Numerical Solution of First-Order Ordinary Differential Equation Using Euler and RK4 Methods

User Guide

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Requirements

For running this application on your desktop computer, you must install JRE (Java Runtime Environment) on your computer.

Brief Description of Methods

Euler Method

From any point on a curve, you can find an approximation of a nearby point on the curve by moving a short distance along a line tangent to the curve.

$$y'(t) = f(t, y(t)), \qquad y(t_0) = y_0, \qquad (1)$$

Starting with the differential equation (1), we replace the derivative *y*' by the finite differences approximation

$$y'(t) \approx \frac{y(t+h) - y(t)}{h}, \qquad (2)$$

which when rearranged yields the following formula

$$y(t+h) \approx y(t) + hy'(t)$$

and using (1) gives:

$$y(t+h) \approx y(t) + hf(t, y(t)). \qquad (3)$$

This formula is usually applied in the following way. First we choose a step size h, and we construct the sequence t_0, $t_1 = t_0 + h$, $t_2 = t_0 + 2h$, … Then we denote by y_n a numerical estimate of the exact solution $y(t_n)$. Finally, motivated by (3), we compute these estimates by the following recursive scheme

$$y_{n+1} = y_n + hf(t_n, y_n). \qquad (4)$$

This is the ***Euler Method.*** The method is named after *Leonhard Euler* who described it in 1768.

The Euler method is an example of an explicit method. This means, the new value y_{n+1} is defined in terms of states that are already known, like y_n.

Runge-Kutta Method (RK4)

Let an initial value problem be specified as follows:

$$y'(t) = f(t, y(t)), \qquad y(t_0) = y_0,$$

Here y is an unknown function (scalar or vector) of time t, which we would like to approximate; we are told that y', the rate at which y changes, is a function of t and of y itself. At the initial time, t_0, the corresponding y value is y_0. The function f and the data y_0 and t_0 are given.

Now, pick a step-size $h > 0$ and define :

$$y_{n+1} = y_n + \tfrac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right),$$
$$t_{n+1} = t_n + h$$

for $n = 0, 1, 2, 3, \ldots$, using

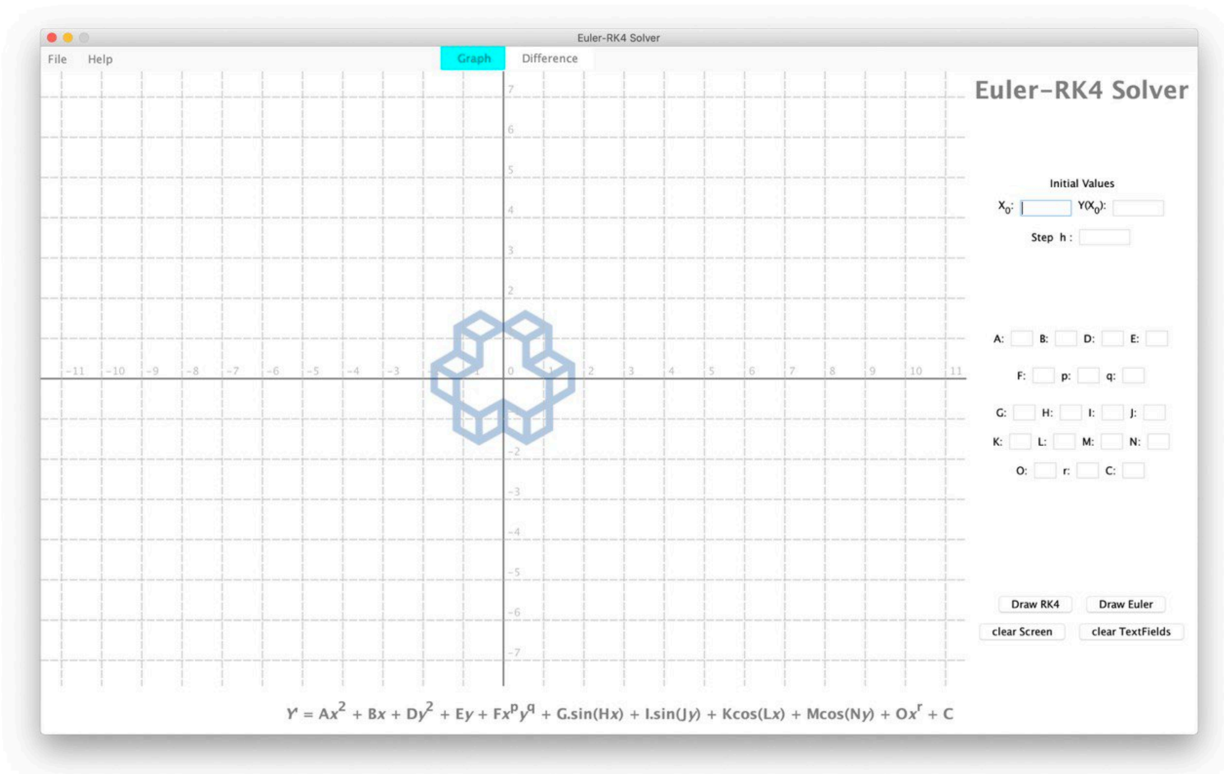
$$k_1 = h\ f(t_n, y_n),$$
$$k_2 = h\ f\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right),$$
$$k_3 = h\ f\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right),$$
$$k_4 = h\ f\left(t_n + h, y_n + k_3\right).$$

Here y_{n+1} is the RK4 approximation of $y(t_{n+1})$, and the next value (y_{n+1}) is determined by the present value (y_n) plus the weighted average of four increments, where each increment is the product of the size of the interval, h, and an estimated slope specified by function f on the right-hand side of the differential equation.

How To Use The Application

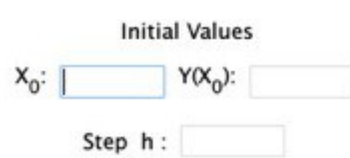
For using this application first download .jar file.

After running the Application You can see The Main Worksheet



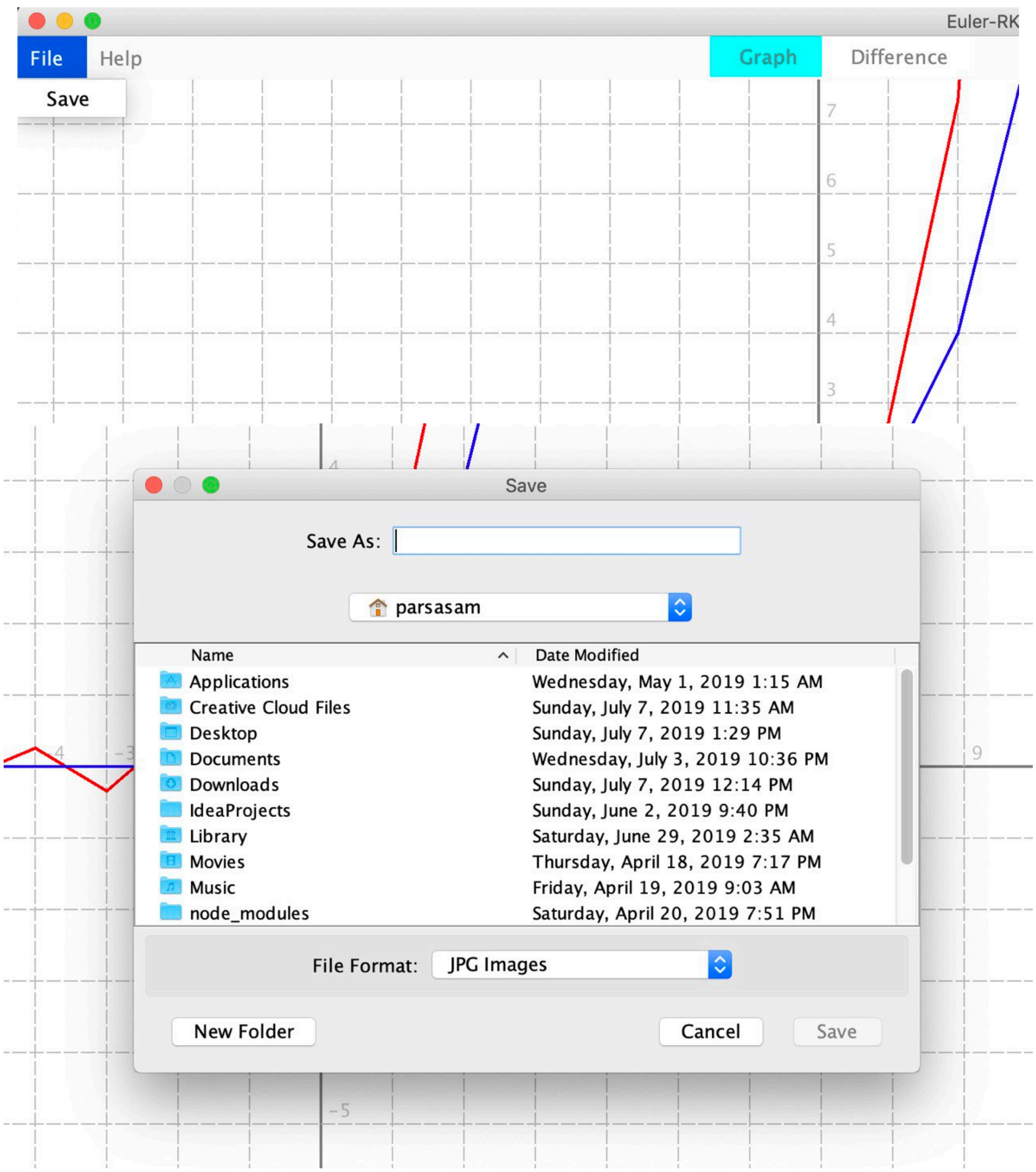
On the bottom of the scene you can find a function with Coefficients from A to r that can be set in the Text Fields on the right side.

Also, there are text boxes to input Initial values and step for the Methods



On the bottom right Corner there are four buttons for drawing Function using Euler and RK4 methods, clearing the Screen and Clearing the values that has been set to the Coefficients.

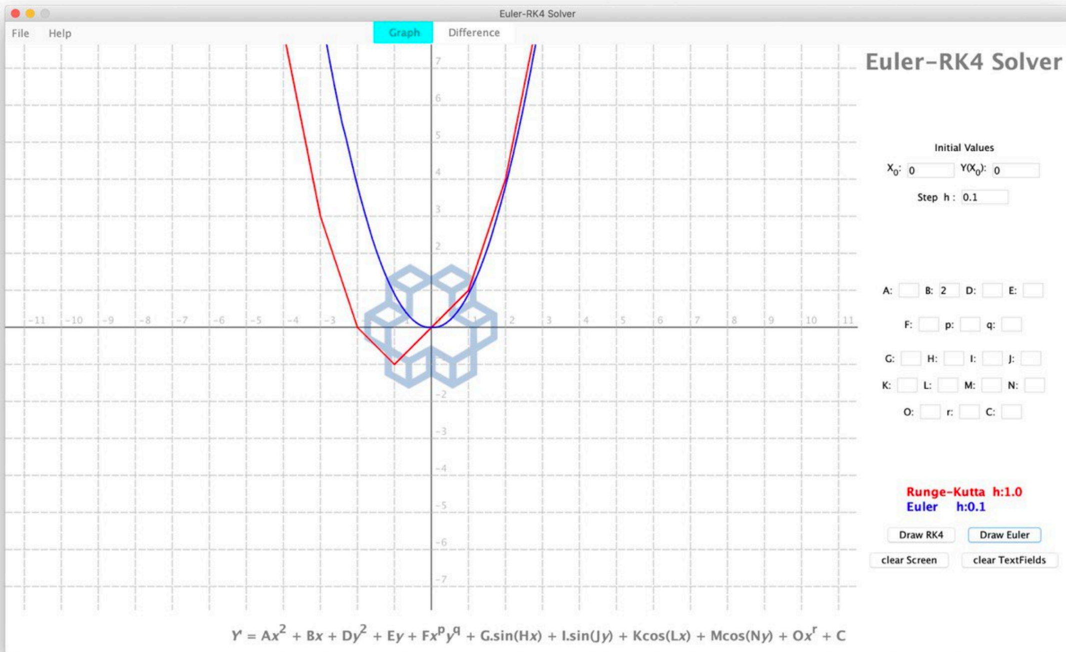
You can save a picture of the Scene On the Top left corner of the Application by choosing : File > Save

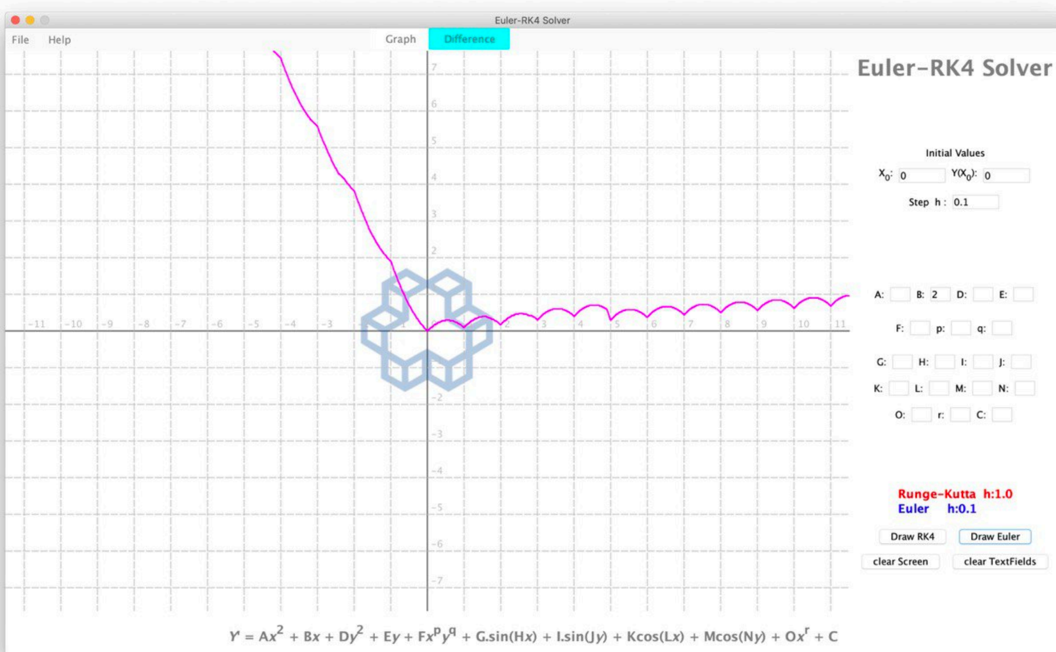


Example 1 .

Drawing function ***y'= 2x*** using RK4 (Red Graph) with Step 1 and Euler (Blue Graph) with Step 0.1

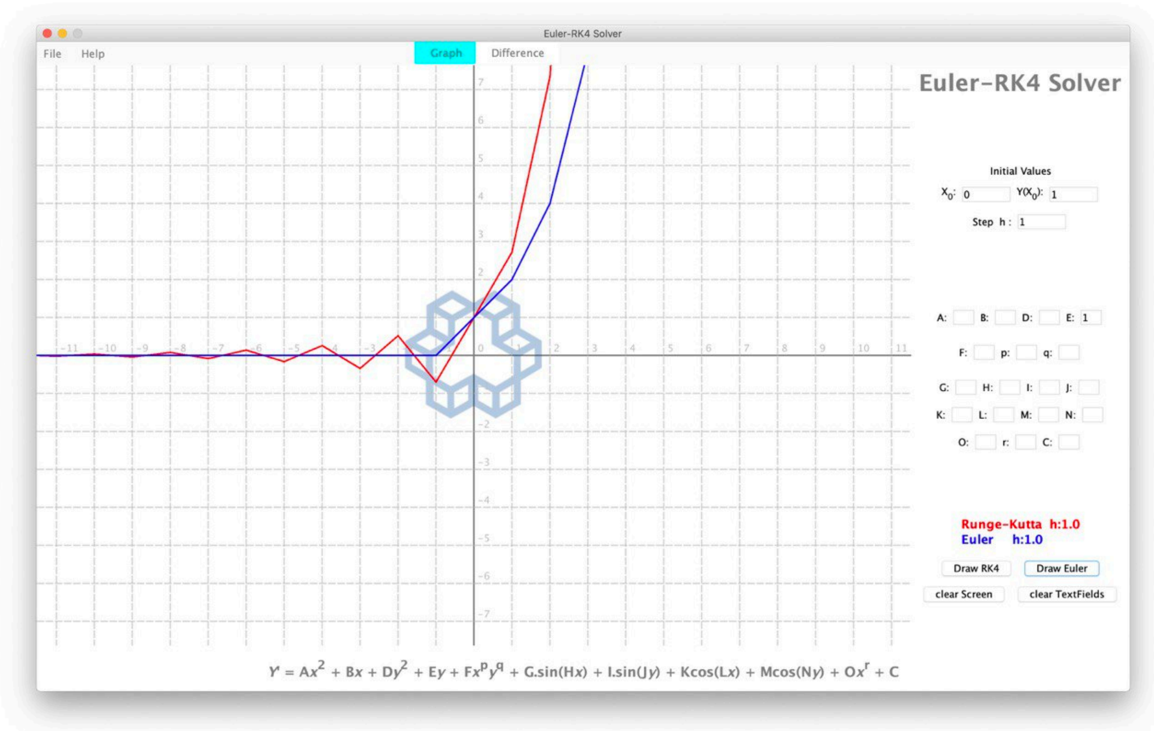
On the ***Graph*** Tab you can See the functions and on the ***Differences*** Tab there is a function that represents Differences of the drawn functions.

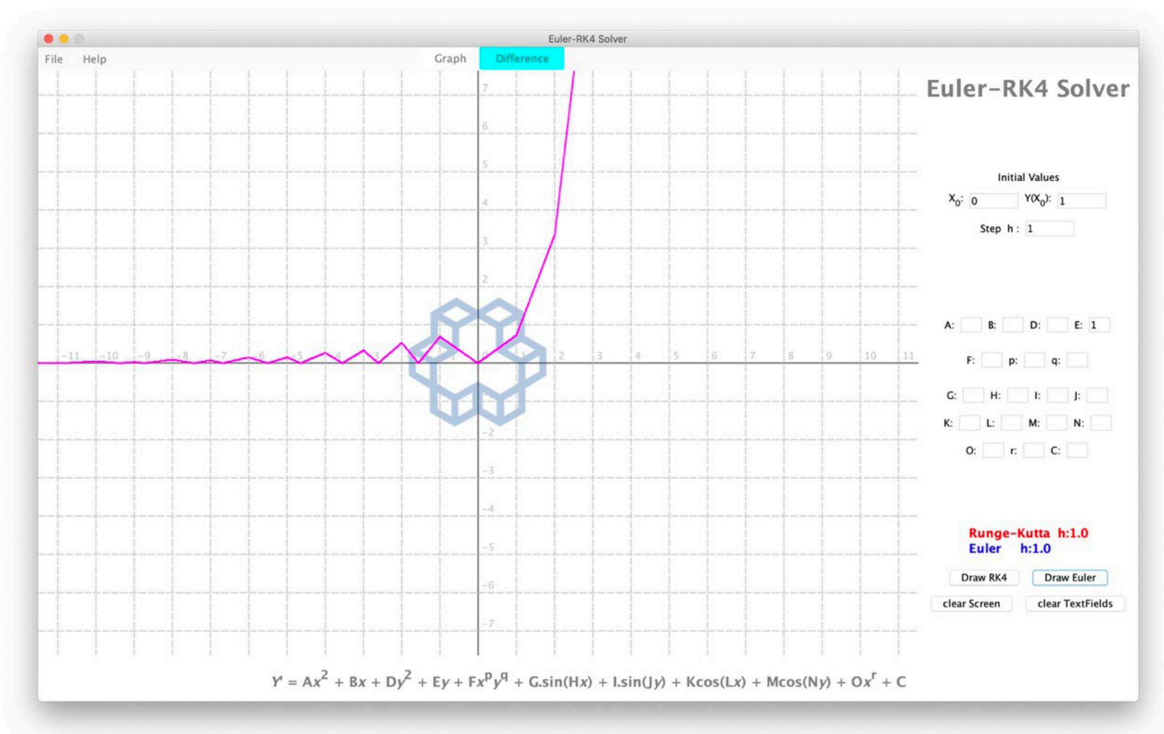




Example 2 .

Function ***y′=y*** has been drawn using RK4 (Red Graph) Step 1 and Euler (Blue Graph) Step 1.





About Us

User guide is written by Parsa Samadnejad
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