Graphical Models 111 (2020) 101081

Contents lists available at ScienceDirect

Graphical Models

journal homepage: www.elsevier.com/locate/gmod

A review on crowd simulation and modeling

Shanwen Yang [a,b], Tianrui Li [a,b,*], Xun Gong [a,b], Bo Peng [a,b], Jie Hu [a,b]

[a] *Institute of Artificial Intelligence, School of Information Science and Technology, Southwest Jiaotong University, Chengdu, 611756, China*

[b] *National Engineering Laboratory of Integrated Transportation Big Data Application Technology, Southwest Jiaotong University, Chengdu, 611756, China*

ARTICLE INFO

Keywords:
Crowd simulation
Interactive group
Emotion contagion

ABSTRACT

Crowd simulation has emerged in the last decade as a widely used method of visual effects, computer games, and urban planning, etc. The improvement of hardware performance and the urgent need of special effect lead to an unprecedented wave of crowd simulation studies. This paper reports a review of crowd simulation models from traditional methods to recent methods (e.g. group simulation, emotion contagion). Traditional models can simulate general crowd dynamics which have the advantages of both microscopic and macroscopic models. The recent studies of crowd simulation from group simulation to social psychology crowds are possible to simulate realistic crowds. The purpose of this review is to introduce commonly used crowd simulation methods for newcomers to this field by making a systematic literature review, discussions and analysis of different models. The results reveal the traditional models can simulate most of the normal crowds, but lack expressiveness for special groups which needs to be solved urgently in the current applications, particularly on visual effects and urban planning. Group simulation and emotion contagion could improve the simulation realism, but it also needs to be improved in computation cost and model optimization. Also, future research directions are suggested aiming to develop new applications focused on more realistic, natural and efficient crowd simulation.

1. Introduction

Crowd simulation is a technique to simulate the motion dynamics of virtual individuals. The earliest crowd simulation system "Boids" was proposed by Reynolds [1,2] in 1987, which was an artificial life project to simulate the flocking behavior of birds. Since then, crowd simulation has been studied by many researches and has been widely used in the fields of visual effects [3–6], computer games [7,8], and urban planning [9,10], etc. Although this field has gained a lot of research progress and is developing rapidly, the influence of locomotion, sensory abilities and a series of psychological factors make individual behavior become complex in different situations. Due to high computational complexity of such heterogeneous crowds, there exist many different challenges which limit the realism in crowd simulation. With the development of computer equipment, understanding and controlling human behavior has become a hot topic. It is of great significance to study how to simulate realistic people to improve the authenticity of visual effects, enhance the immersion of virtual reality, and ameliorate the rationality of urban planning and the efficiency of emergency evacuation.

1.1. Related surveys and challenges

In [11], a crowd was defined as "a large group of individuals share information in the same environment alone or in a group". The key criteria of a crowd included crowd size, crowd density, specific time, crowd collectivity and crowd novelty [12].

In [13], Ijaz et al. gave a survey on hybrid crowd simulations, and the real-time crowd rendering techniques were reviewed in [14–17]. An overview of crowd simulation and its application was provided in [18], which mainly introduced the related works of CAD & CG laboratory in Zhejiang University. The state-of-the-art models were presented and compared in [19]. Surveys of example-based and position-based crowd simulation approaches can be found in [20] and [21]. The introduction of different crowd simulation methods was shown in [22]. Efficient algorithms to animate, control, and author human-like agents were described in [23] to help researchers to design realistic digital humans and their interactions with environments. A deep understanding of state-of-art methods to simulate realistic crowds were provided in [24].

During the past 20 years, researchers have proposed various techniques for crowd simulation, e.g. path planning [25–30], behavior modeling [31–34], navigation graphs [35], emergency evacuation [36–39], hybrid modeling [40–43] and biomechanical models [44,45]. The urgent need for simulating a variety of crowds is limited by the great need for manpower and hardware resources [46]. The performance of the algorithm, such as rendering strategy and group generation scheme, needs to be improved. At the same time, it is necessary to consider the reality and accuracy of virtual groups.

* Corresponding author.
E-mail address: trli@swjtu.edu.cn (T. Li).

https://doi.org/10.1016/j.gmod.2020.101081
Received 21 October 2019; Received in revised form 8 June 2020; Accepted 23 July 2020
Available online 27 July 2020
1524-0703/

Table 1
Major categories of crowd simulation models.

Models	Methods
Microscopic models	Rule-based models [1,2,47,48] Force-based models [49–55] Velocity-based models [56–64] Agent-based models [65–82] Vision-based models [83–87]
Macroscopic models	Continuum models [88–93] Aggregate dynamics [94] Potential field-based models [28,95–100]
Mesoscopic models	Dynamic group behavior [101–113] Interactive group formation [114–133] Social Psychological crowds [134–149]

In this paper, we track the current widely used models which are representative or frontier. Section 2 briefly introduces the research methodology used in this paper. Then we review mainstream traditional crowd simulation methods include microscopic models in Section 3 and macroscopic models in Section 4. We switch focus to recent crowd simulations in Section 5, which includes dynamic group structure, adaptive group formation, and social psychology crowds. Section 6 provides the comparisons and analysis of the major approaches. Conclusions and development tendency are provided in Section 7.

2. Methodology

2.1. Literature collection

This paper uses the literature review method to trace the relevant literature in the crowd simulation. In order to have a comprehensive review of the relevant works of literature, a research panel is formed firstly. The authors all have relevant knowledge and practical expertise in this domain, and actively participate in this research. Based on the online research bases, e.g. Google Scholar, Web of Science (WOS) and SCOPUS, we use the keywords, e.g. “Crowd Simulation”, “Crowd Animation”, “Collision Avoidance”, “Group Behavior”, “Crowd Dynamics”, to search the papers relative to crowd simulation. And we use the words “Literature Review”, “Survey”, “State of the Art”, “Overview”, etc, to search the books and papers on the progress of crowd simulation. We search for relevant papers mainly published on ACM Transactions on Graphics, IEEE Transactions on Visualization and Computer Graphics, IEEE Virtual Reality, Eurographics, Computer Graphics Forum, etc. and then add them into this survey. Through the preliminary analysis of the literature and the discussion related to the research topic, we identify gaps in the literature and the relationships that could better integrate crowd simulation.

2.2. Literature classification

Through the above investigation and research, crowd simulation has experienced the evolution from microscopic models to macroscopic models. Recently, to simulate interactive and realistic crowds, researchers have come up with mesoscopic crowd simulations which can be classified as simulations considering pedestrian dynamics, human-computer interaction, and social psychological factors. Collective behaviors can be generated by using such methods, e.g. group behavior, formation deformation, and emotional contagion. Table 1 depicts these mainstream crowd simulation models proposed in recent decades.

Individuals in “boids” system [1,2,47,48] follow some corresponding rules to realize their movements. Social force [49–52] is the foundation of a force-based model which simulates crowd dynamics through the interaction between individuals and obstacles. Velocity-based models [56–63,150] are developed mainly to generate collision-free trajectories and inter-agent avoidance. These models are now used in some game engines like Unity 3D and Unreal Game Engine. Agent-based models [65–68] can be regarded as the combinations of other microscopic models. To model the human vision, the vision-based models [83–85] are proposed for interpersonal collision avoidance.

Macroscopic models include continuum models [88–93], aggregate dynamic model [94] and potential field models [95–97]. The continuum model mainly considers to reflect the flow characteristics of the crowd and uses continuum dynamics theory to simulate large-scale crowds. And the aggregation behavior of a dense crowd is simulated using the aggregate dynamic model in which agents are viewed as the unity of discrete individuals and continuous entity. However, the field-based models apply various fields to direct crowd motion.

Taking crowd dynamics into account, psychological and physiological characteristics of a crowd are extremely important in human daily life. The group simulation includes the dynamic group behavior models [101–113], interactive group formations [114–128,130–133], and social psychological crowd simulations [134–149,151–155]. This paper will detail these models in the following sections.

3. Microscopic models

Microscopic models are also called “Bottom-Up” methods, which focus on low-level behavioral details and individual features [13]. In this category, individuals are considered as discrete objects whose motion is influenced by their neighbors and the obstacles. On the other hand, the collision avoidance is the local interactions with the surrounding environment, and the combination of these local behaviors produces individual final movement.

3.1. Rule-based models

Early in the 1980s, Reynolds has come up with a distributed “boids” system to simulate flocks, herds, and schools of fishes [1,2]. In this approach, each boid has the attribute of orientation and has no life cycle. As shown in Fig. 1, each boid acts according to the behavioral rules (Separation, Alignment, and Cohesion). Boids system can be seen as a rule-based model, and other crowd simulation models using “Boids” system can be found in [47,48]. Rule-based models are the earliest models employed to simulate the flock dynamics of animal crowds, but they are not suitable to simulate human crowds due to the simple behavior rules.

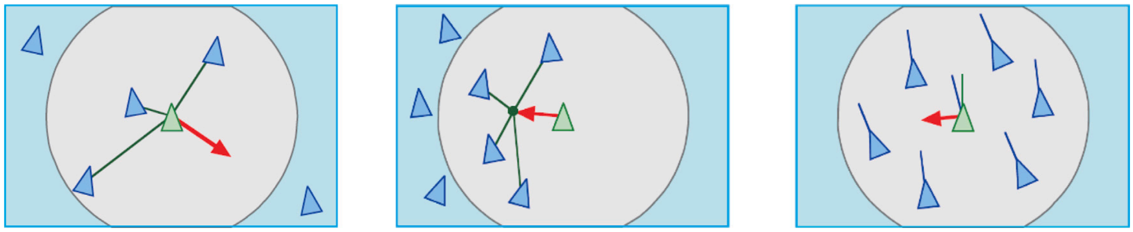
3.2. Force-based models

In 2000, the famous Social Force Model (SFM) [49,50] was presented by Helbing et al. to study the mechanisms and dynamical features of escape crowd panic. Inspired by the interaction forces, SFM is the earliest and widely used approach for simulating human dynamics. It was the first time that physical criteria were applied to microscopic human crowd simulation, and many subsequent studies have been based on SFM or inspired by it. Individual behavior is influenced by socio-psychological and physical forces. The actual movement of an individual depends on the desired velocity and its interaction with the environment. As shown in Eq. (1), the acceleration [49] term of an individual i can be represented as:

$$m_i \frac{d\boldsymbol{v}_i}{dt} = m_i \frac{v_i^0(t)\boldsymbol{e}_i^0(t) - \boldsymbol{v}_i(t)}{\tau_i} + \sum_{j(\neq i)} \boldsymbol{f}_{ij} + \sum_{W} \boldsymbol{f}_{iW}, \tag{1}$$

where m_i and $\boldsymbol{v}_i$ represent the mass and current velocity of individual i, respectively. v_i^0 and $\mathbf{e}_i^0$ represent the desired walking speed and target direction, respectively. $\boldsymbol{f}_{ij}$ and $\boldsymbol{f}_{iW}$ are the interaction forces of individual i with other individual j and with the walls, respectively. τ_i is the update time-step, as shown in Fig. 2.

At *SIGGRAPH* 2007, Pelechano et al. developed a high-density autonomous crowds (HiDAC) model [51]. By applying psychological and geometrical rules, HiDAC can control individual local motion and



(a) separation (b) cohesion (c) alignment

Fig. 1. A flock of birds simulated by boids system [2].

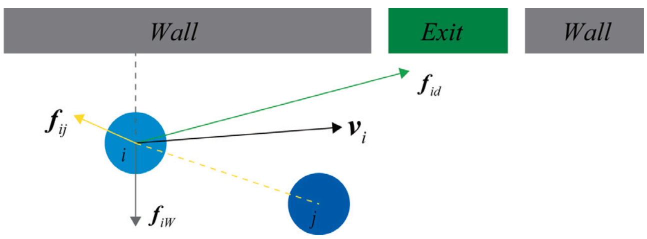


Fig. 2. Illustration of Social Force Model [49].

global way-finding in a high-density dynamic environment. A wide variety of emergent behaviors can be simulated by HiDAC model.

Saboia and Goldenstein improved the social force model by introducing a mobile grid to allow individuals to adjust their preferred velocity, and it can generate softer and more coherent trajectories [52]. Karamouzas et al. introduced the evasive force to construct the predictive avoidance model to help individual to predict future collisions and make efficient collision avoidance [53]. Ji et al. presented a velocity-perception-based social force model (VPBS) [54] in which the personal space and the relative velocities between individuals were considered to predict the future interactions and reduces oscillations. Wagoum et al. proposed an efficient crowd simulation for evacuation assistant integrated with cellular automata [55].

3.3. Velocity-based models

The most widely used velocity-based collision avoidance models were developed by the research group in University of North Carolina, which include Velocity Obstacle (VO) [56], Reciprocal Velocity Obstacle (RVO) [57], Optimal Reciprocal Collision Avoidance (ORCA) [58, 59] and Hybrid Reciprocal Velocity Obstacle (HRVO) [60] models. Velocity-based models consider the neighbor information to make decisions. The global movement of individuals is a combination of local interactions, and these models do not need to represent the environment (e.g. environment grid division).

3.3.1. Reciprocal velocity obstacle model

The explanation of VO space [56] is "the velocity obstacle consists of the velocities of the robot that will cause a collision between the robot and the obstacles at some future time". As shown in Fig. 3(a), the VO space, $VO_B^A(v_B)$ of agent B to agent A is represented by the color of dark gray, which is defined in [56] as Eq. (2):

$$VO_B^A(v_B) = \{v_A | \lambda(p_A, v_A - v_B) \cap B \oplus -A \neq \emptyset\}, \tag{2}$$

where v_A and v_B are current velocity of agent A and B, respectively. However, in this approach, the velocity offset is too large and it will cause oscillation phenomenon.

By extending the VO space [56], the RVO collision avoidance for real-time navigation was presented in [57], as shown in Fig. 3(b). The idea of RVO model is to average the current velocity v_A and the velocities inside the VO space $VO_B^A(v_B)$. The RVO space can be seemed as the VO space translated with its apex lies at $(v_A - v_B)/2$, which is defined in [57] as Eq. (3):

$$RVO_B^A(v_A, v_B) = v'_A | 2v'_A - v_A \in VO_B^A(v_B). \tag{3}$$

To avoid collision with each other, agents A and B should choose the new velocity v'_A and v'_B both outside each other's RVO space.

3.3.2. Optimal reciprocal collision avoidance

In RVO model, individuals must select their velocities inside the RVO space to avoid collision. To find the optimal velocities for multiple individuals, the OCRA approach was proposed in [58]. The idea of OCRA is to "share the responsibility", as shown in Fig. 3(c). Two individuals select their optimal velocity v_A^{opt} and v_B^{opt}. u is the closest point from $v_A^{opt} - v_B^{opt}$ to the boundary of VO space. n is the outward normal of the VO boundary at $(v_A^{opt} - v_B^{opt}) + u$. The OCRA space for individual A is as Eq. (4):

$$ORCA^{\tau}_{A|B} = \{v \mid (v - (v_A^{opt} + \frac{1}{2}u)) \cdot n \geq 0\}. \tag{4}$$

The permitted velocity for individual A is the half-plane pointing in the direction of n starting at the point $v_A^{opt} + \frac{1}{2}u$. The OCRA space of individual B is defined symmetrically.

3.3.3. Hybrid reciprocal velocity obstacle

The HRVO proposed in [60] is an improvement of VO [56] and ROV [57] models. The VO and RVO models both allow agents to choose a velocity outside the obstacle space on the left or on the right. However, it is not sufficient due to the reciprocity between robots. It will sometimes cause undesirable oscillations. Based on the above-mentioned models, to enlarge the RVO space, the edge on one side is replaced by the edge on the same side of VO space [60]. From Fig. 3(d), the HRVO is represented by the color of dark gray and it was defined in [60], as shown in Eq. (5):

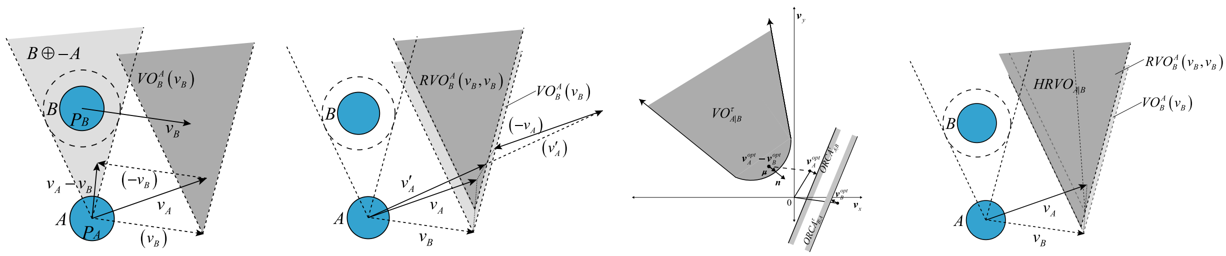
$$HRVO_{A_i} = \bigcup_{A_i \in \mathcal{A}} HRVO_{A_i|A_j} \cup \bigcup_{O_j \in \mathcal{O}} VO_{A_i|O_j}. \tag{5}$$

3.3.4. Other velocity-based models

Paris model [61] uses the VO space to conduct linear extrapolation and the velocity of the agent is adjusted in the non-obstacle area. They also showed the Tangent model [62] which can predict trajectories of an agent and the obstacles. The free-collision movement was realized by adjusting agent's acceleration. Combining the ORCA collision-avoidance [58] and external physical forces, the velocity-based model was used to simulate multi-agent physical interactions [63]. The emergency behaviors such as collisions, pushing, deceleration and resistive forces were simulated at an interactive rate. They further extended their work by using finite state machines to simulate more complex physical interactions in dense crowds [64].

3.4. Agent-based models

Agent-based models provide individual abilities of perception, decision-making and action [23]. In addition, agent-based models can be combined with path planning or some macroscopic models. As a



(a) VO space [56] (b) RVO space [57] (c) ORCA [58, 59] (d) HRVO [60]

Fig. 3. Different velocity obstacles [56–60].

result, these models [7,27,33] have high scalability and can simulate more complicated human behaviors in both local and global aspects.

Karamouzas et al. [65] proposed an optimization integrator "Implicit Crowds" to make compatible with the nonlinear, anticipatory forces and physics-based animation in multi-agent system. Incorporating dissipation functions, this approach is insensitive to the parameters and can anticipate crowd dynamics, as shown in Fig. 4(a). A position-based dynamic system (PBD) [66,67] was developed by Weiss et al. based on the Lagrangian formulation. The per-agent behaviors was simulated by computing the short-range and long-range collision avoidance. PBD can generate collective behaviors, e.g. seamlessly passing each other, lane formation and subgroups, as shown in Fig. 4(b). In order to avoid collisions, Guy et al. [68] constructed a highly parallel multi-agent based crowd simulation "ClearPath". The VO model [56] was extended by taking into account the discrete time interval to construct free collision truncated cone (FVO). Using the KD-tree and quadratic optimization function can solve heterogeneous agents, high-level path planning and support collision interactions, as shown in Fig. 4(c). Yeh et al. introduced a novel concept of composite agent [69]. Based on the external and internal states, each agent can extend its influence over other agents, and it is capable of simulating various emergent behaviors as shown in Fig. 4(d).

Wong et al. [70] constructed a framework to simulate the cooperative behaviors of agents. The cooperative task was sequentially and parallelly decomposed and the desired forces were updated to manually create event chains to simulate various cooperative behaviors (e.g. "pushing", "pulling with handles", "pulling with ropes", and "carrying"). However, the user-defining process needs complex and complicated operations which are not efficient. Chen and Wong developed a multi-agent cooperation system [71] in which the roles of worker and pedestrians are not interchangeable. However, pedestrians should avoid the workers while the two are interchangeable in [70].

The capsule-shaped "Torso Crowds" was presented in [72] based on the recorded real-world human data. This method can simulate torso-twisting and side-stepping behaviors. A perceptual shoulder motion simulation was proposed in [73] by using the secondary motions (shoulder motion), which is capable of increasing the global perceptual level of crowd animations. The "ProactiveCrowd" [74] can simulate the gap-seeking and following behaviors, etc., which is also capable of mapping intuitive steering strategies into realistic behavior models using real-world data.

A density-based filter for dense crowd simulation was shown in [75], in which the SFM [49] and RVO [56] models were introduced for local movement to ensure agents' trajectories correspond to the fundamental diagram. In [76], the environment was divided into walk-able regions, and the density information as well as the distance information were used for agents avoiding congestions.

Liu et al. [77] proposed a "CityFlow" model to simulate the pedestrian traffic dynamics, unidirectional and bidirectional flow, where flow through bottlenecks was generated by using a utility maximization. Based on the SFM model [49], a mixed interaction model was developed in [78] to simulate the interactions between vehicles and pedestrians. Based on the experiment of human participants, a realistic following behavior model was presented in [31], which is similar to the "Car-Following" models used in traffic flow research. Other agent-based crowd simulations such as the crowd behavior models were outlined in [79]. The path planning approaches were discussed in [80], and the collision avoidance can be found in [81,82].

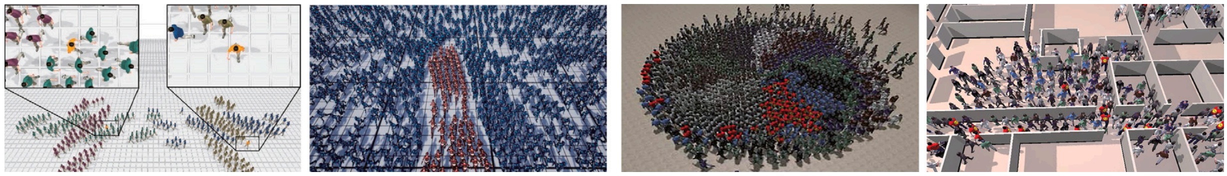
3.5. Vision-based models

Ondřej et al. studied the vision information in crowd simulation. The vision-based models are variations of the velocity-based models, which can better simulate the perception-action loops. The individual movement was computed by evaluating the bearing angle and the time-to-collision of each pixel. Their synthetic vision-based crowd controlling model was proposed in [83], where the time derivative of bearing-angle $\dot{\alpha}$ between an agent and obstacle was determined to adjust the rotational movement, and the time-to-interaction tti was calculated for collision avoidance, as shown in Fig. 5(a). In this approach, the environmental information is the image acquired by the virtual camera, and each pixel of the image stores the velocity control information. By extending the work in [83], a density-adaptive synthetic-vision based steering approach (DAVIS) was illustrated in [84]. A gradient-based steering model was proposed in [85], where a cost function was shown for individuals to evaluate the future collisions. The gradient of this cost function was able to guide individuals' motions. An optimal flow based approach was developed in [87], in which the optimal flow was segmented and processed to extract the visual features. Based on local minimization of control functions, this approach can produce various behaviors, e.g. following, avoiding and reaching.

A context-aware probabilistic motion prediction method was presented in [156], in which the collision probability field was introduced to predict agent's future collision, and the "WarpDriver" algorithm was used to improve the computational efficiency. Although this method is not vision-based, the function of the collision probability field is similar to the tti used in [83]. As shown in Fig. 5(b), the "perceiving agent" was defined as space–time "*projected trajectory*". Then the collision probability field was constructed to evaluate the future collisions of the agent to the environment. By modifying the "*projection trajectory*", "WarpDriver" can improve the quality of crowd simulation.

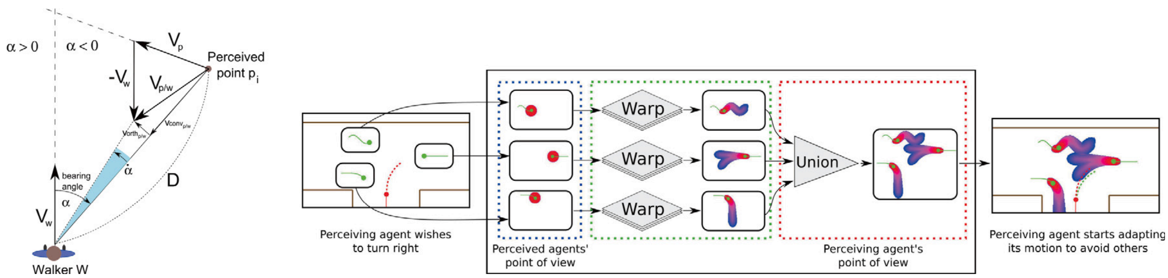
4. Macroscopic models

While simulating large-scale and dense scenario, crowds are considered as a unified and continuous entity, and its movement is governed by potential fields or fluid dynamics, etc. Crowd path planning and collision avoidance are both governed by the global problem solver. These models do not focus on the underlying details, so that each virtual agent does not consider the individual level interactions between others and the environment.



(a) Implicit crowds [65] (b) PBD crowds [66] (c) ClearPath [68] (d) Composite agents [69]

Fig. 4. Agent-based crowd simulation.



(a) Synthetic vision [83] (b) Collision probability field of "WrapDriver" algorithm [156]

Fig. 5. Vision-based simulation and "WarpDriver".

4.1. Continuum model

To describe the pedestrian flow dynamics, Hughes proposed to use the continuum theory to describe pedestrian flow [88]. Based on the hypothesis of pedestrian properties, the solution of the flow equations can be extended to multiple types and applied to model a crowd of individual behaviors.

Based on the theory in [88], a real-time, large-scale crowd simulation method called "Continuum Crowds" was illustrated by Treuille et al. which can be seen as the first time that macroscopic model was developed to simulate large-scale crowd scenario [89]. Without agent-based dynamics, this method is designed for large groups with common goals. The formulation of continuum crowds yields a set of dynamical potential fields (e.g. density fields, speed fields, and discomfort fields) and the overall unit cost function [89] for a group of individuals is calculated by Eq. (6):

$$\int_P C ds, \qquad where \qquad C \equiv \frac{\alpha f + \beta + \gamma g}{f}, \tag{6}$$

where each term in the numerator is the integral with respect to individual path length, travel time and crowd density. α, β and γ are weights of these terms.

The optimal path planning is calculated by the Eikonal equation [89] as shown in Eq. (7):

$$\|\phi(\boldsymbol{x})\| = C, \tag{7}$$

where the measurement of the cost function C is the direction of the gradient, and individual moves in the direction opposite the gradient of this function. The overall algorithm flow is shown in Fig. 6(a), which can generate interesting moving patterns such as lane formation, vortex formation, etc. and can simulate large-scale army retreating as shown in Fig. 6(b).

A crowd simulation approach for complex environments was developed in [90] based on the continuum model. By introducing an environmental structure discretization solution, the additional discomfort field was constructed by using the proposed density conversion to generate smooth trajectories. A continuum traffic simulation was shown in [91] by adapting single-lane continuum flow in which the multi-lane traffic simulation was generated through the lane change approach. A continuum model for crowd turbulence was illustrated in [92] by incorporating an inter-personal stress and acceleration constraints, which can simulate crowd turbulence with close correspondence to the real-world situations. A continuum dynamic-based escaping crowds was simulated in [93], where an offline occlusion culling technique was used to speed up the simulation.

4.2. Aggregate dynamics model

If the continuum model is influenced by the continuous dynamics, the aggregate dynamic model is inspired by the fluid dynamics. Based on a dual representation [94], the compressibility of the crowd was evaluated based on crowd density which was both represented as discrete agents and a single continuous system. This approach was called "UIC", and capable of simulating hundreds of thousands of crowds at an interactive rate, as shown in Fig. 7(a).

The UIC projection in the aggregate dynamic system can be seen as a constrained continuum model in which the density field of agents and the velocity field satisfy the relationship [94] of Eq. (8):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0. \tag{8}$$

Then, a correction of $\widetilde{\boldsymbol{v}}$ and ρ needs to be made to maintain the UIC constraints, which is given [94] as Eq. (9):

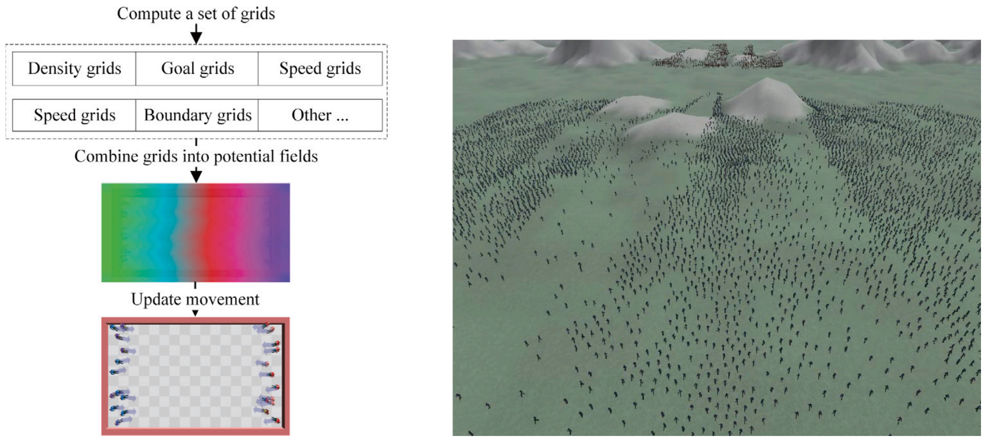
$$\boldsymbol{v} = v_{max} \frac{\widetilde{\boldsymbol{v}} - \nabla \rho}{\|\widetilde{\boldsymbol{v}} - \nabla \rho\|}. \tag{9}$$

When the simulation begins, each agent will be assigned a preferred velocity and yield different density and velocity fields. Based on the UIC projection, the large-scale crowd simulation can be generated by decoupling the local collision avoidance, as shown in Fig. 7(b).

4.3. Potential field models

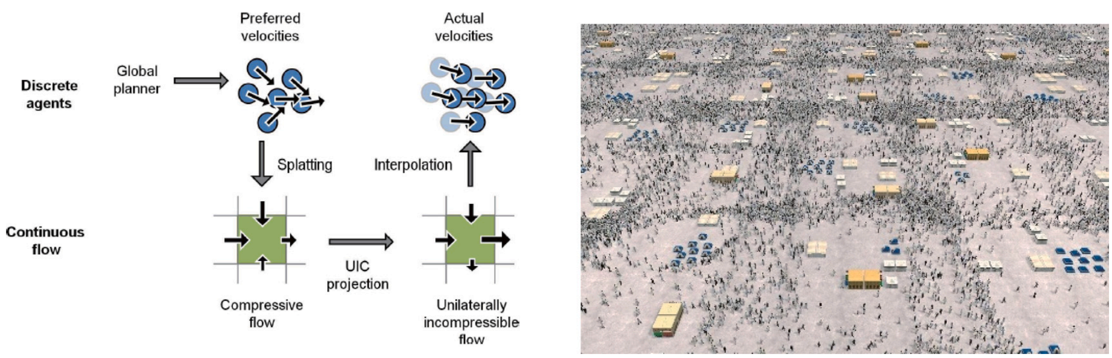
Considering the mobility of the human crowd, another important category of simulating large-scale crowds is potential-based models. Macroscopic models (i.e. continuum model and potential field) need to divide the environment into a set of grids and construct dynamic fields.

The "Flow Tiles" was proposed in [95] for constructing divergence-free velocity fields. This approach can improve the controllability and efficiency of flow-like crowd simulation. However, stitching the finite number of template flow tiles is not interactive.



(a) algorithm overview (b) hundreds of thousands of armies

Fig. 6. The overall algorithm flow of continuum crowds [89].



(a) algorithm overview (b) simulation result

Fig. 7. Aggregate dynamics [94].

The interactive navigation field method was developed for directing one or more agents' movement by integrating user-defined guidance fields [96]. The guidance fields can be designed by a user-input or sketching on an interface to generate a smooth, collision-free navigation field, as shown in Fig. 8(b). Guidance fields such as the sketched path and video footage are composited with the navigation field for users to edit on either global level or individual level behavior (e.g. lane formation, vortices and group behaviors). Each component satisfies Eq. (10) can successfully generate convincing trajectories and individual animations [96].

$$\|s\mathbf{a} - G(X)\| = 1. \tag{10}$$

The coordinated crowds with topological scene analysis were shown in [97]. The environmental topology was represented as a Reeb Graph by computing the harmonic field. It can generate globally coordinated crowd simulation by maximizing the capacity of each path and minimizing congestions.

Based on the continuum model [89], a real-time crowd simulator integrating agent and field methods was illustrated in [98]. In this approach, the simulation environment is divided into three classes as shown in Table 2. During the simulation, the fields constructed by linear interpolation can help agents to gain local information from environmental grids and help them to perform path planning. This approach can produce large-scale smooth and realistic crowd simulations, and it is more efficient compared with other existing works.

An adjustment method of navigation field was presented in [99], in which the moving directions and crowd density information were collected by using the crowd monitors placed at the corners. Then the navigation field was constructed to lead individuals to walk away from the congestions by automatically adjusting guidance paths. Other potential field models can be found in [28,100].

Table 2
Potential fields defined in [98].

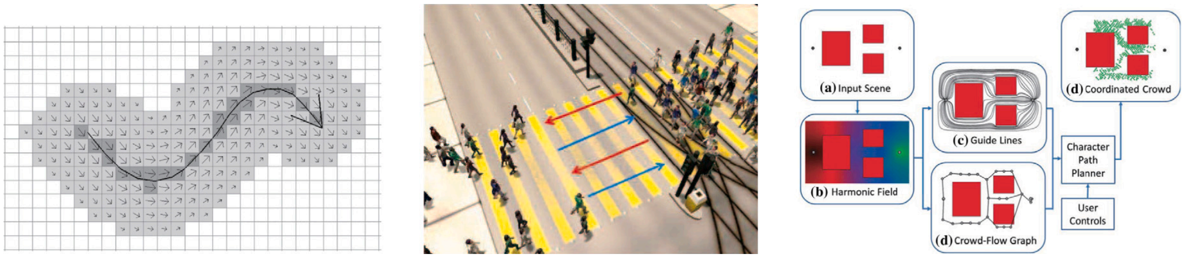
Type of objects	Type of fields
Static objects (obstacles)	Static field
Dynamic objects (moving cars)	Dynamic field
Abstract objects (environmental events)	Abstract field

5. Mesoscopic crowd simulation models

The macro model can simulate thousands of people, while the micro model is based on individuals, and the simulation of large-scale crowd movement is less efficient. Recently, researchers begin to focus on mesoscopic models (e.g. group simulations) which can be divided into (1) dynamic group behaviors such as the social relationships among individuals in dynamic groups, (2) interactive group formation in the RTS games and manipulating schemes, and (3) social-psychological crowds in emergency evacuation and parades.

5.1. Dynamic group behaviors

Group dynamics is a theory trying to obtain the general rules of the group phenomenon through dynamic analysis. It is defined as "a system



(a) Guidance field (b) simulation with a interactive stroke (c) simulation from video streams

Fig. 8. Guidance field and the simulation results of [96].

of behaviors and psychological processes occurring within a social group (intragroup dynamics), or between social groups (intergroup dynamics) [157]". The term of group dynamics was first used in [158] by Lewin to describe how groups and individuals interact with each other. A social group can be defined as a unity with two or more individuals who interact and share information with each other. It is not only a simple collection or aggregate of individuals, but also exhibits social cohesion, such as people walking side by side, or people waiting in a line.

There are many methods have been proposed for crowd simulation in different kinds of scenarios, as we all know that people often walk in groups, e.g. friends, families, colleagues. These group behaviors make human crowd with certain characteristics. Based on the empirical study of the walking behavior of pedestrian social groups [54], the analysis results of empirical data show that the pedestrian walking pattern is with pairs or triples of individuals.

The approach developed in [101] was used to simulate and evaluate the local behavior of small groups. By investigating the intra-group and inter-group relationships, the authors considered three walking patterns: (1) "line-abreast formation", (2) "V-like formation" and (3) "river-like formation", as shown in Fig. 9(a). The candidate pattern was generated by patterns 1 and 3. Inspired by this work, a slot-based group agent representation was proposed in [102] to simulate group formation transition.

A dynamic group behavior model was illustrated in [103] by using the least effort principle to realize coherent group navigation and provide inter-group and intra-group maintenance, as shown in Fig. 9(b). In this model, the first clear definition of a group is represented by the transitive closure of individuals' positions and velocities [103] as shown in Eq. (11):

$$(a \sim b) \equiv (\|\mathbf{p}_a - \mathbf{p}_b\| < \epsilon_p \wedge \|\mathbf{v}_a - \mathbf{v}_b\| < \epsilon_v). \tag{11}$$

This equation satisfies that when individuals are in the same group, they have roughly the same position and velocity. Similar to [101], the group was regarded as a big group agent and the improved ORCA [58, 59] was applied for agent-group and group-group collision avoidance. Based on the principle of least effort, the agents in this model were divided into leaders, followers, and isolated agents. By computing the leader's velocity, the followers will calculate their following velocities to generate interactive group behavior with members leaving and joining groups.

To model the walking behaviors of small pedestrian groups, the "Social Groups and Navigation (SGN)" was presented in [104]. Integrated with a SFM [54] and a vision-based model [83], the leader (closest to the goal) and the last member (farthest away from the goal) are reassigned at each time-step, and the members will compute a path to follow the leader, as shown in Fig. 9(c). The coordination model can rebuild group coherence when groups lose their coherence. SGN model can simulate the coherence and social group behaviors based on the real-world data.

In [101,103,104], the small groups were represented as a "big-group agent" to investigate the individual–group interactions. Bruneau et al. studied the going through and going around behaviors [105] with different density and types of groups. They used a VR-based method to study the real-world situations and then constructed a model for virtual human decision-making, as shown in Fig. 9(d). Both the going through and going around behavior were realized by PME (Principle of Minimum Energy) and the RVO model [57]. Simulation results showed that the interactions between an individual and the group he/she is facing are influenced by the group density, moving direction and their appearances. A unified velocity-based group modeling was shown in [106] to simulate groups with diverse properties, e.g. social groups, marches, tourists and guides, as shown in Fig. 9(e).

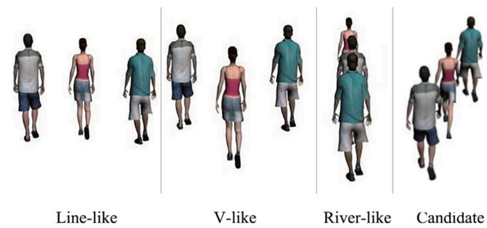
An emergency evacuation with small social groups was presented in [107], and a density-based grouping method was proposed in [108] to simulate the social relationship and group leaders. An agent-based parametric model was developed in [109] to model the group structure, by using intra-group and inter-group matrixes to control the groups. In [110], the "Car Follow" model [31] was integrated with collision avoidance to provide individuals with the ability to perceive and represent the environment, which can generate dynamic velocity waves, group behavior and lane formation. Other dynamic group behavior simulation can be found in [35,111–113].

5.2. Interactive crowd formations

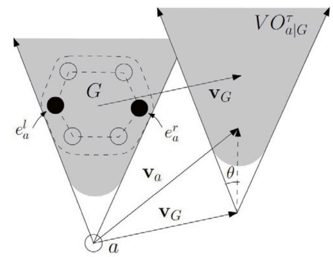
The interactive crowd control technology has the potential to be used in computer games and visual effects. These algorithms can be applied in RTS games such as StarCraft II and Command & Conquer. Also in the visual effects, these interactive formation generation can be adopted in visual effects to simulate wars and parade, etc. In this subsection, we will give a brief overview of the corresponding interactive formation control algorithms.

Early in 2004, inspired by image processing and word processing, an interactive real-time crowd manipulation framework "CrowdBrush" was constructed in [114]. This method allows the designer to edit the crowds in a two-dimensional screen as shown in Fig. 10(a). Operations such as creating or deleting crowd members, changing crowd appearances and generating convincing animations can be generated by the selection of $2D$ screen and mapping with the $3D$ entities.

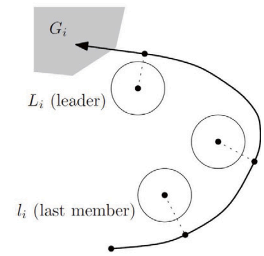
To capture the real-world human motion data, "*Motion Patches*" approach was presented in [115]. By analyzing the geometric attributes and the regularities of the environment, each *motionpatch* was annotated with motion data to form a directed graph, as shown in Fig. 10(b). The overall environment construction is realized by stitching motion patches and it allows the user to interactively manipulate motion patches. By storing dynamical objects (e.g. pedestrians, animals) and static objects (e.g. obstacles), the concept of "motion patches" was extended as "*Crowd Patches*" [116]. Because the trajectories of moving objects ware pre-computed, the process of connecting small areas of pre-computed simulations can break the limitations of classic simulations on environment dimensions and provide a user-guided design technique. Based on the "crowd patches", the factor of walking companions were taken into account in [117] to enhance the



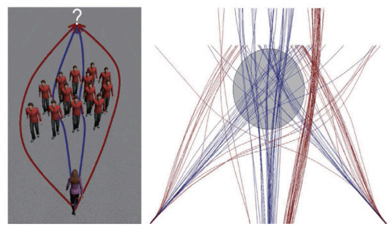
(a) Group walking patterns [101]



(b) Agent-group interactions [103]



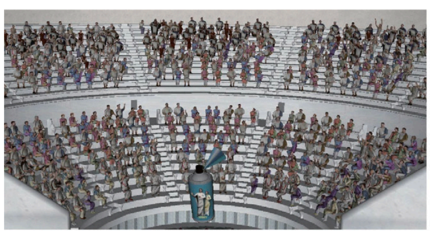
(c) SGN groups [104]



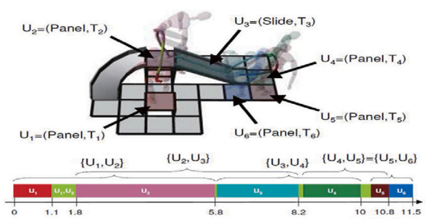
(d) Going through/around behavior [105]

(e) Unified groups [105]

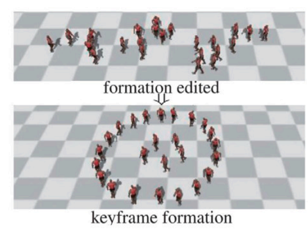
Fig. 9. Dynamic group behaviors.



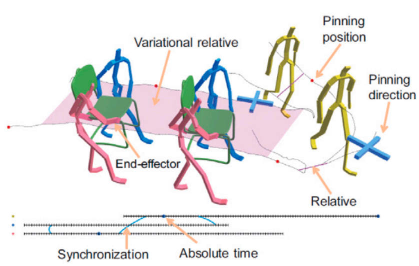
(a) Crowd brush [114]



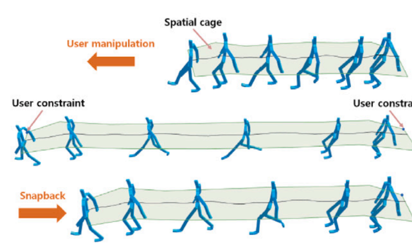
(b) Motion patches [115]



(c) Spectral-based group formation [121]



(d) Synchronized path editing [122]



(e) Cage-based motion editing in [123]

Fig. 10. Interactive group manipulation.

abilities of "crowd patches". Meanwhile the rule-based method [1] was used to control the interactions within the walking companions. The "Crowd Sculpting" was proposed by Jordao et al. [118] which can edit the spatial and the temporal crowd motion through intuitive gestures (e.g. stretching, bending, cutting and merging gestures) in a populated scenario. Inspired by the "Crowd Patch" [116], the group was represented by a graph-based structure in which the nodes and the edges represent groups and their connections. By extending Mutable Elastic Models [119], the user interactions can impact the topological, geometrical and temporal attributes of the graph structure.

The group motion editing approach was shown in [120], which provided the user the ability to design a group motion by pinning or dragging. Based on a graph-based representation, the agent's position, neighborhood and trajectory information are encoded in graph vertices and edges. It also allows users to split a single group motion into multiple clips or stitch multiple group motions into a longer one.

A spectral-based formation control was developed in [121]. By associating with a group formation, the adjacency relationships among individuals were represented as a graph. The formation transition was realized by interpolating the extracted Laplacian matrices between the source and target formations, as shown in Fig. 10(c).

A synchronized motion editing framework was constructed by Kim et al. [122]. It allows users to synchronously edit motion path in space and time. The manipulation is integrated into a curve editing process to produce the corresponding paths. Then the full-body motion along the deformed path is adjusted to avoid foot-sliding artifacts, as shown in Fig. 10(d).

For large-scale crowd animation manipulation, the "Cage-based Editing" method was illustrated in [123] which allows users to edit crowd animation and complex interactions intuitively with real-time performance. Each "cage" contains several characters and it could be edited in spatial and temporal. Combined with as-rigid-as-possible deformations, this approach can efficiently generate desired motion clips, as shown in Fig. 10(e).

The relative works in group formation studied by the Computer Graphics and Interactive Media Lab in University of Houston include [124–126]. In their preliminary research [124], the "Formation Sketching" model was developed to allow users to sketching boundaries to generate smooth group formations, as shown in Fig. 11(a). Based on the near-optimal sampling unit, the desired number of boundary agent b will be estimated by using the number of points on the initial boundary b', the square root of the number of points insider the initial boundary f' and the total number of the formation agents n [124], as shown in Eq. (12):

$$b^2 + (2 \times (n + \frac{\sqrt{f'}}{b'})) \times b + n^2 = 0. \tag{12}$$

To handle the problem that users cannot control formation during agents' path planning [124], Gu and Deng constructed a model in [125] which allows three types of formation input (brush, texture and sketched) to generate the target formation, as shown in Fig. 11(b). This model provides the ability for users to control group formation both in the local and global formation transition. Based on different user input and control information, it can generate freestyle group formation with no collisions.

A collective crowd formation transform was illustrated in [126] which was incorporated with runtime feedback. By using the Kuhn Munkres algorithm, the formation shape is transformed with least-effort pair assignment, and the affinity propagation and Delaunay triangulation are applied into sub-groups clustering. During the matching process, to minimize the overall disorder and local structure, the effort $\triangle E(i, j)$ between start point s_i to target point t_j is represented by two weighted components: 1) the weighted length of $\overline{s_i t_j}$; 2) the weighted difference between the mean length of the edges δs_i and δt_j (edges connect one point to its nearest point set) associated with s_i and t_j [126], as shown in Eq. (13):

$$\triangle E(i, j) = \lambda \| \overline{s_i t_j}^2 \| + (1 - \lambda) |\delta_{s_i} - \delta_{s_j}|^2. \tag{13}$$

Based on the social force model, this approach can realize macroscopic and microscopic behaviors with collision avoidance, as shown in Fig. 11(c).

The interactive formation control methods [127,128] in complex environment were proposed by Henry et al. In [127], a deformable mesh scheme was developed by using as-rigid-as-possible grids to represent group formation, as shown in Fig. 12. Three control schemes (point-based, line-based and area-based) were used to overcome the limitation of a multi-touch system. The formation of deformation was estimated by evaluating the finger movement from a multi-touch device. The environmental potential field can guide the groups to interactively avoid collisions and adjust their behaviors adaptively. To enhance this system, a modified Eikonal function [89] was used for the complex environment to allow agents to behave specially. The improved total cost function [128] can be represented by Eq. (14):

$$C \equiv \frac{\alpha f + \beta + \gamma g}{f \times TravelSpeed(x)}. \tag{14}$$

For more detail, please refer to [89,128].

The crowd dynamic tool ADAPTA was developed in [129] for users to design and author functional, purposeful human characters. Based on interchangeable components, ADAPTA allows a user to author individual decision-making and interactions. A geometry-constrained framework for crowd formation was constructed in [130]. The shape-constrained flock animation was generated in [131]. A real-time sketching crowd simulation was proposed in [132], and other constraint-based group formation methods can be found in [133].

5.3. Social psychological crowds

Personality traits and emotion contagion theories are two important factors affect human behavior [134]. The former includes OCEAN model [151] and the PEN model [152,153]. OCEAN model [151] is also called "the big five personalities" which contains five orthogonal dimensions of human personality space: Openness, Conscientiousness, Extraversion, Agreeableness and Neuroticism. PEN model [152,153] contains three major factors which categorize personality into Psychoticism, Extraversion, and Neuroticism. By defining the degree of each personality component either using OCEAN or PEN, the emotion contagion can be further achieved. The emotion contagion models used in crowd simulation include OCC (Ortony, Clore, Collins) [154], PAD (Pleasure Arousal Dominance) [155], ASCRIBE [135] and ESCAPES [136] models. The OCC model [154] divides the emotions into 22 categories and offers the prospect of accounting for variations in the emotions of different individuals and cultures. Due to the ease of implementation on a computer, the OCC model becomes the common basis of emotion modeling in crowd simulation. PAD model [155] describes that emotion has three dimensions (Pleasurei–Activation–Dominance) and it can effectively explain most of the variation in the other 42 emotion scales through the values of these three dimensions. ASCRIBE model [135] is a multi-agent-based continuous group emotion contagion model which considers emotions as a collective entity. ESCAPES [136] incorporates four key features (e.g. agent types, emotion, information, and behavioral interactions) to reproduce the phenomenon in evacuation scenarios (e.g. People forget their entrance, First-time visitors, Heightened emotions, Herding behavior, Pre-evacuation delay, Families gather before exiting, Authorities calm people).

As shown in Fig. 13(a), the PEN [153] personality-based crowd simulation was shown in [137] to model heterogeneous crowd behaviors. Based on the user study, the crowd behaviors were described as *Aggressive*, *Shy*, *Assertive*, *Tense*, *Impulsive* and *Active*. By randomly choosing the simulation parameters, a mapping relationship between these parameters and the PEN personality was drivers, as shown in Eq. (15), to generate the corresponding behaviors.

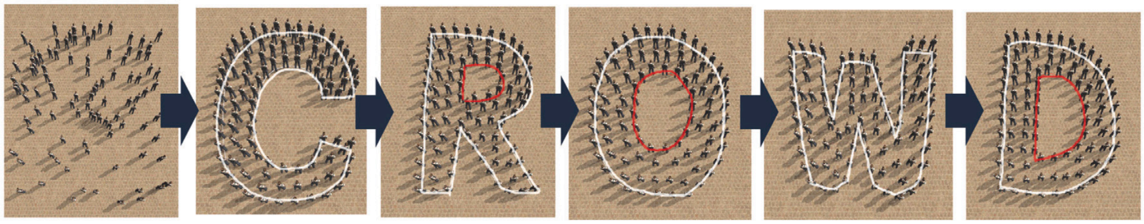
$$\begin{pmatrix} Psychoticism \\ Extraversion \\ Neuroticism \end{pmatrix} = A_{pen} \begin{pmatrix} \frac{1}{13.5}(Neighbor\ Dist - 15) \\ \frac{1}{49.5}(Max.\ Neighbors - 10) \\ \frac{1}{14.5}(Planning\ Horiz. - 30) \\ \frac{1}{0.85}(Radius - 0.8) \\ \frac{1}{0.5}(Pref.\ Speed - 1.4) \end{pmatrix}. \tag{15}$$

The GAS-based (General Adaptation Syndrome) crowds was presented in [138] to simulate interactive crowd dynamics. The accumulated stress ψ from a stressor was computed for agent's psychological behavior, as shown in Fig. 13(b). Inspired by [137], the mapping relationship was integrated with the real-world data, and the stress factors can be coupled with personalities to generate various behaviors, e.g. opposing group behavior, street crossing, evacuation, Shibuya crossing.

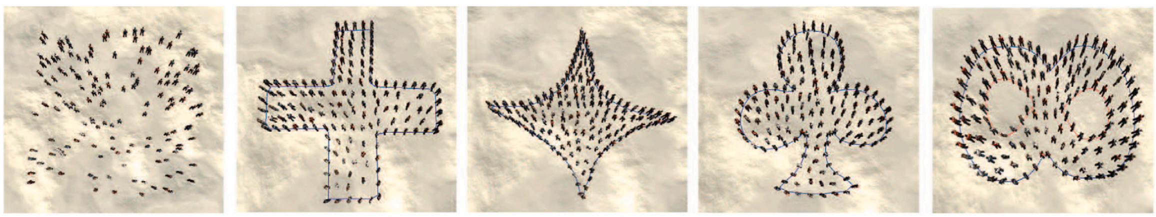
Psychological crowds integrating OCEAN personality traits and HiDAC [51] model was simulated in [139]. To allow user to define groups with different psychological characteristics, this model assigns each individual with different personality traits based on parameter setting. The definition of agent's personality [139] is shown in Eq. (16):

$$\pi = \langle \psi_O, \psi_C, \psi_E, \psi_A, \psi_N \rangle. \tag{16}$$

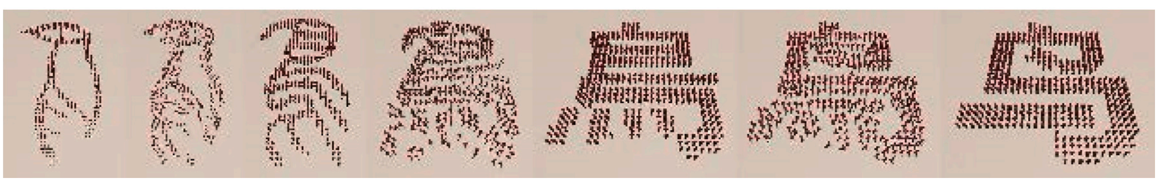
Through the mapping relationship and low-level parameters, the non-uniformity and realistic crowd behavior is realized and frees users of the tedious tasks. Agent behaviors, e.g. leadership, communication, pushing, waiting, can be generated by personality-to-behavior mapping. Durupınar et al. constructed a parametric psychology model [140] to simulate different type behaviors in crowd simulation, e.g. the leadership, the trained behavior, panic behavior. They also incorporated



(a) formation sketching for stylized groups [124]



(b) collective group formation transform [125]



(c) collective group formation transform [126]

Fig. 11. Group formation sketching.

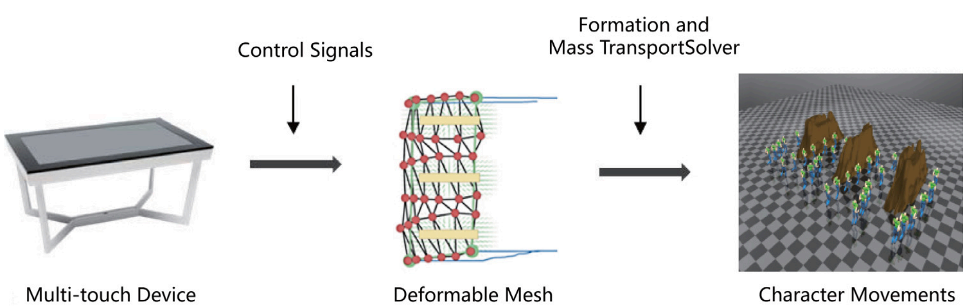
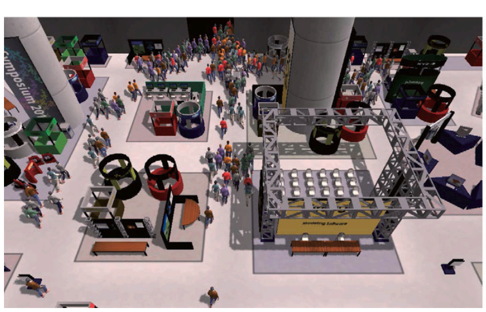
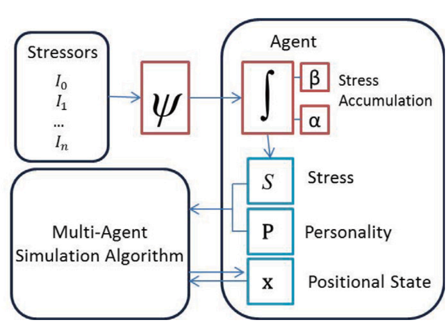


Fig. 12. Illustration of interactive formation control methods [127,128].



(a) Personality-based heterogeneous crowds [137]

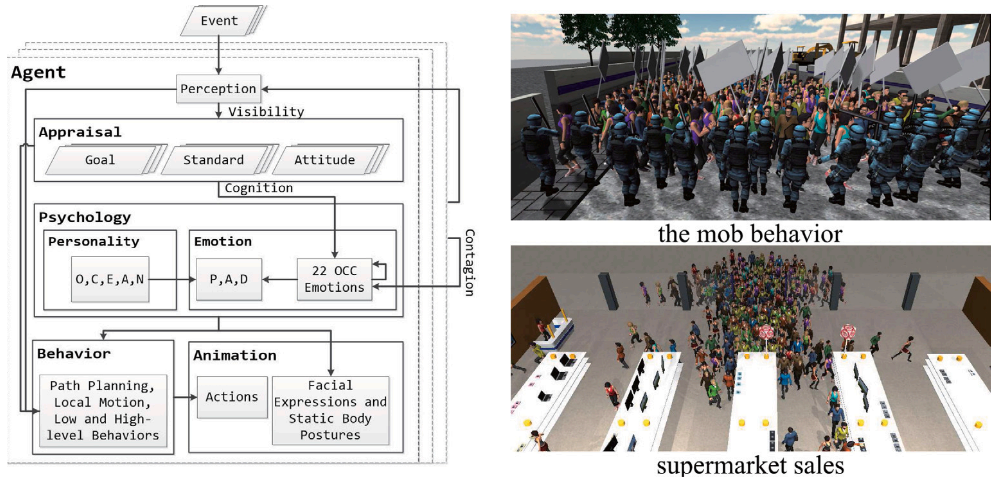


(b) GAS-based stress crowds [138]

Fig. 13. Personality trait-based crowd simulation.

OCEAN personality model and OCC-PAD mapping relationship for a psychological parameters crowd simulation [140] to model the span of personality and emotion from the audience to mobs as shown in Fig. 14.

Related works on psychological crowd simulation of Zhejiang University include [141–144]. In [141], the physiology, psychology and physics factors were integrated into a “CubeP” model to simulate realistic crowd behaviors and conformed to real-world scenarios. The



(a) Agent components (b) Mobs and supermarket sales

Fig. 14. Emotional crowds simulated in [140].

enhanced emotion contagion model was used to leverage the relationship between physical strength and emotion state. Also, in [142], the improved SIR (Susceptible–Infectious–Recovered) model was introduced into “CubeP”. As shown in Fig. 15(a), these two models can predict the changes in different factors.

Inspired by the social psychology and biological immune system, another work [143] of their team was proposed to simulate collective behaviors based on emotion evolution, as shown in Fig. 15(c). Individual can deal with the dynamic changing environment. To model the generation and contagion of panic emotion under multi-hazard circumstances, Xu et al. [144] illustrated a novel method to simulate different types of emergency circumstances. This approach can generate various evacuation strategies. The emotion contagion is introduced to construct Emotional Reciprocal Velocity Obstacles. By classifying different types of hazards, the influence on individuals reacts differently.

Based on the epidemiological SIR model, the “CA-SIRS (Cellular Automaton SIR)” model was presented in [145]. The individual emotion states were classified into susceptible, infected and recovered. On the other hand, the emotion contagion strength was interpolated within the influence range. Recently, in [146], the stress element was introduced to simulate the behavior of ready to help others for a just cause. Mao et al. studied [147] on the emotion contagion and considered the third-party’s effect among groups. A trust-based emotional contagion network (Trust-ECN) and an emotional contagion speed computation model (HECS-CM) were illustrated in [148] to model and guide the emergency evacuation. The crowd behavior evolution model with emotion contagion in political rallies was proposed in [149].

6. Comparison of different methods

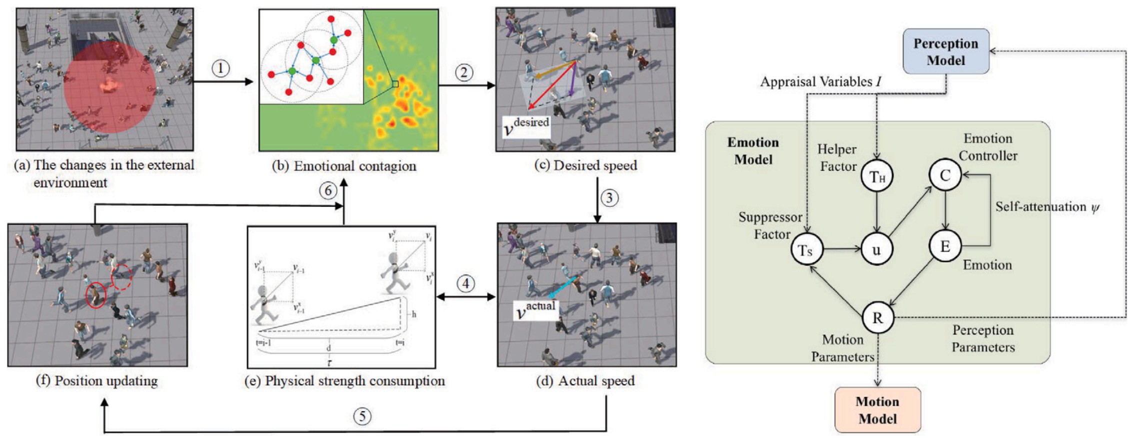
In order to evaluate the performance of different methods, it is desirable to establish uniform evaluation criteria. Berseth et al. [159] investigated the effect of the model parameters on steer algorithm’s performance to optimize an algorithm’s parameters for a range of objectives. However, it is quite difficult to establish uniform evaluation criteria for different categories of models. We collect and analyze the information provided by the papers describing the respective model and focus on the ability of different models to simulate realistic crowds, e.g. crowd dynamics and group dynamics. Besides, since we do not know all the implementation details and parameter dimensions across different approaches, the performance of simple and basic models (e.g. microscopic model, macroscopic models, dynamic groups) is evaluated based on the experiments with the same simulation scenario (all the experiments are conducted in a 2D plane using Unity 3D with different scales of crowds. The experimental platform is a desktop machine with an Nvidia GTX 1060, Intel(R) Core(TM) i7-8750H 4-core processor, 16 GB of memory, and Windows 10.). In addition, the performance of more complex models (e.g. group formations, social psychological crowds) is approximated based on the complexity of their mathematical structures. The comparison of these models is described below.

6.1. *Crowd dynamics*

With the increase of crowd density, pedestrian traffic flow will appear different self-organizing phenomena, e.g. lane-formation, bidirectional flow. Based on these self-organizing behaviors, this paper compares the ability of different models to express crowd dynamics. As shown in Table 3, we use “✓” to represent the model which is able to simulate corresponding behaviors, whereas “×” represents not and “–” means that it is unknown whether it is possible to simulate the corresponding behaviors.

Microscopic models, especially the velocity-based models [57–60], have high scalability which can simulate most of the micro phenomenon, e.g. leaderships, group, pressure and lane formation, as shown in Table 3. They focus on individual characteristics (e.g. individual step size, gender, personal space and visual range), and are controlled by low-level collision avoidance. Different individuals have various walking patterns under specific situations. The factors, e.g. individuals’ own conditions, emergency events, public facilities, will have a lot effect on their microscopic behaviors. To some extent, microscopic models are more consistent with the real-world pedestrian behavior (if social psychology is not taken into account).

With the continuous development of hardware technology, macroscopic models have an advantage in the performance of the overall crowd movement. Inspired by the original continuum model [88], Treuille et al. [89] proposed the continuum crowds to simulate the macroscopic and holistic crowd behavior such as vortices and crossing flow. However, it is unable to display the micro-level of individual movement details. The continuum model used in complex scenario [90] mainly solves the complex environmental representation and it also cannot model some micro-behaviors. To simulate the extremely high-density crowds, the aggregate dynamics model [94] considering the overall crowds has both compressibility and incompressibility which can produce realistic macro behaviors among large-scale crowds. Different from the above two models, the potential-based models can generate different kinds of behaviors by using user interface. For example, the guidance field in [96] can translate the user input information into group formation while maintaining both macro- and micro-level behaviors. Integrating continuum model and potential fields, the



(a) Overview of "CubeP" model [141]

(b) Emotion evolution model [143]

Fig. 15. Emotional crowds simulated in [140,143].

Table 3
The abilities of different models to simulate crowd dynamics (**LF**: Lane-Formation, **CF**: Crossing Flow, **Vortices**, **Blocking**, **S & GW**: Stop-and-Go Waves, **FS**: Fast is Slow, **Turbulence**).

	Name	LF	CF	Vortices	Blocking	S & GW	FS	Turbulence
Rule	Boids [1,2]	×	×	×	×	×	×	×
Force	SFM [49,50]	✓	✓	✓	–	✓	✓	✓
Velocity	VO [56]	✓	✓	✓	×	–	×	×
	RVO [57]	✓	✓	✓	✓	×	×	×
	ORCA [58]	✓	✓	✓	✓	×	×	×
	HRVO [60]	✓	✓	✓	✓	×	×	×
	Paris [61]	✓	✓	✓	×	–	×	×
	Tangent [62]	✓	✓	✓	✓	×	×	×
Agent	HiDAC [51]	✓	✓	✓	✓	×	–	×
	Cooperation [70]	✓	×	×	✓	×	×	×
	CityFlow [77]	✓	✓	✓	–	✓	–	×
	ImplicitCrowd [65]	✓	✓	✓	✓	–	×	✓
	ProactiveCrowd [74]	✓	✓	✓	–	✓	×	×
	ClearPath [68]	✓	✓	✓	✓	✓	–	–
	PBD [66]	✓	✓	✓	✓	–	×	–
Vision	Synthetic Vision [83]	✓	✓	✓	–	✓	–	×
	DAVIS [84]	✓	✓	✓	×	–	×	×
	Gradient [85]	✓	✓	✓	×	×	×	×
Continuum	Flow [88]	×	×	×	×	–	×	×
	Continuum [89]	✓	✓	✓	×	✓	–	×
	Complex [90]	✓	✓	×	–	×	×	×
	Turbulence [92]	✓	×	×	×	✓	–	✓
Aggregate	Aggregate [94]	✓	✓	✓	×	✓	–	✓
Potential	Guidance [96]	✓	✓	✓	×	×	–	–
	Potential-Agent [98]	×	✓	✓	–	✓	–	×
	Adjustment [99]	✓	✓	×	×	×	×	×

method in [98] will construct different kinds of potentials to realize collision-free trajectories. However, the ability to maintain macro- and micro-characteristics is reduced, which is the same as the model in [99].

6.2. Group dynamics

In the field of group dynamics, the research objects are the nature of groups, the law of group development, the relationship between groups and individuals, and between groups. As shown in Table 4, the study of mesoscopic group behaviors can help researchers to better understand human decision-making, behavior pattern and even diseases spread, etc. Group dynamics in crowd simulations can be simply divided into two categories, e.g. inter-group dynamics and intra-group dynamics. From these two aspects, this paper evaluates the group dynamics in mesoscopic crowd simulation models mentioned in Section 5.

From Table 4, we can find that most of the dynamic group behavior models focus on inter-group interactions such as group avoidance, group-individual interactions and group coherence. This is because the goal of these models is to maintain group coherence and integrity. For example, when avoiding other groups, a group/individual usually walks around them instead of going through. The interactive crowd formation pays less attention to group dynamics. These methods concern about the interactive user manipulations on group formations in spatial and temporal. Agents in this category hardly consider the group integrity (but sub-group coherence) and walk alone with collision-free trajectories when achieving their destinations. Social psychological models are novel models that consider to integrate social psychological factors (e.g. emotion contagion and personality traits) to simulate the influence of social-psychological factors on crowd dynamics. So the tendency to pay particular attention to inter-group or intra-group behavior cannot be found in Table 4.

Table 4
Ability of simulating group dynamics with mesoscopic crowd simulation models (**GA**: Group Avoidance, **GI**: Group–individual Interaction, **InterE**: Inter-group Emotion Contagion. **LF**: Leader–Follower, **GF**: Group Formation, **GS**: Group Structure, **GC**: Group Cohesion, **GCo**: Group Cooperation, **PB**: Peer Behavior, **IntraE**: Intra-group Emotion Contagion).

	Method	Inter-Group			Intra-Group						
		GA	GI	InterE	LF	GF	GS	GC	GCo	PB	IntraE
Dynamic Groups	Small Group [101]	×	×	×	×	✓	✓	✓	×	×	×
	Dynamic Group [103]	✓	✓	×	×	×	✓	✓	×	×	×
	Sociality Group [104]	✓	✓	×	✓	×	×	✓	×	×	×
	Through Around [105]	✓	✓	×	✓	×	×	✓	×	×	×
	Group Structure [109]	×	×	×	✓	×	✓	✓	×	×	×
	Lemercier et al. [110]	✓	✓	×	✓	×	×	✓	×	×	×
	Velocity-based [106]	✓	✓	×	✓	✓	✓	✓	✓	×	×
Crowd Formation	CrowdBrush [114]	×	×	×	×	×	×	✓	×	×	×
	CrowdSculpting [118]	✓	×	×	×	×	×	✓	×	×	×
	CrowdPatches [116]	✓	✓	×	×	×	×	✓	×	×	×
	SynchronizedEditing [122]	✓	✓	×	×	✓	×	✓	×	×	×
	Inter-Mnipulation [123]	✓	✓	×	✓	✓	×	✓	✓	×	×
	CollectiveFormation [126]	×	✓	×	×	✓	✓	✓	×	×	×
	FreestyleFormation [125]	×	×	×	×	✓	✓	×	×	×	×
	FormationSketching [124]	×	✓	×	×	✓	✓	✓	×	×	×
	Inter-Formation [128]	✓	×	×	×	✓	✓	✓	×	×	×
	Spectral-based [121]	✓	×	×	×	✓	✓	✓	×	×	×
Social Crowds	OCEAN-HiDAC [139]	×	×	×	✓	×	×	×	✓	✓	×
	HeterogeneousCrowds [137]	×	×	×	×	×	×	×	×	×	×
	AudienceToMobs [140]	×	×	×	×	×	×	×	✓	×	✓
	UnexpectedHazard [144]	×	✓	×	×	×	✓	×	×	×	✓
	P-P-P Factors [142]	×	×	✓	×	×	×	×	×	×	✓
	EmotionEvolution [143]	×	✓	×	✓	×	×	×	×	×	✓
	Third-Party [147]	×	✓	✓	×	×	×	×	×	×	✓
	DiversityBehavior [146]	✓	✓	✓	✓	×	✓	×	×	×	✓

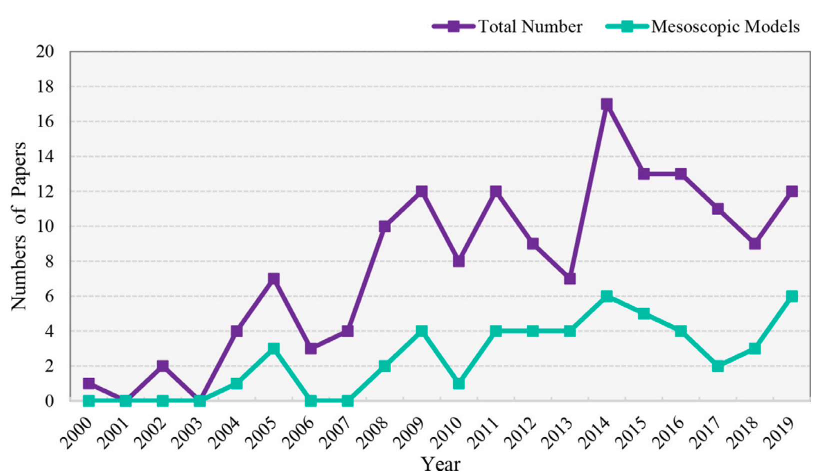
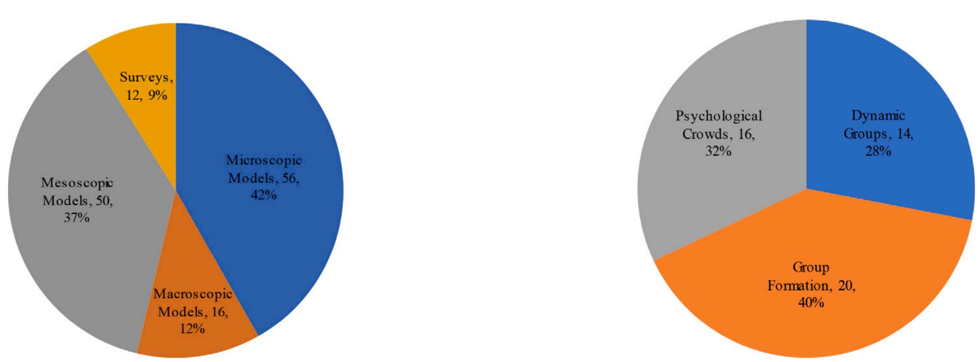


Fig. 16. The number of different types of papers published per year.



(a) The proportion of all types of papers (b) The proportion of mesoscopic crowd simulation

Fig. 17. The proportion of different types of papers.

On the other hand, we analyze the number of papers published in the field of crowd simulation from 2000 to 2020, and the results of a statistical analysis can be seen from Figs. 16 and 17. There are in total about 160 papers published in these years (from 2000 to 2020), and the research trend of the mesoscopic models is roughly the same as the overall research trend and shows an upward trend as illustrated in Fig. 16. After our statistics and analysis, we find that the researches of group behavior date back to 2004 and other methods can be found in the next several years. The group structure was illustrated in [109] in 2010 and after that, the OCEAN personality traits model was then used in [139] to integrate social psychological factors in 2011. We can also find that although the number of papers in crowd simulation is decreasing in 2013, the number of mesoscopic crowd simulation does not decrease and it accounts for more than half of the total number of crowd simulation researches. Fig. 17(a) shows the proportion of different simulation models. The most widely studied models are microscopic models and mesoscopic groups which is because the microscopic models are simple and scalable. Most of the mesoscopic groups are the combinations of microscopic models and formation rules or psychological models. Besides, the research of interactive group formation began in 2004 [114], whereas dynamic group simulation and social crowds both appeared in 2005 [35,134]. The number of social psychological and interactive group formation is still increasing, which means that it can be seen these research topics would become hot topics in computer graphics community.

6.3. Model applicability and performance

We test three different scales (small, middle and large) of crowds in the same 2D plane to qualitatively evaluate the simulation performance of the representative microscopic, macroscopic and dynamic group simulation models. The performance of the interactive formation simulation and the social psychological crowd simulation is approximated based on their mathematical structures, as shown in Table 5. Even if the methods are of the same category, the results of simulation are quite different. For example, although the HiDAC model [51] and the ClearPath model [68] are both agent-based models, the former sometimes cannot simulate high-density crowds, whereas the latter is a parallel algorithm which has the ability to solve this problem. Moreover, due to the differences in their frameworks, they also have different computing costs.

Macroscopic models have obvious advantages in simulating large-scale crowds. Some microscopic models are also good at simulating large populations, e.g. Implicit Crowd [65], Clear Path [68] and PBD [66] models. As for continuum [88] and the aggregate dynamics [94] models, both of them can simulate large-scale crowds, but the computational efficiencies are influenced a lot by the division of environmental grid precision. In order to model the realistic crowds, these models perform relatively well in small-scale population. However, with the increasing number of individuals, the computational efficiency of these models decreases rapidly. Especially in psychological crowds, the model should take psychological factors (personality traits and emotion contagion) into account, and perform path planning and collision avoidance at the same time. The degradation of performance in interactive crowd formation is caused by user input control. Therefore, the large-scale scenario is not suitable for interactive manipulation.

7. Conclusion

This paper reviewed the models used in crowd simulation. We compared the models in different categories and provided a broad overview of the current literature on crowd simulation models of the last decades. The simulation results generated by the traditional models (e.g. microscopic and macroscopic models) sometimes are not as realistic as the real-world human behavior. In general, people's movements are unpredictable and effected by their own internal personality traits. Therefore, even in the same dangerous situation, their reactions will be quite different. Furthermore, humans have very strong social attributes. Their behaviors are not only affected by a single factor, but by a combination of many factors (e.g. obstacle, events, peers, emotions). To understand crowd behaviors, more efforts need to be made in data collection, data analysis and other aspects.

The breaks and delays will affect the user/audience experience to some extent. As a result, the efficiency and performance of crowd simulation models are still needed to be further improved, especially in the real-time or cloud-based scenarios. In the future, with the wide applications of 5G technology and the rise of remote rendering systems, rendering technology (e.g. real-time rendering, global illumination) will be further improved and problems, e.g. rendering quality, efficiency and transferring, will be resolved.

In the current research process, there are several problems and challenges when simulating realistic crowds. Firstly, when an emergency occurs, it is difficult to obtain the actual escape behavior of the crowd, especially in large-scale emergency evacuations. To reconstruct such an emergency situation, researchers try to collect trajectory data through real-world experiments [160]. However, the experimental scene (such as the corridor and the hall) is too simple and small, which could not reflect the real-world situations. Besides, participants knew it was an emergency drill and their behaviors might differ from which in real emergency.

For the above problems, our analysis of the future research direction is as follows.

- **Data collection**: It is feasible for researchers to get real-world data from video sequences obtained from different cameras in one scene, and extract each camera's crowd motion data to recompute the overall motion state of the real-world scene through analysis and processing. Different pieces of fragmented data are integrated to obtain the complete required data.
- **Cognitive science**: To model more autonomous and intelligent crowds, cognitive-based crowds will become a promising research topic with the development of artificial intelligence by integrating with social psychological and physical lows. More general solutions of emergency evacuation and training systems are needed in the future.
- **Behavior simulation**: As for per-character animation, the learning-based and intelligence-based methods [161–165] consider to use the learning framework to automatically generate character animations. From this point of view, the learning framework can be integrated into crowd simulation models. On the one hand, the collected data from real-world can be used for a supervised learning framework. As for crowd simulation, the character's animation is pre-defined and the training model can just focus on generating crowd moving trajectories. On the other hand, by choosing the important features, e.g. moving speed, moving direction, obstacles, start point and end point through the unsupervised learning framework, it is also able to train an adaptive model for simulations.
- **Computational efficiency**: From Table 5, we can find that different crowd simulation methods have different performance. To simulate large-scale realistic crowds, it is important to consider how to solve computational resources. It is feasible to adopt distributed parallel computing [166,167] and deep-learning scheme into crowd simulation.

From the review, it can be seen that new directions of modeling are opening up with the field of crowd simulation. In addition, traditional crowd simulation models have solved the problems of how characters move, react and decide. It focuses on the computational speed but not on the reality of the crowd. Besides, researches at present have laid the groundwork and provided ideas of new research directions (psychological, emotional and intelligent crowds), but the feasibility and scalability of these models still need to be improved. Both types of models lack to

Table 5
Performance of models in different scales of crowds (**S**: Small Scale, individuals < 200, **M**: Middle Scale, 200 < individuals < 500, **L**: Large Scale, 500 < individuals < 1000).

	Model	S	M	L		Model	S	M	L
Micro	Boids [1,2]	–	–	––	Group	DynamicGroup [103]	++	–	––
Micro	SFM [49,50]	–	–	––	Group	SicialityGroup [104]	–	––	––
Micro	RVO [57]	++	–	––	Group	CrowdSculpting [118]	++	–	––
Micro	ORCA [58]	++	++	–	Formation	Inter-Manipulation [123]	++	–	––
Micro	HRVO [60]	++	–	–	Formation	CollectiveFormation [126]	++	++	–
Micro	HiDAC [51]	–	––	––	Formation	FreestyleFormation [125]	++	–	––
Micro	ImplicitCrowd [65]	++	++	++	Formation	Inter-Formation [128]	++	–	–
Micro	ClearPath [68]	++	++	++	Psycho	OCEAN-HiDAC [139]	–	––	––
Micro	PBD [66]	++	++	++	Psycho	Heterogeneous [137]	++	–	––
Micro	SyntheticVision [83]	++	–	––	Psycho	AudienceToMobs [140]	–	––	––
Macro	Continuum [88]	++	++	++	Psycho	UnexpectedHazard [144]	–	––	––
Macro	Aggregate [94]	++	++	++	Psycho	P-P-P Factors [142]	––	––	––
Macro	Guidance Field [96]	++	++	++	Psycho	Third-Party [147]	++	–	–
Macro	Potential-Agent [98]	++	++	++	Psycho	DiversityBehavior [146]	––	––	––

a certain extent either on realistic behavior or computational efficiency. Combining data analysis and artificial intelligence will bring forward a model that is balanced in both aspects. Our future research will look into the combination of crowd simulation with real-world data and achieve possible computational efficiency.

CRediT authorship contribution statement

Shanwen Yang: Writing - original draft, Methodology, Software. **Tianrui Li:** Conceptualization, Supervision, Writing - review & editing. **Xun Gong:** Resources, Investigation. **Bo Peng:** Resources, Investigation. **Jie Hu:** Investigation, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

This work is supported by the National Key R&D Program of China (No. 2019YFB2101802) and the National Natural Science Foundation of China (Nos. 61773324, 61876158).

References

[1] C.W. Reynolds, Flocks, herds and schools: A distributed behavioral model, ACM SIGGRAPH Comput. Graph. 21 (4) (1987) 25–34, http://dx.doi.org/10.1145/37402.37406, URL http://doi.acm.org/10.1145/37402.37406.
[2] C.W. Reynolds, Steering behaviors for autonomous characters, in: Game Developers Conference, Vol. 1999, Citeseer, 1999, pp. 763–782.
[3] S. Gustafson, H. Arumugam, P. Kanyuk, M. Lorenzen, MURE: Fast agent based crowd simulation for VFX and animation, in: ACM SIGGRAPH 2016 Talks, SIGGRAPH '16, ACM, New York, USA, 2016, pp. 56:1–56:2, http://dx.doi.org/10.1145/2897839.2927396, URL http://doi.acm.org/10.1145/2897839.2927396.
[4] D. Ryu, P. Kanyuk, Rivers of rodents: An animation-centric crowds pipeline for ratatouille, in: ACM SIGGRAPH 2007 Sketches, SIGGRAPH '07, ACM, New York, USA, 2007, http://dx.doi.org/10.1145/1278780.1278859, URL http://doi.acm.org/10.1145/1278780.1278859.
[5] P. Kanyuk, L.J.W. Park, E. Weihrich, Headstrong, hairy, and heavily clothed: Animating crowds of scotsmen, in: ACM SIGGRAPH 2012 Talks, SIGGRAPH '12, ACM, New York, USA, 2012, p. 52:1, http://dx.doi.org/10.1145/2343045.2771822, URL http://doi.acm.org/10.1145/2343045.2771822.
[6] P. Kanyuk, C. Lawrence, Brain springs: Fast physics for large crowds on WALL·e, in: ACM SIGGRAPH 2008 Talks, SIGGRAPH '08, ACM, New York, USA, 2008, p. 26:1, http://dx.doi.org/10.1145/1401032.1401066, URL http://doi.acm.org/10.1145/1401032.1401066.
[7] B. Ulicny, D. Thalmann, Towards interactive real-time crowd behavior simulation, Comput. Graph. Forum 21 (4) (2002) 767–775, http://dx.doi.org/10.1111/1467-8659.00634, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/1467-8659.00634.
[8] D. Wolinski, S. J. Guy, A.-H. Olivier, M. Lin, D. Manocha, J. Pettré, Parameter estimation and comparative evaluation of crowd simulations, Comput. Graph. Forum 33 (2) (2014) 303–312, http://dx.doi.org/10.1111/cgf.12328, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/cgf.12328.
[9] T. Feng, L.-F. Yu, S.-K. Yeung, K. Yin, K. Zhou, Crowd-driven mid-scale layout design, ACM Trans. Graph. 35 (4) (2016) 132:1–132:14, http://dx.doi.org/10.1145/2897824.2925894, URL http://doi.acm.org/10.1145/2897824.2925894.
[10] C.D.T. Mathew, P.R. Knob, S.R. Musse, D.G. Aliaga, Urban walkability design using virtual population simulation, Comput. Graph. Forum 38 (1) (2019) 455–469, http://dx.doi.org/10.1111/cgf.13585, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/cgf.13585.
[11] R. Shepherd, C. Clegg, M. Robinson, Understanding Crowd Behaviours, Volume 2: Supporting Theory and Evidence, 2010.
[12] B. Zhou, X. Tang, H. Zhang, X. Wang, Measuring crowd collectiveness, IEEE Trans. Pattern Anal. Mach. Intell. 36 (8) (2014) 1586–1599, http://dx.doi.org/10.1109/TPAMI.2014.2300484.
[13] K. Ijaz, S. Sohail, S. Hashish, A survey of latest approaches for crowd simulation and modeling using hybrid techniques, in: 17th UKSIMAMSS International Conference on Modelling and Simulation, 2015, pp. 111–116.
[14] A. Beacco, N. Pelechano, C. Andújar, A survey of real-time crowd rendering, Comput. Graph. Forum 35 (8) (2016) 32–50, http://dx.doi.org/10.1111/cgf.12774, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/cgf.12774.
[15] G. Ryder, A.M. Day, Survey of real-time rendering techniques for crowds, Comput. Graph. Forum 24 (2) (2005) 203–215, http://dx.doi.org/10.1111/j.1467-8659.2005.00844.x, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1467-8659.2005.00844.x.
[16] G. Ryder, A. Day, Survey of techniques for rendering real-time virtual humans, in: Computer Graphics Forum, Vol. 24, (2) 2005, pp. 203–215.
[17] M.A.B.M. Azahar, M.S. Sunar, D. Daman, A. Bade, Survey on real-time crowds simulation, in: Z. Pan, X. Zhang, A. El Rhalibi, W. Woo, Y. Li (Eds.), Technologies for E-Learning and Digital Entertainment, Springer Berlin Heidelberg, Berlin, Heidelberg, 2008, pp. 573–580.
[18] M.-L. Xu, H. Jiang, X.-G. Jin, Z. Deng, Crowd simulation and its applications: Recent advances, J. Comput. Sci. Tech. 29 (5) (2014) 799–811, http://dx.doi.org/10.1007/s11390-014-1469-y.
[19] D.C. Duives, W. Daamen, S.P. Hoogendoorn, State-of-the-art crowd motion simulation models, Transp. Res. C 37 (2013) 193–209, http://dx.doi.org/10.1016/j.trc.2013.02.005, URL http://www.sciencedirect.com/science/article/pii/S0968090X13000351.
[20] T. Pejsa, I. Pandzic, State of the art in example-based motion synthesis for virtual characters in interactive applications, Comput. Graph. Forum 29 (1) (2010) 202–226, http://dx.doi.org/10.1111/j.1467-8659.2009.01591.x, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1467-8659.2009.01591.x.
[21] J. Bender, M. Müller, M.A. Otaduy, M. Teschner, M. Macklin, A survey on position-based simulation methods in computer graphics, Comput. Graph. Forum 33 (6) (2014) 228–251, http://dx.doi.org/10.1111/cgf.12346, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/cgf.12346.
[22] D. Thalmann, Crowd simulation, in: Wiley Encyclopedia of Computer Science and Engineering, American Cancer Society, 2007, http://dx.doi.org/10.1002/9780470050118.ecse676, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/9780470050118.ecse676.
[23] M. Kapadia, N. Pelechano, J. Allbeck, N. Badler, Virtual crowds: Steps toward behavioral realism, in: Synthesis Lectures on Visual Computing: Computer Graphics, Animation, Computational Photography, and Imaging, Vol. 7(4), 2015, pp. 1–270, http://dx.doi.org/10.2200/S00673ED1V01Y201509CGR020.
[24] M. Kapadia, N. Pelechano, J. Allbeck, N. Badler, Simulating Heterogeneous Crowds with Interactive Behaviors, CRC Press, 2016.
[25] A. Sud, E. Andersen, S. Curtis, M. Lin, D. Manocha, Real-time path planning for virtual agents in dynamic environments, in: ACM SIGGRAPH 2008 Classes, SIGGRAPH '08, ACM, New York, USA, 2008, pp. 55:1–55:9, http://dx.doi.org/10.1145/1401132.1401206, URL http://doi.acm.org/10.1145/1401132.1401206.

[26] A. Sud, E. Andersen, S. Curtis, M.C. Lin, D. Manocha, Real-time path planning in dynamic virtual environments using multiagent navigation graphs, IEEE Trans. Vis. Comput. Graphics 14 (3) (2008) 526–538, http://dx.doi.org/10.1109/TVCG.2008.27.

[27] S. Paris, S. Donikian, N. Bonvalet, Environmental abstraction and path planning techniques for realistic crowd simulation, Comput. Animat. Virtual Worlds 17 (3–4) (2006) 325–335, http://dx.doi.org/10.1002/cav.136, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/cav.136.

[28] B. Liu, A. Hao, Real-time path planning in emergency using non-uniform safety fields, in: 2014 IEEE Virtual Reality (VR), 2014, pp. 93–94, http://dx.doi.org/10.1109/VR.2014.6802067.

[29] R. Gayle, A. Sud, E. Andersen, S.J. Guy, M.C. Lin, D. Manocha, Interactive navigation of heterogeneous agents using adaptive roadmaps, IEEE Trans. Vis. Comput. Graphics 15 (1) (2009) 34–48, http://dx.doi.org/10.1109/TVCG.2008.84.

[30] M. Kapadia, S. Singh, W. Hewlett, P. Faloutsos, Egocentric affordance fields in pedestrian steering, in: Proceedings of the 2009 Symposium on Interactive 3D Graphics and Games, ACM, 2009, pp. 215–223, http://dx.doi.org/10.1145/1507149.1507185.

[31] S. Lemercier, A. Jelic, R. Kulpa, J. Hua, J. Fehrenbach, P. Degond, C. Appert-Rolland, S. Donikian, J. Pettré, Realistic following behaviors for crowd simulation, Comput. Graph. Forum 31 (2pt2) (2012) 489–498, http://dx.doi.org/10.1111/j.1467-8659.2012.03028.x.

[32] S.J. Guy, J. van den Berg, W. Liu, R. Lau, M.C. Lin, D. Manocha, A statistical similarity measure for aggregate crowd dynamics, ACM Trans. Graph. 31 (6) (2012) 190:1–190:11, http://dx.doi.org/10.1145/2366145.2366209, URL http://doi.acm.org/10.1145/2366145.2366209.

[33] M. Sung, M. Gleicher, S. Chenney, Scalable behaviors for crowd simulation, Comput. Graph. Forum 23 (3) (2004) 519–528, http://dx.doi.org/10.1111/j.1467-8659.2004.00783.x, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1467-8659.2004.00783.x.

[34] A. Lerner, E. Fitusi, Y. Chrysanthou, D. Cohen-Or, Fitting behaviors to pedestrian simulations, in: Proceedings of the 2009 ACM SIGGRAPH/Eurographics Symposium on Computer Animation, SCA '09, ACM, New York, NY, USA, 2009, pp. 199–208, http://dx.doi.org/10.1145/1599470.1599496, URL http://doi.acm.org/10.1145/1599470.1599496.

[35] Y.-C. Lai, S. Chenney, S. Fan, Group motion graphs, in: Proceedings of the 2005 ACM SIGGRAPH/Eurographics Symposium on Computer Animation, SCA '05, ACM, New York, NY, USA, 2005, pp. 281–290, http://dx.doi.org/10.1145/1073368.1073409, URL http://doi.acm.org/10.1145/1073368.1073409.

[36] M.L. Chu, P. Parigi, K. Law, J.-C. Latombe, Modeling social behaviors in an evacuation simulator, Comput. Animat. Virtual Worlds 25 (3–4) (2014) 373–382, http://dx.doi.org/10.1002/cav.1595, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/cav.1595.

[37] G. Berseth, M. Usman, B. Haworth, M. Kapadia, P. Faloutsos, Environment optimization for crowd evacuation, Comput. Animat. Virtual Worlds 26 (3–4) (2015) 377–386, http://dx.doi.org/10.1002/cav.1652, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/cav.1652.

[38] B. Haworth, M. Usman, G. Berseth, M. Kapadia, P. Faloutsos, On density–flow relationships during crowd evacuation, Comput. Animat. Virtual Worlds 28 (3–4) (2017) e1783, http://dx.doi.org/10.1002/cav.1783, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/cav.1783, e1783 cav.1783.

[39] B. Haworth, M. Usman, G. Berseth, M. Khayatkhoei, M. Kapadia, P. Faloutsos, CODE: Crowd-optimized design of environments, Comput. Animat. Virtual Worlds 28 (6) (2017) e1749, http://dx.doi.org/10.1002/cav.1749, e1749 cav.1749.

[40] A. Golas, R. Narain, S. Curtis, M.C. Lin, Hybrid long-range collision avoidance for crowd simulation, IEEE Trans. Vis. Comput. Graphics 20 (7) (2014) 1022–1034, http://dx.doi.org/10.1109/TVCG.2013.235.

[41] J. Sewall, D. Wilkie, M.C. Lin, Interactive hybrid simulation of large-scale traffic, ACM Trans. Graph. 30 (6) (2011) 135:1–135:12, http://dx.doi.org/10.1145/2070781.2024169, URL http://doi.acm.org/10.1145/2070781.2024169.

[42] M. Haciomeroglu, O. Barut, C.Y. Ozcan, H. Sever, A GPU-assisted hybrid model for real-time crowd simulations, Comput. Graph. 37 (7) (2013) 862–872, http://dx.doi.org/10.1016/j.cag.2013.05.006, URL http://www.sciencedirect.com/science/article/pii/S0097849313000800.

[43] S. Singh, M. Kapadia, B. Hewlett, G. Reinman, P. Faloutsos, A modular framework for adaptive agent-based steering, in: Symposium on Interactive 3D Graphics and Games, ACM, 2011, pp. 141–150, http://dx.doi.org/10.1145/1944745.1944769, PAGE–9.

[44] S. Singh, M. Kapadia, G. Reinman, P. Faloutsos, Footstep navigation for dynamic crowds, Comput. Animat. Virtual Worlds 22 (2–3) (2011) 151–158, http://dx.doi.org/10.1002/cav.403.

[45] G. Berseth, M. Kapadia, P. Faloutsos, Robust space-time footsteps for agent-based steering, Comput. Animat. Virtual Worlds (2015).

[46] D. Thalmann, H. Grillon, J. Maim, B. Yersin, Challenges in crowd simulation, in: 2009 International Conference on CyberWorlds, 2009, pp. 1–12, http://dx.doi.org/10.1109/CW.2009.23.

[47] C. Hartman, B. Beneš, Autonomous boids, Comput. Animat. Virtual Worlds 17 (3–4) (2006) 199–206, http://dx.doi.org/10.1002/cav.123, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/cav.123.

[48] A.R.D. Silva, W.S. Lages, L. Chaimowicz, Boids that see: Using self-occlusion for simulating large groups on GPUs, Comput. Entertain. 7 (4) (2010) 51:1–51:20, http://dx.doi.org/10.1145/1658866.1658870.

[49] D. Helbing, I. Farkas, T. Vicsek, Simulating dynamical features of escape panic, Nature 407 (6803) (2000) 487.

[50] D. Helbing, L. Buzna, A. Johansson, T. Werner, Self-organized pedestrian crowd dynamics: Experiments, simulations, and design solutions, Transp. Sci. 39 (1) (2005) 1–24, http://dx.doi.org/10.1287/trsc.1040.0108, URL https://pubsonline.informs.org/doi/abs/10.1287/trsc.1040.0108.

[51] N. Pelechano, J.M. Allbeck, N.I. Badler, Controlling individual agents in high-density crowd simulation, in: Proceedings of the 2007 ACM SIGGRAPH/Eurographics Symposium on Computer Animation, SCA '07, Eurographics Association, Aire-la-Ville, Switzerland, Switzerland, 2007, pp. 99–108, URL http://dl.acm.org/citation.cfm?id=1272690.1272705.

[52] P. Saboia, S. Goldenstein, Crowd simulation: applying mobile grids to the social force model, Vis. Comput. 28 (10) (2012) 1039–1048, http://dx.doi.org/10.1007/s00371-012-0731-y.

[53] I. Karamouzas, P. Heil, P. Beek, M.H. Overmars, A predictive collision avoidance model for pedestrian simulation, in: Proceedings of the 2nd International Workshop on Motion in Games, MIG '09, Springer-Verlag, Berlin, Heidelberg, 2009, pp. 41–52, http://dx.doi.org/10.1007/978-3-642-10347-6_4.

[54] Q. Ji, F. Wang, T. Zhu, VPBS: A velocity-perception-based SFM approach for crowd simulation, in: 2016 International Conference on Virtual Reality and Visualization (ICVRV), 2016, pp. 317–324, http://dx.doi.org/10.1109/ICVRV.2016.59.

[55] A.U.K. Wagoum, M. Chraibi, J. Mehlich, A. Seyfried, A. Schadschneider, Efficient and validated simulation of crowds for an evacuation assistant, Comput. Animat. Virtual Worlds 23 (1) (2012) 3–15, http://dx.doi.org/10.1002/cav.1420, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/cav.1420.

[56] P. Fiorini, Z. Shiller, Motion planning in dynamic environments using velocity obstacles, Int. J. Robot. Res. 17 (7) (1998) 760–772, http://dx.doi.org/10.1177/027836499801700706.

[57] J. van den Berg, M. Lin, D. Manocha, Reciprocal Velocity Obstacles for real-time multi-agent navigation, in: 2008 IEEE International Conference on Robotics and Automation, 2008, pp. 1928–1935, http://dx.doi.org/10.1109/ROBOT.2008.4543489.

[58] J. van den Berg, S.J. Guy, M. Lin, D. Manocha, Reciprocal n-body collision avoidance, in: C. Pradalier, R. Siegwart, G. Hirzinger (Eds.), Robotics Research, Springer Berlin Heidelberg, Berlin, Heidelberg, 2011, pp. 3–19.

[59] J. Snape, J. van den Berg, S.J. Guy, D. Manocha, Smooth and collision-free navigation for multiple robots under differential-drive constraints, in: 2010 IEEE/RSJ International Conference on Intelligent Robots and Systems, 2010, pp. 4584–4589, http://dx.doi.org/10.1109/IROS.2010.5652073.

[60] J. Snape, J. v. d. Berg, S.J. Guy, D. Manocha, The hybrid reciprocal velocity obstacle, IEEE Trans. Robot. 27 (4) (2011) 696–706, http://dx.doi.org/10.1109/TRO.2011.2120810.

[61] S. Paris, J. Pettré, S. Donikian, Pedestrian reactive navigation for crowd simulation: a predictive approach, Comput. Graph. Forum 26 (3) (2007) 665–674, http://dx.doi.org/10.1111/j.1467-8659.2007.01090.x, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1467-8659.2007.01090.x.

[62] J. Pettré, J. Ondřej, A.-H. Olivier, A. Cretual, S. Donikian, Experiment-based modeling, simulation and validation of interactions between virtual walkers, in: Proceedings of the 2009 ACM SIGGRAPH/Eurographics Symposium on Computer Animation, SCA '09, ACM, New York, USA, 2009, pp. 189–198, http://dx.doi.org/10.1145/1599470.1599495.

[63] S. Kim, S.J. Guy, D. Manocha, Velocity-based modeling of physical interactions in multi-agent simulations, in: Proceedings of the 12th ACM SIGGRAPH/Eurographics Symposium on Computer Animation, SCA '13, ACM, New York, USA, 2013, pp. 125–133, http://dx.doi.org/10.1145/2485895.2485910.

[64] S. Kim, S.J. Guy, K. Hillesland, B. Zafar, A.A.-A. Gutub, D. Manocha, Velocity-based modeling of physical interactions in dense crowds, Vis. Comput. 31 (5) (2015) 541–555, http://dx.doi.org/10.1007/s00371-014-0946-1.

[65] I. Karamouzas, N. Sohre, R. Narain, S.J. Guy, Implicit crowds: Optimization integrator for robust crowd simulation, ACM Trans. Graph. 36 (4) (2017) 136:1–136:13, http://dx.doi.org/10.1145/3072959.3073705.

[66] T. Weiss, A. Litteneker, C. Jiang, D. Terzopoulos, Position-based multi-agent dynamics for real-time crowd simulation, in: Proceedings of the ACM SIGGRAPH / Eurographics Symposium on Computer Animation, SCA '17, ACM, New York, USA, 2017, pp. 27:1–27:2, http://dx.doi.org/10.1145/3099564.3108160.

[67] T. Weiss, A. Litteneker, C. Jiang, D. Terzopoulos, Position-based real-time simulation of large crowds, Comput. Graph. 78 (2019) 12–22, http://dx.doi.org/10.1016/j.cag.2018.10.008, URL http://www.sciencedirect.com/science/article/pii/S0097849318301699.

[68] S.J. Guy, J. Chhugani, C. Kim, N. Satish, M. Lin, D. Manocha, P. Dubey, Clearpath: Highly parallel collision avoidance for multi-agent simulation, in: Proceedings of the 2009 ACM SIGGRAPH/Eurographics Symposium on Computer Animation, SCA '09, ACM, New York, USA, 2009, pp. 177–187, http://dx.doi.org/10.1145/1599470.1599494.

[69] H. Yeh, S. Curtis, S. Patil, J. van den Berg, D. Manocha, M. Lin, Composite agents, in: Proceedings of the 2008 ACM SIGGRAPH/Eurographics Symposium on Computer Animation, SCA '08, Eurographics Association, Aire-la-Ville, Switzerland, Switzerland, 2008, pp. 39–47, URL http://dl.acm.org/citation.cfm?id=1632592.1632599.

[70] S.-K. Wong, Y.-H. Chou, H.-Y. Yang, A framework for simulating agent-based cooperative tasks in crowd simulation, in: Proceedings of the ACM SIGGRAPH Symposium on Interactive 3D Graphics and Games, I3D '18, ACM, New York, USA, 2018, pp. 11:1–11:10, http://dx.doi.org/10.1145/3190834.3190839.

[71] H. Chen, S.-K. Wong, Transporting objects by multiagent cooperation in crowd simulation, Comput. Animat. Virtual Worlds 29 (3–4) (2018) e1826, http://dx.doi.org/10.1002/cav.1826, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/cav.1826, e1826 cav.1826.

[72] S.A. Stüvel, N. Magnenat-Thalmann, D. Thalmann, A.F. v. d. Stappen, A. Egges, Torso crowds, IEEE Trans. Vis. Comput. Graphics 23 (7) (2017) 1823–1837, http://dx.doi.org/10.1109/TVCG.2016.2545670.

[73] L. Hoyet, A.-H. Olivier, R. Kulpa, J. Pettré, Perceptual effect of shoulder motions on crowd animations, ACM Trans. Graph. 35 (4) (2016) 53:1–53:10, http://dx.doi.org/10.1145/2897824.2925931, URL http://doi.acm.org/10.1145/2897824.2925931.

[74] L. Luo, C. Chai, J. Ma, S. Zhou, W. Cai, Proactivecrowd: Modelling proactive steering behaviours for agent-based crowd simulation, Comput. Graph. Forum 37 (1) (2018) 375–388, http://dx.doi.org/10.1111/cgf.13303, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/cgf.13303.

[75] A. Best, S. Narang, S. Curtis, D. Manocha, Densesense: Interactive crowd simulation using density-dependent filters, in: Proceedings of the ACM SIGGRAPH/Eurographics Symposium on Computer Animation, SCA '14, Eurographics Association, Aire-la-Ville, Switzerland, Switzerland, 2014, pp. 97–102, URL http://dl.acm.org/citation.cfm?id=2849517.2849534.

[76] W.G. van Toll, A.F. Cook I.V., R. Geraerts, Real-time density-based crowd simulation, Comput. Animat. Virtual Worlds 23 (1) (2012) 59–69, http://dx.doi.org/10.1002/cav.1424, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/cav.1424.

[77] S. Liu, S. Lo, J. Ma, W. Wang, An agent-based microscopic pedestrian flow simulation model for pedestrian traffic problems, IEEE Trans. Intell. Transp. Syst. 15 (3) (2014) 992–1001, http://dx.doi.org/10.1109/TITS.2013.2292526.

[78] Q. Chao, Z. Deng, X. Jin, Vehicle–pedestrian interaction for mixed traffic simulation, Comput. Animat. Virtual Worlds 26 (3–4) (2015) 405–412, http://dx.doi.org/10.1002/cav.1654, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/cav.1654.

[79] L. Luo, S. Zhou, W. Cai, M.Y.H. Low, F. Tian, Y. Wang, X. Xiao, D. Chen, Agent-based human behavior modeling for crowd simulation, Comput. Animat. Virtual Worlds 19 (3–4) (2008) 271–281, http://dx.doi.org/10.1002/cav.238, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/cav.238.

[80] F. Lamarche, S. Donikian, Crowd of virtual humans: a new approach for real time navigation in complex and structured environments, Comput. Graph. Forum 23 (3) (2004) 509–518, http://dx.doi.org/10.1111/j.1467-8659.2004.00782.x, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1467-8659.2004.00782.x.

[81] J. Bruneau, J. Pettré, EACS: Effective avoidance combination strategy, Comput. Graph. Forum 36 (8) (2017) 108–122, http://dx.doi.org/10.1111/cgf.13066, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/cgf.13066.

[82] R. Kulpa, A.-H. Olivierxs, J. Ondřej, J. Pettré, Imperceptible relaxation of collision avoidance constraints in virtual crowds, ACM Trans. Graph. 30 (6) (2011) 138:1–138:10, http://dx.doi.org/10.1145/2070781.2024172, URL http://doi.acm.org/10.1145/2070781.2024172.

[83] J. Ondřej, J. Pettré, A.-H. Olivier, S. Donikian, A synthetic-vision based steering approach for crowd simulation, ACM Trans. Graph. 29 (4) (2010) 123:1–123:9, http://dx.doi.org/10.1145/1778765.1778860, URL http://doi.acm.org/10.1145/1778765.1778860.

[84] R. Hughes, J. Ondřej, J. Dingliana, DAVIS: Density-adaptive synthetic-vision based steering for virtual crowds, in: Proceedings of the 8th ACM SIGGRAPH Conference on Motion in Games, MIG '15, ACM, New York, USA, 2015, pp. 79–84, http://dx.doi.org/10.1145/2822013.2822030.

[85] T.B. Dutra, R. Marques, J. Cavalcante-Neto, C.A. Vidal, J. Pettré, Gradient-based steering for vision-based crowd simulation algorithms, Comput. Graph. Forum 36 (2) (2017) 337–348, http://dx.doi.org/10.1111/cgf.13130, arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.1111/cgf.13130.

[86] M. Moussaïd, D. Helbing, G. Theraulaz, How simple rules determine pedestrian behavior and crowd disasters, Proc. Natl. Acad. Sci. 108 (17) (2011) 6884–6888.

[87] A. López, F.c. Chaumette, E. Marchand, J. Pettré, Character navigation in dynamic environments based on optical flow, Comput. Graph. Forum 38 (2) (2019) 181–192, http://dx.doi.org/10.1111/cgf.13629, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/cgf.13629.

[88] R.L. Hughes, A continuum theory for the flow of pedestrians, Transp. Res. B 36 (6) (2002) 507–535, http://dx.doi.org/10.1016/S0191-2615(01)00015-7.

[89] A. Treuille, S. Cooper, Z. Popović, Continuum crowds, ACM Trans. Graph. 25 (3) (2006) 1160–1168, http://dx.doi.org/10.1145/1141911.1142008.

[90] H. Jiang, W. Xu, T. Mao, C. Li, S. Xia, Z. Wang, Continuum crowd simulation in complex environments, in: CAD/GRAPHICS 2009 Extended papers from the 2009 Sketch-Based Interfaces and Modeling Conference Vision, Modeling & Visualization, Comput. Graph. 34 (5) (2010) 537–544, http://dx.doi.org/10.1016/j.cag.2010.05.013.

[91] J. Sewall, D. Wilkie, P. Merrell, M.C. Lin, Continuum traffic simulation, Comput. Graph. Forum 29 (2) (2010) 439–448, http://dx.doi.org/10.1111/j.1467-8659.2009.01613.x, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1467-8659.2009.01613.x.

[92] A. Golas, R. Narain, M.C. Lin, Continuum modeling of crowd turbulence, Phys. Rev. E 90 (2014) 042816, http://dx.doi.org/10.1103/PhysRevE.90.042816.

[93] O. Oğuz, A.A. n, T.Y. lmaz, U. Güdükbay, Emergency crowd simulation for outdoor environments, Comput. Graph. 34 (2) (2010) 136–144, http://dx.doi.org/10.1016/j.cag.2009.12.004, URL http://www.sciencedirect.com/science/article/pii/S0097849309001563.

[94] R. Narain, A. Golas, S. Curtis, M.C. Lin, Aggregate dynamics for dense crowd simulation, ACM Trans. Graph. 28 (5) (2009) 122:1–122:8, http://dx.doi.org/10.1145/1618452.1618468.

[95] S. Chenney, Flow tiles, in: Proceedings of the 2004 ACM SIGGRAPH/Eurographics Symposium on Computer Animation, SCA '04, Eurographics Association, Goslar Germany, Germany, 2004, pp. 233–242, http://dx.doi.org/10.1145/1028523.1028553.

[96] S. Patil, J. van den Berg, S. Curtis, M.C. Lin, D. Manocha, Directing crowd simulations using navigation fields, IEEE Trans. Vis. Comput. Graphics 17 (2) (2011) 244–254, http://dx.doi.org/10.1109/TVCG.2010.33.

[97] A. Barnett, H.P.H. Shum, T. Komura, Coordinated crowd simulation with topological scene analysis, Comput. Graph. Forum 35 (6) (2016) 120–132, http://dx.doi.org/10.1111/cgf.12735, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/cgf.12735.

[98] G. Lu, L. Chen, W. Luo, Real-time crowd simulation integrating potential fields and agent method, ACM Trans. Model. Comput. Simul. 26 (4) (2016) 28:1–28:16, http://dx.doi.org/10.1145/2885496.

[99] T.-Y. Tsai, S.-K. Wong, Y.-H. Chou, G.-W. Lin, Directing virtual crowds based on dynamic adjustment of navigation fields, Comput. Animat. Virtual Worlds 29 (1) (2018) e1765, http://dx.doi.org/10.1002/cav.1765, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/cav.1765, e1765 cav.1765.

[100] X. Wang, X. Jin, Z. Deng, L. Zhou, Inherent noise-aware insect swarm simulation, Comput. Graph. Forum 33 (6) (2014) 51–62, http://dx.doi.org/10.1111/cgf.12277, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/cgf.12277.

[101] I. Karamouzas, M. Overmars, Simulating and evaluating the local behavior of small pedestrian groups, IEEE Trans. Vis. Comput. Graphics 18 (3) (2012) 394–406, http://dx.doi.org/10.1109/TVCG.2011.133.

[102] F.A. Rojas, H.S. Yang, Immersive human-in-the-loop HMD evaluation of dynamic group behavior in a pedestrian crowd simulation that uses group agent-based steering, in: Proceedings of the 12th ACM SIGGRAPH International Conference on Virtual-Reality Continuum and Its Applications in Industry, VRCAI '13, ACM, New York, USA, 2013, pp. 31–40, http://dx.doi.org/10.1145/2534329.2534336.

[103] L. He, J. Pan, S. Narang, D. Manocha, Dynamic group behaviors for interactive crowd simulation, in: Proceedings of the ACM SIGGRAPH/Eurographics Symposium on Computer Animation, SCA '16, Eurographics Association, Goslar Germany, Germany, 2016, pp. 139–147.

[104] N. Jaklin, A. Kremyzas, R. Geraerts, Adding sociality to virtual pedestrian groups, in: Proceedings of the 21st ACM Symposium on Virtual Reality Software and Technology, VRST '15, ACM, New York, USA, 2015, pp. 163–172, http://dx.doi.org/10.1145/2821592.2821597.

[105] J. Bruneau, A. Olivier, J. Pettré, Going through, going around: A study on individual avoidance of groups, IEEE Trans. Vis. Comput. Graphics 21 (4) (2015) 520–528, http://dx.doi.org/10.1109/TVCG.2015.2391862.

[106] Z. Ren, P. Charalambous, J. Bruneau, Q. Peng, J. Pettré, Group modeling: A unified velocity-based approach, Comput. Graph. Forum 36 (8) (2017) 45–56, http://dx.doi.org/10.1111/cgf.12993.

[107] R. Xie, Z. Yang, Y. Niu, Y. Zhang, Simulation of small social group behaviors in emergency evacuation, in: Proceedings of the 29th International Conference on Computer Animation and Social Agents, CASA '16, ACM, New York, USA, 2016, pp. 71–77, http://dx.doi.org/10.1145/2915926.2919325.

[108] Y. Li, H. Liu, G. peng Liu, L. Li, P. Moore, B. Hu, A grouping method based on grid density and relationship for crowd evacuation simulation, Physica A 473 (2017) 319–336, http://dx.doi.org/10.1016/j.physa.2017.01.008.

[109] F. Qiu, X. Hu, Modeling group structures in pedestrian crowd simulation, Simul. Model. Pract. Theory 18 (2) (2010) 190–205, http://dx.doi.org/10.1016/j.simpat.2009.10.005.

[110] S. Lemercier, J.-M. Auberlet, Towards more behaviours in crowd simulation, Comput. Animat. Virtual Worlds 27 (1) (2016) 24–34, http://dx.doi.org/10.1002/cav.1629.

[111] F. Yang, J. Shabo, A. Qureshi, C. Peters, Do you see groups?: The impact of crowd density and viewpoint on the perception of groups, in: Proceedings of the 18th International Conference on Intelligent Virtual Agents, IVA '18, ACM, New York, USA, 2018, pp. 313–318, http://dx.doi.org/10.1145/3267851.3267877.

[112] C. Peters, C. Ennis, Modeling groups of plausible virtual pedestrians, IEEE Comput. Graph. Appl. 29 (4) (2009) 54–63, http://dx.doi.org/10.1109/MCG.2009.69.

[113] S.I. Park, F. Quek, Y. Cao, Simulating and animating social dynamics: embedding small pedestrian groups in crowds, Comput. Animat. Virtual Worlds 24 (3–4) (2013) 155–164, http://dx.doi.org/10.1002/cav.1512, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/cav.1512.

[114] B. Ulicny, P.d.H. Ciechomski, D. Thalmann, Crowdbrush: Interactive authoring of real-time crowd scenes, in: Proceedings of the 2004 ACM SIGGRAPH/Eurographics Symposium on Computer Animation, SCA '04, Eurographics Association, Goslar Germany, Germany, 2004, pp. 243–252, http://dx.doi.org/10.1145/1028523.1028555.

[115] K.H. Lee, M.G. Choi, J. Lee, Motion patches: Buildings blocks for virtual environments annotated with motion data, in: ACM SIGGRAPH 2005 Sketches, SIGGRAPH '05, ACM, New York, USA, 2005, http://dx.doi.org/10.1145/1187112.1187169, URL http://doi.acm.org/10.1145/1187112.1187169.

[116] B. Yersin, J. Maïm, J. Pettré, D. Thalmann, Crowd patches: Populating large-scale virtual environments for real-time applications, in: Proceedings of the 2009 Symposium on Interactive 3D Graphics and Games, I3D '09, ACM, New York, USA, 2009, pp. 207–214, http://dx.doi.org/10.1145/1507149.1507184, URL http://doi.acm.org/10.1145/1507149.1507184.

[117] Y. Yuan, Z. Wu, M. Zhou, Populating virtual environments with crowd patches and rule-based method, in: Proceedings of the 11th ACM SIGGRAPH International Conference on Virtual-Reality Continuum and Its Applications in Industry, VRCAI '12, ACM, New York, USA, 2012, pp. 239–246, http://dx.doi.org/10.1145/2407516.2407574, URL http://doi.acm.org/10.1145/2407516.2407574.

[118] K. Jordao, J. Pettré, M. Christie, M.-P. Cani, Crowd sculpting: A space-time sculpting method for populating virtual environments, in: Computer Graphics Forum, Vol. 33, Wiley Online Library, 2014, pp. 351–360.

[119] A. Milliez, M. Wand, M.-P. Cani, H.-P. Seidel, Mutable elastic models for sculpting structured shapes, Comput. Graph. Forum 32 (2pt1) (2013) 21–30, http://dx.doi.org/10.1111/cgf.12022, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/cgf.12022.

[120] T. Kwon, K.H. Lee, J. Lee, S. Takahashi, Group motion editing, ACM Trans. Graph. 27 (3) (2008) 80:1–80:8, http://dx.doi.org/10.1145/1360612.1360679, URL http://doi.acm.org/10.1145/1360612.1360679.

[121] S. Takahashi, K. Yoshida, T. Kwon, K.H. Lee, J. Lee, S.Y. Shin, Spectral-based group formation control, Comput. Graph. Forum 28 (2) (2009) 639–648, http://dx.doi.org/10.1111/j.1467-8659.2009.01404.x.

[122] M. Kim, K. Hyun, J. Kim, J. Lee, Synchronized multi-character motion editing, ACM Trans. Graph. 28 (3) (2009) 79:1–79:9, http://dx.doi.org/10.1145/1531326.1531385.

[123] J. Kim, Y. Seol, T. Kwon, J. Lee, Interactive manipulation of large-scale crowd animation, ACM Trans. Graph. 33 (4) (2014) 83:1–83:10, http://dx.doi.org/10.1145/2601097.2601170.

[124] Q. Gu, Z. Deng, Formation sketching: An approach to stylize groups in crowd simulation, in: Proceedings of Graphics Interface 2011, GI '11, Canadian Human-Computer Communications Society, School of Computer Science, University of Waterloo, Waterloo, Ontario, Canada, 2011, pp. 1–8.

[125] Q. Gu, Z. Deng, Generating freestyle group formations in agent-based crowd simulations, IEEE Comput. Graph. Appl. 33 (1) (2013) 20–31, http://dx.doi.org/10.1109/MCG.2011.87.

[126] M. Xu, Y. Wu, Y. Ye, I. Farkas, H. Jiang, Z. Deng, Collective crowd formation transform with mutual information–based runtime feedback, Comput. Graph. Forum 34 (1) (2015) 60–73, http://dx.doi.org/10.1111/cgf.12459.

[127] J. Henry, H.P.H. Shum, T. Komura, Environment-aware real-time crowd control, in: Proceedings of the ACM SIGGRAPH/Eurographics Symposium on Computer Animation, SCA '12, Eurographics Association, Goslar Germany, Germany, 2012, pp. 193–200.

[128] J. Henry, H.P.H. Shum, T. Komura, Interactive formation control in complex environments, IEEE Trans. Vis. Comput. Graphics 20 (2) (2014) 211–222, http://dx.doi.org/10.1109/TVCG.2013.116.

[129] A. Shoulson, N. Marshak, M. Kapadia, N.I. Badler, ADAPT: The agent developmentand prototyping testbed, IEEE Trans. Vis. Comput. Graphics 20 (7) (2014) 1035–1047, http://dx.doi.org/10.1109/TVCG.2013.251.

[130] L. Zheng, J. Zhao, Y. Cheng, H. Chen, X. Liu, W. Wang, Geometry-constrained crowd formation animation, Comput. Graph. 38 (2014) 268–276, http://dx.doi.org/10.1016/j.cag.2013.10.035.

[131] J. Xu, X. Jin, Y. Yu, T. Shen, M. Zhou, Shape-constrained flock animation, Comput. Animat. Virtual Worlds 19 (3–4) (2008) 319–330, http://dx.doi.org/10.1002/cav.231, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/cav.231.

[132] L.R. Montana, S. Maddock, Sketching for real-time control of crowd simulations, in: Proceedings of the Conference on Computer Graphics & Visual Computing, CGVC '17, Eurographics Association, Goslar Germany, Germany, 2017, pp. 81–88, http://dx.doi.org/10.2312/cgvc.20171282.

[133] P. Zhang, H. Liu, Y.-h. Ding, Crowd simulation based on constrained and controlled group formation, Vis. Comput. 31 (1) (2015) 5–18, http://dx.doi.org/10.1007/s00371-013-0900-7.

[134] T. Sakuma, T. Mukai, S. Kuriyama, Psychological model for animating crowded pedestrians, Comput. Animat. Virtual Worlds 16 (3–4) (2005) 343–351, http://dx.doi.org/10.1002/cav.105, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/cav.105.

[135] T. Bosse, R. Duell, Z.A. Memon, J. Treur, C.N. van der Wal, Agent-based modeling of emotion contagion in groups, Cogn. Comput. 7 (1) (2015) 111–136, http://dx.doi.org/10.1007/s12559-014-9277-9.

[136] J. Tsai, N. Fridman, E. Bowring, M. Brown, S. Epstein, G. Kaminka, S. Marsella, A. Ogden, I. Rika, A. Sheel, M.E. Taylor, X. Wang, A. Zilka, M. Tambe, ESCAPES: Evacuation simulation with children, authorities, parents, emotions, and social comparison, in: The 10th International Conference on Autonomous Agents and Multiagent Systems - Volume 2, AAMAS '11, International Foundation for Autonomous Agents and Multiagent Systems, Richland, SC, 2011, pp. 457–464.

[137] S.J. Guy, S. Kim, M.C. Lin, D. Manocha, Simulating heterogeneous crowd behaviors using personality trait theory, in: Proceedings of the 2011 ACM SIGGRAPH/Eurographics Symposium on Computer Animation, SCA '11, ACM, New York, USA, 2011, pp. 43–52, http://dx.doi.org/10.1145/2019406.2019413.

[138] S. Kim, S.J. Guy, D. Manocha, M.C. Lin, Interactive simulation of dynamic crowd behaviors using general adaptation syndrome theory, in: Proceedings of the ACM SIGGRAPH Symposium on Interactive 3D Graphics and Games, I3D '12, ACM, New York, USA, 2012, pp. 55–62, http://dx.doi.org/10.1145/2159616.2159626, URL http://doi.acm.org/10.1145/2159616.2159626.

[139] F. Durupinar, N. Pelechano, J. Allbeck, U. Gudukbay, N.I. Badler, How the ocean personality model affects the perception of crowds, IEEE Comput. Graph. Appl. 31 (3) (2011) 22–31, http://dx.doi.org/10.1109/MCG.2009.105.

[140] F. Durupınar, U. Güdükbay, A. Aman, N.I. Badler, Psychological parameters for crowd simulation: From audiences to mobs, IEEE Trans. Vis. Comput. Graphics 22 (9) (2016) 2145–2159, http://dx.doi.org/10.1109/TVCG.2015.2501801.

[141] M. Xu, C. Li, P. Lv, W. Chen, Z. Deng, B. Zhou, D. Manocha, A unified crowd simulation model revealing relationships among" physiology-psychology-physics" factors, 2017, arXiv preprint arXiv:1801.00216.

[142] M. Xu, C. Li, P. Lv, W. Chen, D. Manocha, Z. Deng, B. Zhou, Crowd simulation model integrating "Physiology-Psychology-Physics" factors, 2018, CoRR.

[143] H. Jiang, Z. Deng, M. Xu, X. He, T. Mao, Z. Wang, An emotion evolution based model for collective behavior simulation, in: Proceedings of the ACM SIGGRAPH Symposium on Interactive 3D Graphics and Games, I3D '18, ACM, New York, USA, 2018, pp. 10:1–10:6, http://dx.doi.org/10.1145/3190834.3190844.

[144] M. Xu, X. Xie, P. Lv, J. Niu, H. Wang, C. Li, R. Zhu, Z. Deng, B. Zhou, Crowd behavior simulation with emotional contagion in unexpected multihazard situations, IEEE Transactions on Systems, Man, and Cybernetics: Systems (2019) 1–15, http://dx.doi.org/10.1109/TSMC.2019.2899047.

[145] L. Fu, W. Song, W. Lv, S. Lo, Simulation of emotional contagion using modified SIR model: A cellular automaton approach, Physica A 405 (2014) 380–391, http://dx.doi.org/10.1016/j.physa.2014.03.043, URL http://www.sciencedirect.com/science/article/pii/S0378437114002386.

[146] Y. Mao, Z. Li, Y. Li, W. He, Emotion-based diversity crowd behavior simulation in public emergency, Vis. Comput. (2018) http://dx.doi.org/10.1007/s00371-018-1568-9.

[147] Y. Mao, S. Yang, Z. Li, Y. Li, Personality trait and group emotion contagion based crowd simulation for emergency evacuation, Multimedia Tools Appl. (2018) http://dx.doi.org/10.1007/s11042-018-6069-3.

[148] G. Zhang, D. Lu, H. Liu, Strategies to utilize the positive emotional contagion optimally in crowd evacuation, IEEE Trans. Affect. Comput. (2018) 1, http://dx.doi.org/10.1109/TAFFC.2018.2836462.

[149] P. Lv, Z. Zhang, C. Li, Y. Guo, B. Zhou, M. Xu, Crowd behavior evolution with emotional contagion in political rallies, IEEE Trans. Comput. Soc. Syst. 6 (2) (2019) 377–386, http://dx.doi.org/10.1109/TCSS.2018.2878461.

[150] J. Pettré, D. Wolinski, A.-H. Olivier, Velocity-based models for crowd simulation, in: U. Weidmann, U. Kirsch, M. Schreckenberg (Eds.), Pedestrian and Evacuation Dynamics 2012, Springer International Publishing, Cham, 2014, pp. 1065–1078.

[151] J.S. Wiggins, The Five-Factor Model of Personality: Theoretical Perspectives, Guilford Press, 1996.

[152] H.J. Eysenck, Dimensions of Personality, Vol. 5, Transaction Publishers, 1950.

[153] S.B.G. Eysenck, H.J. Eysenck, The place of impulsiveness in a dimensional system of personality description, Br. J. Soc. Clin. Psychol. 16 (1) (1977) 57–68, http://dx.doi.org/10.1111/j.2044-8260.1977.tb01003.x.

[154] A. Ortony, G.L. Clore, A. Collins, The Cognitive Structure of Emotions, Cambridge University Press, 1988, http://dx.doi.org/10.1017/CBO9780511571299.

[155] A. Mehrabian, Pleasure-arousal-dominance: A general framework for describing and measuring individual differences in temperament, Curr. Psychol. 14 (4) (1996) 261–292, http://dx.doi.org/10.1007/BF02686918.

[156] D. Wolinski, M.C. Lin, J. Pettré, Warpdriver: Context-aware probabilistic motion prediction for crowd simulation, ACM Trans. Graph. 35 (6) (2016) 164:1–164:11, http://dx.doi.org/10.1145/2980179.2982442, URL http://doi.acm.org/10.1145/2980179.2982442.

[157] K.L. Dion, Group cohesion: From "field of forces" to multidimensional construct, Group Dyn.: Theory Res. Pract. 4 (1) (2000) 7.

[158] K. Lewin, The research center for group dynamics at massachusetts institute of technology, Sociometry 8 (2) (1945) 126–136.

[159] G. Berseth, M. Kapadia, B. Haworth, P. Faloutsos, Steerfit: Automated parameter fitting for steering algorithms, in: Proceedings of the ACM SIGGRAPH/Eurographics Symposium on Computer Animation, SCA '14, Eurographics Association, Goslar, DEU, 2015, pp. 113–122.

[160] N. Hu, J. Zhong, J.T. Zhou, S. Zhou, W. Cai, C. Monterola, Guide them through: An automatic crowd control framework using multi-objective genetic programming, Appl. Soft Comput. 66 (2018) 90–103, http://dx.doi.org/10.1016/j.asoc.2018.01.037, URL http://www.sciencedirect.com/science/article/pii/S1568494618300437.

[161] X. Wei, W. Lu, L. Zhu, W. Xing, Learning motion rules from real data: Neural network for crowd simulation, Neurocomputing 310 (2018) 125–134, http://dx.doi.org/10.1016/j.neucom.2018.05.022, URL http://www.sciencedirect.com/science/article/pii/S0925231218305599.

[162] A. Bera, S. Kim, D. Manocha, Online parameter learning for data-driven crowd simulation and content generation, Comput. Graph. 55 (2016) 68–79, http://dx.doi.org/10.1016/j.cag.2015.10.009, URL http://www.sciencedirect.com/science/article/pii/S0097849315001703.

[163] J.L. Vermeulen, A. Hillebrand, R. Geraerts, A comparative study of k-nearest neighbour techniques in crowd simulation, Comput. Animat. Virtual Worlds 28 (3–4) (2017) e1775, http://dx.doi.org/10.1002/cav.1775, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/cav.1775, e1775 cav.1775.

[164] M. Oshita, Agent navigation using deep learning with agent space heat map for crowd simulation, Comput. Animat. Virtual Worlds 30 (3–4) (2019) e1878, http://dx.doi.org/10.1002/cav.1878, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/cav.1878, e1878 cav.1878.

[165] X.B. Peng, G. Berseth, K. Yin, M. Van De Panne, DeepLoco: Dynamic locomotion skills using hierarchical deep reinforcement learning, ACM Trans. Graph. 36 (4) (2017) http://dx.doi.org/10.1145/3072959.3073602.

[166] X. Zhao, L. Lyu, C. Lyu, C. Ji, A new parallel simulation method for massive crowd, in: 2018 International Conference on Identification, Information and Knowledge in the Internet of Things, Procedia Comput. Sci. 147 (2019) 283–287, http://dx.doi.org/10.1016/j.procs.2019.01.265, URL http://www.sciencedirect.com/science/article/pii/S1877050919302881.

[167] A. Malinowski, P. Czarnul, Multi-agent large-scale parallel crowd simulation with NVRAM-based distributed cache, J. Comput. Sci. 33 (2019) 83–94, http://dx.doi.org/10.1016/j.jocs.2019.04.004, URL http://www.sciencedirect.com/science/article/pii/S1877750318307221.