Chapter 4 (Comms Systems and Link Budget) - Exercise Solutions

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1. We start by noting that CNR $\approx$ SNR for a modulated signal. Thus

$$\mathrm{CNR}_{dB} = \mathrm{S}_{dB} - \mathrm{N}_{dB}$$

where S_{dB} is the signal strength, and therefore

$$\mathrm{S}_{dB} = \mathrm{CNR}_{dB} + \mathrm{N}_{dB} = 15dB + 4dB = 19dB.$$

It must be measured immediately on exiting the receiver, before it passes through any further transmission media or other loads.

2. First, we calculate

$$\begin{aligned}
\mathrm{L}_{dB} &= -\mathrm{G}_{Tx\,dB} - \mathrm{G}_{Rx\,dB} - 20\log\lambda + 20\log d + 22 \\
&= -4\mathrm{dB} - 4\mathrm{dB} - 20\log\left(\frac{3\times 10^8}{800\times 10^6}m\right) + 20\log(2000m) + 22 \\
&= 88.54\mathrm{dB}
\end{aligned}$$

From before, we need 19 dB exiting the receiver. Since we then also lose 88.54 dB via propagation, we therefore require a *minimum* Tx power of $19\mathrm{dB} + 88.54\mathrm{dB} = 107.54\mathrm{dB}$.

We could alternatively calculate using the link margin M_L equation

$$\mathrm{M_L} = \mathrm{EIRP} - \mathrm{L_{Path}} + \mathrm{G_{Rx}} - \mathrm{TH_{Rx}}.$$

where $\mathrm{EIRP} = \mathrm{P}_{Tx\,dB} + \mathrm{G}_{Tx\,dB}$. Setting the link margin = 0 in order to find the minimum Tx power, we can then solve for $\mathrm{P}_{Tx\,dB}$.

$$\begin{aligned}
\mathrm{P_{Tx\ \ dB}} &= 96.52\mathrm{dB} - 4\mathrm{dB} + 15\mathrm{dB} \\
&= 107.54\mathrm{dB}
\end{aligned}$$

as before.

3. We are being asked to calculate

$$N = kT_0BF$$

where

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{600}{290} = 3.069.$$

Thus

$$N = (1.38 \times 10^{-23}\text{J K}^{-1})(290\text{K})(100 \times 10^3\text{Hz})(3.069) = 1.228 \times 10^{-15}\text{J Hz} = -149.1\text{dB}.$$

4. To find the max communication distance, we will solve for d in the link margin equation, while setting the link margin = 0. We begin with

$$\mathrm{M_L} = \mathrm{EIRP} - \mathrm{L_{Path}} + \mathrm{G_{Rx}} - \mathrm{TH_{Rx}}$$

and substitute in the given values as

$$0 = \mathrm{P_{Tx}} + \mathrm{G_{Tx}} - 20\log\left(\frac{3 \times 10^8}{4\pi d \times 10 \times 10^9}\right) + 28 - (-90).$$

Solving for d, we obtain $d = 0.0379$ m.

5. Ultimately, we want to find the signal to noise (SNR) given by

$$\mathrm{SNR} = \mathrm{RSL} - \mathrm{N}.$$

Start then by calculating the RSL:

$$\mathrm{RSL} = \mathrm{EIRP} - \mathrm{PL} + \mathrm{G_R}.$$

The receiver gain G_R is calculated as

$$\mathrm{G_R} = \mathrm{G_{Rx}} - \mathrm{L_{radome}} = 35\text{dB} - 2\text{dB} = 32\text{dB}.$$

Next, find the EIRP.

$$\mathrm{EIRP_{dB}} = \mathrm{P_{dB}} + \mathrm{G_{Tx\,dB}} - \mathrm{L_{radome}} = -10\text{dBm} + 35\text{dB} - 2\text{dB} = 23\text{dBm}$$

and the allowable path loss PL is

$$\mathrm{PL} = \mathrm{L_{Free\ Space\,dB}} + \mathrm{FM_{dB}} + \mathrm{L_{Misc\,dB}}.$$

In this case, we have no $\mathrm{L_{Misc\,dB}}$ term, and $\mathrm{L_{Free\ Space\,dB}} = -20\log\left(\frac{\lambda}{4\pi d}\right) = 125.62\text{dB}$. Thus

$$\mathrm{PL} = 125.62 + 10 = 135.62\text{dB}.$$

Revisiting the equation for RSL, we now have

$$\mathrm{RSL} = \mathrm{EIRP} - \mathrm{PL} + \mathrm{G_R} = 23\mathrm{dBm} - 135.62\mathrm{dB} + 33\mathrm{dB} = -79.62\mathrm{dBm}.$$

Before we are ready to find SNR, we must calculate also

$$\mathrm{N_{dBm}} = 10\log\left(kT_0BF\right) = -174\mathrm{dBm} + 10\log B + \mathrm{F}_{dB}$$

where F = 7dB and B is the noise equivalent bandwidth. We treat $B \approx \mathrm{BW} = 50 \times 10^6\mathrm{Hz}$. So $10\log B \approx 10\log(50 \times 10^6) = 76.99\mathrm{dB}$. Finally then, the SNR is

$$\mathrm{SNR_{dB}} = -79.62 - (-174 + 76.99 + 7) = 10.39\mathrm{dBm}.$$

6. (a)

$$\mathrm{RSL_{dB}} = \mathrm{SNR_{dB}} + \mathrm{N_{dB}} = 12\mathrm{dB} + (-174\mathrm{dBm/Hz}) + 10\log B\mathrm{dB\ Hz} + \mathrm{F_{dB}}$$

where again we use BW ≈ B. We then obtain

$$\mathrm{RSL_{dB}} = 12\mathrm{dB} - 174\mathrm{dBm/Hz} + 76.99\mathrm{dB\ Hz} + 7\mathrm{dB} = -78.01\mathrm{dB}.$$

(b) The acceptable path loss PL is

$$\mathrm{PL_{dB}} = \mathrm{EIRP_{dB}} - \mathrm{RSL}_{dB} + \mathrm{G}_{R\,\mathrm{dB}} = \mathrm{P_{Tx\ \ dB}} + \mathrm{G_{Tx\ \ dB}} - \mathrm{RSL}_{dB} + \mathrm{G}_{R\,\mathrm{dB}} = 0 + 30 + 78.01 + 30 = 138.01\mathrm{dB}.$$

(c)

$$\mathrm{PL_{dB}} = \mathrm{L_{Free\ Space\,dB}} + \mathrm{FM}_{dB} + \mathrm{L_{Misc}} = -20\log\left(\frac{8.53 \times 10^{-4}}{d}\right) + 10 + 0.$$

We have that PL = 138.01dB from the previous question, so we can just solve for d now, i.e.

$$-20\log\left(\frac{8.53 \times 10^{-4}}{d}\right) = 138.01\mathrm{dB} - 10\mathrm{dB}$$

and thus

$$d = 2142.64\mathrm{m}.$$