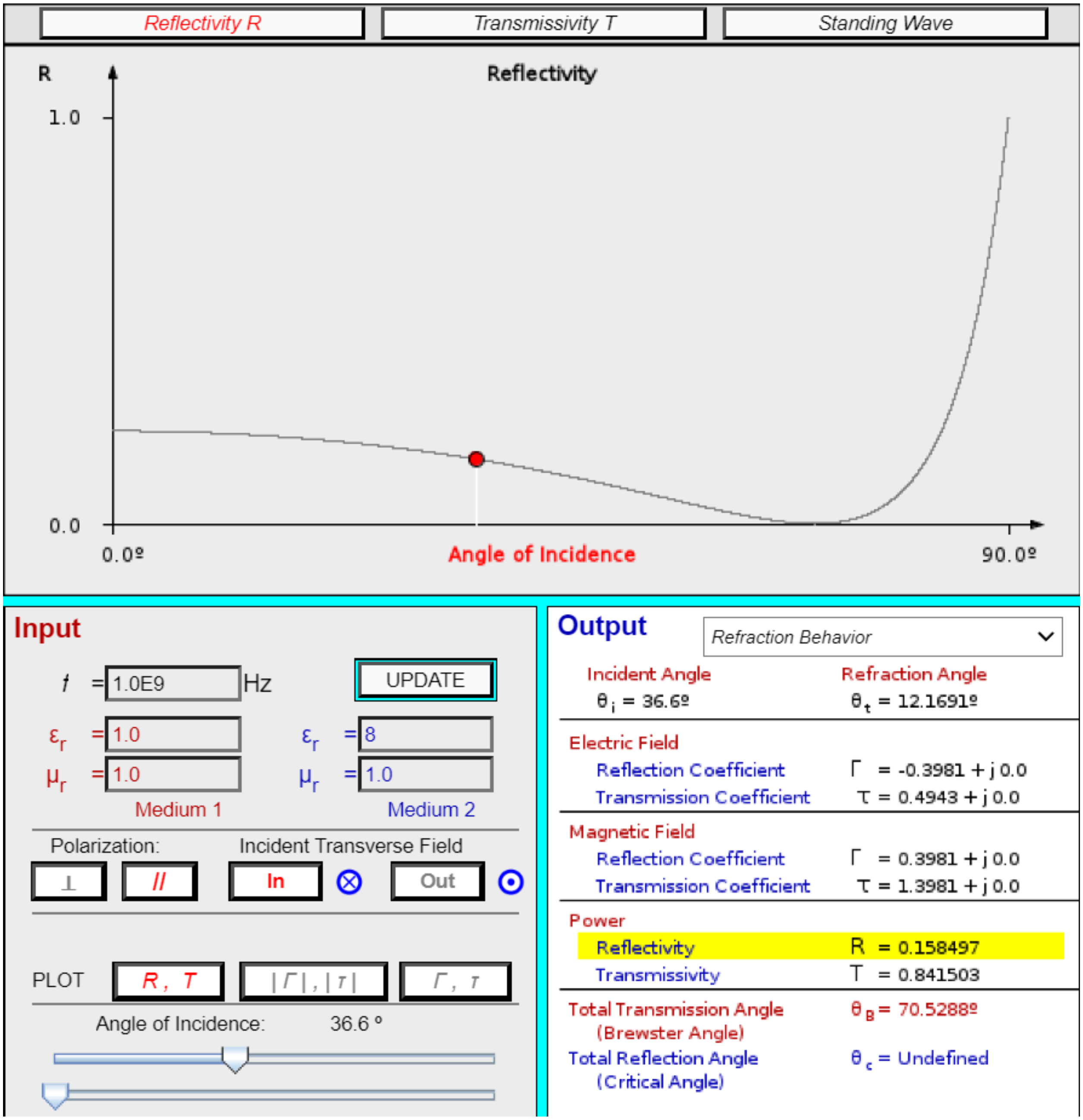

Figure 1.3 - TM Reflectivity vs. Incident Angle

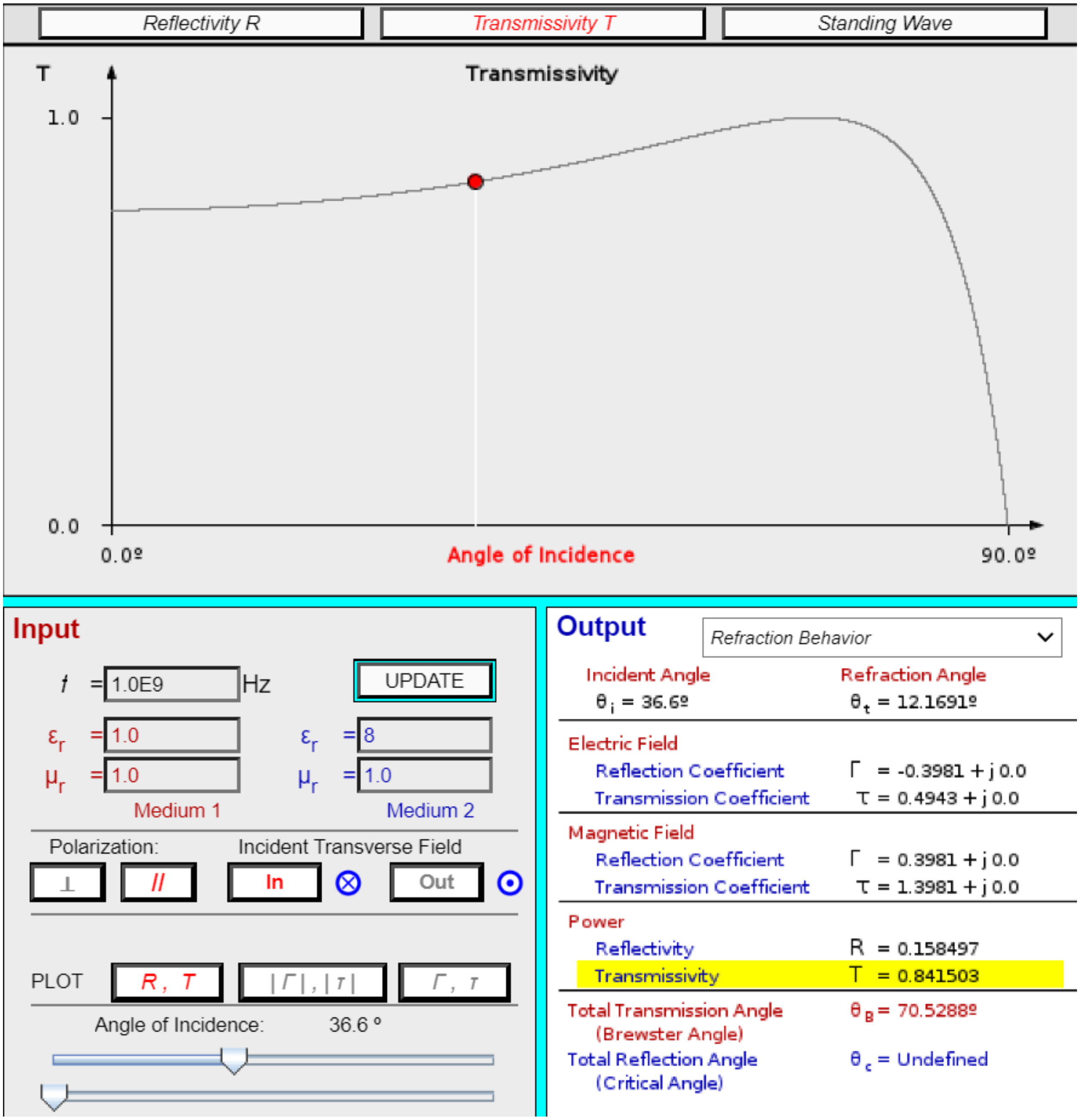

Figure 1.4 - TM Transmissivity vs. Incident Angle

*This section was created using the works of the Module 8.3 app (F. Ulaby, 2020)

Brewster's Angle

$$\theta_B = arctan(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}) = arctan(\sqrt{8}) = 1.2309\, rad = 70.5287°$$

70	19.40428	0.597109	0.402891	0.00016	0.99984
70.5	19.46762	0.604509	0.395491	4.88E-07	1
71	19.52945	0.612027	0.387973	0.000135	0.999865

Table 1.1 - Reflectivity and Transmitivity for TE and TM Polarization from 70° to 71°

As seen in table 1.1, the reflectivity of the TM wave (R(TM)) is very close to 0 and the transmissivity (T(TM)) is equal to 1 at an incident angle of 70.5°. This difference is so small that it is safe to assume that the Brewster angle in our simulation is infact at 70.52° which proves that our tabulated results align with the theory.

Numerical Solution for TM Polarization

$$\theta_i = 60° = \frac{3}{\pi} \qquad \omega = 2\pi f = 2\pi \times 1\times10^9 = 6.28\times10^9 \text{ rad/s}$$

$$\eta_1 = 120\pi\,\Omega \qquad k_1 = \omega\sqrt{\mu_0\varepsilon_0} = 20.95 \text{ rad/m}$$

$$\eta_2 = \frac{120\pi}{\sqrt{8}}\,\Omega \qquad k_2 = \omega\sqrt{\mu_0\varepsilon_r\varepsilon_0} = 59.27 \text{ rad/m}$$

$$\theta_t = \arcsin\left(\frac{k_1}{k_2}\sin\theta_i\right) = 0.311 = 17.825$$

$$\Gamma_{TM} = \frac{(\eta_2)\cos(\theta_t) - \eta_1\cos\theta_i}{(\eta_2)\cos(\theta_t) + \eta_1\cos\theta_i} \qquad \tau_{TM} = \frac{2(\eta_2)\cos\theta_i}{(\eta_2)\cos(\theta_t) + \eta_1\cos\theta_i}$$

$$\Gamma_{TM} = \frac{\left(\frac{120\pi}{\sqrt{8}}\right)\cos(17.825) - 120\pi\cos 60}{\left(\frac{120\pi}{\sqrt{8}}\right)\cos(17.825) + 120\pi\cos 60} = -0.19534 \qquad \tau_{TM} = \frac{2\left(\frac{120\pi}{\sqrt{8}}\right)\cos 60}{\left(\frac{120\pi}{\sqrt{8}}\right)\cos(17.825) + 120\pi\cos 60} = 0.42261$$

$$k_i = x k_1 \sin\theta_i - z k_1 \cos\theta_i = x 20.95 \sin(60) - z\, 59.27\cos(60) = 18.143x + 10.475z \text{ rad/m}$$

$$k_r = x k_1 \sin\theta_r - z k_1 \cos\theta_r = 18.14x - 10.48z \text{ rad/m}$$

$$k_t = x k_2 \sin\theta_t - z k_2 \cos\theta_t = 18.14x + 56.422z \text{ rad/m}$$

$$\tilde{H}_i = \hat{y} H_{i0} e^{-jk_i R} = \hat{y} e^{-j(18.14x + 10.48z)} \text{ A/m}$$

$$\tilde{H}_r = \Gamma_{TM}\tilde{H}_i = -\hat{y}\,0.1953 e^{-j(18.14x - 10.48z)}$$

$$\tilde{H}_t = \tau_{TM}\tilde{H}_i = \hat{y}\,0.4226 e^{-j(18.14x + 56.422z)}$$

$$\tilde{E}_i = -\eta_1 \hat{k}_i \times \tilde{H}_i = -120\pi\frac{(18.14\hat{x} + 10.48\hat{z})}{\sqrt{18.14^2 + 10.48^2}} \times \hat{y} e^{-j(18.14x + 10.48z)} = (188.5\hat{x} - 326.4\hat{z}) e^{-j(18.14x + 10.48z)} \text{ V/m}$$

$$\tilde{E}_r = -\eta_1 \hat{k}_r \times \tilde{H}_r = -120\pi\frac{(18.14\hat{x} - 10.48\hat{z})}{\sqrt{18.14^2 + 10.48^2}} \times -\hat{y}\,0.1953 e^{-j(18.14x - 10.48z)} = (-36.82\hat{x} + 63.75\hat{z}) e^{-j(18.14x - 10.48z)} \text{ V/m}$$

$$\tilde{E}_t = -\eta_2 \hat{k}_t \times \tilde{H}_t = -\frac{120\pi}{\sqrt{8}}\frac{(18.14\hat{x} + 56.422\hat{z})}{\sqrt{18.14^2 + 56.42^2}} \times \hat{y}\,0.4226 e^{-j(18.14x + 56.422z)} = (53.61\hat{x} - 17.25\hat{z}) e^{-j(18.14x + 56.422z)} \text{ V/m}$$

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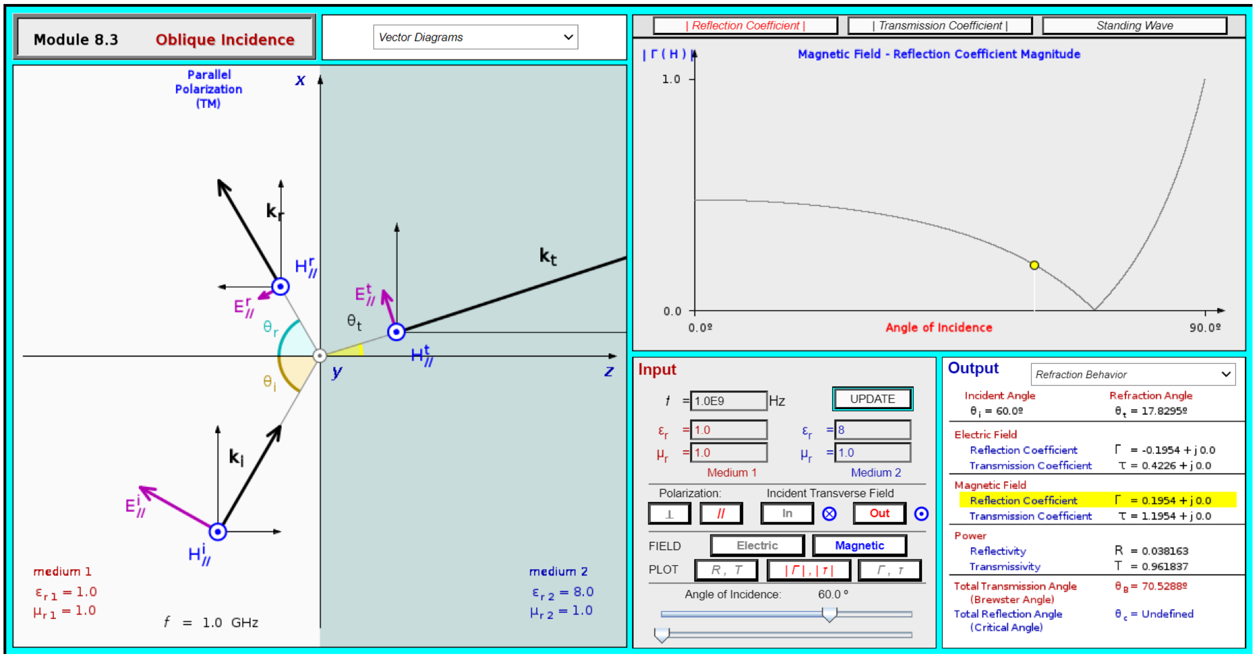

Figure 1.5 - Verification of TM Polarization with Module 8.3 App

Discussion

As seen above, this report presents results on the reflection and transmission of both perpendicular (TE) and parallel (TM) polarizations. The data is organized into Table 1.0 detailing each value of reflectivity (R) and transmissivity (T), which is visually represented in Figure 1.0 Figures 1.1-1.4 further validate these findings using the Module 8.3 app (F. Ulaby, 2020).

Additionally, we confirm the validity of our results by finding Brewster's Angle using the following formula:

$$\theta_B = arctan(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}) = arctan(\sqrt{8}) = 1.2309\, rad = 70.5287°$$

Where we conclude that the values found in Table 1.0 are what we expected to see.

Moreover, we proceed to do a detailed analytical calculation of the phasors for a TM polarized incident wave. Where we confirm this calculation using the aforementioned Module 8.3 app (F. Ulaby, 2020).

Finally, taking the results from Table 1.0 and Figure 1.0, we analyse the relationships between the reflected and transmitted waves for both TE and TM polarizations. Although the data was derived from the given formulas, it is noteworthy that these results are consistent with the relationships:

$$\tau_{TE} = \Gamma_{TE} + 1 \qquad \tau_{TM} = (\Gamma_{TM} + 1)\frac{cos\theta_i}{cos\theta_t}$$

Additionally, our graphical results (Figure 1.0) are in alignment with those generated by the Module 8.3 appilcation (F. Ulaby, 2020).

Conclusion

The derivation of the tabulated results commenced with the establishment of essential formulae to calculate each coefficient. This endeavor was streamlined using a Python code, crafted in alignment with the algorithm outlined in the Algorithm section.

These results were subsequently visualized in Figure 1.0 utilizing the matplotlib library in Python. To ensure precision, the results were cross-referenced with the Module 8.3 app (F. Ulaby, 2020).

For a comprehensive analysis, an incident angle of 60° was examined. The study parameters encompassed an incident wave traveling through air at a frequency of 1 GHz and H_{io}=1. Initial computations involved parameters like wavenumbers across different media and angular frequency, in addition to ascertaining the reflected angle. This paved the way for advanced calculations related to TM reflectivity and transmissivity, enabling the assessment of magnitudes of the reflected and transmitted waves. The wavenumbers from these waves were subsequently integrated into the phasor formula, coupled with coefficients, to determine the magnetic field phasors, which were then converted to their electric equivalents.

To encapsulate, the results were adeptly organized using the DataFrame function from the pandas library, facilitating a direct export to an Excel file. Barring the Python code, the remainder of the report was accomplished with minimal challenges. Moving forward, I intend to explore MATLAB for subsequent assignments, as my experience with Python has proven it to be more efficient than Excel.

References

- H. Schriemer, "Assignment 2," University of Ottawa, Ottawa, Ontario, Canada, Oct. 23, 2023
- F. Ulaby, "Module 8.3." University of Michigan, Dept. of EECS. [Online]. Available: https://em8e.eecs.umich.edu/jsmodules/ch8/mod8_3.html. Accessed on: Oct. 23, 2023.

Appendix
Tables

Incident Angle	Transmission Angle	R (TE)	T (TE)	R (TM)	T (TM)
0	0	0.228094	0.771906	0.228094	0.771906
0.5	0.176775	0.228107	0.771893	0.228082	0.771918
1	0.353538	0.228143	0.771857	0.228045	0.771955
1.5	0.530277	0.228205	0.771795	0.227984	0.772016
2	0.706981	0.228291	0.771709	0.227898	0.772102
2.5	0.883638	0.228402	0.771598	0.227787	0.772213
3	1.060236	0.228537	0.771463	0.227652	0.772348
3.5	1.236763	0.228697	0.771303	0.227492	0.772508
4	1.413208	0.228882	0.771118	0.227308	0.772692
4.5	1.589559	0.229091	0.770909	0.227098	0.772902
5	1.765804	0.229325	0.770675	0.226865	0.773135
5.5	1.94193	0.229585	0.770415	0.226606	0.773394
6	2.117928	0.229869	0.770131	0.226323	0.773677
6.5	2.293783	0.230178	0.769822	0.226014	0.773986
7	2.469486	0.230513	0.769487	0.225681	0.774319
7.5	2.645024	0.230872	0.769128	0.225323	0.774677
8	2.820385	0.231257	0.768743	0.22494	0.77506
8.5	2.995557	0.231668	0.768332	0.224532	0.775468
9	3.170529	0.232104	0.767896	0.224099	0.775901
9.5	3.345289	0.232565	0.767435	0.223641	0.776359
10	3.519825	0.233053	0.766947	0.223158	0.776842

10.5	3.694125	0.233566	0.766434	0.222649	0.777351
11	3.868178	0.234106	0.765894	0.222115	0.777885
11.5	4.041971	0.234671	0.765329	0.221556	0.778444
12	4.215494	0.235263	0.764737	0.220971	0.779029
12.5	4.388733	0.235882	0.764118	0.220361	0.779639
13	4.561678	0.236527	0.763473	0.219725	0.780275
13.5	4.734316	0.237199	0.762801	0.219063	0.780937
14	4.906635	0.237899	0.762101	0.218375	0.781625
14.5	5.078625	0.238625	0.761375	0.217662	0.782338
15	5.250272	0.239379	0.760621	0.216922	0.783078
15.5	5.421566	0.240161	0.759839	0.216157	0.783843
16	5.592494	0.240971	0.759029	0.215365	0.784635
16.5	5.763045	0.241808	0.758192	0.214547	0.785453
17	5.933206	0.242674	0.757326	0.213703	0.786297
17.5	6.102967	0.243569	0.756431	0.212832	0.787168
18	6.272314	0.244493	0.755507	0.211934	0.788066
18.5	6.441237	0.245445	0.754555	0.21101	0.78899
19	6.609723	0.246427	0.753573	0.210058	0.789942
19.5	6.777761	0.247439	0.752561	0.20908	0.79092
20	6.945339	0.24848	0.75152	0.208075	0.791925
20.5	7.112445	0.249552	0.750448	0.207042	0.792958
21	7.279067	0.250654	0.749346	0.205983	0.794017
21.5	7.445193	0.251787	0.748213	0.204895	0.795105
22	7.610812	0.252952	0.747048	0.20378	0.79622
22.5	7.775911	0.254147	0.745853	0.202638	0.797362
23	7.94048	0.255375	0.744625	0.201468	0.798532

23.5	8.104506	0.256634	0.743366	0.200269	0.799731
24	8.267977	0.257926	0.742074	0.199043	0.800957
24.5	8.430881	0.259251	0.740749	0.197788	0.802212
25	8.593207	0.260609	0.739391	0.196505	0.803495
25.5	8.754943	0.262	0.738	0.195194	0.804806
26	8.916077	0.263426	0.736574	0.193854	0.806146
26.5	9.076598	0.264886	0.735114	0.192485	0.807515
27	9.236493	0.26638	0.73362	0.191087	0.808913
27.5	9.39575	0.26791	0.73209	0.189661	0.810339
28	9.554359	0.269475	0.730525	0.188205	0.811795
28.5	9.712307	0.271077	0.728923	0.18672	0.81328
29	9.869582	0.272715	0.727285	0.185206	0.814794
29.5	10.02617	0.274389	0.725611	0.183662	0.816338
30	10.18207	0.276101	0.723899	0.182088	0.817912
30.5	10.33725	0.277851	0.722149	0.180485	0.819515
31	10.49172	0.27964	0.72036	0.178852	0.821148
31.5	10.64546	0.281467	0.718533	0.177189	0.822811
32	10.79845	0.283333	0.716667	0.175496	0.824504
32.5	10.95069	0.28524	0.71476	0.173773	0.826227
33	11.10216	0.287187	0.712813	0.17202	0.82798
33.5	11.25285	0.289174	0.710826	0.170236	0.829764
34	11.40276	0.291204	0.708796	0.168422	0.831578
34.5	11.55186	0.293275	0.706725	0.166578	0.833422
35	11.70015	0.295389	0.704611	0.164703	0.835297
35.5	11.84762	0.297546	0.702454	0.162797	0.837203
36	11.99425	0.299746	0.700254	0.160861	0.839139

36.5	12.14003	0.301992	0.698008	0.158894	0.841106
37	12.28495	0.304282	0.695718	0.156897	0.843103
37.5	12.429	0.306618	0.693382	0.154869	0.845131
38	12.57217	0.309	0.691	0.15281	0.84719
38.5	12.71445	0.311429	0.688571	0.150721	0.849279
39	12.85582	0.313906	0.686094	0.148601	0.851399
39.5	12.99627	0.316431	0.683569	0.146451	0.853549
40	13.13579	0.319004	0.680996	0.14427	0.85573
40.5	13.27438	0.321628	0.678372	0.142059	0.857941
41	13.41202	0.324302	0.675698	0.139818	0.860182
41.5	13.54869	0.327027	0.672973	0.137546	0.862454
42	13.68439	0.329804	0.670196	0.135245	0.864755
42.5	13.8191	0.332633	0.667367	0.132914	0.867086
43	13.95282	0.335516	0.664484	0.130554	0.869446
43.5	14.08553	0.338453	0.661547	0.128164	0.871836
44	14.21723	0.341445	0.658555	0.125746	0.874254
44.5	14.34789	0.344493	0.655507	0.123299	0.876701
45	14.47751	0.347597	0.652403	0.120824	0.879176
45.5	14.60608	0.350759	0.649241	0.118321	0.881679
46	14.73359	0.353979	0.646021	0.115791	0.884209
46.5	14.86003	0.357258	0.642742	0.113234	0.886766
47	14.98538	0.360597	0.639403	0.110651	0.889349
47.5	15.10964	0.363997	0.636003	0.108042	0.891958
48	15.23279	0.367458	0.632542	0.105409	0.894591
48.5	15.35482	0.370983	0.629017	0.102751	0.897249
49	15.47573	0.374571	0.625429	0.10007	0.89993

49.5	15.5955	0.378223	0.621777	0.097367	0.902633
50	15.71412	0.381941	0.618059	0.094642	0.905358
50.5	15.83158	0.385726	0.614274	0.091897	0.908103
51	15.94787	0.389578	0.610422	0.089132	0.910868
51.5	16.06298	0.393498	0.606502	0.08635	0.91365
52	16.1769	0.397488	0.602512	0.083551	0.916449
52.5	16.28962	0.401548	0.598452	0.080737	0.919263
53	16.40112	0.40568	0.59432	0.077909	0.922091
53.5	16.51141	0.409885	0.590115	0.075069	0.924931
54	16.62047	0.414163	0.585837	0.07222	0.92778
54.5	16.72828	0.418516	0.581484	0.069362	0.930638
55	16.83485	0.422945	0.577055	0.066499	0.933501
55.5	16.94015	0.427451	0.572549	0.063632	0.936368
56	17.04418	0.432035	0.567965	0.060764	0.939236
56.5	17.14693	0.436698	0.563302	0.057898	0.942102
57	17.2484	0.441442	0.558558	0.055037	0.944963
57.5	17.34856	0.446267	0.553733	0.052184	0.947816
58	17.44741	0.451175	0.548825	0.049341	0.950659
58.5	17.54494	0.456167	0.543833	0.046514	0.953486
59	17.64115	0.461244	0.538756	0.043706	0.956294
59.5	17.73602	0.466407	0.533593	0.04092	0.95908
60	17.82954	0.471658	0.528342	0.038163	0.961837
60.5	17.92171	0.476998	0.523002	0.035438	0.964562
61	18.01252	0.482429	0.517571	0.032751	0.967249
61.5	18.10195	0.48795	0.51205	0.030107	0.969893
62	18.19	0.493565	0.506435	0.027514	0.972486

62.5	18.27666	0.499273	0.500727	0.024976	0.975024
63	18.36192	0.505077	0.494923	0.022503	0.977497
63.5	18.44578	0.510977	0.489023	0.0201	0.9799
64	18.52822	0.516976	0.483024	0.017777	0.982223
64.5	18.60924	0.523074	0.476926	0.015542	0.984458
65	18.68882	0.529272	0.470728	0.013405	0.986595
65.5	18.76697	0.535573	0.464427	0.011376	0.988624
66	18.84367	0.541977	0.458023	0.009467	0.990533
66.5	18.91891	0.548486	0.451514	0.00769	0.99231
67	18.9927	0.555102	0.444898	0.006057	0.993943
67.5	19.06501	0.561825	0.438175	0.004582	0.995418
68	19.13584	0.568658	0.431342	0.003281	0.996719
68.5	19.2052	0.575601	0.424399	0.00217	0.99783
69	19.27306	0.582656	0.417344	0.001266	0.998734
69.5	19.33942	0.589825	0.410175	0.00059	0.99941
70	19.40428	0.597109	0.402891	0.00016	0.99984
70.5	19.46762	0.604509	0.395491	4.88E-07	1
71	19.52945	0.612027	0.387973	0.000135	0.999865
71.5	19.58975	0.619665	0.380335	0.000589	0.999411
72	19.64852	0.627423	0.372577	0.001392	0.998608
72.5	19.70576	0.635304	0.364696	0.002574	0.997426
73	19.76145	0.643309	0.356691	0.004169	0.995831
73.5	19.81559	0.65144	0.34856	0.006213	0.993787
74	19.86818	0.659697	0.340303	0.008745	0.991255
74.5	19.91921	0.668083	0.331917	0.011809	0.988191
75	19.96868	0.676599	0.323401	0.015452	0.984548

75.5	20.01657	0.685247	0.314753	0.019723	0.980277
76	20.06289	0.694027	0.305973	0.024679	0.975321
76.5	20.10763	0.702943	0.297057	0.030381	0.969619
77	20.15078	0.711994	0.288006	0.036894	0.963106
77.5	20.19234	0.721183	0.278817	0.044291	0.955709
78	20.23231	0.730511	0.269489	0.052651	0.947349
78.5	20.27068	0.73998	0.26002	0.062061	0.937939
79	20.30745	0.749592	0.250408	0.072615	0.927385
79.5	20.34261	0.759347	0.240653	0.08442	0.91558
80	20.37617	0.769247	0.230753	0.097588	0.902412
80.5	20.4081	0.779294	0.220706	0.112248	0.887752
81	20.43843	0.789489	0.210511	0.128538	0.871462
81.5	20.46713	0.799834	0.200166	0.146611	0.853389
82	20.4942	0.810331	0.189669	0.166638	0.833362
82.5	20.51966	0.82098	0.17902	0.188808	0.811192
83	20.54348	0.831783	0.168217	0.213327	0.786673
83.5	20.56567	0.842742	0.157258	0.240427	0.759573
84	20.58622	0.853857	0.146143	0.270366	0.729634
84.5	20.60514	0.865132	0.134868	0.303429	0.696571
85	20.62243	0.876566	0.123434	0.339937	0.660063
85.5	20.63807	0.888162	0.111838	0.380245	0.619755
86	20.65207	0.89992	0.10008	0.424754	0.575246
86.5	20.66442	0.911842	0.088158	0.473912	0.526088
87	20.67514	0.92393	0.07607	0.528224	0.471776
87.5	20.6842	0.936184	0.063816	0.588258	0.411742
88	20.69162	0.948606	0.051394	0.654656	0.345344

88.5	20.69739	0.961198	0.038802	0.728145	0.271855
89	20.70151	0.97396	0.02604	0.80955	0.19045
89.5	20.70399	0.986893	0.013107	0.899811	0.100189
90	20.70481	1	0	1	0

Table 1.0 - Reflectivity and Transmitivity for TE and TM Polarization

70	19.40428	0.597109	0.402891	0.00016	0.99984
70.5	19.46762	0.604509	0.395491	4.88E-07	1
71	19.52945	0.612027	0.387973	0.000135	0.999865

Table 1.1 - Reflectivity and Transmitivity for TE and TM Polarization from 70° to 71°

Figures

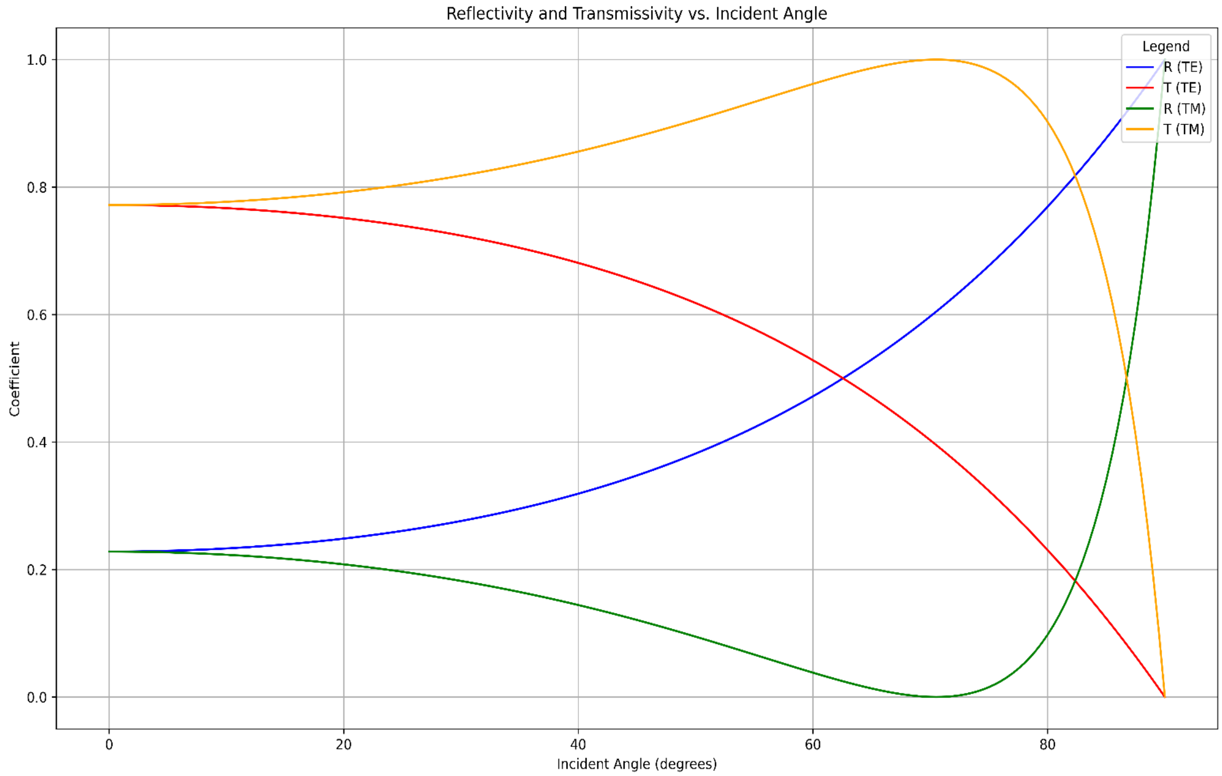

Figure 1.0 - Reflectivity and Transmissivity vs. Incident Angle

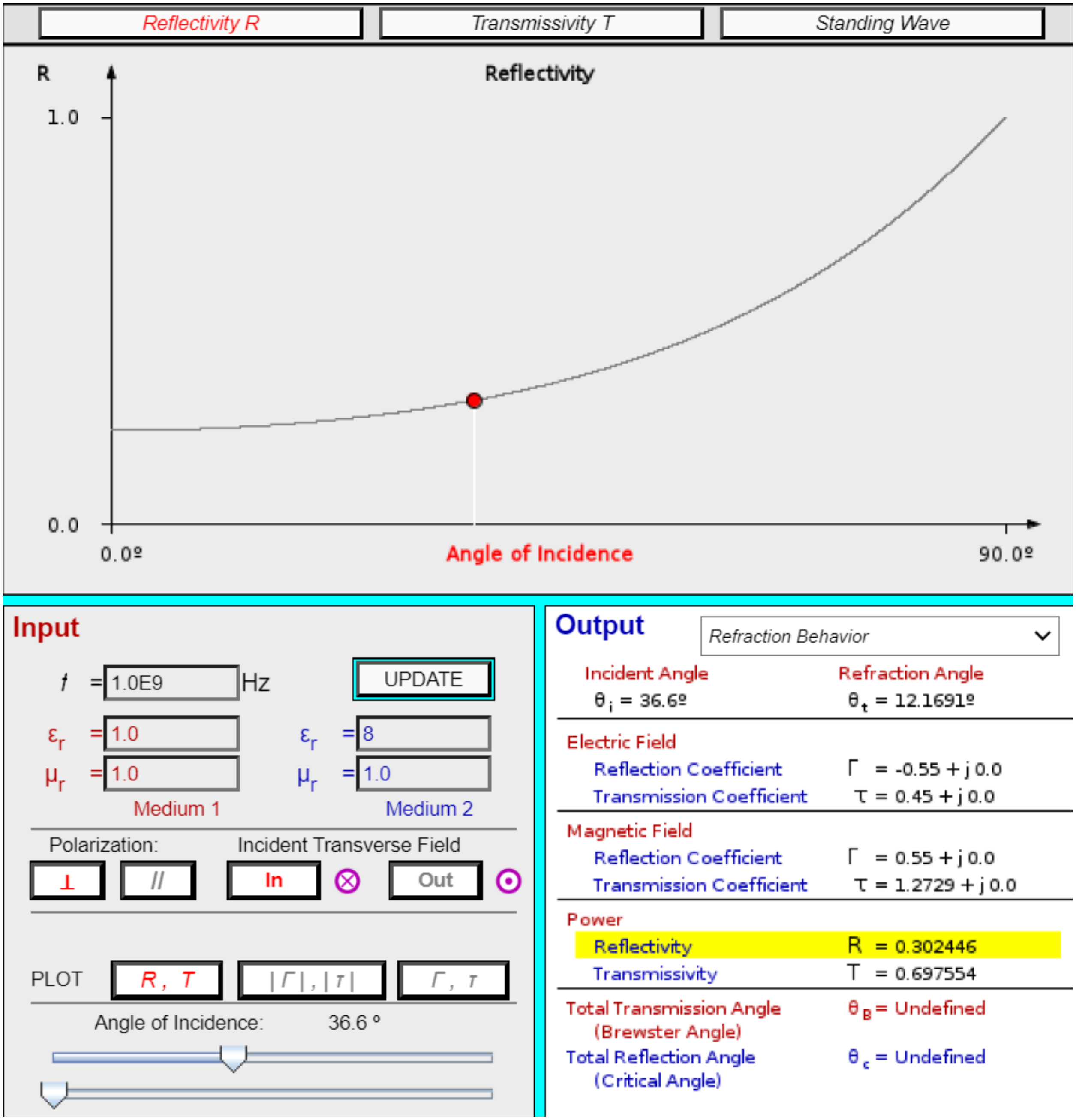

Figure 1.1 - TE Reflectivity vs. Incident Angle

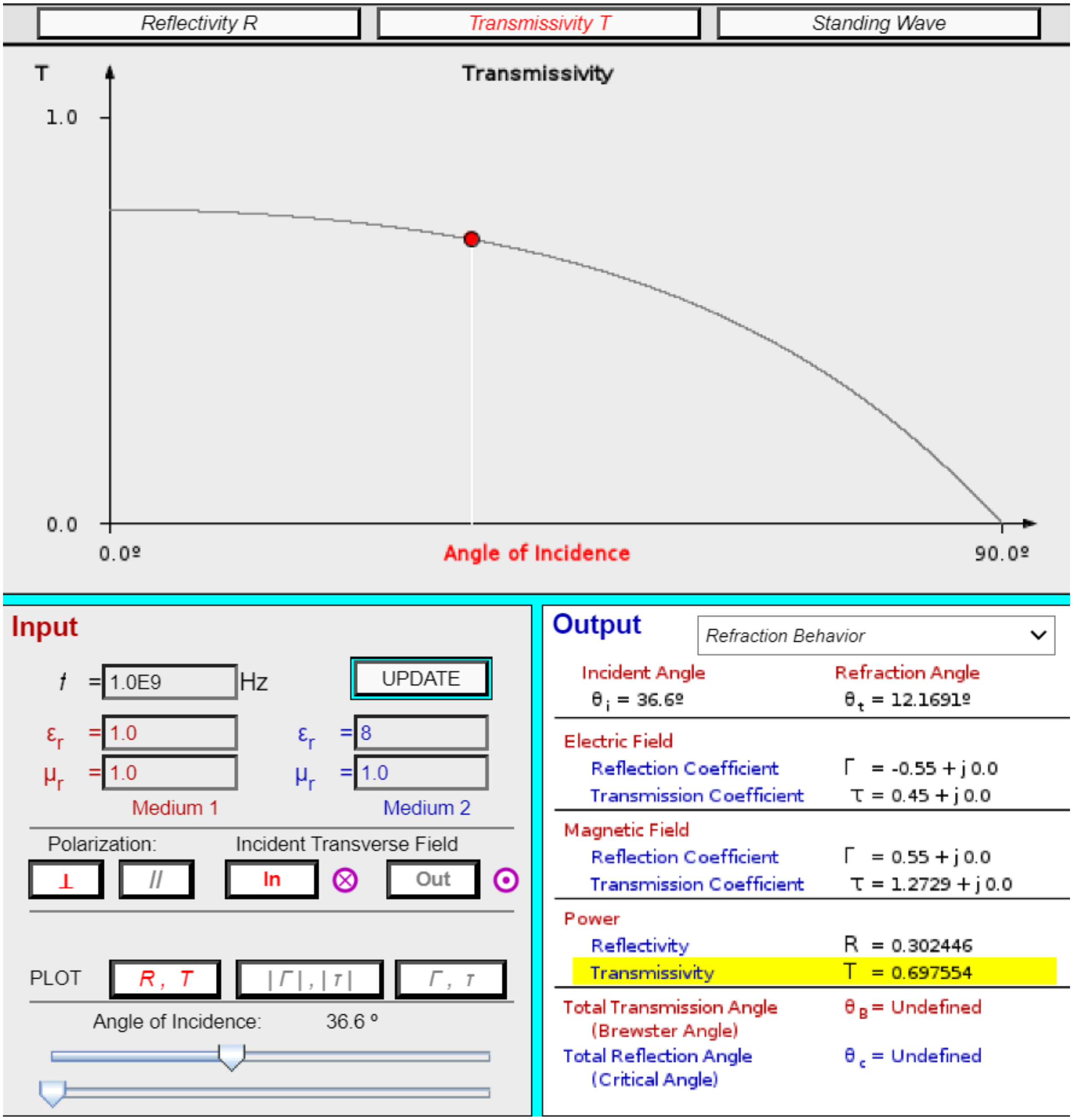

Figure 1.2 - TE Transmissivity vs. Incident Angle

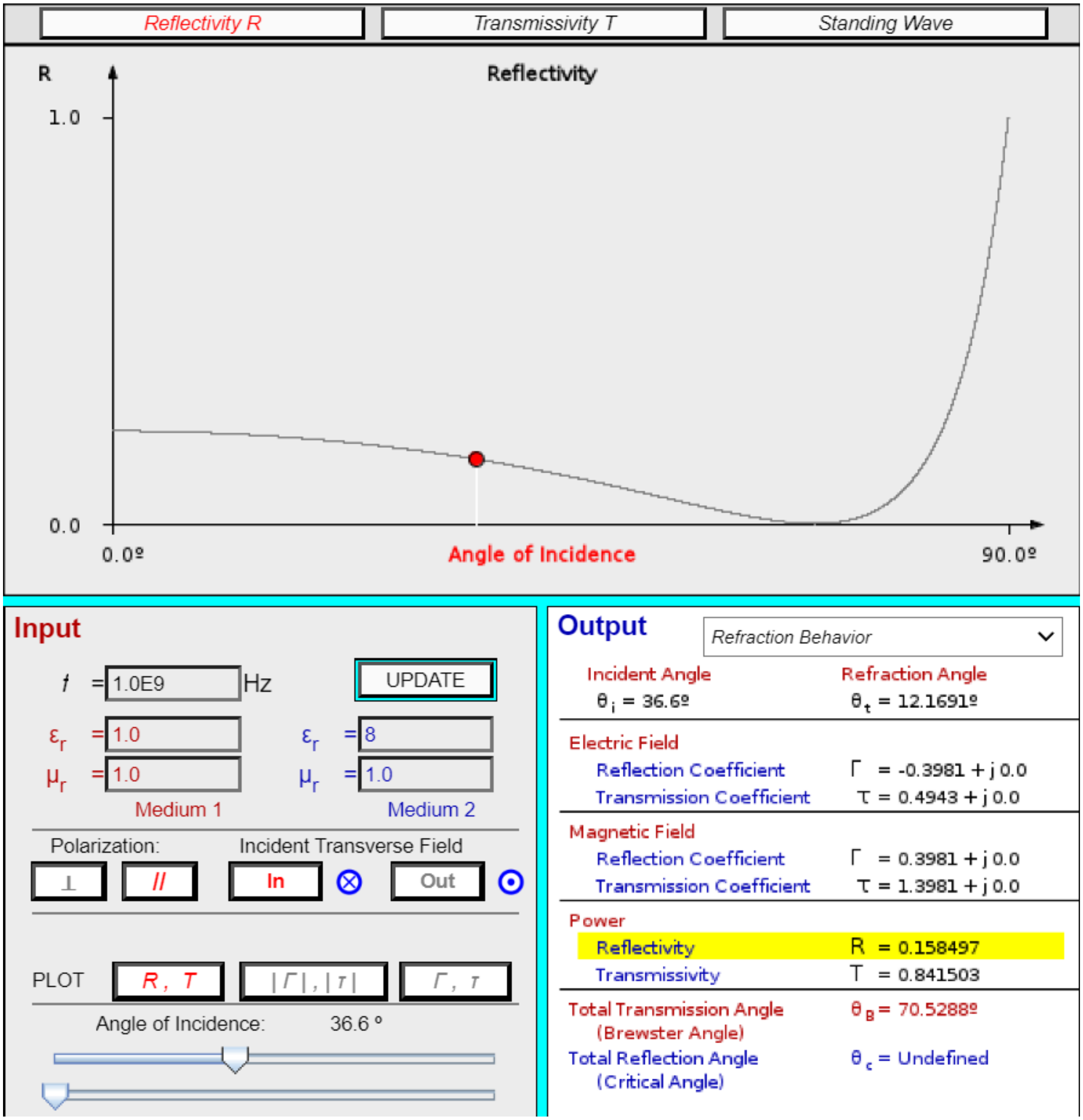

Figure 1.3 - TM Reflectivity vs. Incident Angle

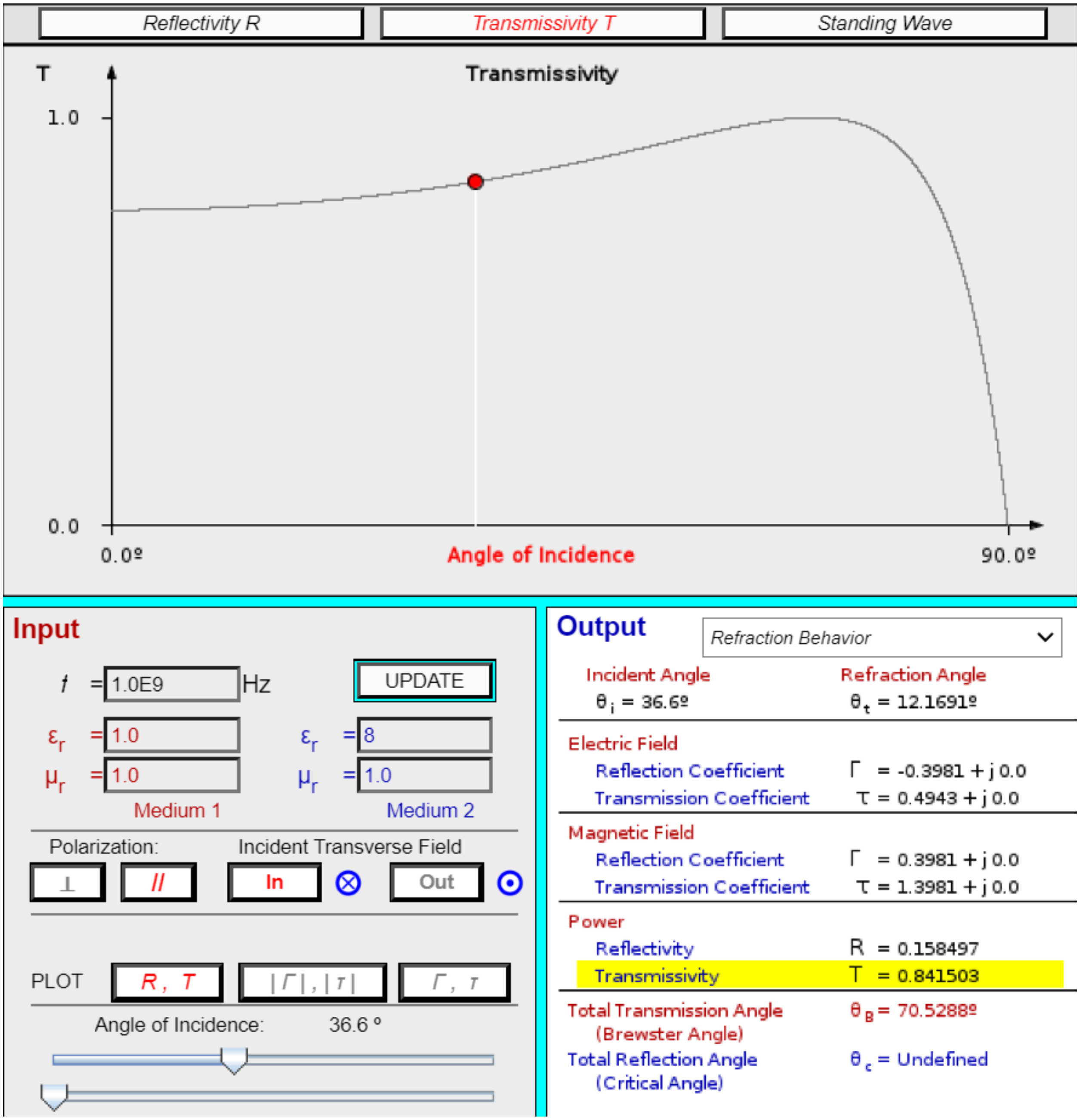

Figure 1.4 - TM Transmissivity vs. Incident Angle

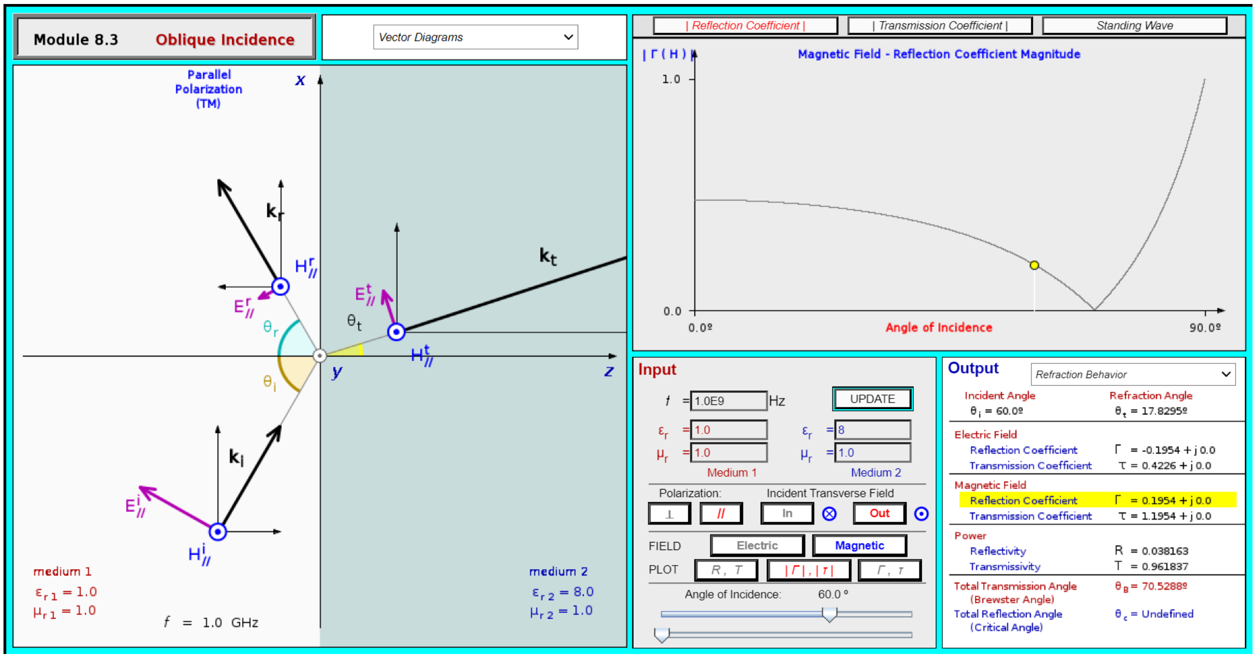

Figure 1.5 - Verification of TM Polarization with Module 8.3 App

Code Appendix

The following code was used to create Table 1.0 and Figure 1.0.

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

def compute_data(n2_value, incident_angle_deg):
    # Constants
    n1 = 120 * np.pi

    # Formulas
    incident_angle_rad = np.radians(incident_angle_deg)
    transmission_angle_rad = np.arcsin(np.sin(incident_angle_rad) /
np.sqrt(8))
    transmission_angle_deg = np.degrees(transmission_angle_rad)


    Γ_TE = (n2_value * np.cos(incident_angle_rad) - n1 *
np.cos(transmission_angle_rad)) / (n2_value * np.cos(incident_angle_rad) +
n1 * np.cos(transmission_angle_rad))

    τ_TE = (2 * n2_value * np.cos(incident_angle_rad)) / (n2_value *
np.cos(incident_angle_rad) + n1 * np.cos(transmission_angle_rad))


    Γ_TM = (n2_value * np.cos(transmission_angle_rad) - n1 *
np.cos(incident_angle_rad)) / (n2_value * np.cos(transmission_angle_rad) +
n1 * np.cos(incident_angle_rad))

    τ_TM = (2 * n2_value * np.cos(incident_angle_rad)) / (n2_value *
np.cos(transmission_angle_rad) + n1 * np.cos(incident_angle_rad))

    T_TE_squared = τ_TM**2 * (n1 * np.cos(transmission_angle_rad) /
(n2_value * np.cos(incident_angle_rad)))

    R_TE = Γ_TE**2
    T_TE = 1 - R_TE

    R_TM = Γ_TM**2
```

```
    T_TM = 1 - R_TM

    return {
        "n1": n1,
        "n2": n2_value,
        "Incident Angle": incident_angle_deg,
        "Transmission Angle": transmission_angle_deg,
        "R (TE)": R_TE,
        "T (TE)": T_TE,
        "R (TM)": R_TM,
        "T (TM)": T_TM,
        "Incident Angle in Rads": incident_angle_rad,
        "Transmission Angle (Rads)": transmission_angle_rad,
        "Γ (TE)": Γ_TE,
        "τ (TE)": τ_TE,
        "T (TE).1": T_TE_squared,
        "Γ (TM)": Γ_TM,
        "τ (TM)": τ_TM
    }

def plot_reflectivity_transmissivity(n2_value, incident_angles):

    data_list = [compute_data(n2_value, angle) for angle in
incident_angles]
    df = pd.DataFrame(data_list)


    incident_angle_degrees = df["Incident Angle"]
    Rte = df["R (TE)"]
    Tte = df["T (TE)"]
    Rtm = df["R (TM)"]
    Ttm = df["T (TM)"]

    # Plotting
    plt.figure(figsize=(16, 9))
    plt.plot(incident_angle_degrees, Rte, label='R (TE)', color='blue')
    plt.plot(incident_angle_degrees, Tte, label='T (TE)', color='red')
    plt.plot(incident_angle_degrees, Rtm, label='R (TM)', color='green')
    plt.plot(incident_angle_degrees, Ttm, label='T (TM)', color='orange')
    plt.title('Reflectivity and Transmissivity vs. Incident Angle')
```

```
    plt.xlabel('Incident Angle (degrees)')
    plt.ylabel('Coefficient')
    plt.legend(loc='upper right', title='Legend')
    plt.grid(True)
    plt.savefig("reflectivity_transmissivity_graph.png", dpi=600)  # Save
figure as PNG
    plt.show()


n2_value = (120 * np.pi)/np.sqrt(8)
incident_angles = np.arange(0, 90.5, 0.5)  # Angles from 0 to 90 in steps
of 0.5 degrees
plot_reflectivity_transmissivity(n2_value, incident_angles)

data_list = [compute_data(n2_value, angle) for angle in incident_angles]
df = pd.DataFrame(data_list)


condensed_data = df[["Incident Angle", "Transmission Angle", "R (TE)", "T
(TE)", "R (TM)", "T (TM)"]]

# Save the full data
output_file_path = "computed_data.xlsx"
df.to_excel(output_file_path, index=False, engine='openpyxl')
print(df)
# Save the condensed data
condensed_file_path = "condensed_data.xlsx"
condensed_data.to_excel(condensed_file_path, index=False,
engine='openpyxl')
```