ANALYSIS AND SIMULATION OF THE TARGET-ATTACKER AND THE TARGET-ATTACKER-DEFENDER PROBLEMS_Results and Discussion

By Mostafa Ali Rushdi

WORD COUNT	2778	TIME SUBMITTED	19-AUG-2017 03:39AM
		PAPER ID	31178437

Chapter 8: OPTIMAL HEADING ANGLE SIMULATION

In this chapter, we address the active target defense differential game where an Attacker missile pursues a Target aircraft. A Defender missile is fired by the Targets wingman in order to intercept the Attacker before it reaches the aircraft. Thus, a team is formed by the Target and the Defender which cooperate to maximize the distance between the Target aircraft and the point where the Attacker missile is intercepted by the Defender missile, while the Attacker tries to minimize said distance. The results shown here extend previous work. We consider here the case where the Defender is faster than the Attacker. The solution to this differential game provides optimal heading angles for the Target and the Defender team to maximize the terminal separation between Target and Attacker and it also provides the optimal heading angle for the Attacker to minimize the said distance.

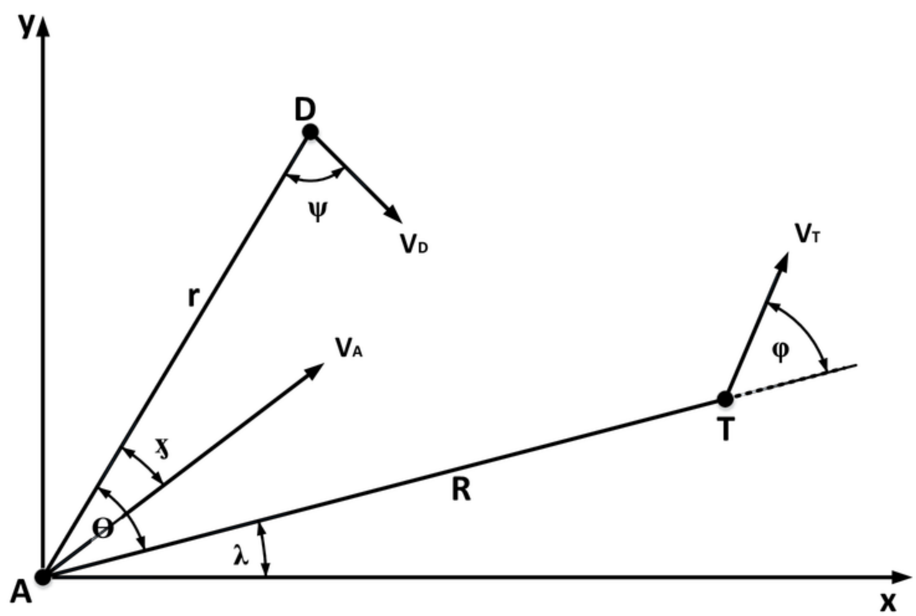

Figure 8.1: Scenario for the Attacker (A), Target (T), and Defender (D). The origin is arbitrary situated at the Attacker position (A).

Figure 8.1 illustrates the scenario for the TAD game. The variables R and r are the separations between the Target and Attacker, respectively, and hence denote the radial polar coordinates of the Target and Defender when the origin of coordinates is arbitrarily situated at the position of the Attacker, The speeds of the Attacker, Target and Defender are denoted by $\boldsymbol{V_A}$, $\boldsymbol{V_T}$, $\boldsymbol{V_D}$ with corresponding magnitudes V_A, V_T and V_D, and polar angles

$$Ang\ \boldsymbol{V_A} = \hat{\chi} = \lambda + \theta - \chi \tag{8.1}$$

$$Ang\ \boldsymbol{V_T} = \hat{\phi} = \lambda + \phi \tag{8.2}$$

$$Ang\ \boldsymbol{V_D} = \hat{\psi} = \lambda + \theta - (\pi - \psi) \tag{8.3}$$

The derivatives $\dot{R}$ and $\dot{r}$ can be obtained as differences in radial speeds, while $\dot{\theta}$ can be obtained in terms of the difference in circumferential speeds. The end points of the distance R are moving at a speed V_T at an angle ϕ with respect to the radial directions and a speed V_A at an angle $(\theta - \chi)$ with respect to the radial directions. Hence;

$$\begin{aligned} \dot{R} &= V_t \cos\phi - V_A \cos(\theta - \chi) \\ &= V_A(\alpha \cos\phi - \cos(\theta - \chi)) \end{aligned} \tag{8.4}$$

where $\alpha = \frac{V_t}{V_A}$ Similarly, we obtain

$$\begin{aligned} \dot{r} &= -V_A \cos\chi - V_D \cos\psi \\ &= V_A(-\cos\chi - \gamma \cos\psi) \end{aligned} \tag{8.5}$$

where $\gamma = \frac{V_D}{V_A}$ In the following, we will deliberately differ from Garcia et.al. [47] We will not use <u>reduced</u> equations in which $V_A = V_D = 1$. Our equations will look dimensionally homogeneous, and we will allow $\gamma = \dfrac{V_D}{V_A}$ to differ from 1.

The circumferential speeds are

$$\begin{aligned} R\dot{\lambda} &= V_T \sin\phi - V_A \sin(\theta - \chi) \\ \dot{\lambda} &= V_A[\frac{\alpha}{R}\sin\phi - \frac{1}{R}\sin(\theta - \chi)] \\ r(\theta + \lambda)^{\cdot} &= -V_D \sin\psi + V_A \sin\chi \end{aligned}$$

Hence, one obtains

$$\dot{\lambda} = V_A[\frac{\alpha}{R}\sin\phi - \frac{1}{R}\sin(\theta - \chi)] \tag{8.6}$$

$$\dot{\theta} + \dot{\lambda} = V_A[-\frac{\gamma}{r}\sin\psi + \frac{1}{r}\sin\chi] \tag{8.7}$$

Now, subtract (8.7) minus (8.6) to obtain

$$\dot{\theta} = V_A[-\frac{\alpha}{R}\sin\phi + \frac{1}{R}\sin(\theta - \chi) - \frac{\gamma}{r}\sin\psi + \frac{1}{r}\sin\chi] \tag{8.8}$$

Equations (8.4), (8.5), (8.8) reduce to equations (8.6) of Garcia et.al [47] when we set both V_A and $\gamma = \dfrac{V_D}{V_A}$ equal to 1.

The system dynamics are given by (8.4), (8.5), and (8.8) for $0 < t < t_f$, together with the initial conditions

$$R(t_0) = R_0,$$
$$r(t_0) = r_0,$$
$$\theta(t_0) = \theta_0,$$

The objective of the Target-Defender team is to maximize the separation between the Target and the Attacker at the interception time $R(t_f)$, where the terminal time t_f is free, such that $r(t_f) = r_c$. The objective of the Attacker is to minimize the same distance $R(t_f)$. This can be expressed as

$$max_{\phi,\psi}\ min_{\chi}\ J = \int_{t_0}^{t_f} \dot{R}\,dt$$

The Hamiltonian is

$$\begin{aligned} H = &\cos(\theta-\chi) - \alpha\cos\phi \\ &+ [\alpha\cos\phi - \cos(\theta-\chi)]\,\lambda_R \\ &- [\cos\chi + \cos\psi]\,\lambda_r \\ &+ [-\frac{\alpha}{R}\sin\phi + \frac{1}{R}\sin(\theta-\chi) - \frac{1}{r}\sin\psi + \frac{1}{r}\sin\chi]\,\lambda_\theta \end{aligned} \tag{8.9}$$

$$\begin{aligned} H = &-(1-\lambda_R)[\alpha\cos\phi - \cos(\theta-\chi)] \\ &- [\cos\chi + \beta\cos\psi]\,\lambda_R \\ &+ [-\frac{\alpha}{R}\sin\phi + \frac{1}{R}\sin(\theta-\chi) - \frac{\beta}{r}\sin\psi + \frac{1}{r}\sin\chi]\,\lambda_\theta \end{aligned}$$

the co-state equations

$$-\frac{\partial H}{\partial R} = \dot{\lambda}_R = \frac{\lambda_\theta}{R^2}(\sin(\theta-\chi) - \alpha\sin(\phi)) \tag{8.10}$$

$$-\frac{\partial H}{\partial r} = \dot{\lambda}_r = \frac{\lambda_\theta}{r^2}(\sin(\chi) - \alpha\sin(\psi)) \tag{8.11}$$

$$-\frac{\partial H}{\partial \theta} = \dot{\lambda}_\theta = (1-\lambda_R)\sin(\theta-\chi) - \frac{\lambda_\theta}{R}\cos(\theta-\chi) \tag{8.12}$$

the terminal conditions

$$\begin{aligned} r(t_f) &= r_c \\ \lambda_R(t_f) &= 0 \\ \lambda_\theta(t_f) &= 0 \\ \alpha^2 + 2(\alpha + \cos\theta(t_f))\lambda_R(t_f) - 1 &= 0 \end{aligned} \tag{8.13}$$

We now find the optimal heading angles ψ^*, ϕ^* and χ^*. Since these angles are known to be positive or negative angles (ranging from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$), it suffices to determine the tangent

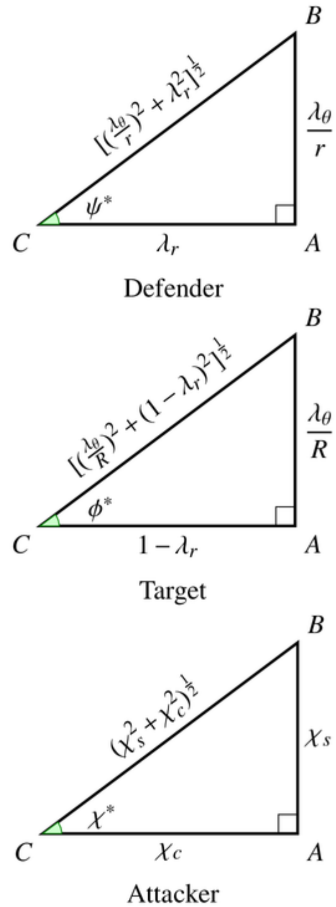

Figure 8.2: Right-angled triangles that define the optimal headings ψ^*, ϕ^* and χ^* for the Defender, Target, and Attacker, respectively

$\tan a$ of each angle a and then determine the cosine and sine from

$$\cos a = [1 + \tan^2 a]^{\frac{-1}{2}} \tag{8.14}$$

$$\sin a = \tan a[1 + \tan^2 a]^{\frac{-1}{2}} \tag{8.15}$$

We obtain ψ^* by partially differentiating the Hamiltonian (8.9) w.r.t. ψ and equating the derivative $\frac{\partial H}{\partial \psi}$ to zero, namely

$$\frac{\partial H}{\partial \psi} = \beta \lambda_r \sin\psi - \frac{\beta}{r}\lambda_\theta \cos\psi = 0 \tag{8.16}$$

Hence the optimal heading ψ^* is given by:

$$\tan\psi^* = \frac{\lambda_\theta / r}{\lambda_r} \tag{8.17}$$

Figure 8.2 shows that ψ^* is a (positive or negative) acute angle in a right-angled triangle with an opposite side equal to (λ_θ/r), an adjacent side equal to λ_r and a hypotenuse equal to $[(\frac{\lambda_\theta}{r})^2+\lambda_r^2]^{\frac{1}{2}}$. Using Fig.8.2 or relations (8.14) and (8.15) one obtains

$$\cos\psi^* = \lambda_r[(\frac{\lambda_\theta}{r})^2+\lambda_r^2]^{\frac{-1}{2}} = \frac{r\lambda_r}{\sqrt{\lambda_\theta^2+r^2\lambda_r^2}} \tag{8.18}$$

$$\sin\psi^* = (\frac{\lambda_\theta}{r})[(\frac{\lambda_\theta}{r})^2+\lambda_r^2]^{\frac{-1}{2}} = \frac{\lambda_\theta}{\sqrt{\lambda_\theta^2+r^2\lambda_r^2}} \tag{8.19}$$

We further compute the second partial derivative of the Hamiltonian w.r.t. ψ, that is

$$\begin{aligned}\frac{\partial^2 H}{\partial\psi^2} &= \beta\lambda_r\cos\psi+\beta\frac{\lambda_\theta}{r}\sin\psi \\ &= [\beta r\lambda_r^2+\beta\frac{\lambda_\theta^2}{r}][\lambda_\theta^2+r^2\lambda_r^2]^{\frac{-1}{2}} > 0\end{aligned} \tag{8.20}$$

The fact that $\frac{\partial^2 H}{\partial\psi^2} > 0$ means that the extremum obtained by (8.16) is a minimum, i.e., the optimal value ψ^* minimizes the cost $(-J)$.

We now obtain ϕ^* by partially differentiating Hamiltonian (8.9) w.r.t. ϕ and equating the partial derivative $\frac{\partial H}{\partial\phi}$ to zero, namely

$$\frac{\partial H}{\partial\phi} = \alpha(1-\lambda_R)\sin\phi-\frac{\alpha}{R}\lambda_\theta\cos\phi = 0 \tag{8.21}$$

Hence the optimal heading ϕ^* is given by

$$\tan\phi^* = \frac{(\lambda_\theta/R)}{1-\lambda_R} \tag{8.22}$$

Figure 8.2 shows that ϕ^* is a positive or negative acute angle in a right-angled triangle with an opposing side of length (λ_θ/R), an adjacent side of length $(1-\lambda_R)$, and a hypotenuse of length $[(\lambda_\theta/R)^2+(1-\lambda_R)^2]^{\frac{1}{2}}$.

Using Fig.8.2 or relations (8.14) and (8.15), we obtain

$$\cos\phi^* = (1-\lambda_R)[(\lambda_\theta/R)^2+(1-\lambda_R)^2]^{\frac{-1}{2}} = \frac{R(1-\lambda_R)}{\sqrt{\lambda_\theta^2+R^2(1-\lambda_R)^2}} \tag{8.23}$$

$$\sin\phi^* = (\lambda_\theta/R)[(\lambda_\theta/R)^2+(1-\lambda_R)^2]^{\frac{-1}{2}} = \frac{\lambda_\theta}{\sqrt{\lambda_\theta^2+R^2(1-\lambda_R)^2}} \tag{8.24}$$

The second partial derivative of the Hamiltonian w.r.t. ϕ is given by

$$\begin{aligned}\frac{\partial^2 H}{\partial\phi^2} &= \alpha(1-\lambda_R)\cos\phi+\frac{\alpha}{R}\lambda_\theta\sin\phi \\ &= [\alpha R(1-\lambda_R)^2+\frac{\alpha}{R}\lambda_\theta^2][\lambda_\theta^2+R^2(1-\lambda_R)^2]^{\frac{-1}{2}} > 0\end{aligned} \tag{8.25}$$

The fact that $\frac{\partial^2 H}{\partial \phi^2} > 0$ means that the extrema obtained by (8.21) is minimum, i.e., the optimal heading ϕ^* minimizes the cost $(-J)$.

Finally, we obtain the optimal heading χ^* by partially differentiating the Hamiltonian (8.9) w.r.t. χ and equating the partial derivative $\frac{\partial H}{\partial \chi}$ to zero, namely

$$\frac{\partial H}{\partial \chi} = (1-\lambda_R)\sin(\theta-\chi) + \lambda_r \sin\chi - \frac{\lambda_\theta}{R}\cos(\theta-\chi) + \frac{\lambda_\theta}{r}\cos\chi = 0 \tag{8.26}$$

Using the trigonometric expansions:

$$\sin(\theta-\chi) = \sin\theta\ \cos\chi - \cos\theta\ \sin\chi \tag{8.27}$$

$$\cos(\theta-\chi) = \cos\theta\ \cos\chi + \sin\theta\ \sin\chi \tag{8.28}$$

We can rewrite (8.26) in the form

$$\frac{\partial H}{\partial \chi} = \chi_s \cos\chi - \chi_c \sin\chi = 0 \tag{8.29}$$

where

$$\chi_s = (1-\lambda_R)\sin\theta - \frac{\lambda_\theta}{R}\cos\theta + \frac{\lambda_\theta}{r} \tag{8.30}$$

$$\chi_c = (1-\lambda_R)\cos\theta + \frac{\lambda_\theta}{R}\sin\theta - \lambda_r \tag{8.31}$$

The optimal heading χ^* is obtained from (8.29) as

$$\tan\chi^* = \frac{\chi_s}{\chi_c} \tag{8.32}$$

Figure 8.2 shows that χ^* is a positive or negative acute angle in a right-angled triangle with an opposing leg of length χ_s, an adjacent leg of length χ_c, and a hypotenuse of length $[\chi_s^2+\chi_c^2]^{\frac{1}{2}}$. Using Fig. 8.2 or relations (8.14) and (8.15), we obtain

$$\cos\chi^* = \chi_c\ [\chi_s^2+\chi_c^2]^{\frac{-1}{2}} = \frac{\chi_c}{\sqrt{\chi_s^2+\chi_c^2}} \tag{8.33}$$

$$\sin\chi^* = \chi_s\ [\chi_s^2+\chi_c^2]^{\frac{-1}{2}} = \frac{\chi_s}{\sqrt{\chi_s^2+\chi_c^2}} \tag{8.34}$$

The second derivative of the Hamiltonian w.r.t. χ is obtained from (8.29) as

$$\begin{aligned} \frac{\partial^2 H}{\partial \chi^2} &= -\chi_s \sin\chi - \chi_c \cos\chi \\ &= -[\chi_s^2+\chi_c^2](\chi_s^2+\chi_c^2)^{\frac{-1}{2}} \\ &= -(\chi_s^2+\chi_c^2)^{\frac{1}{2}} > 0 \end{aligned} \tag{8.35}$$

Equation (8.35) demonstrates definitely that $\frac{\partial^2 H}{\partial \chi^2}$ is negative. The corresponding claim via

Eq.(28) of Garcia et. al. [47] is not complete. The fact that $\frac{\partial^2 H}{\partial \chi^2} < 0$ means that the extremum obtained via (8.29) is a maximum, i.e., the optimal heading χ^* maximizes the cost $(-J)$.

In passing, we note that (8.26) can be simplified via (8.18), (8.19), (8.23) and (8.24) to give:

$$\begin{aligned}\frac{\partial H}{\partial \chi} &= [(\frac{\lambda_\theta}{R})^2 + (1-\lambda_R)^2]^{\frac{1}{2}}[\cos\phi\sin(\theta-\chi) - \sin\phi\cos(\theta-\chi)] + [(\frac{\lambda_\theta}{r})^2 + \lambda_r^2]^{\frac{1}{2}}[\cos\psi\sin\chi + \sin\psi\cos\chi] \\ &= [(\frac{\lambda_\theta}{R})^2 + (1-\lambda_R)^2]^{\frac{1}{2}}\sin(\theta-\chi-\phi) + [(\frac{\lambda_\theta}{r})^2 + \lambda_r^2]^{\frac{1}{2}}\sin(\chi+\psi) = 0\end{aligned} \tag{8.36}$$

The expressions for the optimal heading angles (8.17), (8.22), and (8.32) are to be used in the numerical solution of the Two-Point Boundary Value Problem (TPBVP). That solution is found by substituting the optimal headings into the state equations (8.4, 8.5, 8.8), and the co-state equations (8.10, 8.11, 8.12) with the terminal conditions (8.13). We solve this TPBVP using the "bvp4c" which is a matlab function for solving Two-Point Boundary Value Problems. This method may not be the best way to solve optimal control problems. The results include some error and the final time isn't correct and exceed t_f, so the trajectories in Fig. 8.5 didn't stop at the correct time and the shape is incorrect.

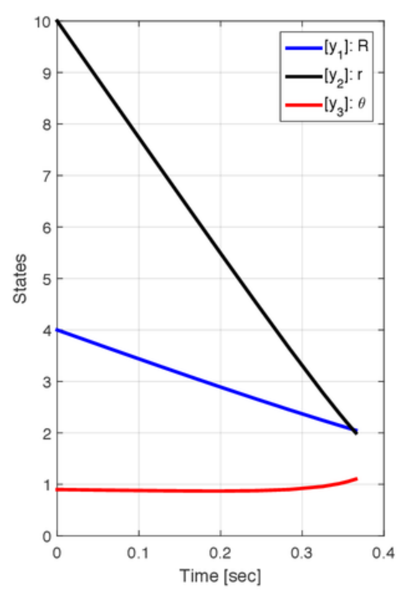

Figure 8.3: States

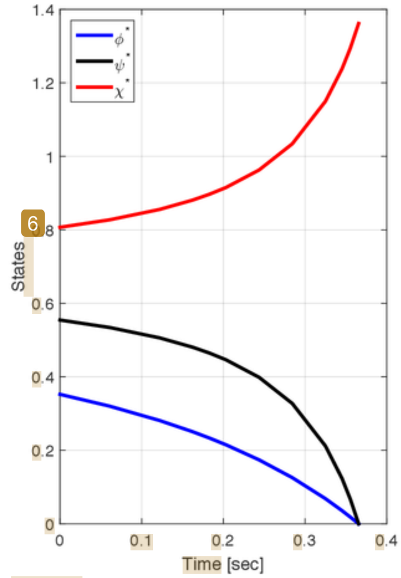

Figure 8.4: Optimal heading angles

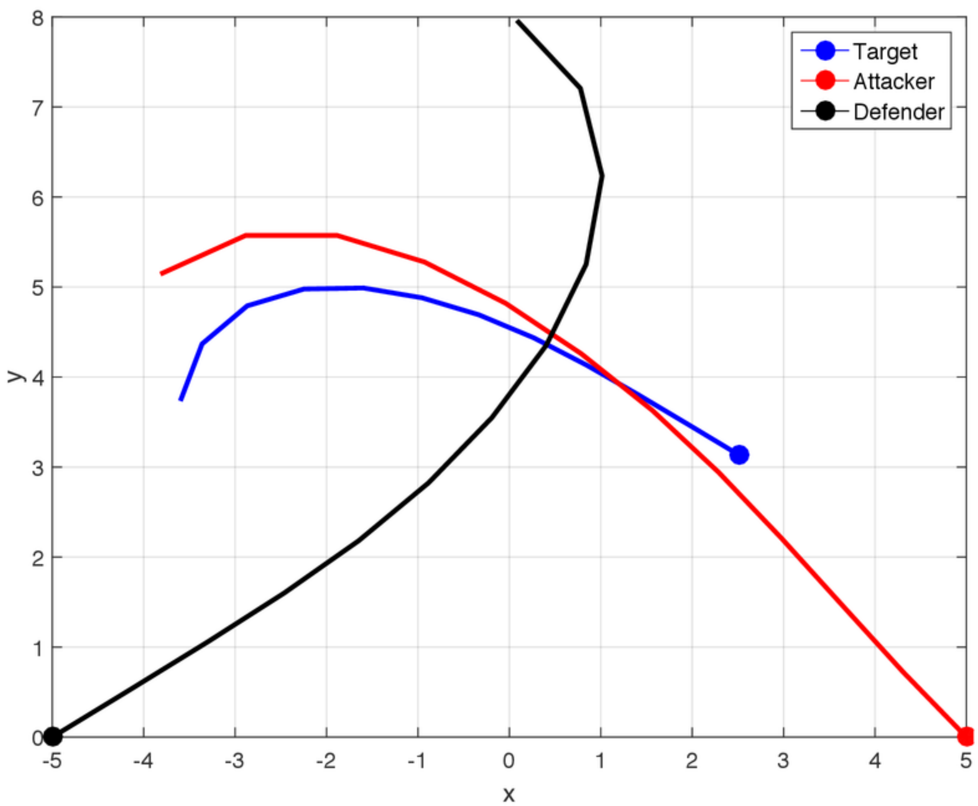

Figure 8.5: Optimal trajectories

Part IV

CONCLUSION AND FUTURE WORK

Chapter 9: CONCLUSION AND FUTURE WORK

9.1 TA problem

In this part, we search for a path that the Target can move on to escape from the Attacker. All the evasion techniques depend on the time of the turn that the Target makes when it detects the Attacker (Missile) and the objective is to maximize the Missile acceleration till the Missile power bleed. We choose the escaping trajectory as a polynomial with unknown coefficients, then decide the values of these coefficients so as to make the Missile exert a maximum acceleration to bleed its power as fast as possible before it reaches the Target. Optimization is achieved via the techniques of Monte-Carlo simulation and genetic algorithms. We establish a Graphic User Interface (GUI) Guidance toolbox containing our guidance law and several types of maneuvers. This toolbox is an open-source program for the development and addition of other guidance laws and maneuvers. So anybody can add some guidance laws or maneuvers as a future work. Then A novel paradigm of games played by humans versus computers is introduced and exemplified by a game handling the sophisticated and complex task of solving our classical pursuit-evasion problem that has many automated algorithmic solutions based on mathematically-elaborate game-theoretic techniques. The proposed strategy requests a small number of ordinary people to play a computer game, which is constructed to result in a correct solution and (as incentive to players) is enjoyable. The preliminary results obtained via our first game version Evasion-1 are reasonable and encouraging. In fact, human players unknowingly obtained a target acceleration of a Barrel-Roll shape, one of the best known policies for practical periodic maneuvers. The results suggest the production of enhanced versions of the game with more desirable outcomes. Specifically, in our next version (Evasion-2), the plane will no longer be destined to be intercepted by the missile, since we will impose the restriction that the missile has a limited reservoir of fuel that could be drained as a result of clever target maneuver. The ultimate goal is to test human brain capability to work collectively to produce a new guidance law which could possibly compete in accuracy with the existing ones, while surpassing them in the simplicity of the way it is produced.

9.2 TAD problem

This part offered a unified analytic solution of the TAD problem in which an Attacker is pursuing a Target while attempting to evade a Defender. The thesis reviews and extends the work that has recently appeared in [47, 48, 49], beside making the following new contributions:

1. The thesis covers all cases for the ratio γ of the Attacker's speed w.r.t the Defender's speed. It treats the case of a slow Defender $\gamma > 1$ for the first time, and presents this case along with the case of a fast Defender ($\gamma < 1$) discussed in [48] and the case of a similar Defender ($\gamma = 1$), discussed earlier in [47, 49].

2. The thesis demonstrates a striking increase in complexity when $\gamma \neq 1$ compared with the case $\gamma = 1$. It also demonstrates some sort of *duality* between the two cases of $(\gamma < 1)$ and $(\gamma > 1)$.

3. The thesis develops novel analytic expressions for the Voronoi diagrams for bordering the escape regions when $(\gamma < 1)$ or $(\gamma > 1)$. These expressions are more complex than the ones obtained in [3] for $(\gamma = 1)$, and reduce to it as a limiting case.

4. The thesis offers a tutorial exposition of the TAD problem, uses simple arguments of plane geometry to develop the necessary Apollonius circle, utilizes equalities rather than inequalities in developing Voronoi diagram, and pays careful attention to the inadvertent inclusion of extraneous solutions so as to justify their subsequent rejection.

5. The thesis supplements its analysis with extensive computations for the critical speed ratio, Voronoi diagrams, and the optimal interception points. The results obtained encompass all possible values of γ, and they reduce to the already available results for $\gamma = 1$. Results for the trajectories and optimal interception points obtained agree with those obtained by the numerical solution of a two-point boundary value problem (TPBVP) utilizing Pontryagin's Maximum Principle [48].

Some possible extensions of the current work that warrant further exploration include:

1. Further analysis of the quartic equation obtained for the Voronoi diagram when $\gamma \neq 1$, with an aim to *split* it into two factors representing the rejected and accepted branches of the diagram.

2. Investigation of the sixth-degree complex polynomial equation for the optimal interception angle to get some *insight* about its six roots, and to find a better way for *selecting* desirable root.

3. Relaxation of some of the assumption used in this study. In particular, it is very interesting to consider the possibility of *variable* rather than constant speeds the three agents.

4. Addition of an element of *uncertainty* to the computation. For example, we might assume that the initial positions of the three agents are not deterministic but *stochastic* or *fuzzy*.

5. Extension of the current work to a more general situation involving several Targets, several Attackers and/or several Defenders.

ANALYSIS AND SIMULATION OF THE TARGET-ATTACKER AND THE TARGET-ATTACKER-DEFENDER PROBLEMS_Results and Discussion

ORIGINALITY REPORT

11%

SIMILARITY INDEX

PRIMARY SOURCES

1	Garcia, Eloy, David W. Casbeer, and Meir Pachter. "Active Target defense differential game with a fast Defender", 2015 American Control Conference (ACC), 2015. Crossref	194 words — 7%
2	奥山 直弥. "高次元重力理論とその宇宙論への応用 :博士論文", Waseda University, 2006. Publications	24 words — 1%
3	Garcia, Eloy, David Casbeer, Khanh D. Pham, and Meir Pachter. "Cooperative Aircraft Defense from an Attacking Missile using Proportional Navigation", AIAA Guidance Navigation and Control Conference, 2015. Crossref	23 words — 1%
4	www.freepatentsonline.com Internet	18 words — 1%
5	innovexpo.itee.uq.edu.au Internet	13 words — < 1%
6	psopt.googlecode.com Internet	12 words — < 1%
7	www.iafastro.net Internet	11 words — < 1%
8	Burton, . "Further Aerodynamic Topics for Wind Turbines", Wind Energy Handbook Burton/Wind Energy Handbook, 2011.	10 words — < 1%