Problem 6.

Let f and g be functions with appropriate typing.

Base Case: For the base case of tip we have

$$\begin{aligned} \text{mapST g (mapST f Tip)} &= \text{mapST g Tip} \\ &= \text{Tip} \\ &= \text{mapST (g.f) Tip} \end{aligned}$$

Inductive Step: Let t = (SNode t1 a t2) be a STree and assume for any smaller STree the property holds. If we note that $t1$ and $t2$ are smaller than t and therefore satisfy the property, we can compute

$$\begin{aligned} \text{mapST g (mapST f t)} &= \text{mapST g (mapST f (SNode t1 a t2)} \\ &= \text{mapST g (SNode (mapST f t1) (f a) (mapST f t2))} \\ &= \text{SNode (mapST g (mapST f t1)) (g (f a)) (mapST g (mapST f t2))} \\ &= \text{SNode (mapST (g.f) t1) ((g.f) a) (mapST (g.f) t2)} \\ &= \text{mapST (g.f) (SNode t1 a t2)} \\ &= \text{mapST (g.f) t} \end{aligned}$$

thus we are done.