

Reiknigreind - Reikniverkefni 2

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27. apríl 2014

Monte Carlo Spilari

Niðurstöður: Þegar Monte Carlo spilaði við Random, þá vann Monte Carlo 46 skipti af 60, svo Monte Carlo þefur töluvert mikið forskot á móti random spilara.

Hinsvegar, þá fer Monte Carlo vs. Monte Carlo Plus 15–5–40, þ.e. Monte Carlo Plus vann 40 skipti af 60, eða $2/3$, og 5 jafntefli. Það er töluvert forskot, en oftast þegar Monte Carlo vann, þá var það vegna þess að hann var kominn með svikamyllu, þ.e. þannig að sama hvað Monte Carlo Plus gerði, þá mundi Monte Carlo vinna.

Heuristic

Ég ákvað að prófa probabilistic heuristic, sem metur hag gildi (utility value) ástands útfrá því hve langt það er frá sigri/tapi, þ.e. gefur hátt utility fyrir skjótan sigur, (og því ætti hann að velja það að blokka/sigra ef það er í boði), en lægra fyrir sigur sem er lengra í burtu. Einnig er gefin mikil refsing ef núverandi spilari er búinn að tapa, en mikill ávinningur ef hann er búinn að vinna.

Þegar Heuristic keppir við Monte Carlo þá sigrar það í 54 skipti af 60, sem er betra en gengið hjá Monte Carlo Plus Þegar það keppir við Monte Carlo Plus, þá er staðan hinsvegar jöfn, 30 – 30, svo það virðist sem það sé jafn gott og Monte Carlo Plus, en það kemur heim og saman við það að Heuristic er að hegða sér mjög líkt Monte Carlo Plus.

Þegar Heuristic keppir við Board Inversion Heuristic, þá fer það 57 – 3 fyrir Heuristic, svo Heuristic virðist vera betra mat á stöðunni en board inversion, með w sem w sem gefið er í kynningunni.

Þegar Board Inversion Heuristic keppir við Monte Carlo, þá fer það 49–11 fyrir Monte Carlo.

Reinforcement Learning

Við forritið var bætt við $TD(\lambda)$ útfærslu, sem var unnin út frá $TD(0)$ matlab lausninni sem við fengum gefna. Þegar $\lambda = 0$, þá hagar það sér eins og $TD(0)$, og skilar álíka win-rate og það gerði fyrir $TD(0)$, sem bendir til þess að útfærslan sé rétt. $TD(1)$ hagar sér eins og Monte Carlo skv. fræðunum, en með $\lambda = 1$ fæ ég mjög álíkt winrate og Monte-Carlo gegn random, en hinsvegar þegar að ég læt það keppa við Monte Carlo, þá virðist það ganga mun verr, en það sigrar aðeins 13 skipti af 60. Það er sennilega vegna þess að í Monte-Carlo er ég að taka meðaltal út frá stöðunni eins og hún er núna, en í Board Inversion Heuristic með $\lambda = 1$ er ég í raun ekki að taka mið út frá stöðunni eins og hún er, heldur er ég bara að byggja á því sem ég hef séð áður. Þegar $\lambda = 0.7$, þá er þetta ennþá verra, en á móti Monte Carlo, þá fer leikurinn 11–3–46, þ.e. Board Inversion Heuristic sigrar 11 leiki af 60, og það eru 3 jafntefli. Það verður að teljas frekar slappt, svo hugsanlega er galli í implementationinu á $TD(\lambda)$

Board Inversion Heuristic mundi sennilega standa sig betur ef það væri þjálfað í fleiri episode, en hjá mér eru það aðeins 2000 episode sem það fer í gegnum, en það er sökum þess hve lengi þetta tekur að keyra. Einnig væri hægt að tune-a gildin á λ , α og γ í forritinu til þess að fá betri niðurstöður.

N-tuples voru implementuð á svipaðan hátt og þau voru gerð í sýnidæmi, þ.e. þau voru generateuð þannig að þau væru eins og snákar í borðinu, en ekki random eins og stungið var uppá. Þau voru síðan notuð til að þjálfá vigtir fyrir þau tuple.

Þjálfun á spilaranum fyrir N-tuples tók rosalega langan tíma, en það er sennilega vegna þess að það var ekki útfært með fylkjum, heldur með einfaldari hlutum í python. Það var því ekki þjálfað fyrir fleiri en 2000 episode.

Þar sem að þetta eru heldur fá episode (talað er um 100 milljón episode í greininni), þá stóð N-tuples sig frekar illa á móti Monte-Carlo Plus (selective útgáfunni), en staðan var 4–56 Monte-Carlo Plus í vil. Þar sem að Monte-Carlo Plus var mjög álíkt mínu Heuristic, þá áætluð við að það muni fara svipað gegn því.

Það verður að teljast heldur slappt, en eins og bent hefur verið á, þá mundi niðurstaðan vera mjög ólík ef þjálfuð væru fleiri episode. Sennilega væri hægt að bæta þessa niðurstöðu með því að láta vigtirnar vera þjálfðar gegn Monte-Carlo Plus spilara, frekar en random eins og gert er, því þá mundi N-tuples miða frekar að að vinna þann spilara.

Hvað ég hef lært

Eftir að hafa unnið þetta verkefni hef ég núna mun betri skilning á hvernig TD og Monte Carlo hermanirnar virka, sérstaklega eftir að maður útfærði það sjálfur. Einnig veitti útfærslan á N-tuples mér innsýn yfir í hvernig má búa til sniðugt subspace af featurespace, til þess að geta ráðið við hversu stórt það verður.

Viðauki

../connect4.py

```
1 from random import choice
2 from pylab import *
3 from util import *
4
5 class connect4(object):
6     #Creates new game
7     def __init__(self, state = None, currPlayer = None,
8         alpha=0.5, w = givenw, ntups = None, verbose = False,
9         shape = (7,6)):
10         self.shape = shape
11         self.state = zeros(shape) if state is None else
12             state
13         self.win = None
14         self.alpha = alpha
15         self.w = w
16         self.currPlayer = 1 if currPlayer is None else
17             currPlayer
18         self.color = True
19         self.ntups = ntups
20         self.verbose = verbose
21
22     def getFeatures(self):
23         if self.ntups is not None:
24             ks = []
25             N = 0
26             for tup in self.ntups:
27                 k = N
28                 N += 3**len(tup)
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25         for i,(x,y) in enumerate(tup):
26             s = self.state[x,y]
27             s = 1 if s < 0 else 2 if s > 0 else 0
28             k += s*3**i #3 possible states
29             ks.append(k)
30             phi = zeros((N,))
31             phi[ks] = 1
32     else:
33         cop = copy(self.state)
34         phi = cop.flatten()
35     return phi
36
37 def makeNtups(self,n=8,numtups=70):
38     #if self.verbose:
39     print("Creating_%d_%d-tuples" % (numtups,n))
40     ntups = []
41     for i in range(numtups):
42         if self.verbose:
43             print("Creating_tuple_%d" %(i+1,))
44         x,y = randint(0,self.shape[0]), randint(0,self.
45             shape[1])
46         snake = []
47         for _ in range(n):
48             snake.append((x,y))
49             while (x,y) in snake:
50                 minx = -1 if x-1 >= 0 else 0 #Inclusive
51                 maxx = 2 if x+1 < self.shape[0] else 1
52                 #Exclusive
53                 miny = -1 if y-1 >= 0 else 0
54                 maxy = 2 if y+1 < self.shape[1] else 1
55                 x += randint(minx,maxx)
56                 y += randint(miny,maxy)
57             ntups.append(snake)
58     self.ntups = ntups
59
60 def copy(self):
61     return connect4(state = copy(self.state),
62                     currPlayer = self.currPlayer,
63                     shape = self.shape,

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63         alpha = self.alpha ,
64         w = self.w,
65         ntups = self.ntups ,
66         verbose = self.verbose)
67
68
69 #Make a legal move
70 def makeRandomMove(self):
71     legalMoves = self.getLegalMoves()
72     move = choice(legalMoves)
73     return self.simulate(move)
74
75 def getLegalMoves(self):
76     legal = lambda x: self.isLegal(x)
77     return list(filter(legal ,range(0 ,self.shape[1])) )
78
79
80 def isLegal(self ,move):
81     t = transpose(self.state)
82     fi = find(t[move]==0)
83     return len(fi) > 0
84
85
86 # 0 $\leq$ action $\leq$ shape[0]
87 # colour = -1 $\wedge$ 1
88 def simulate(self ,action ,colour = None):
89     if colour is None:
90         colour = self.currPlayer
91         self.currPlayer *= -1
92
93     t = transpose(self.state)
94     indicesWhereEmpty = find(t[action]==0)
95     row = argmax(indicesWhereEmpty)
96     column = action
97     self.state[row][column] = colour
98     self.state = transpose(t)
99     self.win = self.checkwin(self.state)
100     return self.state , self.win
101
102 def rollout(self ,move):

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103         sumReward = 0
104         simmove = self.copy()
105         state, win = simmove.simulate(move)
106         for i in range(5):
107             simulation = simmove.copy()
108             while win is None:
109                 state, win = simulation.makeRandomMove()
110             sumReward += win
111         return sumReward * currPlayer
112
113     def rolloutPlus(self, move):
114         sumReward = 0
115         player = self.currPlayer
116         simmove = self.copy()
117         state, win = simmove.simulate(move)
118         for i in range(5):
119             simulation = simmove.copy()
120             while win is None:
121                 if simulation.currPlayer == player:
122                     obvMove = simulation.obviousMove()
123                     if obvMove is not None:
124                         state, win = simulation.simulate(
125                             obvMove)
126                 else:
127                     state, win = simulation.
128                         makeRandomMove()
129             else:
130                 state, win = simulation.makeRandomMove
131                 ()
132             sumReward += win
133         return sumReward * currPlayer
134
135     def winningMove(self):
136         legalMoves = self.getLegalMoves()
137         for move in legalMoves:
138             simulation = self.copy()
139             state, win = simulation.simulate(move)
140             if win == self.currPlayer:
141                 return move

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140         return None
141
142     def blockMove(self):
143         legalMoves = self.getLegalMoves()
144         for move in legalMoves:
145             simulation = self.copy()
146             simulation.currPlayer = -1*self.currPlayer
147             state, win = simulation.simulate(move)
148             if win == -1*self.currPlayer:
149                 return move
150         return None
151
152
153     def obviousMove(self):
154         winMove = self.winningMove()
155         if winMove is not None:
156             return winMove
157         blockMove = self.blockMove()
158         if blockMove is not None:
159             return blockMove
160         return None
161
162     def learnWFromTd(self, eps = 0.1, alpha = 0.01,
163                    numEpisodes = 10000, lamb = 0, gamma=
164                    1, useNtups = True):
165         if useNtups:
166             self.makeNtups()
167         n = len(self.getFeatures())
168         w = zeros(n,)
169         ntups = self.ntups
170         e = zeros((n,2))
171         for episode in range(numEpisodes):
172             trainee = connect4(w = w, ntups=ntups)
173             phi = zeros((n,2))
174             counter = 0
175             while trainee.win is None:
176                 if self.verbose:
177                     print(trainee)
178                     print("Learning_episode_%d_of_%d" %(
179                         episode, numEpisodes))

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178         counter += 1
179         legalMoves = trainee.getLegalMoves()
180         if (rand(1)[0] < eps):
181             move = choice(legalMoves)
182         else:
183             move = trainee.boardInversionHeuristic(
184                 trainee)
185             player = trainee.currPlayer
186             playerInd = (player+1)//2
187             trainee.simulate(move)
188             phinew = player*trainee.getFeatures()
189             delta = 0 + gamma*logsig(dot(w,phinew)) -
190                 logsig((dot(w,phi[:,playerInd])))
191             e[:,playerInd] = gamma*lamb*e[:,playerInd]\
192                 +dlogsig(dot(w,phi[:,
193                     playerInd]))*transpose(
194                     phi[:,playerInd])
195
196             if(counter > 2):
197                 w += alpha*delta*e[:,playerInd]
198                 phi[:,playerInd] = phinew
199             if trainee.win == 1:
200                 reward = 1
201             elif trainee.win == -1:
202                 reward = 0
203             else:
204                 reward = 0.5
205
206             deltapos = reward - logsig(dot(w,phi[:,0]))
207             deltaneq = (1-reward) - logsig(dot(w,phi[:,1]))
208             w += alpha*deltapos*e[:,0]
209             w += alpha*deltaneq*e[:,1]
210             if not self.verbose:
211                 print("Learning_episode_%d_of_%d" %(episode
212                     , numEpisodes),end="\r")
213
214     self.w = w
215     return w

```



```

213 def boardInversionHeuristic(self, sim):
214     features = sim.getFeatures()
215     w = self.w
216     legalMoves = sim.getLegalMoves()
217     if len(legalMoves) == 0:
218         return float("-inf") #Don't want to make
                illegal move.
219     phi = zeros((len(w), len(legalMoves)))
220     for i, move in enumerate(legalMoves):
221         cp = sim.copy()
222         cp.simulate(move)
223         phi[:, i] = cp.getFeatures()
224     bestMove = legalMoves[argmax(sim.currPlayer*tansig(
                dot(w, phi)))]
225     return bestMove
226
227
228 def monteCarloPlay(self):
229     legalMoves = self.getLegalMoves()
230     rewards = [self.rollout(move) for move in
                legalMoves]
231     bestMove = legalMoves[argmax(rewards)]
232     return self.simulate(bestMove)
233
234 def monteCarloPlayPlus(self):
235     obvMove = self.obviousMove()
236     if obvMove is not None:
237         return self.simulate(obvMove)
238     legalMoves = self.getLegalMoves()
239     rewards = [self.rolloutPlus(move) for move in
                legalMoves]
240     bestMove = legalMoves[argmax(rewards)]
241     return self.simulate(bestMove)
242
243
244 def heuristic(self, sim):
245     #Our heuristic here is very similar to
246     #The one used by the monte carlo player,
247     #Except that we use distance from the win
248     #As a metric as well, such that a

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249 #win in the next move is much better than a win
250 #Two moves later, and a loss in the next move
251 #Is worse than a loss two moves later
252 #And we average over 7 games.
253 #This would cause us to block if possible,
254 #and win if possible (how much depends on alpha)
255 #but also block in the better way if possible
256 #alpha controls how fast the function drops
257 alpha = self.alpha
258 #alpha = 0.9
259 reward = 0
260 player = currPlayer
261 if sim.win is not None:
262     #Large win or punishment for win or loss
263     return sim.win*currPlayer*100
264 for i in range(5):
265     simulation = sim.copy()
266     win = None
267     count = 0
268     while win is None:
269         state, win = simulation.makeRandomMove()
270         count += 1
271     #We exponent alpha to the power of moves.
272     #Since it is < 1, it becomes smaller
273     #The further away we are.
274     reward += 10*win*(alpha**count)
275 return reward*currPlayer
276
277
278 def heuristicPlay(self, heuristicFunc = None):
279     if heuristicFunc is None:
280         heuristicFunc = self.heuristic
281     legalMoves = self.getLegalMoves()
282     simulation = self.copy()
283     heuristicVals = []
284     for move in legalMoves:
285         sim = simulation.copy()
286         state, win = sim.simulate(move)
287         heuristicVals.append(heuristicFunc(sim))
288

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289         bestMove = legalMoves[argmax( heuristicVals )]
290         return self.simulate(bestMove)
291
292
293
294
295
296     def __str__( self ):
297         oldOptions = get_printoptions()
298         if self.color:
299             set_printoptions(linewidth = 100,formatter = {"
300                 float":colorPrinter})
301             BLUE = '\033[34m'
302             ENDC = '\033[0m'
303             header = "\n_"
304             header += BLUE + "_0"
305             for i in range(1,self.shape[1]-1):
306                 header += " _ _ _ %d" %(i ,)
307             header += " _ _ _ " + str(self.shape[1]-1) + ENDC
308             header += " _ ] \n \n"
309             end = "\n"
310         else:
311             header = ""
312             end = ""
313         s = header + str(self.state) + end
314         set_printoptions(**oldOptions)
315         return s
316
317     # 0 is draw, 1,-1 is win for that colour, None is game
not over
318     def checkwin(self , state):
319         s = state.flatten()
320         for winpos in winningLocations:
321             posSum = sum(s[winpos])
322             if abs(posSum) == 4:
323                 return sign(posSum)
324
325         if all(state,1)[0]:
326             return 0

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327         return None
328
329 def resultsToString(results):
330     st = ""
331     for color in [-1,0,1]:
332         st += colors[color] + ":_" + str(results[color]) + \
333             "_" if color in results else ""
334     return st
335
336 def colorPrinter(x):
337     GREEN = '\033[32m'
338     YELLOW = '\033[33m'
339     BLACK = '\033[30m'
340     BLUE = '\033[34m'
341     RED = '\033[31m'
342     ENDC = '\033[0m'
343
344     START = "" if x == 0 else BLUE if x < 0 else RED
345     ADDSPACE = '_' if x >= 0 else ""
346     if x == 0:
347         return "___"
348     return START + ADDSPACE + ("%0f." % x) + ENDC
349
350
351 if __name__ == "__main__":
352     color = True
353     GREEN = '\033[32m'
354     BLUE = '\033[34m'
355     RED = '\033[31m'
356     ENDC = '\033[0m'
357
358     colors = {-1: BLUE+"Blue"+ENDC, 0: "Ties", 1: RED+"Red"
359               +ENDC}
360     def getInput(currPlayer):
361         col = int(input(colors[currPlayer] + "_player, _
362                     enter_column:_"))
363         while col not in range(7):
364             print("Incorrect_move!")
365             col = int(input(colors[currPlayer] + "_player, _
366                             enter_column:_"))

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364         return col
365
366
367     play = True
368     numPlayers = -1
369     verbose = bool(int(input("Display_board?_1/0:_").split
370         ()[-1]))
371     dispTrain = bool(int(input("Display_Training?_1/0:_").
372         split()[-1]))
373     while numPlayers not in range(3):
374         numPlayers = int(input("Enter_number_of_players:_")
375             .split()[-1])
376     if numPlayers < 2:
377         posopponents = ["random", "MC", "MCPlus", "Heuristic",
378             "BoardInv", "N-tups"]
379         opponentChoices = {}
380         print("Available_opponents:")
381         for i, opponent in enumerate(posopponents):
382             print("%d._%s" % (i, opponent))
383         opponentChoices[-1] = posopponents[int(input("Pick_
384             opponent_for_")+
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)
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split
()
[-1])
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```

385         print(opponentChoices[1] + "_chosen")
386
387     results = {-1: 0, 0: 0, 1: 0}
388     totalRoundsToPlay = 60
389     playedRounds = 0
390     interactive = False
391     startingPlayer = 1
392     w = None
393     ntups = None
394     while play:
395         c4 = connect4(currPlayer = startingPlayer, verbose=
            dispTrain)
396         if w is not None:
397             c4.w = w
398             c4.ntups = ntups
399         elif "BoardInv" or "N-tups" in opponentChoices.
            values():
400             print("TD_chosen, _learning_w")
401             episodes = int(input("Enter_episodes_for_TD: _")
                .split()[-1])
402             print(episodes)
403             lamb = float(input("Enter_lambda_for_TD: _").
                split()[-1])
404             print(lamb)
405             if "N-tups" in opponentChoices.values():
406                 w = c4.learnWFromTd(numEpisodes = episodes,
                    lamb = lamb)
407                 ntups = c4.ntups
408             else:
409                 w = c4.learnWFromTd(numEpisodes = episodes,

```

```

410                                     lamb = lamb, useNtups =
411                                     False)
412 #c4.w = givenw
413 boardInv = lambda : c4.heuristicPlay(heuristicFunc
414                                     = c4.boardInversionHeuristic)
415 opponentActions = {"random": c4.makeRandomMove,
416                   "MC": c4.monteCarloPlay,
417                   "MCPlus": c4.monteCarloPlayPlus,
418                   "Heuristic": c4.heuristicPlay,
419                   "BoardInv": boardInv,
420                   "N-tups": boardInv}
421 win = None
422 while win is None:
423     if verbose:
424         print(c4)
425         print("Score:_")
426         print(resultsToString(results))
427     currPlayer = c4.currPlayer
428     if numPlayers == 0:
429         if verbose:
430             print("Method:_")
431             print(resultsToString(opponentChoices))
432         state, win = opponentActions[
433             opponentChoices[currPlayer]]()
434     elif numPlayers == 1:
435         if currPlayer == 1:
436             col = getInput(currPlayer)
437             state, win = c4.simulate(col)
438         else:
439             state, win = opponentActions[
440                 opponentChoices[currPlayer]]()
441     else:
442         col = getInput(currPlayer)
443         state, win = c4.simulate(col)
444
445     if verbose:
446         print(c4)
447     else:

```

```

446         print(resultsToString(results))
447     if win != 0:
448         print( colors[win] + "_player_wins!")
449     else:
450         print(RED + "Tie!" +ENDC)
451     results[win] += 1
452     playedRounds += 1
453     if interactive:
454         play = input("Play_Again,_Y/N?_") not in ["N", "n", "no"]
455     else:
456         play = playedRounds < totalRoundsToPlay
457         if playedRounds == totalRoundsToPlay/2:
458             print("Swapping_players")
459             startingPlayer = -1
460 print("Final_score:_")
461 print(resultsToString(results))
462 if opponentChoices:
463     print("Methods:_")
464     print(resultsToString(opponentChoices))

```

../util.py

```

1 from pylab import *
2
3 def logsig(n):
4     return 1/(1+exp(-n))
5
6 def dlogsig(n):
7     a = logsig(n)
8     return a*(1-a)
9
10 def tansig(n):
11     return tanh(n)
12
13 givenw = [3,4,5,5,4,3,4,6,8,8,6,4,5,8,11,11,8,5,7,10,13,
14           13,10,7,5,8,11,11,8,5,4,6,8,8,6,4,3,4,5,5,4,3]
15
16 winningLocations = array(
17 [[1, 2, 3, 4],

```


18	[2, 3, 4, 5],
19	[3, 4, 5, 6],
20	[7, 8, 9, 10],
21	[8, 9, 10, 11],
22	[9, 10, 11, 12],
23	[13, 14, 15, 16],
24	[14, 15, 16, 17],
25	[15, 16, 17, 18],
26	[19, 20, 21, 22],
27	[20, 21, 22, 23],
28	[21, 22, 23, 24],
29	[25, 26, 27, 28],
30	[26, 27, 28, 29],
31	[27, 28, 29, 30],
32	[31, 32, 33, 34],
33	[32, 33, 34, 35],
34	[33, 34, 35, 36],
35	[37, 38, 39, 40],
36	[38, 39, 40, 41],
37	[39, 40, 41, 42],
38	[1, 7, 13, 19],
39	[7, 13, 19, 25],
40	[13, 19, 25, 31],
41	[19, 25, 31, 37],
42	[2, 8, 14, 20],
43	[8, 14, 20, 26],
44	[14, 20, 26, 32],
45	[20, 26, 32, 38],
46	[3, 9, 15, 21],
47	[9, 15, 21, 27],
48	[15, 21, 27, 33],
49	[21, 27, 33, 39],
50	[4, 10, 16, 22],
51	[10, 16, 22, 28],
52	[16, 22, 28, 34],
53	[22, 28, 34, 40],
54	[5, 11, 17, 23],
55	[11, 17, 23, 29],
56	[17, 23, 29, 35],
57	[23, 29, 35, 41],

```

58 | [ 6,12,18,24],
59 | [12,18,24,30],
60 | [18,24,30,36],
61 | [24,30,36,42],
62 | [ 4, 9,14,19],
63 | [ 5,10,15,20],
64 | [10,15,20,25],
65 | [ 6,11,16,21],
66 | [11,16,21,26],
67 | [16,21,26,31],
68 | [12,17,22,27],
69 | [17,22,27,32],
70 | [22,27,32,37],
71 | [18,23,28,33],
72 | [23,28,33,38],
73 | [24,29,34,39],
74 | [ 3,10,17,24],
75 | [ 2, 9,16,23],
76 | [ 9,16,23,30],
77 | [ 1, 8,15,22],
78 | [ 8,15,22,29],
79 | [15,22,29,36],
80 | [ 7,14,21,28],
81 | [14,21,28,35],
82 | [21,28,35,42],
83 | [13,20,27,34],
84 | [20,27,34,41],
85 | [19,26,33,40]]) - 1
86
87
88 def winningPosistion():
89     C = zeros((69,42));
90     for i in range(size(location,0)):
91         C[i,winningLocations[i,:]] = 1;
92     return C,winningLocations

```