MLR_HW_BaseballSalary_CMPE188

March 2, 2025

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1. Perform full EDA on Salary data and determine the candidate features for a multiple linear regression model.(hint: you may want to use log transformation on the salary column and use it instead of Salary provided in the data). Use the log salary as the output and the rest of the variables as inputs. (you can ignore the categorical data columns).

[10]:
```
import pandas as pd
import matplotlib.pyplot as plt
from pandas import set_option
from pandas import read_csv
from sklearn.preprocessing import StandardScaler
from sklearn.preprocessing import Normalizer
from sklearn.linear_model import LinearRegression
from sklearn.feature_selection import RFE
from numpy import set_printoptions, log, argmax
import seaborn as sns
from pandas.plotting import scatter_matrix
import statsmodels.api as sm
```

[11]:
```
filename = 'Baseball_salary.csv'
data = read_csv(filename)
set_printoptions(precision=3)
data = data.drop(['Unnamed: 0','League', 'Division', 'NewLeague'], axis=1)
data.head(5)
```

[11]:
```
   AtBat  Hits  HmRun  Runs  RBI  Walks  Years  CAtBat  CHits  CHmRun  CRuns
CRBI  CWalks  \
0    293    66      1    30   29     14      1     293     66       1     30
29      14
1    315    81      7    24   38     39     14    3449    835      69    321
414     375
2    479   130     18    66   72     76      3    1624    457      63    224
266     263
3    496   141     20    65   78     37     11    5628   1575     225    828
838     354
4    321    87     10    39   42     30      2     396    101      12     48
46      33
```

```
   PutOuts  Assists  Errors  Salary
0      446       33      20     NaN
1      632       43      10   475.0
2      880       82      14   480.0
3      200       11       3   500.0
4      805       40       4    91.5
```

[12]:
```
print(data.isnull().sum())
```

```
AtBat       0
Hits        0
HmRun       0
Runs        0
RBI         0
Walks       0
Years       0
CAtBat      0
CHits       0
CHmRun      0
CRuns       0
CRBI        0
CWalks      0
PutOuts     0
Assists     0
Errors      0
Salary     59
dtype: int64
```

[13]:
```
# Clean the data by dropping rows with null salary
data = data.dropna(subset=['Salary'])
print(data.isnull().sum())
```

```
AtBat      0
Hits       0
HmRun      0
Runs       0
RBI        0
Walks      0
Years      0
CAtBat     0
CHits      0
CHmRun     0
CRuns      0
CRBI       0
CWalks     0
PutOuts    0
Assists    0
Errors     0
```

```
Salary      0
dtype: int64
```

[14]:
```
data['Log_Salary'] = log(data['Salary'])
array = data.values
Y1 = data['Log_Salary']
X1 = data.drop(['Salary', 'Log_Salary'], axis=1)
X1names = X1.columns
data.head(5)
```

[14]:
```
   AtBat  Hits  HmRun  Runs  RBI  Walks  Years  CAtBat  CHits  CHmRun  CRuns
CRBI  CWalks  \
1    315    81      7    24   38     39     14    3449    835      69    321
414     375
2    479   130     18    66   72     76      3    1624    457      63    224
266     263
3    496   141     20    65   78     37     11    5628   1575     225    828
838     354
4    321    87     10    39   42     30      2     396    101      12     48
46      33
5    594   169      4    74   51     35     11    4408   1133      19    501
336     194

   PutOuts  Assists  Errors  Salary  Log_Salary
1      632       43      10   475.0         6.2
2      880       82      14   480.0         6.2
3      200       11       3   500.0         6.2
4      805       40       4    91.5         4.5
5      282      421      25   750.0         6.6
```

4. Standardize the data by removing the mean and making the standard deviation equal to one (use from sklearn.preprocessing import StandardScaler, , and look at an example on how it is used)
5. Normalize the features by scaling them to a range between 0 and 1. Use the normalize object in scikit learn library to perform normalization on the data. Read the documentation from the preprocessing library documentation and look at the sample code given in the documentation as a guide on how to perform normalization.

[15]:
```
data_norm = X1.copy()

# Normalize
norm_scaler = Normalizer().fit(data_norm)
data_norm = norm_scaler.transform(data_norm)
# add output to normalized data
data_norm = pd.DataFrame(data_norm, columns=X1names, index=X1.index)
X1_norm = data_norm.copy()
data_norm['Log_Salary'] = Y1
```

```
data_stand = X1.copy()

# Standardize
stand_scaler = StandardScaler().fit(data_stand)
data_stand = stand_scaler.transform(data_stand)
# add output to standardized data
data_stand = pd.DataFrame(data_stand, columns=X1names, index=X1.index)
X1_stand = data_stand.copy()
data_stand['Log_Salary'] = Y1
data_objects = ((data_norm, 'data_norm'), (data_stand, 'data_stand'), (data,␣
↪"data_raw"))
```

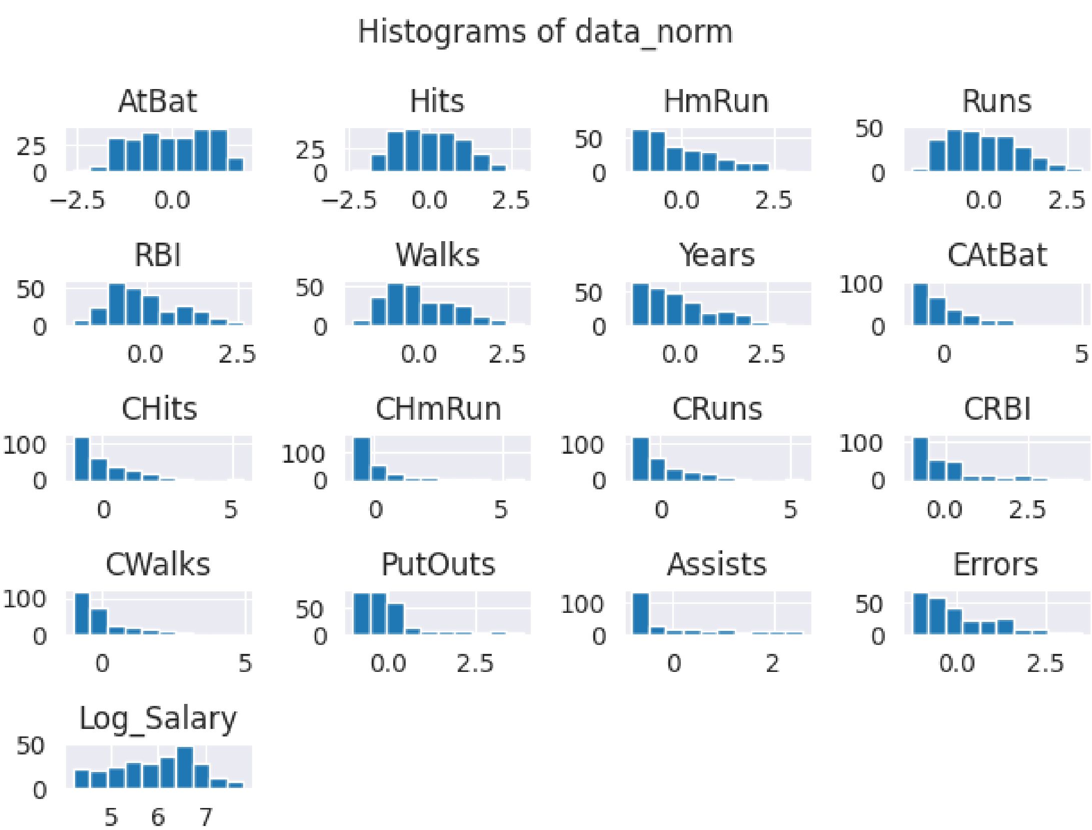
[16]:
```
# Descriptive stats
set_option('display.width', 100)
set_option('display.precision', 1)
for data, name in data_objects:
    print(f"Data: {name}")
    print(data.describe())
```

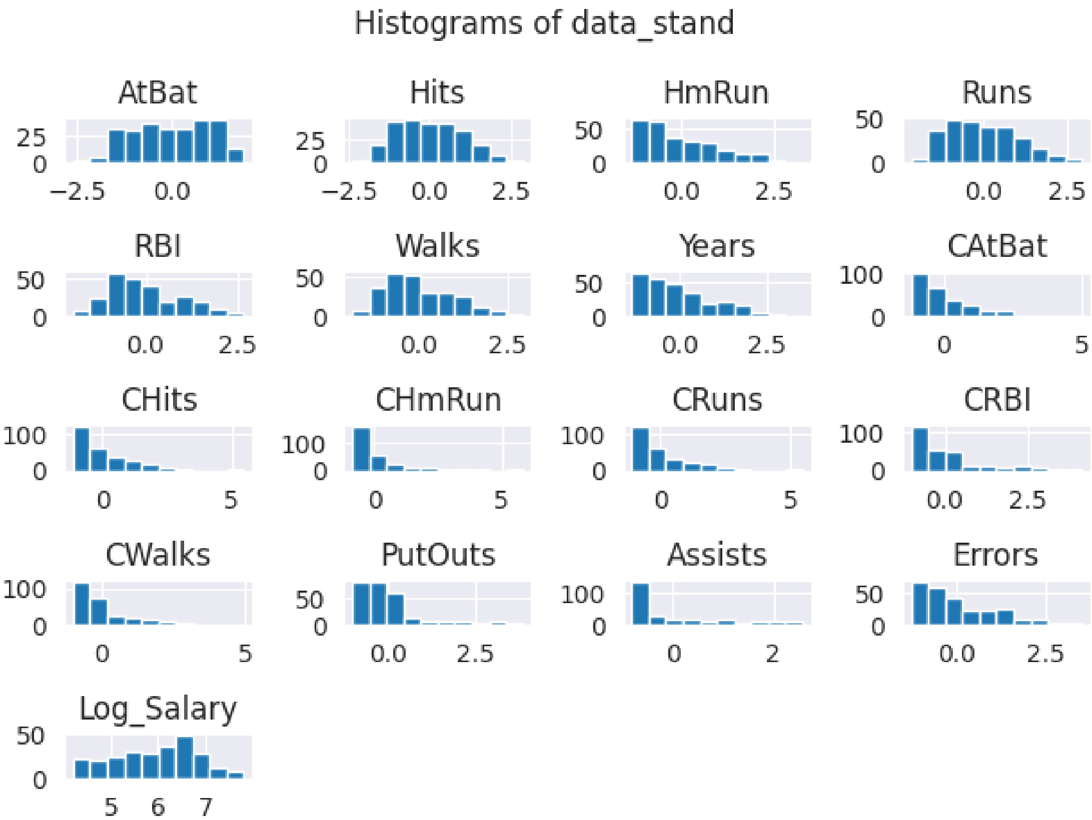
```
Data: data_norm
          AtBat     Hits    HmRun     Runs      RBI    Walks    Years   CAtBat
CHits   CHmRun  \
count  2.6e+02  2.6e+02  2.6e+02  2.6e+02  2.6e+02  2.6e+02  2.6e+02  2.6e+02
2.6e+02  2.6e+02
mean   2.4e-01  6.2e-02  6.3e-03  3.1e-02  2.9e-02  2.3e-02  3.0e-03  8.5e-01
2.2e-01  2.1e-02
std    1.7e-01  4.4e-02  6.9e-03  2.4e-02  2.2e-02  2.0e-02  1.7e-03  1.4e-01
4.5e-02  1.3e-02
min    1.4e-02  3.5e-03  0.0e+00  0.0e+00  0.0e+00  0.0e+00  6.6e-04  2.7e-02
5.7e-03  0.0e+00
25%    9.8e-02  2.6e-02  1.9e-03  1.3e-02  1.2e-02  9.3e-03  2.0e-03  8.4e-01
2.0e-01  1.0e-02
50%    1.9e-01  5.0e-02  3.9e-03  2.5e-02  2.2e-02  1.7e-02  2.5e-03  9.1e-01
2.4e-01  1.9e-02
75%    3.3e-01  8.8e-02  8.4e-03  4.3e-02  3.9e-02  3.0e-02  3.6e-03  9.3e-01
2.5e-01  3.1e-02
max    6.5e-01  2.0e-01  3.7e-02  1.2e-01  1.1e-01  1.0e-01  1.3e-02  9.5e-01
3.1e-01  6.1e-02

          CRuns     CRBI   CWalks  PutOuts  Assists   Errors  Log_Salary
count  2.6e+02  2.6e+02  2.6e+02  2.6e+02  2.6e+02  2.6e+02       263.0
mean   1.1e-01  1.0e-01  8.1e-02  1.7e-01  7.1e-02  5.7e-03         5.9
std    2.7e-02  3.2e-02  3.1e-02  1.8e-01  1.2e-01  7.2e-03         0.9
min    2.9e-03  4.3e-03  1.4e-03  0.0e+00  0.0e+00  0.0e+00         4.2
25%    9.8e-02  7.6e-02  6.0e-02  4.7e-02  3.9e-03  1.3e-03         5.2
50%    1.2e-01  1.0e-01  7.9e-02  1.1e-01  2.3e-02  3.4e-03         6.1
75%    1.3e-01  1.3e-01  1.0e-01  2.5e-01  9.0e-02  7.3e-03         6.6
max    1.9e-01  1.7e-01  1.8e-01  9.9e-01  9.2e-01  5.3e-02         7.8
```

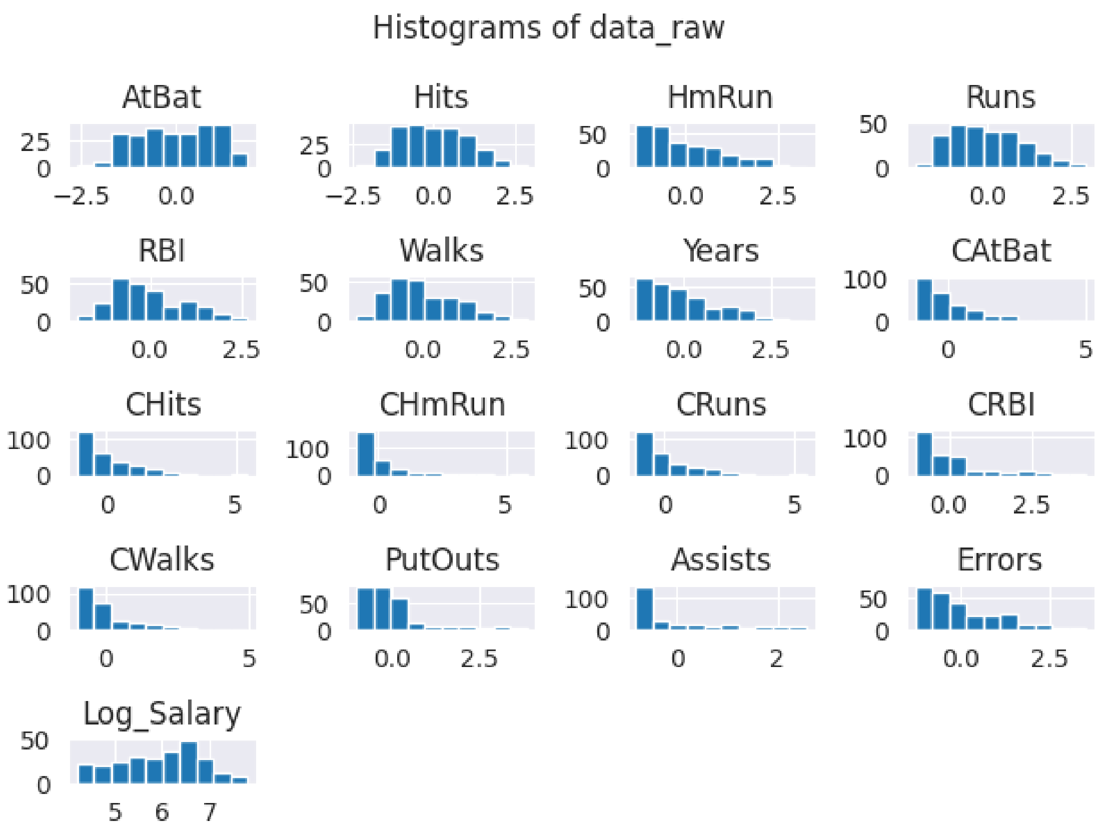
```
Data: data_stand
          AtBat     Hits    HmRun     Runs      RBI    Walks    Years   CAtBat
CHits   CHmRun  \
count  2.6e+02  2.6e+02  2.6e+02  2.6e+02  2.6e+02  2.6e+02  2.6e+02  2.6e+02
2.6e+02  2.6e+02
mean   1.0e-17  5.7e-17  3.4e-17 -5.1e-17  1.2e-16  1.7e-18 -5.4e-17  6.1e-17
6.8e-17  5.4e-17
std    1.0e+00  1.0e+00  1.0e+00  1.0e+00  1.0e+00  1.0e+00  1.0e+00  1.0e+00
1.0e+00  1.0e+00
min   -2.6e+00 -2.4e+00 -1.3e+00 -2.1e+00 -2.0e+00 -1.9e+00 -1.3e+00 -1.2e+00
-1.1e+00 -8.4e-01
25%   -8.2e-01 -8.1e-01 -7.6e-01 -8.3e-01 -8.3e-01 -8.4e-01 -6.9e-01 -8.0e-01
-7.9e-01 -6.6e-01
50%    6.4e-02 -1.1e-01 -3.0e-01 -1.1e-01 -1.7e-01 -1.9e-01 -2.7e-01 -3.2e-01
-3.2e-01 -3.6e-01
75%    8.3e-01  7.5e-01  7.3e-01  7.2e-01  7.6e-01  7.3e-01  5.6e-01  5.4e-01
5.1e-01  2.8e-01
max    1.9e+00  2.9e+00  3.2e+00  3.0e+00  2.7e+00  2.9e+00  3.5e+00  5.0e+00
5.5e+00  5.8e+00

          CRuns     CRBI   CWalks  PutOuts  Assists   Errors  Log_Salary
count  2.6e+02  2.6e+02  2.6e+02  2.6e+02    263.0  2.6e+02       263.0
mean   3.4e-17  4.1e-17  1.1e-16  7.4e-17      0.0  1.0e-16         5.9
std    1.0e+00  1.0e+00  1.0e+00  1.0e+00      1.0  1.0e+00         0.9
min   -1.1e+00 -1.0e+00 -9.8e-01 -1.0e+00     -0.8 -1.3e+00         4.2
25%   -7.7e-01 -7.3e-01 -7.2e-01 -6.3e-01     -0.8 -8.5e-01         5.2
50%   -3.4e-01 -3.1e-01 -3.3e-01 -2.4e-01     -0.5 -2.4e-01         6.1
75%    4.1e-01  2.9e-01  2.6e-01  1.1e-01      0.5  6.7e-01         6.6
max    5.5e+00  4.1e+00  5.0e+00  3.9e+00      2.6  3.5e+00         7.8
Data: data_raw
        AtBat   Hits  HmRun   Runs    RBI  Walks  Years   CAtBat   CHits  CHmRun
CRuns    CRBI  \
count  263.0  263.0  263.0  263.0  263.0  263.0  263.0    263.0   263.0   263.0
263.0   263.0
mean   403.6  107.8   11.6   54.7   51.5   41.1    7.3   2657.5   722.2    69.2
361.2   330.4
std    147.3   45.1    8.8   25.5   25.9   21.7    4.8   2286.6   648.2    82.2
331.2   323.4
min     19.0    1.0    0.0    0.0    0.0    0.0    1.0     19.0     4.0     0.0
2.0     3.0
25%    282.5   71.5    5.0   33.5   30.0   23.0    4.0    842.5   212.0    15.0
105.5    95.0
50%    413.0  103.0    9.0   52.0   47.0   37.0    6.0   1931.0   516.0    40.0
250.0   230.0
75%    526.0  141.5   18.0   73.0   71.0   57.0   10.0   3890.5  1054.0    92.5
497.5   424.5
max    687.0  238.0   40.0  130.0  121.0  105.0   24.0  14053.0  4256.0   548.0
2165.0  1659.0
```

	CWalks	PutOuts	Assists	Errors	Salary	Log_Salary
count	263.0	263.0	263.0	263.0	263.0	263.0
mean	260.3	290.7	118.8	8.6	535.9	5.9
std	264.1	279.9	145.1	6.6	451.1	0.9
min	1.0	0.0	0.0	0.0	67.5	4.2
25%	71.0	113.5	8.0	3.0	190.0	5.2
50%	174.0	224.0	45.0	7.0	425.0	6.1
75%	328.5	322.5	192.0	13.0	750.0	6.6
max	1566.0	1377.0	492.0	32.0	2460.0	7.8

[17]:
```python
# Histograms
for data, name in data_objects:
    data_stand.hist()
    plt.suptitle(f"Histograms of {name}")
    plt.tight_layout()
    plt.show()
```




Histograms of data_stand
AtBat
Hits
HmRun
Runs
RBI
Walks
Years
CAtBat
CHits
CHmRun
CRuns
CRBI
CWalks
PutOuts
Assists
Errors
Log_Salary

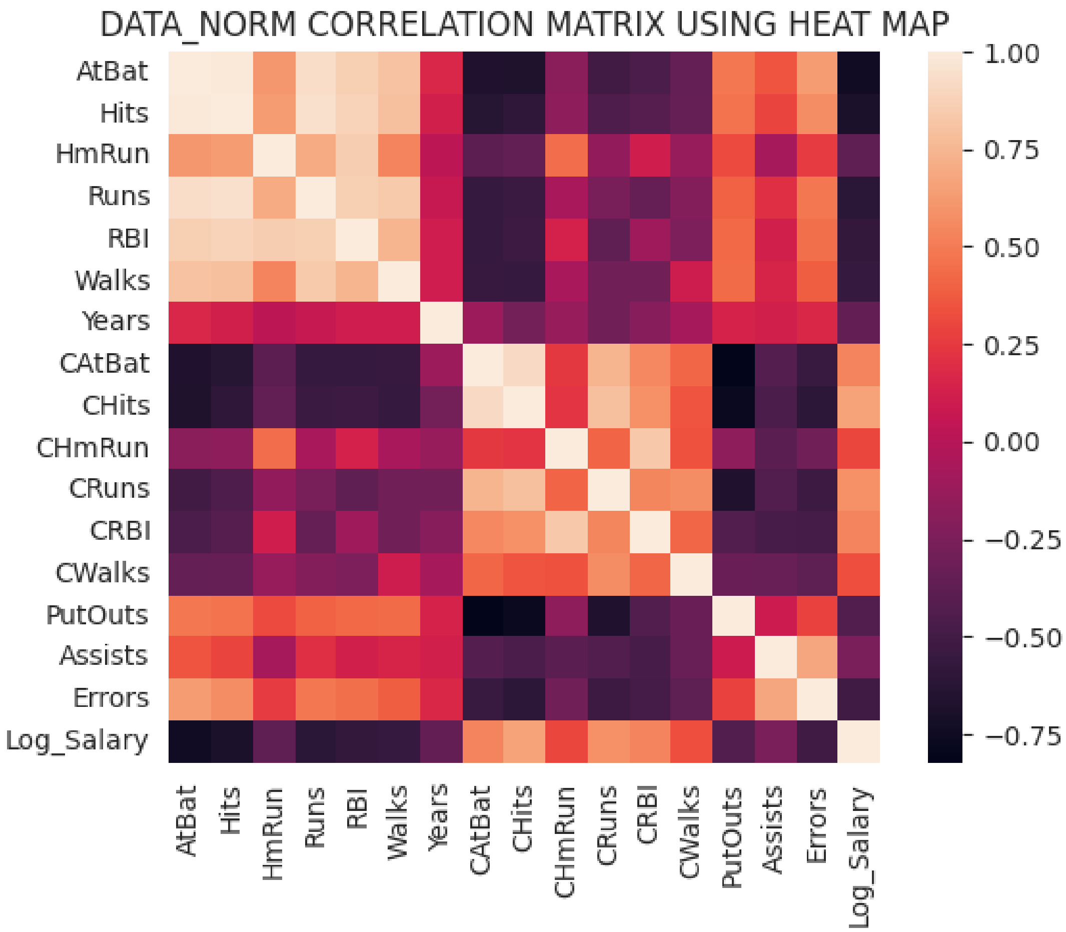


[18]:
```python
for data, name in data_objects:
    plt.figure() # new plot
    #plt.tight_layout()
    corMat = data_norm.corr(method='pearson')
    print(corMat)
    ## plot correlation matrix as a heat map
    sns.heatmap(corMat, square=True)
    plt.yticks(rotation=0)
    plt.xticks(rotation=90)
    plt.title(f"{name.upper()} CORRELATION MATRIX USING HEAT MAP")
    plt.show()

    ## scatter plot of all data
    plt.figure()
    # # The output overlaps itself, resize it to display better (w padding)
    scatter_matrix(data_norm)
    plt.tight_layout(pad=0.1)
    plt.show()
```

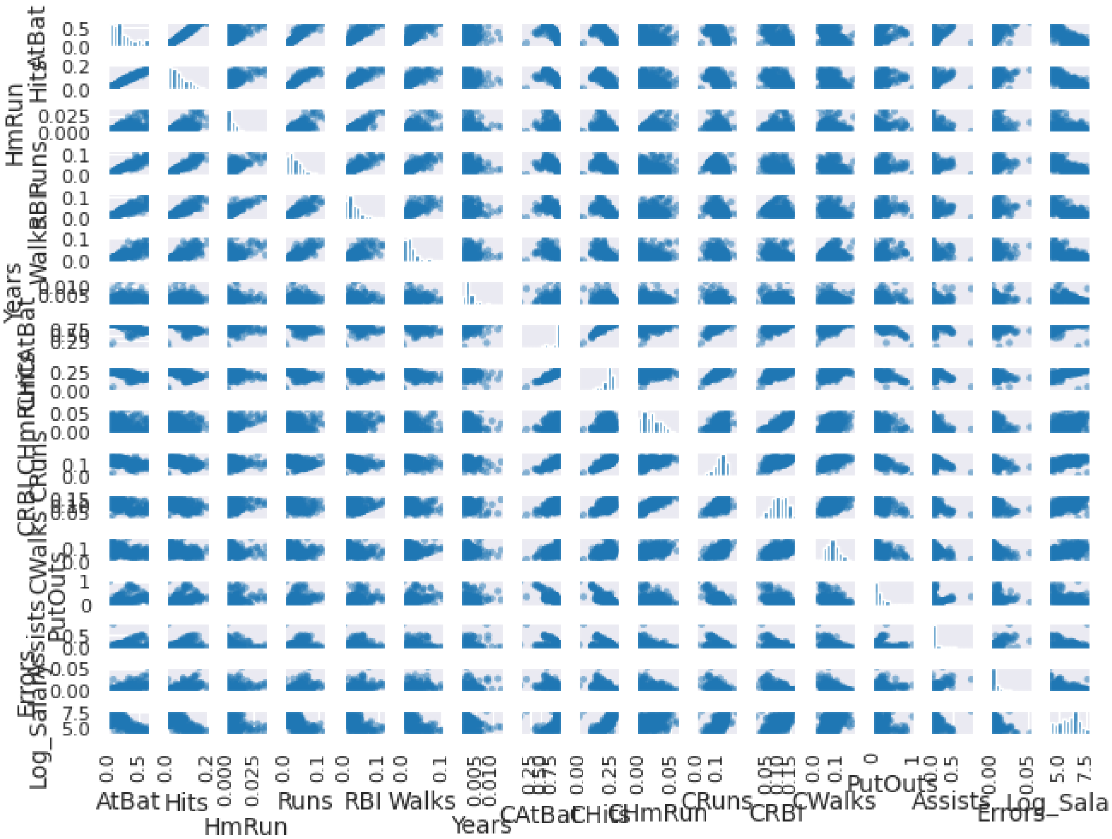
```
               AtBat   Hits     HmRun      Runs   RBI     Walks     Years   CAtBat   CHits  \
CHmRun   CRuns   \
```

```
AtBat          1.0   1.0  6.1e-01  9.3e-01  0.9  8.0e-01  1.6e-01    -0.7   -0.7
-1.9e-01   -0.5
Hits           1.0   1.0  6.4e-01  9.4e-01  0.9  7.9e-01  1.2e-01    -0.6   -0.6
-1.7e-01   -0.4
HmRun          0.6   0.6  1.0e+00  7.0e-01  0.9  5.3e-01  2.8e-02    -0.4   -0.4
4.4e-01   -0.2
Runs           0.9   0.9  7.0e-01  1.0e+00  0.9  8.4e-01  6.9e-02    -0.6   -0.5
-6.4e-02   -0.3
RBI            0.9   0.9  8.6e-01  8.6e-01  1.0  7.4e-01  1.1e-01    -0.6   -0.5
1.4e-01   -0.4
Walks          0.8   0.8  5.3e-01  8.4e-01  0.7  1.0e+00  1.1e-01    -0.6   -0.6
-6.6e-02   -0.3
Years          0.2   0.1  2.8e-02  6.9e-02  0.1  1.1e-01  1.0e+00    -0.1   -0.3
-1.4e-01   -0.3
CAtBat        -0.7  -0.6 -3.8e-01 -5.7e-01 -0.6 -5.5e-01 -1.1e-01     1.0    0.9
2.4e-01    0.7
CHits         -0.7  -0.6 -3.6e-01 -5.4e-01 -0.5 -5.6e-01 -2.8e-01     0.9    1.0
2.3e-01    0.8
CHmRun        -0.2  -0.2  4.4e-01 -6.4e-02  0.1 -6.6e-02 -1.4e-01     0.2    0.2
1.0e+00    0.4
CRuns         -0.5  -0.4 -1.5e-01 -2.7e-01 -0.4 -3.0e-01 -2.9e-01     0.7    0.8
4.0e-01    1.0
CRBI          -0.5  -0.4  1.2e-01 -3.4e-01 -0.1 -3.0e-01 -2.0e-01     0.5    0.6
8.3e-01    0.5
CWalks        -0.3  -0.3 -1.3e-01 -2.1e-01 -0.2  9.8e-02 -7.2e-02     0.4    0.4
3.4e-01    0.6
PutOuts        0.5   0.5  3.1e-01  4.0e-01  0.4  4.4e-01  1.4e-01    -0.8   -0.8
-1.7e-01   -0.7
Assists        0.3   0.3 -7.2e-02  2.1e-01  0.1  1.5e-01  1.2e-01    -0.4   -0.5
-3.9e-01   -0.4
Errors         0.6   0.6  2.6e-01  4.8e-01  0.4  3.9e-01  1.7e-01    -0.5   -0.6
-3.0e-01   -0.5
Log_Salary    -0.7  -0.7 -3.7e-01 -6.2e-01 -0.6 -5.6e-01 -3.6e-01     0.5    0.7
3.0e-01    0.6

            CRBI   CWalks  PutOuts  Assists  Errors  Log_Salary
AtBat       -0.5 -3.5e-01  4.8e-01  3.5e-01     0.6        -0.7
Hits        -0.4 -3.4e-01  4.6e-01  2.9e-01     0.6        -0.7
HmRun        0.1 -1.3e-01  3.1e-01 -7.2e-02     0.3        -0.4
Runs        -0.3 -2.1e-01  4.0e-01  2.1e-01     0.5        -0.6
RBI         -0.1 -2.4e-01  4.3e-01  1.2e-01     0.4        -0.6
Walks       -0.3  9.8e-02  4.4e-01  1.5e-01     0.4        -0.6
Years       -0.2 -7.2e-02  1.4e-01  1.2e-01     0.2        -0.4
CAtBat       0.5  4.1e-01 -8.2e-01 -4.2e-01    -0.5         0.5
CHits        0.6  3.5e-01 -7.7e-01 -4.6e-01    -0.6         0.7
CHmRun       0.8  3.4e-01 -1.7e-01 -3.9e-01    -0.3         0.3
CRuns        0.5  5.7e-01 -6.7e-01 -4.3e-01    -0.5         0.6
CRBI         1.0  4.2e-01 -4.3e-01 -4.8e-01    -0.5         0.5
```

```
CWalks        0.4  1.0e+00 -3.3e-01 -3.3e-01     -0.4          0.3
PutOuts      -0.4 -3.3e-01  1.0e+00  9.0e-02      0.3         -0.4
Assists      -0.5 -3.3e-01  9.0e-02  1.0e+00      0.7         -0.3
Errors       -0.5 -3.8e-01  2.8e-01  6.8e-01      1.0         -0.5
Log_Salary    0.5  3.3e-01 -4.3e-01 -2.6e-01     -0.5          1.0
```

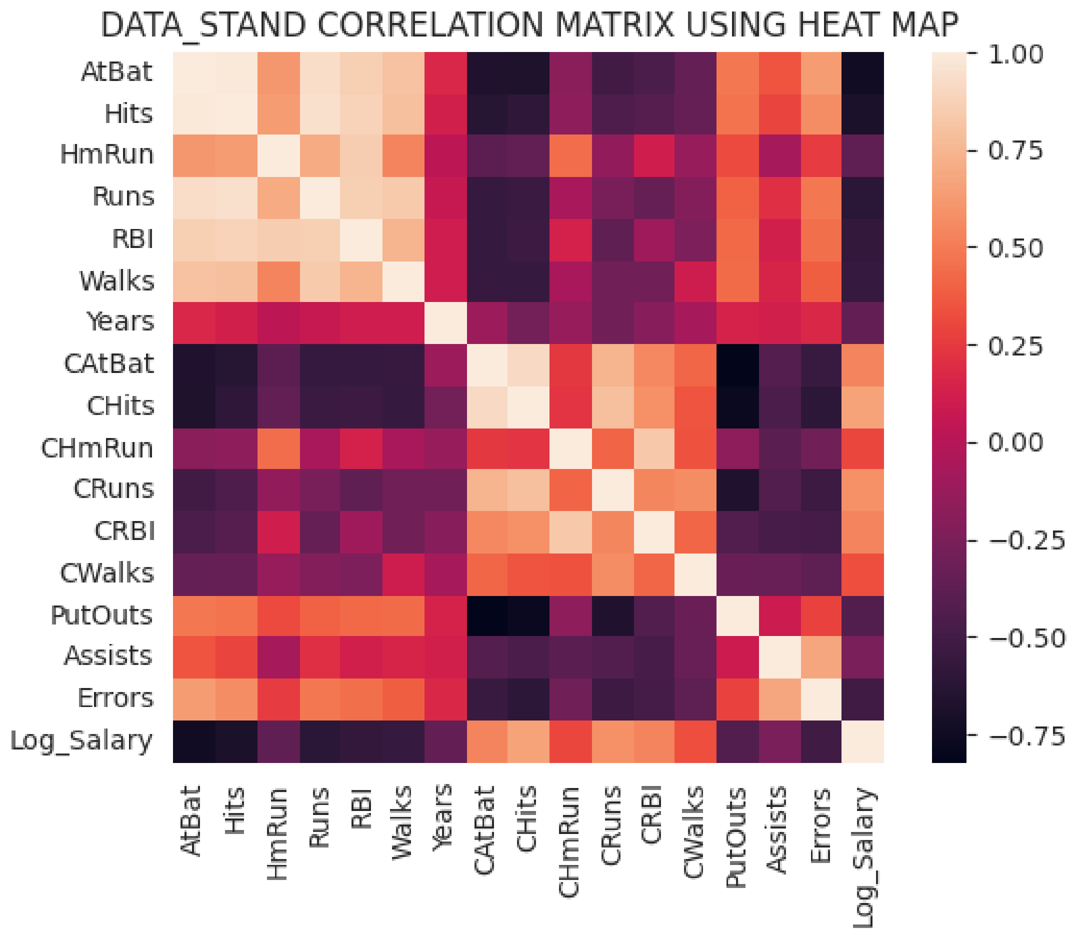



```
<Figure size 640x480 with 0 Axes>
```

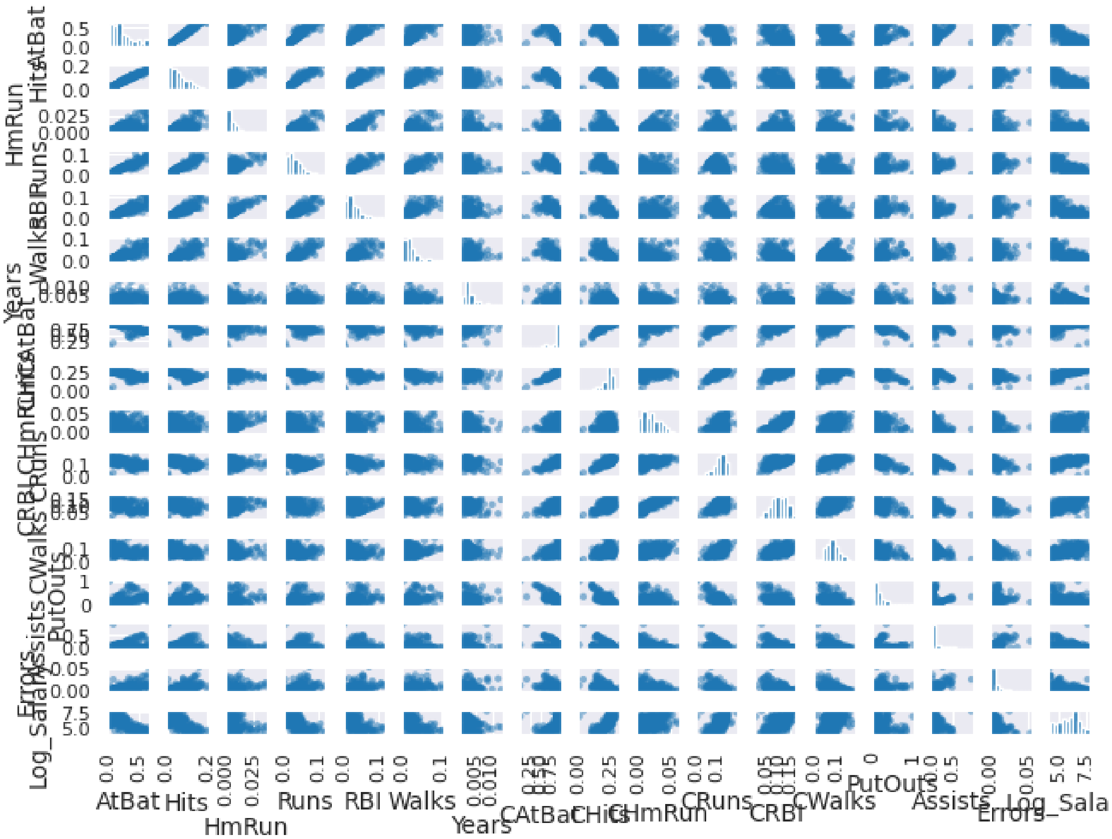


```
                AtBat  Hits    HmRun     Runs  RBI    Walks    Years  CAtBat  CHits
CHmRun   CRuns  \
AtBat             1.0   1.0  6.1e-01  9.3e-01  0.9  8.0e-01  1.6e-01    -0.7   -0.7
-1.9e-01   -0.5
Hits              1.0   1.0  6.4e-01  9.4e-01  0.9  7.9e-01  1.2e-01    -0.6   -0.6
-1.7e-01   -0.4
HmRun             0.6   0.6  1.0e+00  7.0e-01  0.9  5.3e-01  2.8e-02    -0.4   -0.4
4.4e-01   -0.2
Runs              0.9   0.9  7.0e-01  1.0e+00  0.9  8.4e-01  6.9e-02    -0.6   -0.5
-6.4e-02   -0.3
RBI               0.9   0.9  8.6e-01  8.6e-01  1.0  7.4e-01  1.1e-01    -0.6   -0.5
1.4e-01   -0.4
Walks             0.8   0.8  5.3e-01  8.4e-01  0.7  1.0e+00  1.1e-01    -0.6   -0.6
-6.6e-02   -0.3
Years             0.2   0.1  2.8e-02  6.9e-02  0.1  1.1e-01  1.0e+00    -0.1   -0.3
-1.4e-01   -0.3
CAtBat           -0.7  -0.6 -3.8e-01 -5.7e-01 -0.6 -5.5e-01 -1.1e-01     1.0    0.9
2.4e-01    0.7
CHits            -0.7  -0.6 -3.6e-01 -5.4e-01 -0.5 -5.6e-01 -2.8e-01     0.9    1.0
2.3e-01    0.8
CHmRun           -0.2  -0.2  4.4e-01 -6.4e-02  0.1 -6.6e-02 -1.4e-01     0.2    0.2
```

```
1.0e+00    0.4
CRuns        -0.5  -0.4 -1.5e-01 -2.7e-01 -0.4 -3.0e-01 -2.9e-01     0.7    0.8
4.0e-01    1.0
CRBI         -0.5  -0.4  1.2e-01 -3.4e-01 -0.1 -3.0e-01 -2.0e-01     0.5    0.6
8.3e-01    0.5
CWalks       -0.3  -0.3 -1.3e-01 -2.1e-01 -0.2  9.8e-02 -7.2e-02     0.4    0.4
3.4e-01    0.6
PutOuts       0.5   0.5  3.1e-01  4.0e-01  0.4  4.4e-01  1.4e-01    -0.8   -0.8
-1.7e-01   -0.7
Assists       0.3   0.3 -7.2e-02  2.1e-01  0.1  1.5e-01  1.2e-01    -0.4   -0.5
-3.9e-01   -0.4
Errors        0.6   0.6  2.6e-01  4.8e-01  0.4  3.9e-01  1.7e-01    -0.5   -0.6
-3.0e-01   -0.5
Log_Salary   -0.7  -0.7 -3.7e-01 -6.2e-01 -0.6 -5.6e-01 -3.6e-01     0.5    0.7
3.0e-01    0.6

            CRBI   CWalks  PutOuts  Assists  Errors  Log_Salary
AtBat       -0.5 -3.5e-01  4.8e-01  3.5e-01     0.6        -0.7
Hits        -0.4 -3.4e-01  4.6e-01  2.9e-01     0.6        -0.7
HmRun        0.1 -1.3e-01  3.1e-01 -7.2e-02     0.3        -0.4
Runs        -0.3 -2.1e-01  4.0e-01  2.1e-01     0.5        -0.6
RBI         -0.1 -2.4e-01  4.3e-01  1.2e-01     0.4        -0.6
Walks       -0.3  9.8e-02  4.4e-01  1.5e-01     0.4        -0.6
Years       -0.2 -7.2e-02  1.4e-01  1.2e-01     0.2        -0.4
CAtBat       0.5  4.1e-01 -8.2e-01 -4.2e-01    -0.5         0.5
CHits        0.6  3.5e-01 -7.7e-01 -4.6e-01    -0.6         0.7
CHmRun       0.8  3.4e-01 -1.7e-01 -3.9e-01    -0.3         0.3
CRuns        0.5  5.7e-01 -6.7e-01 -4.3e-01    -0.5         0.6
CRBI         1.0  4.2e-01 -4.3e-01 -4.8e-01    -0.5         0.5
CWalks       0.4  1.0e+00 -3.3e-01 -3.3e-01    -0.4         0.3
PutOuts     -0.4 -3.3e-01  1.0e+00  9.0e-02     0.3        -0.4
Assists     -0.5 -3.3e-01  9.0e-02  1.0e+00     0.7        -0.3
Errors      -0.5 -3.8e-01  2.8e-01  6.8e-01     1.0        -0.5
Log_Salary   0.5  3.3e-01 -4.3e-01 -2.6e-01    -0.5         1.0
```

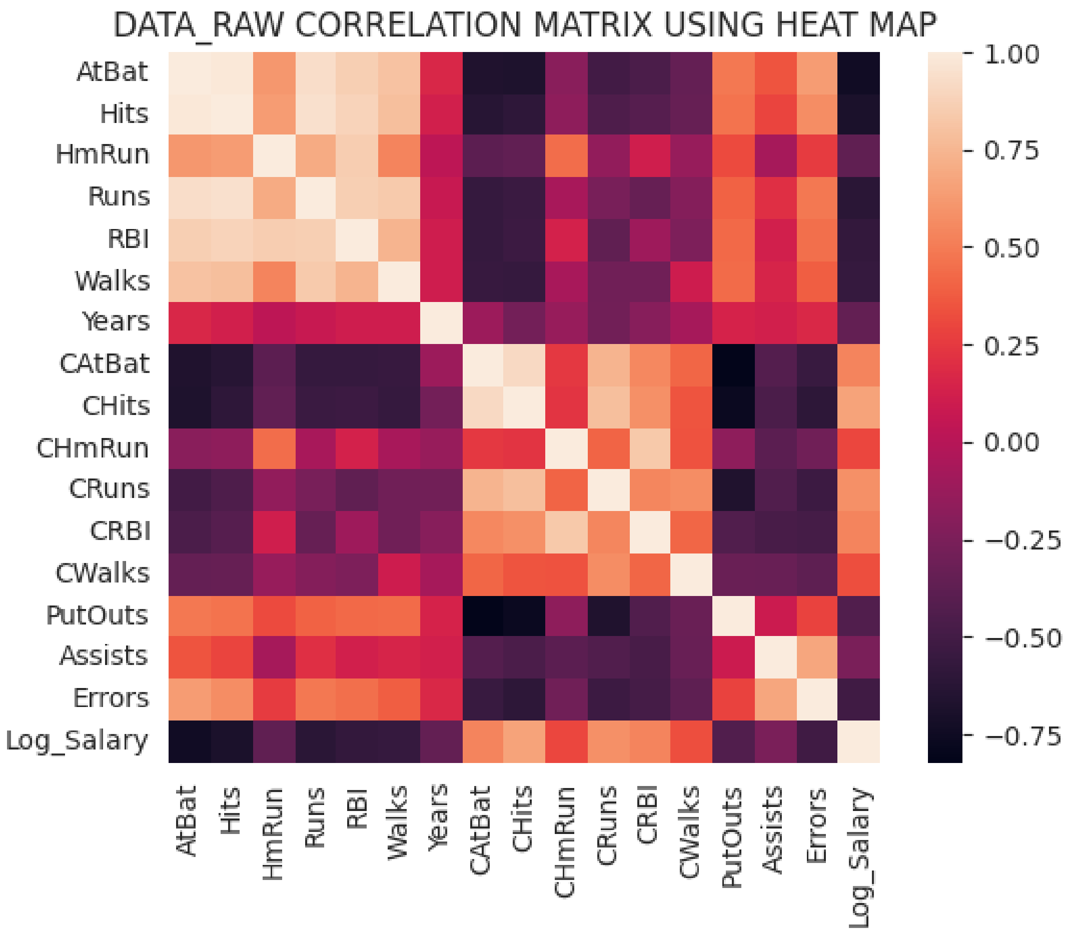



```
<Figure size 640x480 with 0 Axes>
```

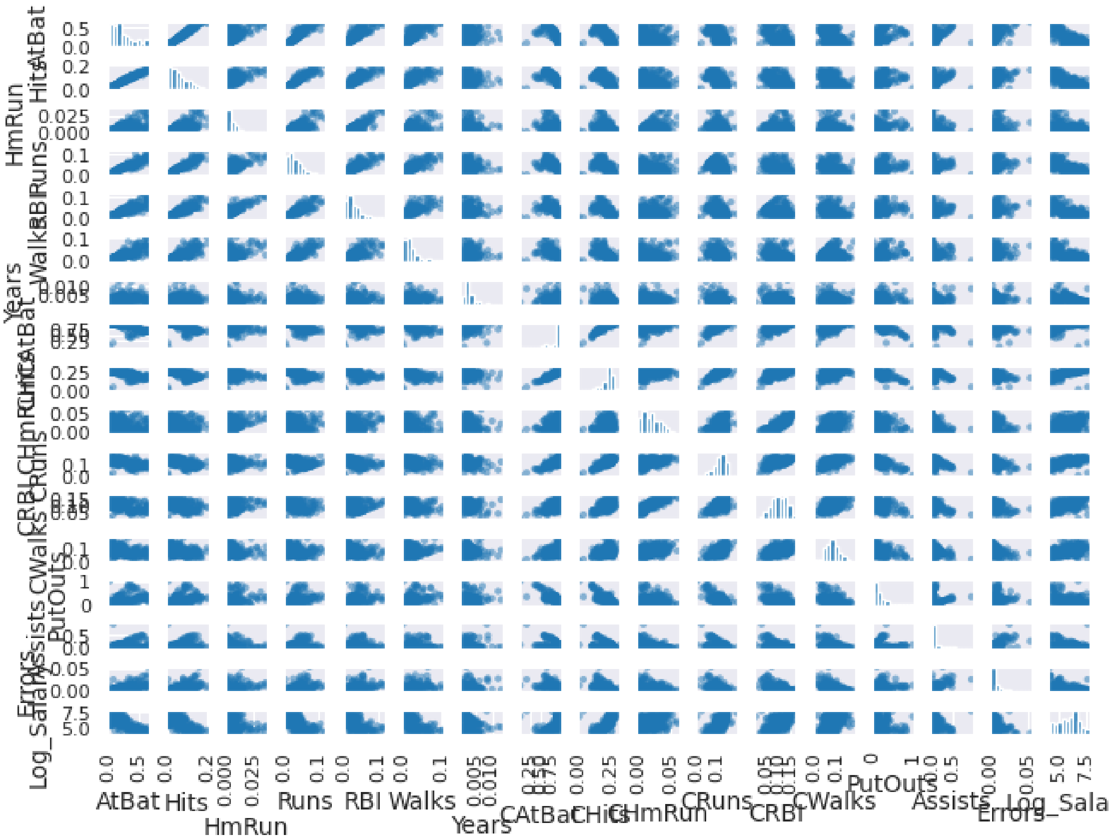


	AtBat	Hits	HmRun	Runs	RBI	Walks	Years	CAtBat	CHits	CHmRun	CRuns \
AtBat	1.0	1.0	6.1e-01	9.3e-01	0.9	8.0e-01	1.6e-01	-0.7	-0.7	-1.9e-01	-0.5
Hits	1.0	1.0	6.4e-01	9.4e-01	0.9	7.9e-01	1.2e-01	-0.6	-0.6	-1.7e-01	-0.4
HmRun	0.6	0.6	1.0e+00	7.0e-01	0.9	5.3e-01	2.8e-02	-0.4	-0.4	4.4e-01	-0.2
Runs	0.9	0.9	7.0e-01	1.0e+00	0.9	8.4e-01	6.9e-02	-0.6	-0.5	-6.4e-02	-0.3
RBI	0.9	0.9	8.6e-01	8.6e-01	1.0	7.4e-01	1.1e-01	-0.6	-0.5	1.4e-01	-0.4
Walks	0.8	0.8	5.3e-01	8.4e-01	0.7	1.0e+00	1.1e-01	-0.6	-0.6	-6.6e-02	-0.3
Years	0.2	0.1	2.8e-02	6.9e-02	0.1	1.1e-01	1.0e+00	-0.1	-0.3	-1.4e-01	-0.3
CAtBat	-0.7	-0.6	-3.8e-01	-5.7e-01	-0.6	-5.5e-01	-1.1e-01	1.0	0.9	2.4e-01	0.7
CHits	-0.7	-0.6	-3.6e-01	-5.4e-01	-0.5	-5.6e-01	-2.8e-01	0.9	1.0	2.3e-01	0.8
CHmRun	-0.2	-0.2	4.4e-01	-6.4e-02	0.1	-6.6e-02	-1.4e-01	0.2	0.2		

```
1.0e+00    0.4
CRuns        -0.5  -0.4 -1.5e-01 -2.7e-01 -0.4 -3.0e-01 -2.9e-01     0.7    0.8
4.0e-01    1.0
CRBI         -0.5  -0.4  1.2e-01 -3.4e-01 -0.1 -3.0e-01 -2.0e-01     0.5    0.6
8.3e-01    0.5
CWalks       -0.3  -0.3 -1.3e-01 -2.1e-01 -0.2  9.8e-02 -7.2e-02     0.4    0.4
3.4e-01    0.6
PutOuts       0.5   0.5  3.1e-01  4.0e-01  0.4  4.4e-01  1.4e-01    -0.8   -0.8
-1.7e-01   -0.7
Assists       0.3   0.3 -7.2e-02  2.1e-01  0.1  1.5e-01  1.2e-01    -0.4   -0.5
-3.9e-01   -0.4
Errors        0.6   0.6  2.6e-01  4.8e-01  0.4  3.9e-01  1.7e-01    -0.5   -0.6
-3.0e-01   -0.5
Log_Salary   -0.7  -0.7 -3.7e-01 -6.2e-01 -0.6 -5.6e-01 -3.6e-01     0.5    0.7
3.0e-01    0.6

            CRBI   CWalks  PutOuts  Assists  Errors  Log_Salary
AtBat       -0.5 -3.5e-01  4.8e-01  3.5e-01     0.6        -0.7
Hits        -0.4 -3.4e-01  4.6e-01  2.9e-01     0.6        -0.7
HmRun        0.1 -1.3e-01  3.1e-01 -7.2e-02     0.3        -0.4
Runs        -0.3 -2.1e-01  4.0e-01  2.1e-01     0.5        -0.6
RBI         -0.1 -2.4e-01  4.3e-01  1.2e-01     0.4        -0.6
Walks       -0.3  9.8e-02  4.4e-01  1.5e-01     0.4        -0.6
Years       -0.2 -7.2e-02  1.4e-01  1.2e-01     0.2        -0.4
CAtBat       0.5  4.1e-01 -8.2e-01 -4.2e-01    -0.5         0.5
CHits        0.6  3.5e-01 -7.7e-01 -4.6e-01    -0.6         0.7
CHmRun       0.8  3.4e-01 -1.7e-01 -3.9e-01    -0.3         0.3
CRuns        0.5  5.7e-01 -6.7e-01 -4.3e-01    -0.5         0.6
CRBI         1.0  4.2e-01 -4.3e-01 -4.8e-01    -0.5         0.5
CWalks       0.4  1.0e+00 -3.3e-01 -3.3e-01    -0.4         0.3
PutOuts     -0.4 -3.3e-01  1.0e+00  9.0e-02     0.3        -0.4
Assists     -0.5 -3.3e-01  9.0e-02  1.0e+00     0.7        -0.3
Errors      -0.5 -3.8e-01  2.8e-01  6.8e-01     1.0        -0.5
Log_Salary   0.5  3.3e-01 -4.3e-01 -2.6e-01    -0.5         1.0
```




```
<Figure size 640x480 with 0 Axes>
```



[19]:
```
NUM_FEATURES = 16
model = LinearRegression()
rfe = RFE(model, n_features_to_select = NUM_FEATURES)
fit = rfe.fit(X1, Y1)
print("Num Features:", fit.n_features_)
print("Selected Features:", fit.support_)
print("Feature Ranking:", fit.ranking_)
score = rfe.score(X1,Y1)
print("Model Score with selected features is: ", score)
```

```
Num Features: 16
Selected Features: [ True  True  True  True  True  True  True  True  True  True
True  True
  True  True  True  True]
Feature Ranking: [1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1]
Model Score with selected features is:  0.5426879134785905
```

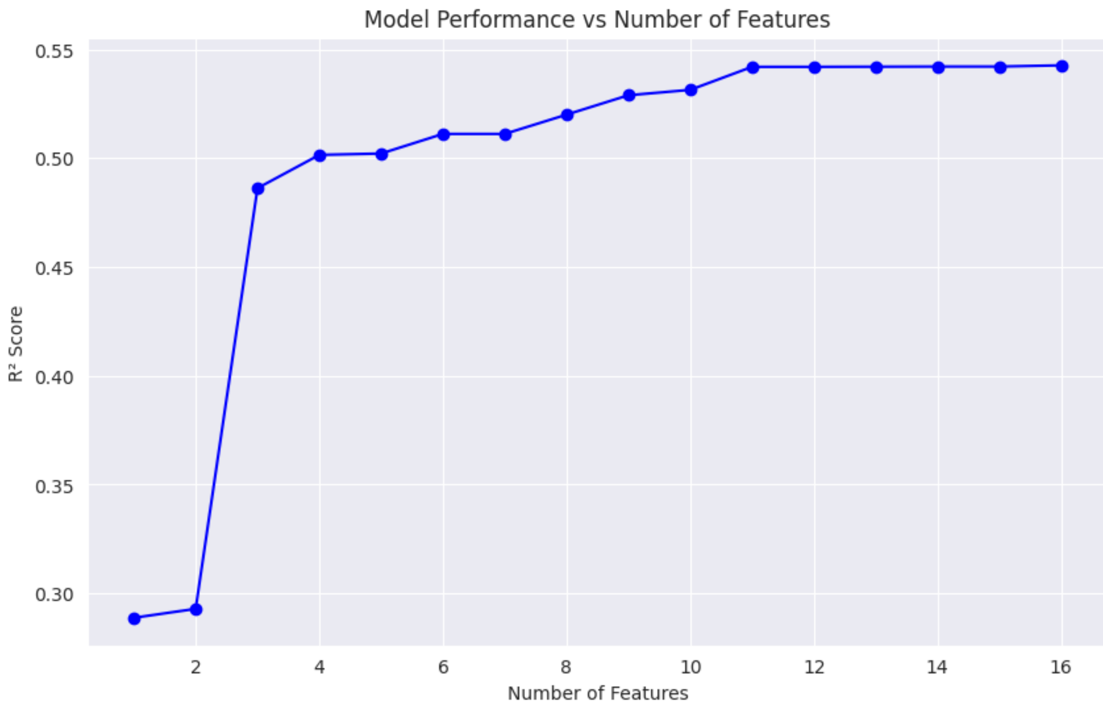
3. For feature selection start with a large number of features and monitor the performance measures. Pick the number of features based on the performance measure when there is a significant change and stop when you do not see a major improvement.

[20]:
```python
def determine_optimal_number_of_features(X, Y):
    feature_counts = range(1, X.shape[1] + 1)
    scores = []
    print(feature_counts)
    for num_features in feature_counts:
        model = LinearRegression()
        rfe = RFE(model, n_features_to_select = num_features)
        fit = rfe.fit(X, Y)
        print("Num Features:", fit.n_features_)
        print("Selected Features:", fit.support_)
        print("Feature Ranking:", fit.ranking_)
        scores.append(rfe.score(X,Y))
    # Plot results
    plt.figure(figsize=(10, 6))
    plt.plot(feature_counts, scores, 'b-', marker='o')
    plt.xlabel('Number of Features')
    plt.ylabel('R² Score')
    plt.title('Model Performance vs Number of Features')
    plt.grid(True)
    plt.show()
    best_num_features = feature_counts[argmax(scores)]
    print(f"Optimal number of features: {best_num_features}")
    print(f"Best score: {max(scores):.4f}")
    return feature_counts, scores
print("Raw Data")
feature_counts, scores = determine_optimal_number_of_features(X1, Y1)
```

```
Raw Data
range(1, 17)
Num Features: 1
Selected Features: [False False False False False False  True False False False
False False
 False False False False]
Feature Ranking: [ 6  3  5  7 12  4  1 16 14 15  8 13  9 11 10  2]
Num Features: 2
Selected Features: [False False False False False False  True False False False
False False
 False False False  True]
Feature Ranking: [ 5  2  4  6 11  3  1 15 13 14  7 12  8 10  9  1]
Num Features: 3
Selected Features: [False  True False False False False  True False False False
False False
 False False False  True]
Feature Ranking: [ 4  1  3  5 10  2  1 14 12 13  6 11  7  9  8  1]
Num Features: 4
Selected Features: [False  True False False False  True  True False False False
False False
 False False False  True]
```

```
Feature Ranking: [ 3  1  2  4  9  1  1 13 11 12  5 10  6  8  7  1]
Num Features: 5
Selected Features: [False  True  True False False  True  True False False False
False False
 False False False  True]
Feature Ranking: [ 2  1  1  3  8  1  1 12 10 11  4  9  5  7  6  1]
Num Features: 6
Selected Features: [ True  True  True False False  True  True False False False
False False
 False False False  True]
Feature Ranking: [ 1  1  1  2  7  1  1 11  9 10  3  8  4  6  5  1]
Num Features: 7
Selected Features: [ True  True  True  True False  True  True False False False
False False
 False False False  True]
Feature Ranking: [ 1  1  1  1  6  1  1 10  8  9  2  7  3  5  4  1]
Num Features: 8
Selected Features: [ True  True  True  True False  True  True False False False
True False
 False False False  True]
Feature Ranking: [1 1 1 1 5 1 1 9 7 8 1 6 2 4 3 1]
Num Features: 9
Selected Features: [ True  True  True  True False  True  True False False False
True False
  True False False  True]
Feature Ranking: [1 1 1 1 4 1 1 8 6 7 1 5 1 3 2 1]
Num Features: 10
Selected Features: [ True  True  True  True False  True  True False False False
True False
  True False  True  True]
Feature Ranking: [1 1 1 1 3 1 1 7 5 6 1 4 1 2 1 1]
Num Features: 11
Selected Features: [ True  True  True  True False  True  True False False False
True False
  True  True  True  True]
Feature Ranking: [1 1 1 1 2 1 1 6 4 5 1 3 1 1 1 1]
Num Features: 12
Selected Features: [ True  True  True  True  True  True  True False False False
True False
  True  True  True  True]
Feature Ranking: [1 1 1 1 1 1 1 5 3 4 1 2 1 1 1 1]
Num Features: 13
Selected Features: [ True  True  True  True  True  True  True False False False
True  True
  True  True  True  True]
Feature Ranking: [1 1 1 1 1 1 1 4 2 3 1 1 1 1 1 1]
Num Features: 14
Selected Features: [ True  True  True  True  True  True  True False  True False
```

```
True  True
  True  True  True  True]
Feature Ranking: [1 1 1 1 1 1 1 3 1 2 1 1 1 1 1 1]
Num Features: 15
Selected Features: [ True  True  True  True  True  True  True False  True  True
True  True
  True  True  True  True]
Feature Ranking: [1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1]
Num Features: 16
Selected Features: [ True  True  True  True  True  True  True  True  True  True
True  True
  True  True  True  True]
Feature Ranking: [1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1]
```




```
Optimal number of features: 16
Best score: 0.5427
```

[21]:
```
# Because 16 is the best, we will look for the lowest number of features with a␣
↪similar score
def determine_features_within_threshold(_feature_counts, _scores):
    best_score = max(_scores)
    best_score_index = argmax(_scores)
    threshold = best_score * 0.99
    best_num_features = _feature_counts[best_score_index]
```

```
    i = best_score_index
    for i in range(best_score_index, 0, -1):
        if _scores[i] < threshold:
            print(f"Lowest number of features within 1% of best score:␣
↪{_feature_counts[i]}")
            break
print("Raw Data")
determine_features_within_threshold(feature_counts, scores)
```

```
Raw Data
Lowest number of features within 1% of best score: 10
```

[22]:
```
def stepwise_selection(X, y,
                       initial_list=[],
                       threshold_in=0.01,
                       threshold_out = 0.05,
                       verbose=True):
        """ Perform a forward-backward feature selection
        based on p-value from statsmodels.api.OLS
        Arguments:
            X - pandas.DataFrame with candidate features
            y - list-like with the target
            initial_list - list of features to start with (column names of X)
            threshold_in - include a feature if its p-value < threshold_in
            threshold_out - exclude a feature if its p-value > threshold_out
            verbose - whether to print the sequence of inclusions and exclusions
        Returns: list of selected features
        Always set threshold_in < threshold_out to avoid infinite looping.
        See https://en.wikipedia.org/wiki/Stepwise_regression for the details
        """
        included = list(initial_list)
        while True:
            changed=False
            # forward step
            excluded = list(set(X.columns)-set(included))
            new_pval = pd.Series(index=excluded)
            for new_column in excluded:
                model = sm.OLS(y, sm.add_constant(pd.
↪DataFrame(X[included+[new_column]]))).fit()
                new_pval[new_column] = model.pvalues[new_column]
            best_pval = new_pval.min()
            if best_pval < threshold_in:
                best_feature = new_pval.idxmin()
                included.append(best_feature)
                changed=True
                if verbose:
```

```
                    print('Add  {:30} with p-value {:.6}'.format(best_feature,␣
↪best_pval))

            # backward step
            model = sm.OLS(y, sm.add_constant(pd.DataFrame(X[included]))).fit()
            # use all coefs except intercept
            pvalues = model.pvalues.iloc[1:]
            worst_pval = pvalues.max() # null if pvalues is empty
            if worst_pval > threshold_out:
                changed=True
                worst_feature = pvalues.idxmax()
                included.remove(worst_feature)
                if verbose:
                    print('Drop {:30} with p-value {:.6}'.format(worst_feature,␣
↪worst_pval))
            if not changed:
                break
        return included
```

[23]:
```
result = stepwise_selection(X1, Y1)
```

```
Add  CRuns                          with p-value 1.88938e-29
Add  Hits                           with p-value 2.94819e-11
Add  Years                          with p-value 0.00578972
```

[24]:
```
print(result)
```

```
['CRuns', 'Hits', 'Years']
```

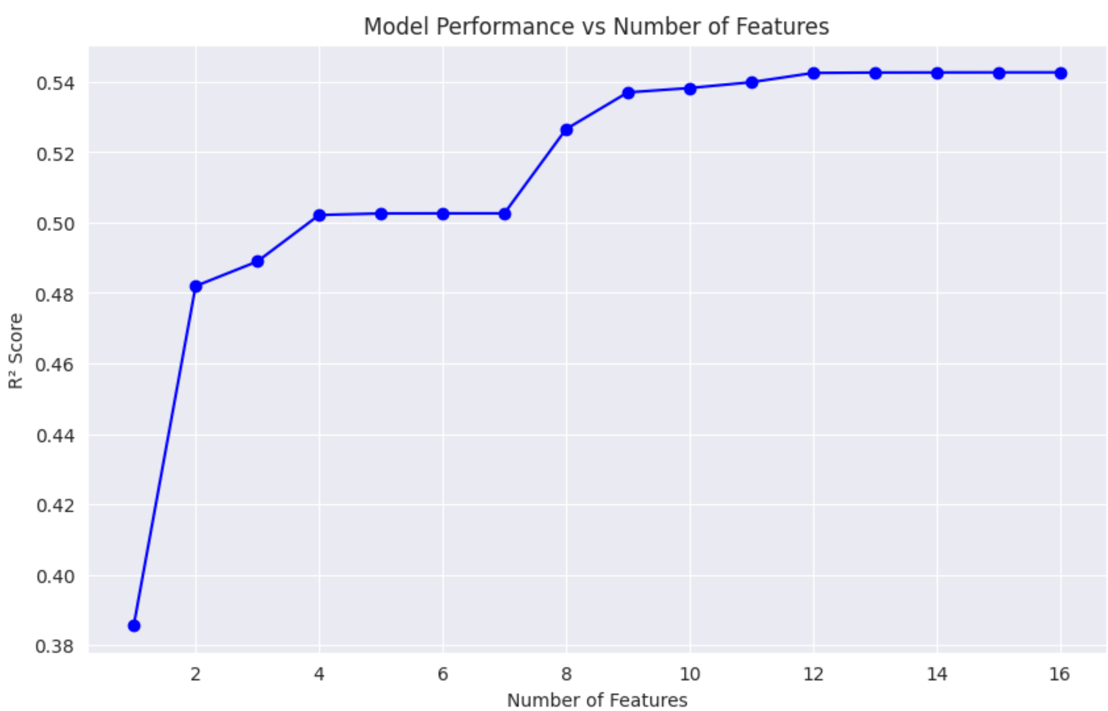
2. Build a multiple linear regression model using the RFE and the stepwise methods. (Standardized Data)

[25]:
```
print("Standard Data")
stand_feature_counts, stand_scores =␣
 ↪determine_optimal_number_of_features(X1_stand, Y1)
determine_features_within_threshold(stand_feature_counts, stand_scores)
result = stepwise_selection(X1_stand, Y1)
print('resulting features:')
print(result)
```

```
Standard Data
range(1, 17)
Num Features: 1
Selected Features: [False False False False False False False False False False
True False
 False False False False]
Feature Ranking: [ 3  2 10 13 16  8  4  5  6 15  1 14  7  9 11 12]
Num Features: 2
Selected Features: [False  True False False False False False False False False
```

```
True False
 False False False False]
Feature Ranking: [ 2  1  9 12 15  7  3  4  5 14  1 13  6  8 10 11]
Num Features: 3
Selected Features: [ True  True False False False False False False False False
True False
 False False False False]
Feature Ranking: [ 1  1  8 11 14  6  2  3  4 13  1 12  5  7  9 10]
Num Features: 4
Selected Features: [ True  True False False False False  True False False False
True False
 False False False False]
Feature Ranking: [ 1  1  7 10 13  5  1  2  3 12  1 11  4  6  8  9]
Num Features: 5
Selected Features: [ True  True False False False False  True  True False False
True False
 False False False False]
Feature Ranking: [ 1  1  6  9 12  4  1  1  2 11  1 10  3  5  7  8]
Num Features: 6
Selected Features: [ True  True False False False False  True  True  True False
True False
 False False False False]
Feature Ranking: [ 1  1  5  8 11  3  1  1  1 10  1  9  2  4  6  7]
Num Features: 7
Selected Features: [ True  True False False False False  True  True  True False
True False
  True False False False]
Feature Ranking: [ 1  1  4  7 10  2  1  1  1  9  1  8  1  3  5  6]
Num Features: 8
Selected Features: [ True  True False False False  True  True  True  True False
True False
  True False False False]
Feature Ranking: [1 1 3 6 9 1 1 1 1 8 1 7 1 2 4 5]
Num Features: 9
Selected Features: [ True  True False False False  True  True  True  True False
True False
  True  True False False]
Feature Ranking: [1 1 2 5 8 1 1 1 1 7 1 6 1 1 3 4]
Num Features: 10
Selected Features: [ True  True  True False False  True  True  True  True False
True False
  True  True False False]
Feature Ranking: [1 1 1 4 7 1 1 1 1 6 1 5 1 1 2 3]
Num Features: 11
Selected Features: [ True  True  True False False  True  True  True  True False
True False
  True  True  True False]
Feature Ranking: [1 1 1 3 6 1 1 1 1 5 1 4 1 1 1 2]
```

```
Num Features: 12
Selected Features: [ True  True  True False False  True  True  True  True False
True False
  True  True  True  True]
Feature Ranking: [1 1 1 2 5 1 1 1 1 4 1 3 1 1 1 1]
Num Features: 13
Selected Features: [ True  True  True  True False  True  True  True  True False
True False
  True  True  True  True]
Feature Ranking: [1 1 1 1 4 1 1 1 1 3 1 2 1 1 1 1]
Num Features: 14
Selected Features: [ True  True  True  True False  True  True  True  True False
True  True
  True  True  True  True]
Feature Ranking: [1 1 1 1 3 1 1 1 1 2 1 1 1 1 1 1]
Num Features: 15
Selected Features: [ True  True  True  True False  True  True  True  True  True
True  True
  True  True  True  True]
Feature Ranking: [1 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1]
Num Features: 16
Selected Features: [ True  True  True  True  True  True  True  True  True  True
True  True
  True  True  True  True]
Feature Ranking: [1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1]
```



```
Optimal number of features: 16
Best score: 0.5427
Lowest number of features within 1% of best score: 9
Add  CRuns                          with p-value 1.88938e-29
Add  Hits                           with p-value 2.94819e-11
Add  Years                          with p-value 0.00578972
resulting features:
['CRuns', 'Hits', 'Years']
```

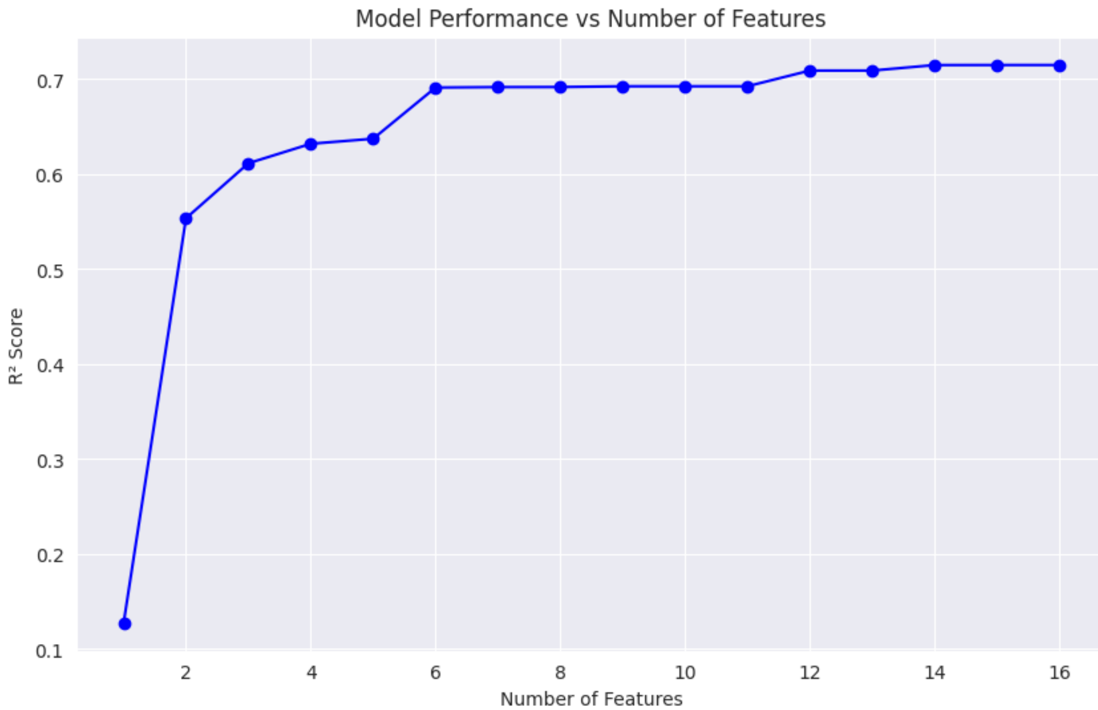
2. Build a multiple linear regression model using the RFE and the stepwise methods. (Normalized Data)

[26]:
```python
print("Normalized Data")
norm_feature_counts, norm_scores =␣
 ↪determine_optimal_number_of_features(X1_norm, Y1)
determine_features_within_threshold(norm_feature_counts, norm_scores)
result = stepwise_selection(X1_norm, Y1)
print('resulting features:')
print(result)
```

```
Normalized Data
range(1, 17)
Num Features: 1
Selected Features: [False False False False False False  True False False False
False False
 False False False False]
Feature Ranking: [ 9  2 10  8  7 11  1  6  5 13  3  4 15 14 12 16]
Num Features: 2
Selected Features: [False  True False False False False  True False False False
False False
 False False False False]
Feature Ranking: [ 8  1  9  7  6 10  1  5  4 12  2  3 14 13 11 15]
Num Features: 3
Selected Features: [False  True False False False False  True False False False
True False
 False False False False]
Feature Ranking: [ 7  1  8  6  5  9  1  4  3 11  1  2 13 12 10 14]
Num Features: 4
Selected Features: [False  True False False False False  True False False False
True  True
 False False False False]
Feature Ranking: [ 6  1  7  5  4  8  1  3  2 10  1  1 12 11  9 13]
Num Features: 5
Selected Features: [False  True False False False False  True False  True False
True  True
 False False False False]
Feature Ranking: [ 5  1  6  4  3  7  1  2  1  9  1  1 11 10  8 12]
```

```
Num Features: 6
Selected Features: [False  True False False False False  True  True  True False
True  True
 False False False False]
Feature Ranking: [ 4  1  5  3  2  6  1  1  1  8  1  1 10  9  7 11]
Num Features: 7
Selected Features: [False  True False False  True False  True  True  True False
True  True
 False False False False]
Feature Ranking: [ 3  1  4  2  1  5  1  1  1  7  1  1  9  8  6 10]
Num Features: 8
Selected Features: [False  True False  True  True False  True  True  True False
True  True
 False False False False]
Feature Ranking: [2 1 3 1 1 4 1 1 1 6 1 1 8 7 5 9]
Num Features: 9
Selected Features: [ True  True False  True  True False  True  True  True False
True  True
 False False False False]
Feature Ranking: [1 1 2 1 1 3 1 1 1 5 1 1 7 6 4 8]
Num Features: 10
Selected Features: [ True  True  True  True  True False  True  True  True False
True  True
 False False False False]
Feature Ranking: [1 1 1 1 1 2 1 1 1 4 1 1 6 5 3 7]
Num Features: 11
Selected Features: [ True  True  True  True  True  True  True  True  True False
True  True
 False False False False]
Feature Ranking: [1 1 1 1 1 1 1 1 1 3 1 1 5 4 2 6]
Num Features: 12
Selected Features: [ True  True  True  True  True  True  True  True  True False
True  True
 False False  True False]
Feature Ranking: [1 1 1 1 1 1 1 1 1 2 1 1 4 3 1 5]
Num Features: 13
Selected Features: [ True  True  True  True  True  True  True  True  True  True
True  True
 False False  True False]
Feature Ranking: [1 1 1 1 1 1 1 1 1 1 1 1 3 2 1 4]
Num Features: 14
Selected Features: [ True  True  True  True  True  True  True  True  True  True
True  True
 False  True  True False]
Feature Ranking: [1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 3]
Num Features: 15
Selected Features: [ True  True  True  True  True  True  True  True  True  True
True  True
```

```
  True  True  True False]
Feature Ranking: [1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2]
Num Features: 16
Selected Features: [ True  True  True  True  True  True  True  True  True  True
True  True
  True  True  True  True]
Feature Ranking: [1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1]
```




```
Optimal number of features: 16
Best score: 0.7149
Lowest number of features within 1% of best score: 11
Add  AtBat                          with p-value 2.04387e-46
Add  CRuns                          with p-value 1.68308e-09
Add  Years                          with p-value 1.1962e-06
Add  CRBI                           with p-value 0.00123202
Add  Assists                        with p-value 0.00102767
Add  PutOuts                        with p-value 0.00233182
Add  CHits                          with p-value 0.000270681
Add  CAtBat                         with p-value 9.77072e-05
resulting features:
['AtBat', 'CRuns', 'Years', 'CRBI', 'Assists', 'PutOuts', 'CHits', 'CAtBat']
```

Normalization Impact: - Substantially increases best score from 0.527 to 0.7149 (35%). - Gives additional features which weren't significant in other methods. The top feature aligns with raw data. - Appears to better represent the data by finding additional features leading to a better overall score.

Standardized Impact: - Same top 3 features as raw, same best score. - Chose feature 11 as highest (different from raw)

```
[ ]:
```