

Fall 2024

CS 302 Data Structures and Algorithm Analysis

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Indiana State University

Homework II: Asymptotic Analysis

Name:

Fall 2024

Data Structures and Algorithm Analysis

Problem 1. Consider two non-negative functions $T_1(N)$ and $T_2(N)$. We know that $T_1(N)$ is O(F(N)) and $T_2(N)$ is O(F(N)) for some non-negative function F(N).

For each of the statements below, answer with True or False. If the statement is True, you must explain why using the formal definition of O. In other words, you must find constants c and N_0 such that the definition of O is satisfied.

If the statement if False, you must give a counter example for which the definition would not be satisfied. In other words, give examples for $T_1(N)$, $T_2(N)$, and F for which the definion of O is not satisfied.

(a) (5 points)

 $T_1(N)$. $C_1(1)$

$$\overline{T_2(N)}$$
 is $O(1)$

$$\Upsilon_{2}(n) \quad \text{is} \quad O(F(n)), \quad \Upsilon_{2}(n) \leq C_{1} \cdot F(n)$$

$$\Upsilon_{3}(n) \quad \text{is} \quad O(F(n)), \quad \Upsilon_{3}(n) \leq C_{2} \cdot F(n)$$

$$\frac{C_{1} \cdot F(n)}{C_{3} \cdot F(n)} = \frac{C_{1}}{C_{3}} \quad \text{is} \quad O(1),$$

$$\frac{C_{1}}{C_{3}} \leq c.1 \qquad \text{True}.$$

(b) (5 points)

 $T_1(N)$ is $O(T_2(N))$

$$T_{1}(n)$$
 is $D(T_{2}(n))$ $T_{1}(n)$ is $D(F(n))$
 $T_{1}(n) \leq C.T_{2}(n)$ for all $n \geq n_{p}$. $T_{2}(n)$ is $D(F(n))$
 $F_{1}(n) \leq C.T_{2}(n)$ for all $n \geq n_{p}$. $F_{2}(n)$ is $F_{2}(n)$
 $F_{3}(n) \leq C.F(n)$
 $F_{3}(n) \leq C.F(n)$
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Problem 2. (10 points) Mohammad claims that when dealing with asymptotic analysis, the base in an exponentiation does not matter. Specifically, Mohammad claims that 10^N is Θ (2^N).

Do you think that Mohammad's claim is correct? Support your answer using the formal definition of Θ (also that of Ω and O).

Hint: This is easier than it looks.

Hint: Can you express 10^N as a function of 2^N ?

has
$$|0_{h} \otimes (J_{h})$$

 $|\cdot| \mathcal{I}_{h} \otimes |0_{h} \otimes (J_{h})$
 $|\cdot| \mathcal{I}_{h} \otimes |0_{h} \otimes |0_{h} \otimes |0_{h}|$ for all $|h| \leq p$
 $|0_{h} = \mathcal{I}_{h} (|od^{3}r|_{0}) \subseteq (oustant)$
 $|0_{h} = |0_{h}| (|od^{3}r|_{0}) \subseteq (oustant)$

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Problem 3. For each of the following code fragments, find the exact number of times that the statement sum++ would execute, and then give your Θ analysis of its runtime. You do not have to prove your Θ analysis, you can simply state it.

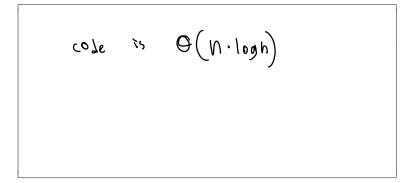
(a) (5 points)

Hint: Look carefully at the loop conditions and iterator variables.

```
code is O(N^3)
```

(b) (5 points) For the code fragment below, you may assume that n is a power of 2.

```
for(int i = n; i > 0; i = i / 2) { _____ (o_j h) 
for(int j = 0; j < n; j++) { _____ (n) 
sum++; } 
}
```



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Problem 4. Given a string of characters, you are given the task of finding whether the characters in that string are unique or not. In other words, you are to find whether any character in that string appears more than once, if so, then the characters are not unique, otherwise, they are. Here's a couple of examples:

1. Consider the string $\{a,b,c,k,l,a,n\}$. The characters here are not unique since a appears more than once

2. Consider the string $\{a,k,l\}$. The characters in this string are unique since each of them appear exactly once in the string.

Constraint 1: In all of the questions below, we will restrict our attention to lower case alphabet characters, i.e., a through z only. The problem wouldn't change much if we drop this constraint, but it just makes it nicer.

Constraint 2: In all of the questions below, you are only allowed to use lists (i.e., arrays), you should not use any other data structure (e.g., hash map, set, etc.).

 Hint : In all of the questions below, you may assume that the length of your input string is n.

(a) (5 points) Give a naive algorithm to check whether the characters in a string are unique. You can write this in any language you would like or a simple pseudocode.

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(b) (10 points) What is the asymptotic runtime of your naive algorithm above? Show your work. You should be able to count the number of times your most frequent line is executed, then propose an asymptotic runtime using O

```
\begin{cases} \Theta_{\Gamma}(i-0; i < h^{-1}; i+0) & \longleftarrow f(h) \\ f_{W}(j_{w}(i+2; j < h) +1+1) & \longleftarrow f(h) \\ \text{if } (string(i) = 1 string(i)) \\ \text{if } \\ \text{if } (string(i) = 2 string(i)) \\ \text{if } \\ \text{if } \\ \text{if } (string(i) = 2 string(i)) \\ \text{if } \\ \text{i
```

(c) (5 points) Can you suggest an algorithm that does better than the naive solution? What is the asymptotic runtime of your improved algorithm?

Hint: You might find that using another array can be helpful.

 ${\it Hint:}$ How do we represent characters in programming languages?

Hint: You should be able to do this in $\Theta(n)$.

hostmap = {}

for i in rang(u):

If string (i) in hostmap:

resturn total

mistanap (i) = string (i)

refurn true.

F(n) is $\Theta(n)$

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