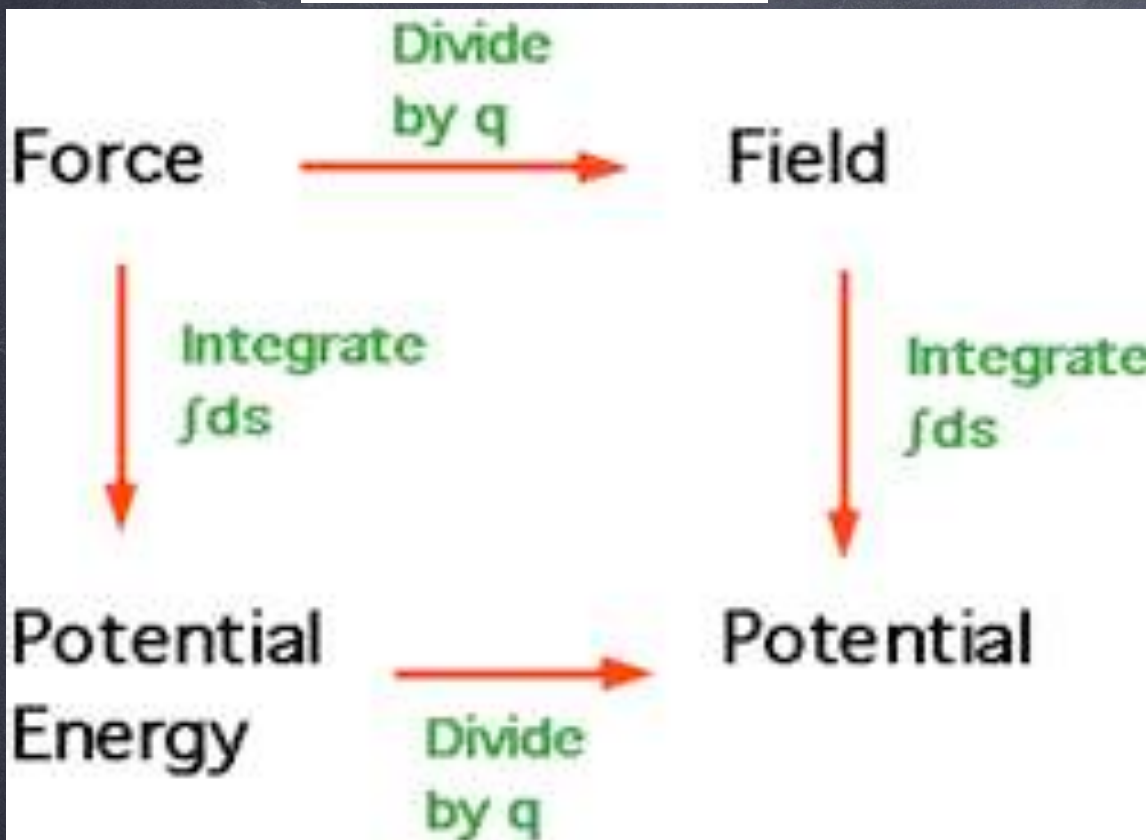
The big picture: Electric Force, Field, Potential Energy and Potential

$$\vec{F}_{elec} = q_o \vec{E}$$



$$V \equiv \frac{PE_{elec}}{q_2}$$

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

$$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$$

Physics 2B Master Class in Electricity & Magnetism

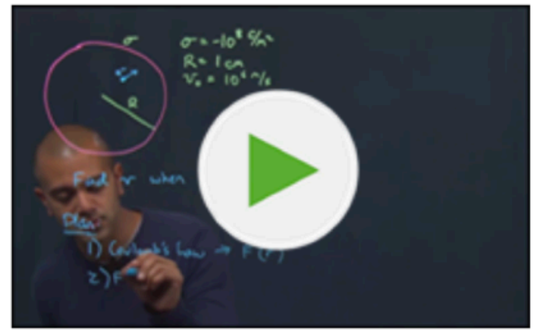
Welcome to the Physics 2B Master Class in Electricity & Magnetism video site.

Before watching each video, please download the supplementary solution so that you can follow along with the algebra which will be skimmed over in the video. You are encouraged to pause the video at the problem statement, read the problem a few times, and attempt it on your own to see where you get stuck. Please be sure to leave feedback about each video so that we can gear the videos towards whatever you find most useful. We hope these will be a great tool for you throughout the course!

Coulomb's Law [13:05]

Solution Feedback

Coulomb's Law and Charged Shells [9:34]

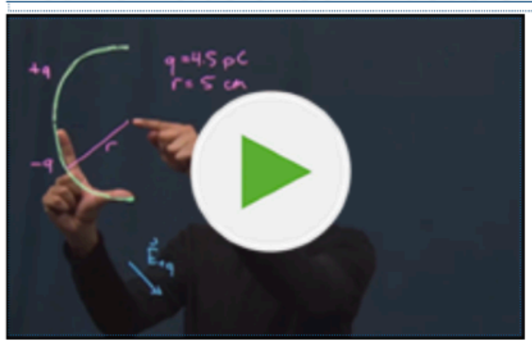


Solution Feedback

Electric Fields of Point Charges [10:45]

Solution Feedback

Electric Fields of Continuous Charge [13:59]



Solution Feedback

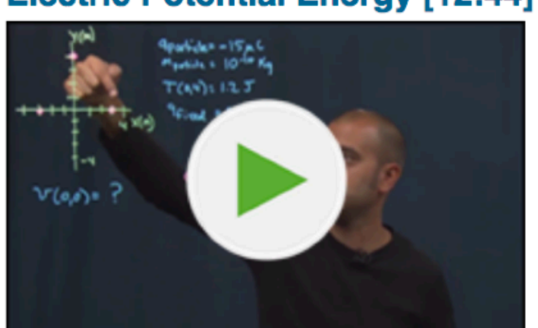
Gauss' Law 1 [11:52]

Solution Feedback

Gauss' Law II [9:27]

Solution Feedback

Electric Potential Energy [12:44]

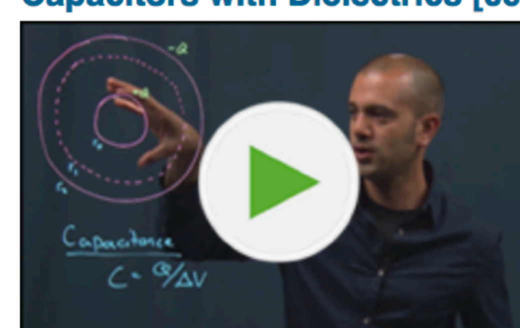


Solution Feedback

Calculating Electric Fields from Electric Potentials [09:09]

Solution Feedback

Capacitors with Dielectrics [09:32]

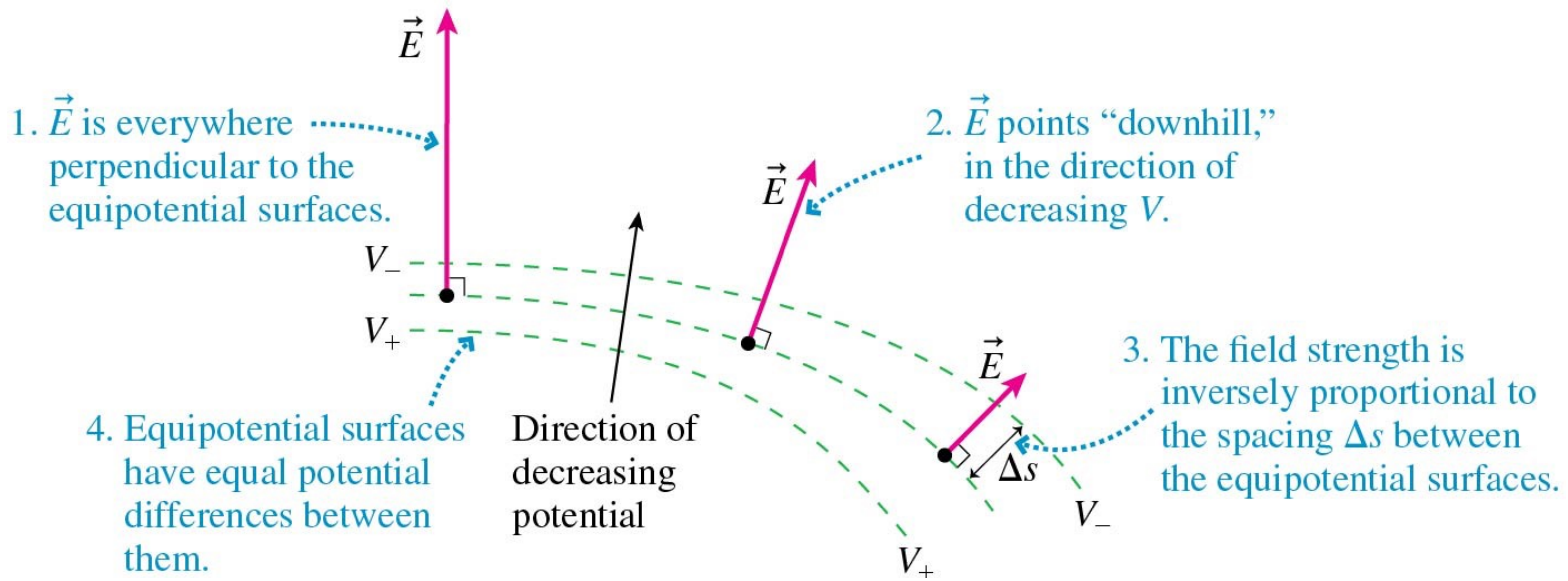


Solution Feedback

The Geometry of Potential and Field

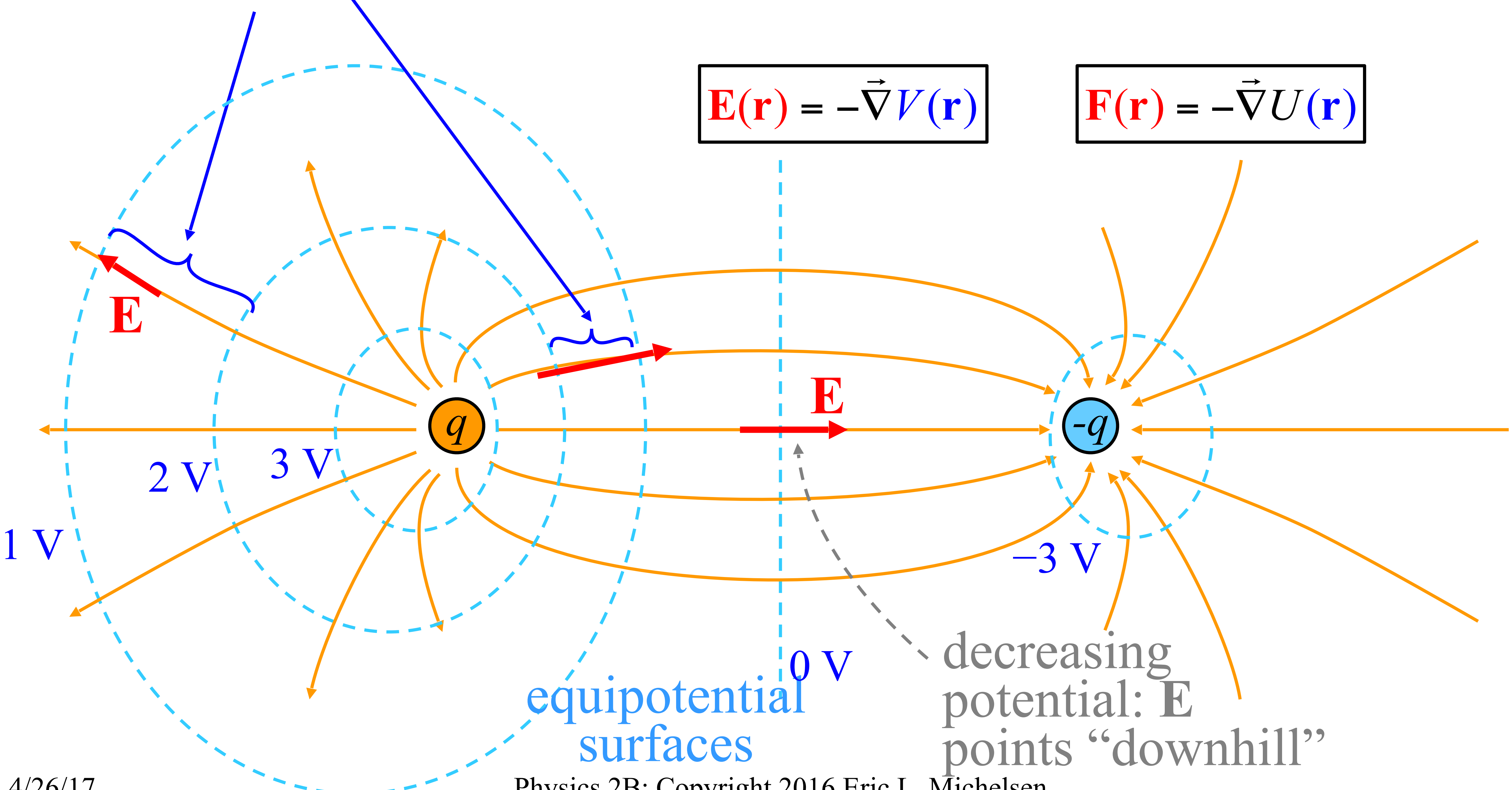
- In three dimensions, we can find the electric field from the electric potential as

$$\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k} = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right)$$



Relationship between E-field and V-field

- **E** points between equipotentials
- **E** is perpendicular to equipotentials
  - Tighter equipotential surfaces = stronger **E** (more volts/meter)
  - Wider equipotential surfaces = weaker **E** (less volts/meter)



iClicker question 8-1

To know all of the electrostatic properties of a region of space, I need ...

A the electric field, **E**(**r**)

B the potential (i.e. voltage) field, *V*(**r**)

C both fields

D either field

E *V* at one point

Finding the Potential from the Electric Field

- The potential difference between two points in space is

$$\Delta V = V_f - V_i = -\int_{s_i}^{s_f} E_s\, ds = -\int_i^f \vec{E} \cdot d\vec{s}$$

  where s is the position along a line from point i to point f.
- We can find the potential difference between two points if we know the electric field.
- Thus a graphical interpretation of the equation above is

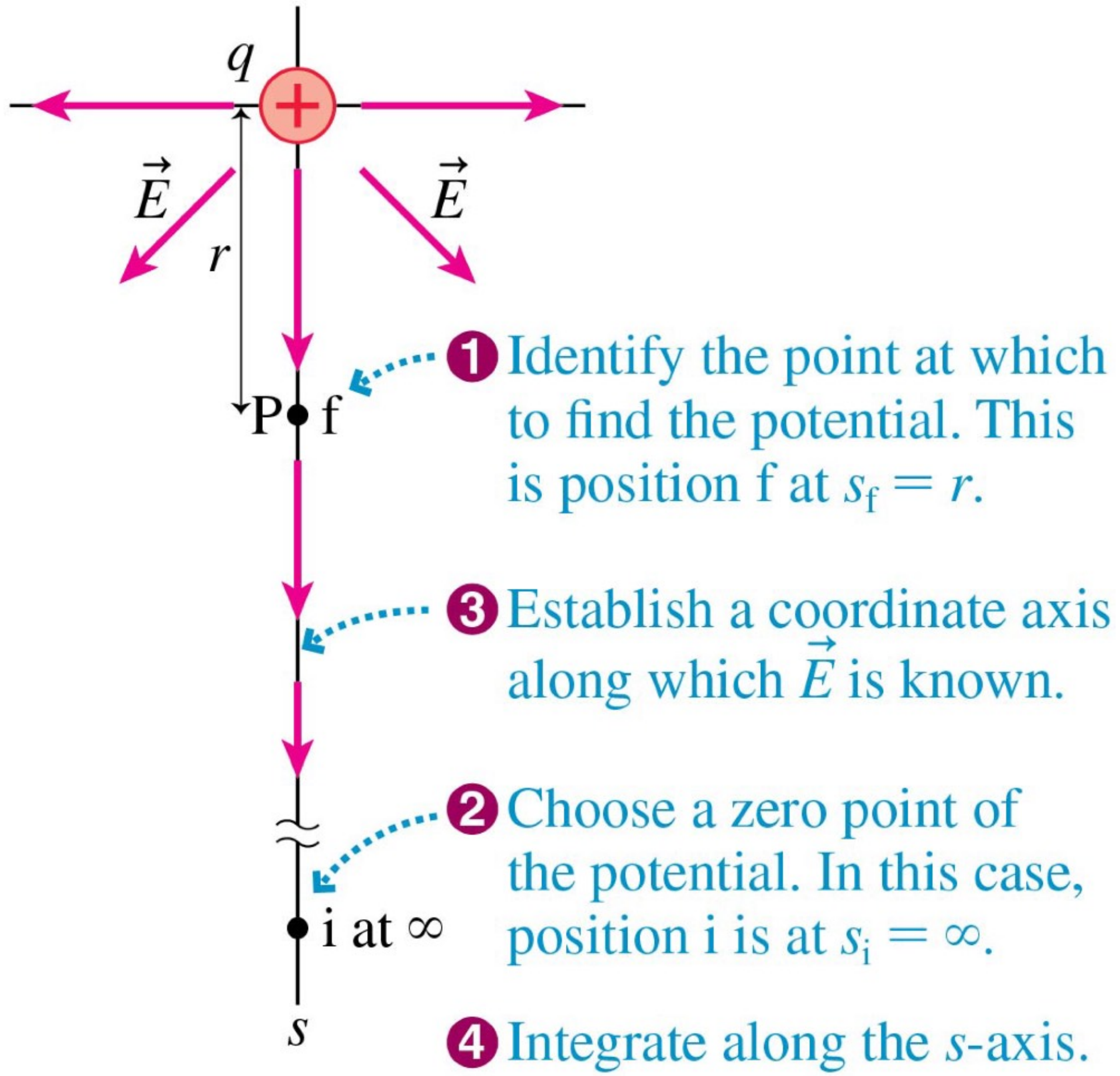
$$V_f = V_i - (\text{area under the } E_s\text{-versus-}s \text{ curve between } s_i \text{ and } s_f)$$

Finding the Potential of a Point Charge

$$E_s = \frac{1}{4\pi\epsilon_0}\frac{q}{s^2}$$

$$V(r) = V(\infty) + \frac{q}{4\pi\epsilon_0}\int_r^\infty \frac{ds}{s^2}$$

$$V_{\text{point charge}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$



Example: potential of charged disk

- For example, in Chapter 23 we found that the electric field on the z-axis from a charged disk was:

$$E_{disk} = \frac{\sigma}{2\varepsilon_o}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

- If we wanted to find the electric potential we can turn to:

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

- Evaluating the right side we get:

$$\int_i^f \vec{E} \cdot d\vec{s} = \frac{\sigma}{2\varepsilon_o}\int_\infty^z \left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right) dz$$

Example: potential of charged disk

- The first term is easy, but the second term needs a u-substitution, let u = (z^2+R^2)

$$\int_i^f \vec{E}\cdot d\vec{s} = \frac{\sigma}{2\varepsilon_o}\left(z-\sqrt{z^2+R^2}\right)_\infty^z$$

$$\int_i^f \vec{E}\cdot d\vec{s} = \frac{\sigma}{2\varepsilon_o}\left(z-\sqrt{z^2+R^2}\right)$$

Turning back to our equation for potential:

$$V_f - V_i = -\frac{\sigma}{2\varepsilon_o}\left(z-\sqrt{z^2+R^2}\right)$$

- We usually define a place (like V_i to be zero at infinite distance), yielding:

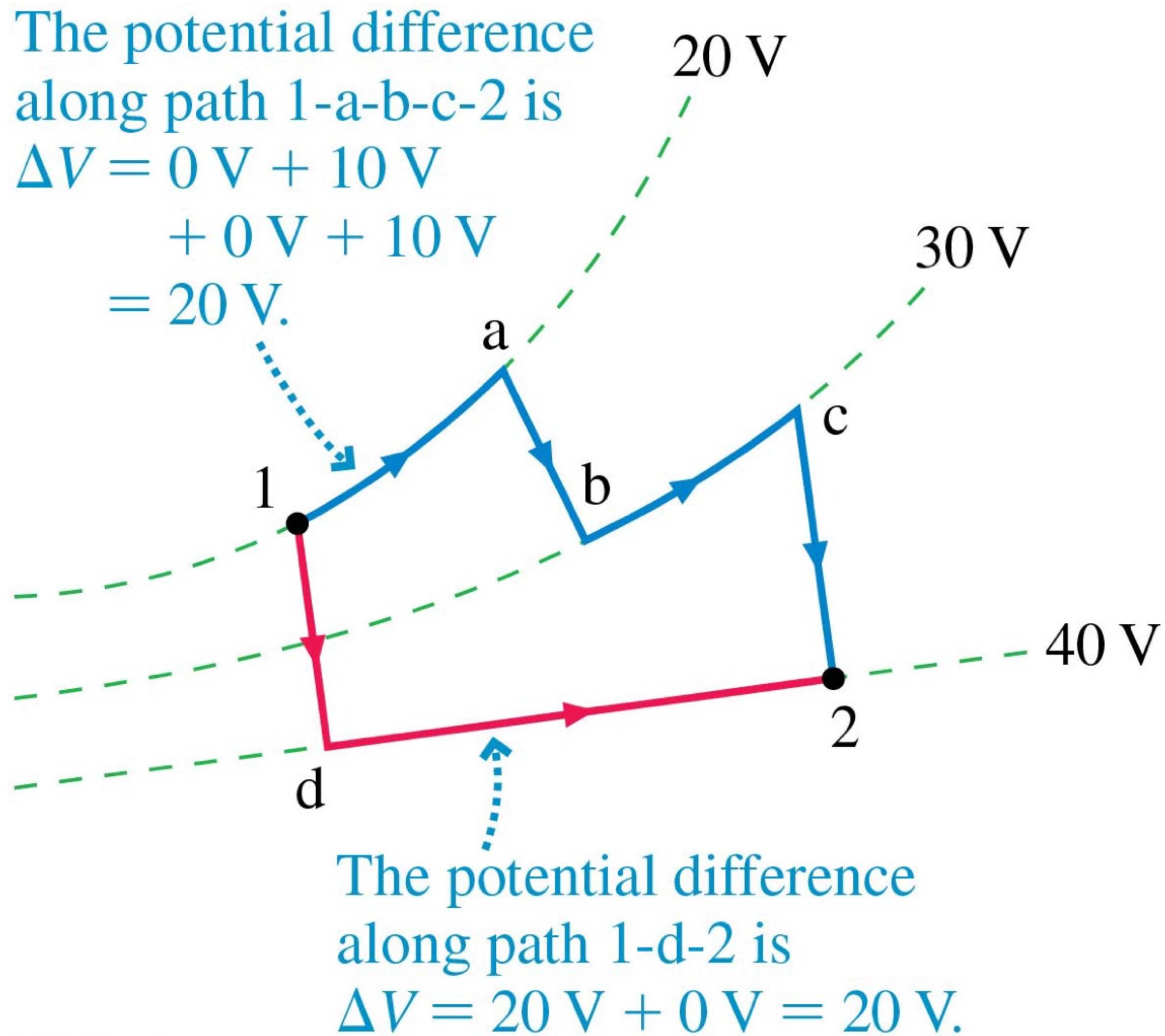
$$V = \frac{\sigma}{2\varepsilon_o}\left(\sqrt{z^2+R^2}-z\right)$$

Kirchhoff's Loop Law

- For any path that starts and ends at the same point:

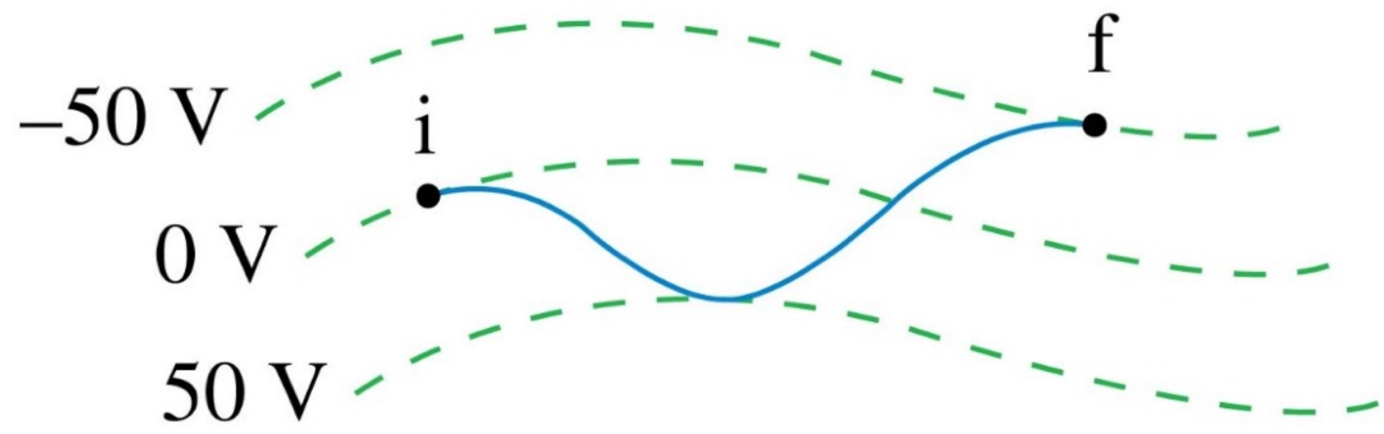
$$\Delta V_{\text{loop}} = \sum_i (\Delta V)_i = 0$$

- The sum of all the potential differences encountered while moving around a loop or closed path is zero.
- This statement is known as **Kirchhoff's loop law**.



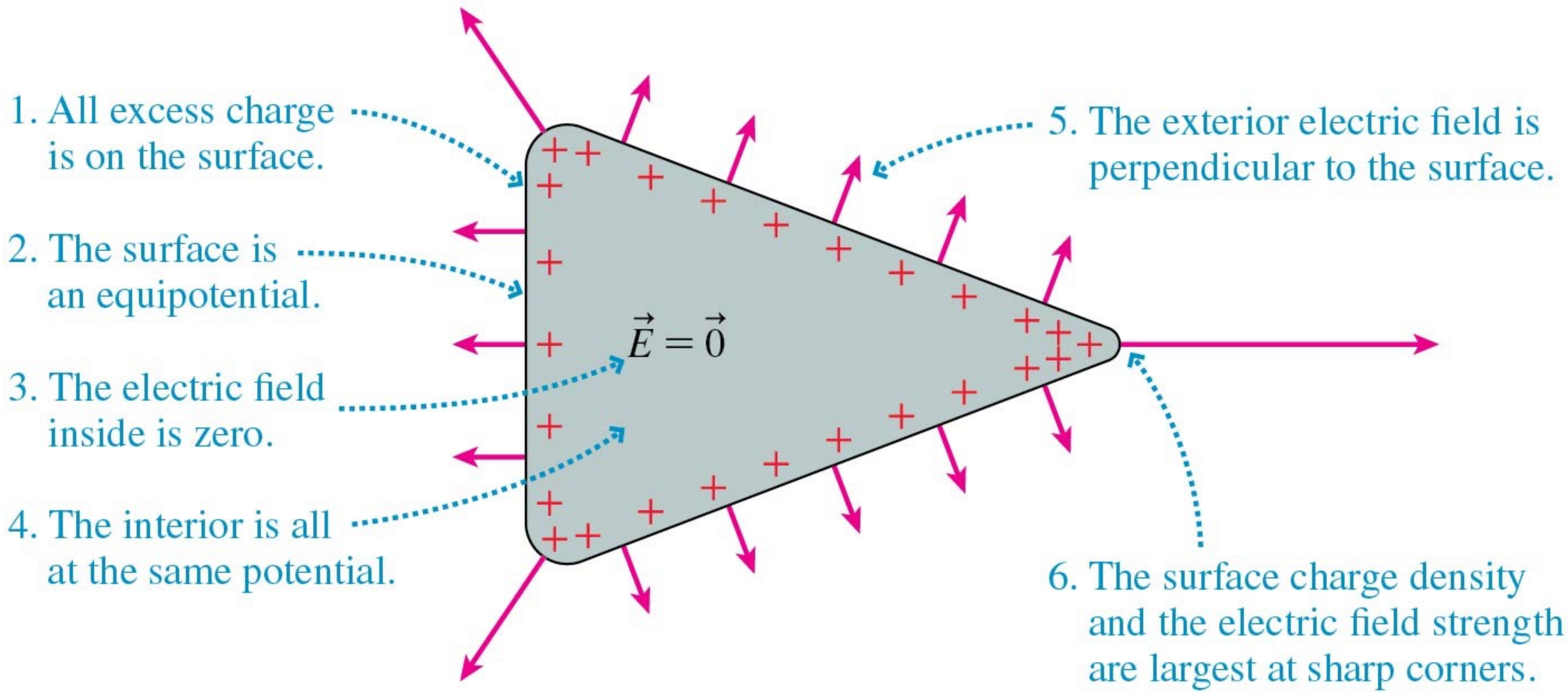
iClicker question 8-2

A particle follows the trajectory shown from initial position i to final position f. The potential difference ΔV *is*



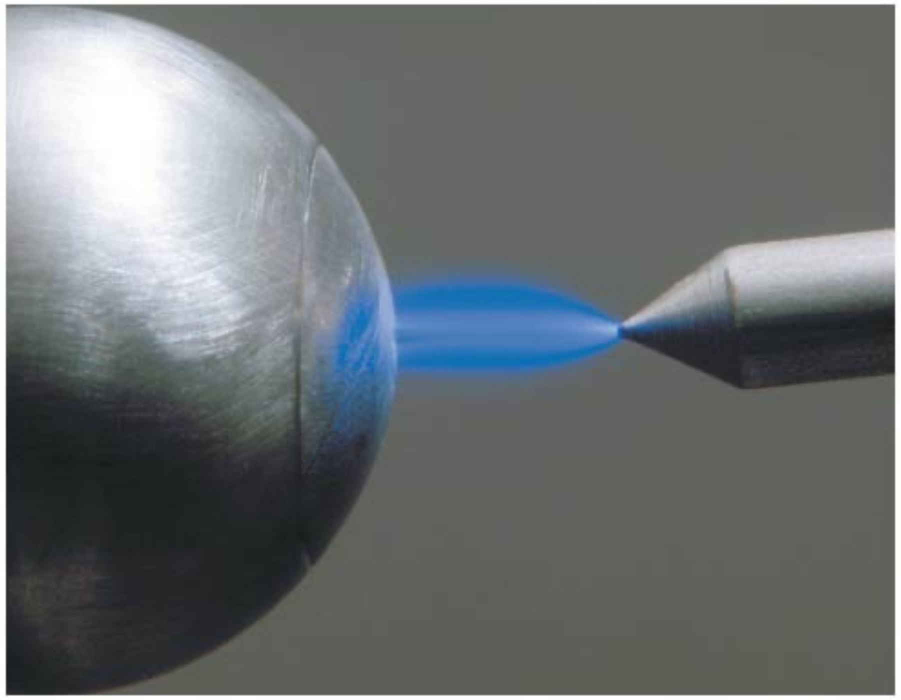
A. 100 V
B. 50 V
C. 0 V
D. –50 V
E. –100 V

A Conductor in Electrostatic Equilibrium



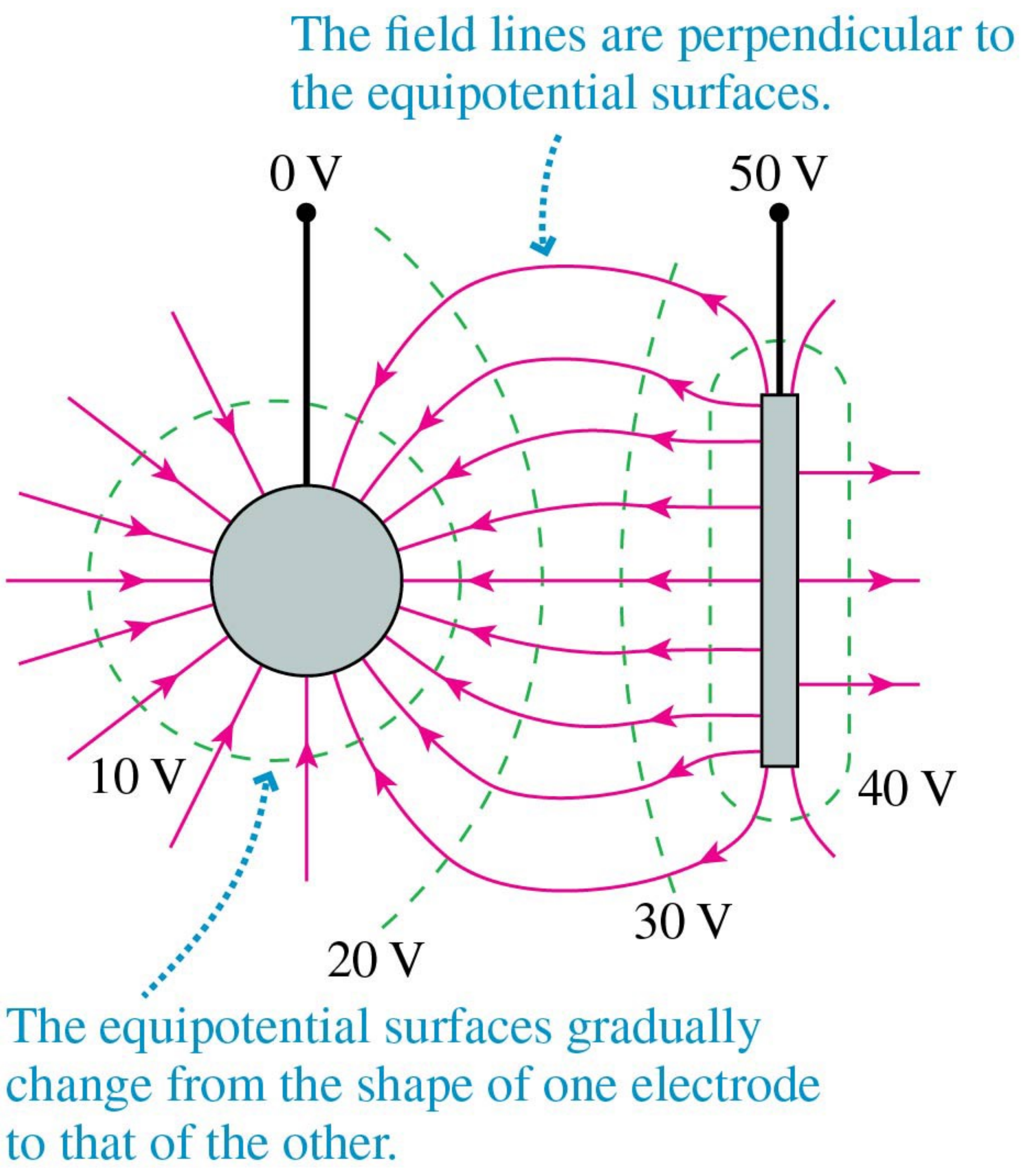
A Conductor in Electrostatic Equilibrium

- When a conductor is in equilibrium:
  - All excess charge sits on the surface.
  - The surface is an equipotential.
  - The electric field inside is zero.
  - The external electric field is perpendicular to the surface at the surface.
  - The electric field is strongest at sharp corners of the conductor's surface.



A corona discharge occurs at pointed metal tips where the electric field can be very strong.

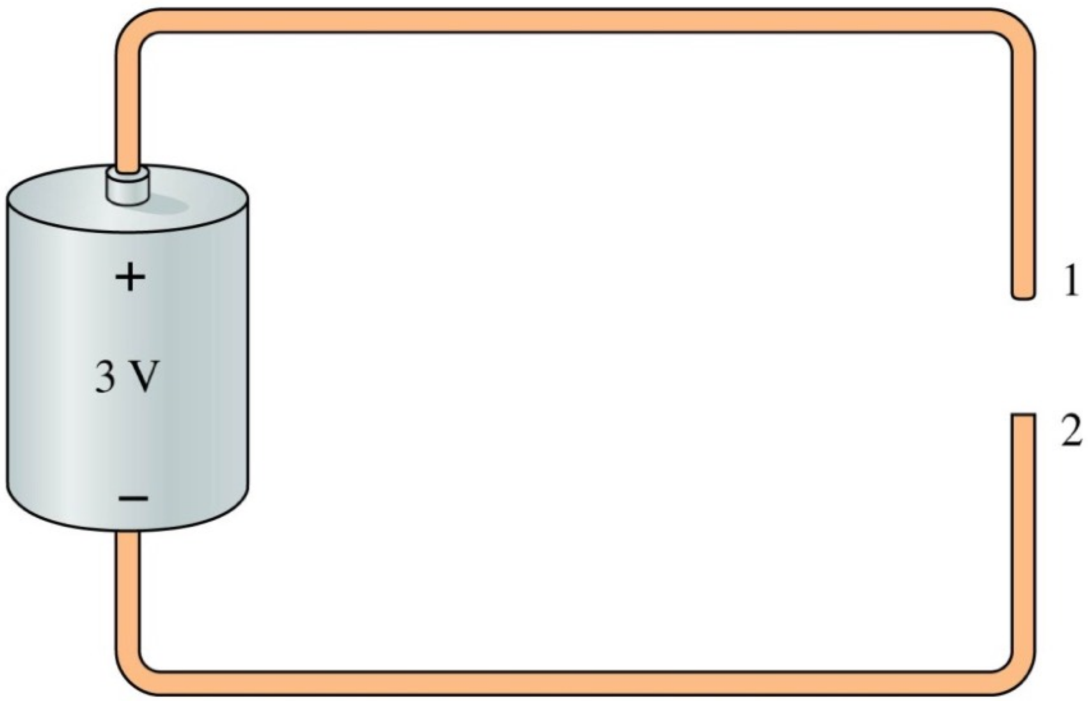
A Conductor in Electrostatic Equilibrium



- The figure shows a negatively charged metal sphere near a flat metal plate.
- Since a conductor surface must be an equipotential, the equipotential surfaces close to each electrode roughly match the shape of the electrode.

iClicker question 8-3

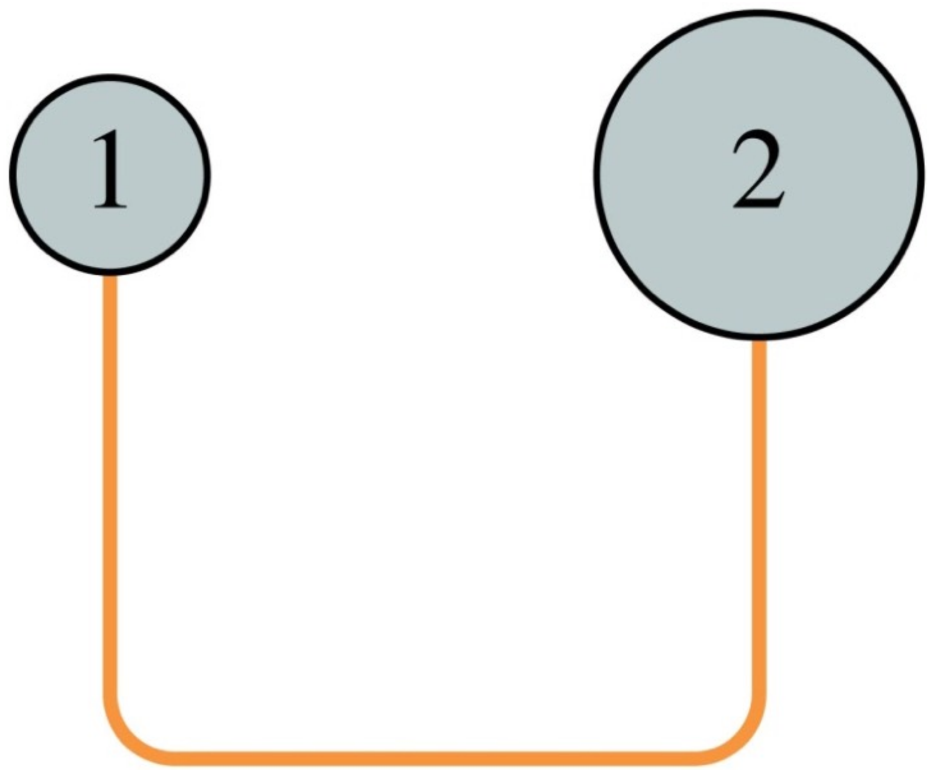
Metal wires are attached to the terminals of a 3 V battery. What is the potential difference between points 1 and 2?



A. 6 V
B. 3 V
C. 0 V
D. Undefined.
E. Not enough information to tell.

iClicker question 8-4

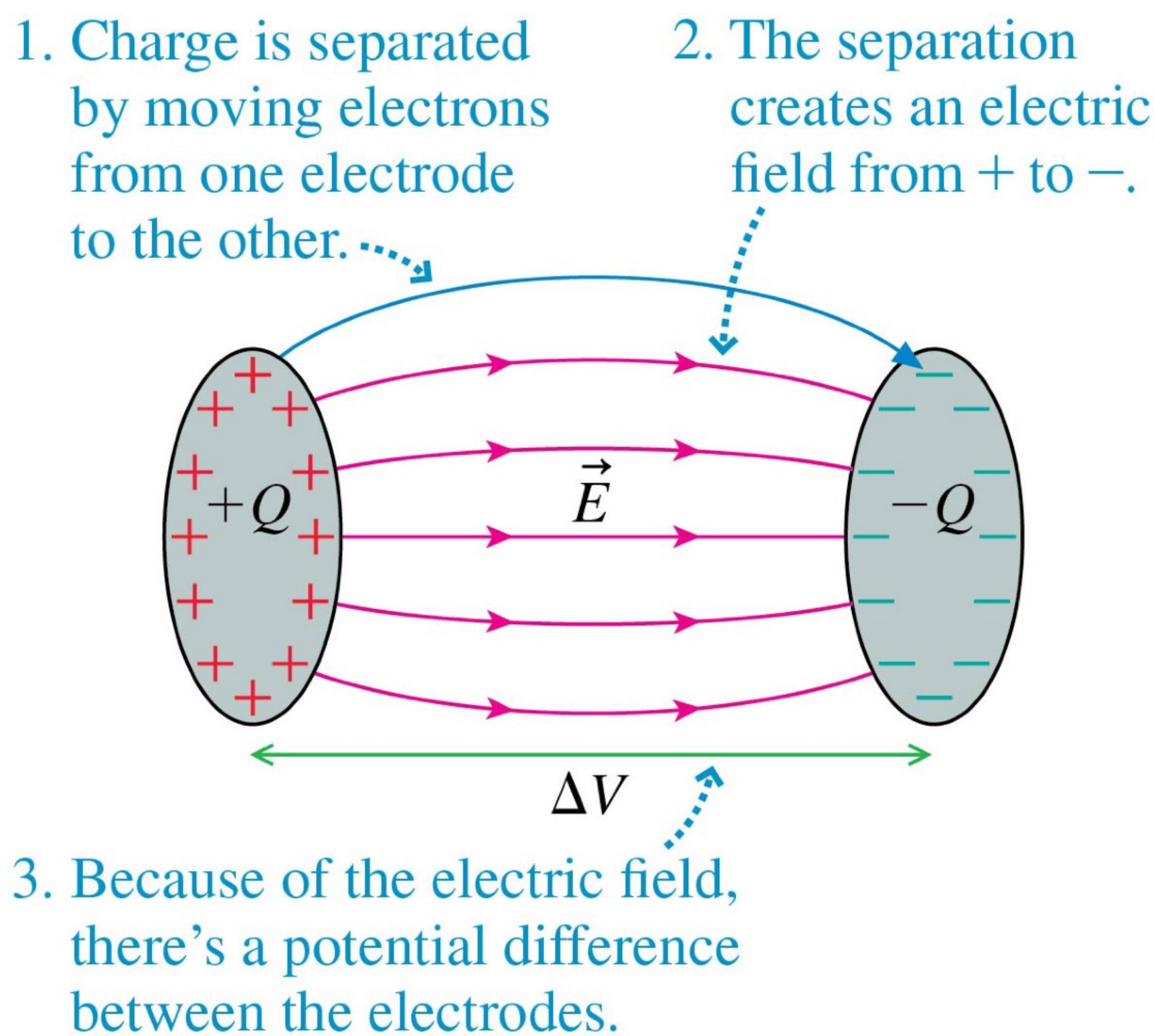
Metal spheres 1 and 2 are connected by a metal wire. What quantities do spheres 1 and 2 have in common?



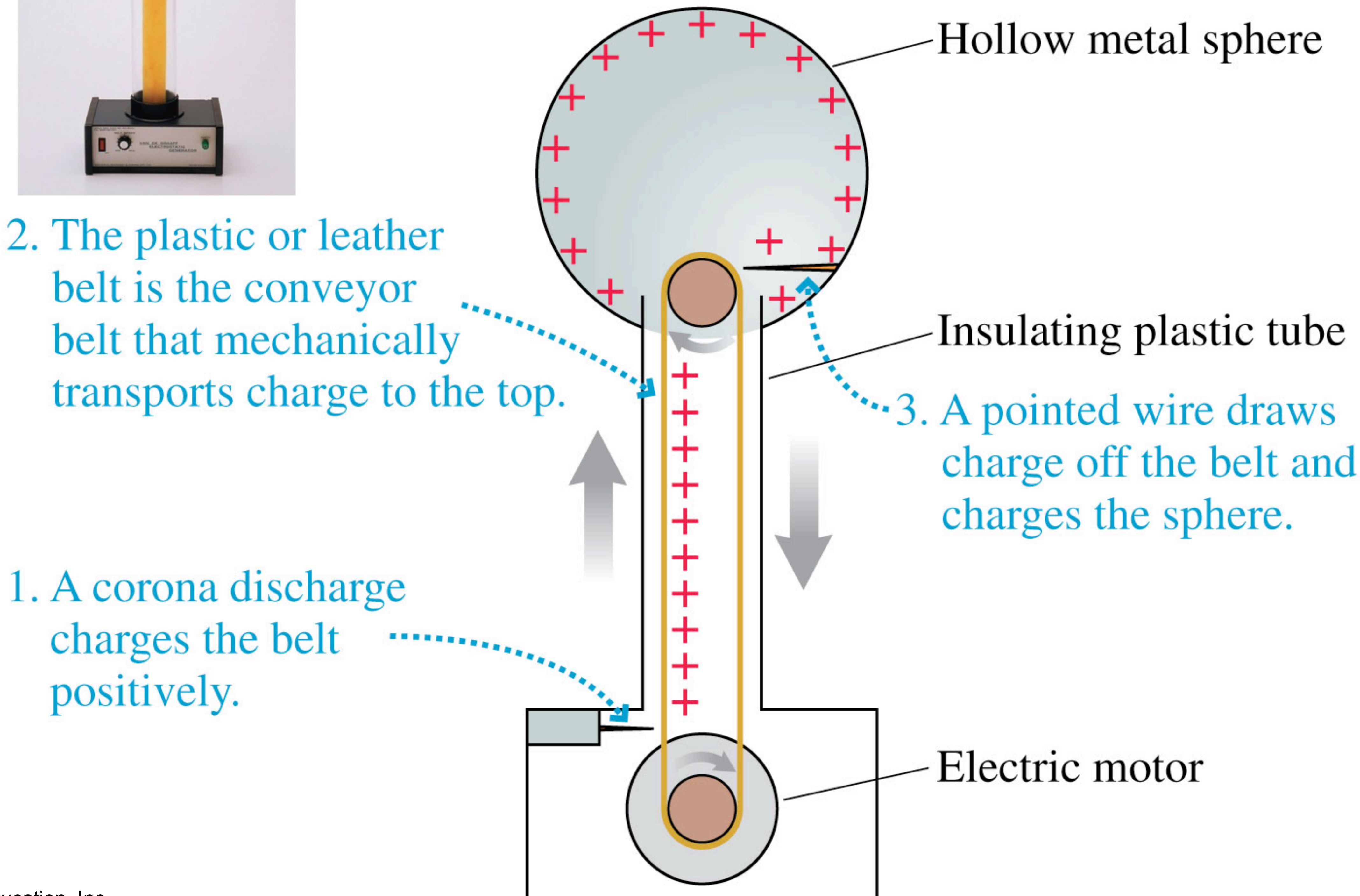
A. Same potential
B. Same electric field
C. Same charge
D. Both A and B
E. Both A and C

Sources of Electric Potential

- A *separation of charge* creates an electric potential difference.
- Shuffling your feet on the carpet transfers electrons from the carpet to you, creating a potential difference between you and other objects in the room.
- This potential difference can cause sparks.



Van de Graaff Generator



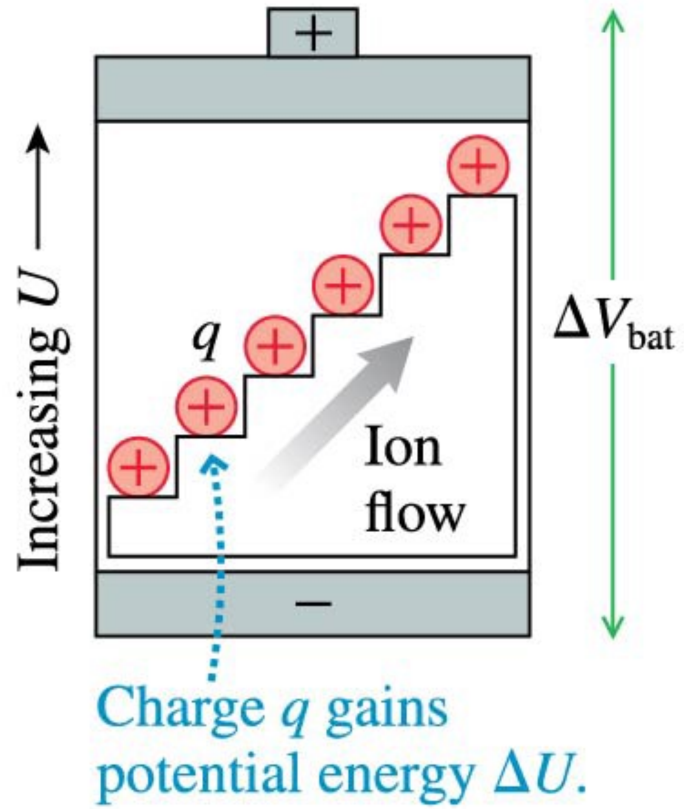
Charge escalator model of a battery

MODEL 26.1

Charge escalator model of a battery

A battery uses chemical reactions to separate charge.

- The charge escalator "lifts" positive charges from the negative terminal to the positive terminal. This requires *work*, with the energy being supplied by the chemical reactions.
- The work done *per charge* is called the **emf** of the battery: $\mathcal{E} = W_{\text{chem}}/q$.
- The charge separation creates a potential difference ΔV_{bat} between the terminals. An *ideal battery* has $\Delta V_{\text{bat}} = \mathcal{E}$.



- Limitations: $\Delta V_{\text{bat}} < \mathcal{E}$ if current flows through the battery. In most cases, the difference is small and a battery can be considered ideal.

Batteries and emf

- **emf** is the work done per charge to pull positive and negative charges apart.
- In an ideal battery, this work creates a potential difference $\Delta V_{\text{bat}} = \mathcal{E}$ between the positive and negative terminals.
- This is called the terminal voltage.

A battery constructed to have an emf of $1.5\ \text{V}$ creates a $1.5\ \text{V}$ potential difference between its positive and negative terminals.

iClicker question 8-5

The charge escalator in a battery does 4.8×10^{-19} J of work for each positive ion that it moves from the negative to the positive terminal. What is the battery's emf?

$e = 1.6 \times 10^{-19}$ C for an ion

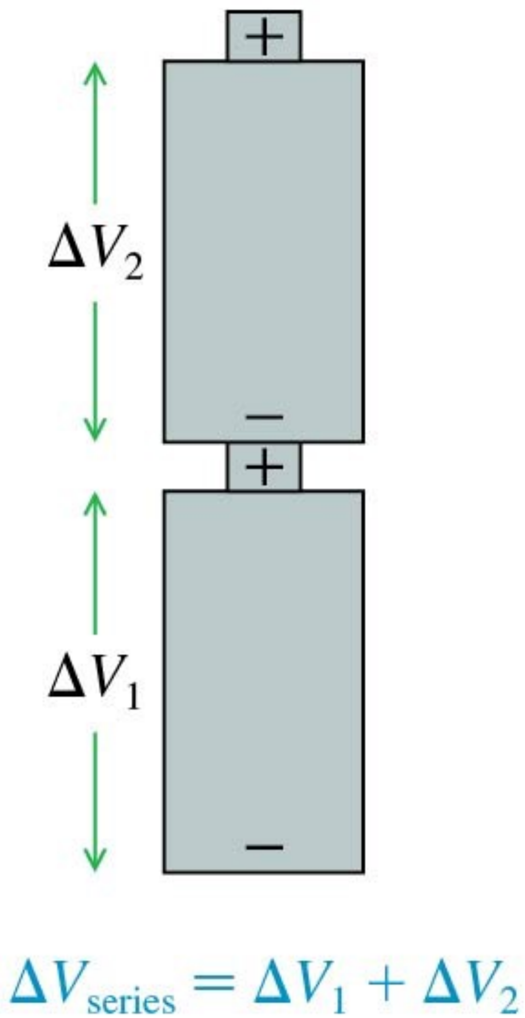
A. 9 V
B. 4.8 V
C. 3 V
D. 4.8×10^{-19} V
E. I have no idea.

- The total potential difference of batteries in series is simply the sum of their individual terminal voltages:

$$\Delta V_{\text{series}} = \Delta V_1 + \Delta V_2 + \cdots$$

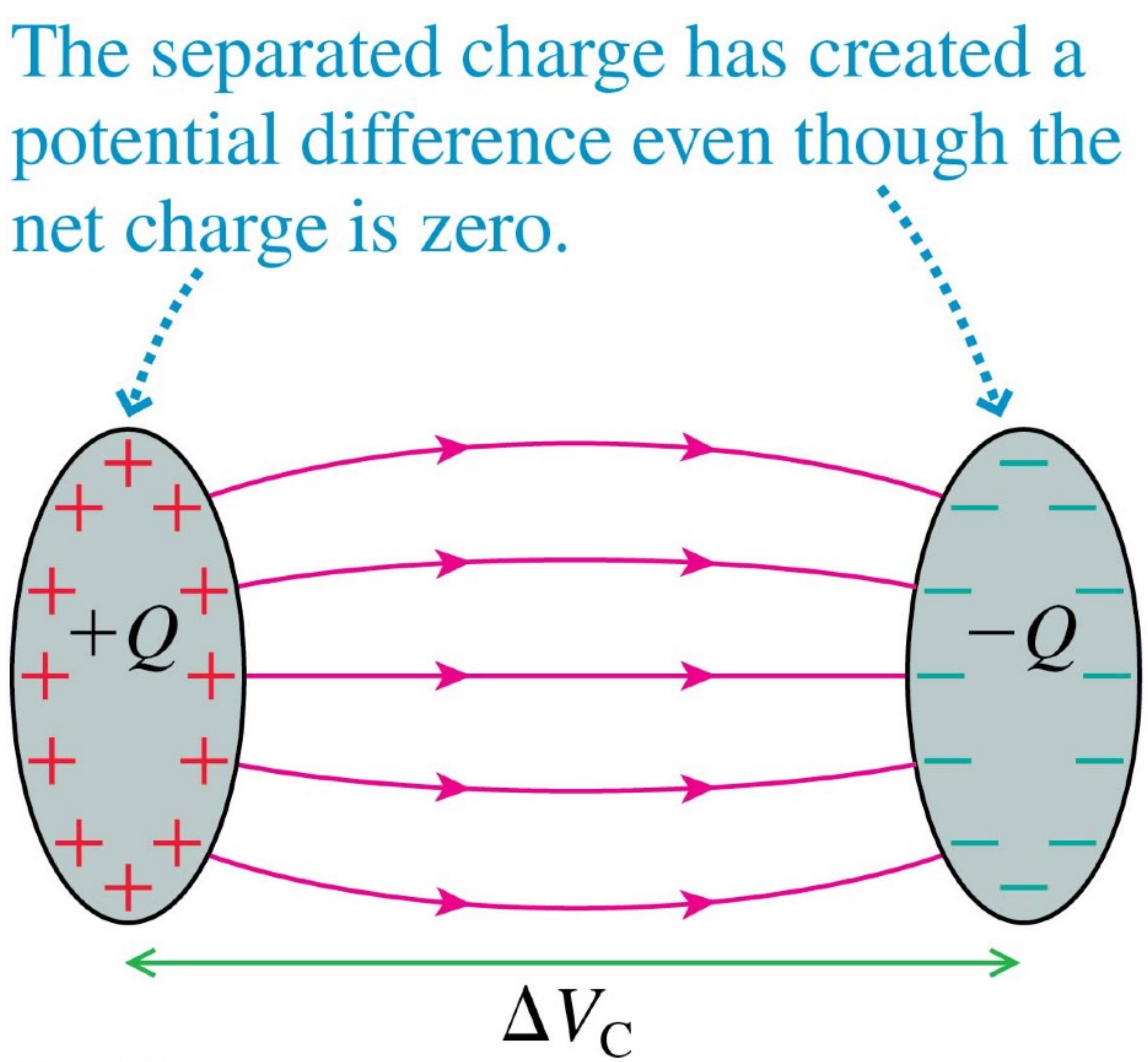
- Flashlight batteries are placed in series to create twice the potential difference of one battery.
- For this flashlight:

$$\begin{aligned}\Delta V_{\text{series}} &= \Delta V_1 + \Delta V_2\\ &= 1.5\text{ V} + 1.5\text{ V}\\ &= 3.0\text{ V}\end{aligned}$$



- X-ray picture of flashlight
- In vacuum,
  X-rays from tape!

Capacitance and Capacitors



- The figure shows two arbitrary electrodes charged to $\pm Q$.
- There is a potential difference ΔV_C that is directly proportional to Q.

- The ratio of the charge Q to the potential difference ΔV_C is called the **capacitance** C:

$$C \equiv \frac{Q}{\Delta V_C} = \frac{\epsilon_0 A}{d} \qquad \text{(parallel-plate capacitor)}$$

Capacitance and Capacitors

- Capacitance is a purely *geometric* property of two electrodes because it depends only on their surface area and spacing.
- The SI unit of capacitance is the **farad**:

$$1 \text{ farad} = 1 \text{ F} = 1 \text{ C/V}$$

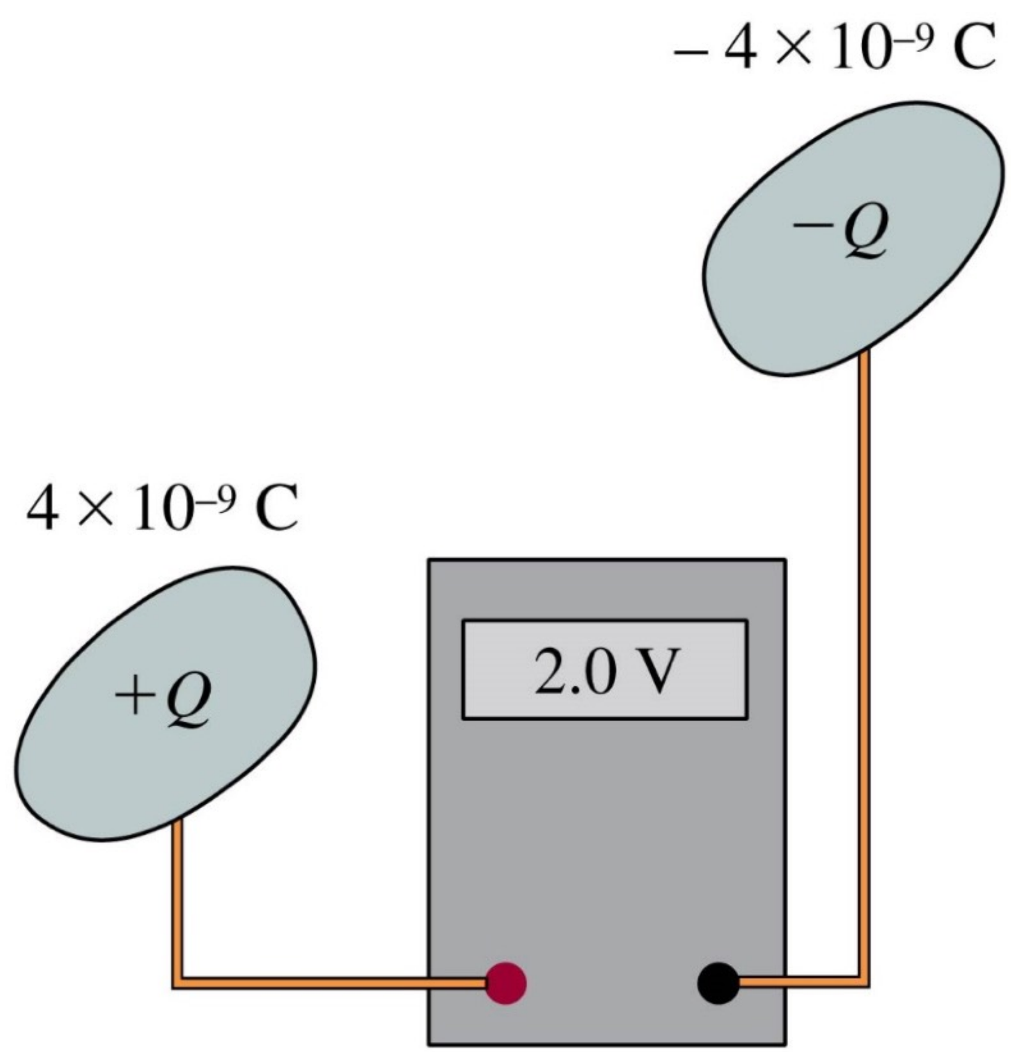
- **The charge on the capacitor plates is directly proportional to the potential difference between the plates:**

$$Q = C\Delta V_{\mathrm{C}} \qquad \text{(charge on a capacitor)}$$

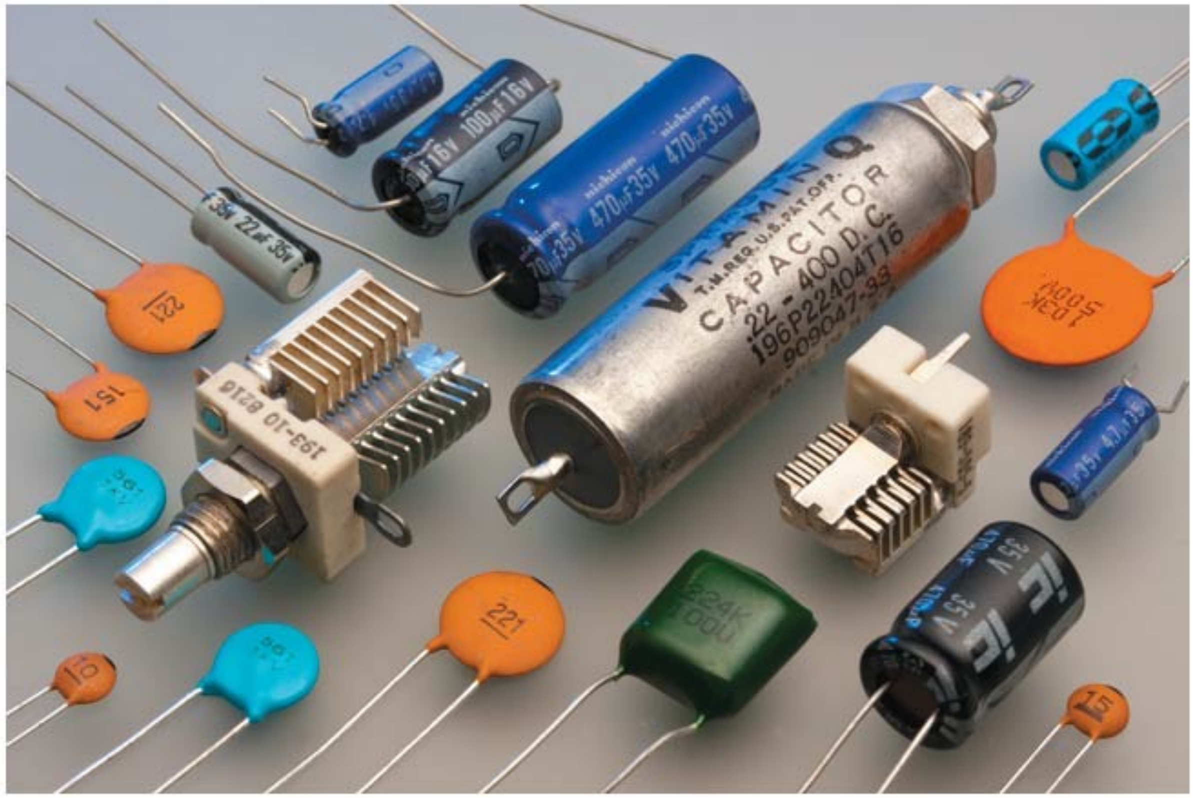
iClicker question 8-6

What is the capacitance of these two electrodes?

A. 8 nF
B. 4 nF
C. 2 nF
D. 1 nF
E. Some other value



Capacitance and Capacitors

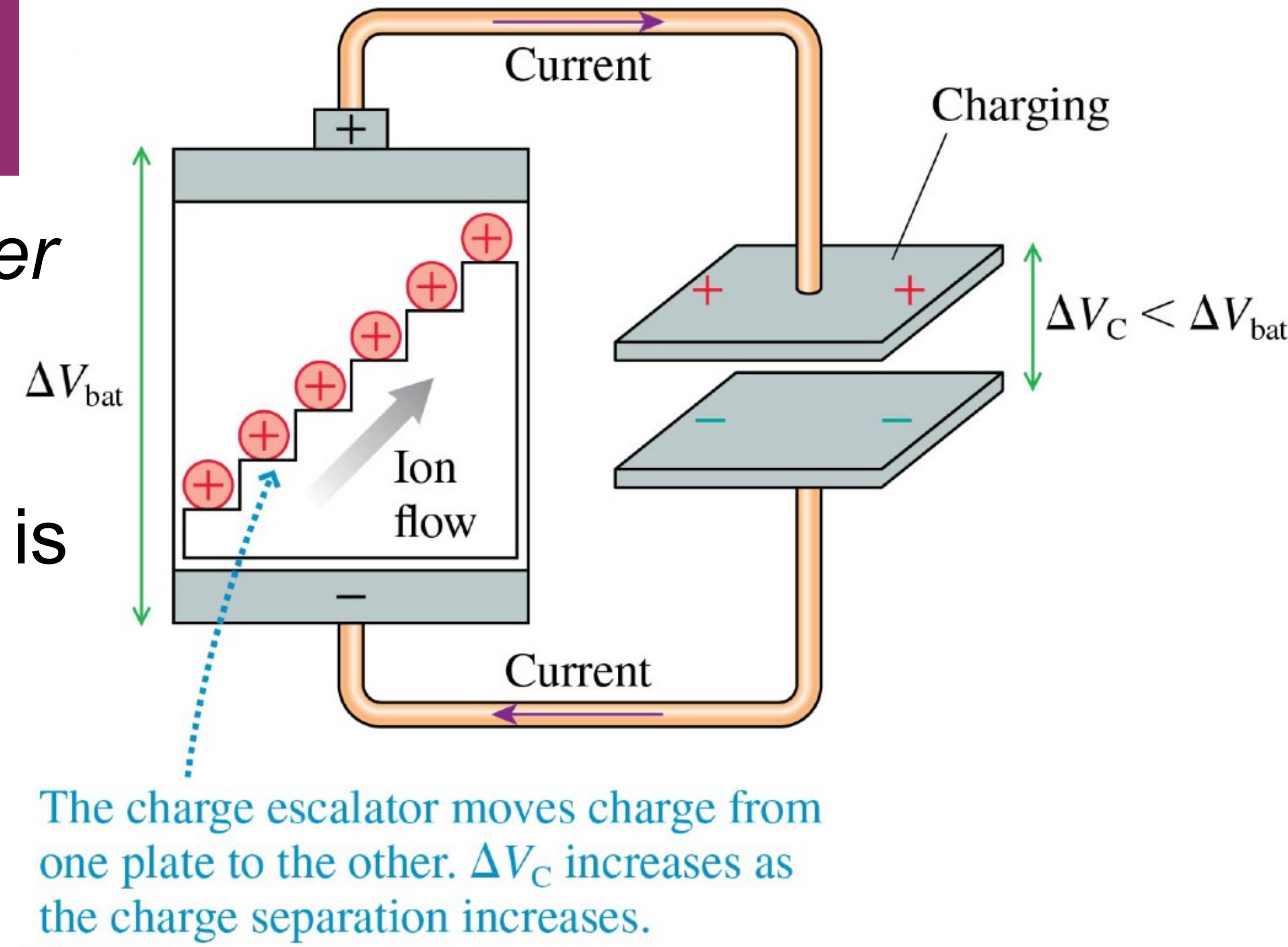


Capacitors are important elements in electric circuits. They come in a variety of sizes and shapes.

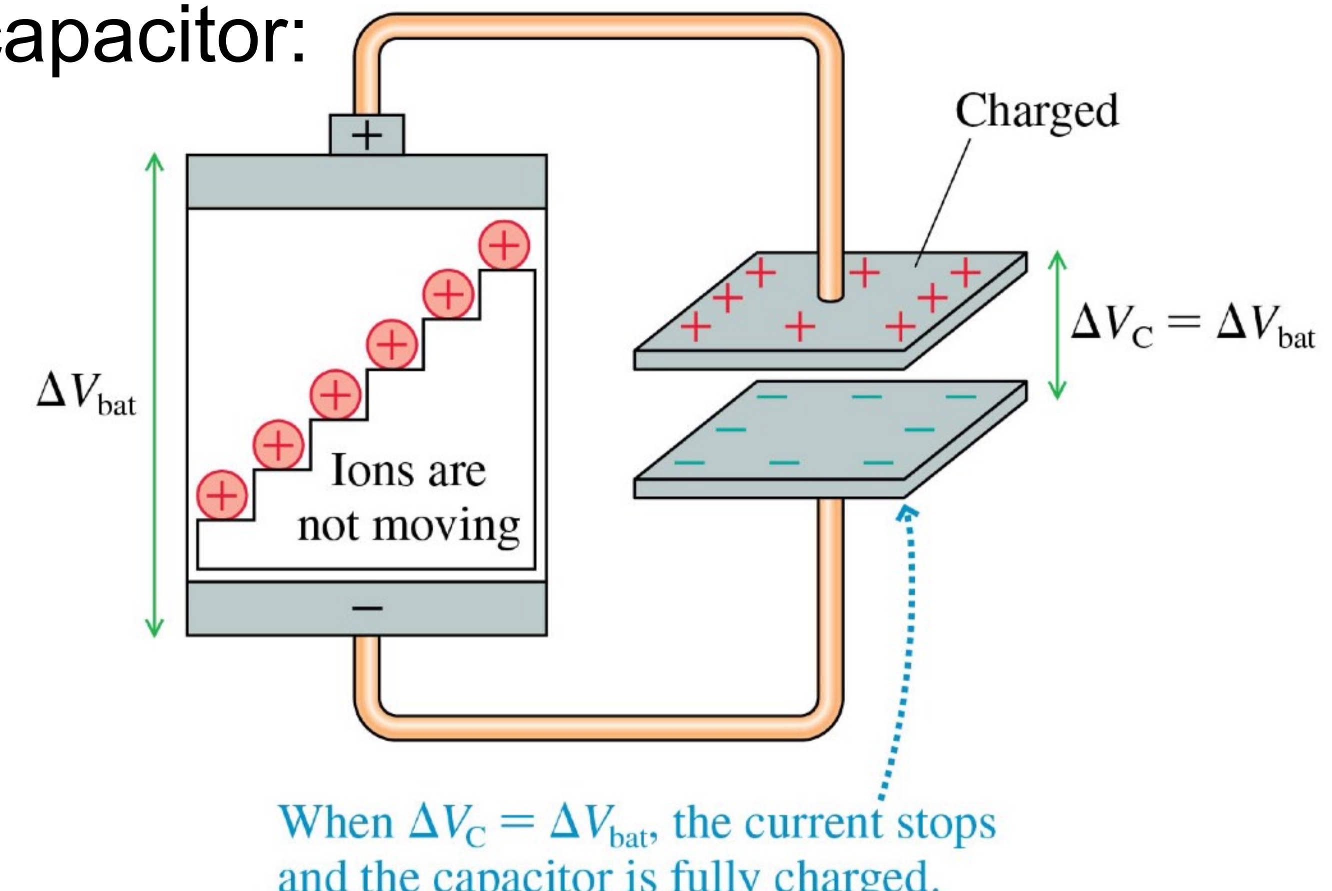
The keys on most computer keyboards are capacitor switches. Pressing the key pushes two capacitor plates closer together, increasing their capacitance.

Charging a Capacitor

- The figure shows a capacitor *just after* it has been connected to a battery:
- Current will flow in this manner for a nanosecond or so until the capacitor is fully charged.



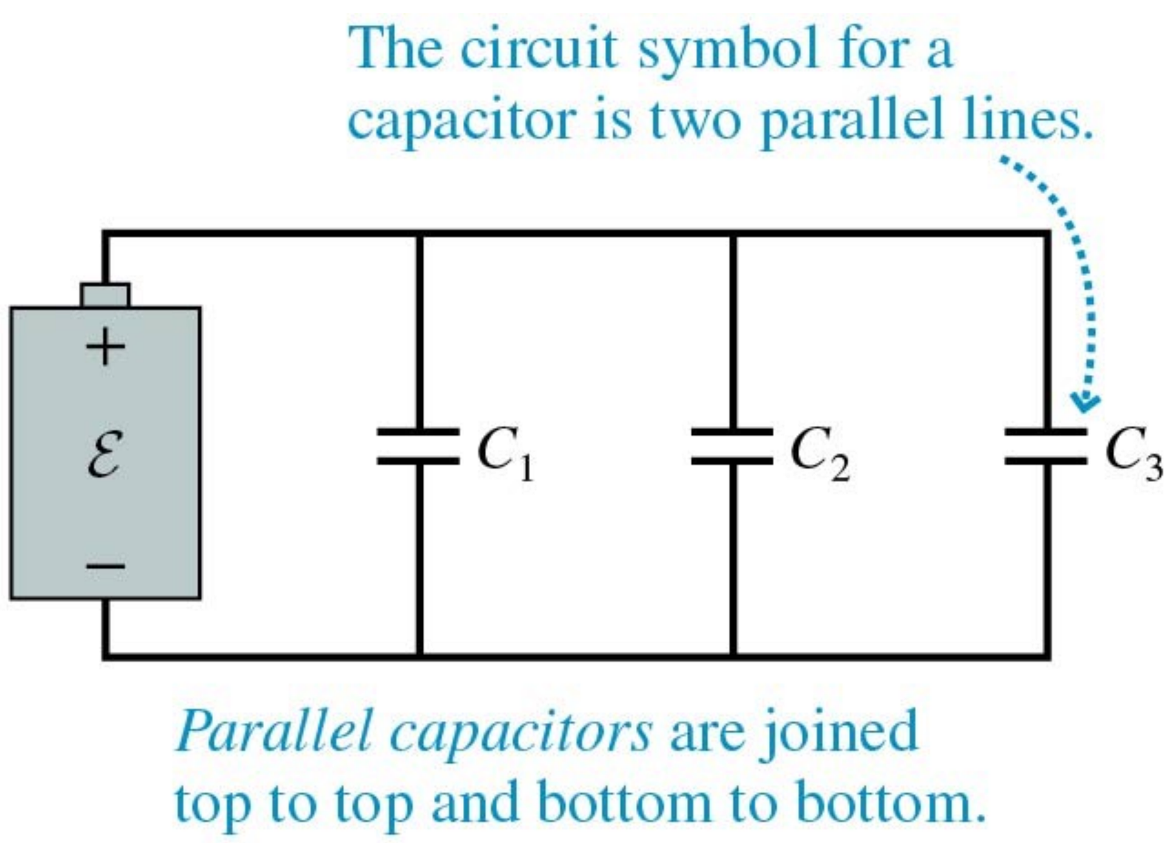
- The figure shows a *fully charged* capacitor:
- Now the system is in electrostatic equilibrium.

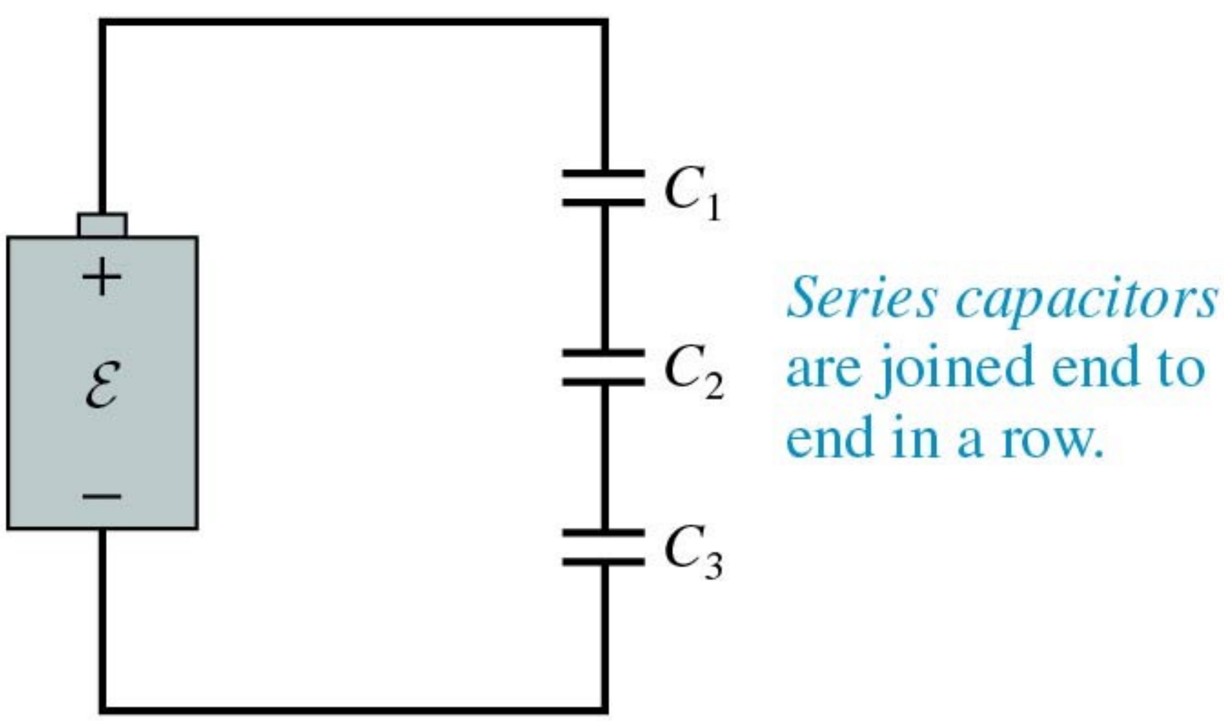


- Capacitance always refers to the charge per voltage on a ***fully charged*** capacitor.

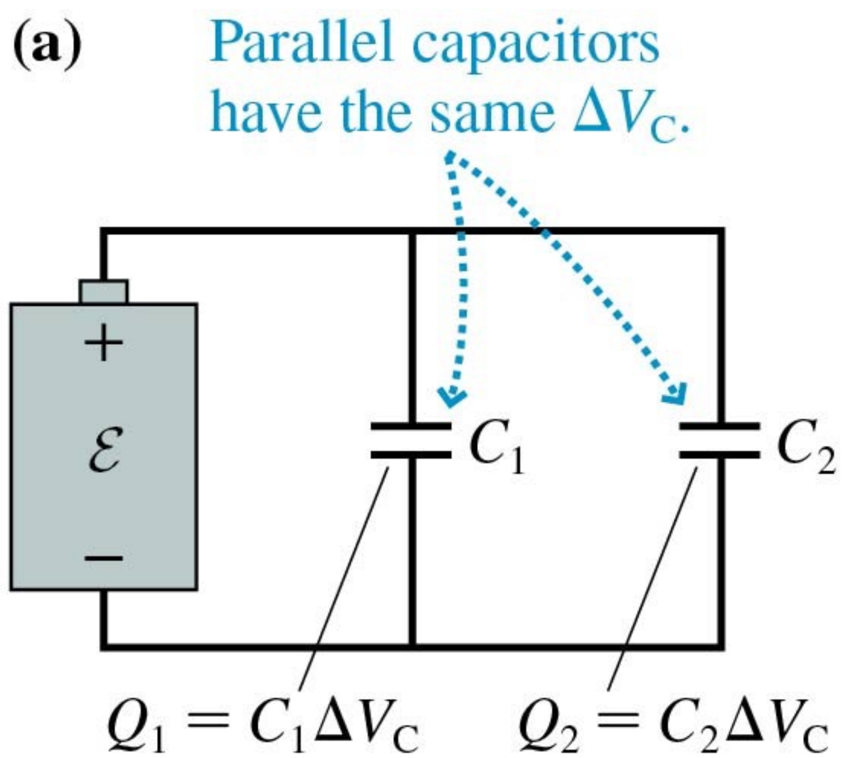
Combinations of Capacitors

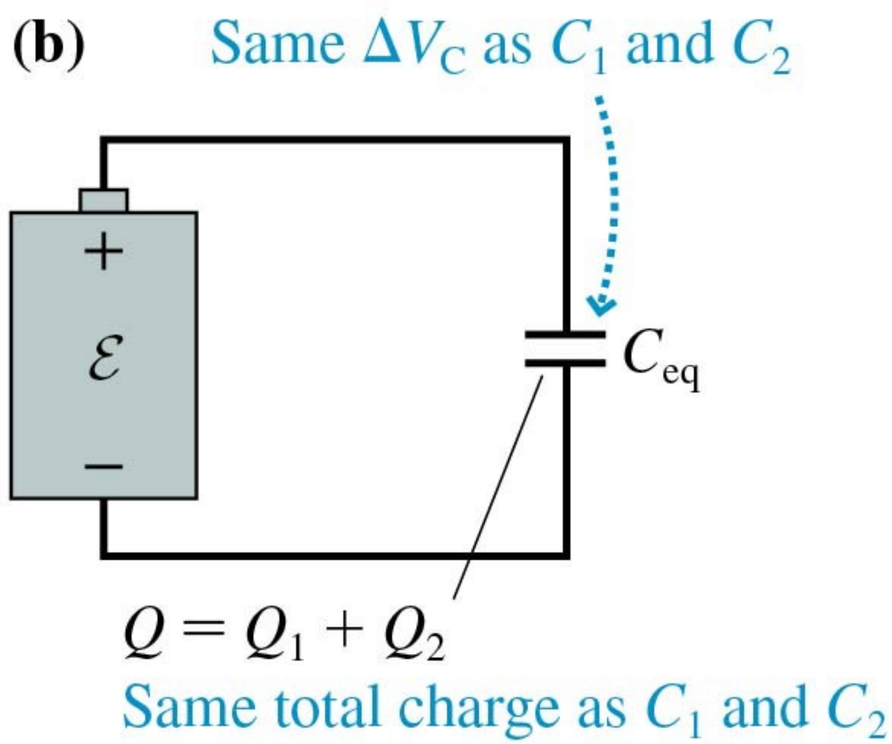
- In practice, two or more capacitors are sometimes joined together.
- The circuit diagrams below illustrate two basic combinations: **parallel capacitors** and **series capacitors**.





Capacitors Combined in Parallel





- Consider two capacitors C_1 and C_2 connected in parallel.
- The total charge drawn from the battery is $Q = Q_1 + Q_2$.
- In figure (b) we have replaced the capacitors with a single "equivalent" capacitor:

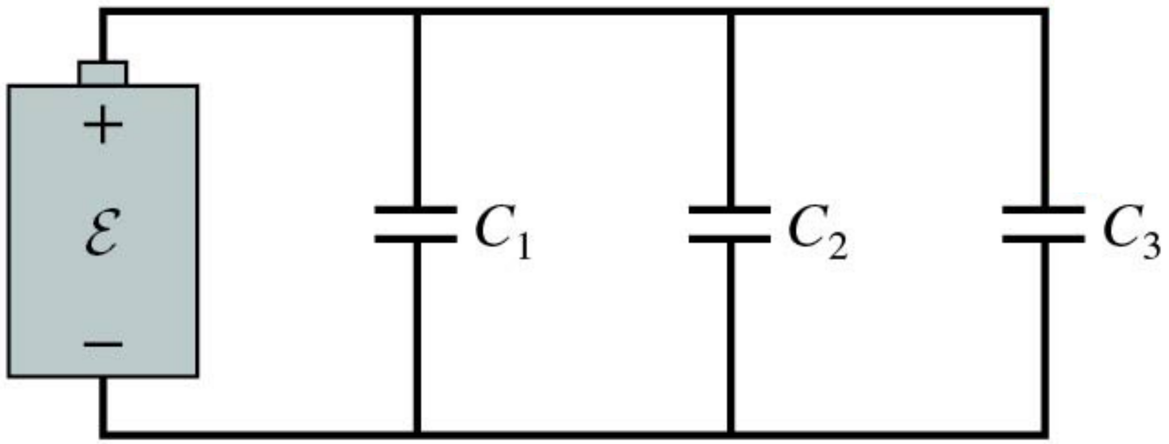
$$C_{eq} = \frac{Q}{\Delta V_C} = \frac{Q_1 + Q_2}{\Delta V_C} = \frac{Q_1}{\Delta V_C} + \frac{Q_2}{\Delta V_C}$$

$$C_{eq} = C_1 + C_2$$

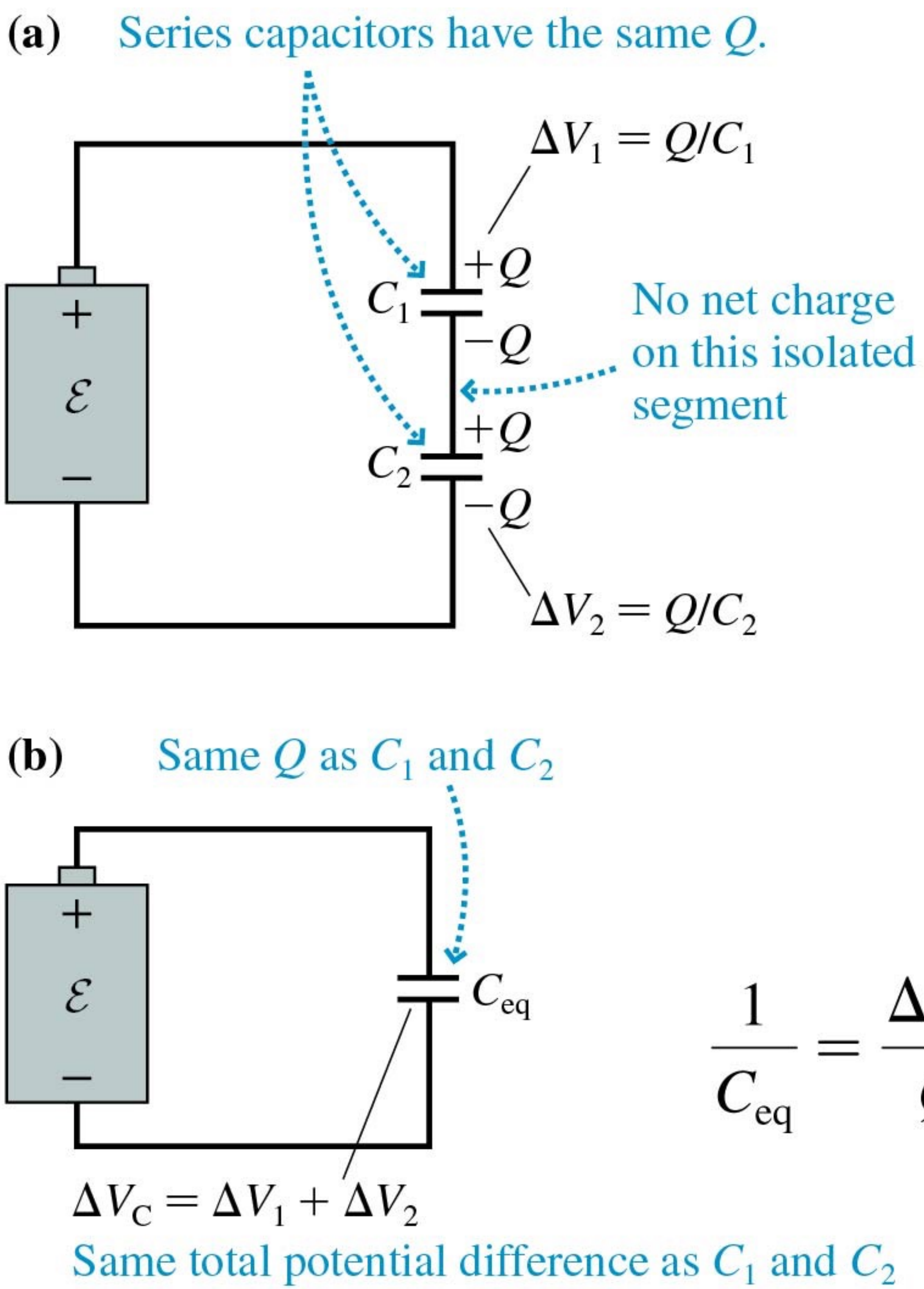
Capacitors Combined in Parallel

- If capacitors C_1, C_2, C_3, … are in parallel, their equivalent capacitance is:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel capacitors)}$$



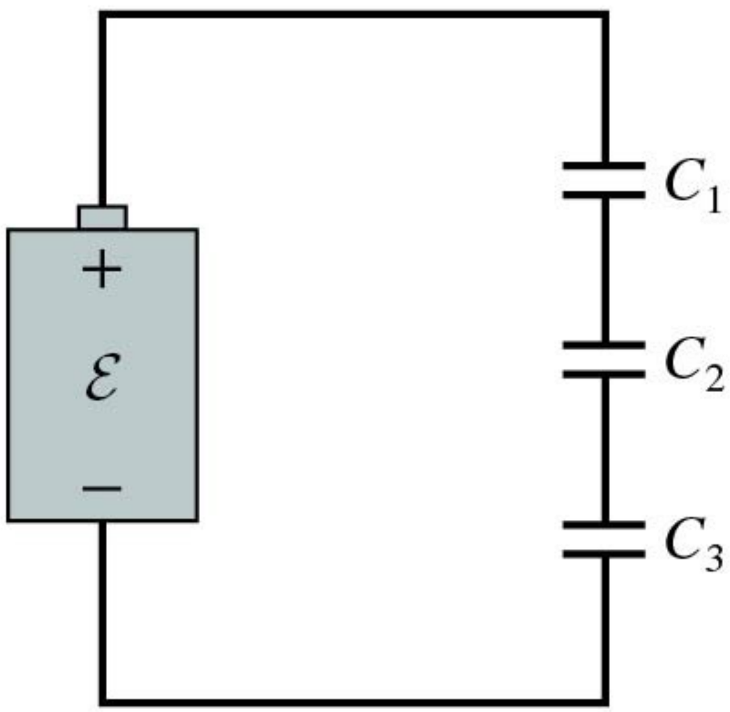
Capacitors Combined in Series



- Consider two capacitors C_1 and C_2 connected in series.
- The total potential difference across both capacitors is $\Delta V_C = \Delta V_1 + \Delta V_2$.
- The inverse of the equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{\Delta V_C}{Q} = \frac{\Delta V_1 + \Delta V_2}{Q} = \frac{\Delta V_1}{Q} + \frac{\Delta V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

Capacitors Combined in Series



- If capacitors C_1, C_2, C_3, … are in series, their equivalent capacitance is

$$C_{\mathrm{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\right)^{-1} \qquad \text{(series capacitors)}$$

iClicker question 8-7

The equivalent capacitance is

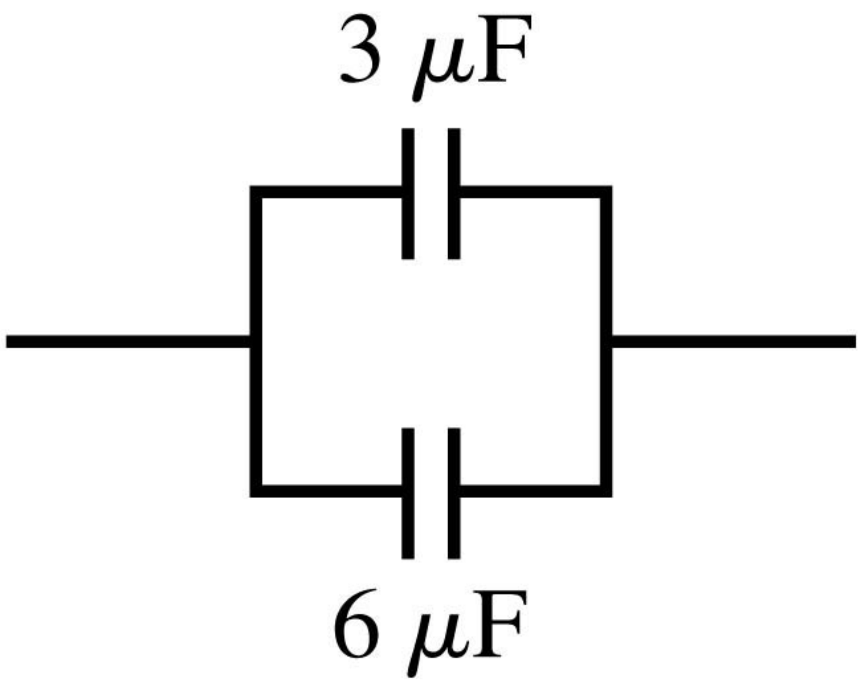
A. 9 μF

B. 6 μF

C. 3 μF

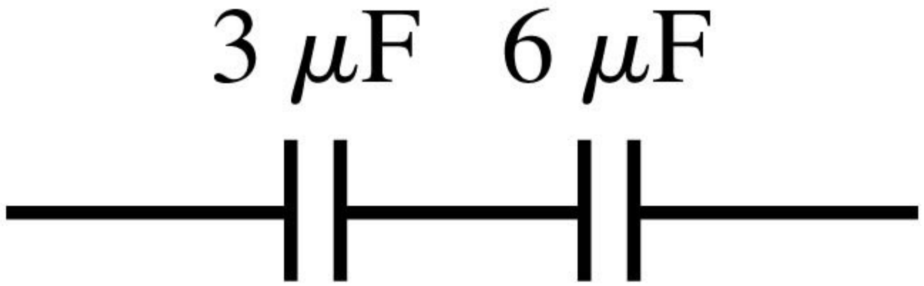
D. 2 μF

E. 1 μF



iClicker question 8-8

The equivalent capacitance is



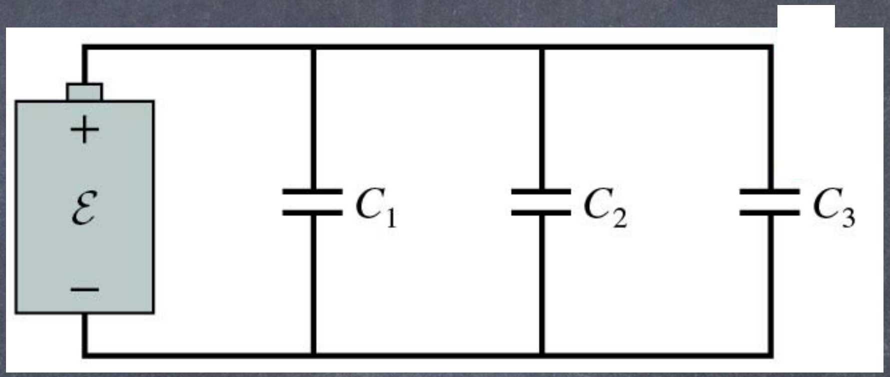
A. 9 μF
B. 6 μF
C. 3 μF
D. 2 μF
E. 1 μF

Capacitors

Remember when you have multiple capacitors in a circuit that:

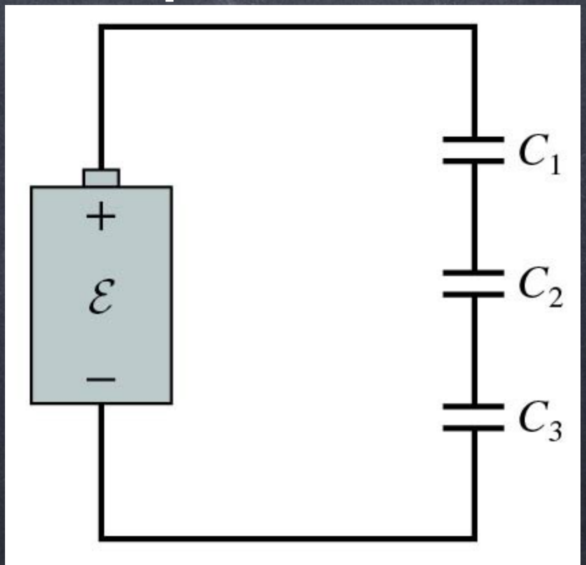
- Capacitors in **<u>parallel</u>** all have **<u>same potential differences</u>**
- The equivalent capacitance of the parallel capacitors also will have the same potential difference.

$$C_{\mathrm{eq}} = C_1 + C_2 + C_3 + \cdots$$



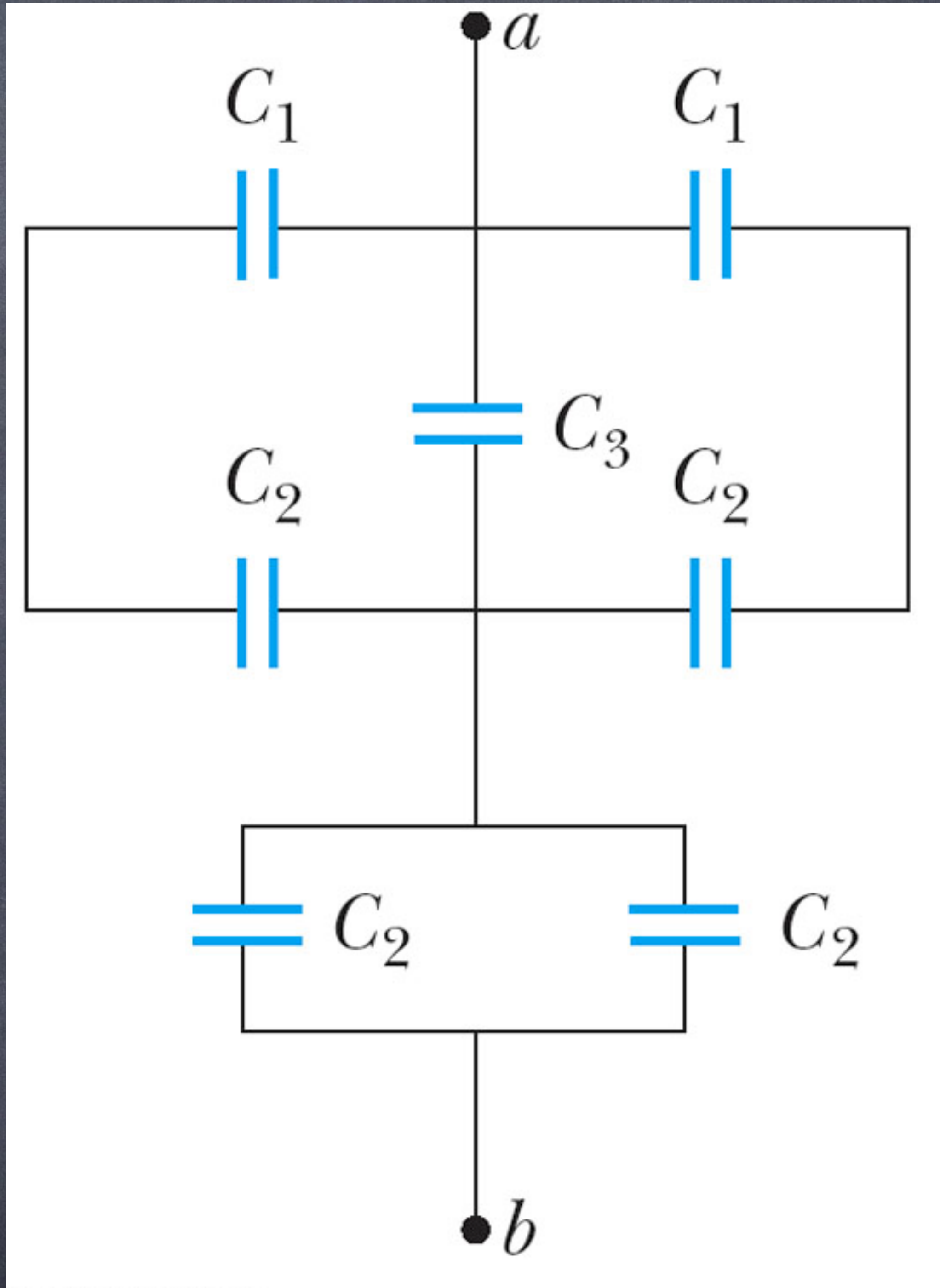
- Capacitors in **<u>series</u>** all have **<u>same charge</u>**
- The equivalent capacitor of the series capacitors also will have the same charge.

$$C_{\mathrm{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\right)^{-1}$$



Capacitor network, example

- Find the equivalent capacitance between points a and b in the group of capacitors connected in series as shown in the figure to the right (take $C_1 = 5.00\ \mu F$, $C_2 = 10.0\ \mu F$, and $C_3 = 2.00\ \mu F$). If the potential difference between points a and b is 60.0V, what is the charge stored on C_3?



- Answer
- Reduce the circuit by equivalent capacitance to find Q_3.
- Start with the C_1 and C_2 in series on the top.

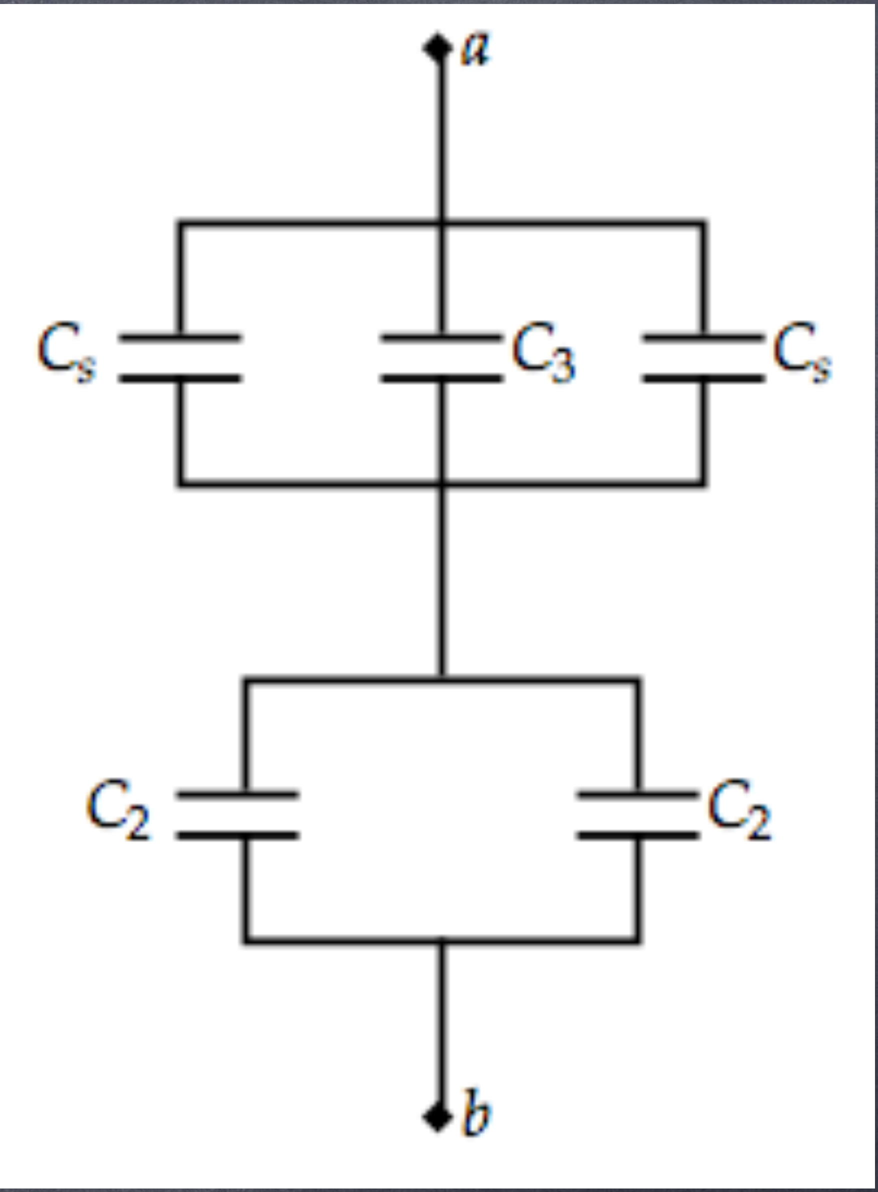
Capacitor network, example

- Answer
- Combine the two top capacitors that are in series (left and right are the same) to form an equivalent C_S:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5\mu\text{F}} + \frac{1}{10\mu\text{F}}$$

$$\frac{1}{C_S} = \frac{2}{10\mu\text{F}} + \frac{1}{10\mu\text{F}} = \frac{3}{10\mu\text{F}}$$

$$C_S = \frac{10\mu\text{F}}{3} = 3.33\mu\text{F}$$



- Next, combine the top three that are in parallel:

$$C_{\text{P1}} = C_S + C_3 + C_S$$

$$C_{\text{P1}} = 3.33\mu\text{F} + 2.00\mu\text{F} + 3.33\mu\text{F} = 8.66\mu\text{F}$$

Capacitor network, example

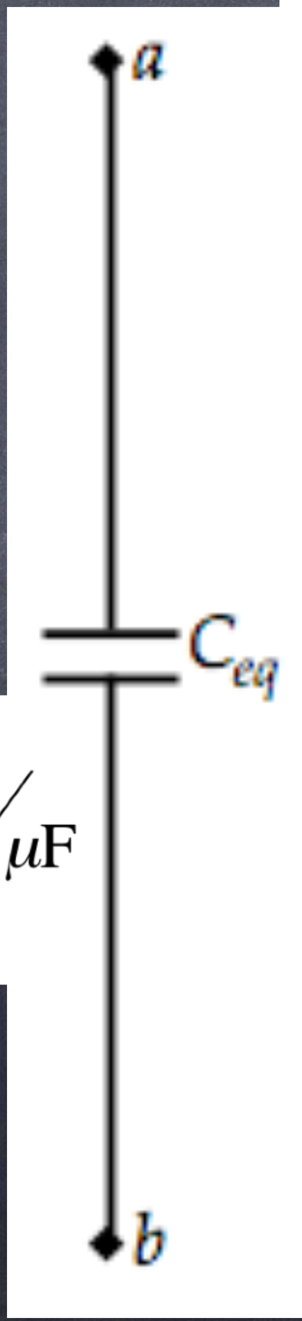
- Answer

- Next, combine the bottom two that are in parallel:

$$C_{P2} = C_2 + C_2$$

$$C_{P2} = 10.0\,\mu\text{F} + 10.0\,\mu\text{F} = 20.0\,\mu\text{F}$$

- Finally, combine the remaining two capacitors that are in series:



$$\frac{1}{C_{eq}} = \frac{1}{C_{P1}} + \frac{1}{C_{P2}} = \frac{1}{8.66\,\mu\text{F}} + \frac{1}{20\,\mu\text{F}}$$

$$\frac{1}{C_{eq}} = (0.1155)\,{}^{1}\!/_{\mu\text{F}} + (0.0500)\,{}^{1}\!/_{\mu\text{F}} = (0.1655)\,{}^{1}\!/_{\mu\text{F}}$$

$$C_{eq} = 6.04\,\mu\text{F}$$

Capacitor network, example

- Answer

- Next, we need to find the total charge stored on the equivalent capacitor:

$$Q_{eq} = C_{eq}(\Delta V_{ab})$$

$$Q_{eq} = 6.04\,\mu\text{F}(60\text{V}) = 362\,\mu\text{C}$$

- Looking back at the two equivalent capacitors in series (C_{p1} and C_{p2}), we see that they must have the same amount of charge:

$$Q_{eq} = Q_{P1} = Q_{P2} = 362\,\mu\text{C}$$

- This means that they potential difference across C_{p1} is:

$$\Delta V_{P1} = \frac{Q_{P1}}{C_{P1}} \qquad \Delta V_{P1} = \frac{362\,\mu\text{C}}{8.66\,\mu\text{F}} = 41.8\text{V}$$

Capacitor network, example

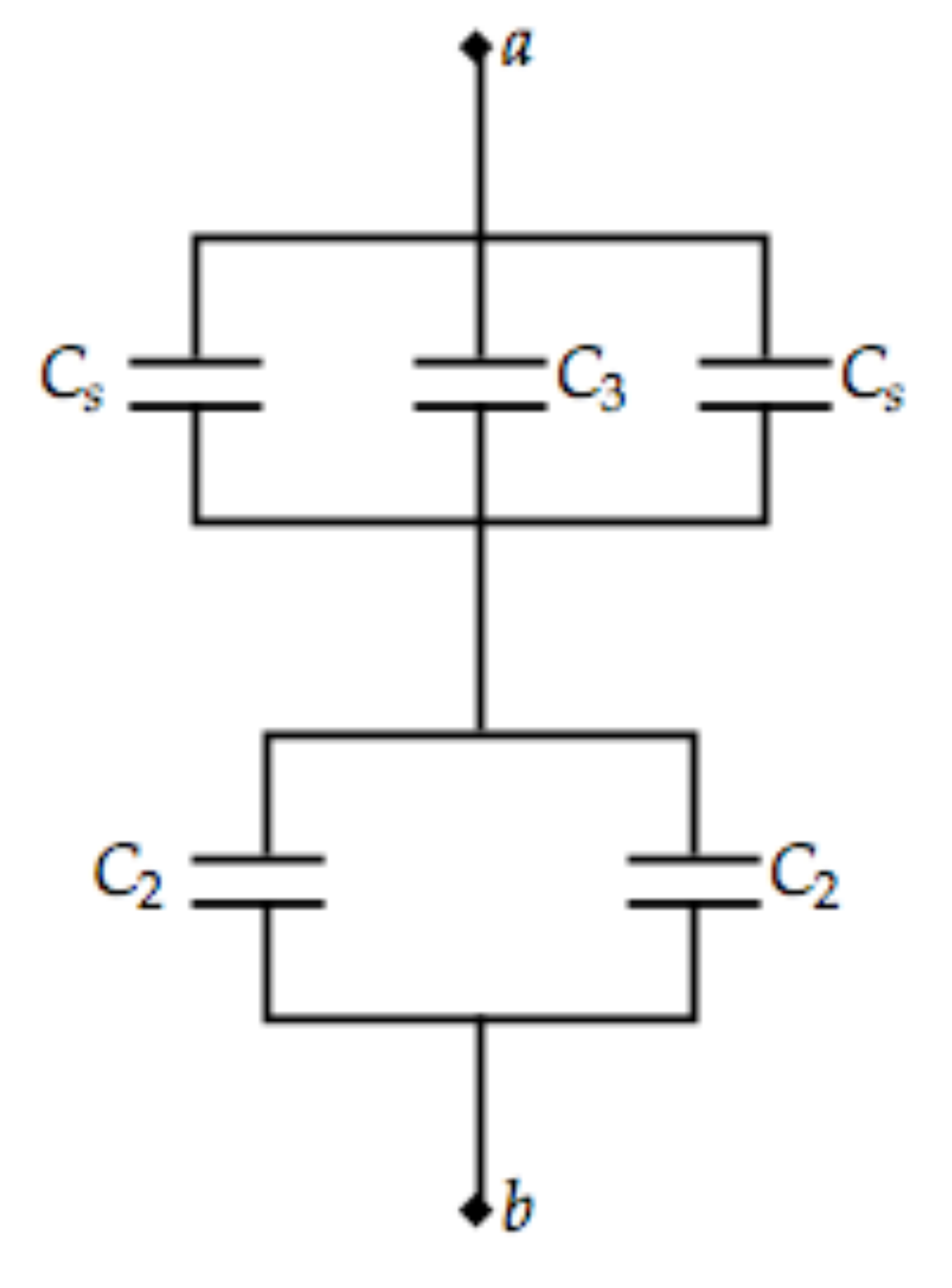
- Answer
- We turn back to the three top capacitors in parallel (C_S, C_3, and C_S).
- We note that all three of these capacitors must have the same potential difference as each other (and C_{p1}).

$$\Delta V_{C3} = 41.8\text{V}$$

- This means that the charge on C_3 is:

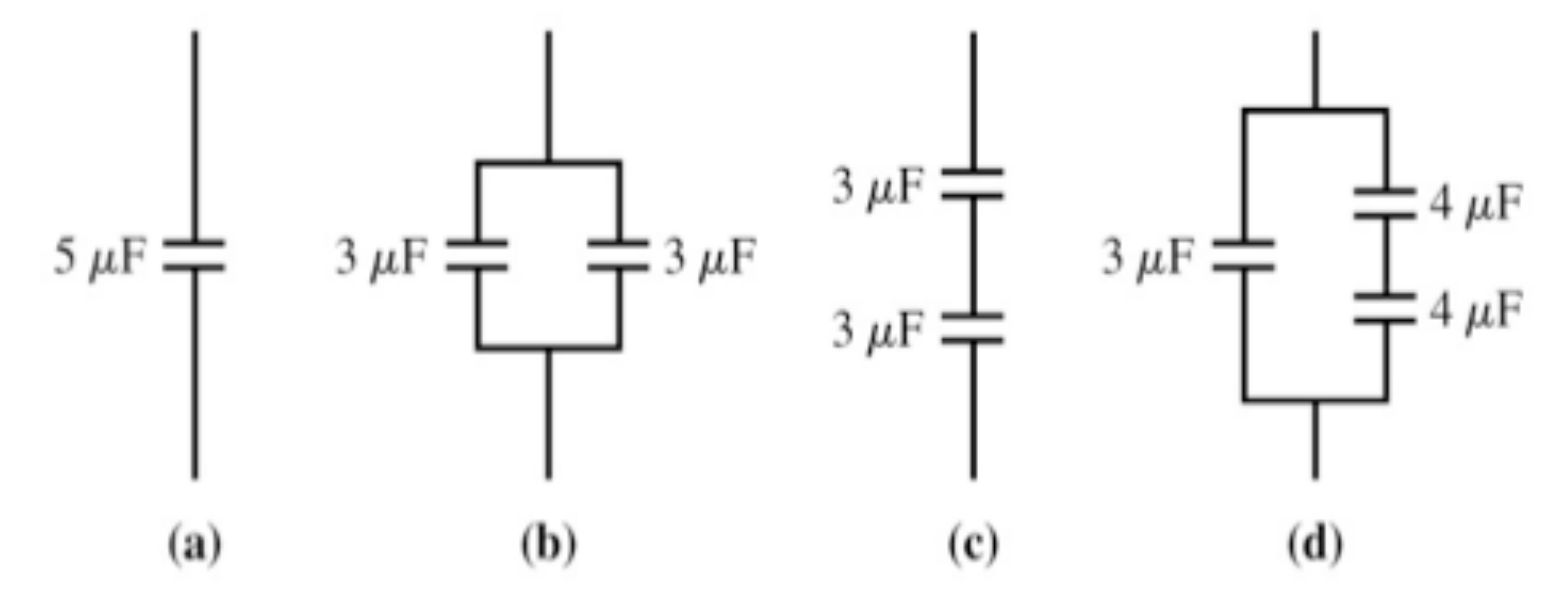
$$Q_3 = C_3\left(\Delta V_{C3}\right)$$

$$Q_3 = 2.00\mu\text{F}\left(41.8\text{V}\right) = 83.6\mu\text{C}$$



iClicker take home Question

- Rank in order, from largest to smallest the equivalent capacitance C_a to C_d of the circuits a to d.



- A) $C_d > C_b > C_a > C_c$.
- B) $C_d > C_b = C_c > C_a$.
- C) $C_a > C_b = C_c > C_d$.
- D) $C_b > C_a = C_d > C_c$.
- E) $C_c > C_a = C_d > C_b$.

Energy stored in a Capacitor

- Suppose you have a given capacitor charged to a potential difference ΔV with a charge ±q on either plate. How much energy would it take to transfer an small amount of charge dq?

- Start with the relationship between work and electric potential:

$$dW = (\Delta V)\,dq$$

$$dW = \left(\frac{q}{C}\right)dq$$

- If we wanted to calculate the entire work done to fully charge the capacitor in this manner (from q = 0 to q = Q) we have:

$$W = \int dW = \int_0^Q \left(\frac{q}{C}\right)dq$$

Energy of a Capacitor

- Since C merely depends on the geometry of the capacitor, we have:

$$W = \frac{1}{C}\int_0^Q (q)\,dq \qquad W = \frac{1}{C}\left(\frac{Q^2}{2}\right)$$

- Since the electric force is conservative, the energy stored (U_{elec}) in the capacitor is equivalent to the work put in:

$$\text{Energy stored} = \tfrac{1}{2}Q(\Delta V) = \tfrac{1}{2}C(\Delta V)^2 = \frac{Q^2}{2C}$$

- The main use of a capacitor is to store and then discharge energy.

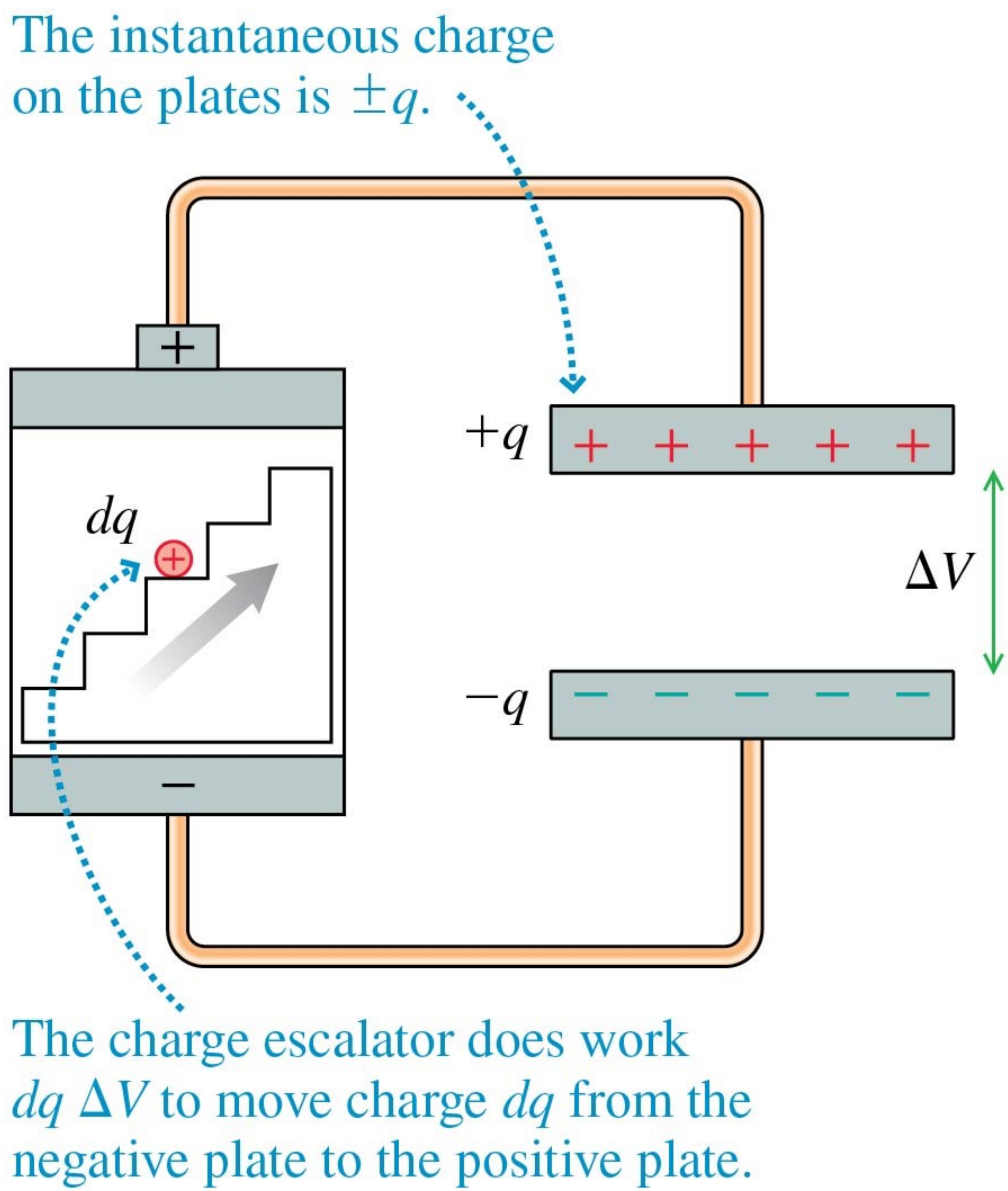
The Energy Stored in a Capacitor

- The figure shows a capacitor being charged.
- As a small charge *dq* is lifted to a higher potential, the potential energy of the capacitor increases by

$$dU = dq\,\Delta V = \frac{q\,dq}{C}$$

- The total energy transferred from the battery to the capacitor is

$$U_C = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$



write eq on blackboard!

The Energy Stored in a Capacitor

- Capacitors are important elements in electric circuits because of their ability to store energy.
- The charge on the two plates is $\pm q$ and this charge separation establishes a potential difference $\Delta V = q/C$ between the two electrodes.
- In terms of the capacitor's potential difference, the potential energy stored in a capacitor is

$$U_C = \frac{Q^2}{2C} = \frac{1}{2}C(\Delta V_C)^2$$

write eq on blackboard!

The Energy Stored in a Capacitor

- A capacitor can be charged slowly but then can release the energy very quickly.
- An important medical application of capacitors is the *defibrillator*.

- A heart attack or a serious injury can cause the heart to enter a state known as *fibrillation* in which the heart muscles twitch randomly and cannot pump blood.
- A strong electric shock through the chest completely stops the heart, giving the cells that control the heart's rhythm a chance to restore the proper heartbeat.