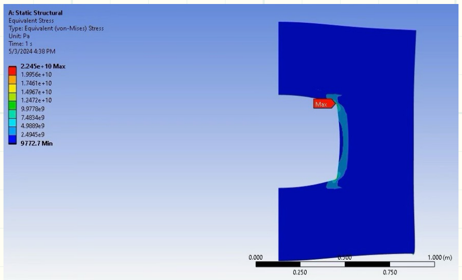
Eric Lee | ME 478 FEA | HW2

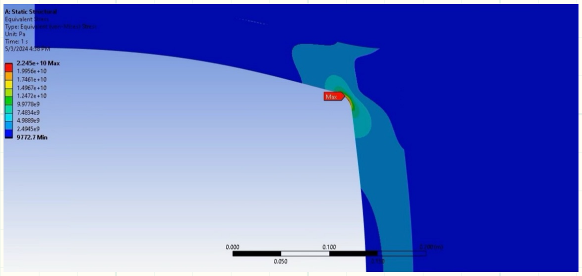
Problem 9.15

Assumptions:

- Plane strain
- mesh size (nominal): 3mm
- mesh size (interface): 1mm
- Symmetry line on left side of object
- Pressure vessel fixed on bottom right corner
- Von Mises stress

Solved:





The maximum stress is at the filleted corner of the steel liner

Problem 10.4

$$X(s) = a_1 + a_2 s + a_3 s^2 + a_4 s^3$$

$$\begin{cases} X(-1) = X_1 = \emptyset = a_1 - a_2 + a_3 - a_4 \\ X(-1/2) = X_2 = \frac{L}{4} = a_1 - \frac{1}{2}a_2 + \frac{1}{4}a_3 - \frac{1}{8}a_4 \\ X(1/2) = X_3 = \frac{3L}{4} = a_1 + \frac{1}{2}a_2 + \frac{1}{4}a_3 + \frac{1}{8}a_4 \\ X(1) = X_4 = L = a_1 + a_2 + a_3 + a_4 \end{cases}$$

$$\underbrace{\begin{bmatrix} \emptyset \\ \frac{1}{4}L \\ \frac{3}{4}L \\ L \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & -\frac{1}{2} & \frac{1}{4} & -\frac{1}{8} \\ 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{8} \\ 1 & 1 & 1 & 1 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}}_{A}$$

$$A = B^{-1}X$$

$$A = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} \frac{2}{3}(X_2 + X_3) - \frac{1}{6}(X_1 + X_4) \\ \frac{4}{3}(X_3 - X_2) + \frac{1}{6}(X_1 - X_4) \\ \frac{2}{3}(X_1 + X_4 - X_2 - X_3) \\ \frac{4}{3}(X_2 - X_3) + \frac{2}{3}(X_4 - X_1) \end{bmatrix} = \begin{bmatrix} \frac{1}{2}L \\ \frac{1}{2}L \\ \emptyset \\ \emptyset \end{bmatrix}$$

$$\frac{dX}{ds} = a_2 + 2\cancelto{\emptyset}{a_3} s + 3\cancelto{\emptyset}{a_4} s^2$$

$$\boxed{\frac{dX}{ds} = [J] = \frac{L}{2}}$$

$$\underline{X} = \underline{N}\,\underline{C}$$

$$X(s) = \left[\frac{2}{3}(X_2 + X_3) - \frac{1}{6}(X_1 + X_4)\right] + \left[\frac{4}{3}(X_3 - X_2) + \frac{1}{6}(X_1 - X_4)\right]s + \left[\frac{2}{3}(X_1 + X_4 - X_2 - X_3)\right]s^2 + \left[\frac{4}{3}(X_2 - X_3) + \frac{2}{3}(X_4 - X_1)\right]s^3$$

$$X(s) = \left(-\frac{1}{6} + \frac{1}{6}s + \frac{2}{3}s^2 - \frac{2}{3}s^3\right)X_1 + \left(\frac{2}{3} - \frac{4}{3}s - \frac{2}{3}s^2 + \frac{4}{3}s^3\right)X_2 + \left(\frac{2}{3} + \frac{4}{3}s - \frac{2}{3}s^2 - \frac{4}{3}s^3\right)X_3 + \left(-\frac{1}{6} - \frac{1}{6}s + \frac{2}{3}s^2 + \frac{2}{3}s^3\right)X_4$$

$$\boxed{\begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{6} + \frac{1}{6}s + \frac{2}{3}s^2 - \frac{2}{3}s^3 \\ \frac{2}{3} - \frac{4}{3}s - \frac{2}{3}s^2 + \frac{4}{3}s^3 \\ \frac{2}{3} + \frac{4}{3}s - \frac{2}{3}s^2 - \frac{4}{3}s^3 \\ -\frac{1}{6} - \frac{1}{6}s + \frac{2}{3}s^2 + \frac{2}{3}s^3 \end{bmatrix}}$$

$$B = \frac{dN}{dx}$$

$$\boxed{\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{6}\frac{ds}{dx} + \frac{4}{3}s\frac{ds}{dx} - 2s^2\frac{ds}{dx} \\ -\frac{4}{3}\frac{ds}{dx} - \frac{4}{3}s\frac{ds}{dx} + 4s^2\frac{ds}{dx} \\ \frac{4}{3}\frac{ds}{dx} - \frac{4}{3}s\frac{ds}{dx} - 4s^2\frac{ds}{dx} \\ -\frac{1}{6}\frac{ds}{dx} + \frac{4}{3}s\frac{ds}{dx} + 2s^2\frac{ds}{dx} \end{bmatrix}}$$

Problem 10.15

$n_{int}=2$: $w_1=w_2=1$ $\quad s_1=-\frac{1}{\sqrt{3}}$ $\quad s_2=\frac{1}{\sqrt{3}}$

$n_{int}=3$: $w_1=w_3=\frac{5}{9}$ $\quad w_2=\frac{8}{9}$ $\quad s_1=-\sqrt{\frac{3}{5}}$ $\quad s_2=\varnothing$ $\quad s_3=\sqrt{\frac{3}{5}}$

a) $2.\left[\cos\left(\frac{-1/\sqrt{3}}{2}\right)\cdot 1+\cos\left(\frac{1/\sqrt{3}}{2}\right)\cdot 1\right]$

$3.\left[\cos\left(\frac{-\sqrt{3/5}}{2}\right)\cdot\frac{5}{9}+\cos(\varnothing)\cdot\frac{8}{9}+\cos\left(\frac{\sqrt{3/5}}{2}\right)\cdot\frac{5}{9}\right]$

b) $2.\left[\left(-\frac{1}{\sqrt{3}}\right)^2\cdot 1+\left(\frac{1}{\sqrt{3}}\right)^2\cdot 1\right]$

$3.\left[\left(-\sqrt{\frac{3}{5}}\right)^2\cdot\frac{5}{9}+(\varnothing)^2\cdot\frac{8}{9}+\left(\sqrt{\frac{3}{5}}\right)^2\cdot\frac{5}{9}\right]$

c) $2.\left[\left(-\frac{1}{\sqrt{3}}\right)^4\cdot 1+\left(\frac{1}{\sqrt{3}}\right)^4\cdot 1\right]$

$3.\left[\left(-\sqrt{\frac{3}{5}}\right)^4\cdot\frac{5}{9}+(\varnothing)^4\cdot\frac{8}{9}+\left(\sqrt{\frac{3}{5}}\right)^4\cdot\frac{5}{9}\right]$

d) $2.\left[\frac{\cos(-1/\sqrt{3})}{1-(-1/\sqrt{3})^2}\cdot 1+\frac{\cos(1/\sqrt{3})}{1-(1/\sqrt{3})^2}\cdot 1\right]$

$3.\left[\frac{\cos(-\sqrt{3/5})}{1-(-\sqrt{3/5})^2}\cdot\frac{5}{9}+\frac{\cos(\varnothing)}{1-(\varnothing)^2}\cdot\frac{8}{9}+\frac{\cos(\sqrt{3/5})}{1-(\sqrt{3/5})^2}\cdot\frac{5}{9}\right]$

e) $2.\left[(-1/\sqrt{3})^3\cdot 1+(1/\sqrt{3})^3\cdot 1\right]$

$3.\left[(\sqrt{3/5})^3\cdot\frac{5}{9}+(\varnothing)^2\cdot\frac{8}{9}+(\sqrt{3/5})^3\cdot\frac{5}{9}\right]$

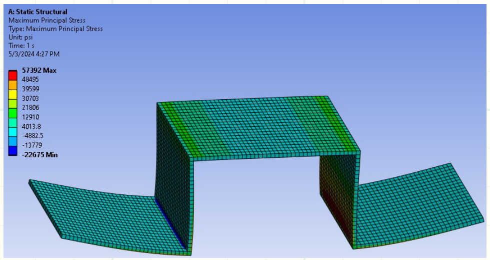
f) $2.\left[\left(-\frac{1}{\sqrt{3}}\right)\cos\left(-\frac{1}{\sqrt{3}}\right)\cdot 1+\left(\frac{1}{\sqrt{3}}\right)\cos\left(\frac{1}{\sqrt{3}}\right)\cdot 1\right]$

$3.\left[\left(-\sqrt{\frac{3}{5}}\right)\cos\left(-\sqrt{\frac{3}{5}}\right)\cdot\frac{5}{9}+(\varnothing)\cos(\varnothing)\cdot\frac{8}{9}+(\sqrt{3/5})\cos(\sqrt{3/5})\cdot\frac{5}{9}\right]$

g) $2.\left[\left(4^{-1/\sqrt{3}}-2\left(-\frac{1}{\sqrt{3}}\right)\right)\cdot 1+\left(4^{1/\sqrt{3}}-2\left(\frac{1}{\sqrt{3}}\right)\right)\cdot 1\right]$

$3.\left[\left(4^{-\sqrt{3/5}}-2(-\sqrt{3/5})\right)\cdot\frac{5}{9}+\left(4^{0}-2(\varnothing)\right)\cdot\frac{8}{9}+\left(4^{\sqrt{3/5}}-2(\sqrt{3/5})\right)\cdot\frac{5}{9}\right]$

Problem 12.6



Using a mesh size of $\frac{1}{2}$ in, the max stress of 57,392 Psi at the bottom outside corner. This is far above the yield stress of 36,259 Psi and it will fail. I recommend increasing the structure thickness, for instance w/ a thickness of Ø.5 in (double the previous thickness) the max stress of 13,3ØZ Psi is well below the yield stress