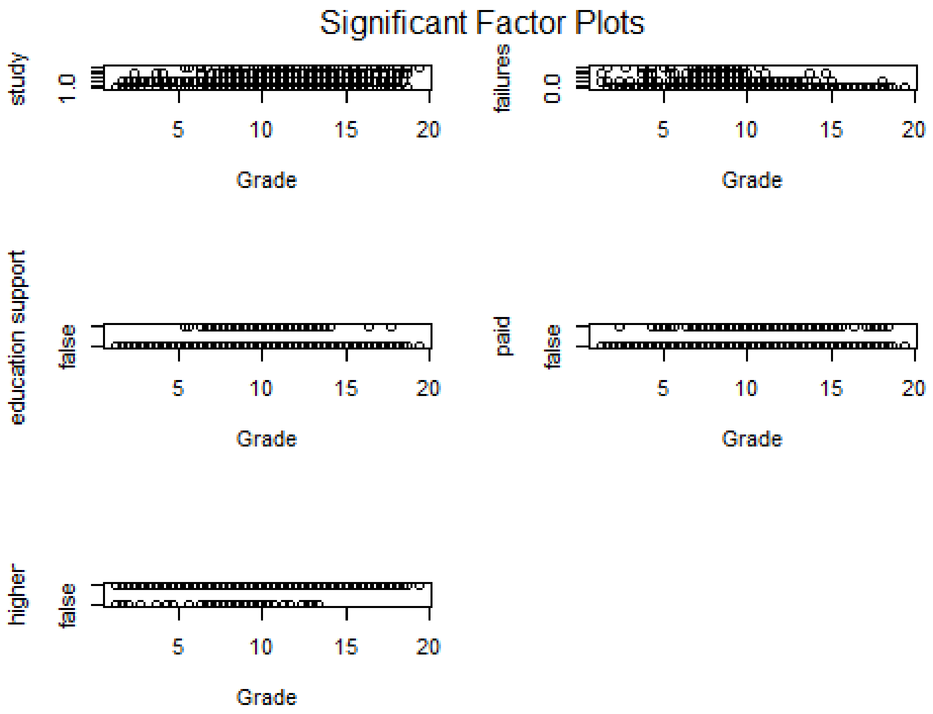

R Notebook: Multiple Regression Model of Student Academic Achievement

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April 6, 2017

Abstract

The interest of this study is in developing a prediction model of student success based on measured factors of success in Mathematics and Portuguese. Multiple regression is applied to develop a regression classifier based on student-provided factors that relate to living conditions and education conditions. Though regression tree modeling would appear at first to be the correct approach, the number and high-cardinality nature of many of the variables in this data makes such an approach less feasible in practice.

```
set.seed(1)
library(car)
library(boot)
c <- read.table("student-por.csv",sep=";",header=TRUE)
c <- data.frame(c)
d <- read.table("student-mat.csv",sep=";",header=TRUE)
d <- data.frame(d)
e <- rbind(c,d)
```

Data Cleaning

Both Math and Porteguese sets are merged, alternate column titles are applied, and all student grades are averaged across three grade entries.

```
names(e) <- c("school","sex","age", "address","family size","parents
cohab.", "mom's education",
             "dad's education","mom's job", "dad's job","reason",
"guardian","travel", "study",
             "failures","education support","family
support","paid","activities", "nursery","higher",
             "internet","romantic","family bond","free
time","social","workday alch.","weekend alch.","health",
             "absences","Grade 1","Grade 2","Grade 3")

Grade <- (e$`Grade 1` + e$`Grade 2` + e$`Grade 3`) / 3
e$`Grade 1` <- NULL
e$`Grade 2` <- NULL
e$`Grade 3` <- NULL
e = cbind(e, Grade)
attach(e)
```

Exploratory Investigation

From initial inspection it is clear education success is quantified by the Grade variable. Results of inspection indicate general normality of this output variable.

The class distributions of explanatory variables, 'dad's job' and 'mom's job', appear to show questionable value by inspection of the summary table. This is indicated by the limited difference between class-levels, except for the vaguely defined class, 'other', showing the survey question isn't well-defined or reliable an indicator.

An inspection of the VIF's (Variance Inflation Factors) of model parameters is performed to check for multicolonarity in the data-set.

```
summary(e)
```

```
##  school   sex          age        address family size parents cohab.
##  GP:772   F:591   Min.   :15.00   R:285   GT3:738     A:121
##  MS:272   M:453   1st Qu.:16.00   U:759   LE3:306     T:923
##                   Median :17.00
##                   Mean   :16.73
##                   3rd Qu.:18.00
##                   Max.   :22.00
##  mom's education dad's education    mom's job      dad's job
##  Min.   :0.000   Min.   :0.000   at_home :194   at_home : 62
##  1st Qu.:2.000   1st Qu.:1.000   health  : 82   health  : 41
##  Median :3.000   Median :2.000   other   :399   other   :584
##  Mean   :2.603   Mean   :2.388   services:239   services:292
##  3rd Qu.:4.000   3rd Qu.:3.000   teacher :130   teacher : 65
##  Max.   :4.000   Max.   :4.000
##         reason      guardian       travel          study
##  course    :430   father:243   Min.   :1.000   Min.   :1.00
##  home      :258   mother:728   1st Qu.:1.000   1st Qu.:1.00
##  other     :108   other : 73   Median :1.000   Median :2.00
##  reputation:248                Mean   :1.523   Mean   :1.97
##                                3rd Qu.:2.000   3rd Qu.:2.00
##                                Max.   :4.000   Max.   :4.00
##     failures      education support family support  paid
activities
##  Min.   :0.0000   no :925           no :404        no :824   no :528

##  1st Qu.:0.0000   yes:119           yes:640        yes:220   yes:516

##  Median :0.0000

##  Mean   :0.2644

##  3rd Qu.:0.0000

##  Max.   :3.0000

##  nursery   higher    internet  romantic   family bond      free time

##  no :209   no : 89   no :217   no :673   Min.   :1.000   Min.   :
1.000
##  yes:835   yes:955   yes:827   yes:371   1st Qu.:4.000   1st
Qu.:3.000
##                                           Median :4.000   Median :
3.000
##                                           Mean   :3.936   Mean   :
3.201
##                                           3rd Qu.:5.000   3rd
Qu.:4.000
##                                           Max.   :5.000   Max.   :
5.000
```

```
##      social      workday alch.   weekend alch.       health
##  Min.   :1.000   Min.   :1.000   Min.   :1.000   Min.   :1.000
##  1st Qu.:2.000   1st Qu.:1.000   1st Qu.:1.000   1st Qu.:3.000
##  Median :3.000   Median :1.000   Median :2.000   Median :4.000
##  Mean   :3.156   Mean   :1.494   Mean   :2.284   Mean   :3.543
##  3rd Qu.:4.000   3rd Qu.:2.000   3rd Qu.:3.000   3rd Qu.:5.000
##  Max.   :5.000   Max.   :5.000   Max.   :5.000   Max.   :5.000
##     absences          Grade
##  Min.   : 0.000   Min.   : 1.333
##  1st Qu.: 0.000   1st Qu.: 9.333
##  Median : 2.000   Median :11.333
##  Mean   : 4.435   Mean   :11.267
##  3rd Qu.: 6.000   3rd Qu.:13.333
##  Max.   :75.000   Max.   :19.333

hist(Grade)
```

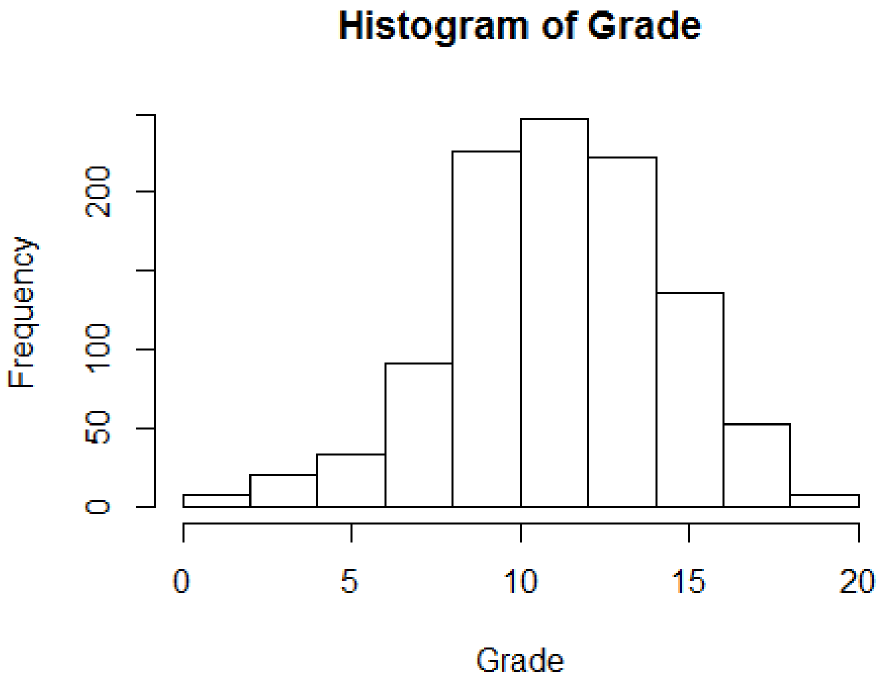

Fig 1: Student-grade frequency distribution.

```
str(e)

## 'data.frame':    1044 obs. of  31 variables:
##  $ school           : Factor w/ 2 levels "GP","MS": 1 1 1 1 1 1 1 1
1 1 ...
##  $ sex              : Factor w/ 2 levels "F","M": 1 1 1 1 1 2 2 1 2
```

```
2 ...
##  $ age              : int  18 17 15 15 16 16 16 17 15 15 ...
##  $ address          : Factor w/ 2 levels "R","U": 2 2 2 2 2 2 2 2 2
2 ...
##  $ family size      : Factor w/ 2 levels "GT3","LE3": 1 1 2 1 1 2 2
1 2 1 ...
##  $ parents cohab.   : Factor w/ 2 levels "A","T": 1 2 2 2 2 2 2 1 1
2 ...
##  $ mom's education  : int  4 1 1 4 3 4 2 4 3 3 ...
##  $ dad's education  : int  4 1 1 2 3 3 2 4 2 4 ...
##  $ mom's job        : Factor w/ 5 levels "at_home","health",..: 1 1
1 2 3 4 3 3 4 3 ...
##  $ dad's job        : Factor w/ 5 levels "at_home","health",..: 5 3
3 4 3 3 3 5 3 3 ...
##  $ reason           : Factor w/ 4 levels "course","home",..: 1 1 3 2
2 4 2 2 2 2 ...
##  $ guardian         : Factor w/ 3 levels "father","mother",..: 2 1 2
2 1 2 2 2 2 2 ...
##  $ travel           : int  2 1 1 1 1 1 1 2 1 1 ...
##  $ study            : int  2 2 2 3 2 2 2 2 2 2 ...
##  $ failures         : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ education support: Factor w/ 2 levels "no","yes": 2 1 2 1 1 1 1 2
1 1 ...
##  $ family support   : Factor w/ 2 levels "no","yes": 1 2 1 2 2 2 1 2
2 2 ...
##  $ paid             : Factor w/ 2 levels "no","yes": 1 1 1 1 1 1 1 1
1 1 ...
##  $ activities       : Factor w/ 2 levels "no","yes": 1 1 1 2 1 2 1 1
1 2 ...
##  $ nursery          : Factor w/ 2 levels "no","yes": 2 1 2 2 2 2 2 2
2 2 ...
##  $ higher           : Factor w/ 2 levels "no","yes": 2 2 2 2 2 2 2 2
2 2 ...
##  $ internet         : Factor w/ 2 levels "no","yes": 1 2 2 2 1 2 2 1
2 2 ...
##  $ romantic         : Factor w/ 2 levels "no","yes": 1 1 1 2 1 1 1 1
1 1 ...
##  $ family bond      : int  4 5 4 3 4 5 4 4 4 5 ...
##  $ free time        : int  3 3 3 2 3 4 4 1 2 5 ...
##  $ social           : int  4 3 2 2 2 2 4 4 2 1 ...
##  $ workday alch.    : int  1 1 2 1 1 1 1 1 1 1 ...
##  $ weekend alch.    : int  1 1 3 1 2 2 1 1 1 1 ...
##  $ health           : int  3 3 3 5 5 5 3 1 1 5 ...
##  $ absences         : int  4 2 6 0 0 6 0 2 0 0 ...
##  $ Grade            : num  7.33 10.33 12.33 14 12.33 ...
```

```
names(e)
```

```
##  [1] "school"            "sex"               "age"
##  [4] "address"           "family size"       "parents cohab."
##  [7] "mom's education"   "dad's education"   "mom's job"
## [10] "dad's job"         "reason"            "guardian"
## [13] "travel"            "study"             "failures"
## [16] "education support" "family support"    "paid"
## [19] "activities"        "nursery"           "higher"
## [22] "internet"          "romantic"          "family bond"
## [25] "free time"         "social"            "workday alch."
## [28] "weekend alch."     "health"            "absences"
## [31] "Grade"

x1 <- e[c(31, 1 : 10)]
pairs(x1)
```

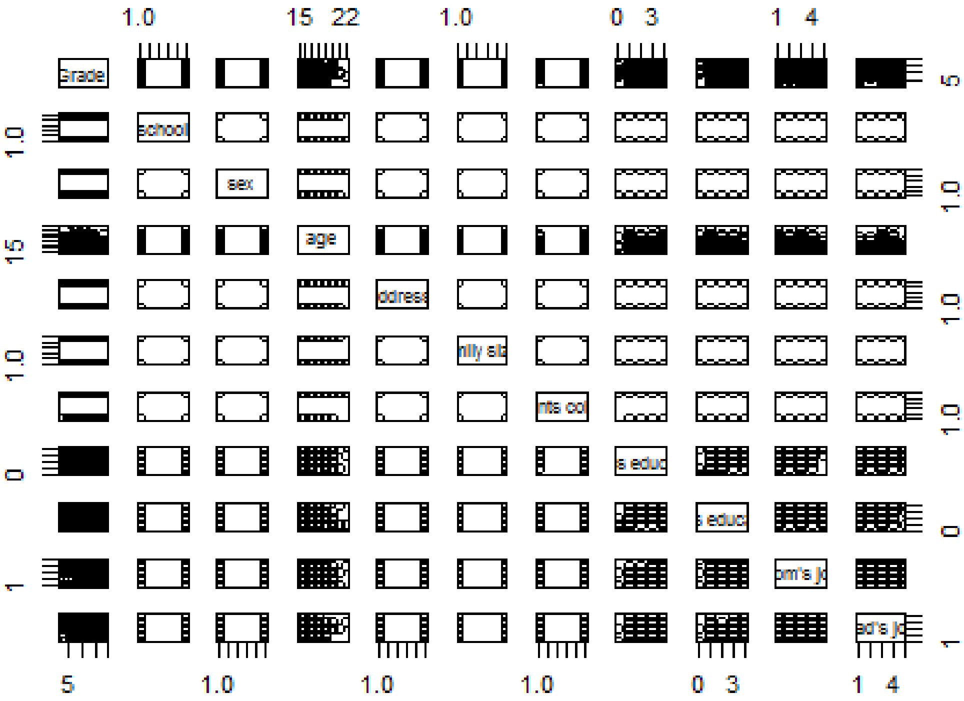

Table 1 : Scatter plot, Grade verses independent predictors 1 through 10.

```
x2 <- e[c(31, 10 : 20)]
pairs(x2)
```

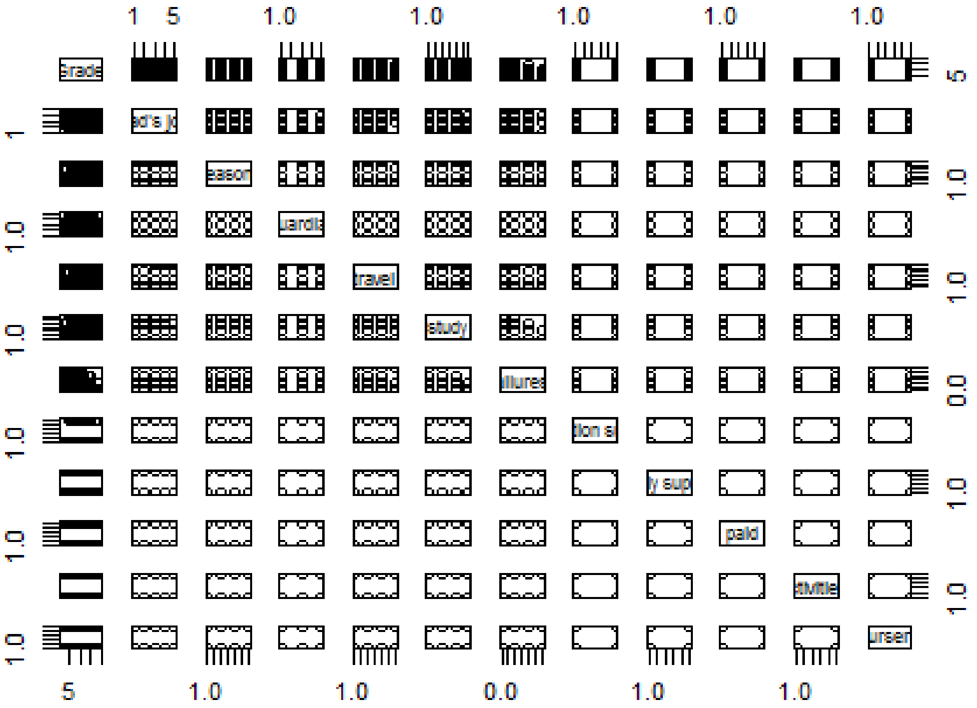

Table 2 : Scatter plot, Grade verses independent predictors 10 through 20.

```
x3 <- e[c(31, 20 : 30)]
pairs(x3)
```

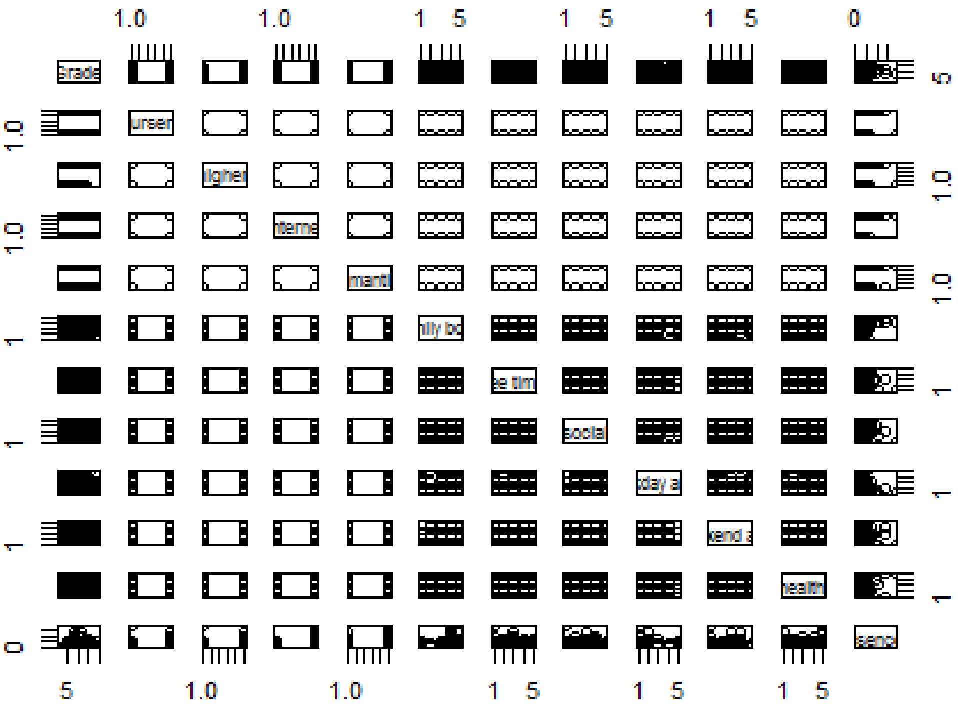

Table 3 : Scatter plot, Grade verses independent predictors 20 through 30.

Model Development

K-fold CV is applied in the fitting of linear models to the training data. Successively, models of lesser complexity are derived (starting with the saturated model), selecting statistically significant predictors that are reported with every model fit. Cross validation indicates a Mean Square Error rate estimate to verify that, in choosing lower complexity models, we are not introducing significant error. Finally, a best-fit model containing significant predictors (showing little difference in MSE from the saturated model), is tested with the clean data to provide an out-of-sample estimate for model performance. The following is a series of progressive model fits performed to find the best possible fit. The saturated model cardinality is 30 variables. 10-fold CV is applied to the MSE estimation of model performance on the test data.

```
names(e)

##  [1] "school"            "sex"               "age"
##  [4] "address"           "family size"       "parents cohab."
##  [7] "mom's education"   "dad's education"   "mom's job"
## [10] "dad's job"         "reason"            "guardian"
## [13] "travel"            "study"             "failures"
## [16] "education support" "family support"    "paid"
## [19] "activities"        "nursery"           "higher"
## [22] "internet"          "romantic"          "family bond"
## [25] "free time"         "social"            "workday alch."
## [28] "weekend alch."     "health"            "absences"
## [31] "Grade"

fit <- glm(Grade~., data = e)
MSE1 <- cv.glm(e, fit, K = 10)$delta[1]
summary(fit)

##
## Call:
## glm(formula = Grade ~ ., data = e)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -10.8517   -1.4833    0.1019    1.8281    7.8999
##
## Coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)             9.718585   1.641229   5.922 4.38e-09 ***
## schoolMS               -0.492338   0.235632  -2.089 0.036919 *  
## sexM                   -0.065729   0.202918  -0.324 0.746068    
## age                     0.030970   0.083072   0.373 0.709372    
## addressU                0.240670   0.221106   1.088 0.276645    
## `family size`LE3        0.369219   0.199709   1.849 0.064783 .  
## `parents cohab.`T       0.023677   0.287473   0.082 0.934375    
## `mom's education`       0.173160   0.126079   1.373 0.169925    
## `dad's education`       0.042871   0.112327   0.382 0.702792    
## `mom's job`health       0.934994   0.442614   2.112 0.034896 *  
## `mom's job`other       -0.020608   0.262211  -0.079 0.937372    
## `mom's job`services     0.524154   0.310235   1.690 0.091426 .  
## `mom's job`teacher     -0.013337   0.410768  -0.032 0.974105    
## `dad's job`health      -0.057531   0.600577  -0.096 0.923704    
## `dad's job`other       -0.065647   0.386378  -0.170 0.865120    
## `dad's job`services    -0.247048   0.404383  -0.611 0.541386    
## `dad's job`teacher      1.133663   0.538623   2.105 0.035562 *  
## reasonhome              0.133123   0.229150   0.581 0.561410    
## reasonother             0.066553   0.311433   0.214 0.830825    
## reasonreputation        0.303609   0.239565   1.267 0.205329    
## guardianmother         -0.220538   0.219213  -1.006 0.314636    
## guardianother           0.217507   0.420273   0.518 0.604896    
## travel                 -0.094595   0.132621  -0.713 0.475841    
## study                   0.418159   0.115143   3.632 0.000296 ***
## failures               -1.476144   0.148519  -9.939  < 2e-16 ***
## `education support`yes -1.398765   0.286959  -4.874 1.27e-06 ***
## `family support`yes    -0.273525   0.188290  -1.453 0.146627    
## paidyes                -0.768545   0.221702  -3.467 0.000549 ***
## activitiesyes           0.097293   0.181423   0.536 0.591887    
## nurseryyes             -0.025260   0.222561  -0.113 0.909661    
## higheryes               1.409229   0.341220   4.130 3.93e-05 ***
## internetyes             0.323375   0.233715   1.384 0.166780    
## romanticyes            -0.448088   0.188898  -2.372 0.017874 *  
## `family bond`           0.102933   0.096962   1.062 0.288680    
## `free time`             0.032757   0.093005   0.352 0.724759    
## social                 -0.218046   0.089048  -2.449 0.014510 *  
## `workday alch.`        -0.117328   0.128143  -0.916 0.360092    
## `weekend alch.`        -0.008548   0.098360  -0.087 0.930768    
## health                 -0.156854   0.063972  -2.452 0.014378 *  
## absences               -0.016961   0.014979  -1.132 0.257773    
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## (Dispersion parameter for gaussian family taken to be 7.662112)
## 
##     Null deviance: 10806.2  on 1043  degrees of freedom
## Residual deviance:  7692.8  on 1004  degrees of freedom
## AIC: 5129.8
## 
## Number of Fisher Scoring iterations: 2
```

A report on statistical significance of saturated model coefficients indicates significant (p < 0.01) predictors of Grade to be study, failures, education, support, paid, and higher.

MSE for 10-Fold CV of fit of saturated model:

```
MSE1

## [1] 7.988418
```

A check for multicolinarity by VIF shows negative results, indicating the potential for linear modeling success (conditioned on all GVIF values being less than 5).

```
vif(fit)

##                          GVIF Df GVIF^(1/(2*Df))
## school               1.457487  1        1.207264
## sex                  1.378085  1        1.173919
## age                  1.444359  1        1.201815
## address              1.322027  1        1.149794
## `family size`        1.125962  1        1.061114
## `parents cohab.`     1.153808  1        1.074155
## `mom's education`    2.738116  1        1.654725
## `dad's education`    2.077963  1        1.441514
## `mom's job`          2.713832  4        1.132916
## `dad's job`          1.890115  4        1.082832
## reason               1.427053  3        1.061060
## guardian             1.472356  2        1.101547
## travel               1.281912  1        1.132215
## study                1.256347  1        1.120869
## failures             1.292695  1        1.136968
## `education support`  1.133132  1        1.064487
## `family support`     1.145957  1        1.070494
## paid                 1.113884  1        1.055407
## activities           1.121040  1        1.058792
## nursery              1.080640  1        1.039538
## higher               1.237127  1        1.112262
## internet             1.225436  1        1.106994
## romantic             1.113774  1        1.055355
## `family bond`       1.115004  1        1.055937
## `free time`         1.252843  1        1.119305
## social              1.433910  1        1.197460
## `workday alch.`     1.857976  1        1.363076
## `weekend alch.`     2.174975  1        1.474780
## health              1.130731  1        1.063359
## absences            1.177898  1        1.085310
```

Second Fit:

A lower complexity model of 5 variables (reported significant), from the saturated model is fitted. All variables included are checked for significance (p < 0.01).

```
fit2 <- glm(Grade ~ study + failures + `education support` + paid +
higher, data = e)
MSE2 <- cv.glm(e, fit2, K = 10)$delta[1]
summary(fit2)
```

```
##
## Call:
## glm(formula = Grade ~ study + failures + `education support` +
##     paid + higher, data = e)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -10.4849   -1.6574    0.0707    1.9185    7.9790
##
## Coefficients:
##                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)              9.2697     0.3692  25.108  < 2e-16 ***
## study                    0.4942     0.1091   4.530 6.57e-06 ***
## failures                -1.6364     0.1415 -11.566  < 2e-16 ***
## `education support`yes  -1.2480     0.2796  -4.463 8.96e-06 ***
## paidyes                 -0.6331     0.2193  -2.887  0.00397 **
## higheryes                1.8935     0.3368   5.622 2.42e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 8.158794)
##
##     Null deviance: 10806.2  on 1043  degrees of freedom
## Residual deviance:  8468.8  on 1038  degrees of freedom
## AIC: 5162.2
##
## Number of Fisher Scoring iterations: 2
```

10-Fold CV MSE estimate of Model II:

```
MSE2

## [1] 8.222127
```

This model shows negligible difference in MSE from the saturated model and contains 26 fewer predictors, thus indicating potential for out-of-sample performance.

```
summary(fit2)

##
## Call:
## glm(formula = Grade ~ study + failures + `education support` +
##     paid + higher, data = e)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -10.4849   -1.6574    0.0707    1.9185    7.9790
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)                9.2697     0.3692  25.108  < 2e-16 ***
## study                      0.4942     0.1091   4.530 6.57e-06 ***
## failures                  -1.6364     0.1415 -11.566  < 2e-16 ***
## `education support`yes  -1.2480     0.2796  -4.463 8.96e-06 ***
## paidyes                   -0.6331     0.2193  -2.887  0.00397 **
## higheryes                  1.8935     0.3368   5.622 2.42e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 8.158794)
##
##     Null deviance: 10806.2  on 1043  degrees of freedom
## Residual deviance:  8468.8  on 1038  degrees of freedom
## AIC: 5162.2
##
## Number of Fisher Scoring iterations: 2

par(mfrow = c(3, 2))

plot(Grade, study)
plot(Grade, failures)
plot(Grade,`education support`, yaxt='n')
```

```
axis(2, labels = c("false","true"), at = c(1, 2))
plot(Grade, paid, yaxt='n')
axis(2, labels = c("false","true"), at = c(1, 2))
plot(Grade, higher, yaxt='n')
axis(2, labels = c("false","true"), at = c(1, 2))
mtext("Significant Factor Plots", side = 3, line = -3, outer = TRUE)
```

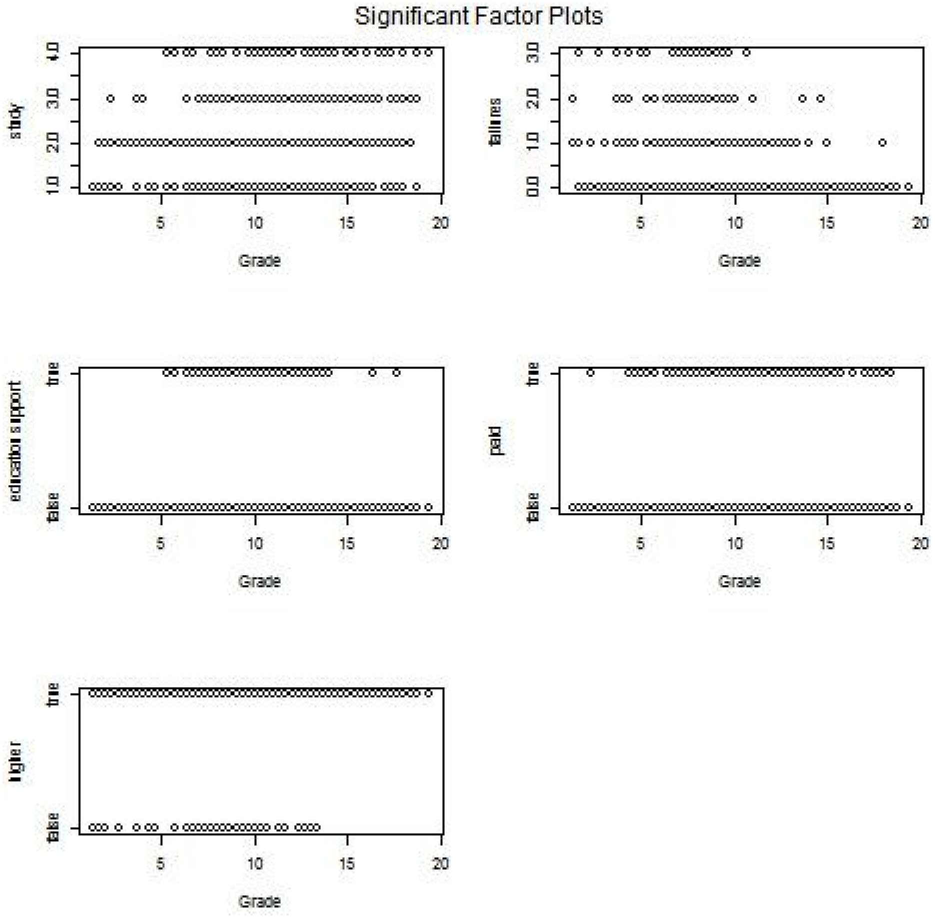

Fig 2: Scatter-plots of Grade verses significant discovered factors.

Results

This results of this work should be seen as a starting point for more advanced studies of success prediction in general education. They may only hold significance for the originating educational department. It appears strictly domain specific, general claims to any general predictive success of any derived models is not generally indicated. This view is surmised from the collected data and available documentation. The intent interpreted is to find a model of specific factors relevant to learning success that are shared between Mathematics an Portuguese, discussed in identical terms: identical variables are chosen for both data sets as collected from student surveys. This indicates an implicit assumption of the study that a uniform learning measure exists between mathematics and language. This assumption appears to be latent in the study variables, as chosen factors are more generally living-condition or non-subject specific. It is apparent from the chosen causal-factors that the data is not sufficiently diverse in academic types to address the generality of their scope. In addition, this study has a limited data quantity relative to the number of the predictors. All inclusions and exclusions of predictors suggested by this regression model should be seen as restricted from any general claims predictive power (in a broader or differing range of academic subjects).

Conclusion

This work could be seen a good use of resources for determining how best to design future studies, specifically, what questions to exclude from study surveys in any following work. Results indicate a reduction in the set of explanatory variables by a factor of 5 in predictive modeling. Certain difficulties are present in interpreting results from the data documentation, indicating restricted application to the listed education categories.