Programming assignment

This is the report of the programming assignment: Finding sparse solution via Orthogonal Matching Pursuit (OMP)

1. Performance Metrics:

Normalized Error is calculated in function 'normalized_error' in omp.py

```
def normalized_error(x, x_):
    return norm(x - x_, ord=2) / norm(x, ord=2)
```

2. Experiment setup:

experiments setup is in omp.py

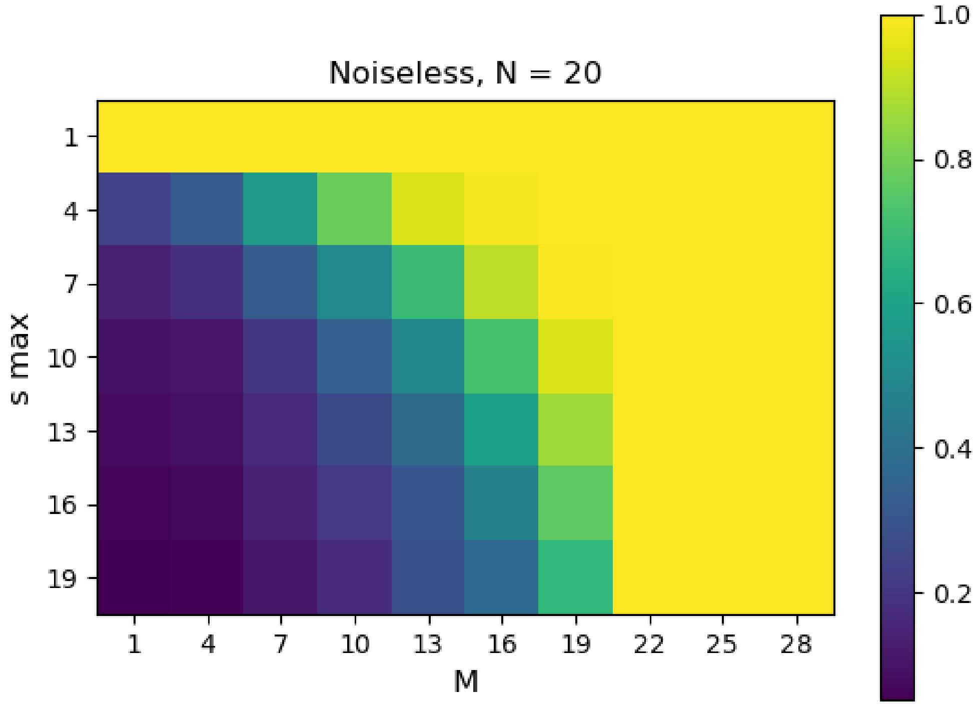
```
def OMP(A, y, iteration, acc_tolerance=0.001):
    """
    Orthogonal Matching pursuit algorithm
    :input A: measurement matrix M*N
    :input y: measurement vector M*1
    :output x: sparse vector  N*1
    """
    r = y    # initialize the residual as y
    M, N = A.shape
    x = np.zeros(N)
    Lambdas = []
    i = 0
    # Control stop interation with norm thresh or sparsity

    while norm(r, ord=2) > acc_tolerance and i < iteration:
        scores = A.T.dot(r)  # Compute the score of each atoms
        Lambda = np.argmax(abs(scores))  # Select the atom with the max score
        Lambdas.append(Lambda)
        An = A[:, Lambdas]  # All selected atoms form a basis
```

```
        # least square solution: x = (A^T A)^(-1) A^T y
        x[Lambdas] = np.linalg.pinv(An).dot(y)
        x = x.reshape(N, -1)
        r = y - A.dot(x)  # Calculate the residual
        i += 1
    return x
```

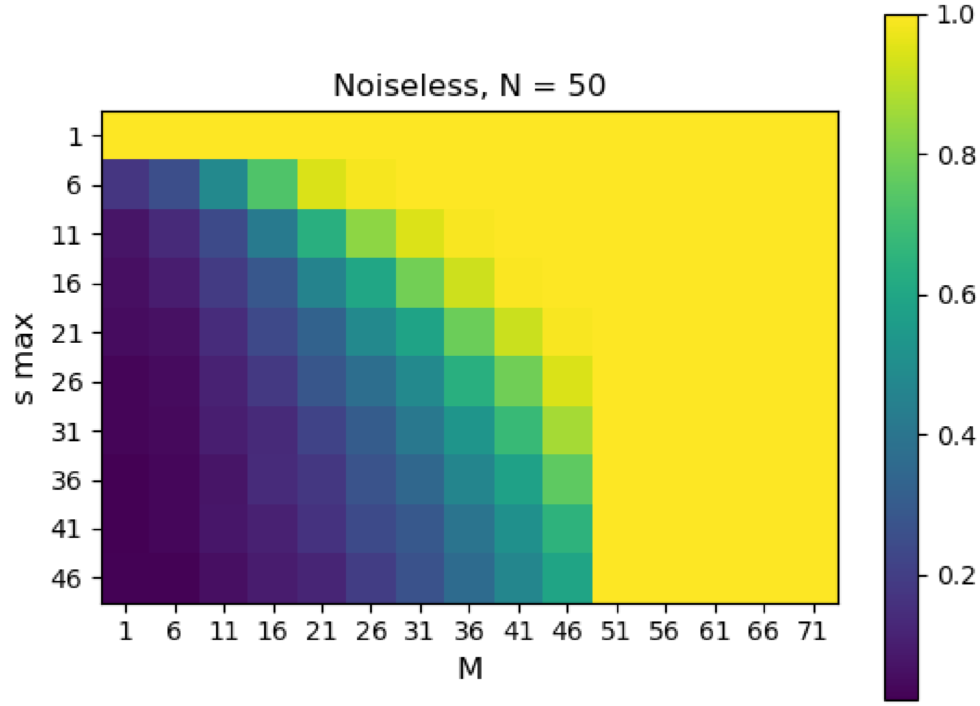
3. Noiseless case:

The noiseless case is in [noiseless.py](). The experiment results are as follows:

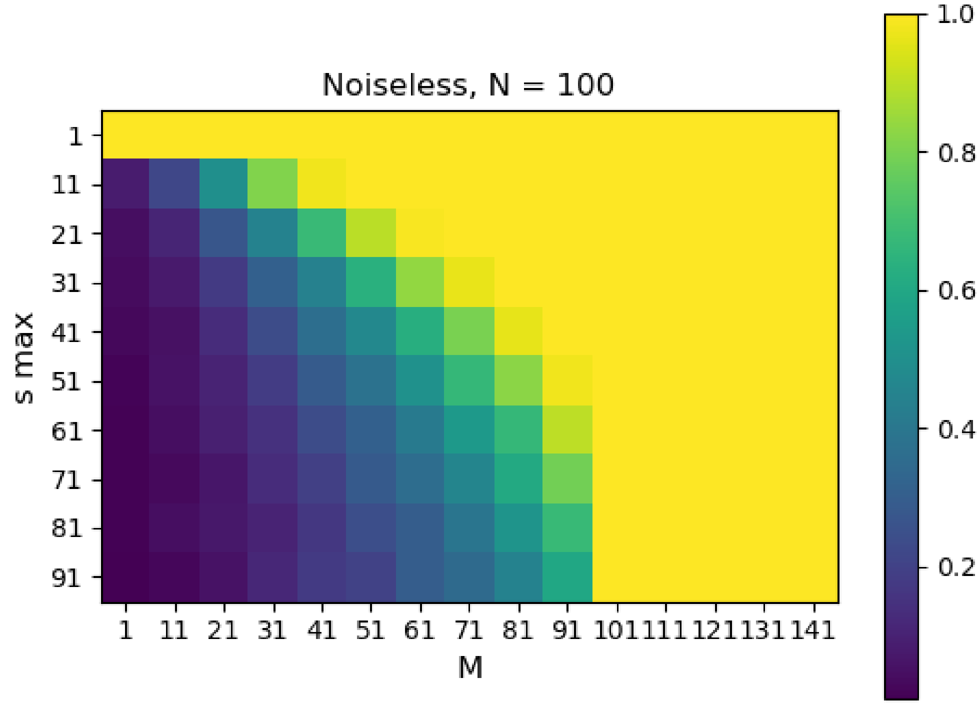
We implement the OMP algorithm for 2000 times with the parameter required in 5.3, with random realizations of A. The noiseless transition map of this scenario is shown below: (color value refers to the success rate)



- N = 20, M from 1 to 30, s from 1 to 20, both with an interval of 3



- N = 50, M and s from 1 to 50, with an interval of 5



- N = 100, M and s from 1 to 100, with an interval of 10

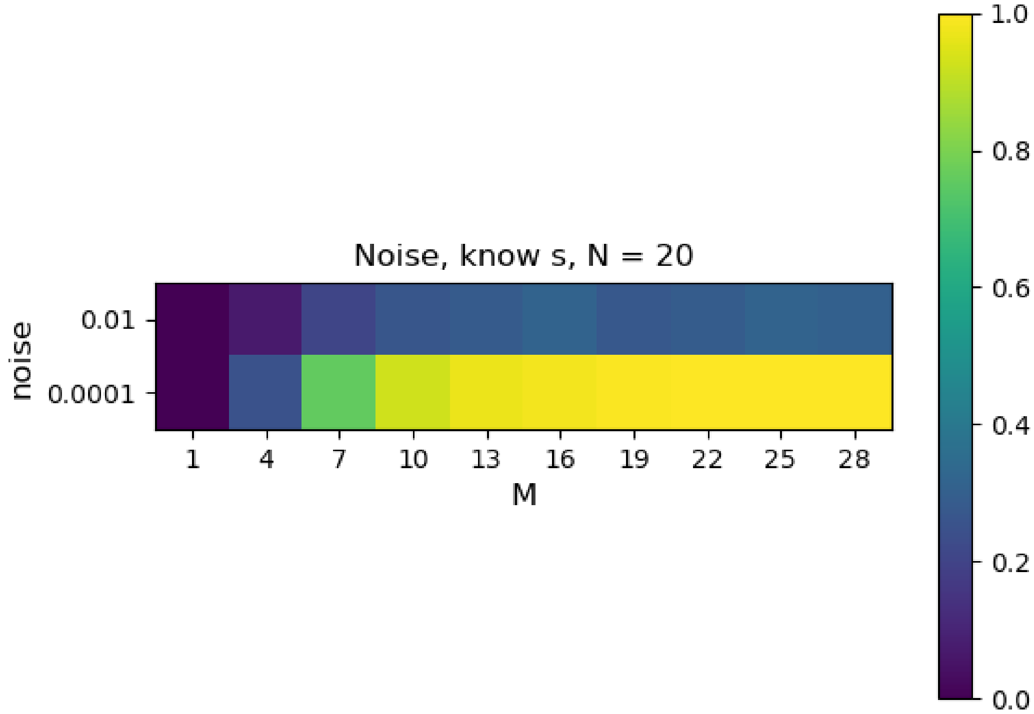
As shown in the figures above, the performance of OMP algorithm is better with the increase of M and decrease of s, which shows that the OMP algorithm prefers sparse matrix s. What's more, we can observe an obvious change near $M=N$, over which the OMP algorithm will remain an 100% success rate.

4. Noisy case:

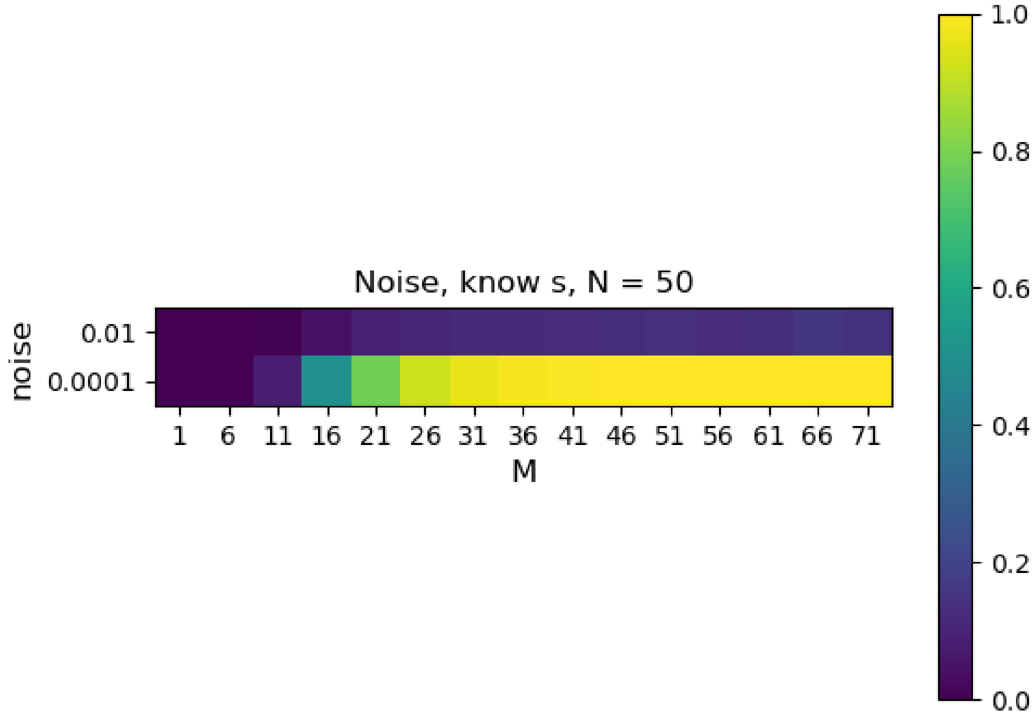
we input parameter 'know_s=True' or 'know_s=False' to distinguish the two requirements. Detailed implementation can be viewed in the code files.

4.1 Noisy case with sparsity s is known

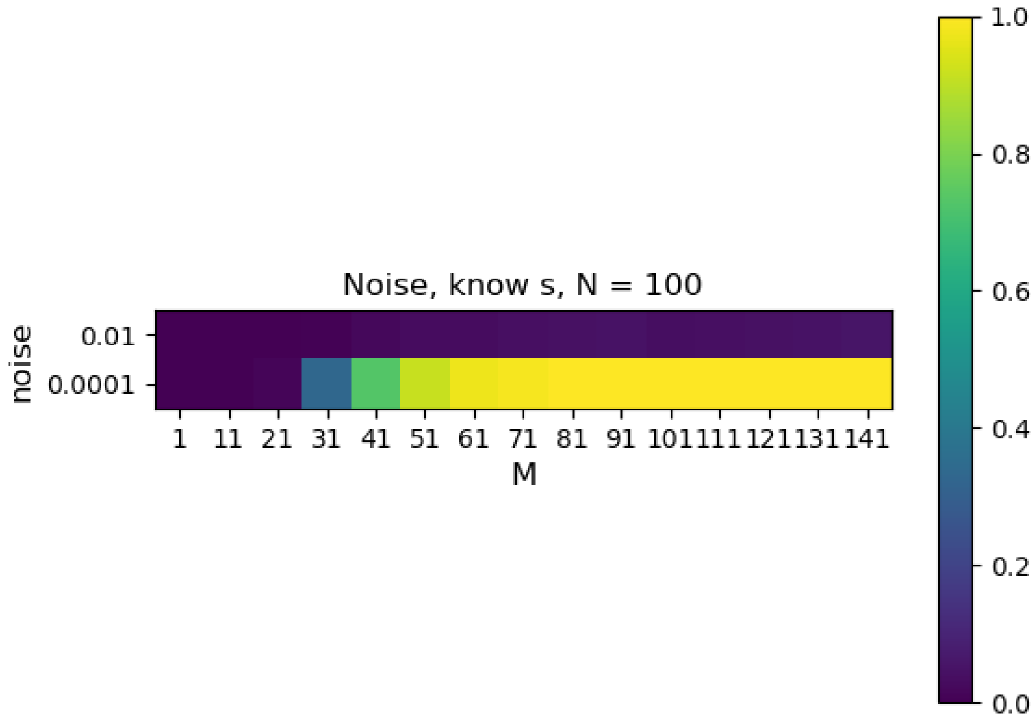
The noisy case with sparsity s is known is written in code file under_noise1.py. Here we run cases with N = 20, 50, 100. And we test the OMP with two different noises: 1e-2 and 1e-5. Considered that OMP algorithm is a sparse solver, we choose s = N // 10. The transition maps of success rate regarding noise and M are shown in figure 4.1.



- N = 20, M = [1, 20, 3], noise = 1e-2, 1e-4



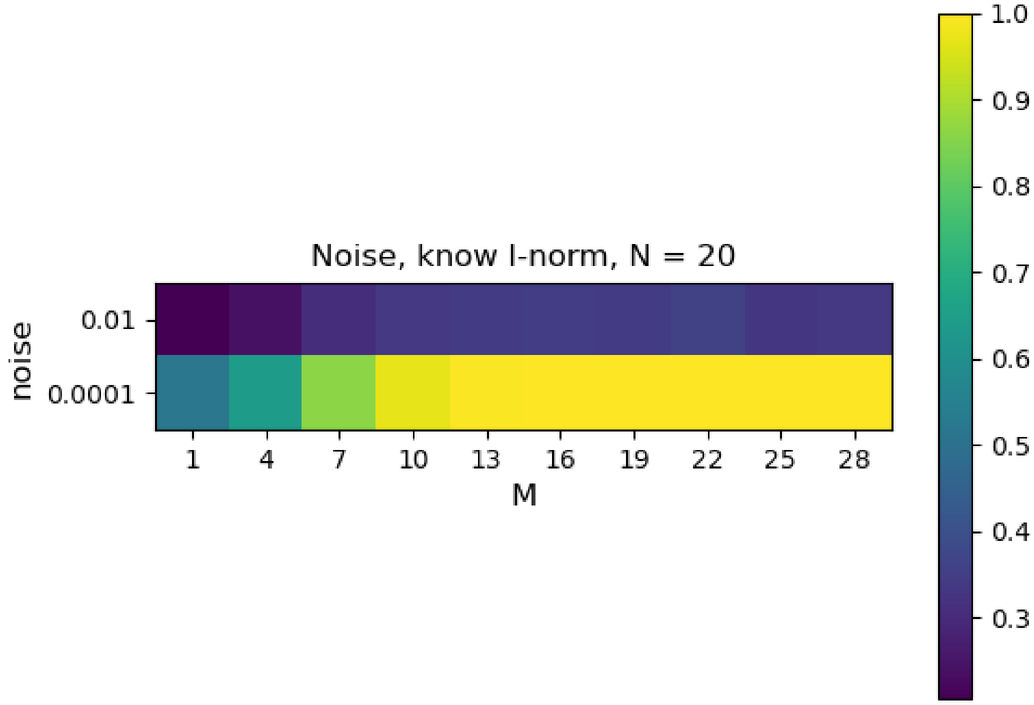
- N = 20, M = [1, 50, 3], noise = 1e-2, 1e-4



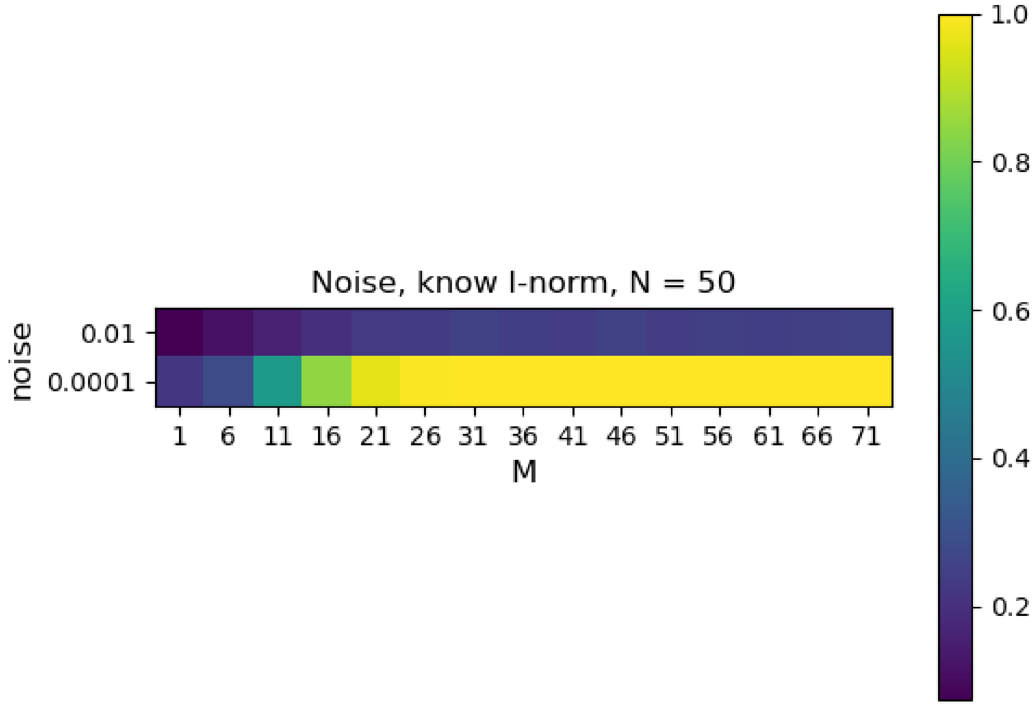
- N = 100, M = [1, 100, 5], noise = 1e-2, 1e-4

4.2 Noisy case with sparsity s is unknown and $||n||_2$ is known.

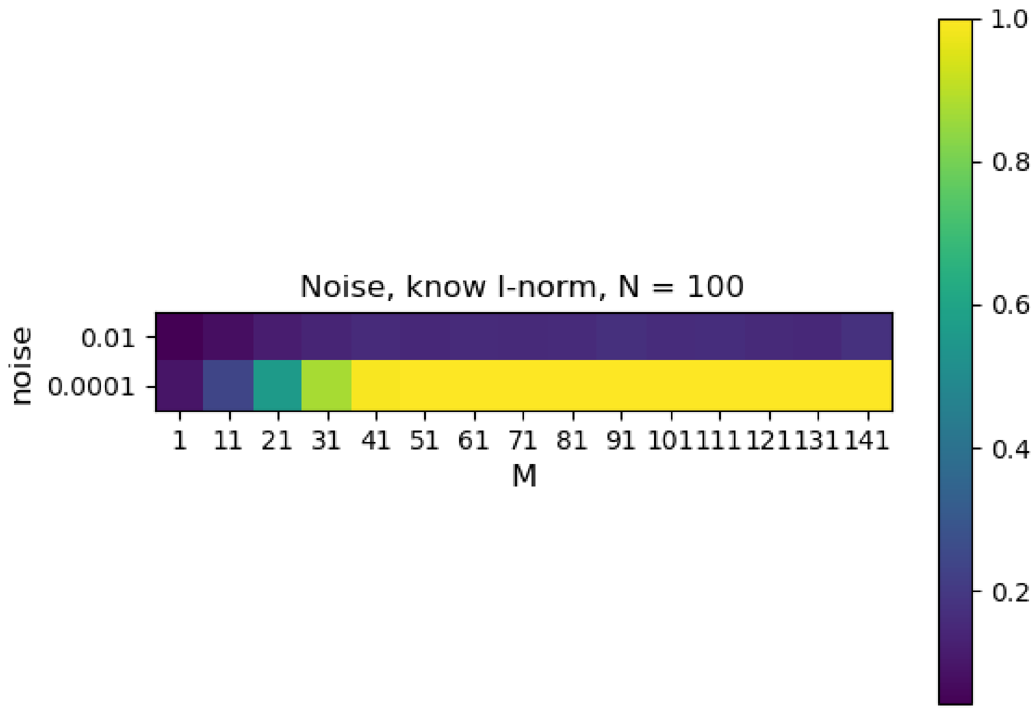
The noisy case with sparsity s is unknown and $||n||_2$ is known is written in code file [under_noise2.py](). Here we run cases with N = 20, 50, 100. And we test the OMP with two different noises: 1e-2 and 1e-5. Considered that OMP algorithm is a sparse solver, we choose s_max = N // 10. The transition maps of success rate regarding noise and M are shown in figure 4.2.



- N = 20, M = [1, 20, 3], noise = 1e-2, 1e-4



- N = 20, M = [1, 50, 3], noise = 1e-2, 1e-4



- N = 100, M = [1, 100, 5], noise = 1e-2, 1e-4

Both 4.1 and 4.2 show that the successful rate increase with the decrease of noise and increase of M

5. Decode a Compressed Signal

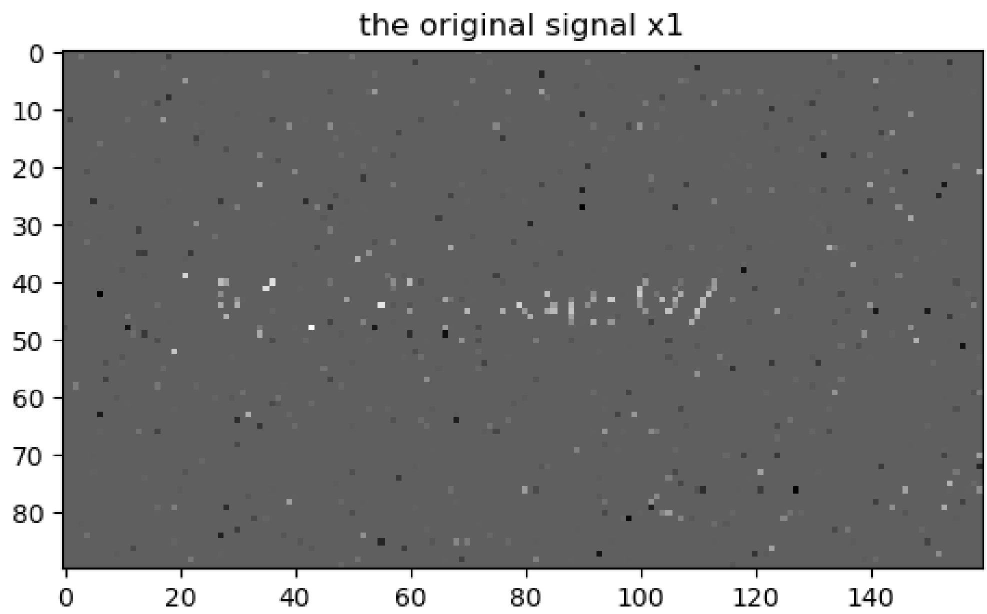
We implement this part with the 'omp.py' function we write in [omp.py](). Implementation is in [decode_message.py]()

5(a) Can You Guess?

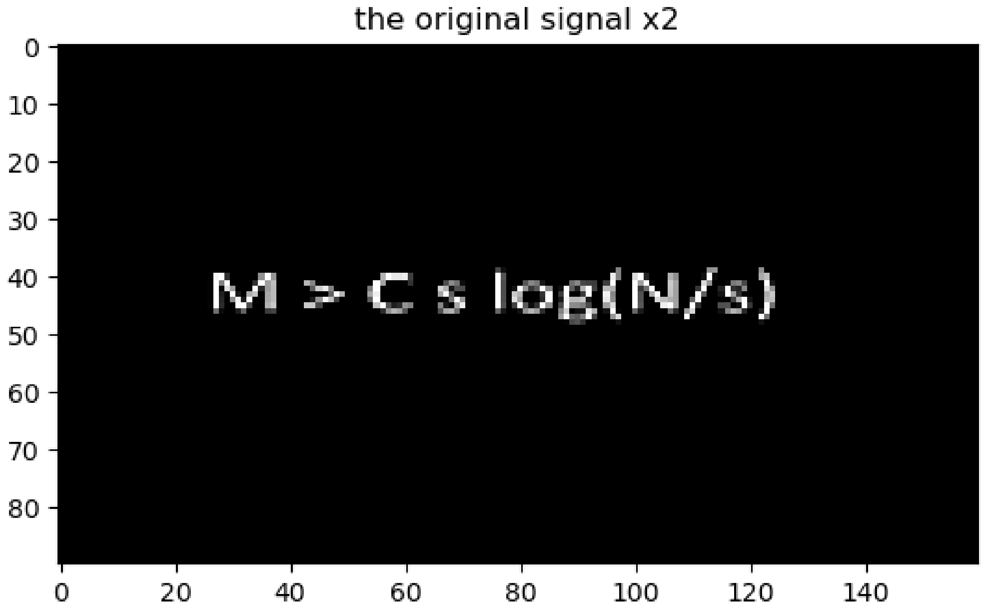
Unfortunately, No.

5(b) Recover the Signal

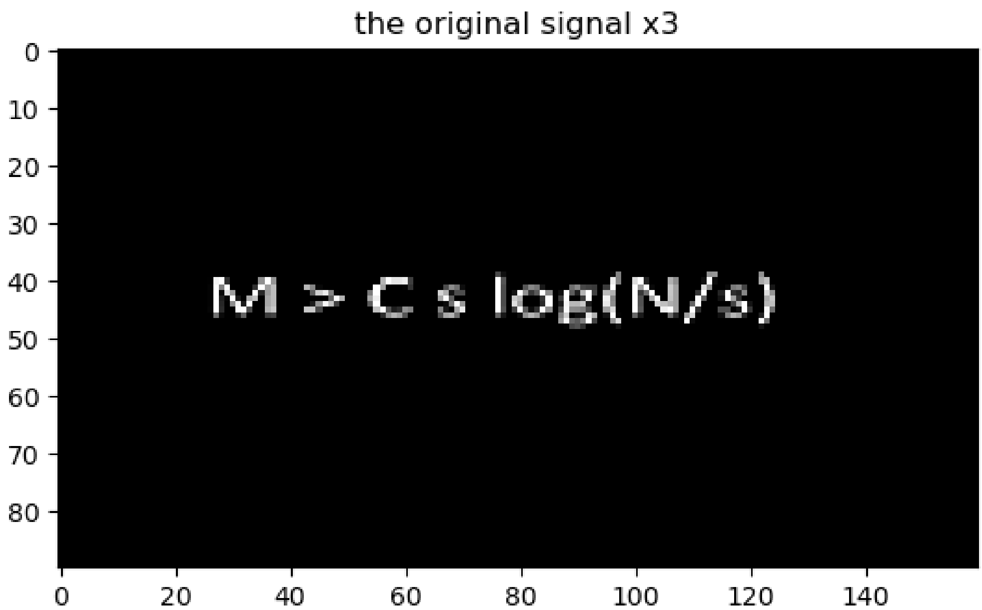
We recovered the original signal from Y_1, Y_2, Y_3 respectively and obtained the X_1, X_2, X_3. The results are shown below:



- A1 size: (960, 14400); Y1 size: (960, 1); X1 size: (14400, 1)



- A2 size: (1440, 14400); Y2 size: (1440, 1); X2 size: (14400, 1)



- A3 size: (2880, 14400); Y2 size: (2880, 1); X2 size: (14400, 1)

5(c) Analysis

We can see that the second and third figures are far more clear than the first one. Since in the second and third cases, the row number of matrix A is larger, and we have concluded empirically that the OMP algorithm works better under large M (row number) and small s(support size) condition.

5(d) Guess the Meaning of the picture

The figure is a formula:

$$M > C_s log(N/s)$$

This formula shows the same pattern as the experiments. For a larger N, you generally need a larger M to recover the x signal.

Worth mentioning, this formula assembles resemble the Shannon theorom:

$$C = Blog_2(1 + S/N)$$

I guess that the function is a theoretical bound of OMP/compressed sensing.