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Due: 12:00 pm 18 October 2022

Problem 1.

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maxb 100 [5, 122, 101, 32, 19]?
| maxb 100 [5, 122, 101, 32, 19] :=  maxb top [5, 122, 101, 32, 19]
  ...match[top := 100]
| = foldr compare FF [5, 122, 101, 32, 19]?
| | foldr compare FF [5, 122, 101, 32, 19]
  := foldr f b []
    ...match fails
  := foldr f b (x:xs)
    ...match [f := compare, b := FF, x := 5, xs := [122, 101, 32, 19]]
| | = f x (foldr f b xs) [f := compare, b := FF, x := 5, xs := [122, 101, 32, 19]]?
| | | compare 5 ( foldr compare FF [122, 101, 32, 19])
| | | := compare m v
  ...match [m := 5, v := (foldr compare FF [122, 101, 32, 19])]
| | | m >= top [m:=5, n:=100] = 5 >= 100
  ...false
| | | otherwise
  ...true
| | | = ssmax m v[m := 5, v := (foldr compare FF [122, 101, 32, 19])]?
| | | | ssmax 5 (foldr compare FF [122, 101, 32, 19])
| | | | := ssmax m (SS n)
  ... forces evaluation
| | | | | foldr compare FF [122, 101, 32, 19]?
| | | | | := foldr f b (x:xs)
  ... match [f:=compare, b:=FF, x=122, xs:=[101, 32, 19]]
| | | | | compare 122 (foldr compare FF [101, 32, 19])
| | | | | := compare m v
  ... match [m:=122, v:=(foldr compare FF [101, 32, 19])]
| | | | | m >= top [m:=122, n:=100] = 122 >= 100
  ...true
| | | | | = SS 100
| | | | | := ssmax m (SS n)
  ...match [m:=5, (SS n):= (SS 100)]
| | | | | = SS (max m n) [m:=5, n:=100]
| | | | | SS (max 5 100)
| | | | | SS 100
| | | | | SS 100
| | | | | SS 100

```

Problem 6.

Let f and g be functions with appropriate typing.

Base Case: For the base case of tip we have

$$\begin{aligned}\text{mapST } g \ (\text{mapST } f \ \text{Tip}) &= \text{mapST } g \ \text{Tip} \\ &= \text{Tip} \\ &= \text{mapST } (g.f) \ \text{Tip}\end{aligned}$$

Inductive Step: Let $t = (\text{SNode } t1 \ a \ t2)$ be a STree and assume for any smaller STree the property holds. If we note that $t1$ and $t2$ are smaller than t and therefore satisfy the property, we can compute

$$\begin{aligned}\text{mapST } g \ (\text{mapST } f \ t) &= \text{mapST } g \ (\text{mapST } f \ (\text{SNode } t1 \ a \ t2)) \\ &= \text{mapST } g \ (\text{SNode } (\text{mapST } f \ t1) \ (f \ a) \ (\text{mapST } f \ t2)) \\ &= \text{SNode } (\text{mapST } g \ (\text{mapST } f \ t1)) \ (g \ (f \ a)) \ (\text{mapST } g \ (\text{mapST } f \ t2)) \\ &= \text{SNode } (\text{mapST } (g.f) \ t1) \ ((g.f) \ a) \ (\text{mapST } (g.f) \ t2) \\ &= \text{mapST } (g.f) \ (\text{SNode } t1 \ a \ t2) \\ &= \text{mapST } (g.f) \ t\end{aligned}$$

thus we are done.