

M-Array Frequency Key Shifting
and Bit Error Rate vs Noise
Group 13

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Introduction: [Shahbaz]

This project demonstrates constructed circuits that perform M-FSK modulation and demodulation based on MATLAB code. In this system we are testing the effect of noise it has on the system to correctly demodulate signal data. It covers the concept of Bit Error Rate (BER) that can be defined as the relationship between the number of bits that were received as an error to the number of bits that were transmitted. It depends on the medium between the transmitter and receiver. High signal to noise ratio signifies good medium, thus resulting in small BER (*Understanding bit-error-rate HOTLink® - Cypress, n.d.*)

External Literature: [Thomas]

The first source for this project is a book called fundamentals of communications systems. This book provides a formula for the carrier signal that uses the carrier frequency as a base frequency instead of a central frequency which helped to simplify the Modulator block. It also suggested using phi to orthogonally spread the wave function to reduce noise impact. Finally it gives a formula for the Signal to Noise Ratio which proved useful in the creation of the Simulink unit

The next source used provides a formula for converting decibels to linear, which combined with the Noise from the prior source, allows us to calculate Gaussian noise to add to the system based on the system's Energy per Bit and a chosen Signal to Noise Ratio. This is really important as it allows testing of the system at different SNR levels, facilitating tests of SNR vs BER.

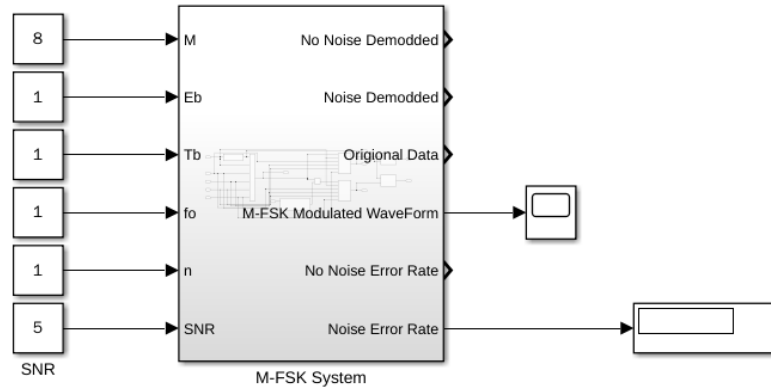
The final source provides a university lab that was the basis for the MATLAB script, used as a base to test the Simulink modulator against.

Framework: [Tingxun]

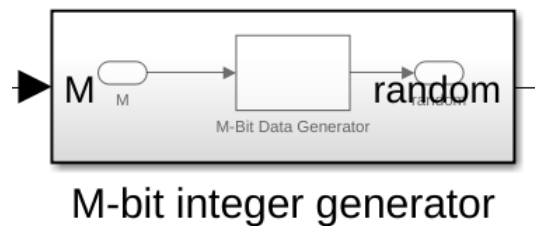
The framework we use is matlab, and the simulation of MFSK is completed through matlab code and matlab simulink. MATLAB is a programming language commonly used for complex computational mathematics through the operation of matrices. Matlab simulink is a graphical, user-interactive MATLAB feature. Usually used for Model-Based Design. Simulate the MFSK system through MATLAB simulink, input different M (2, 4, 8) and different SNR (Signal-to-noise ratio). Different demand results are output when connecting different ports. MATLAB code is a highly readable code file that we use to draw and simulate the examples given in the research paper and add new functions.

Methodology: [Thomas & Tingxun]

This M-FSK modulation testing unit consists of six inputs and 6 outputs. The input for the size of the M-array Frequency Shift Keying unit is required, along with the signal energy per bit, the period per bit, the base frequency and the step size. This unit outputs the demodulated data, with and without noise, the original data signal, the modulated wave signal, and the error rates.



The unit starts with an M bit number generator. The generator will generate a random number between 0 and $2^{\log_2(M)}-1$. This will be used as a source of entropy and a random data source, allowing the unit to test.



M-FSK Modulator: [Thomas]

The units modulator generates waves based on the following function as shown in [1, eq 9.5.2]:

$$u_m(t) = \sqrt{\frac{2\xi_b}{T_b}} * \cos(2\pi f_c t + 2\pi m \Delta f t)$$

This equation can be transformed by the following

$$u_m(t) = \sqrt{\frac{2\xi_b}{T_b}} * \cos(2\pi(f_c + m\Delta f)t)$$

From here we can write our frequency change in regards to the bit frequency, size of M, and the portion of the bit frequency.

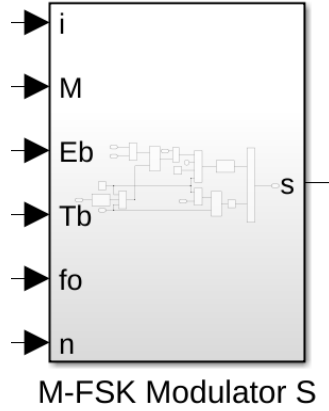
$$\Delta f = \frac{nf_b}{\log_2(M)}$$

So given,

$$f_b = \frac{1}{T_b}$$

Therefore

$$u_m(t) = \sqrt{\frac{2\xi_b}{T_b}} * \cos(2\pi(f_c + \frac{mn}{T_b \log_2(M)})t)$$



This equation produces a frequency modulated wave based on the input data m , labeled as i in figure 4. The waveform can then be spread further to reduce impact of noise on the bit error rate. Modulator Phi spreads the waveform

$$u_m(t)$$

by a factor of

$$\frac{u_m(t)}{\sqrt{\xi_b \log_2(M)}}$$

Noise Generator: [Shahbaz]

After modulation, gaussian white noise is added to the system. The noise can be randomly generated based on the desired SNR and the given energy per bit. This is done by the rearrangement of the formula

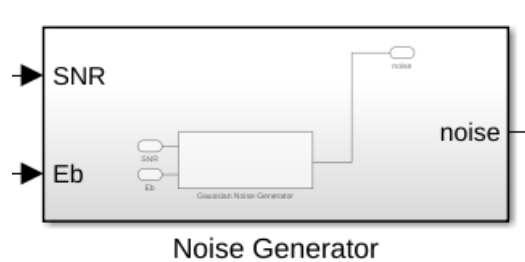
$$SNR = \frac{\xi_b}{N_o}$$

to

$$N_o = \frac{\xi_b}{10^{\frac{SNR}{10}}}$$

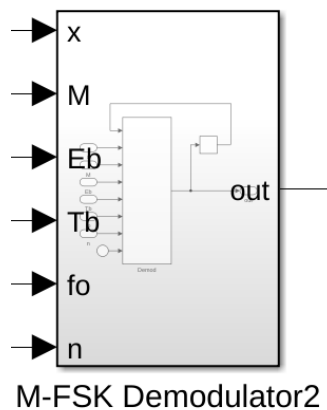
as seen in [2, eq. 2]. With the noise we can generate a random sinusoidal noise function with an amplitude within the specified range of noise.

$$NoiseWave = N_o * \cos(2\pi(rand))$$



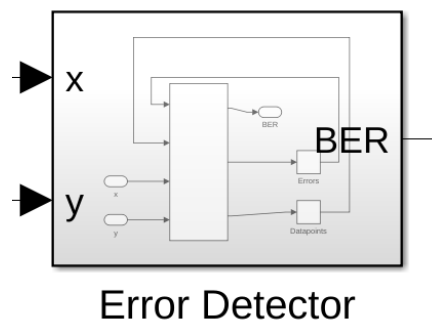
M-FSK Demodulator: [Thomas]

Once noise is applied to the transmitted wave function we can begin to demodulate the waveform. Given that the conditions that create each possible modulated wave are known, the demodulator can perform a noncoherent demodulation. This demodulation method requires the demodulator to generate waveforms identical to each level m in M . These waves are then compared to the input wave, where the most similar wave function is assumed to be correct.



Error Detector: [Shahbaz]

Once the waveform is demodulated the device compares how much of the retrieved data is the same as the original transmitted data. From this the Bit Error Rate is calculated and displayed. Using the average error rate obtained from 100 trials at each level of N and SNR, the relationship between the Bit Error Rate and SNR was then plotted.

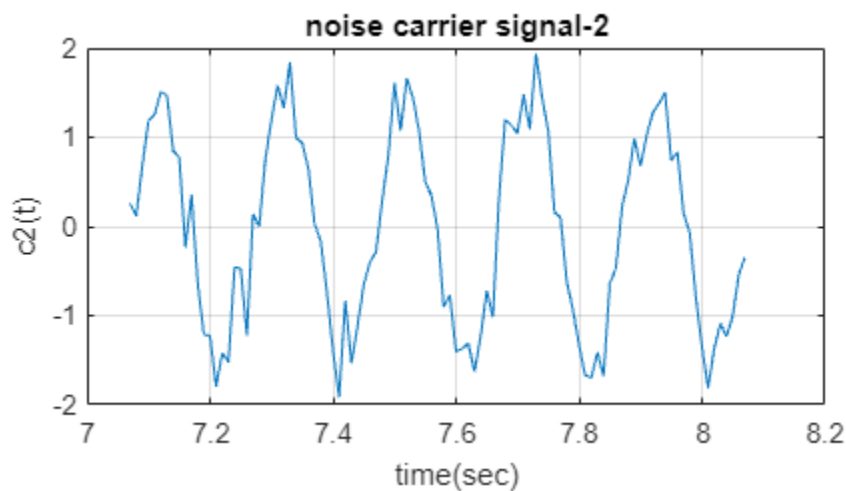
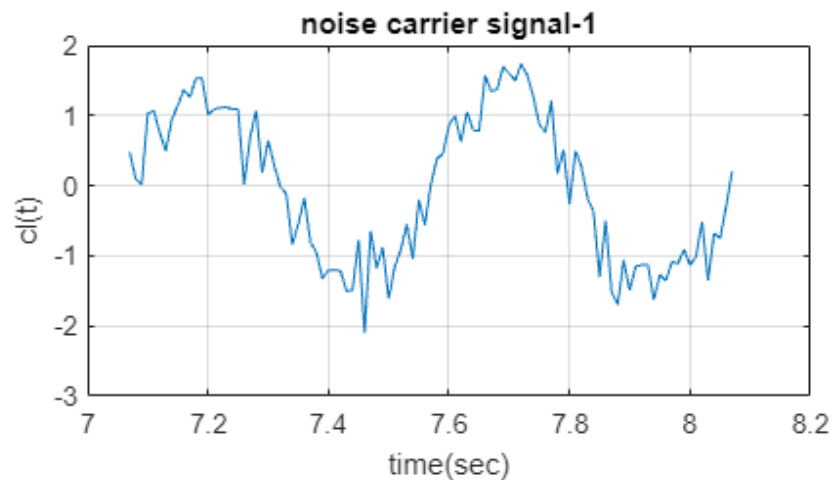


MATLAB Script: [Tingxun]

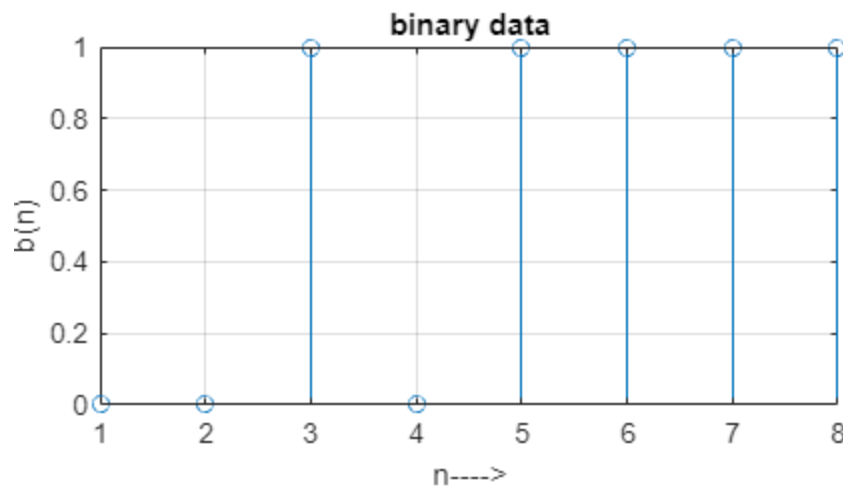
This component system in simulink was based on a simplified version of an FSK modulator simulated in MATLAB scripts, where the Bit Energy is assumed to be 1.

$$\text{carrier signal} = \sqrt{\frac{2}{T_b}} \cos 2\pi f t \quad 0 \leq t \leq T_b$$

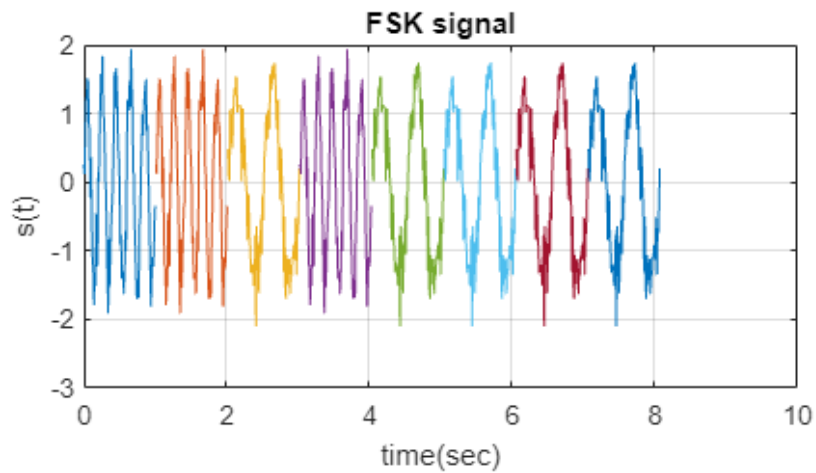
Different carrier signals are generated by different input frequencies. Add Gaussian noise to carrier signals through matlab built-in function $\text{noiseSignal} = \text{awgn}(\text{carrierSignal}, \text{SNR})$.



Then, make the code generate 8 random message signals (BFSK: 0 or 1, 4FSK: 00, 01, 10 or 11, 8FSK: 000, 001, 010, 100, 011, 110, 101 or 111).

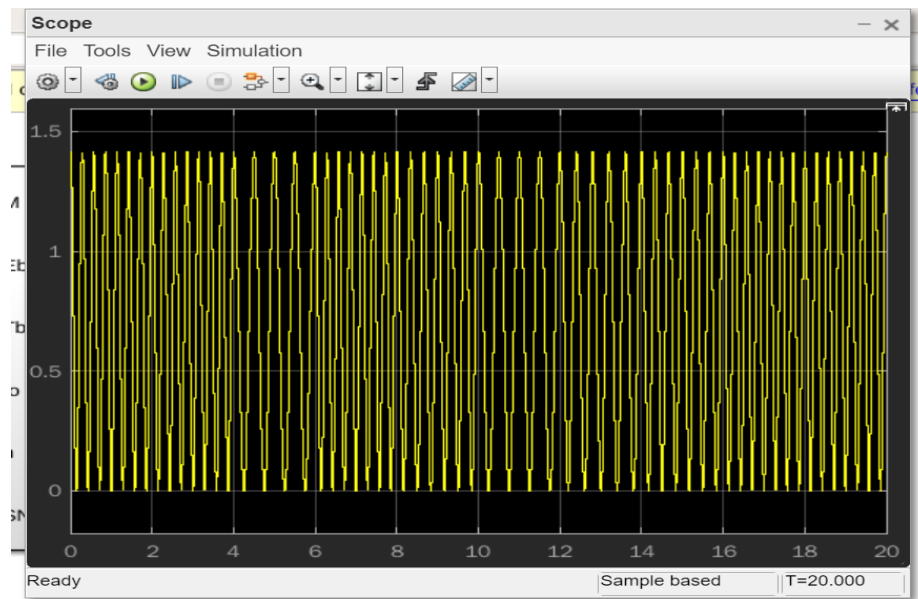


Then multiply the carrier signal and the message signal and add them together to get the FSK modulated signal. Finally, plot the carrier signal, message signal and FSK modulated signal.

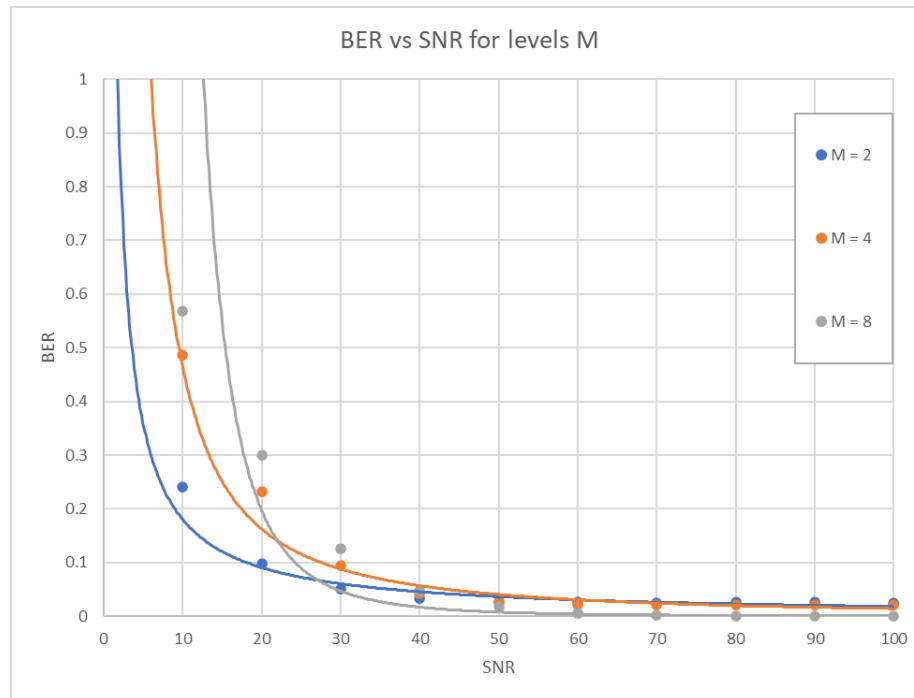


After completing the code of BFSK modulation, write the corresponding code of 4FSK and 8FSK based on it (in the submit file).

Results: [Shahbaz]



SNR		10	20	30	40	50	60	70	80	90	100
BER	M=2	0.240055	0.098226	0.051379	0.033223	0.025797	0.025922	0.025482	0.025687	0.025697	0.025317
	M=4	0.487211	0.232774	0.095042	0.041994	0.026007	0.021364	0.021364	0.021644	0.021794	0.021579
	M=8	0.567551	0.299745	0.126307	0.048236	0.017766	0.005707	0.001734	0.000715	0.00049	0.000395



Plotting the Bit Error Rate vs Signal to Noise Ratio displays the negative exponential relationship between the SNR and BER. As SNR increases the BER approaches 0. The larger M gets, the more a low SNR effects accuracy. However, the increase in accuracy improves at an accelerated rate beyond the knee of the trend respectively.

Conclusion: [Shahbaz]

The Matlab Simulink M-FSK device performed functionally and demonstrated the relationship between the Bit Error Rate and the Signal to Noise Ratio as different levels of M.

Bibliography:

- [1] J. G. Proakis and M. Salehi, *Fundamentals of Communication Systems*. Pearson, 2014.
- [2] "A guide to SDR and DSP using python," *PySDR*. [Online]. Available: <https://pysdr.org/content/noise.html>. [Accessed: 15-Apr-2022].
- [3] n.d., *SRM Institute of Science and Technology lab*. Available: https://webstor.srmist.edu.in/web_assets/srm_mainsite/files/files/comm_lab_manual_final.pdf