Chapter 2 (Electromagnetics and RF Propagation) - Exercise Solutions

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1. (a)

$$\hat{E} = \frac{q}{4\pi r^2 \epsilon}\hat{r} = \frac{5 \text{ C}}{4\pi(1)^2\epsilon_r\epsilon_0}\hat{r} = (16.643 \times 10^9 \text{ N C}^{-1})\hat{r}$$

where $\hat{r}$ is the unit vector pointing from the point charge towards the point of measurement. Note that N C^{-1} =V m^{-1} - this can be checked easily using base SI units.

(b)

$$\hat{E} = \frac{q}{2\pi r\epsilon}\hat{r} = \frac{2 \text{ C}}{2\pi\frac{1}{2}\epsilon_r\epsilon_0}\hat{r} = (8.87 \times 10^8 \text{ N C}^{-1})\hat{r}$$

(c) This question is actually ill-posed, since an infinite plane with charge necessarily has infinite charge. Let us assume then that the value of 1 C should actually be the *charge density*, with units of C m^{-2} for a plane. It is usually written as $\sigma = 1$ C m^{-2}.

We can then use the formula for the electric field $E = \frac{\sigma}{2\epsilon}$, which is derived nicely here. Of specific interest here is that the value of the E field is independent of the distance away from the infinite plane! So the answer is

$$\hat{E} = \frac{1 \text{ C m}^{-2}}{\epsilon_r\epsilon_0}\hat{n} = (112.876 \times 10^9 \text{ N C}^{-1} \text{ m}^{-2})\hat{n}$$

where $\hat{n}$ is the unit vector normal to the surface of the infinite plane charge.

(d) For reasons already pointed out, the answer is the same as for above.

2.

$$\phi_2 = \tan^{-1}\left(\frac{\epsilon_2}{\epsilon_1}\tan\phi_1\right) = \tan^{-1}\left(\frac{81}{1.0006} \times 0.577\right) = 88.77° \text{ (away from the horizontal.)}$$

3. The magnitude of the magnetic field vector B at a distance r from a long, straight current-carrying wire with current I is

$$B = \frac{I\mu_0}{2\pi r} \text{ A m}^{-1}$$

Here, we have

$$B = \frac{1 \times 4\pi \times 10^{-7}}{2\pi} \text{ A m}^{-1} = 2 \times 10^{-6} \text{ A m}^{-1}$$

The direction of the magnetic field follows the usual right hand rule. In vector notation, we would write this as

$$\hat{B} = \frac{I\mu_0}{2\pi r}(\hat{I} \times \hat{r})$$

where $\hat{I}$ (thumb) points in the direction of current flow (positive z axis here) and $\hat{r}$ (index finger) is perpendicular to the wire and points from the wire to the point we're measuring the magnetic field at. The middle finger now points in the direction of the magnetic field.

4. This is a TM wave, in terms specified by the text. Some of the energy is reflected/scattered, and some is refracted/absorbed. The reflected energy is of primary interest generally to CIS personnel. The angle of the reflected wave (let's say θ_1^1) is the same as the angle of the incident wave θ_1. The angle of refraction θ_2, as in the text, is

$$\theta_2 = \cos^{-1}\left[\cos\theta_1\sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}\frac{\mu_{r1}}{\mu_{r2}}}\right] = \cos^{-1}\left[\cos\theta_1\sqrt{\frac{2.7}{1.0006}\frac{1}{2.55}}\right] = \cos^{-1}\left[1.029\cos\theta_1\right].$$

We don't actually have an incident angle, so we must leave this expression as it is. The reflection coefficient ρ allows us to calculate the amplitude of the reflected wave, and the transmission coefficient τ allows us to calculate the amplitude of the refracted wave. Where Z_2 is the effective impedance of the polystyrene and Z_1 is that of the air, then

$$\rho = \frac{Z_2 - Z_{Z1}}{Z_2 + Z_{Z1}}$$

and

$$\tau = \frac{2Z_2}{Z_2 + Z_{Z1}}.$$

But since[1] $Z = Z_0\frac{\mu_{r1}}{n_1}$, where Z_0 is the vacuum wave impedance, and (for TM waves) $Z_{Z1} = Z_1 \sin\theta_1$ then

$$\rho = \frac{Z_0\frac{\mu_{r2}}{n2} - Z_0\frac{\mu_{r1}}{n_1}\sin\theta_1}{Z_0\frac{\mu_{r2}}{n_2} + Z_0\frac{\mu_{r1}}{n1}\sin\theta_1} = \frac{\mu_{r2}n_1 - \mu_{r1}n_2\sin\theta_1}{\mu_{r2}n_1 + \mu_{r1}n_2\sin\theta_1} = \frac{2.55 - 1.586\sin\theta_1}{2.55 + 1.586\sin\theta_1}$$

and

$$\tau = \frac{2Z_0\frac{\mu_{r2}}{n_2}}{Z_0\frac{\mu_{r2}}{n_2} + Z_0\frac{\mu_{r1}}{n_1}\sin\theta_1} = \frac{2n_1\mu_{r2}}{n_1\mu_{r2} + n_2\mu_{r1}\sin\theta_1} = \frac{5.102}{2.5507 + 1.586\sin\theta_1}.$$

Note that this problem does not depend on the frequency of the incident EM wave.

5. For a TE wave, we simply replace $Z_{Z1} = Z_1 \csc\theta_1$ and repeat.

[1] The wave impedance in a non-conductive material is $Z = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_o\mu_r}{\epsilon_0\epsilon_r}} = Z_0\sqrt{\frac{\mu_r}{\epsilon_r}} = Z_0\frac{\mu_r}{n}$

6. (a) The skin depth δ is given by

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi (100 \times 10^6)(0.999)(1.257 \times 10^{-6})(5.7 \times 10^7)}} = 6.67 \times 10^{-6} \text{ m} = 6.67\mu\text{m}.$$

(b) The skin depth is defined as the distance that the EM wave is attenuated by a factor of $\frac{1}{e} = 0.368$. Thus, in order to attenuate 97% of the amplitude, we would require a thickness of $\frac{97}{36.8}\delta = 2.636\delta = 17.582\mu$m.

7. Begin with F defined in terms of AR_r and AR_t, ie

$$F = \frac{\left(1 + AR_t^2 \frac{G_A}{G_B}\right)(1 + AR_r^2) + 4AR_t AR_r \sqrt{\frac{G_A}{G_B}} + \left(1 - AR_t^2 \frac{G_A}{G_B}\right)(1 - AR_r^2)}{2(1 + AR_t^2 \frac{G_A}{G_B})(1 + AR_r^2)}.$$

Once we plug in the values $AR_t = 2$ and $AR_r = 2.5$, we're then interested in taking the limit of F as $G_B \to 0$. Taking the usual step, we rewrite as

$$\lim_{G_B \to 0} F = \lim_{G_B \to 0} \frac{7.25(G_B + 4G_A) + 20\sqrt{G_A}\frac{\sqrt{G_B}}{G_B} - 5.25(G_B - 4G_A)}{7.25(G_B + 8G_A)} = \frac{25}{29}.$$

So the polarisation loss L can be written as $L = \frac{25G_A}{29} = 0.862G_A$.