Chapter 3 (Antenna Fundamentals) - Exercise Solutions

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1. First, the effective area of the aperature is

$$A_e = \eta A_p = 0.65\pi(15 \times 10^{-2}\text{m})^2 = 45.95 \times 10^{-3}\ \text{m}^2.$$

Thus the gain G is

$$G = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi(45.95 \times 10^{-3}\ \text{m}^2)}{(7.5 \times 10^{-3}\ \text{m})^2} = 10265.33.$$

In decibels,

$$10 \log 10265.33 = 40.11\text{dB}.$$

2. As before, we just use the formula

$$G = \frac{4\pi\eta A_p f^2}{c^2} = \frac{4\pi(0.6)0.2\ \text{m}^2(10 \times 10^9\ \text{s}^{-1})}{(3 \times 10^8\ \text{m s}^{-1})^2} = 1675.52 = 32.24\ \text{dB}.$$

3. A loss of 0.5 dB is equivalent to a loss of $10^{-0.05} = 0.891$ in absolute terms. Thus, for this system $P_t = 0.891 P_i$. Then, for $(1 - \rho^2) = 0.891$ we have that $\rho = \pm 0.33$. Taking each value in turn:

$$0.33(Z_1 + Z_0) = Z_1 - Z_0$$
$$\implies Z_1 \approx \frac{1.33}{0.67} Z_0 = 99.25\Omega$$

or

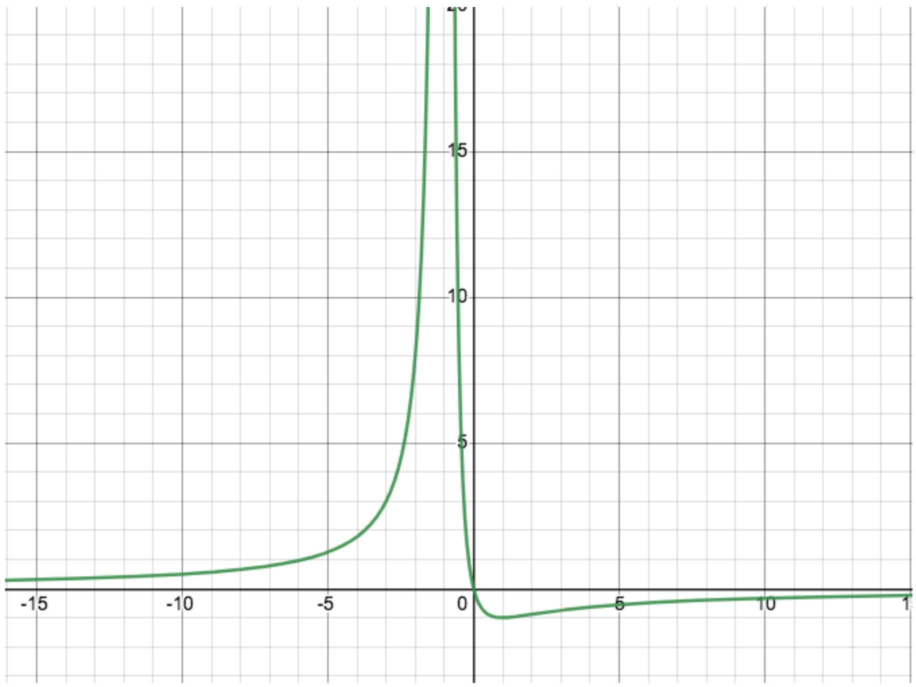
$$-0.33(Z_1 + Z_0) = Z_1 - Z_0$$
$$\implies Z_1 \approx \frac{0.67}{1.33} Z_1 = 25.19\Omega.$$

So as long as the driving point impedance is between 25.19Ω and 99.25Ω, then the system will stay within the 0.5 dB loss value.

4. We can show (by simply rearranging variables) that the loss can be rewritten as

$$L = -\left[1 - \left(\frac{\text{VSWR} - 1}{\text{VSWR} + 1}\right)^2\right].$$

So we simply plot VSWR (x axis) against loss (y axis), noting that there is formally undefined (infinite) loss when VSWR $= -1$.



5. Using the formula

$$F = \frac{(1+\text{AR}_w^2)(1+\text{AR}_r^2) + 4\text{AR}_w\text{AR}_r + (1-\text{AR}_w^2)(1+\text{AR}_r^2)\cos(2[\tau_w - \tau_r])}{2(1+\text{AR}_w^2)(1+\text{AR}_r^2)}$$

Since $\text{AR}_{w,dB} = \text{AR}_{r,dB} = 2db$, then $AR_w = AR_r = 10^{\frac{2}{20}} = 1.259$, and the above equation reduces to

$$F = 0.918 + 0.0256\cos[2(\tau_w - \tau_r)].$$

Where $P_r = FP_i$, the minimum *value* (most loss) of F should therefore occur when $\cos[2(\tau_w - \tau_r)] = -1$, and the *max* (least loss) should occur when $\cos[2(\tau_w - \tau_r)] = 1$. So

$$F_{min} = 0.918 - 0.0256 = 0.8924 = -0.494 \text{ dB}$$

and

$$F_{max} = 0.918 + 0.0256 = 0.9436 = -0.252 \text{ dB}$$

So when the angle between the same axes of each antenna is rotated either 90° or 270°, we experience the maximum loss of -0.494 dB. When 0° or 180°, we experience a minimum loss of -0.252 dB. For all angles in between, the loss will vary sinusoidally between these two values.

6. To get the effective height, h_e, we first must consider the average value of a sin wave. By definition, this is an integral over the period $[0, \pi]$

$$I_{avg} = \frac{I_0}{\pi} \int_0^{\pi} \sin x \text{ dx} = \frac{I_0}{\pi} \big[-\cos x \big]_0^{\pi} = \frac{2}{\pi} I_0.$$

h_e is then given by

$$h_{eff} = \frac{1}{I_0} \int_0^L I_{avg} \text{dx} = \frac{1}{I_0} \int_0^L \frac{2}{\pi} I_0 \text{dx} = \frac{2}{\pi}(L - 0) \approx 0.637L$$

where L is the physical length of the antenna. In this case,

$$L = 0.1 \frac{3 \times 10^8 \text{ m s}^{-1}}{80 \times 10^6 \text{ s}^{-1}} = 0.375 \text{ m}.$$

Thus

$$h_e = 0.637 \times 0.375 = 0.239$$

and the effective area A_e is

$$A_e = \frac{h_e^2 Z_0}{4R_r} = \frac{(0.239)^2 377\Omega}{4 \times 73\Omega} = 0.074 \text{ m}^2.$$

7. We simply use Fig. 3.2 of the text. The lowest point outside of the mainlobe is -80 dB, and its independent of distance. Since

$$100 \text{ dBW} - 80 \text{ dB} = 20 \text{ dBW}$$

we can then calculate the power as

$$P = 10^{\frac{100}{10}} = 10^{10} \text{ W}.$$

Since we were asked for power *density* S, then

$$S = \frac{P}{4\pi d^2} = \frac{10^{10}}{4\pi 2^2} = 198.94 \times 10^6 \text{ W m}^{-2}$$