Robust control system development for VTOL-to-fixed wing flight transition with the EcoSoar UAV

A thesis in Automatic Control

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Abstract

This is wehere we write about the abstract. asdasdasd asd asdasdjhsdkjhdsak dikd sdoj asodj asdoja sdojas do

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Thank **Khalid**, (George), Mr. K, Avory, Wife, etc.

[Lets see if they require rent before adding below]

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1 List of variables

L_L	Lift from left wing
L_R	Lift from right wing
D_L	Drag from left wing
D_R	Drag from right wing
F_L	Lift due to aileron deflection, left side, outside propeller wake
F_{L_T}	Lift due to aileron deflection, left side, inside propeller wake
F_R	Lift due to aileron deflection, right side, outside propeller wake
F_{R_T}	Lift due to aileron deflection, right side, inside propeller wake
D_{α_L}	Drag due to aileron deflection, left side, outside propeller wake
$D_{\alpha_{L,T}}$	Drag due to aileron deflection, left side, inside propeller wake
D_{α_R}	Drag due to aileron deflection, right side, outside propeller wake
$D_{\alpha_{R,T}}$	Drag due to aileron deflection, right side, inside propeller wake
L_{W_L}	Lift from left winglet
L_{W_R}	Lift from right winglet
D_{W_L}	Drag from left winglet
D_{W_R}	Drag from right winglet
S_a	Surface area of one aileron, parts outside propeller wake
S_{a_w}	Surface area of one aileron, area inside propeller wake
c.g.	center of gravity (point)
T_L	Thrust from left propeller
T_R	Thrust from right propeller
S_w	Area of one wing, area outside of propeller wake
S_{w_w}	Area of one wing, area inside propeller wake
S_W	Area of one winglet
x_f	distance in x from c.g. to aileron force
x_T	distance in x from c.g. to thrust force
x_{ac}	distance in x from c.g. to wing aerodynamic center
x_W	distance in x from c.g. to winglet force
y_T	distance in y from c.g. to thrust force
y_f	distance in y from c.g. to aileron force
y_{ac}	distance in y from c.g. to wing aerodynamic center
y_w	width of propeller wake
b	wingspan
z_W	distance in z from c.g. to winglet force

—

2 Background

EcoSoar background, Malawi, lead into VTOL motivation

3 Problem Forumulation

To create a mathematical model of the ecosoar, then implement a model based controller that can handle both hovering, transition into flying and flying.

4 Needed math

4.1 General State Space background

Diff. Eqs, state feedback, observability, controllability. Are obs and contr. applicable on non linear systems? MIMO

4.2 State estimation/observers?

4.3 Feedback Linearization

4.3.1 Mimo: Lie derivatives and brackets

4.3.2 Decoupling matrix

4.4 L1 parameter compensation/estimation

4.5 Reference Frames

NED, World+body frame

4.6 Accellerated reference frames

4.7 Quaternions

Motivate through problems with Euler angles, RPY, etc Rotation matrix Cover basic quaternion math, world/bodyframe conversions

4.7.1 Madgwick filter

4.8 Equations of Motion, in body frame

4.9 Basic aerodynamics

When a wing moves through air two major forces act upon it, a lifting force and a drag force. The lift is primarily due to camber (the non symmetry of the cross section of the wing) and to the angle of attack: the angle at which the incoming air hits the wing. The same goes for drag.

- Bernoulli Equation, center of Pressure, etc
- Basic plate theory
- Aerodynamic center
- Aerodynamic forces and Moments
  - Lift, and moment, due to camber
  - Lift due to angle of attack
  - Drag

  - Forces/Moments due to airflow over Ailerons and Elevators
  - Forces and Moments due to sideslip and roll
  - Forces/Moment due to roll rate, pitch rate and yaw rate
- Propeller efficiency, airspeed, wake velocity

5 Mathematical Model of EcoSoar

5.1 Assumptions

- There is no external wind and so v_∞ is replaced with v which is the aircrafts velocity through the air.
- All forces operate on points in the $x - y$ plane, i.e. no offset in the z-direction (with the notable exception of the winglet forces). Forces may still have a z-component though.
- The propeller wake is unaffected by aircraft velocity, i.e. only determined by propeller speed.
- Airflow outside of wake is only determined by aircraft velocity.
- The aerodynamic centre (a.c.) is static and does not move with changes in speed and pressure. The moment around it, however, will vary and thus account for the effects of the a.c. movement.
- Airflow speed in wake is proportional to propeller rotational speed, $v_w = b_w \omega_i$, where b_w is a constant.
- The principal moments axis align with the symmetry plane; only diagonal elements in the inertia matrix aligned with the body frame coordinate axis.
- Roll moment due to sideslip angle, negligible due to lack of tail with verical offset from c.g.
- Yawing moment due to roll rate is negligible due to the lack of a tail.
- Forces and moments due to acceleration are not explicitly calculated; they are already included in dynamic model since it is based on forces.

5.2 Coordinate system

NED: North East Down. x-axis is essentially the forward direction of the airplane in normal flying mode. z-axis is pointed downwards. y-axis is along the right wing.

5.3 Aircraft force diagram

The aircraft in question is the EcoSoar; a flying wing. It has two control surfaces, one on each wing called elevons. In front of each wing a motor and propeller is mounted providing thrust. The motors also supply an airflow over the control surfaces when the aircraft is not moving through the air.

The elevon forces, F_L, F_{L_T}, D_{a_L} and $D_{a_{L,T}}$, have only been marked on the left wing, but exist symmetrically on the right wing with index R instead of L. The same is true for the drag force, D_R and the winglet forces, L_{W_R} and D_{W_R}, existing symmetrically on the left side. F_L is the force due to elevon deflection over surface area S_a, and F_{L_T} is the force due to elevon deflection over S_{a_w}; the surface in the wake of the propeller. Similarly for the drag, D_{α_L}, $D_{\alpha_{L,T}}$, forces. We also split the wing area into two sections: S_w for the wing area outside the wake, and S_{w_w} for the wing area in the wake. The thrust forces, T_i, are assumed to act at the arrow end of the vector due to the offset in x created by the motor and drive shaft. The lift forces, L_L and L_R, are functions of airflow over wing outside of wake and in the wake, which in turn are functions of aircraft velocity, propeller speed, angle of attack, air density, wing shape, etc.

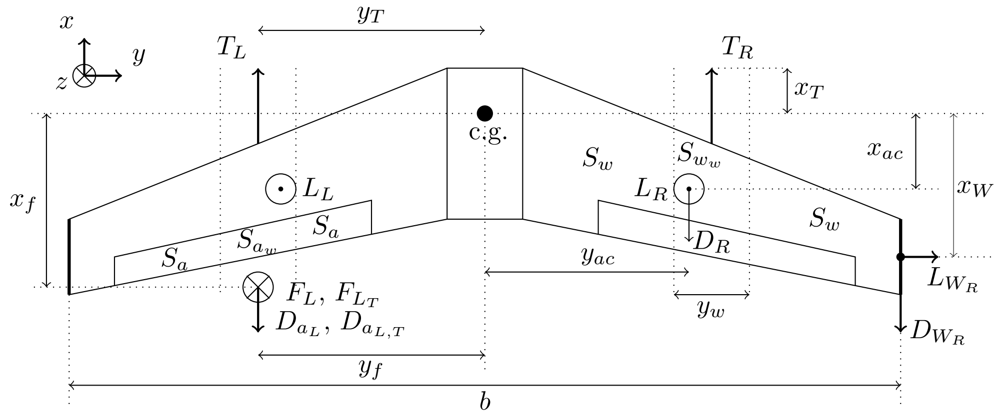

Figure 1: Drawing of the EcoSoar, its propellers with wake and its control surfaces in body frame.

5.3.1 Aerodynamic Lift force

The lift force of a wing depends on several factors. Mainly: geometrical shape, dynamic pressure and angle of attack. The bigger the wing the more lift. The faster air flows the more lift. The further away from hitting the wing straight on, usually more lift.

Important note: Lift is defined as the **Aerodynamic force component perpendicular to the incoming airflow.** This means that, for example, the lift force labeled in figure 1 as L_L **is not the aerodynamic lift but rather the lift force in body frame.** The same argument applies to the drag forces.

Aerodynamic lift is usually modeled as:

$$L = QC_LS \tag{1}$$

where S is a reference surface area of the wing and Q is the dynamic pressure:

$$Q = \frac{1}{2}\rho v^2 \tag{2}$$

ρ is the density of air and v the velocity of the air hitting the wing. C_L is the coefficient of lift, a function of the angle of attack. This coefficient is not linear. Empirical studies have found functions similair to the one in figure 2.

The angle of attack, α, is formally the angle at which the air is hitting the wing relative the body frame x-z axis. If air is aligned with the x-axis α is zero, and if aligned with the z-axis α is 90. (The airflow will obviously be in the negative axis direction since the airplane is flying in the positive direction and hitting the air). Thus, if we assume no wind the angle of attack is only dependent on the aircraft velocity $\bar{v}$. Normally this is modeled as:

$$\alpha = tan^{-1}(\frac{v_z}{v_x}) \tag{3}$$

but in our case the velocity may have any direction in the x-z plane, and is not limited to the right half plane. The polar coordinates for vector $\bar{v}$ in the figure below, figure 3, are:

$$\begin{aligned} v_x &= r\,cos(\alpha) \\ v_y &= r\,sin(\alpha) \\ r &= \sqrt{v_x^2 + v_z^2} \end{aligned} \tag{4}$$

where α is in the interval $[-\pi, \pi]$.

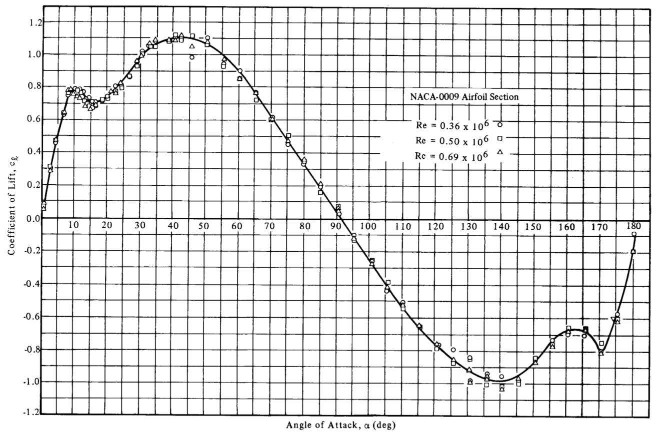

Figure 9. Full Range Section Lift Coefficients for the NACA-0009 Airfoil at Reynolds Numbers of 0.36 x 10^6, 0.50 x 10^6, and 0.69 x 10^6

Figure 2: Coefficient of lift for a symmetrical wing vs angle of attack for the entire 180 degree rotation of the wing. The function is periodic.

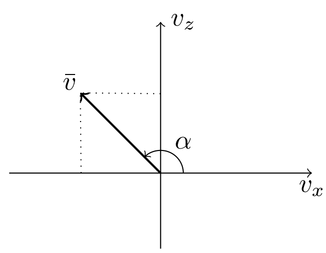

Figure 3: Velocity vector $\bar{v}$ in polar coordinates.

5.3.2 Aerodynamic drag

Similarly to aerodynamic lift the wing will also have an aerodynamic drag force, but parallel to the incoming flow and not perpendicular as lift. The drag is also modeled like so:

$$D = QC_D S \tag{5}$$

where the coefficient of drag, C_D, can also be experimentally obtained as in figure 4.

5.3.3 Lift and drag in body frame

Since the aerodynamic lift and drag forces are relative the incoming airflow, i.e. functions of the angle of attack, α, the body frame Lift and Drag have to transformed:

$$\begin{aligned} L_i &= cos(\alpha)L + sin(\alpha)D \\ D_i &= -sin(\alpha)L + cos(\alpha)D \end{aligned} \tag{6}$$

Note that the above forces are in the direction labeled in figure 1 and not necessarily in the positive axis they act in. From equation 4 we see that

$$\begin{aligned} v_x &= r\,cos(\alpha) \Rightarrow cos(\alpha) = \frac{v_x}{\sqrt{v_x^2 + v_z^2}} \\ v_z &= r\,sin(\alpha) \Rightarrow sin(\alpha) = \frac{v_z}{\sqrt{v_x^2 + v_z^2}} \end{aligned} \tag{7}$$

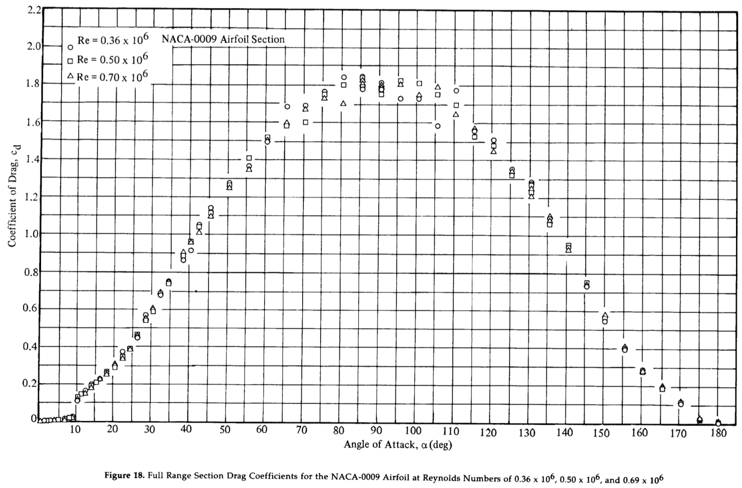

Figure 4: Coefficient of drag for a symmetrical wing vs angle of attack for the entire 180 degree rotation of the wing.

If we substitute in expressions for aerodynamic lift and drag we obtain:

$$
\begin{aligned}
L_i =& \frac{v_x}{\sqrt{v_x^2 + v_z^2}} Q C_L S + \frac{v_z}{\sqrt{v_x^2 + v_z^2}} Q C_D S \\
D_i =& -\frac{v_z}{\sqrt{v_x^2 + v_z^2}} Q C_L S + \frac{v_x}{\sqrt{v_x^2 + v_z^2}} Q C_D S
\end{aligned} \tag{8}
$$

which simplify into:

$$
\begin{aligned}
L_i =& S \frac{1}{2} \rho v^2 \left(\frac{C_L v_x + C_D v_z}{\sqrt{v_x^2 + v_z^2}} \right) \\
D_i =& S \frac{1}{2} \rho v^2 \left(\frac{-C_L v_z + C_D v_x}{\sqrt{v_x^2 + v_z^2}} \right)
\end{aligned} \tag{9}
$$

where $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$.

5.4 Moment due to force

Any force not acting at the center of gravity will produce a moment around c.g. The moment of a force is:

$$
\bar{M} = \bar{r} \times \bar{F} \tag{10}
$$

With our assumptions the above equation can be written as:

$$
\bar{M} = (r_x, r_y, 0) \times (F_x, F_y, F_z) = \begin{bmatrix} F_z r_y \\ -F_z r_x \\ F_y r_x - F_x r_y \end{bmatrix} \tag{11}
$$

5.4.1 Pitching moment

Lift due to camber on a wing acts at 50% of the cord line. Lift due to angle of attack acts at roughly 25% of the cord line. This makes the force acting point, center of pressure, move along the cord line when angle of attack and aircraft velocity changes. A common simplification is to choose the aerodynamic center, a.c., as the point at which Lift acts. It can be shown that placing the a.c. at the 25% cord position makes the moment generated vary little as angle of attack changes, yielding simpler equations. The pitching moment due to lift can now be expressed as:

$$\tau_{y,lift} = \sum_{j\in[w,w_w]} C_{m,y,l} Q S_j l + \cancelto{0}{\sum_{j\in[w,w_w]} C_{m,y,l,\alpha} Q S_j l \alpha} \tag{12}$$

where l is the characteristic length of the wing, usually taken as the mean cord for pitching moments and the wingspan for rolling and yawing moments.

5.5 Equations of motion

The equations of motion can be split into three parts: one part due to gravity, one part due to the rigid body frame being an accelerated reference frame, and one part due to aerodynamic forces.

5.5.1 Attitude

The attitude is represented by a quaternion:

$$\bar{q} = [q_0, q_1, q_2, q_3]^T \tag{13}$$

Any vector, $\bar{r}_w$, in world frame can be represented in the body frame by the conversion:

$$\bar{r}_b = \bar{q}\bar{r}_w\bar{q}^* \tag{14}$$

5.5.2 Gravity

In the case of gravity, $\begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}_w$, the force, $m\bar{g}$, becomes

$$m\dot{\bar{v}}_b = \begin{bmatrix} 2(\bar{q}_1\bar{q}_3 + \bar{q}_0\bar{q}_2) \\ 2(\bar{q}_2\bar{q}_3 - \bar{q}_0\bar{q}_1) \\ (\bar{q}_0{}^2 - \bar{q}_1{}^2 - \bar{q}_2{}^2 + \bar{q}_3{}^2) \end{bmatrix} mg \tag{15}$$

Gravity acts in the center of gravity and creates no moment. Since gravity is in world frame and we want to express it in body frame we need to conjugate the quaternion.

5.5.3 Forces and moments due to accelerated body reference frame

In the case of a rigid body with inertia matrix I, velocity $\bar{v}$ and angular velocity $\bar{\omega}$ relative a fixed frame we have the moment equation:

$$I\dot{\bar{\omega}} = \bar{\tau} - \bar{\omega} \times I\bar{\omega} \tag{16}$$

where τ is the external torque acting on the body.

Similarly, if we include the coriolis force we obtain the forces in our body frame:

$$\bar{F} = \bar{F}_e + 2m\,\bar{\omega} \times \bar{v} \tag{17}$$

where $\bar{F}_e$ is the external forces acting on the body. No other inertial forces are of interest; the linear accelleration is irrelevant since we are not flying inside a linearly accellerating frame, and the other two depend on rotation around a non principal axis.

In our case, with assumptions made, the above equations simplify to:

$$\begin{bmatrix} \dot{\bar{v}} \\ \dot{\bar{\omega}} \end{bmatrix} = \begin{bmatrix} \dot{v_x} \\ \dot{v_y} \\ \dot{v_z} \\ \dot{\omega_x} \\ \dot{\omega_y} \\ \dot{\omega_z} \end{bmatrix} = - \begin{bmatrix} 2(v_z\omega_y - v_y\omega_z) \\ 2(v_x\omega_z - v_z\omega_x) \\ 2(v_y\omega_x - v_x\omega_y) \\ \frac{1}{I_x}(I_z - I_y)\omega_y\omega_z \\ \frac{1}{I_y}(I_x - I_z)\omega_x\omega_z \\ \frac{1}{I_z}(I_y - I_x)\omega_x\omega_y \end{bmatrix} + \texttt{external forces and moments} \tag{18}$$

5.5.4 elevons

Elevon forces are modeled similarly to lift, as well, but with an extra term, δ_i, for the deflection degree in radians:

$$F_i = C_{L_\delta} \sum_{j \in [a, a_w]} Q_{j,i} S_j \delta_i \tag{19}$$

and similairly the drag created by the elevons:

$$D_{\alpha_i} = C_{D_\delta} \sum_{j \in [a, a_w]} Q_{j,i} S_j \delta_i \tag{20}$$

The elevon lift, and drag, coefficient is assumed to be static over all angle of attacks since they will be in the wake of the wing, and the lift and drag are linear in the region of angles the ailerons can deflect.

5.6 Winglets, forces and moments due to sideslip

The aircraft considered here has no tail nor real body. The only surfaces providing relevant forces due to sideslip are the winglets.

The winglets each have an area S_W and are flat plate wings which will generate aerodynamic lift and drag forces due to β providing an angle of attack in their reference frame.

β can be, similarly to α, be expressed in polar coordinates:

$$\begin{aligned} v_x &= r\,cos(\beta) \\ v_y &= r\,sin(\beta) \\ r &= \sqrt{v_x^2 + v_y^2} \end{aligned} \tag{21}$$

The aerodynamic forces, L_β and D_β, from the winglets convert into body forces, directed as in figure 1, similarly to lift and drag from the main wings:

$$\begin{aligned} L_{W_i} &= cos(\beta) L_\beta + sin(\beta) D_\beta \\ D_{W_i} &= -sin(\beta) L_\beta + cos(\beta) D_\beta \end{aligned} \tag{22}$$

where

$$\begin{aligned} L_\beta &= Q C_{L_\beta} S_W \\ D_\beta &= Q C_{D_\beta} S_W \end{aligned} \tag{23}$$

and C_{k_β} are the lift and drag coefficients for each winglet, here assumed to be flat plates. v_z can be assumed to be negligible in the following equations since it will have no measurable effect on the winglets. If we assume v_z to be negligible then the equations simplify into

$$\begin{aligned} L_{W_i} &= \frac{\rho v S_W}{2} \left(C_{L_\beta} v_x + v_y C_{D_\beta} \right) \\ D_{W_i} &= \frac{\rho v S_W}{2} \left(-v_y C_{L_\beta} + v_x C_{D_\beta} \right) \end{aligned} \tag{24}$$

If the sideslip angle, β, is small (which implies v_y small) the lift/drag coefficients can be approximated as

$$\begin{aligned} C_{L_\beta} &\approx \frac{dC_{L_\beta}}{d\beta}\Big|_{\beta=0} \beta \\ C_{D_\beta} &\approx 0 \end{aligned} \tag{25}$$

which results in:

$$\begin{aligned} L_{W_i} &\approx \frac{\rho v S_W}{2} \frac{dC_{L_\beta}}{d\beta}\Big|_{\beta=0} \beta\, v_x \\ D_{W_i} &\approx 0 \end{aligned} \tag{26}$$

This gives us, in body frame, one restoring force from each winglet roughly proportional to the forward velocity squared of the aircraft and the sideslip angle.. The forces have the same magnitude and direction on both sides of the aircraft.

5.6.1 Restoring moment

D_{W_i} are symmetrical on both sides and only create an acceleration in y. They do not generate any moment. L_{W_i}, however, act asymmetrically around c.g. and do generate a moment/torque (note that the force L_{W_R} generates a negative torque around z in our coordinate system):

$$M_z = L_{W_L} x_W + L_{W_R} x_R = -\rho v S_W \frac{dC_{L_\beta}}{d\beta}\Big|_{\beta=0} \beta \, v_x x_W \tag{27}$$

5.6.2 Non linear Forces and Moments

If we do not make the assumption that β is small we instead get the equations:

$$\begin{aligned} M_z =& - \, x_W \rho v S_W \left(C_{L_\beta}(\beta) v_x + v_y C_{D_\beta}(\beta) \right) \\ D_W =& \rho v S_W \left(-v_y C_{L_\beta}(\beta) + v_x C_{D_\beta}(\beta) \right) \end{aligned} \tag{28}$$

where $D_W = D_{W_L} + D_{W_R}$, and the coefficients of lift and drag are as shown in the figures above, but functions of β instead of α as β becomes the effective angle of attack in the winglets reference frame.

5.6.3 Rolling moment due to sideslip

The center of the winglet is slightly offset in the z-direction from the c.g. by distance z_W. Given the forces L_{W_i} the rolling moment due to sideslip, M_x, is simply

$$M_x = z_W (L_{W_L} + L_{W_R}) \tag{29}$$

5.7 Propeller Thrust

Thrust will be modeled as

$$T_i = K_T \omega_i^2 \tag{30}$$

where K_T is the thrust coefficient of the propeller and ω_i the rotational speed of the propeller.

Behind the propellers an area of airflow is generated, called a wake. Given a propeller diameter of d_P, pushing air through at a rate of v_w the volume of air being moved by the propeller during time t is:

$$V_{air} = \int_0^t \frac{d}{2} \pi^2 v_w dt \, . \tag{31}$$

The air has density ρ converting the volume into mass:

$$m_{air} = \rho \, m_{air} \tag{32}$$

If we assume the air to be stationary in front of the propeller and the aircraft standing still the change in momentum in the air is:

$$\Delta p = m_{air} v_{air} = \rho \frac{d\pi^2}{2} v_w^2 t \tag{33}$$

Force due to change in momentum is:

$$Ft = \Delta p \Rightarrow F = \frac{\Delta p}{t} \tag{34}$$

giving us:

$$F = \rho \frac{d\pi^2}{2} v_w^2 \, . \tag{35}$$

The propellers suffer from aerodynamic losses, non linear effect, etc, so normally an efficiency factor is used. TODO: FIND REFERENCE. This results in an equation as:

$$F = K_v v_w^2 \, . \tag{36}$$

If we instead look at the lift generated by the propeller blades we can find the thrust as a function of propeller rotation speed. Assuming the aircraft has no speed and there is no wind we can model the lift from one propeller blade as:

$$L_i = \int_0^{\frac{d}{2}} \frac{\rho}{2}(\omega_i r)^2 C_L(pitch)dr \tag{37}$$

If we also assume a constant pitch along the diameter of the propeller blade the above integral simplifies into:

$$L_i = \frac{\rho d^3}{48} C_L(pitch)\omega_i^2 \tag{38}$$

and thus with n blades we find the total lift, i.e. thrust:

$$T_i = L = \sum_{i \in [1,n]} L_i = nL_i \tag{39}$$

By comparing the equation for thrust due to rotation speed and thrust due to airflow we conclude that:

$$\begin{aligned} K_v v_w^2 =& n\frac{\rho d^3}{48} C_L(pitch)\omega_i^2 \Rightarrow \\ v_w =& b_w \omega_i \end{aligned} \tag{40}$$

and so the assumption that airflow in the wake is proportional to the propeller angular speed holds.

5.8 Restoring moment due to roll rate

If we assume that v_y is negligible then the Lift and Drag forces in body fram simplify into:

$$\begin{aligned} L_i =& S\frac{1}{2}\rho v^2 \left(\frac{C_L v_x + C_D v_z}{\sqrt{v_x^2 + v_z^2}} \right) = S\frac{1}{2}\rho\sqrt{v_x^2 + v_z^2}\,(C_L v_x + C_D v_z) \\ D_i =& S\frac{1}{2}\rho v^2 \left(\frac{-C_L v_z + C_D v_x}{\sqrt{v_x^2 + v_z^2}} \right) = S\frac{1}{2}\rho\sqrt{v_x^2 + v_z^2}\,(-C_L v_z + C_D v_x) \end{aligned} \tag{41}$$

If the plane is experiencing rolling, a rotation about the x-axis with a rotational speed of ω_x then a restoring moment will be created. This is because the air is resisting the wings motion through the air.

The rotational speed will yield an increased local v_z, $v_{z,l}$, along the wing:

$$v_{z,l} = v_z + \omega_x y \tag{42}$$

where y is the distance away from the body along the wing. The change in velocity changes the angle of attack locally. The local angle of attack is denoted α_l.

Lets consider the full moment as the integral of the full moment across both wings. This way the symmetrical lift forces will cancel and only the rolling moment will remain.

$$\begin{aligned} M_x =& -\int_{-b/2}^{b/2} y dL_i(y)dy \\ =& -\frac{\rho}{2}\int_{-b/2}^{b/2} yc(y)\sqrt{v_x^2 + v_{z,l}^2}\,(C_L v_x + C_D v_{z,l})\,dy \end{aligned} \tag{43}$$

If we make the crude assumption that $C_L = a * sin(2 * \alpha)$, and the less crude asumption that $C_D = -b * cos(\alpha) + b$, it is still impossible to find the function $M_x(\omega_x)$ for the entire state space. A term:

$$\int_{-b/2}^{b/2} y\sqrt{(v_1 + \omega y)^2 + v_2^2}dy \tag{44}$$

will remain, and one has to assume $\alpha \in [-\pi/2, \pi/2]$, $v_2 \neq 0$ and/or many of the variables always positive, to find the function without the integral.

A simulator can solve the integral numerically but in order to find smooth analytical expressions for the controller approximations have to be made.

It turns out that the four dimensional function for restoring moment due to roll as a function of roll rate ω_x, v_x and v_z is actually quite linear with respect to ω_x. The remaining function can be closely approximated by a two dimensional polynomial in v_x and v_z allowing for a feedback linearized model to be implemented with very little error.

5.9 Restoring moment due to yaw rate

Similarly the restoring moment due to yaw rate can be obtained from the following integral:

$$M_z = \int_{-b/2}^{b/2} y \left(\frac{1}{2}\rho\sqrt{v_{x,l}^2 + v_z^2}\,(-C_L v_z + C_D v_{x,l}) \right) c(y) dy \tag{45}$$

where $v_{x,l} = v_x - \omega_z y$.

For the controller some linear approximation has to be found, similarly as to restoring moment due to roll rate.

5.10 Rolling moment due to yaw rate

Due to the change in forward velocity a change in lift is also produced, inducing a rolling moment obtained similarily:

$$M_x = \int_{-b/2}^{b/2} y \left(\frac{1}{2}\rho\sqrt{v_{x,l}^2 + v_z^2}\,(C_L v_{x,l} + C_D v_z) \right) c(y) dy \tag{46}$$

where $v_{x,l} = v_x - \omega_z y$.

For the controller some linear approximation has to be found, similarly as to restoring moment due to roll rate.

5.11 Restoring moment due to pitch rate

Due to the absence of a tail this moment will be much smaller than on aircraft with tail. Most of the wing surface area is behind the center of gravity and due to the wing sweep more surface will be at a longer distance from c.g. in x direction. This means that we can approximate the entire wing to be behind the c.g. This aircraft also has a relatively small inertia around the y axis; the aircraft will quickly turn into the wind.

Since we have modeled the lift in such a way that the pitching moment does not vary with angle of attack we can assume here that it is only proportional to the aircraft velocity and ω_y, similarily as in section **??** but with constants instead of coefficients of lift and drag, giving us the following expression directly:

$$M_y = -\omega_y \rho S \sqrt{v_x^2 + v_z^2} C_{D,\omega_y} \frac{x_{\omega_y}^2}{24} = -\omega_y \rho S C_{\omega_y} \sqrt{v_x^2 + v_z^2} \tag{47}$$

where C_{ω_y} is some constant that needs to be approximated experimentally.

5.12 Actuator dynamics

The actuators onboard are not direct term but contain some dynamics. In this thesis they will be modelled as first order systems.

5.12.1 elevon dynamics

Given an input signal u_{δ_i} for elevon i we model the deflection, δ_i, like so:

$$\dot{\delta}_i = K_{\delta_i}(-\delta_i + u_{\delta_i}) \tag{48}$$

5.12.2 Motor dynamics

Given an input signal u_{ω_i} we model the motor dynamics as so:

$$\dot{\omega_i} = K_{\omega_i}(-\omega_i + u_{\omega_i}) \tag{49}$$

5.13 Full equations of motion

We now include all terms to find the full state space equations:

$$\begin{bmatrix} \dot{v_x} \\ \dot{v_y} \\ \dot{v_z} \\ \dot{\omega_x} \\ \dot{\omega_y} \\ \dot{\omega_z} \\ \dot{q_0} \\ \dot{q_1} \\ \dot{q_2} \\ \dot{q_3} \\ \dot{\delta_L} \\ \dot{\delta_R} \\ \dot{\omega_L} \\ \dot{\omega_R} \end{bmatrix} = \begin{bmatrix} 2g(q_1q_3 + q_0q_2) - \sum_i \left(D_i + D_{i,T} + D_{W_i} + D_{\alpha_i} + D_{\alpha_{i,T}}\right) + T_L + T_R \\ 2g(q_2q_3 + q_0q_1) + L_{W_L} + L_{W_R} \\ g(q_0^2 - q_1^2 - q_2^2 + q_3^2) + \sum_i \left(L_i + F_i + F_{i_T}\right) \\ f_1/I_x \\ f_2/I_y \\ f_3/I_z \\ \frac{1}{2}(q_1\omega_x + q_2\omega_y + q_3\omega_z) \\ \frac{1}{2}(q_2\omega_z - q_3\omega_y - q_0\omega_x) \\ \frac{1}{2}(q_3\omega_x - q_0\omega_y - q_1\omega_z) \\ \frac{1}{2}(q_1\omega_y - q_2\omega_x - q_0\omega_z) \\ K_{\delta_L}(-\delta_L + u_{\delta_L}) \\ K_{\delta_R}(-\delta_R + u_{\delta_R}) \\ K_{\omega_L}(-\omega_L + u_{\omega_L}) \\ K_{\omega_R}(-\omega_R + u_{\omega_R}) \end{bmatrix} - \begin{bmatrix} 2(v_z\omega_y - v_y\omega_z) \\ 2(v_x\omega_z - v_z\omega_x) \\ 2(v_y\omega_x - v_x\omega_y) \\ \frac{1}{I_x}(I_z - I_y)\omega_y\omega_z \\ \frac{1}{I_y}(I_x - I_z)\omega_x\omega_z \\ \frac{1}{I_z}(I_y - I_x)\omega_x\omega_y \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{50}$$

where f_1 is given by:

$$\begin{aligned} f_1 =& y_f C_{L_\delta} \sum_{j\in[a,a_W]} S_j\left(Q_{j,L}\delta_L - Q_{j,R}\delta_R\right) \text{ :: diff in ail deflection} \\ & - \frac{\rho}{2}\int_{-b/2}^{b/2} yc(y)\sqrt{v_x^2 + v_{z,l}^2}\left(C_L(\alpha_l)v_x + C_D(\alpha_l)v_{z,l}\right)dy \text{ :: restmom roll} \\ & + \int_{-b/2}^{b/2} y\left(\frac{\rho}{2}\sqrt{v_{x,l}^2 + v_z^2}\left(C_L v_{x,l} + C_D v_z\right)\right)c(y)dy \text{ :: roll due to yaw rate} \end{aligned} \tag{51}$$

, f_2 by:

$$\begin{aligned} f_2 =& x_f C_{L_\delta}\left(\sum_{j\in[a,a_W]} S_j\left(Q_{j,L}\delta_L + Q_{j,R}\delta_R\right)\right) \text{ :: joint ail deflection} \\ & + \sum_{j\in[w,w_w]} C_{m,y,l}Q_jS_jl \text{ :: pitch moment} \\ & - \omega_y\rho\, 2S_wC_{\omega_y}\sqrt{v_x^2 + v_z^2} \text{ :: restmom pitch rate} \end{aligned} \tag{52}$$

and f_3 by:

$$\begin{aligned} f_3 =& y_T\left(T_L - T_R\right) \text{ :: diff in T} \\ & - x_W\rho v S_W\left(C_{L_\beta}(\beta)v_x + v_yC_{D_\beta}(\beta)\right) \text{ :: mom due to sideslip} \\ & + \int_{-b/2}^{b/2} y\left(\frac{\rho}{2}\sqrt{v_{x,l}^2 + v_z^2}\left(-C_L v_z + C_D v_{x,l}\right)\right)c(y)dy \text{ :: restmom yaw rate} \end{aligned} \tag{53}$$

$Q_{j,i}$ is given by $\frac{\rho}{2}v_k^2$ where v_k is the speed in the relevant area, which can be either the aircraft velocity if the surface in question is outside the propeller wake, or the propeller wake air speed given by the left or right propeller speed if inside the propeller wake.

Variables like D_i and $D_{i,T}$ differ by both which area they use and which speed the airflow has since they are separated by if they are on the right or left side, inside or outside the propeller wake, and thus the currect expressions for the airspeed have to be used. The same argument goes for expressions which depend on α.

6 Numerical approximations for Coefficients

Since the functions describing the coefficient of lift and drag are empirically obtained analytical expressions are helpful to obtain. Below are two approximations that closely follow the characteristics of the coefficients in the graphs above. The functions will have to be adapted for the EcoSoar.

$$C_D(\alpha) = 0.2663\alpha^4 - 1.676\alpha^3 + 2.555\alpha^2 + 0.2559\alpha + 0.2 \tag{54}$$

where α is in radians in the interval $[0, \pi]$.

$$\begin{aligned} C_D(\alpha) = 2.93910^{-6}\alpha^3 - 0.0009205\alpha^2 + 0.05943\alpha - 0.00613 \\ +0.85e^{-0.1(\alpha-10)^2-1} + 0.4e^{-0.1(\alpha-5)^2+-1} - 0.15e^{-0.01(\alpha-20)^2+-1} + 0.3 \end{aligned} \tag{55}$$

where α is in degrees in the interval $[0, 90]$. To get the full range mirror and flip it across $\alpha = 90°$. You will need to adjust the 0.3 offset accordingly as well since it is added due to camber.

7 EcoSoar

7.1 CAD model

7.2 modifications to dual engine

3D printing, building, etc

7.3 Parameter estimation/declarations

8 Robust controller development and Implementation

8.1 Simulink

8.2 Gazeebo?

8.3 X-plane 11

9 Results

9.1 Performance in Simulations

9.2 Performance in reality

10 Conclusion