Assignment 4

λ/4 Transformer for Complex Loads

ELG 3106 - Electromagnetic Engineering

Fall 2023

School of Electrical Engineering and Computer Science

University of Ottawa

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Index

Executive Summary

Objective:

This assignment focused on the use of a λ/4 transformer for impedance matching in transmission lines. The goal was to match a line with a lossless characteristic impedance (Z_{01}) and a complex load (Z_L) to a feedline, also with impedance Z_{01}, determining the transformer's impedance (Z_{02}) and load line length (d).

Methodology:

We combined theoretical analysis with verification applications, using a high-resolution Smith Chart and Module 2.7 applets. This approach involved calculating the input impedance at different points along the transmission line and verifying these calculations through the Module 2.7 applets.

Results and Discussion:

The results showed effective impedance matching, with calculated transformer impedances of 161.245155Ω and 15.491933Ω at specific points (d_{max} and d_{min}). These were confirmed by simulations indicating a near-zero reflection coefficient, demonstrating the transformer's efficiency in a range of impedance scenarios.

Conclusion:

The assignment successfully bridged theoretical knowledge with practical application, demonstrating the λ/4 transformer's role in impedance matching. It highlighted the importance of analytical tools like the Smith Chart in electromagnetic engineering, providing a comprehensive understanding of impedance matching techniques.

Introduction

In this assignment, we will be exploring the application of a $\lambda/4$ transformer on its use for impedance matching. The main challenge lies in matching a transmission line with a lossless characteristic impedance, denoted as Z_{01}, and a complex load Z_L, to a feedline also characterized by Z_{01}. Our task is to determine the appropriate lossless characteristic impedance of the transformer, Z_{02}, and the necessary load line length, d, to achieve optimal performance.

The theoretical basis of this assignment is grounded in the concept that the input impedance at the source end of a transmission line can be mathematically determined and used to solve practical engineering problems. Our approach will utilize the high-resolution Smith Chart to find $Z(d_{max})$ and $Z(d_{min})$, as well as their corresponding distances d_{max} and d_{min}. Our findings will be confirmed by simulations with the Module 2.7 applets, to verify these theoretical principles.

In the following report, we will clearly define the task at hand and summarize the relevant theory. The results section will include a summary of our findings, verified by screenshots from the Module 2.7 Design app, showcasing the Set Line/Transformer, Output Data, and Phasor Plots panels. Our discussion will aim to provide a clear understanding of what occurs in each line segment of the process.

Finally, our conclusion will not only restate the purpose of the $\lambda/4$ transformer but also reflect on its effectiveness in impedance matching within transmission systems. This assignment is an opportunity for us to test our theoretical knowledge, enhancing our understanding of key concepts in electromagnetic engineering.

Theory

The input impedance at the source end of a line of length l is given by the following equation:

$$Z_{in} = Z_0 \left\{ \frac{Z_L + jZ_0 tan(\beta l)}{Z_0 + jZ_L tan(\beta l)} \right\}$$

Where, a lossless line of length $l = \frac{\lambda}{4}$ characteristic impedance Z_0, and real load Z_L, have an input impedance of $Z_{in} = \frac{Z_0^2}{Z_L}$ or $Z_{in}Z_L = Z_0^2$. This understanding, when applied to the transmission line shown in Figure 1.0, indicates that Z_d, the input impedance of the load line, needs to be a real number. Since real wave impedances are found at voltage maximum and minimum points, this leads to two distinct solutions.

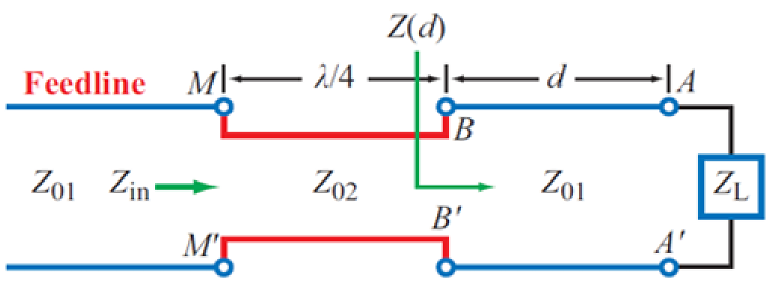


Figure 1.0 - An In-Series $\frac{\lambda}{4}$ Transformer Inserted at Either d_{max} or d_{min}

Since in Figure 1.0, the impedance to be matched is $Z_{in} = Z_{01}$, the solutions for $Z(d)$ must satisfy:

$$Z_{01}Z(d) = Z_{02}^2$$

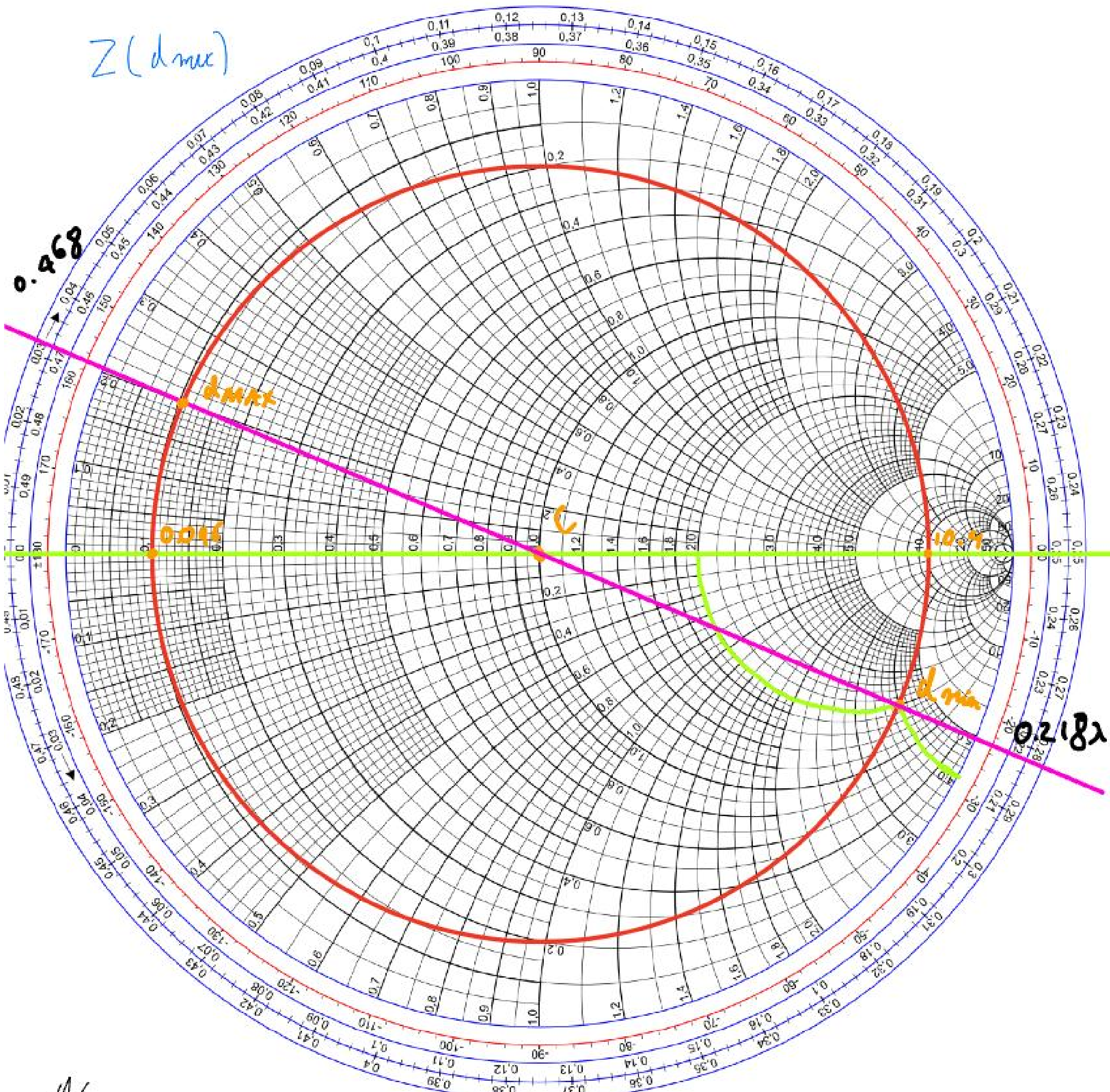
Where $d = dmax$ is the location of a voltage maximum, and $d = dmin$ is the location of a voltage minimum.

*The entirety of this section was created using the works of Dr. H. Schrimer. (Schriemer, 2023)

Flow Chart

- **Start**
- **Identify Load Impedance (Z_L):** The first step is to identify the load impedance that needs to be matched (100-j200Ω).
- **Normalize Load Impedance:** Normalize the load impedance Z_L with respect to the characteristic impedance Z_0 of the line (2-j4Ω).
- **Plot Normalized Load Impedance on Smith Chart:** Locate and plot the normalized load impedance on the Smith chart.
- **Determine Load Admittance (Y_L):** Convert the normalized impedance to normalized admittance, if necessary.
- **Locate Voltage Minima and Maxima Points:**
  - **Find Minimum Voltage Point (d_{min}):** Rotate along the constant SWR circle to locate the point of minimum voltage (minimum impedance).
  - **Mark Z(d_{min}):** Record the impedance at this point as Z(d_{min}).
  - **Find Maximum Voltage Point (dmax):** Continue rotating along the constant SWR circle to locate the point of maximum voltage (maximum impedance).
  - **Mark Z(d_{max}):** Record the impedance at this point as Z(d_{max}).
- **Move λ/4 Distance Along Constant SWR Circle:** Rotate the point representing Z_L *o*r Y_L on the Smith chart by a distance equivalent to a quarter-wavelength along the constant SWR circle. This represents the impedance transformation over a λ/4 line.
- **Read Off New Impedance (Z_{in}):** The point at which you stop represents the input impedance Z_{in} of the λ/4 transformer.
- **Denormalize Z_{in} to Find Z_0 of Transformer:** Multiply Z_{in} by the characteristic impedance Z_0 to find the actual impedance of the λ/4 transformer line.
- **End**

Graphical Results

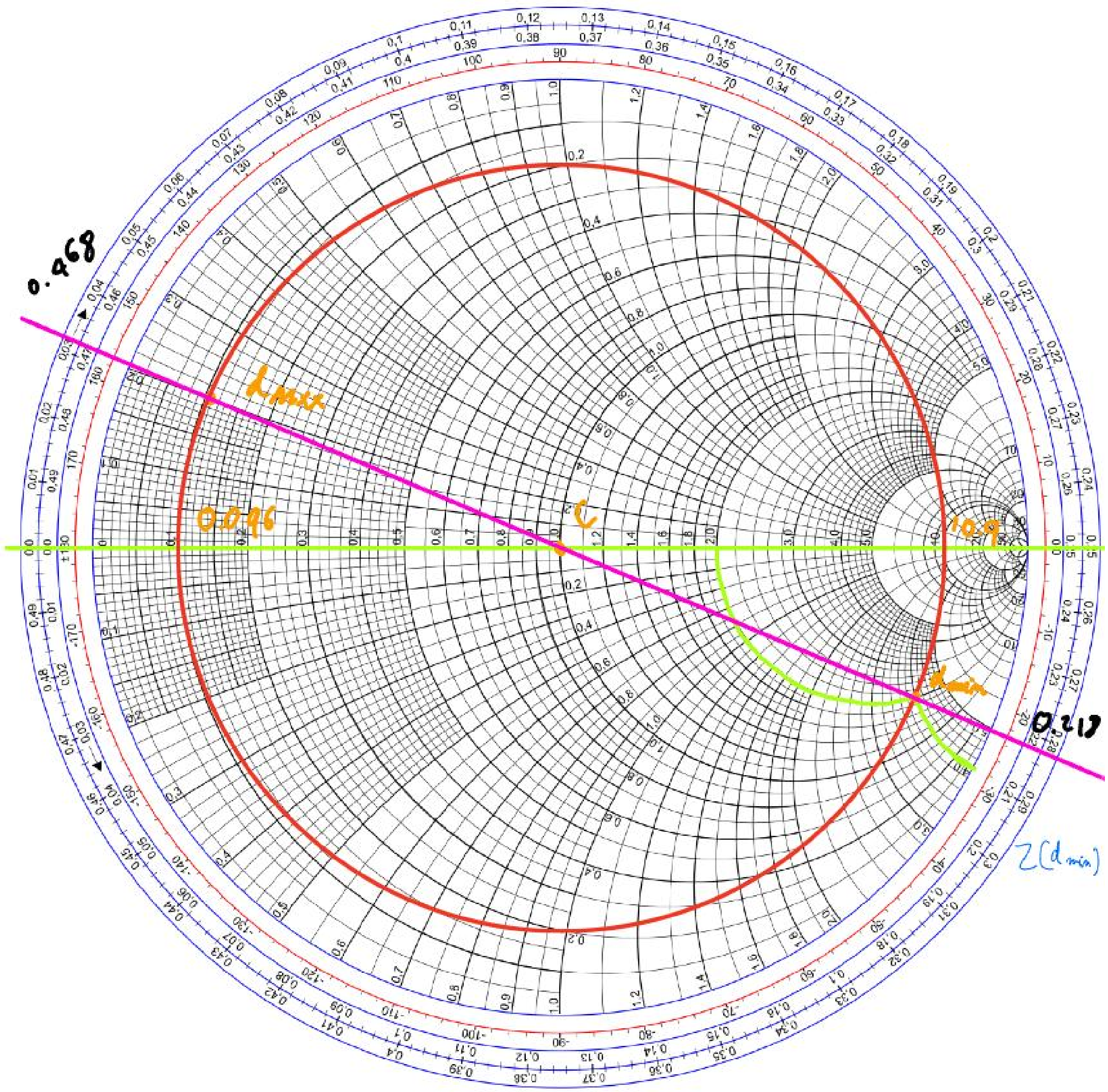


Normalization of ZL: $\frac{100 - j200}{50} = \underline{2 - j4\ \Omega}$

$Z(d_{max}) = 10.4 \times 50 = 520$

$Z_{01}' =$ characteristic impedance $= \sqrt{Z_{in}\, Z(d_{max})} = \sqrt{50 \times 520} = \underline{161.2451\ \Omega}$

Figure 2.0 - Z(dmax) Plotted on Smith Chart and Calculations



$$Z(d_{min}) = 0.096 \times 50 = 4.8$$

$$Z_{0z}' = \sqrt{Z_{in} Z(d_{min})} = \sqrt{50 \times 4.8} = \underline{15.491933\,\Omega}$$

Figure 2.1 - Z(d_{min}) Plotted on Smith Chart and Calculations

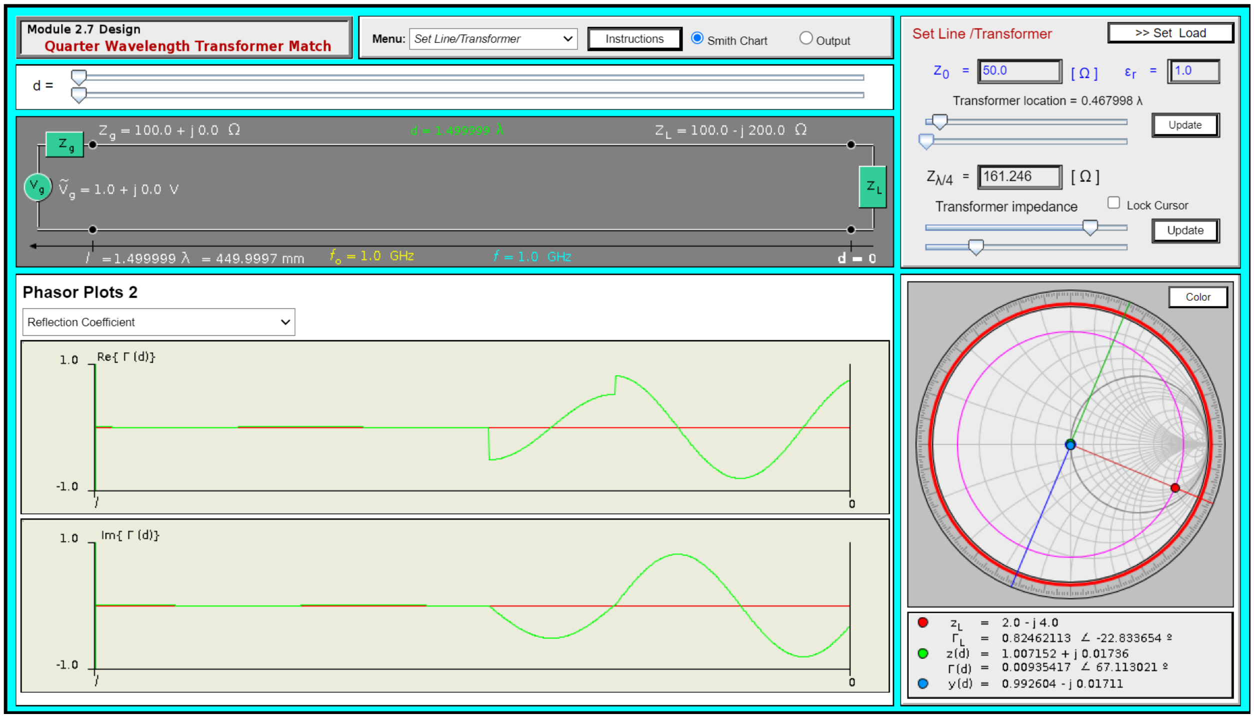


Figure 3.0 - Module 2.7 Design Smith Chart and Phasor Plots for $Z(d_{max})$

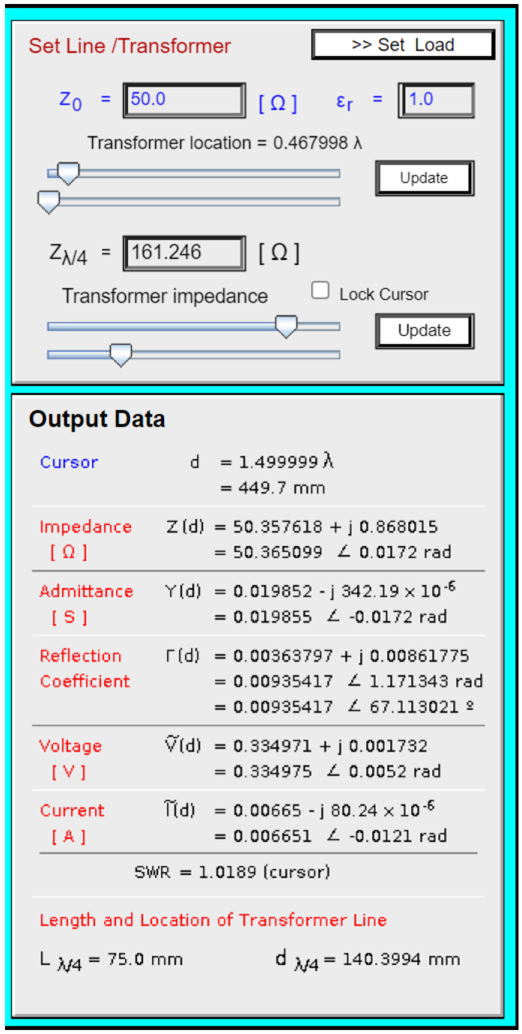


Figure 3.1 - Module 2.7 Design Output Data for $Z(d_{max})$

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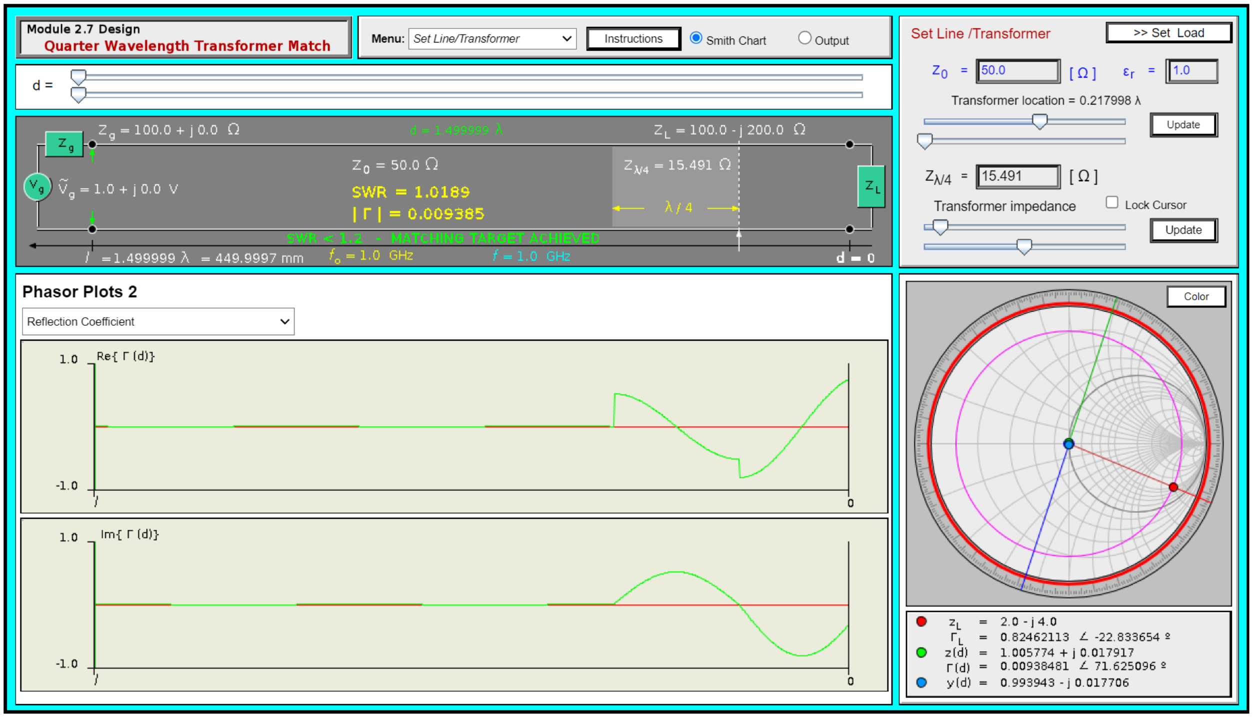

Figure 4.0 - Module 2.7 Design Smith Chart and Phasor Plots for $Z(d_{min})$

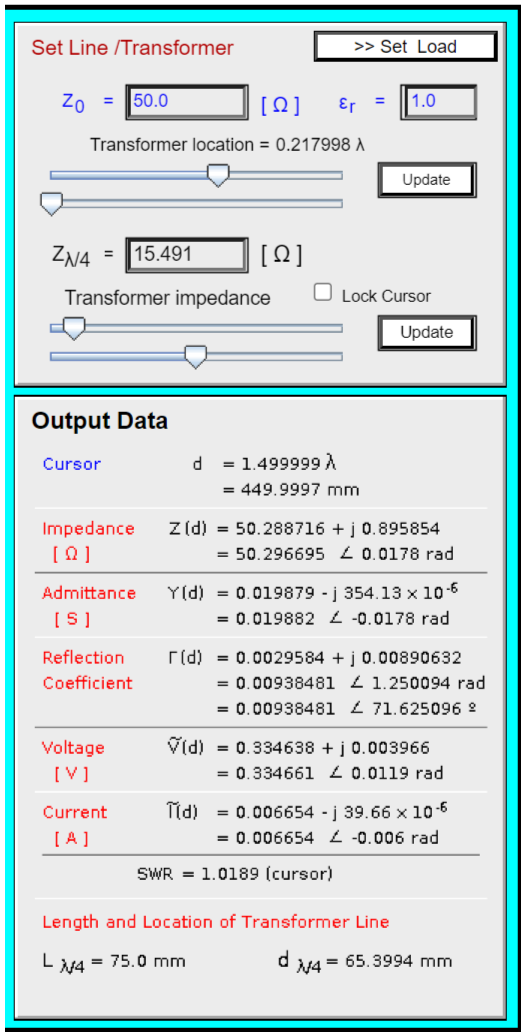

Figure 4.1 - Module 2.7 Design Output Data for $Z(d_{min})$

*This section was created using the works of the Module 2.7 Design app (F. Ulaby, 2020)

Tabulated Results

	For $d_{max} = 0.468$	**For $d_{min} = 0.218$**
Z(d)	520Ω	4.8Ω
$Z_{\lambda/4}$	161.245155Ω	15.491933Ω

Table 1.0 - Tabulated Results for d_{max} and d_{min}

Discussion

Analyzing the results of our experiment with the λ/4 transformer in impedance matching reveals several key insights into its operational dynamics and efficiency. The reflection coefficients, being smaller than 0.01 for both outputs from the Module 2.7 applets, indicate that our Smith chart calculations were correct.

At d_{max}, the point of voltage maximum, the real part of the reflection coefficient reaches its highest value. Conversely, at d_{min}, the point of voltage minimum, this value is at its lowest. This observation is in line with theoretical expectations, where d_{max} and d_{min} represent key points in the impedance-matching process.

By plotting the normalized load impedance, we obtained the values for $Z(d_{max})$ and $Z(d_{min})$, which are crucial for determining the transformer's characteristic impedance. Our results, showed impedance values of 520Ω at d_{max} and 4.8Ω at d_{min}, demonstrating a significant transformation, and highlighting the transformer's ability to match a wide range of impedances.

Moreover, the derived transformer impedances of 161.245155Ω and 15.491933Ω for d_{max} and d_{min} respectively, further validate the effectiveness of our design.

The use of the Module 2.7 applets was a critical component of our methodology, serving as a practical confirmation of our theoretical and chart-based findings. The ability of these simulations to visually and numerically demonstrate the zero reflection coefficient in the feedline provided a compelling verification of our Smith chart calculations.

Conclusion

In conclusion, this assignment has successfully demonstrated the practical application and theoretical understanding of the λ/4 transformer in impedance matching. We started with the challenge of matching a complex load Z_L to a transmission line with a characteristic impedance Z_{01}, using a λ/4 transformer.

Our findings from the Smith Chart analysis, which were verified by the Module 2.7 applets, have been instrumental in confirming the transformer's ability to match impedances efficiently. The reflection coefficients, being significantly low, indicate an excellent impedance match, a critical factor for minimizing signal loss and reflection in transmission lines. The specific impedance values obtained at d_{max} and d_{min}, and their corresponding transformer impedances, highlight the λ/4 transformer's versatility in adapting to different impedance scenarios. This assignment has reinforced the theoretical principles outlined in our study but also provided us with valuable practical experience in using analytical tools such as the Smith Chart.

Overall, the successful completion of this assignment, with its combination of theory, practical analysis, and verification, illustrates the effective use of various tools and methodologies in solving complex engineering problems. It has provided us with a comprehensive understanding of the λ/4 transformer's role in impedance matching and its critical importance in the field of electromagnetic engineering.

References

- H. Schriemer, "Assignment 4," University of Ottawa, Ottawa, Ontario, Canada, Nov. 21, 2023
- F. Ulaby, "Module 2.7. Design" University of Michigan, Dept. of EECS. [Online]. Available: https://em8e.eecs.umich.edu/jsmodules/ch2/mod2_7des.html. Accessed on: Nov. 21, 2023.
- F. Ulaby, "Module 2.7. Tutorial" University of Michigan, Dept. of EECS. [Online]. Available: https://em8e.eecs.umich.edu/jsmodules/ch2/mod2_7tut.html. Accessed on: Nov. 21, 2023.

Appendix
Tables

	For d_{max} = 0.468	**For d_{min} = 0.218**
Z(d)	520Ω	4.8Ω
$Z_{\lambda/4}$	161.245155Ω	15.491933Ω

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Figures

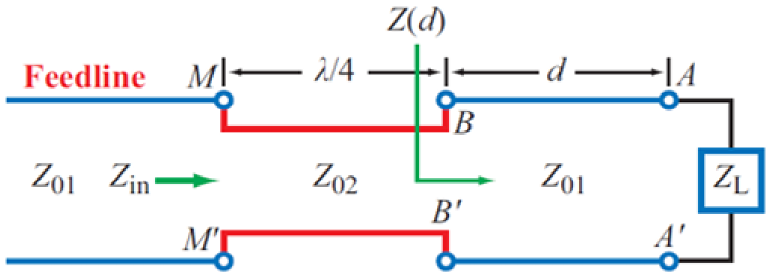
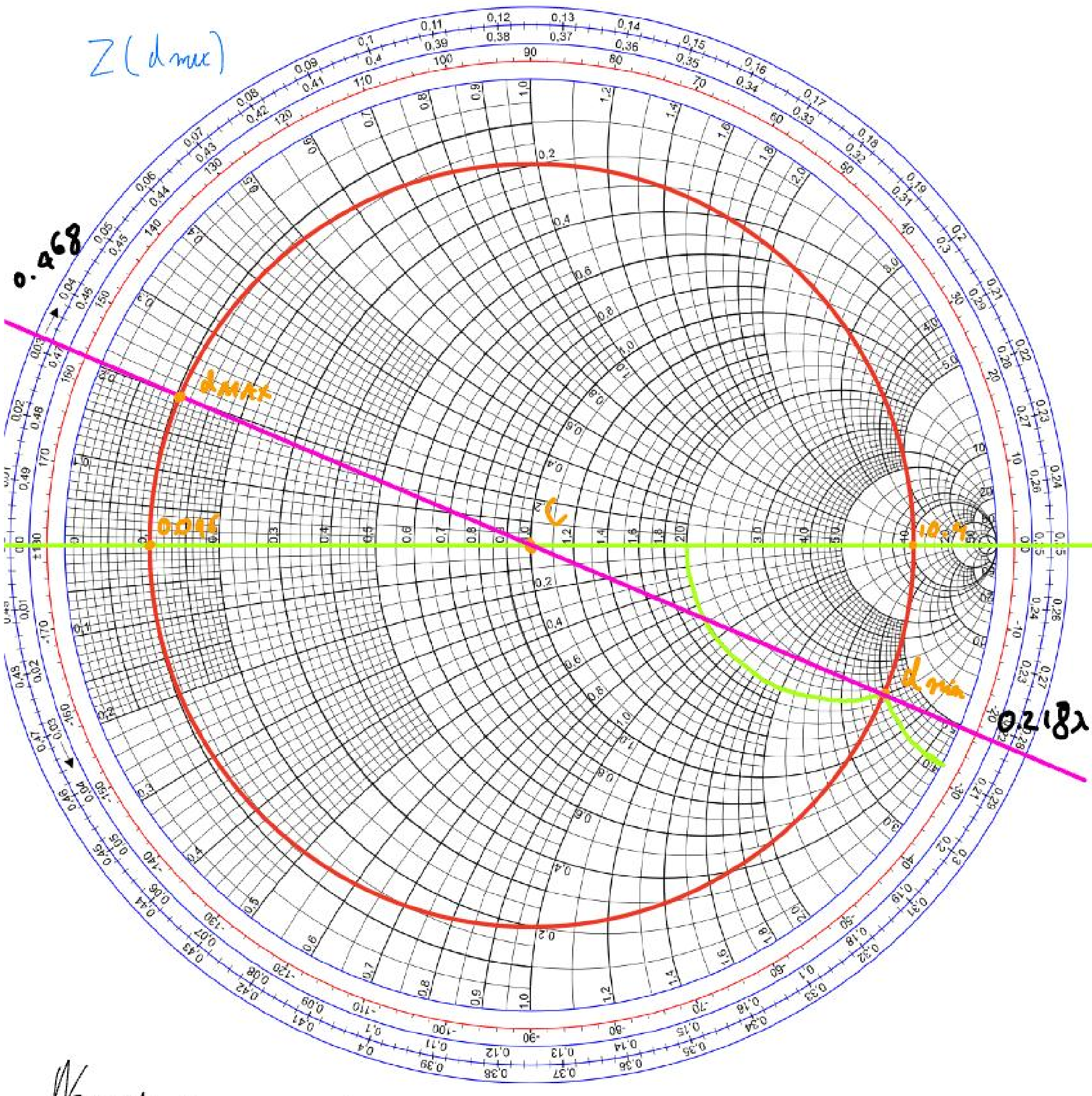


Figure 1.0 - An In-Series $\frac{\lambda}{4}$ Transformer Inserted at Either d_{max} or d_{min}

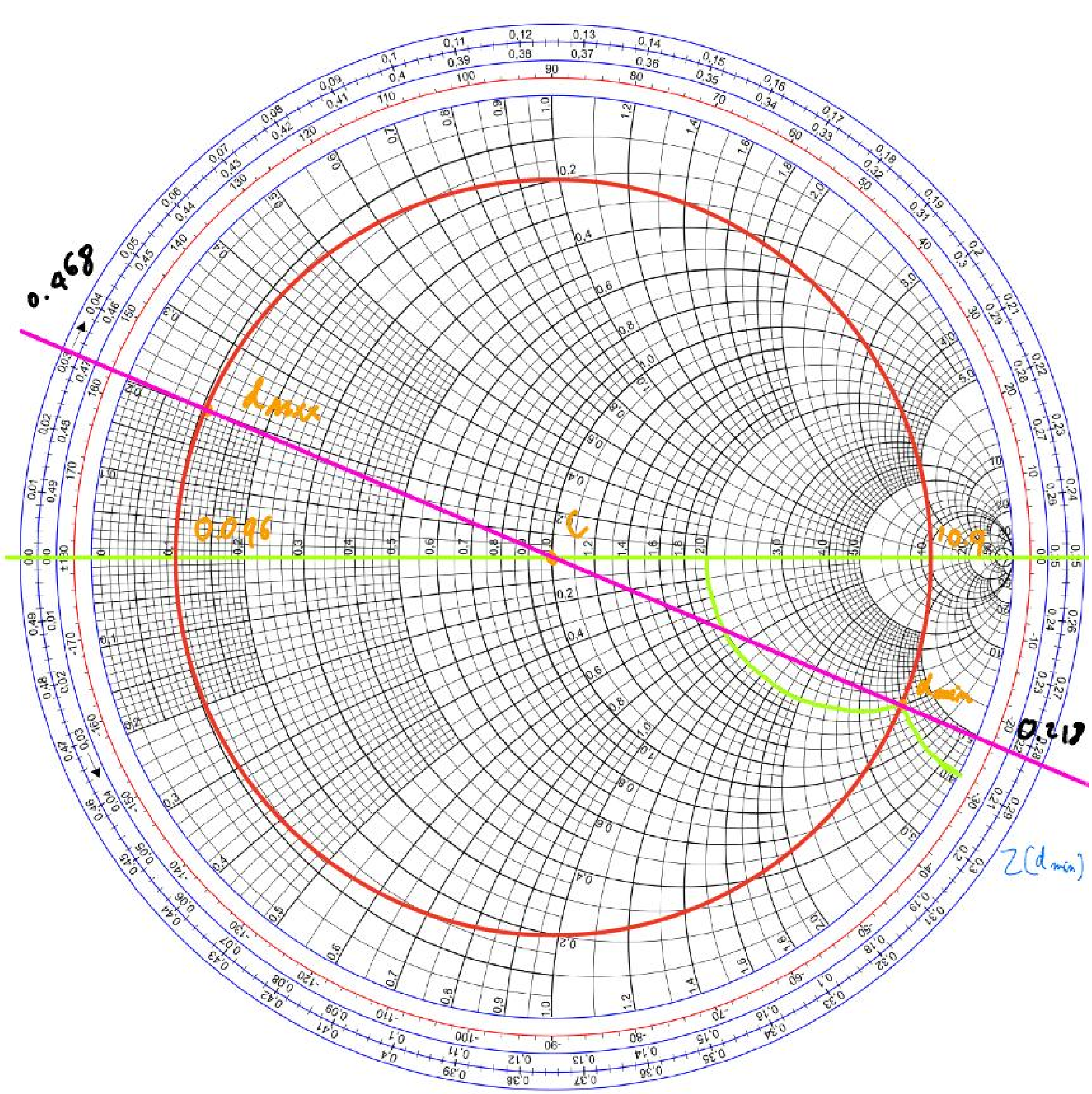


Normalization of Z_L: $\dfrac{160 - j200}{50} = 2 - j4\ \Omega$

$Z(d_{max}) = 10.4 \times 50 = 520$

$Z_{01}' =$ characteristic impedance $= \sqrt{Z_{in}\, Z(d_{max})} = \sqrt{50 \times 520} = 161.2451\ \Omega$

Figure 2.0 - Z(dmax) Plotted on Smith Chart and Calculations



$$Z(d_{min}) = 0.096 \times 50 = 4.8$$

$$Z_{o2}' = \sqrt{Z_{in} Z(d_{min})} = \sqrt{50 \times 4.8} = \underline{15.491933\,\Omega}$$

Figure 2.1 - Z(d_{min}) Plotted on Smith Chart and Calculations

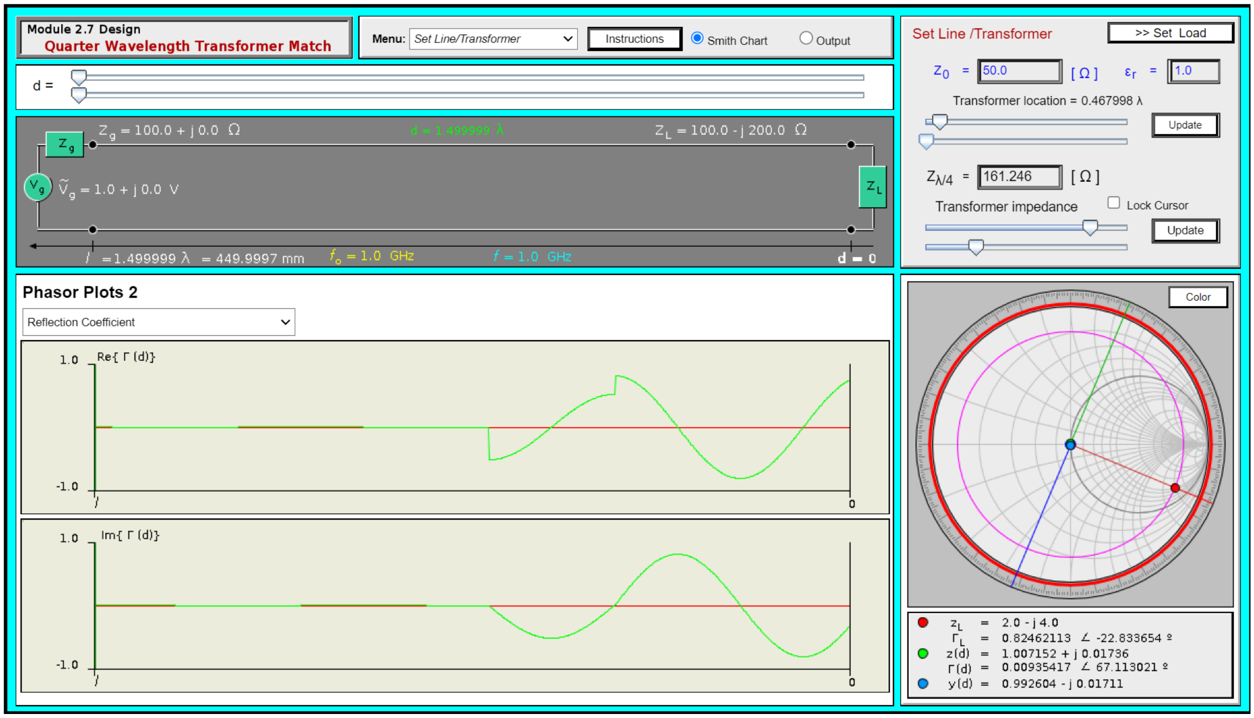


Figure 3.0 - Module 2.7 Design Smith Chart and Phasor Plots for $Z(d_{max})$

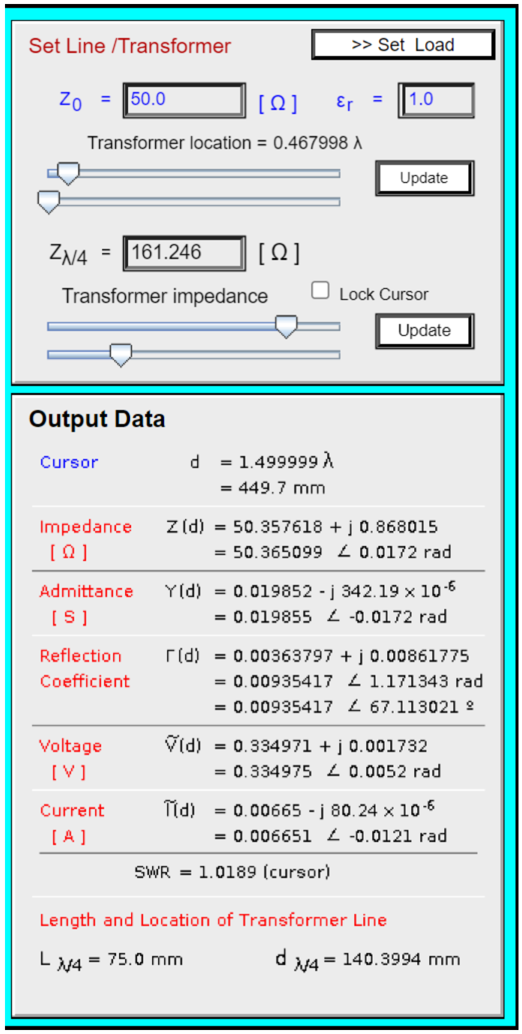


Figure 3.1 - Module 2.7 Design Output Data for $Z(d_{max})$

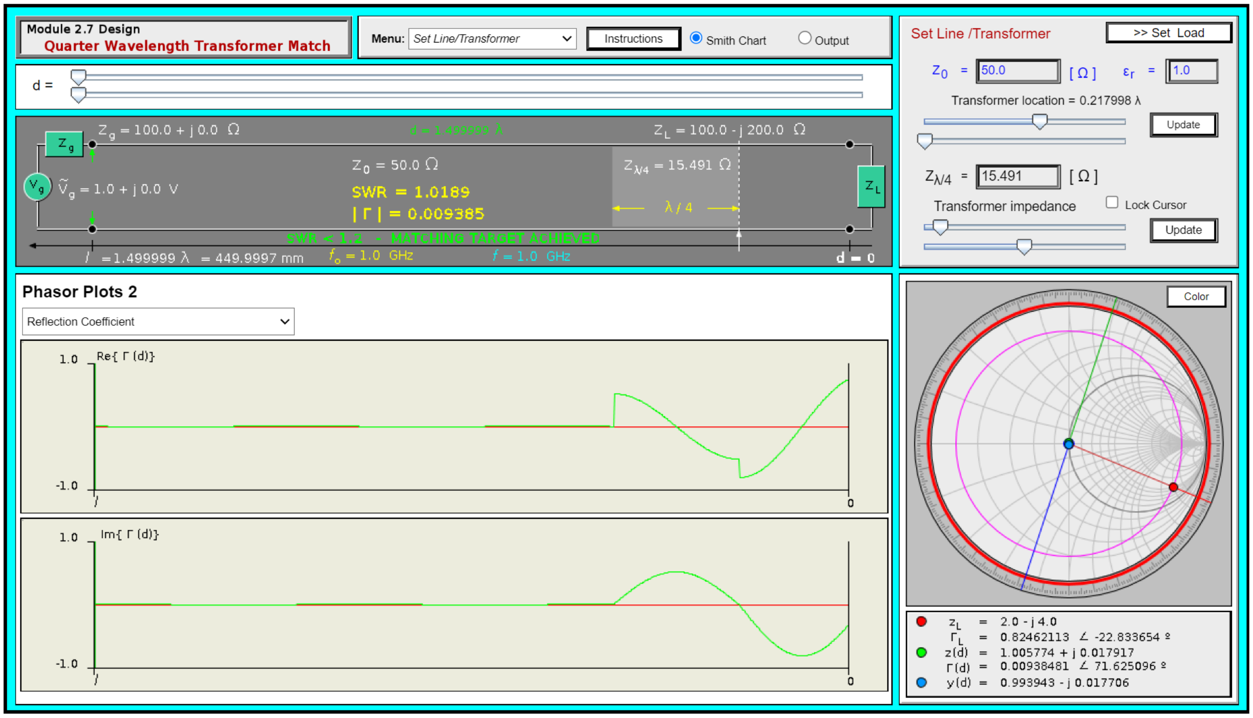


Figure 4.0 - Module 2.7 Design Smith Chart and Phasor Plots for Z(d_{min})

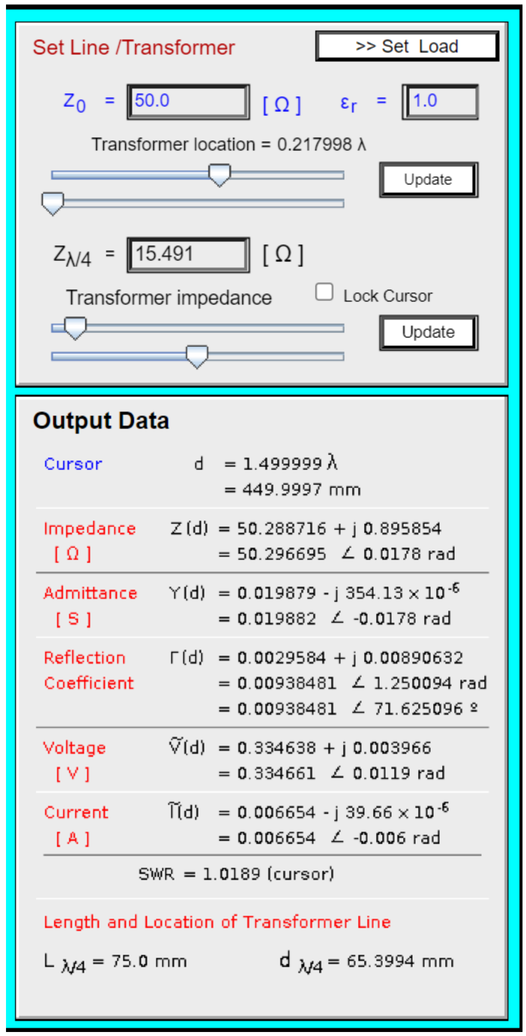


Figure 4.1 - Module 2.7 Design Output Data for $Z(d_{min})$

Code Appendix

This assignment did not require any computational methods.