Assignment 1

ELG 3106 - Electromagnetic Engineering

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Introduction

The primary task at hand is to methodically tabulate the polarization states for various combinations of polarization angles, represented by γ and χ, and to discern the relationships they share with the wave parameters a_x, a_y, and δ.

Our approach will integrate the power of computational tools with theory. Utilizing the Module 7.3 app which will offer a visual representation of polarization states, we can thus validate our calculated parameters against visual models. For the automation, I will use Excel, a robust computational tool, that facilitates this task, allowing me to automate my calculations, and ensuring precision, repeatability, and efficiency in my analysis.

In this report, my objective is to tabulate and understand the polarization states for various combinations of polarization angles. I also aim to bridge the gap between the theoretical equations and their visual representations, offering a comprehensive view of the plane wave's polarization. Through this exploration, I hope to shed light on the complexities of wave polarization, its computational representation, and the insights it offers.\

Let us define a few equations before starting. If we consider an arbitrary electromagnetic wave propagating in the +z direction.
We can represent this using the following phasor representation:

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\tilde{E}_x(z) + \hat{\mathbf{y}}\tilde{E}_y(z)$$

This can converted to the following instantaneous field:

$$\boxed{\mathbf{E}(z,t) = \mathrm{Re}\left\{\tilde{\mathbf{E}}(z)e^{j\omega t}\right\} = \hat{\mathbf{x}}a_x\cos(\omega t - kz + \delta_x) + \hat{\mathbf{y}}a_y\cos(\omega t - kz + \delta_y).}$$

From which we can define the relative polarization:

$$\boxed{\delta = \delta_y - \delta_x\,,}$$

Additionally, we can define the polarization angles and wave parameters by:

$$\boxed{\begin{aligned}\tan(2\gamma) &= \tan(2\psi_0)\cos\delta\,,\\ \sin(2\chi) &= \sin(2\psi_0)\sin\delta\,,\end{aligned}}$$

Finally, we can define the auxiliary angle as:

$$\boxed{\tan\psi_0 = \frac{a_y}{a_x}\,,}$$

Derivation

$$\tilde{E}(z) = \hat{x}\, a_x e^{j\delta_x} e^{-jkz} + \hat{y}\, a_y e^{j\delta_y} e^{-jkz}$$

$$\mathbf{E}(z,t) = \mathrm{Re}\left\{\tilde{\mathbf{E}}(z)e^{j\omega t}\right\} = \hat{\mathbf{x}}a_x \cos\left(\omega t - kz + \delta_x\right) + \hat{\mathbf{y}}a_y \cos\left(\omega t - kz + \delta_y\right)$$

$$\sin(2\chi) = \sin(2\psi)\sin\delta$$

$$\tan(2\gamma) = \tan(2\psi)\cos\delta$$

$$\delta = \arccos\left(\frac{\tan(2\gamma)}{\tan(2\psi)}\right) = \arcsin\left(\frac{\sin(2\chi)}{\sin(2\psi)}\right) = \sin\left(\arccos\left(\frac{\tan(2\gamma)}{\tan(2\psi)}\right)\right)$$

$$\left(\frac{\sin(2\chi)}{\sin(2\psi)}\right)^2 = \sqrt{1 - \frac{\tan(2\gamma)^2}{\tan(2\psi)^2}}$$

$$\frac{\sin(2\chi)^2}{\sin(2\psi)^2} = 1 - \frac{\tan(2\gamma)^2}{\tan(2\psi)^2}$$

$$\sin(2\chi)^2 = \sin(2\psi)^2 - \cos(2\psi)^2\tan(2\gamma)^2 = 1 - \cos(2\psi)^2 - \cos(2\psi)^2\tan(2\gamma)^2$$

$$\frac{1-\sin(2\chi)^2}{1+\tan(2\gamma)^2} = \cos(2\psi)^2$$

$$\boxed{\psi = \frac{1}{2}\arccos\sqrt{\frac{1-\sin(2\chi)^2}{1+\tan(2\gamma)^2}}}$$

$$\boxed{\tan(\psi) = \frac{a_y}{a_x}}$$

$$\tan(2\gamma) = \tan(2\psi_0)\cos\delta$$

$$\cos\delta = \frac{\tan(2\gamma)}{\tan(2\psi_0)} \qquad \arccos\left(\frac{\tan(2\gamma)}{\tan(2\psi)}\right) \begin{matrix}\gamma = -90\\ \psi = 22.5\end{matrix} = 90$$

$$\boxed{\delta = \arccos\left(\frac{\tan(2\gamma)}{\tan(2\psi_0)}\right) = \arcsin\left(\frac{\sin(2\chi)}{\sin(2\psi_0)}\right)}$$

Tabulated Values

x axis	y axis	Polarization	ax	ay	δ	Ψ0	γ	ξ
↓	↓							
45°	-90°	Left Circular Polarization	1	1	90	45	-90°	45°
45°	-45°	Left Circular Polarization	1	1	90	45	-45°	45°
45°	0°	Left Circular Polarization	1	1	90	45	0°	45°
45°	45°	Left Circular Polarization	1	1	90	45	45°	45°
45°	90°	Left Circular Polarization	1	1	90	45	90°	45°
22.5°	-90°	Left Elliptical Polarization	0.414213562	1	90	22.5	-90°	22.5°
22.5°	-45°	Left Elliptical Polarization	1	1	135	45	-45°	22.5°
22.5°	0°	Left Elliptical Polarization	0.414213562	1	90	22.5	0°	22.5°
22.5°	45°	Left Elliptical Polarization	1	1	45	45	45°	22.5°
22.5°	90°	Left Elliptical Polarization	0.414213562	1	90	22.5	90°	22.5°
0°	-90°	Linear Polarization	1	0	0	0	-90°	0°
0°	-45°	Linear Polarization	1	1	180	45	-45°	0°
0°	0°	Linear Polarization	1	0	0	0	0°	0°
0°	45°	Linear Polarization	1	1	0	45	45°	0°
0°	90°	Linear Polarization	1	0	0	0	90°	0°
-22.5°	-90°	Right Elliptical Polarization	0.414213562	1	-90	22.5	-90°	-22.5°
-22.5°	-45°	Right Elliptical Polarization	1	1	-135	45	-45°	-22.5°
-22.5°	0°	Right Elliptical Polarization	0.414213562	1	-90	22.5	0°	-22.5°
-22.5°	45°	Right Elliptical Polarization	1	1	-45	45	45°	-22.5°
-22.5°	90°	Right Elliptical Polarization	0.414213562	1	-90	22.5	90°	-22.5°
-45°	-90°	Right Circular Polarization	1	1	-90	45	-90°	-45°
-45°	-45°	Right Circular Polarization	1	1	-90	45	-45°	-45°
-45°	0°	Right Circular Polarization	1	1	-90	45	0°	-45°
-45°	45°	Right Circular Polarization	1	1	-90	45	45°	-45°
-45°	90°	Right Circular Polarization	1	1	-90	45	90°	-45°

Table 1.0 - Spreadsheet demonstrating all 25 combinations of (χ,γ)

Excel formulas utilized:

For Ψ0:
=DEGREES(0.5*ACOS(SQRT((1-SIN(2*RADIANS(VALUE(LEFT(H3,LEN(H3)-1))))^2)/((1+TAN(2*RADIANS(VALUE(LEFT(I3,LEN(I3)-1))))^2)))))

This formula calculates Ψ. This ratio is determined by trigonometric functions (sine and tangent) applied to values extracted from cells H3 and I3. The values from these cells are first stripped of their last character (the degree symbol), converted to numbers, and then to radians for the trigonometric calculations. The final result is given in degrees.

For δ:
=DEGREES(ASIN(TAN(2*RADIANS(VALUE(LEFT(I3,LEN(I3)-1))))/TAN(2*RADIANS(G3))))

This formula calculates δ. The numerator of the ratio is derived from the value in cell I3 (after stripping its degree symbol and converting to a number), and the denominator is directly taken from cell G3. Both values are converted to radians, doubled, and then their tangents are taken. The final result of the arcsine operation is provided in degrees.

Flow Chart & Waveform Simulations

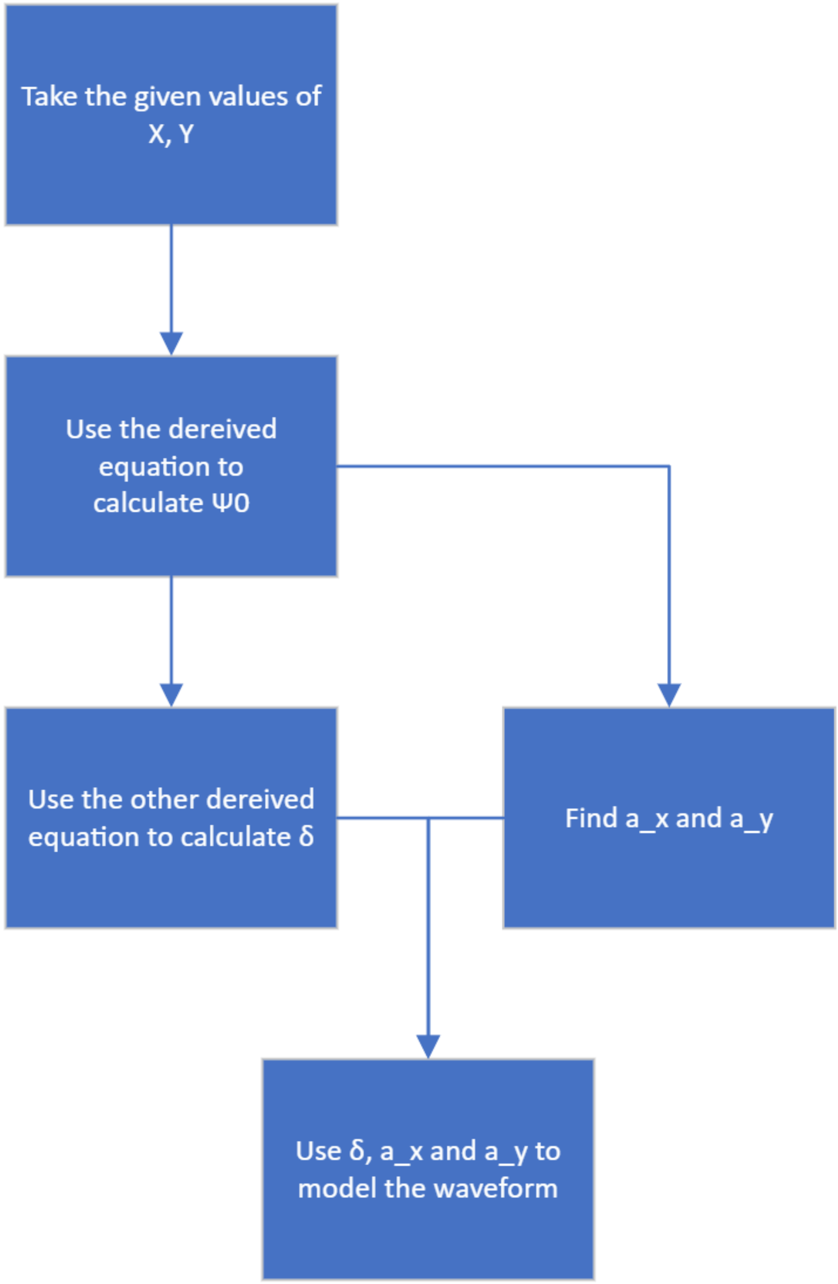


Figure 1.0 - Flowchart Explaining the Algorithm

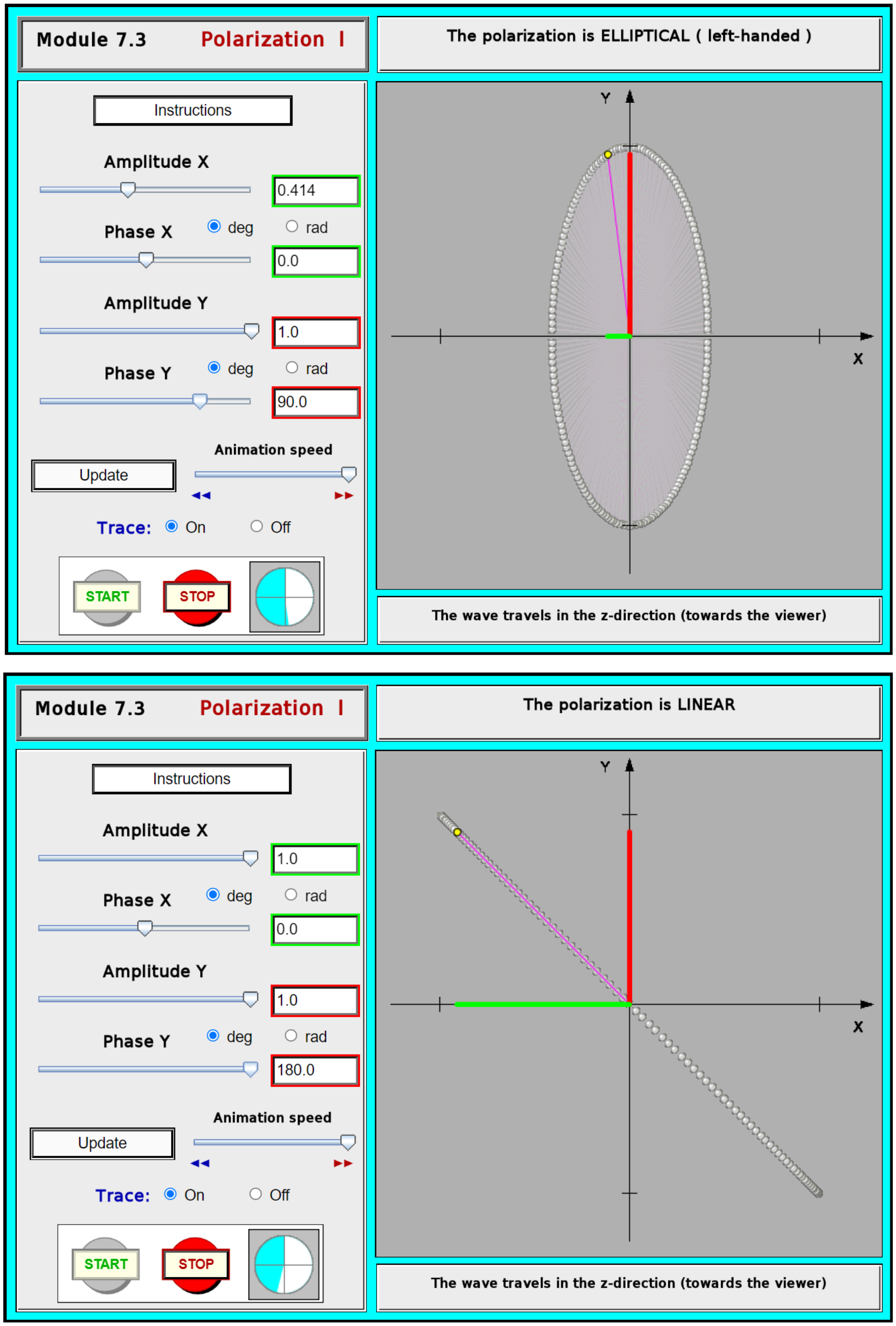
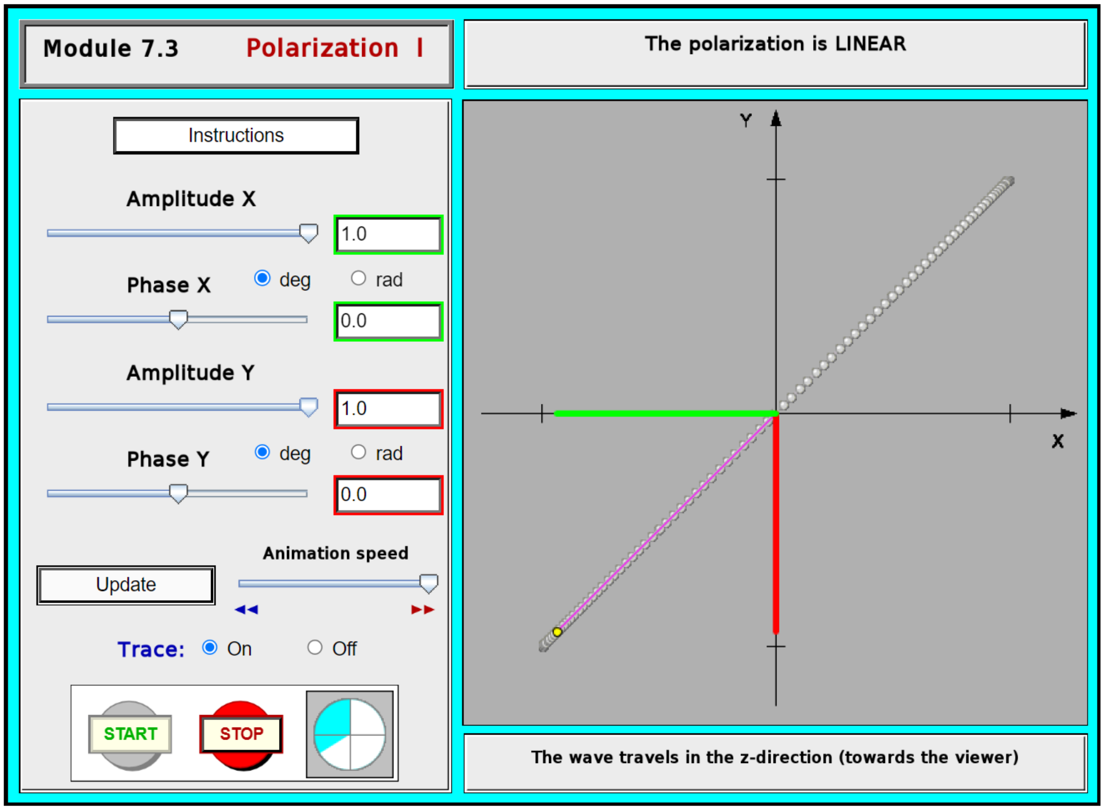


Figure 1.1 - 10 Models of the Waveform (Part I)



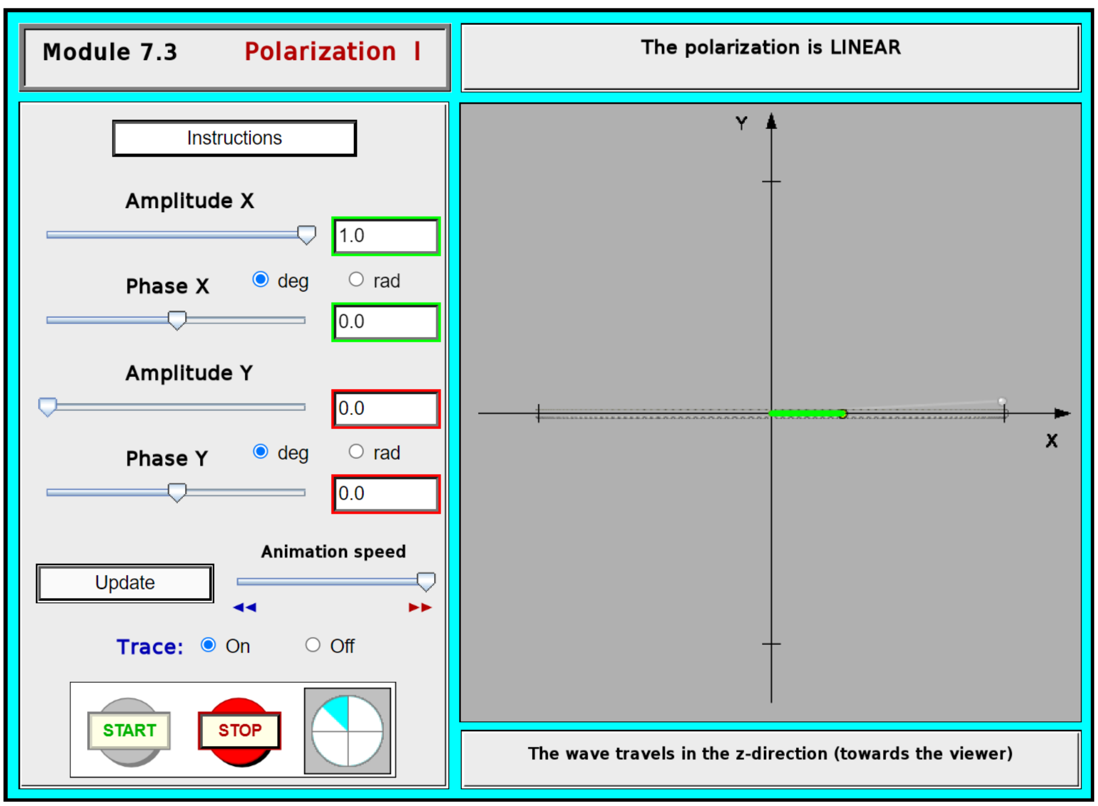
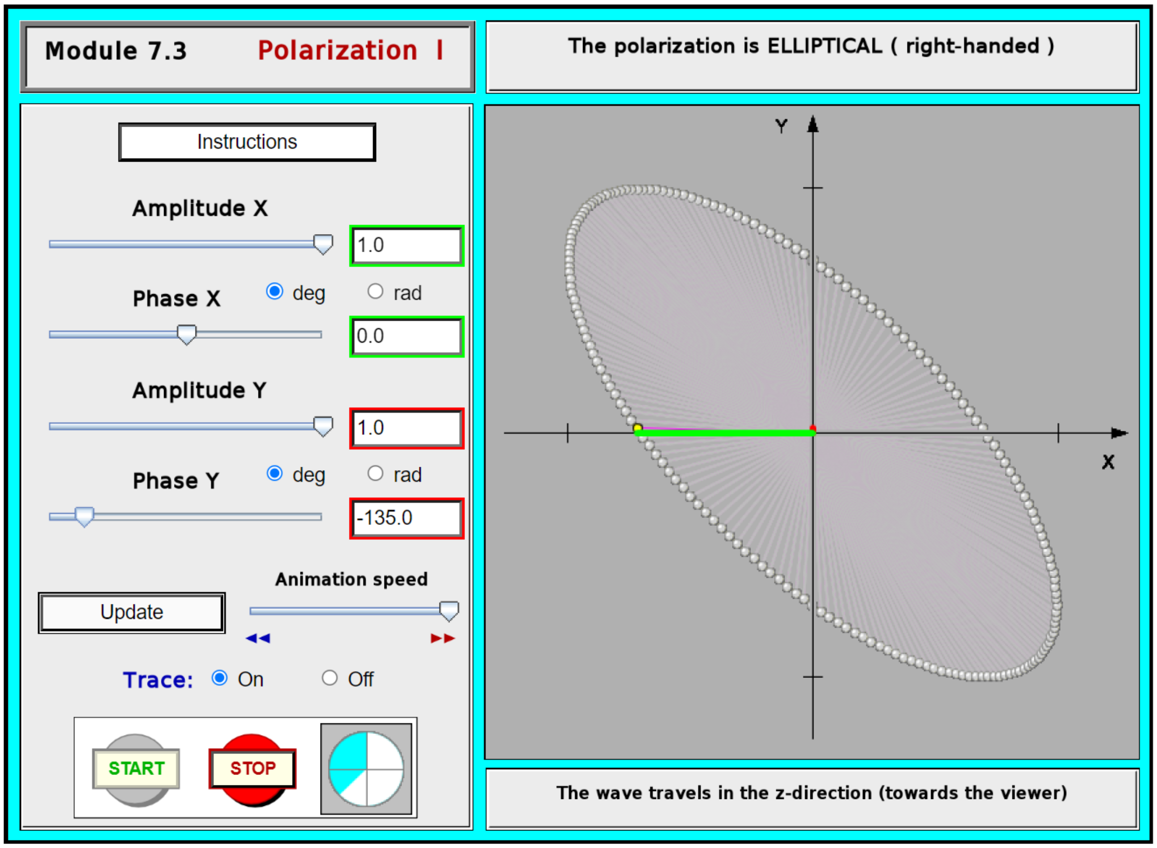


Figure 1.2 - 10 Models of the Waveform (Part II)



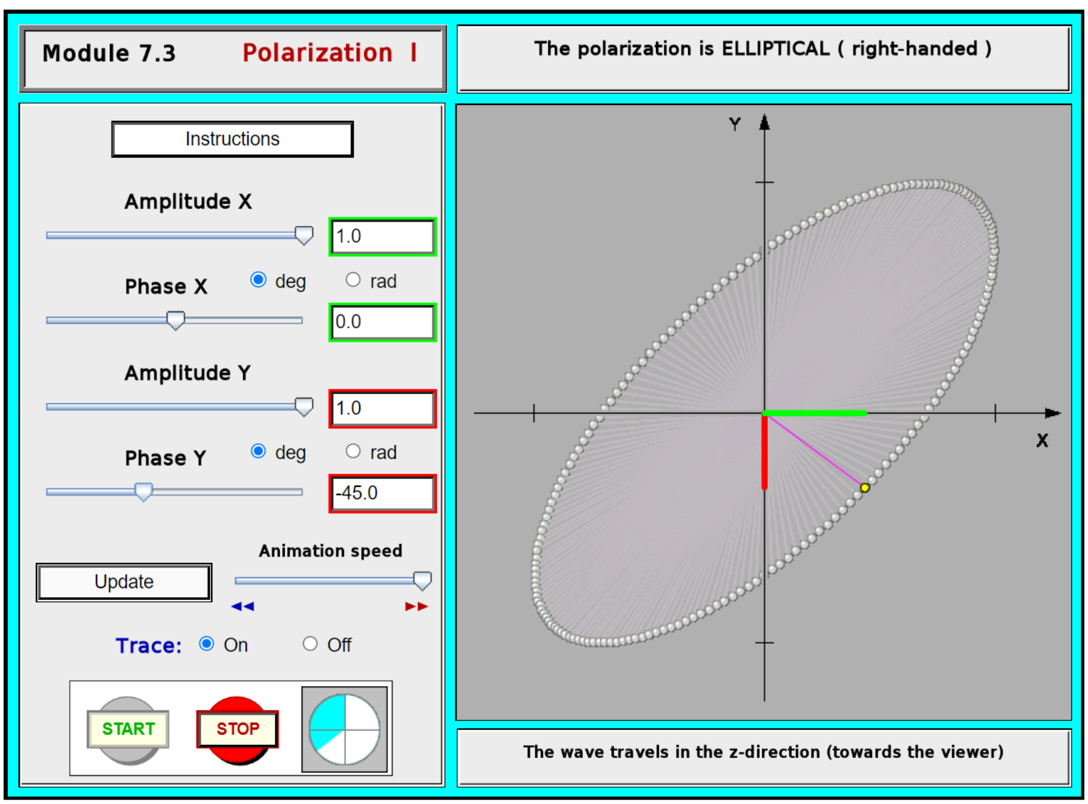
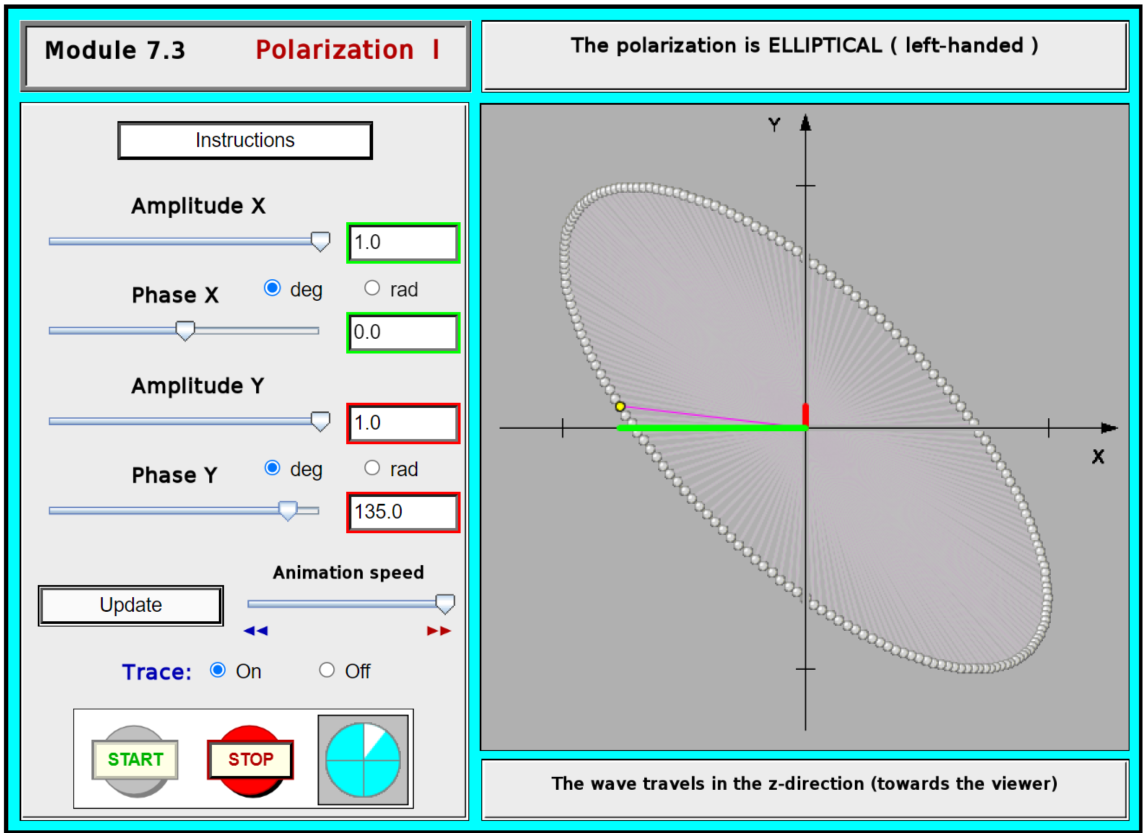


Figure 1.3 - 10 Models of the Waveform (Part III)



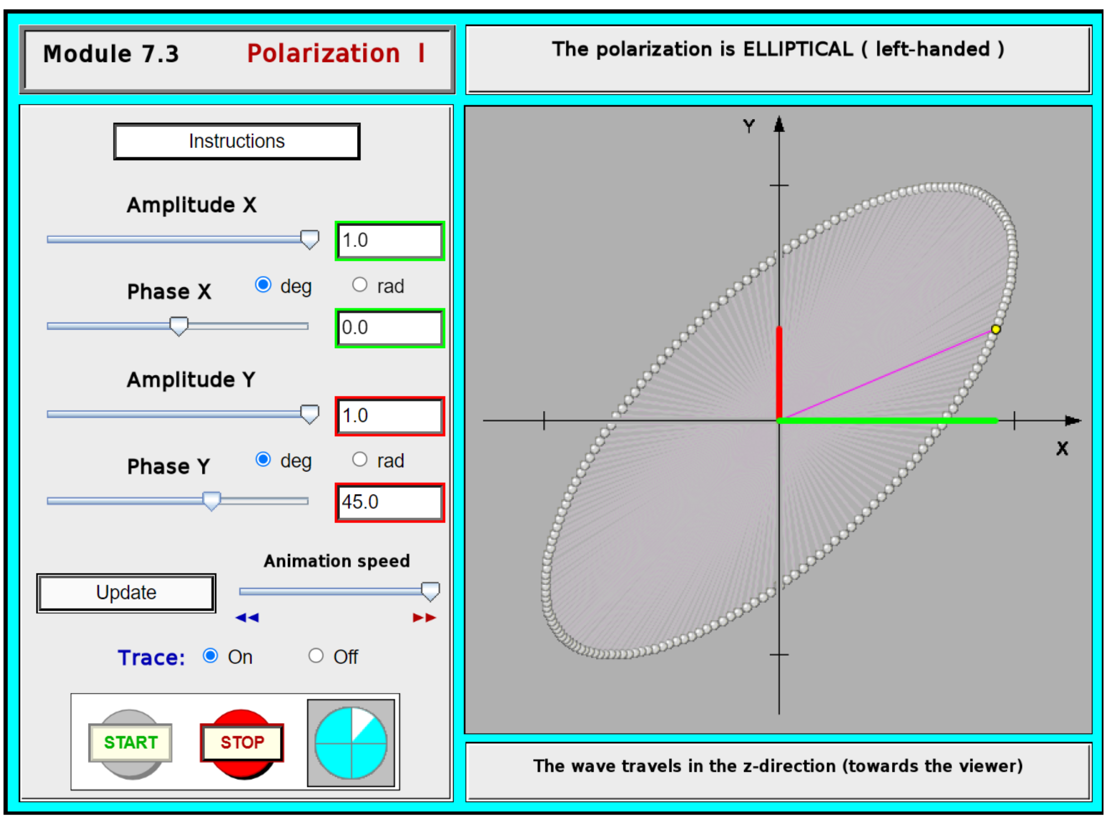
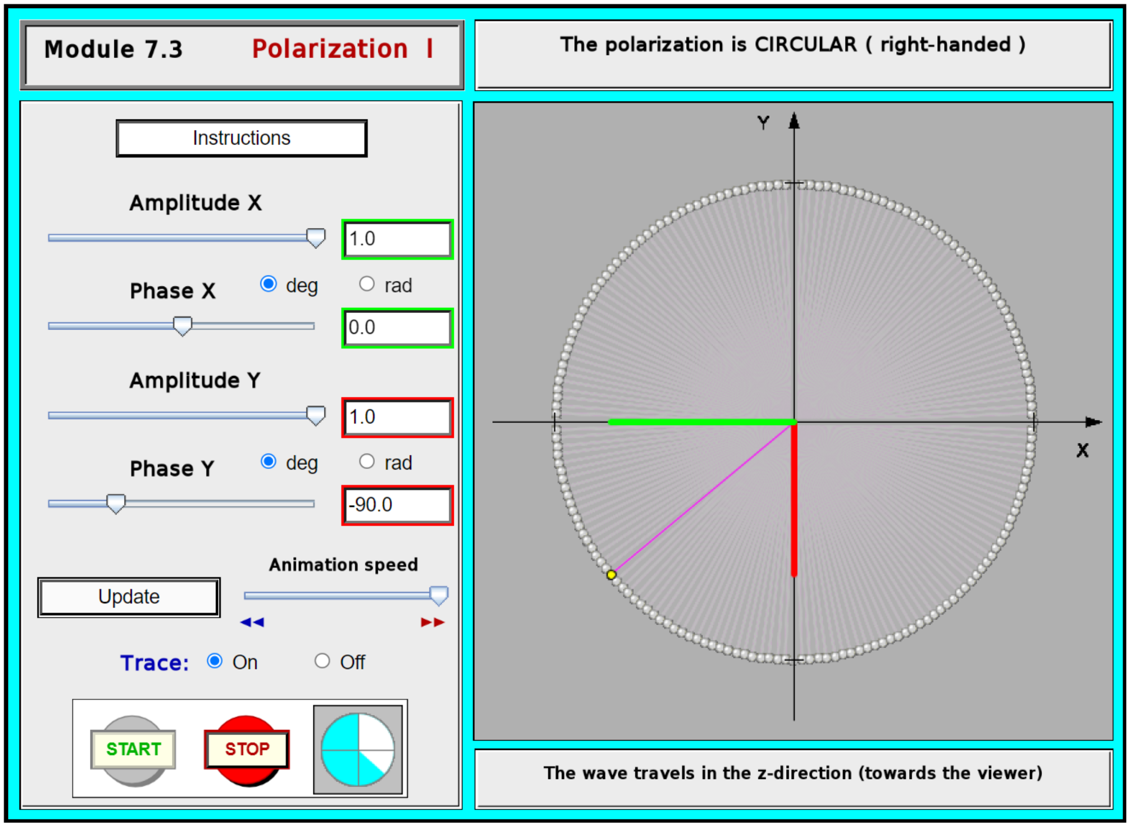


Figure 1.4 - 10 Models of the Waveform (Part IV)



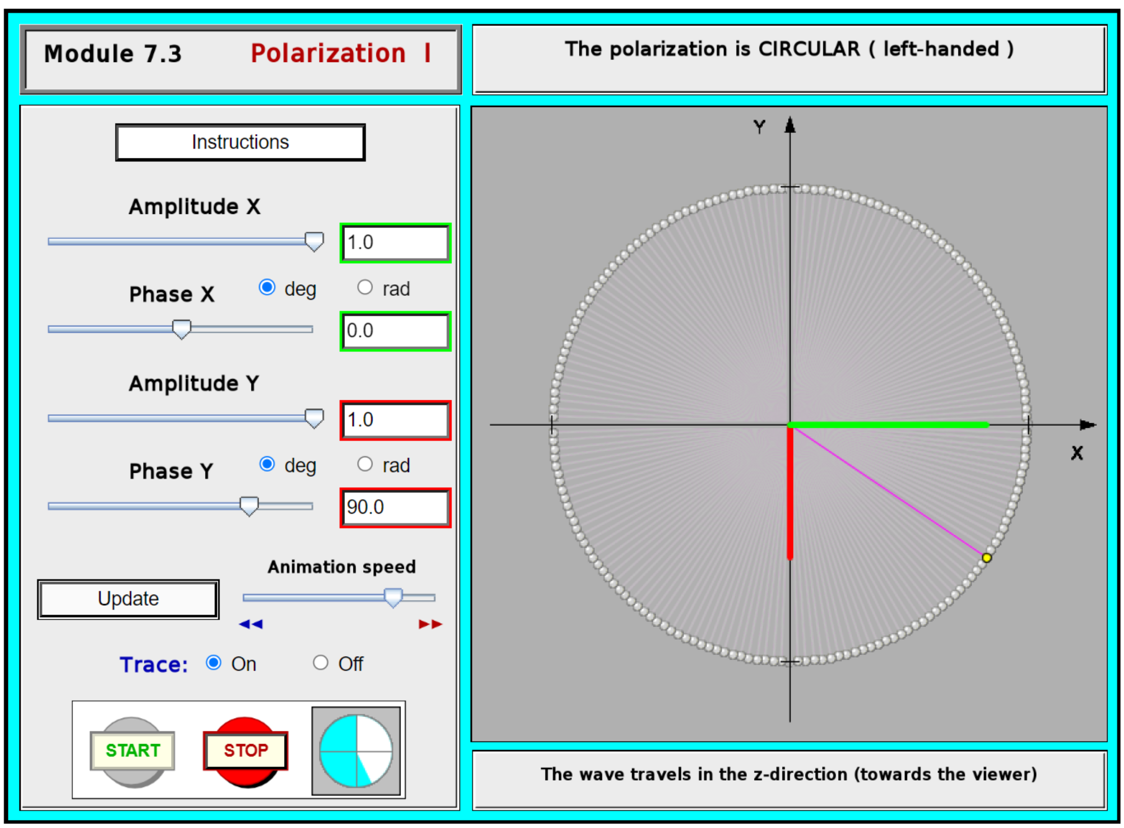


Figure 1.5 - 10 Models of the Waveform (Part V)

Conclusion

In conclusion, the graphical representations provide compelling evidence that, under specific conditions such as δ=90° and $a_x = a_y$, we observe a circular polarization, and when δ=0, the wave demonstrates linear polarization. The relationship between ξ and the magnitudes of a_x and a_y further establishes foundational concepts: at $\xi = \pm 45^{\circ}$, a_x is equivalent to a_y.

At its core, the γ and χ parameters serve as a reflection of the wave's intrinsic characteristics, influenced by the magnitudes a_x and a_y and the phase difference δ. For instance, when δ is at specific values such as 90°, indicating a phase difference of a quarter cycle, and the amplitudes a_x and a_y are equal, the resulting polarization is circular. This is visually represented by the γ and χ parameters tracing a perfect circle.

Conversely, when δ is 0, meaning there's no phase difference, the polarization is linear, causing the y and χ parameters to align along a straight path. The relationship further unfolds as we delve into the nuances of ξ. For angles $\xi = \pm 45$ $\circ$, the magnitudes a_x and a_y become equivalent, leading to an isotropic behaviour in the γ-χ plane. In essence, γ and χ can be envisioned as the visible fingerprints of the more abstract parameters a_x, a_y, and δ.

To derive the equation for ψ0 posed a rigorous challenge. Special cases within the equation presented unique obstacles, particularly when values were contingent upon the tangent of specific angles like 90° or 0°. These instances led to potential errors, either due to undefined operations or divisions by zero.

Finally, although tracking the minus signs might seem trivial, in such precise calculations and simulations, such a task holds great importance. Fortunately, our rigorous computational approach ensured that sign conventions did not pose many significant hurdles in this assignment. The end results, reflected in our simulations, were consistent with expectations.