COMP 4102A: Assignment 3

Instructions for submission: Please write a document (pdf or doc) with your solutions on theory questions. Put your code file and the output result in a zip folder including the answers of the theory questions. Note that the total mark of this assignment is out of 130. You are expected to work on the assignment **individually**.

1 Coding: Calibration (40 points)

The goal of this questions is to implement some code that performs calibration using the method described in the class and in book(ch6.3); by first computing a projection matrix, and then decomposing that matrix to find the extrinsic and intrinsic camera parameters. Use the approach described in the slides. I have given you a program, written in C++ that uses OpenCV, called *projection-template.cpp*. This program takes ten given 3d points and projects them into a 2d image using the given supplied camera calibration matrix, rotation matrix and translation vector. Your goal is to write the two routines that are missing, which are *computeprojectionmatrix* and *decomposeprojectionmatrix*. The first routine computes the projection matrix using the method described in Section 6.3.1 of the book, and the second uses the method in Section 6.3.2 to decompose the projection matrix into a camera calibration matrix, rotation matrix and translation vector. It should be the case that the computed camera matrix, rotation matrix and translation vector are the same (or very similar) to the original versions that were used to create the projected points. This shows that your two routines are working properly. A similar routine provided in Python called *projection-template-python.py*. Submit your program source code and the resulting output file assign3-out created by running this modified program. Or show a screen capture of the output which shows that the computed quantities are the same, or close to the same as the original ones used to create the projection matrix.

2 Theory questions(90 points)

1. Consider two points A and B in a simple stereo system. Point A projects to A_l on the left image, and A_r on the right image. Similarly there is a point B which projects to B_l and B_r. Consider the order of these two points in each image on their epipolar lines. There are two possibilities; either they ordered on the epipolar lines in the same order; for example they appear as A_l, B_l and A_r, B_r, or they are in opposite order, such as B_l, A_l and A_r, B_r. Place the two 3d points A and B in two different locations in a simple stereo diagram which demonstrates these two possibilities. (Draw a different picture for each situation).(20 points)

2. There is a simple stereo system with one camera placed above the other camera in the y direction (not the x direction is as usual) by a distance of b. In such a case there is no rotation between the cameras, only a translation by a vector $T = [0, b, 0]$. First compute the essential matrix E in this case. You are given a point p1 in camera co-ordinates in the first image as $(x1, y1, f)$, and a matching point p2 in the second image where p2 is $(x2, y2, f)$. Write the equation of the epipolar line that contains the matching point p2 in camera co-ordinates in the second image. In this case you are given p1 and you have computed E, and you need to write the equation of the line that contains p2 (the free variables are $x2, y2$) using p1 and the elements of E as the fixed variables. Now repeat the entire process again for the case where $T = [b, b, 0]$ (a translation of 45 degrees to the right in the x,y plane), and finally where $T = [0, 0, b]$ (a translation straight ahead in the Z direction). For the particular case where p1= $(0, 1, f)$ what is the equation of the epipolar line for all three situations? And where p1= $(1, 1, f)$ what is the equation of the epipolar line in these three situations? I want these all equations simplified as far as possible, with all terms grouped and collected. Draw the epipolar lines for all three cases; that is you need to draw the epipolar lines in the right image for the two cases where p1 = $(0, 1, f)$ and p1 = $(1, 1, f)$. I want to have three different diagrams where in each diagram you draw these two epipolar lines. Hint, in each of these three situations where we move the second camera by a different translation vector so this means that we will have three different equations, and three different diagrams.(30 points)

3. In simple stereo Z = f B/d. If the baseline B is 0.5 meter, Z is 2.0 meters, and f is 50 millimeters what is the value of d in mm? Repeat this process for Z = 1 meter, and 0.5 meter to get two more values for d in mm (same f and B).
If we are measuring depth at a given value of Z then we have a certain accuracy in the measurement which depends on how much error there is in computing the disparity. In turn, the error in disparity computation depends on how accurately we can locate a feature, such as a line or corner in the image. In practice, the error in computing the location of a feature is fixed; it does not change regardless of the value of Z for that feature. Assume that this error in disparity d for locating a feature is $+-$ 1mm. In other words, for a computed value of disparity d, the true value is d plus or minus 1mm. Now for each of the three given values of d computed above, compute two new values for Z; Z high when d = d - 1 mm, and z low when d = d + 1mm. Now in each of the three cases above compute the function Zdiff = Z high - Z low. You will now have three values for Z diff. We call these three values Zdiff(2 meters), Zdiff(1 meter) and Zdiff(0.5meter). In these three cases we have cut the value of the value of Z by one half, from 2 meters, to 1 meter to 0.5 meters. Hypothesize a relationship that seems to hold (approximately) on the ratio of the Z diffs when we cut Z in half. Look at the ratio of Zdiff(2 meters)/Zdiff(1 meter) and Zdiff(1 meter)/Zdiff(0.5 meter). If the number these ratios are converging to is not obvious, then repeat the experiment again but compute Zdiff(0.25meter) and consider the ratio which is Zdiff(0.5 meter)/Zdiff(0.25 meter). Guess the obvious number! Another way to say this is to consider the following statements, where X is that same number; If we are doing stereo measurements at a given distance Z we expect a certain accuracy in the measurements ($+ - \Delta$Z). If we now measure at one half the distance of Z, which is Z/2 we expect our accuracy to improve by a factor of X. So when we cut our depth in half, then depth resolution improves by a factor of X. Tell me the number X. You also need to write down each of these ratios properly to get the full marks.(20 points)

4. In Figure 1 on the last page there are three cameras where the distance between the cameras is B, and all three cameras have the same focal length f. The disparity $d_L = x_0 - x_L$, while the disparity $d_R = x_R - x_0$. Show that $|d_L| = |d_R|$. You should prove this relationship holds mathematically by using the appropriate equations. This proof is trivial, but when you write the proof you should use an English sentence to explain why these particular equations are true. (20 points)

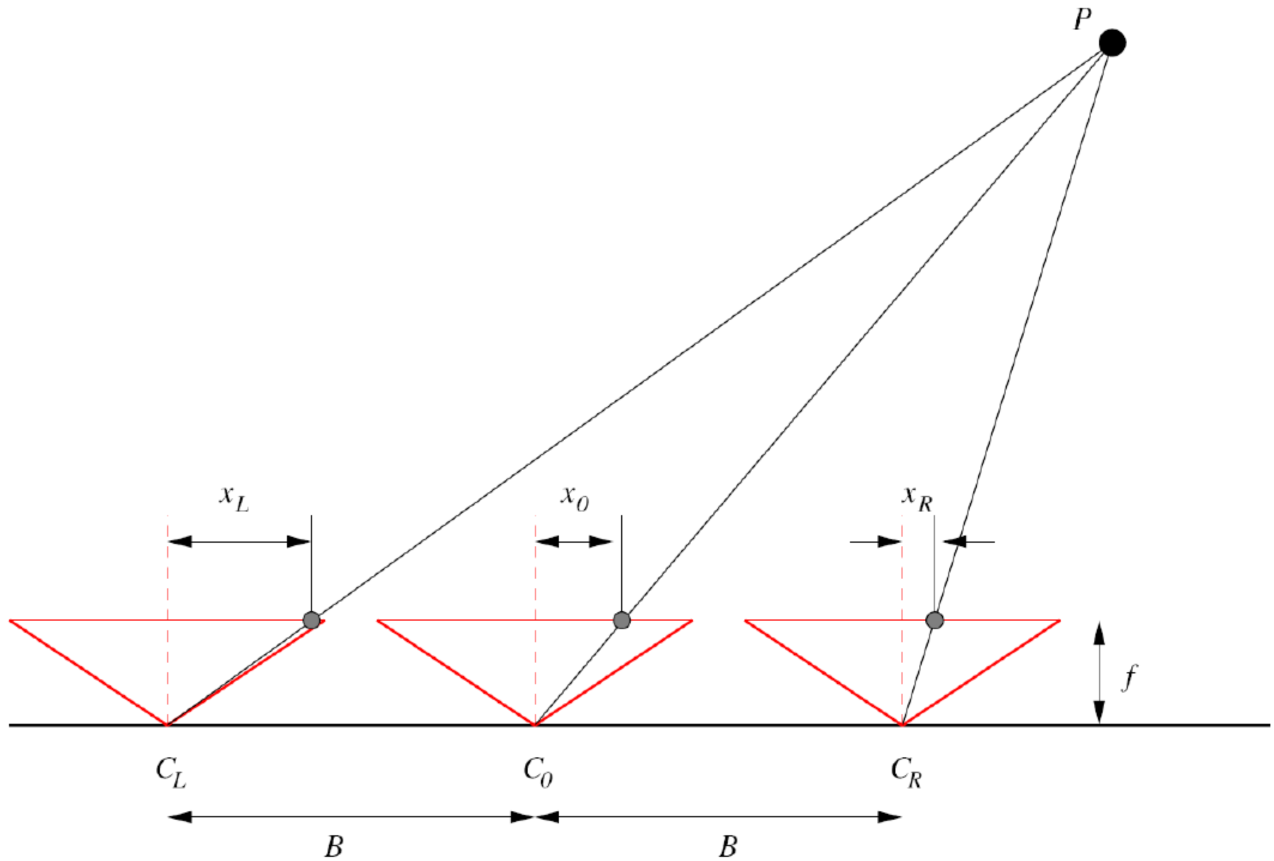

Figure 1