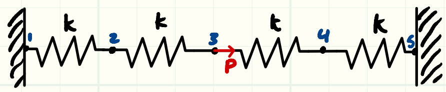
Eric Lee | ME478 FEA | Homework 1

Problem 2.3

$$\begin{bmatrix} F_1 \\ \emptyset \\ P \\ \emptyset \\ F_5 \end{bmatrix} = \begin{bmatrix} k & -k & 0 & 0 & 0 \\ -k & 2k & -k & 0 & 0 \\ 0 & -k & 2k & -k & 0 \\ 0 & 0 & -k & 2k & -k \\ 0 & 0 & 0 & -k & k \end{bmatrix} \begin{bmatrix} \emptyset \\ u_2 \\ u_3 \\ u_4 \\ \emptyset \end{bmatrix}$$



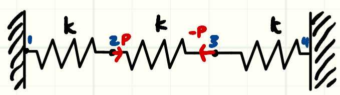
$$\begin{cases} \emptyset = 2ku_2 - ku_3 \rightarrow u_2 = \frac{1}{2}u_3 = \frac{P}{2k} \\ P = -ku_2 + 2ku_3 - ku_4 \rightarrow P = -k(\frac{1}{2}u_3) + 2ku_3 - k(\frac{1}{2}u_3) \rightarrow u_3 = \frac{P}{k} \\ \emptyset = -ku_3 + 2ku_4 \rightarrow u_4 = \frac{1}{2}u_3 = \frac{P}{2k} \end{cases}$$

$$\begin{cases} F_1 = -ku_2 = -\frac{P}{2} \\ F_5 = -ku_4 = -\frac{P}{2} \end{cases}$$

$$\boxed{\begin{bmatrix} -\frac{P}{2} \\ \emptyset \\ P \\ \emptyset \\ -\frac{P}{2} \end{bmatrix} = \begin{bmatrix} k & -k & 0 & 0 & 0 \\ -k & 2k & -k & 0 & 0 \\ 0 & -k & 2k & -k & 0 \\ 0 & 0 & -k & 2k & -k \\ 0 & 0 & 0 & -k & k \end{bmatrix} \begin{bmatrix} \emptyset \\ \frac{P}{2k} \\ \frac{P}{k} \\ \frac{P}{2k} \\ \emptyset \end{bmatrix}}$$

Problem 2.16

$k = 1\emptyset\emptyset \frac{lb}{in}$ $P = 1\emptyset\emptyset lb$



$$\begin{bmatrix} F_1 \\ P \\ -P \\ F_4 \end{bmatrix} = \begin{bmatrix} k & -k & 0 & 0 \\ -k & 2k & -k & 0 \\ 0 & -k & 2k & -k \\ 0 & 0 & -k & k \end{bmatrix} \begin{bmatrix} \emptyset \\ u_2 \\ u_3 \\ \emptyset \end{bmatrix}$$

$$P = 2ku_2 - ku_3$$
$$+2(-P = -ku_2 + 2ku_3)$$
$$-P = 3ku_3$$

$$u_3 = \frac{-P}{3k} = -\frac{1}{3}$$

$$P = 2ku_2 - k\left(\frac{-P}{3k}\right)$$

$$P = 2ku_2 + \frac{P}{3}$$

$$u_2 = \frac{P}{3k} = \frac{1}{3}$$

$$F_1 = -ku_2 = -\frac{100}{3}$$

$$F_2 = -ku_3 = \frac{100}{3}$$

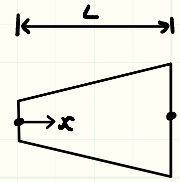
$$\boxed{\begin{bmatrix} -\frac{100}{3} \\ 100 \\ -100 \\ \frac{100}{3} \end{bmatrix} = \begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & 200 & -100 & 0 \\ 0 & -100 & 200 & -100 \\ 0 & 0 & -100 & 100 \end{bmatrix} \begin{bmatrix} \emptyset \\ \frac{1}{3} \\ -\frac{1}{3} \\ \emptyset \end{bmatrix}}$$

Problem 3.53

$N_1 = \frac{x-L}{\varnothing - L} = -\frac{x-L}{L}$

$A(x) = A_0\left(1+\frac{x}{L}\right) \qquad dv = A(x)\,dx$

$N_2 = \frac{x}{L}$



$N = \begin{bmatrix} \frac{L-x}{L} & \frac{x}{L} \end{bmatrix} \qquad B = \frac{d}{dx}N = \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix}$

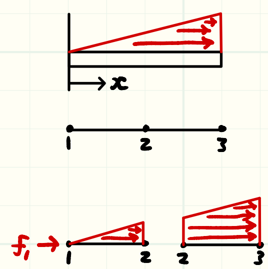
$$k = \int_V B^T E B\, dv = \int_V \begin{bmatrix} -\frac{1}{L} \\ \frac{1}{L} \end{bmatrix} E \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} dv = E\int_V \begin{bmatrix} \frac{1}{L^2} & -\frac{1}{L^2} \\ -\frac{1}{L^2} & \frac{1}{L^2} \end{bmatrix} dv = \frac{E}{L^2}\int_\varnothing^L A(x) \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} dx$$

$$k = \frac{EA_0}{L^2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\int_0^L \left(1+\frac{x}{L}\right)dx = \frac{EA_0}{L^2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\left(x+\frac{x^2}{2L}\right)\Bigg|_\varnothing^L$$

$$\boxed{k = \frac{3EA_0}{2L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}$$

Problem 3.54

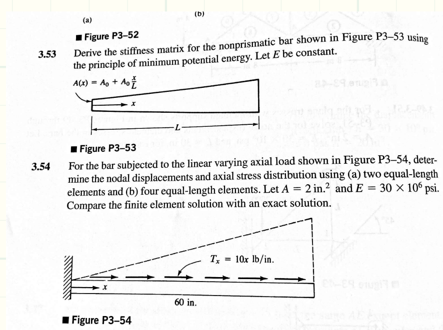
Element 1:



$$k = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$N_1 = \frac{x-30}{-30} \qquad N_2 = \frac{x}{30}$

$N = \begin{bmatrix} \frac{30-x}{30} & \frac{x}{30} \end{bmatrix}$


(a)

(b)

■ Figure P3–52

3.53 Derive the stiffness matrix for the nonprismatic bar shown in Figure P3–53 using the principle of minimum potential energy. Let E be constant.

■ Figure P3–53

3.54 For the bar subjected to the linear varying axial load shown in Figure P3–54, determine the nodal displacements and axial stress distribution using (a) two equal-length elements and (b) four equal-length elements. Let $A = 2$ in.^{2} and $E = 30 \times 10^6$ psi. Compare the finite element solution with an exact solution.

■ Figure P3–54

$$f_e = \begin{bmatrix} f_1 \\ \varnothing \end{bmatrix} + \frac{1}{30}\int_0^L \begin{bmatrix} 30-x \\ x \end{bmatrix}(10x)\,dx = \begin{bmatrix} f_1 + 150\varnothing \\ 3\varnothing\varnothing\varnothing \end{bmatrix}$$

$$\frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} f_{T_1} \\ f_{T_2} \end{bmatrix}$$

Element 2:

$T(x) = 3\varnothing\varnothing + 1\varnothing x$

$$f_T = \int_0^{30} N^T(300+10x)\,dx = \begin{bmatrix} 6000 \\ 7500 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} d_2 \\ d_3 \end{bmatrix} = f_{T_2}$$

$$\frac{AE}{L}\begin{bmatrix} 1 & -1 & \varnothing \\ -1 & 2 & -1 \\ \varnothing & -1 & 1 \end{bmatrix}\begin{bmatrix} \cancel{d_1}\ \varnothing \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} f_{T_1}^{(1)} \\ f_{T_2}^{(1)} + f_{T_2}^{(2)} \\ f_{T_3}^{(2)} \end{bmatrix} = \begin{bmatrix} f_1 + 1500 \\ 9\varnothing\varnothing\varnothing \\ 7500 \end{bmatrix}$$

$$\frac{AE}{L}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 9\varnothing\varnothing\varnothing \\ 75\varnothing\varnothing \end{bmatrix}$$

$$\boxed{d_2 = \varnothing.\varnothing\varnothing825 \text{ in} \qquad d_3 = \varnothing.\varnothing12 \text{ in}}$$

Element 1 Element 2

$B = \frac{d}{dx}N = \begin{bmatrix} \frac{-1}{30} & \frac{1}{30} \end{bmatrix}$

$\sigma = E\varepsilon = EBd$

$\sigma_1 = EB \cdot d = \begin{bmatrix} \frac{-1}{30} & \frac{1}{30} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \boxed{8250 \text{ Psi}}$ ← element 1

$\sigma_1 = EB \cdot d = \begin{bmatrix} \frac{-1}{30} & \frac{1}{30} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \boxed{3750 \text{ Psi}}$ ← element 2