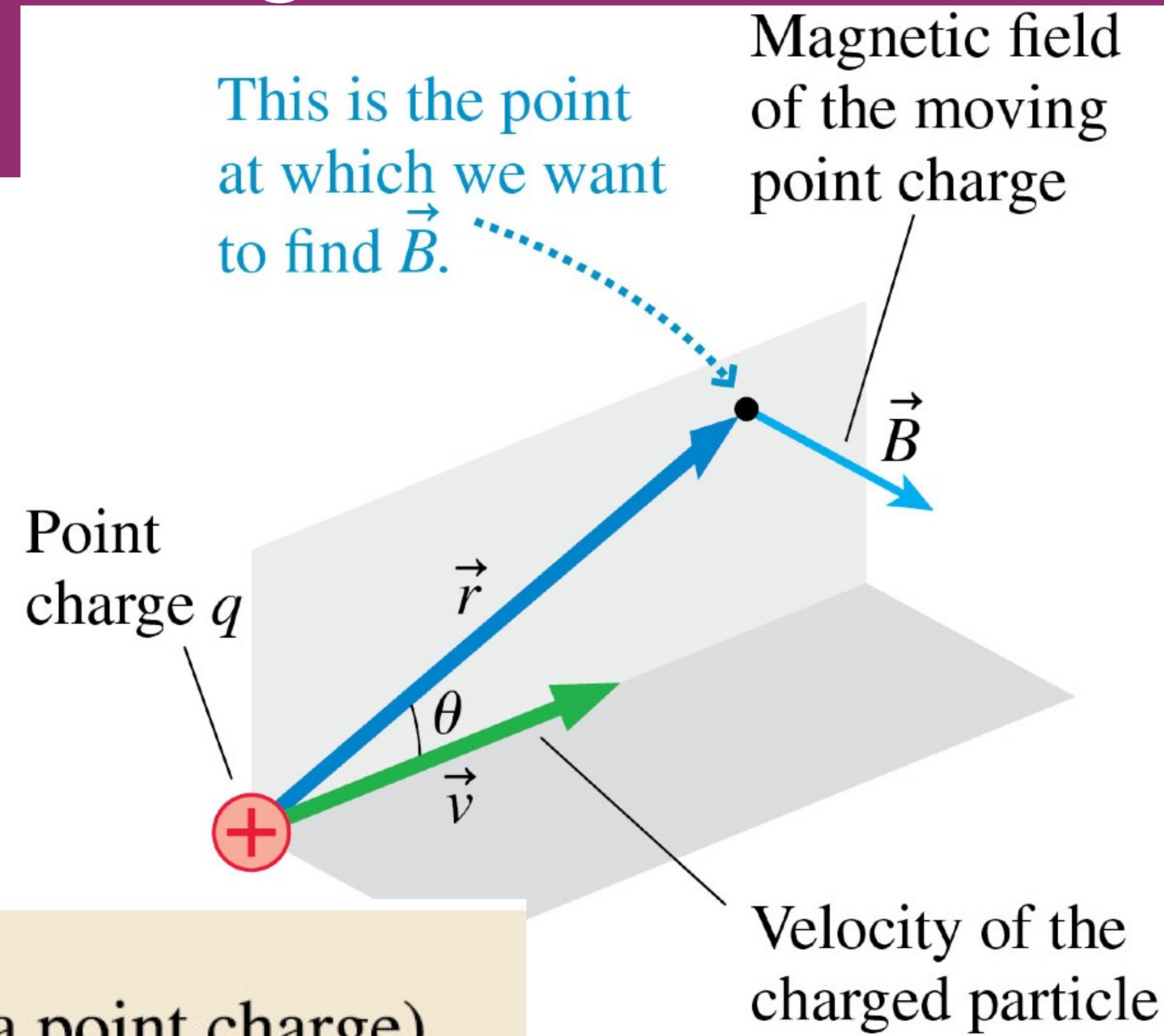
Physics 2B
Lecture 14

The Source of the Magnetic Field: Moving Charges

- The magnetic field of a charged particle q moving with velocity v is given by the **Biot-Savart law**:

$$\vec{B}_{\text{point charge}} = \left(\frac{\mu_0}{4\pi} \frac{qv\sin\theta}{r^2}, \right.$$

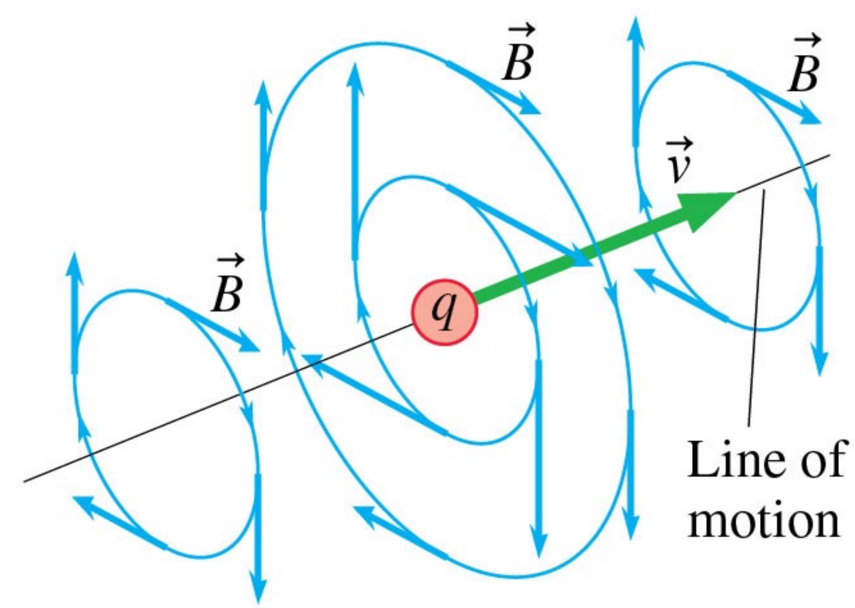
direction given by the right-hand rule



$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2} \quad \text{(magnetic field of a point charge)}$$

$$B_{\text{wire}} = \frac{\mu_0}{2\pi} \frac{I}{r} \quad \text{(long, straight wire)}$$

$$B_{\text{coil center}} = \frac{\mu_0}{2} \frac{NI}{R} \quad (N\text{-turn current loop})$$



PROBLEM-SOLVING STRATEGY 29.1

The magnetic field of a current

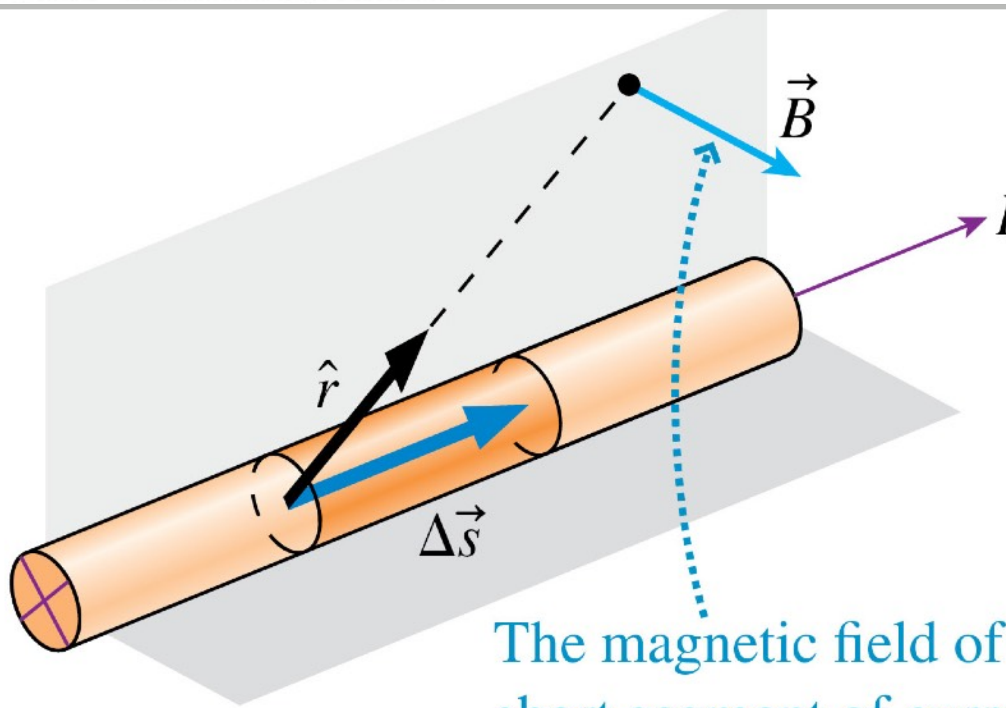
MODEL Model the wire as a simple shape.

VISUALIZE For the pictorial representation:

- Draw a picture, establish a coordinate system, and identify the point P at which you want to calculate the magnetic field.
- Divide the current-carrying wire into small segments for which you *already know* how to determine $\vec{B}$. This is usually, though not always, a division into very short segments of length Δs.
- Draw the magnetic field vector for one or two segments. This will help you identify distances and angles that need to be calculated.

$$\vec{B}_{\text{current segment}} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2}$$

(magnetic field of a very short segment of current)



The magnetic field of the short segment of current is in the direction of $\Delta\vec{s}\times\hat{r}$.

PROBLEM-SOLVING STRATEGY 29.1

The magnetic field of a current

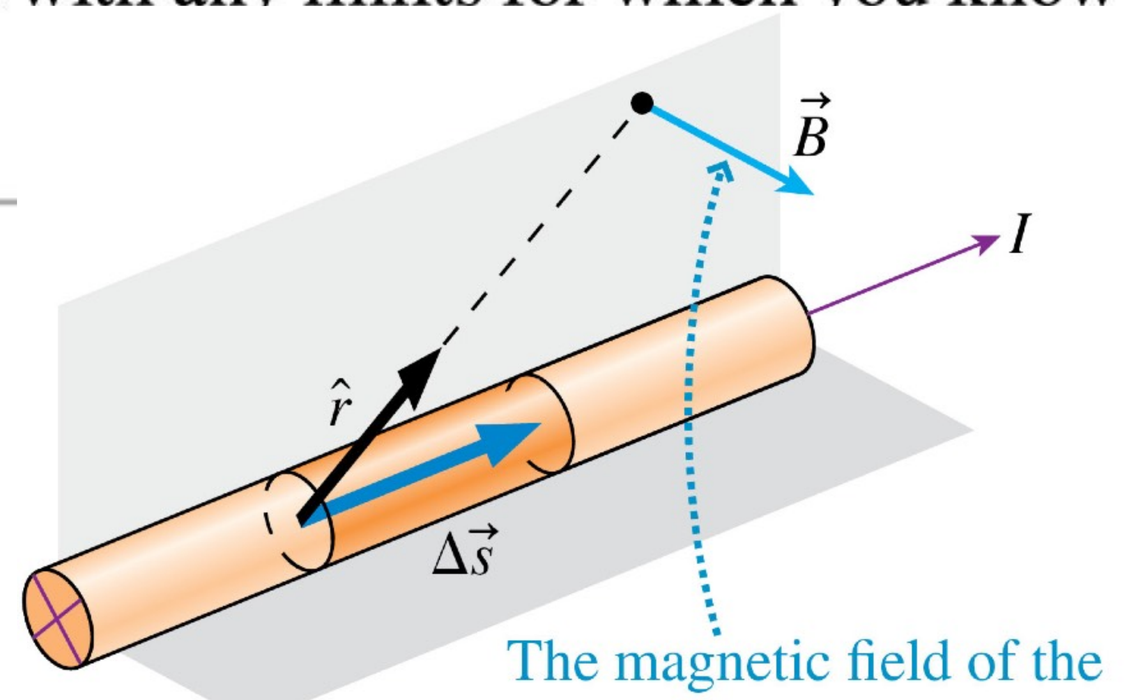
SOLVE The mathematical representation is $\vec{B}_{\text{net}} = \sum \vec{B}_i$.

- Write an algebraic expression for *each* of the three components of $\vec{B}$ (unless you are sure one or more is zero) at point P. Let the (x, y, z)-coordinates of the point remain as variables.
- Express all angles and distances in terms of the coordinates.
- Let $\Delta s \rightarrow ds$ and the sum become an integral. Think carefully about the integration limits for this variable; they will depend on the boundaries of the wire and on the coordinate system you have chosen to use.

ASSESS Check that your result is consistent with any limits for which you know what the field should be.

$$\vec{B}_{\text{current segment}} = \frac{\mu_0}{4\pi}\frac{I\,\Delta\vec{s} \times \hat{r}}{r^2}$$

(magnetic field of a very short segment of current)



The Magnetic Field of a Current

- Examples 29.3 and 29.5 in the book.

- The magnetic field of a long, straight wire carrying current I at a distance r from the wire is

$$B_{\text{wire}} = \frac{\mu_0}{2\pi}\frac{I}{r} \quad \text{(long, straight wire)}$$

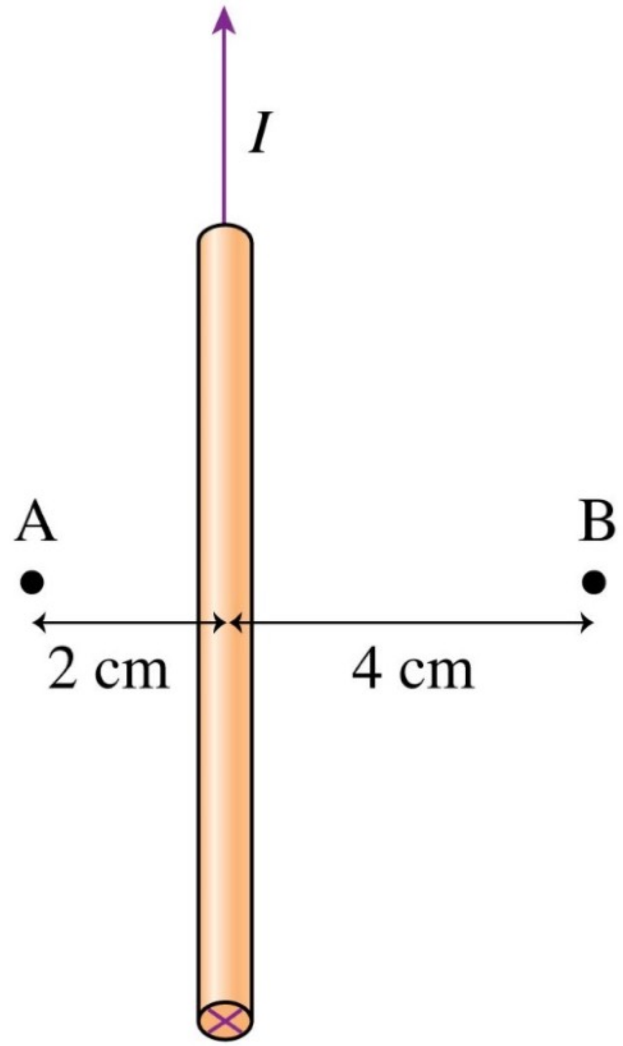
- The magnetic field at the center of a coil of N turns and radius R, carrying a current I is

$$B_{\text{coil center}} = \frac{\mu_0}{2}\frac{NI}{R} \quad (N\text{-turn current loop})$$

- Write in BB

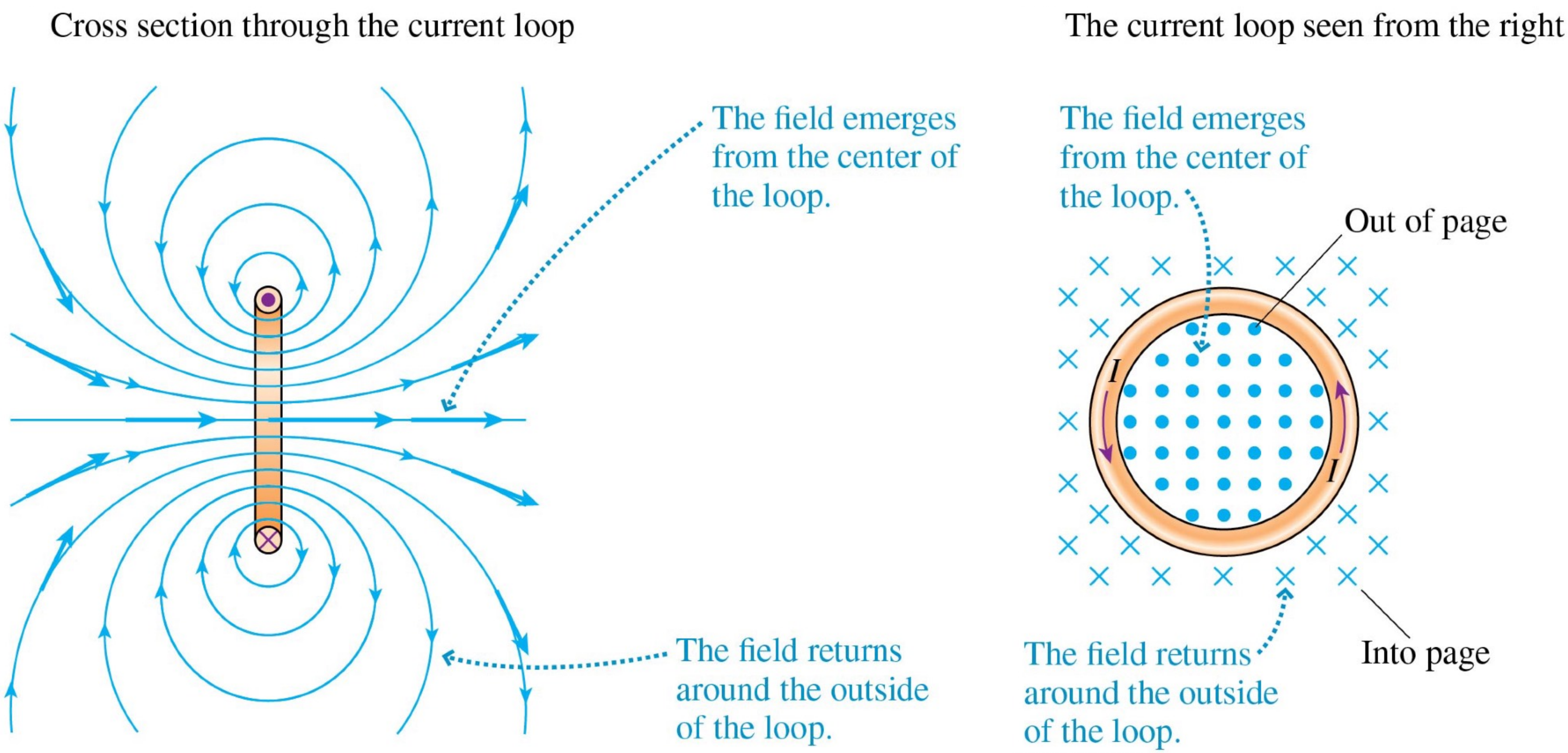
iClicker question #13-5

Compared to the magnetic field at point A, the magnetic field at point B is



A. Half as strong, same direction.
B. Half as strong, opposite direction.
C. One-quarter as strong, same direction.
D. One-quarter as strong, opposite direction.
E. Can't compare without knowing I.

The Magnetic Field of a Current Loop (single)



The Magnetic Field of a Current Loop

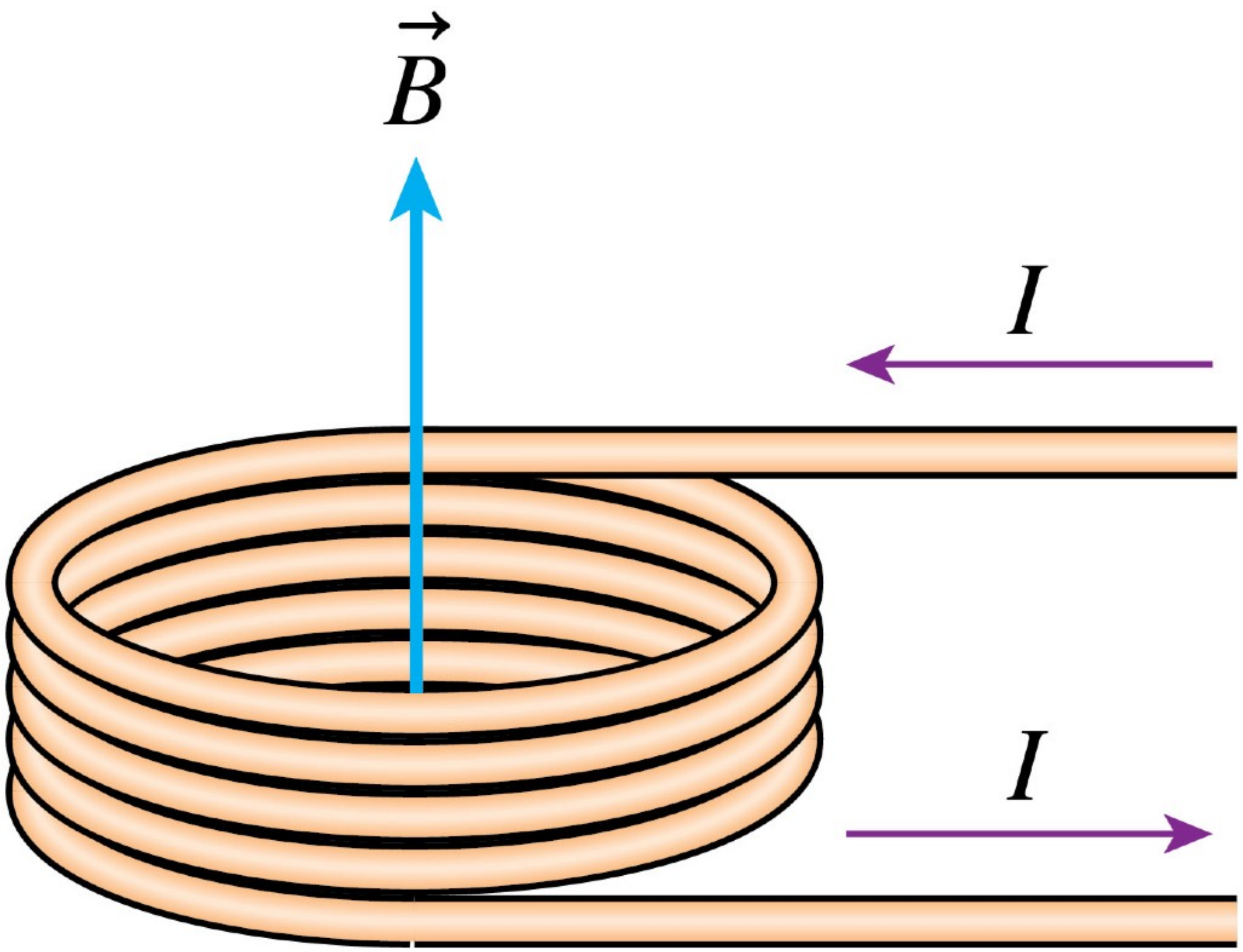
A photo of iron filings

- The magnetic field is revealed by the pattern of iron filings around a current-carrying loop of wire.

Example 29.6 Matching the Earth's Magnetic Field

EXAMPLE 29.6 **Matching the earth's magnetic field**

VISUALIZE **FIGURE 29.18** shows a five-turn coil of wire. The magnetic field is five times that of a single current loop.



Example 29.6 Matching the Earth's Magnetic Field

$$B_{\text{coil center}} = \frac{\mu_0}{2}\frac{NI}{R} \qquad (N\text{-turn current loop})$$

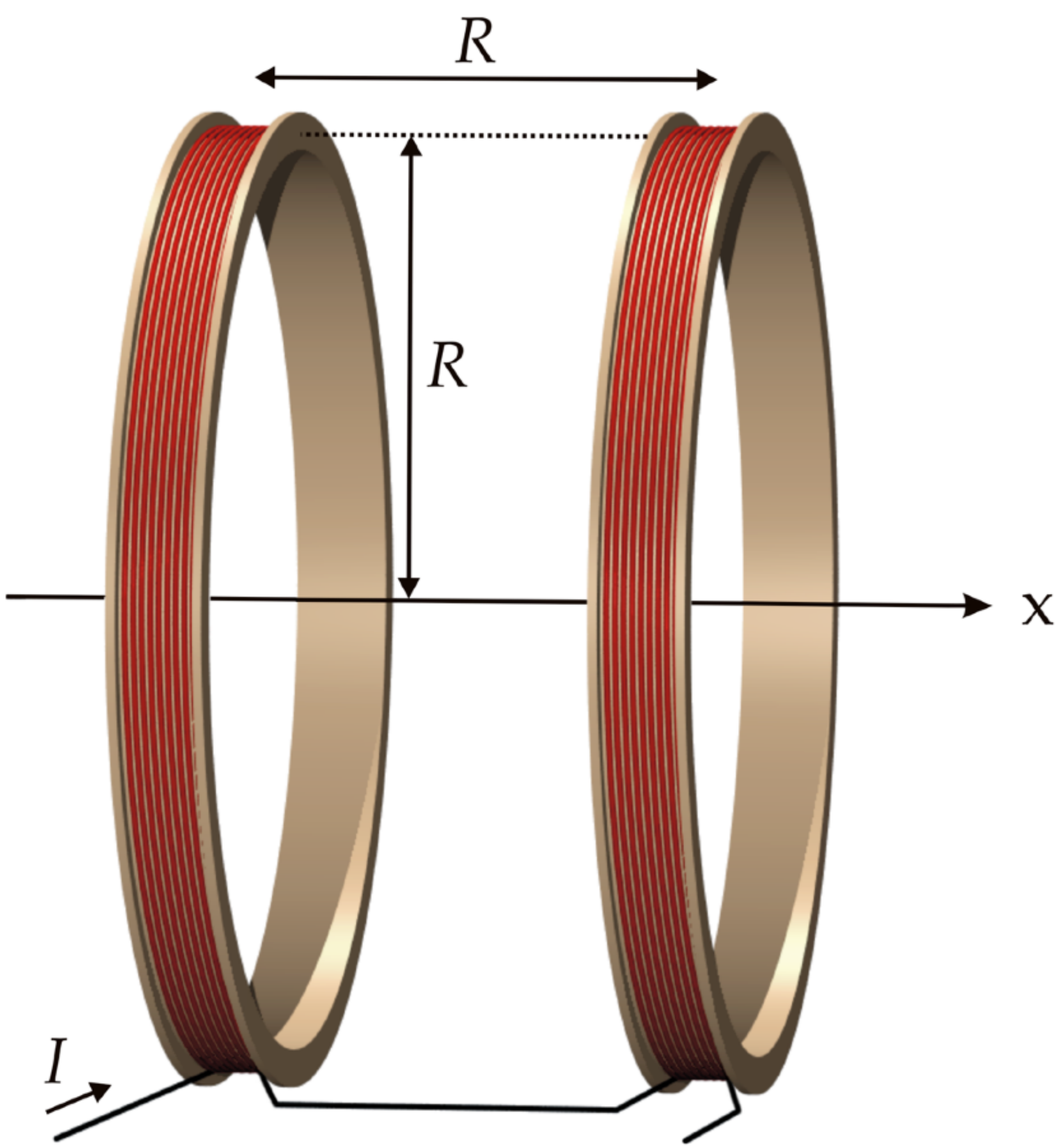
EXAMPLE 29.6 **Matching the earth's magnetic field**

SOLVE The earth's magnetic field, from Table 29.1, is 5×10^{-5} T. We can use Equation 29.8 to find that the current needed to generate a 5×10^{-5} T field is

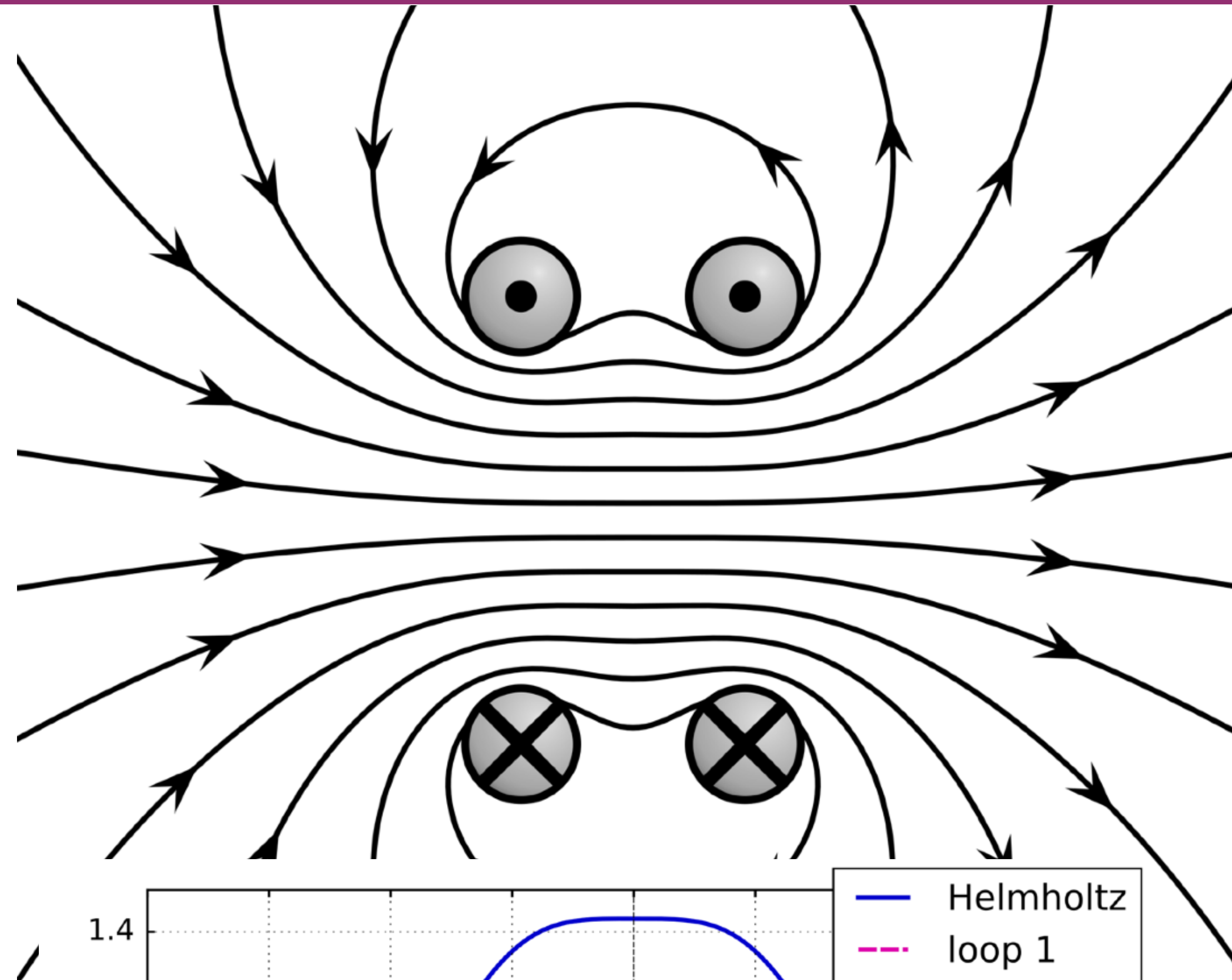
$$I = \frac{2RB}{\mu_0 N} = \frac{2(0.050\ \text{m})(5.0 \times 10^{-5}\ \text{T})}{5(4\pi \times 10^{-7}\ \text{T m/A})} = 0.80\ \text{A}$$

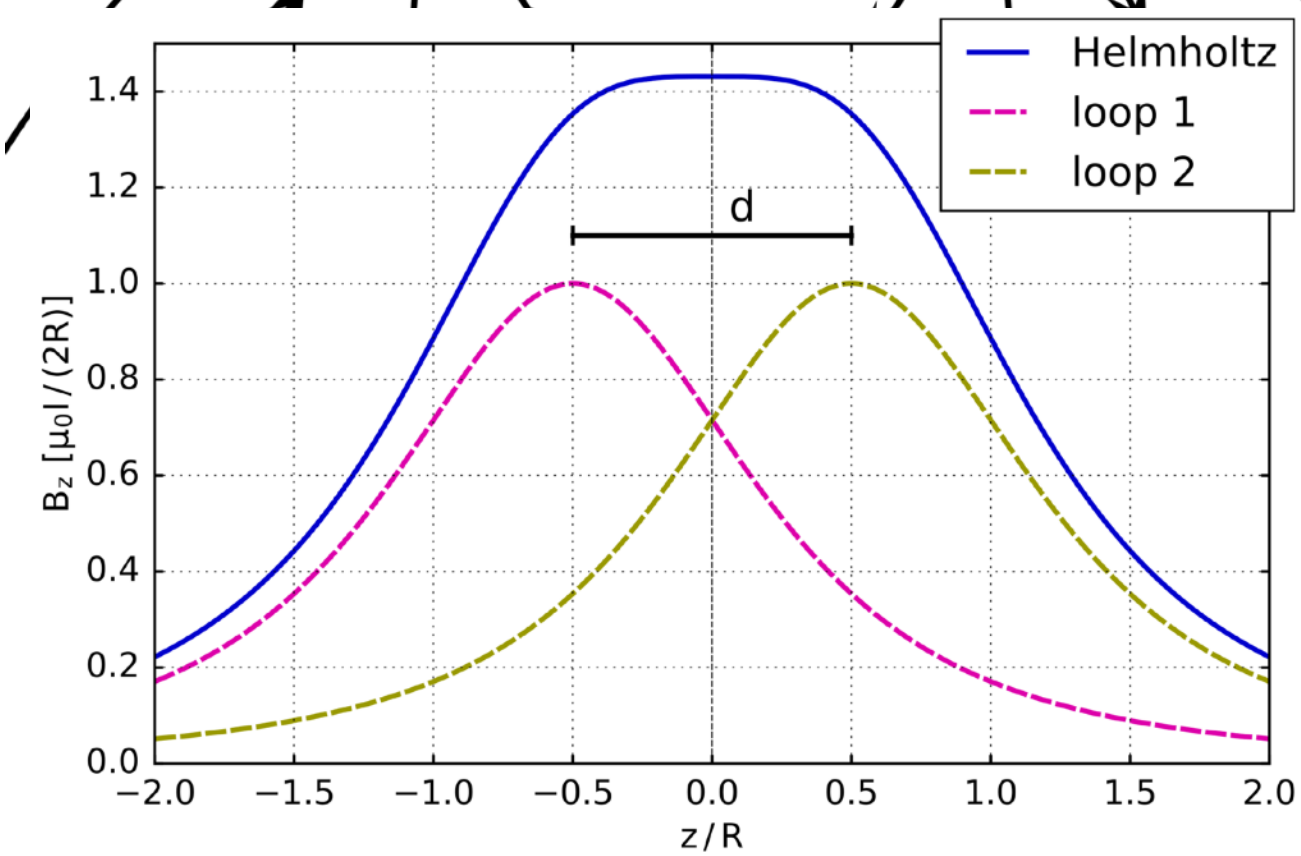
ASSESS A 0.80 A current is easily produced. Although there are better ways to cancel the earth's field than using a simple coil, this illustrates the idea.

Helmholtz coils: nearly uniform magnetic field at center



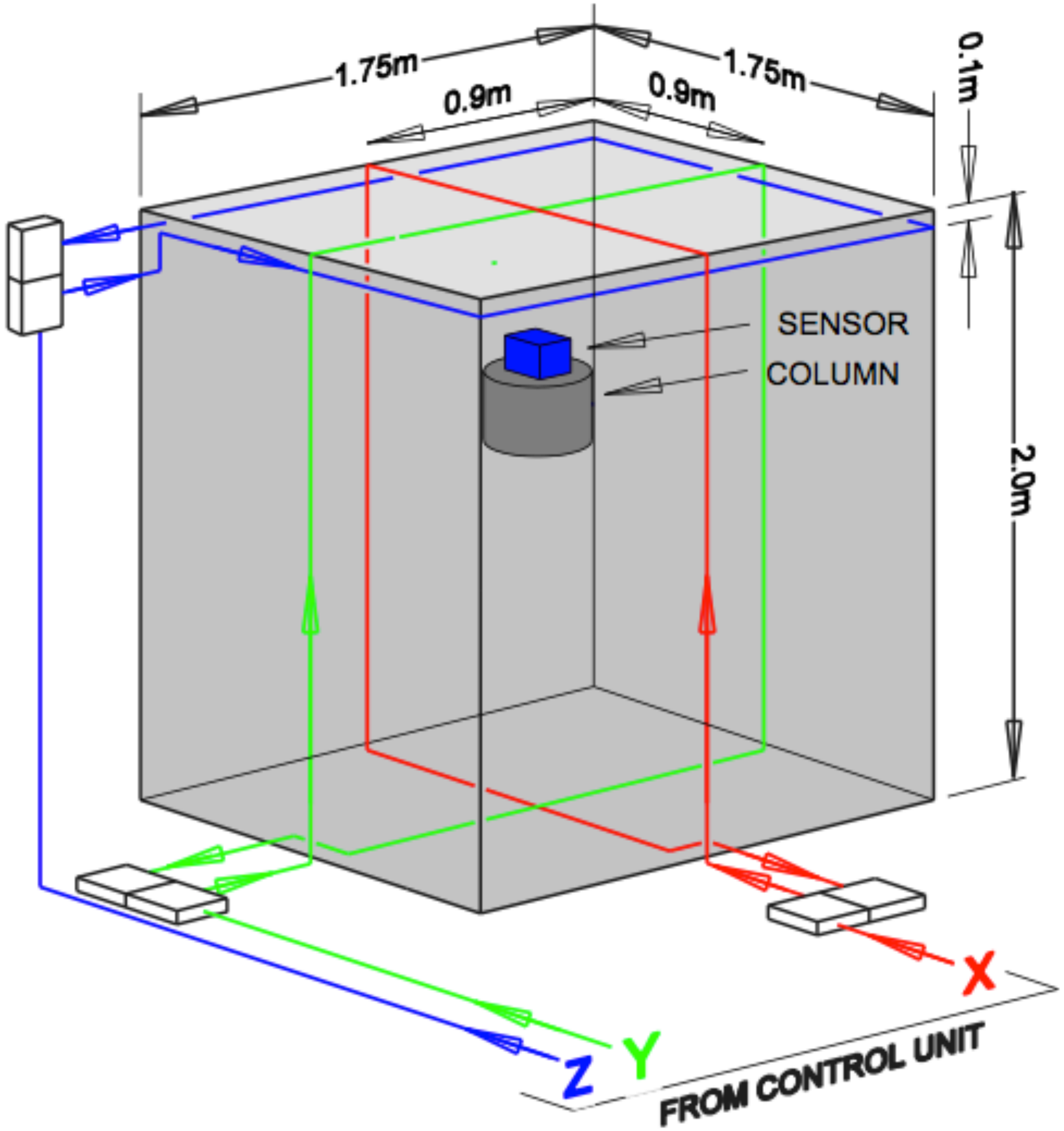
$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 n I}{R}$$

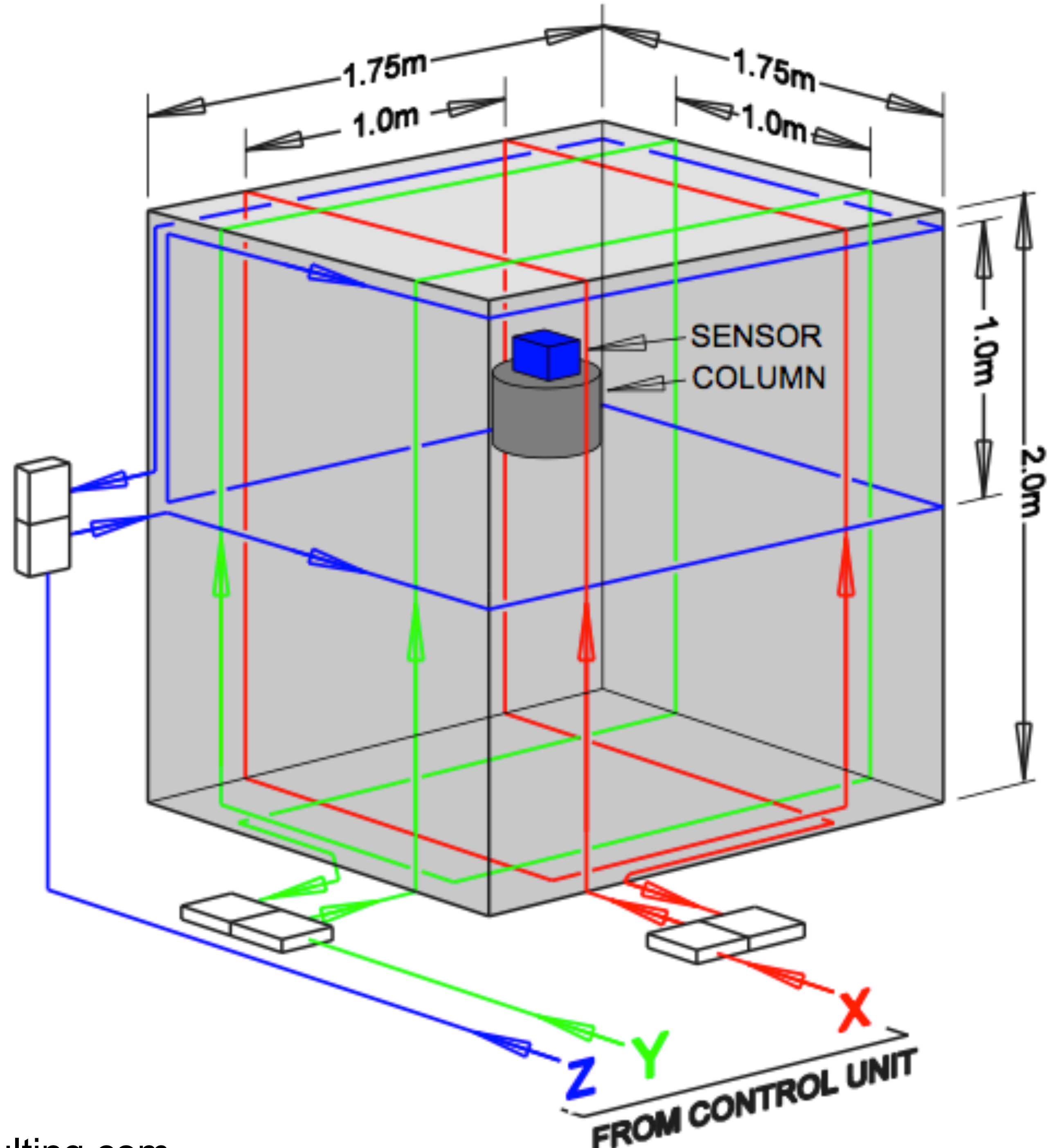




Cancelling the earth's and ambient B fields

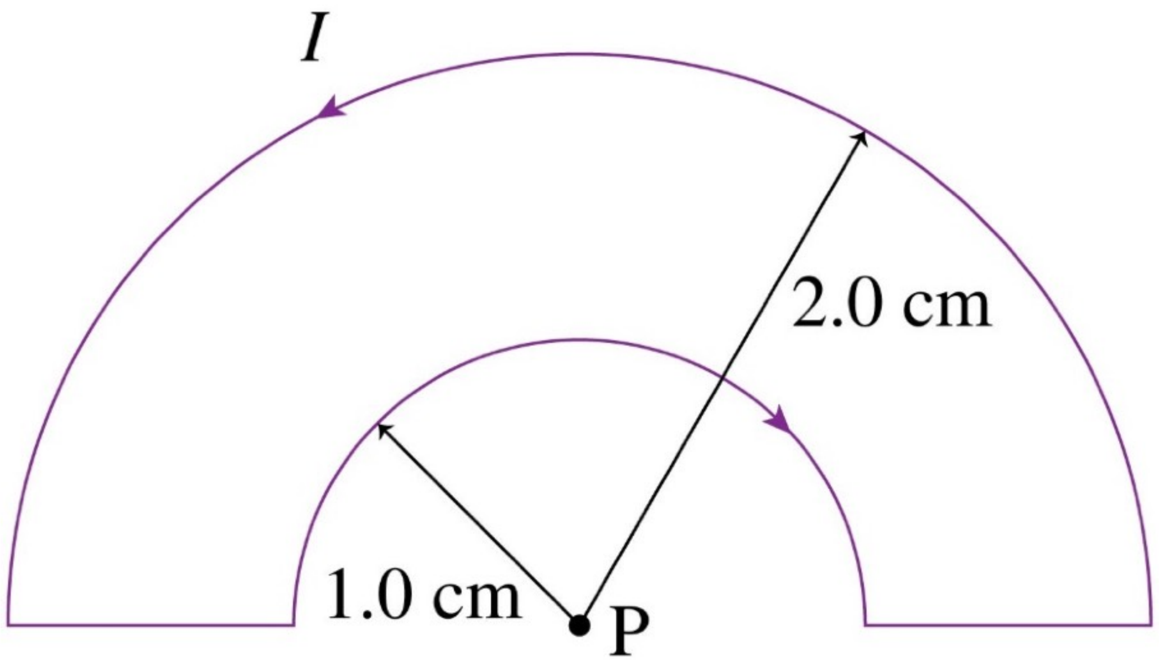
- Electron microscopes, chip nano fabrication, my labs
  - Single coil per axis
  - Pair of coils/axis, Helmoltz





iClicker question #14-1

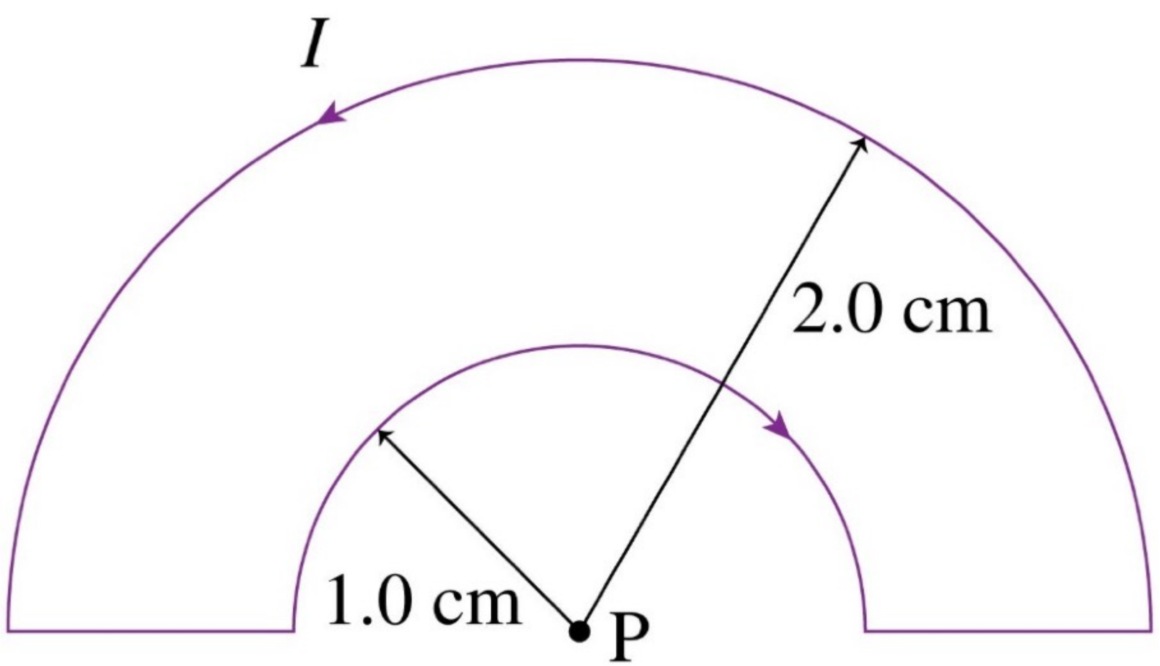
The magnet field at point P is



A. Into the screen.
B. Out of the screen.
C. Zero.

iClicker question #14-1

The magnet field at point P is



✔ **A. Into the screen.**
B. Out of the screen.
C. Zero.

TACTICS BOX 29.2

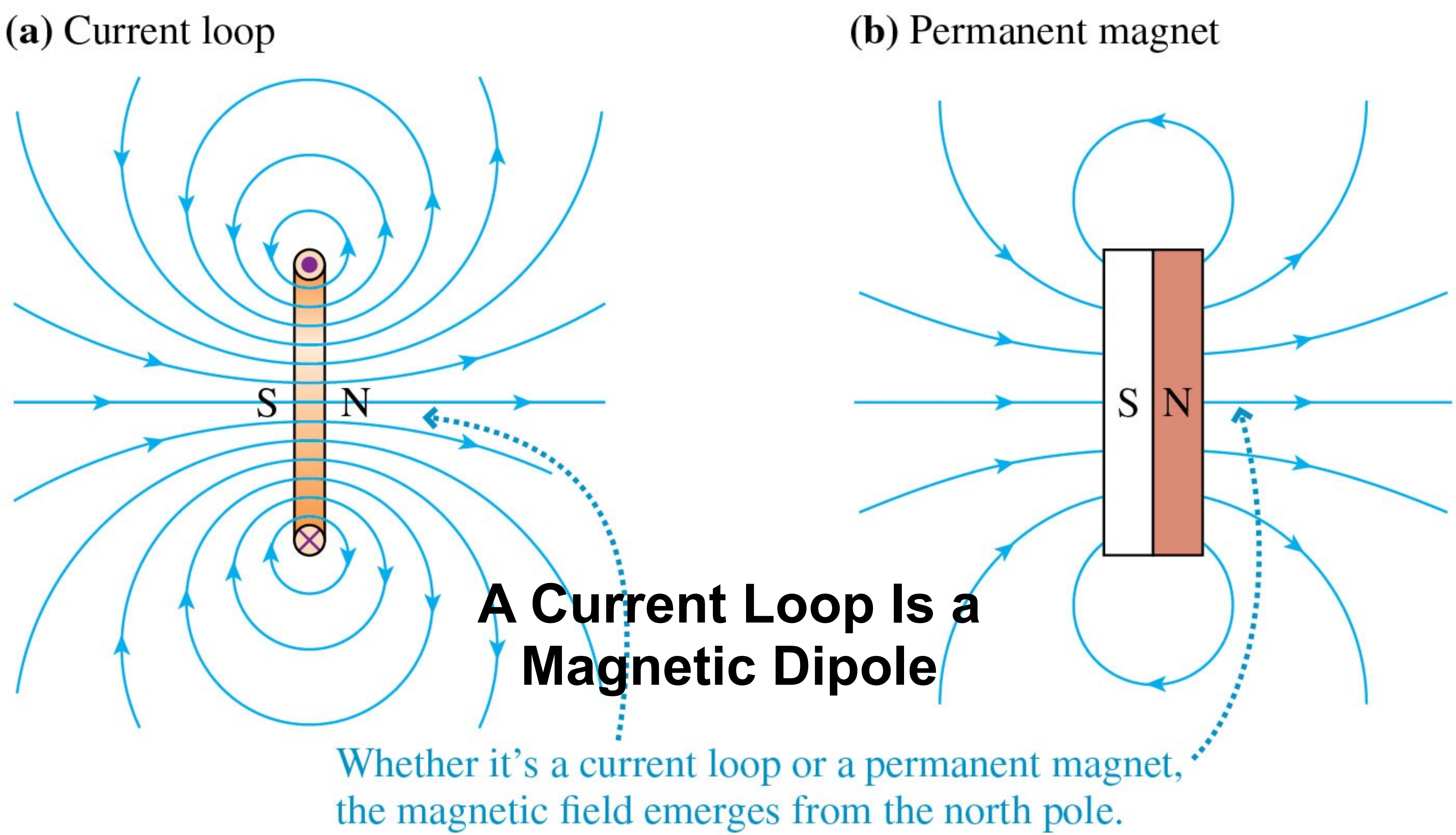
MP

Finding the magnetic field direction of a current loop

Use either of the following methods to find the magnetic field direction:

1. Point your right thumb in the direction of the current at any point on the loop and let your fingers curl through the center of the loop. Your fingers are then pointing in the direction in which $\vec{B}$ leaves the loop.
2. Curl the fingers of your right hand around the loop in the direction of the current. Your thumb is then pointing in the direction in which $\vec{B}$ leaves the loop.

3–20



A Current Loop Is a Magnetic Dipole

iClicker question #14-2

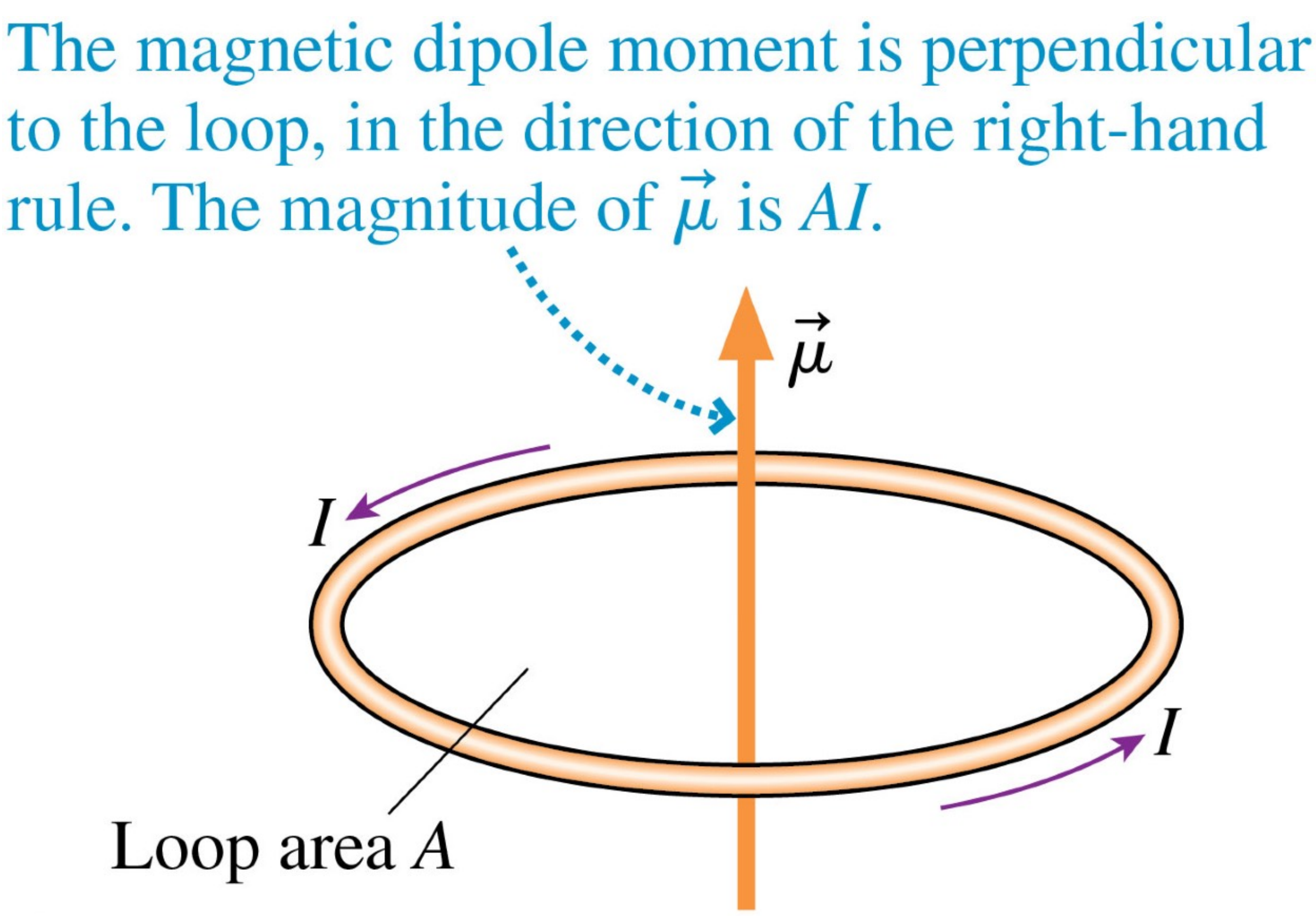
Where is the north magnetic pole of this current loop?

A. Top side.
B. Bottom side.
C. Right side.
D. Left side.
E. Current loops don’t have north poles.

The Magnetic Dipole Moment

- The **magnetic dipole moment** of a current loop enclosing an area A is defined as

$$\vec{\mu} = (AI, \text{ from the south pole to the north pole})$$

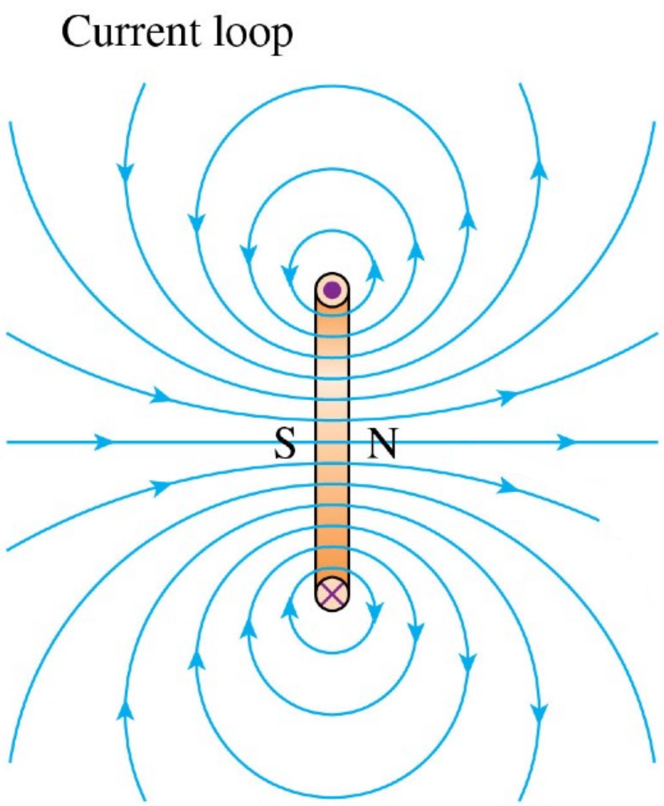


The Magnetic Dipole Moment

$$\vec{\mu} = (AI, \text{ from the south pole to the north pole})$$

- The SI units of the magnetic dipole moment are $\mathrm{A\,m^2}$.
- The on-axis field of a magnetic dipole is

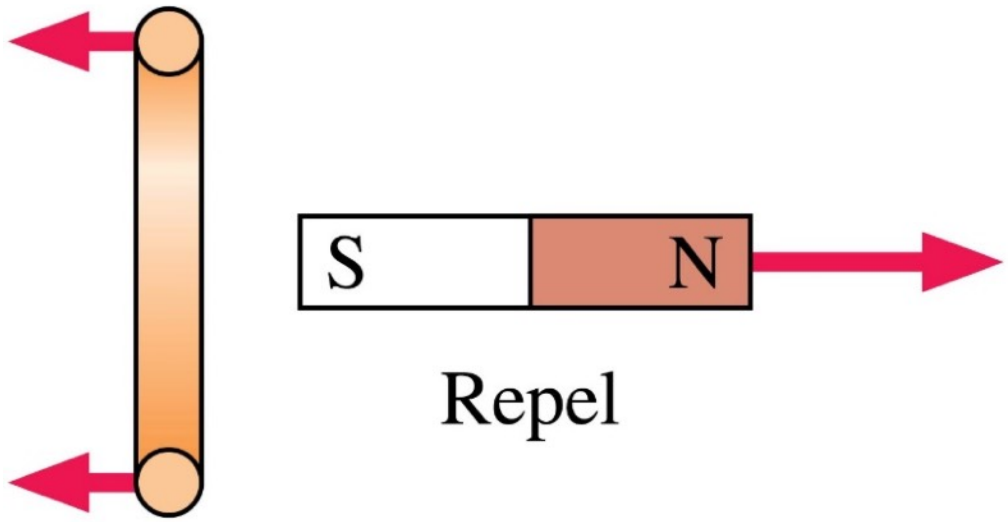
$$\vec{B}_{\text{dipole}} = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{z^3} \quad \text{(on the axis of a magnetic dipole)}$$



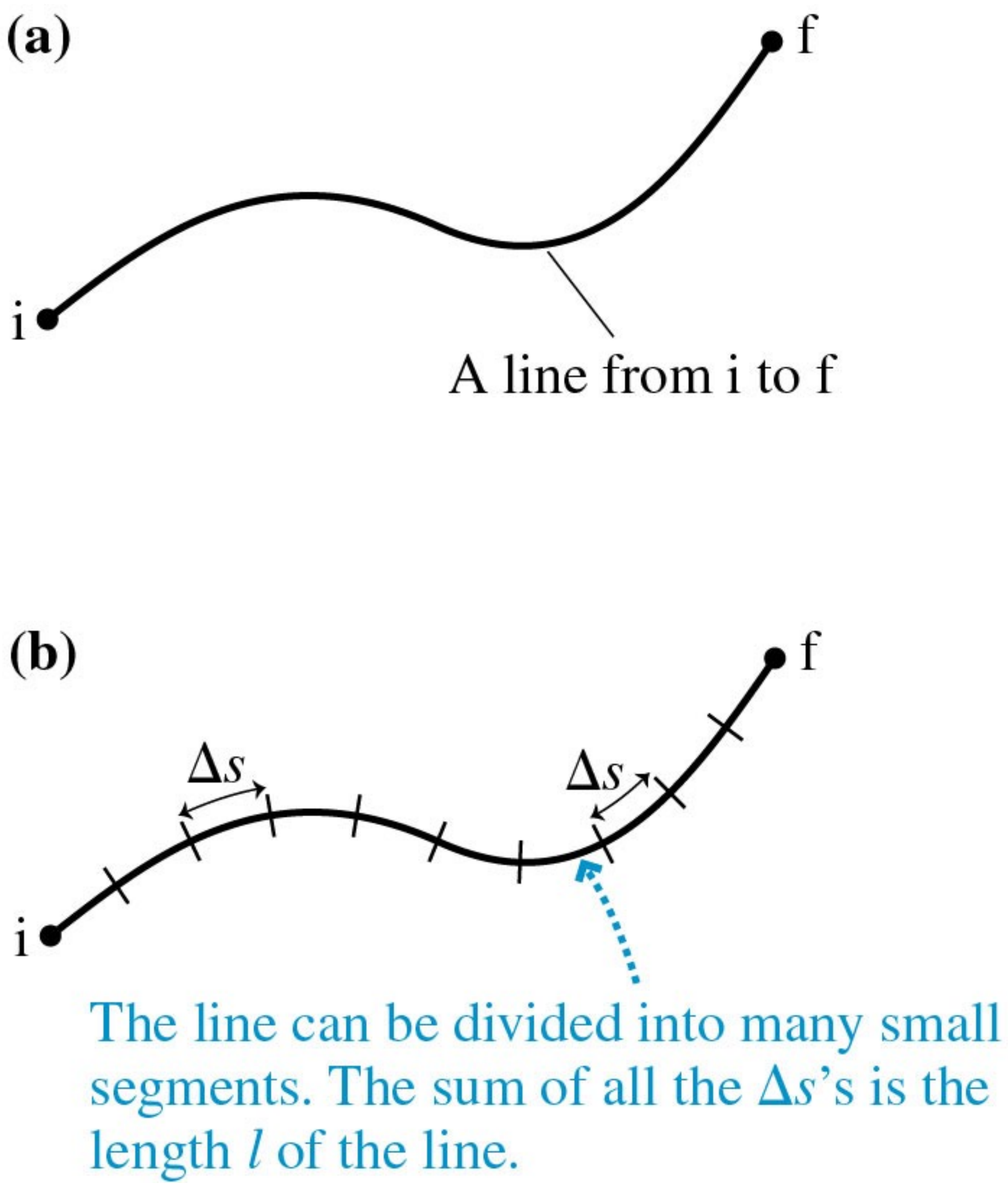
iClicker question #14-3

What is the current direction in the loop?

A. Out at the top, in at the bottom.
B. In at the top, out at the bottom.
C. Either A or B would cause the current loop and the bar magnet to repel each other.

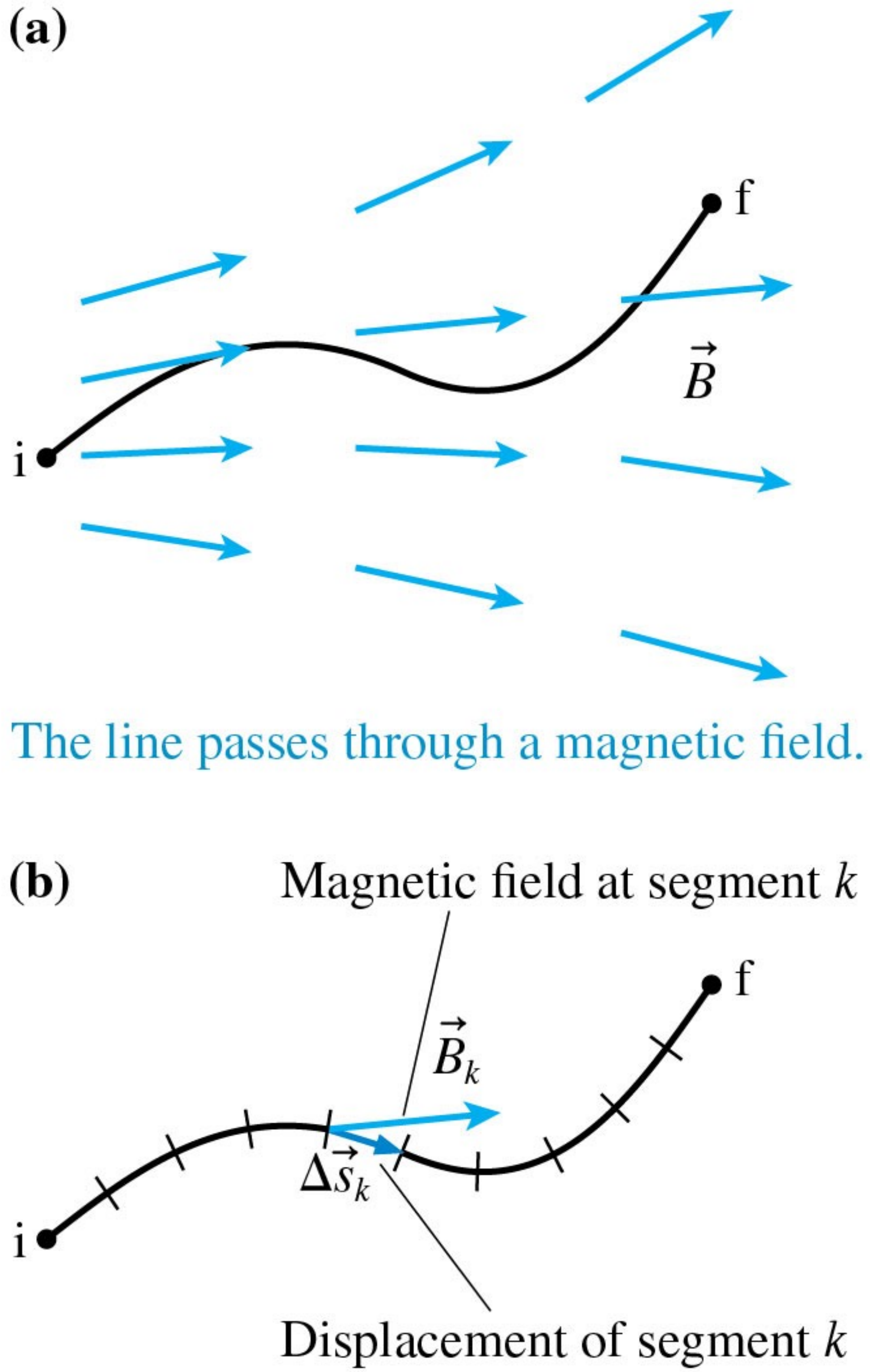


Line Integrals



- Figure (a) shows a curved line from i to f.
- The length l of this line can be found by doing a line integral:

$$l = \sum_k \Delta s_k \rightarrow \int_{\mathrm{i}}^{\mathrm{f}} ds$$



- Figure (a) shows a curved line which passes through a magnetic field $\vec{B}$.
- We can find the line integral of $\vec{B}$ from i to f as measured along this line, in this direction:

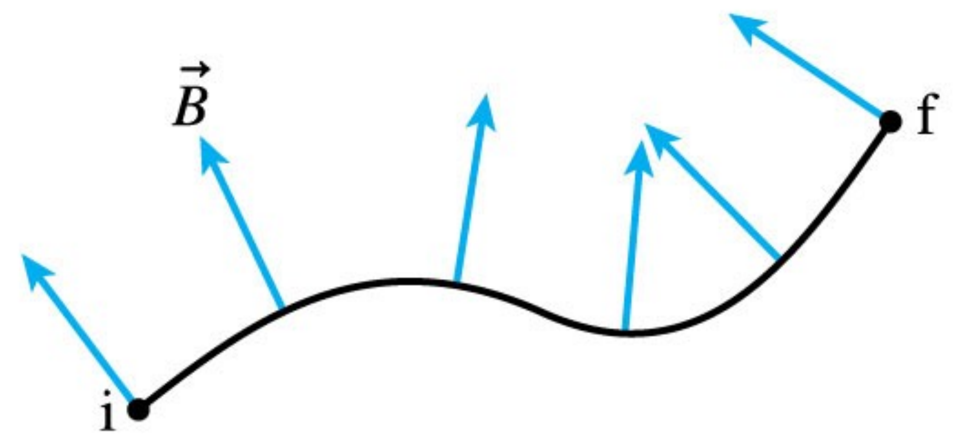
$$\sum_k \vec{B}_k \cdot \Delta\vec{s}_k \rightarrow \int_{\mathrm{i}}^{\mathrm{f}} \vec{B} \cdot d\vec{s}$$

TACTICS BOX 29.3

Evaluating line integrals

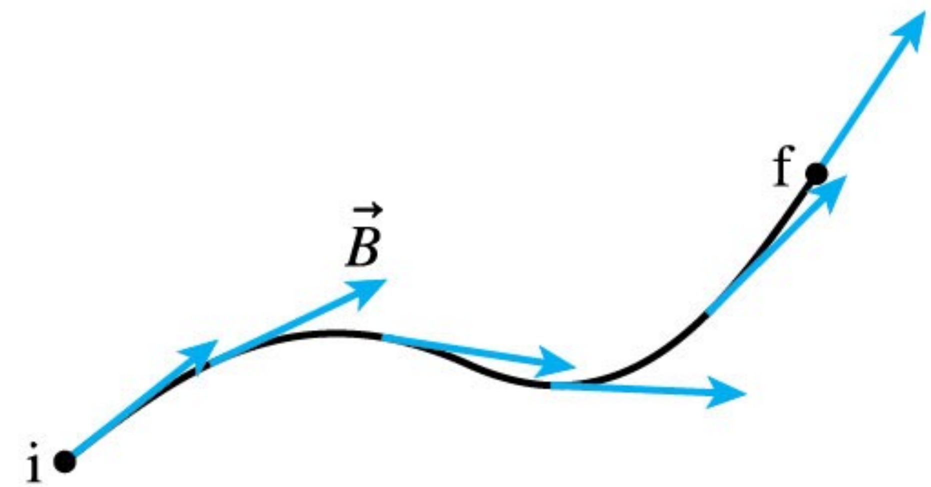
❶ If $\vec{B}$ is everywhere perpendicular to a line, the line integral of $\vec{B}$ is

$$\int_{\mathrm{i}}^{\mathrm{f}} \vec{B} \cdot d\vec{s} = 0$$



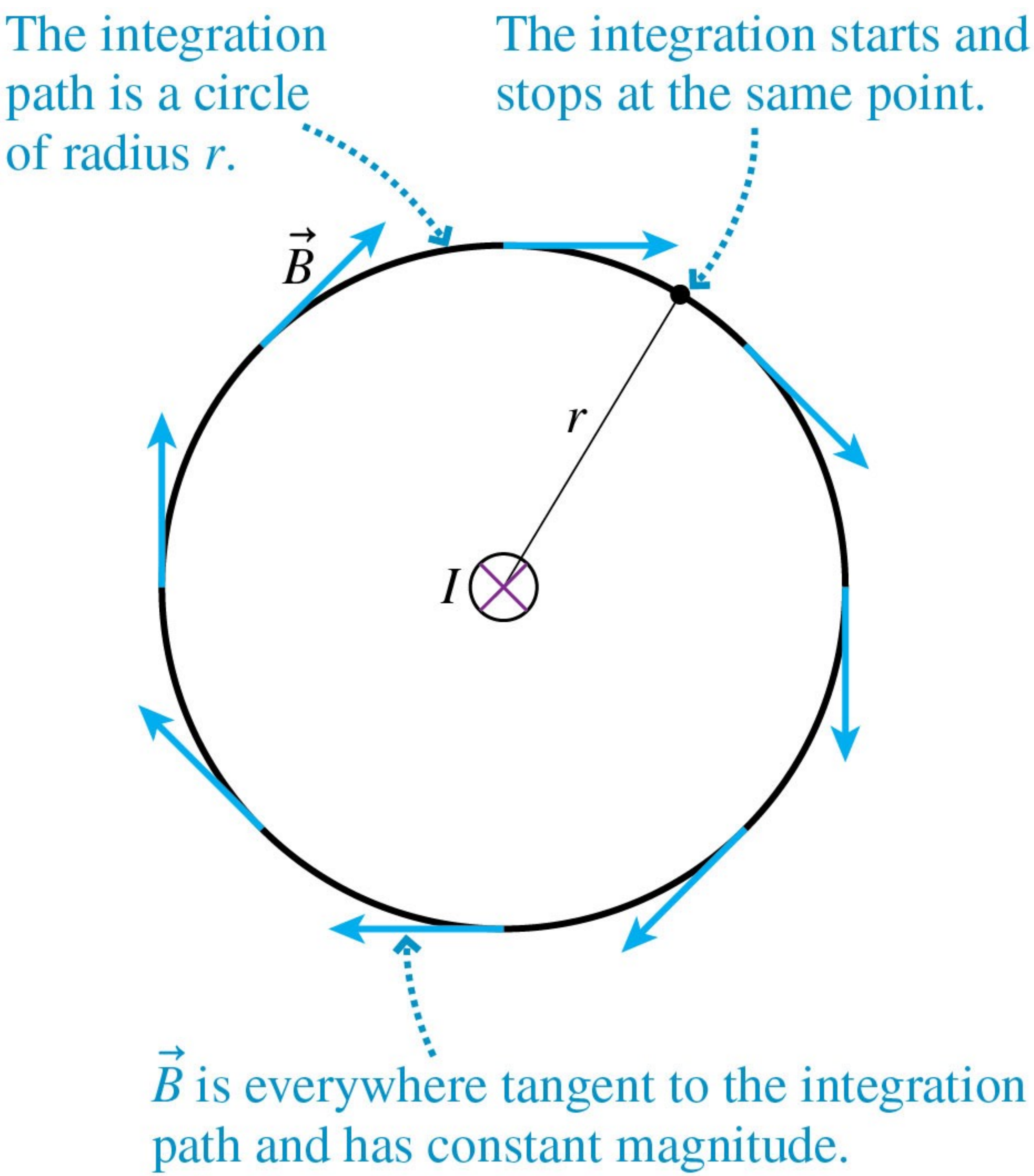
❷ If $\vec{B}$ is everywhere tangent to a line of length l *and* has the same magnitude B at every point, then

$$\int_{\mathrm{i}}^{\mathrm{f}} \vec{B} \cdot d\vec{s} = Bl$$



Exercises 23–24

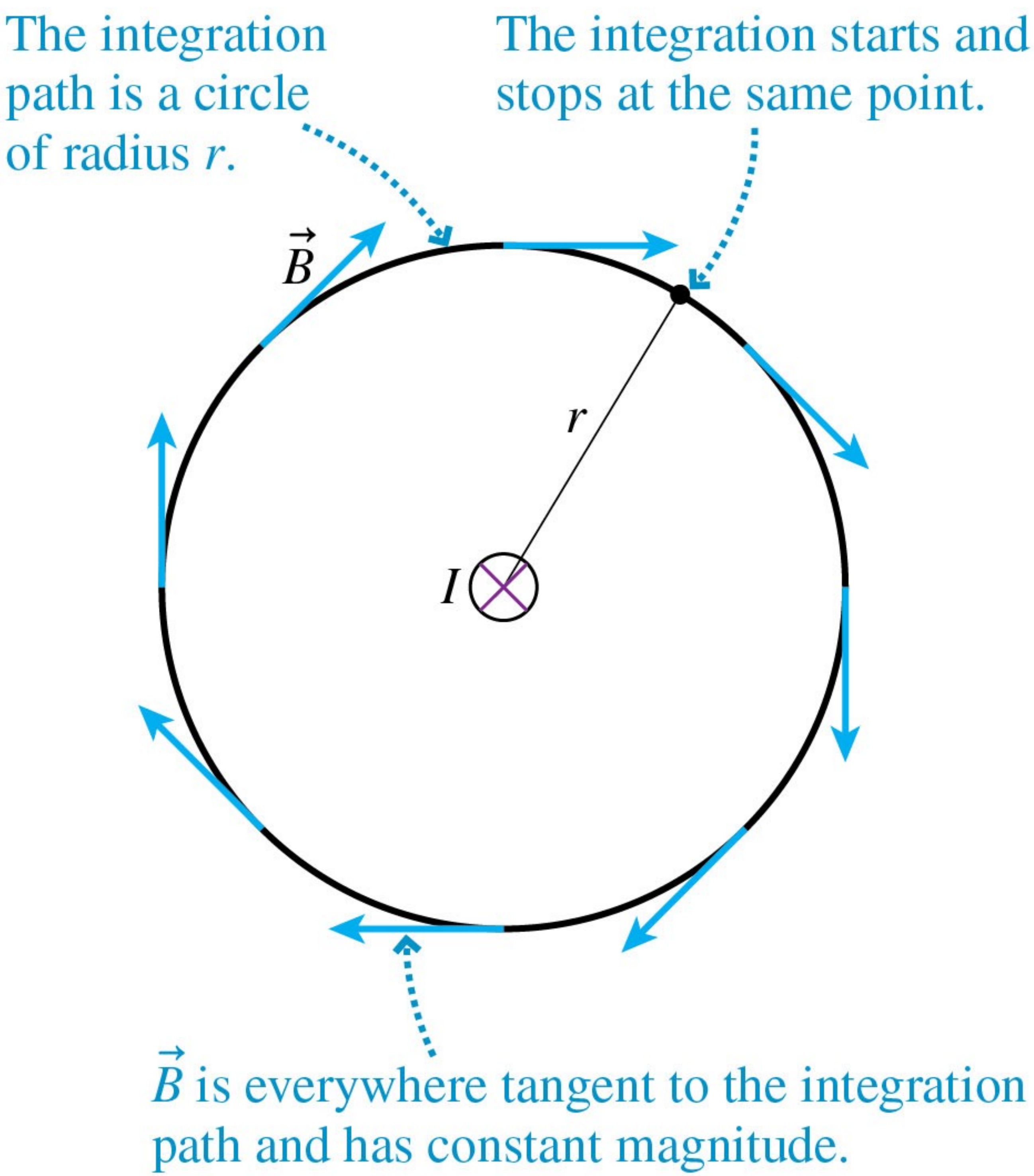
Ampère's Law



- Consider a line integral of $\vec{B}$ evaluated along a circular path all the way around a wire carrying current I.
- This is the line integral around a *closed curve*, which is denoted

$$\oint \vec{B} \cdot d\vec{s}$$

Ampère's Law



- Because $\vec{B}$ is tangent to the circle and of constant magnitude at every point on the circle, we can write

$$\oint \vec{B} \cdot d\vec{s} = Bl = B(2\pi r)$$

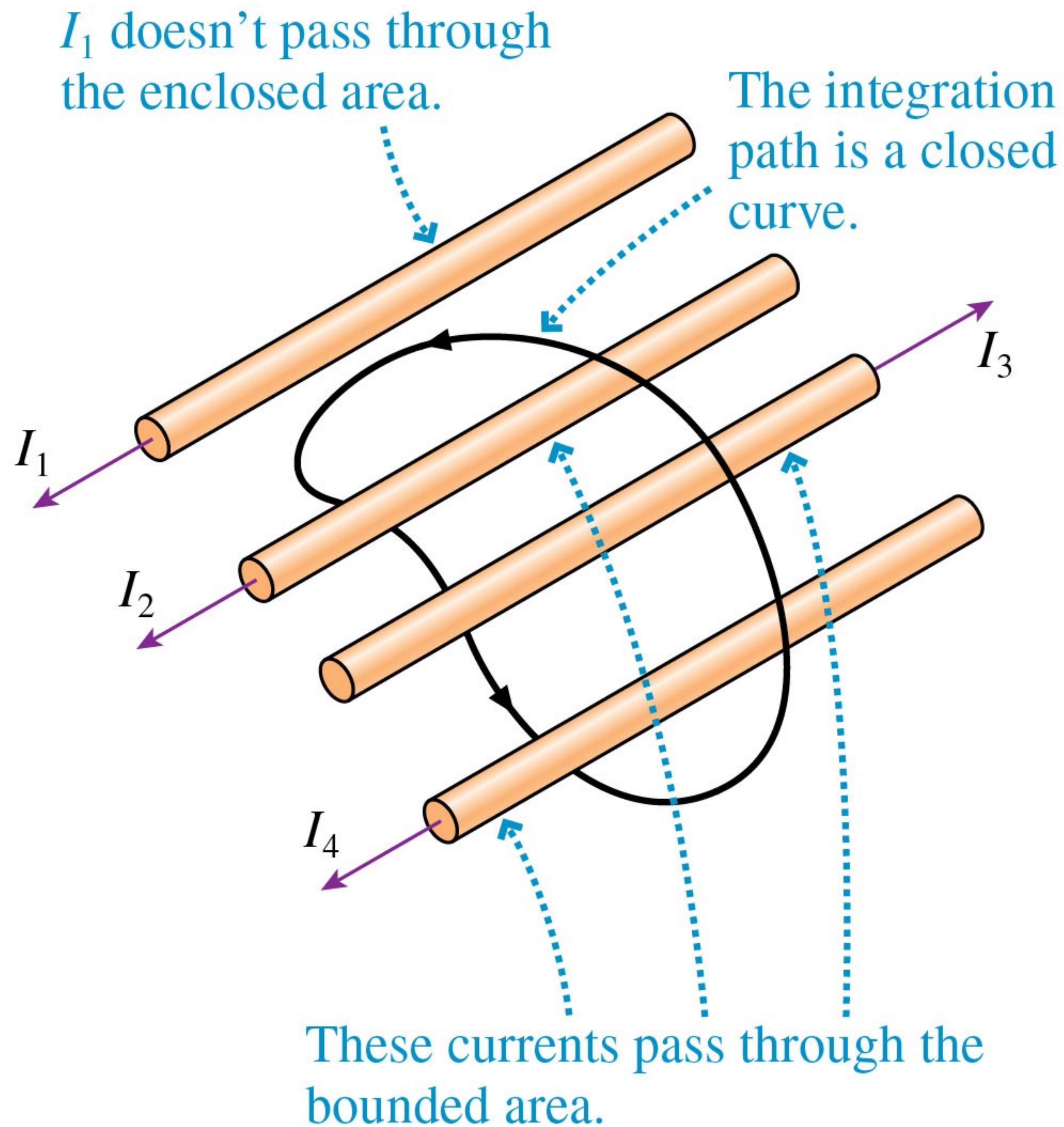
- Here $B = \mu_0 I/2\pi r$ where I is the current *through* this loop, hence

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

Ampère's Law

- Whenever total current I_{through} passes through an area bounded by a *closed curve*, the line integral of the magnetic field around the curve is given by Ampère's law:

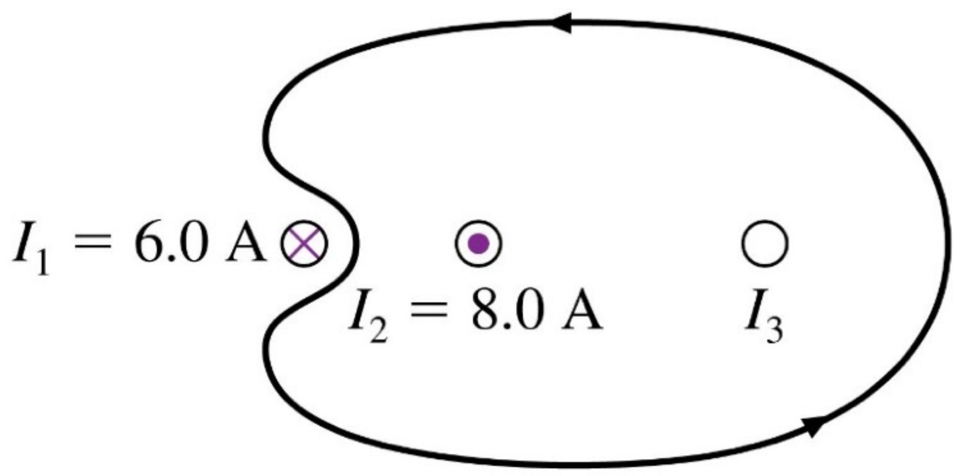
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}}$$



iClicker question #14-4

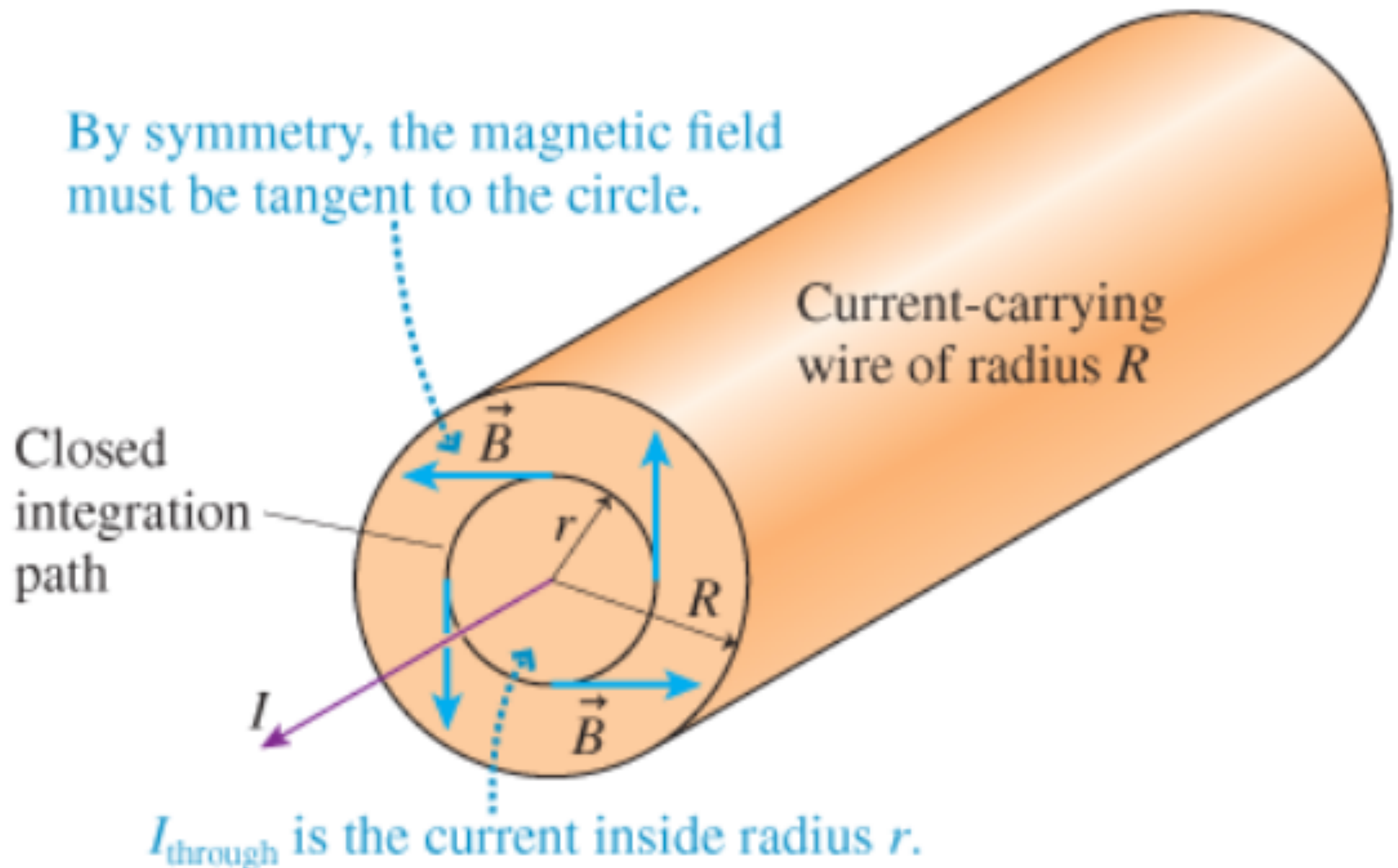
The line integral of B around the loop is $\mu_0 \cdot 7.0\ \text{A}$. Current I_3 is

A. $0\ \text{A}$.
B. $1\ \text{A}$ out of the screen.
C. $1\ \text{A}$ into the screen.
D. $5\ \text{A}$ out of the screen.
E. $5\ \text{A}$ into the screen.



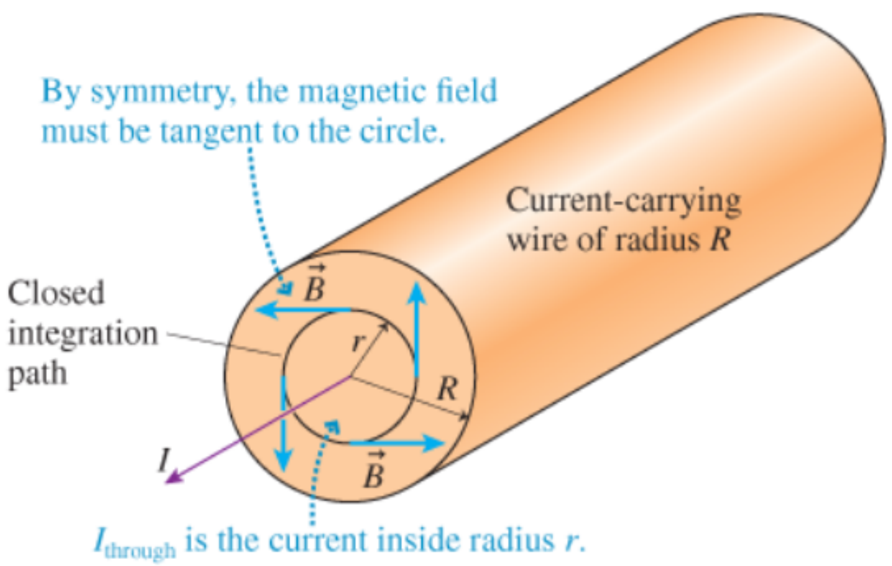
Example 29.8 The magnetic field inside a current-carrying wire

A wire of radius R carries current I. Find the magnetic field *inside* the wire at distance $r < R$ from the axis.



Example 29.8 **The magnetic field inside a current-carrying wire**

A wire of radius R carries current I. Find the magnetic field *inside* the wire at distance $r < R$ from the axis.



For your reference only in lecture slides, not to read in class ;)

SOLVE To find the field strength at radius r, we draw a circle of radius r. The amount of current passing through this circle is

$$I_{\text{through}} = JA_{\text{circle}} = \pi r^2 J$$

where J is the current density. Our assumption of a uniform current density allows us to use the full current I passing through a wire of radius R to find that

$$J = \frac{I}{A} = \frac{I}{\pi R^2}$$

Thus the current through the circle of radius r is

$$I_{\text{through}} = \frac{r^2}{R^2} I$$

Let's integrate $\vec{B}$ around the circumference of this circle. According to Ampère's law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} = \frac{\mu_0 r^2}{R^2} I$$

We know from the symmetry of the wire that $\vec{B}$ is everywhere tangent to the circle *and* has the same magnitude at all points on the circle. Consequently, the line integral of $\vec{B}$ around the circle can be evaluated using Option 2 of Tactics Box 29.3:

$$\oint \vec{B} \cdot d\vec{s} = Bl = 2\pi r B$$

where $l = 2\pi r$ is the path length. If we substitute this expression into Ampère's law, we find that

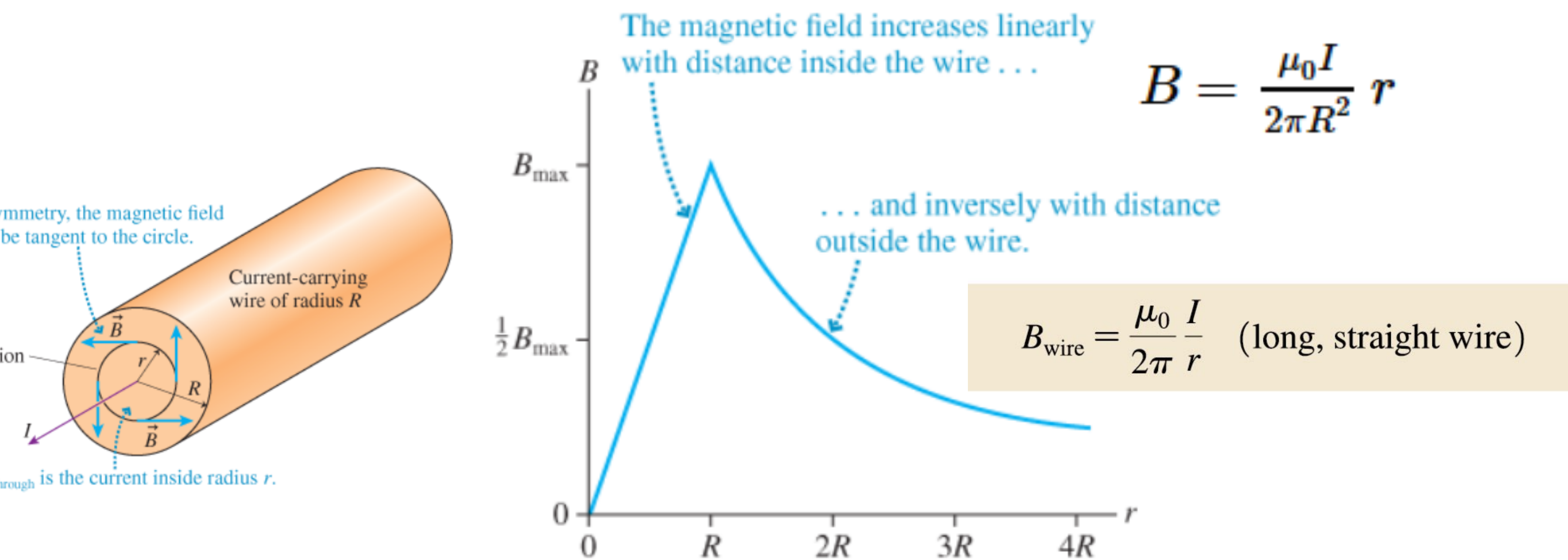
$$2\pi r B = \frac{\mu_0 r^2}{R^2} I$$

Solving for B, we find that the magnetic field strength at radius r *inside* a current-carrying wire is

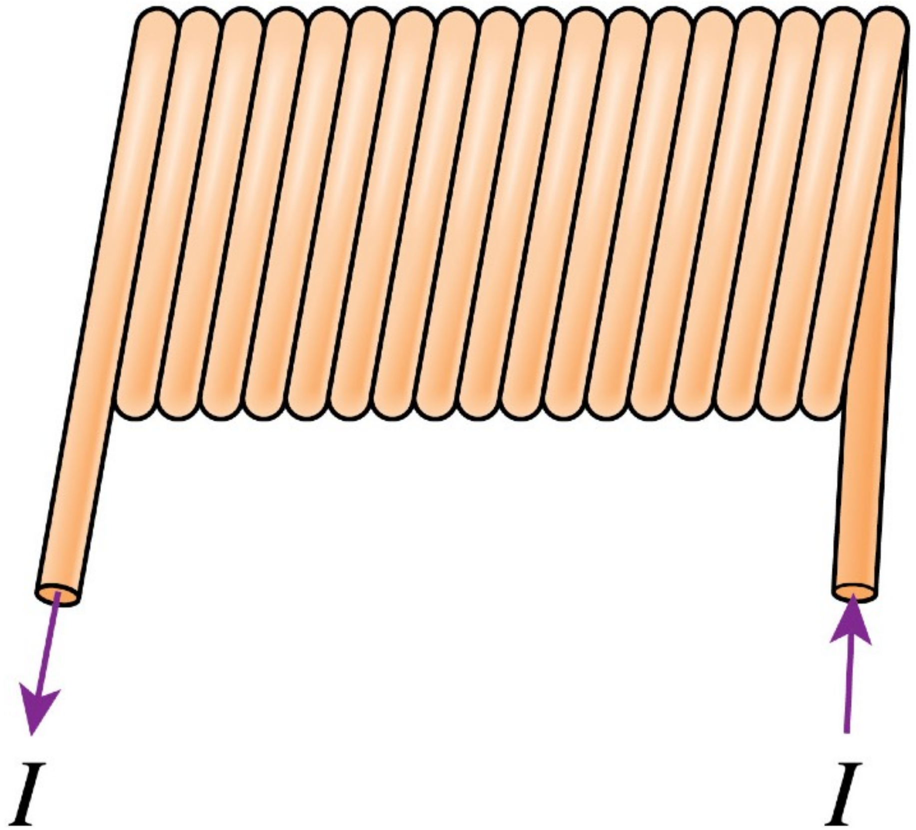
$$B = \frac{\mu_0 I}{2\pi R^2} r$$

ASSESS The magnetic field *inside* a wire increases linearly with distance from the center until, at the surface of the wire, $B = \mu_0 I / 2\pi R$ matches our earlier solution for the magnetic field *outside* a current-carrying wire. This agreement at $r = R$ gives us confidence in our result. The magnetic field strength both inside and outside the wire is shown graphically in Figure 29.27.

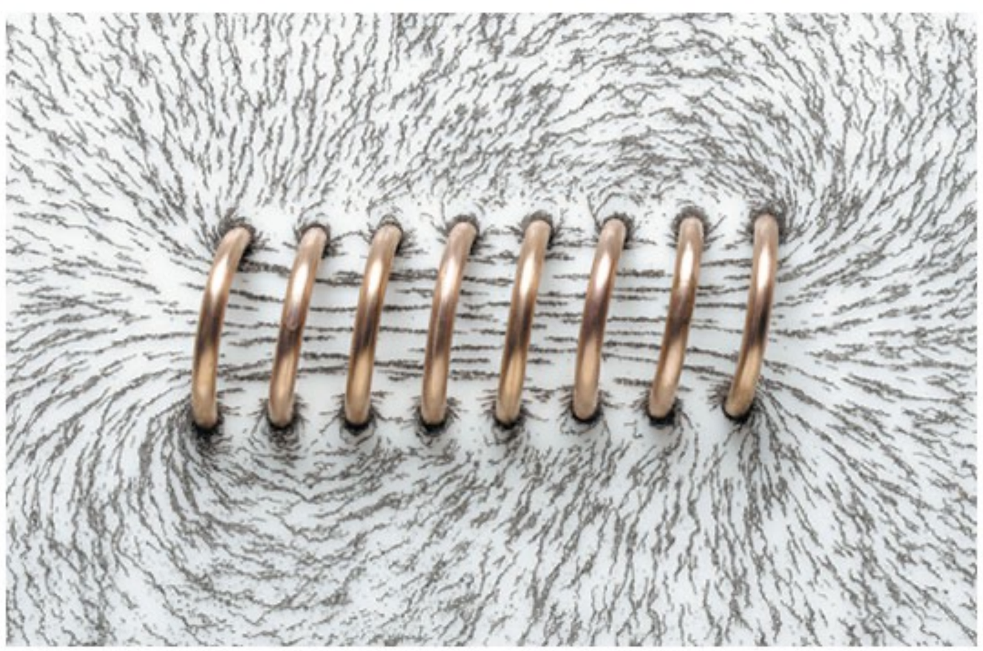
Figure 29.27 Graphical representation of the magnetic field of a current-carrying wire.



Solenoids



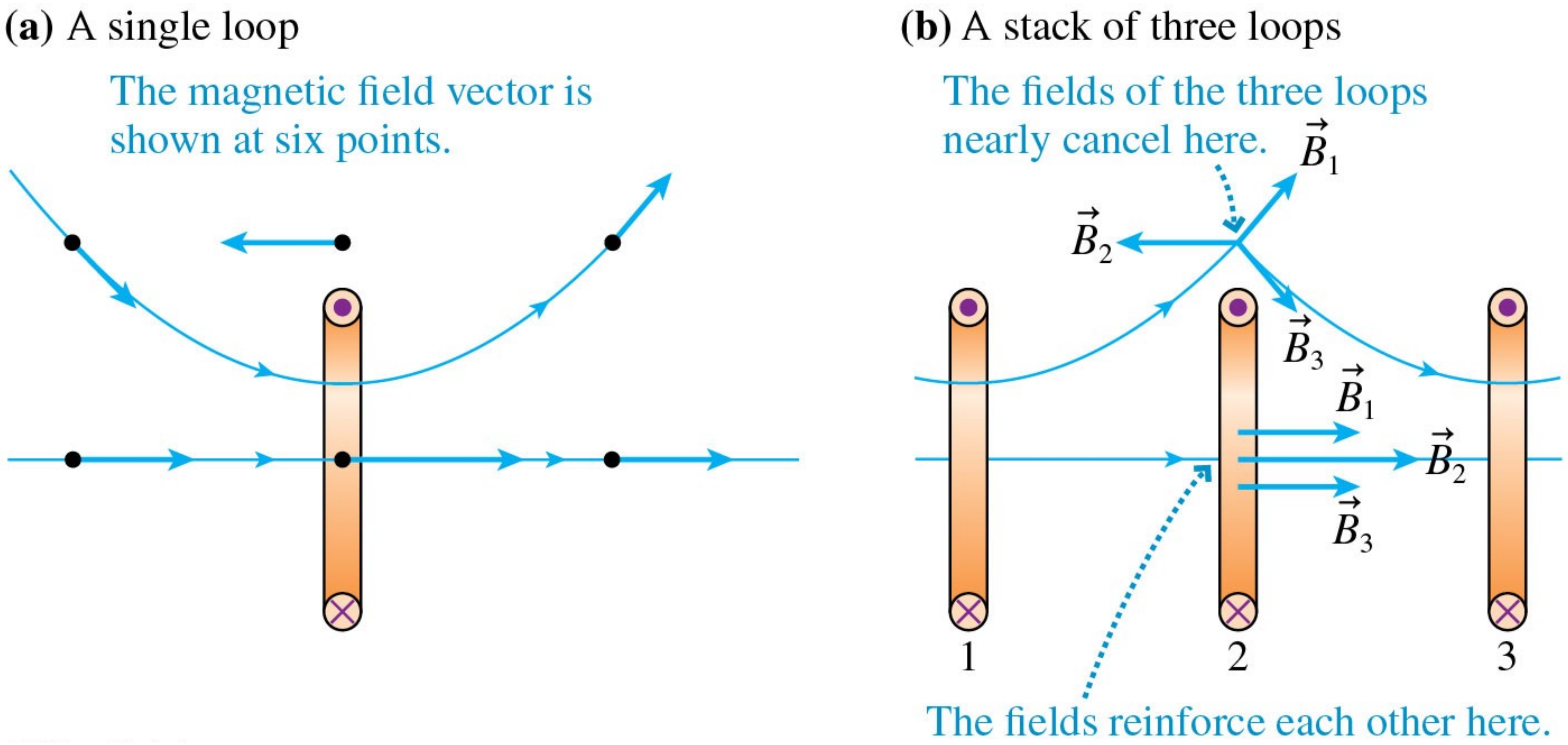
A short solenoid



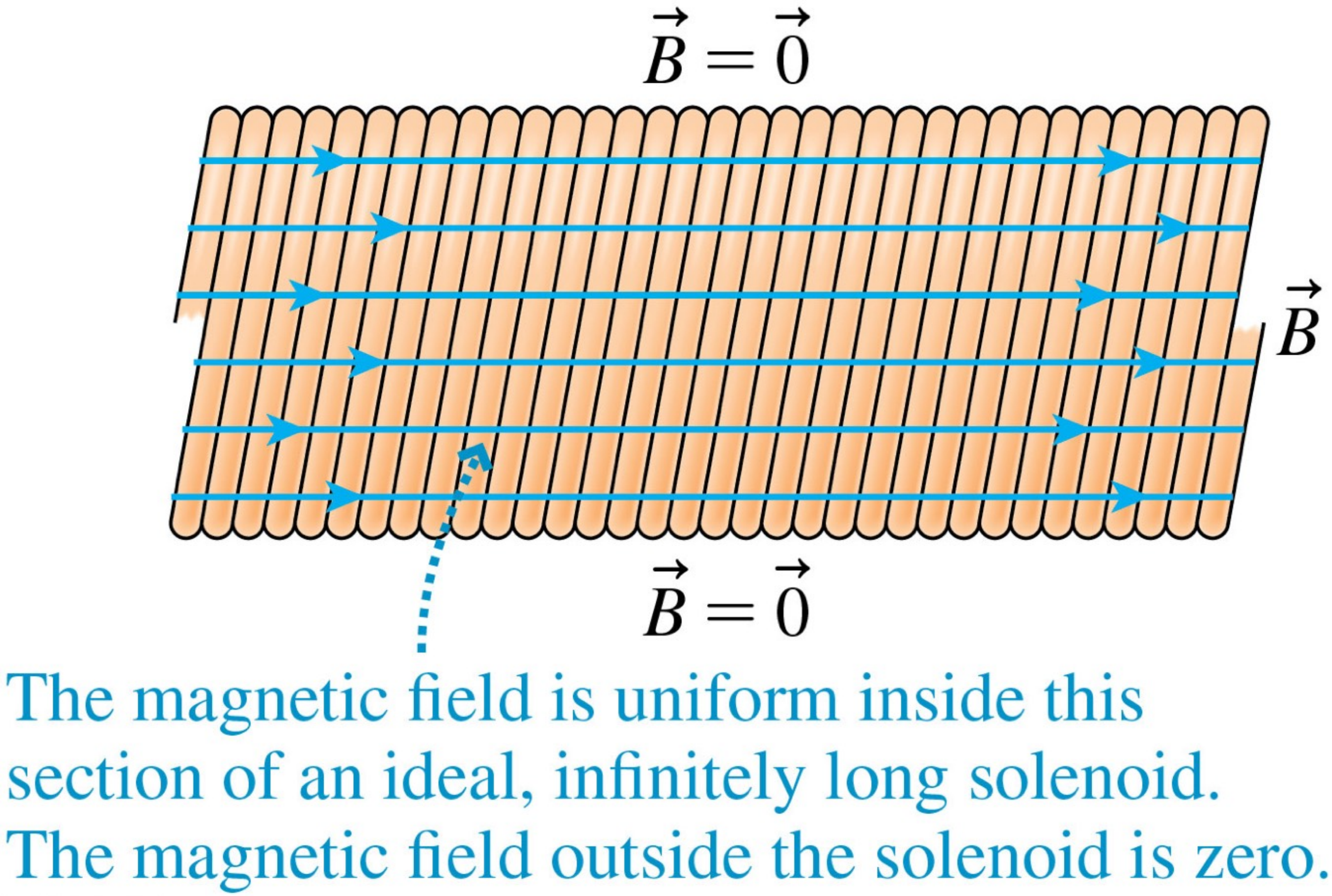
- A **uniform magnetic field** can be generated with a **solenoid**.
- A solenoid is a helical coil of wire with the same current I passing through each loop in the coil.
- Solenoids may have hundreds or thousands of coils, often called *turns*, sometimes wrapped in several layers.
- The magnetic field is strongest and most uniform *inside* the solenoid.

The Magnetic Field of a Solenoid

- With many current loops along the same axis, the field in the center is strong and roughly parallel to the axis, whereas the field outside the loops is very close to zero.

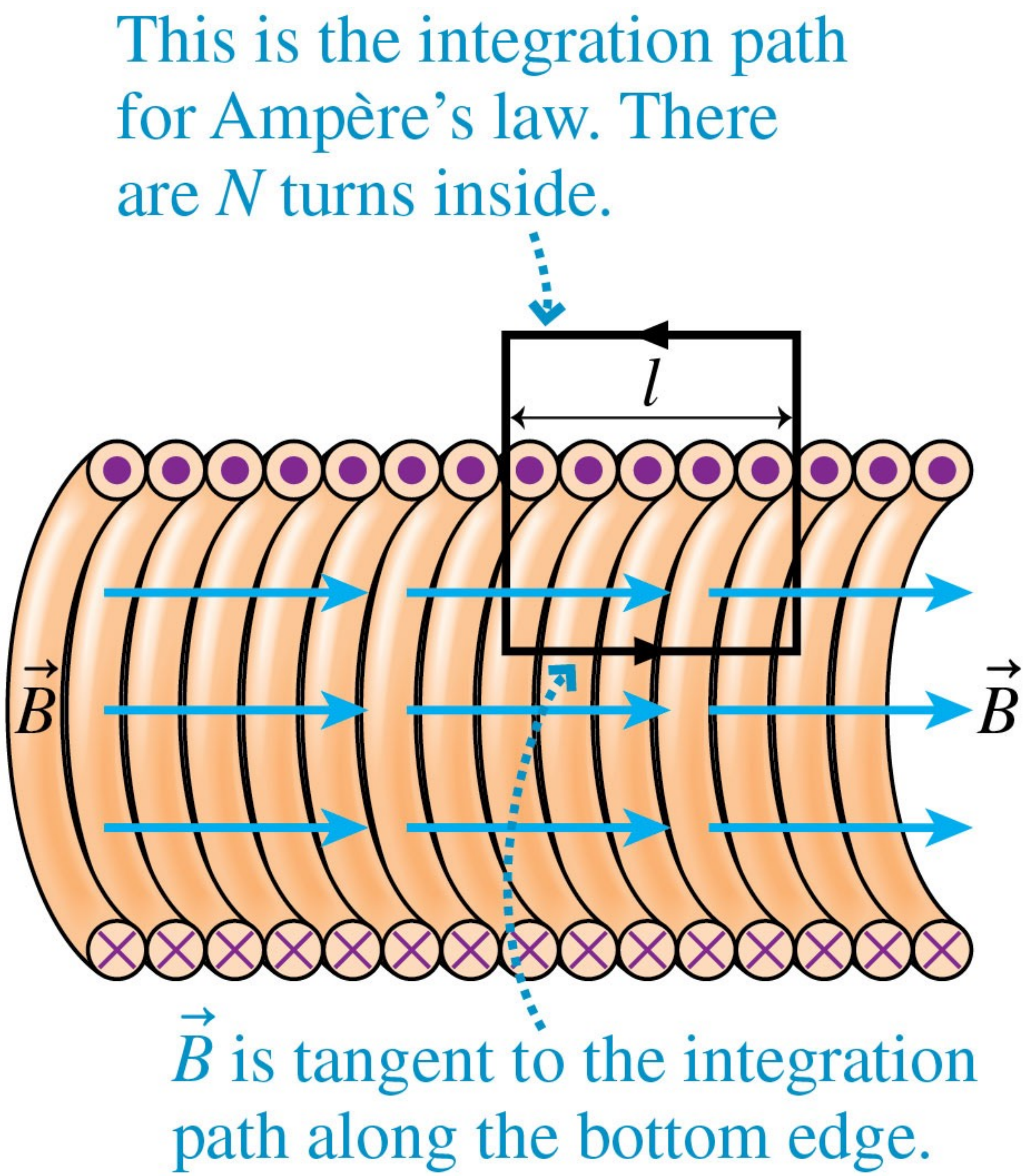


The Magnetic Field of a Solenoid



- No real solenoid is ideal, but a very uniform magnetic field can be produced near the center of a tightly wound solenoid whose length is much larger than its diameter.

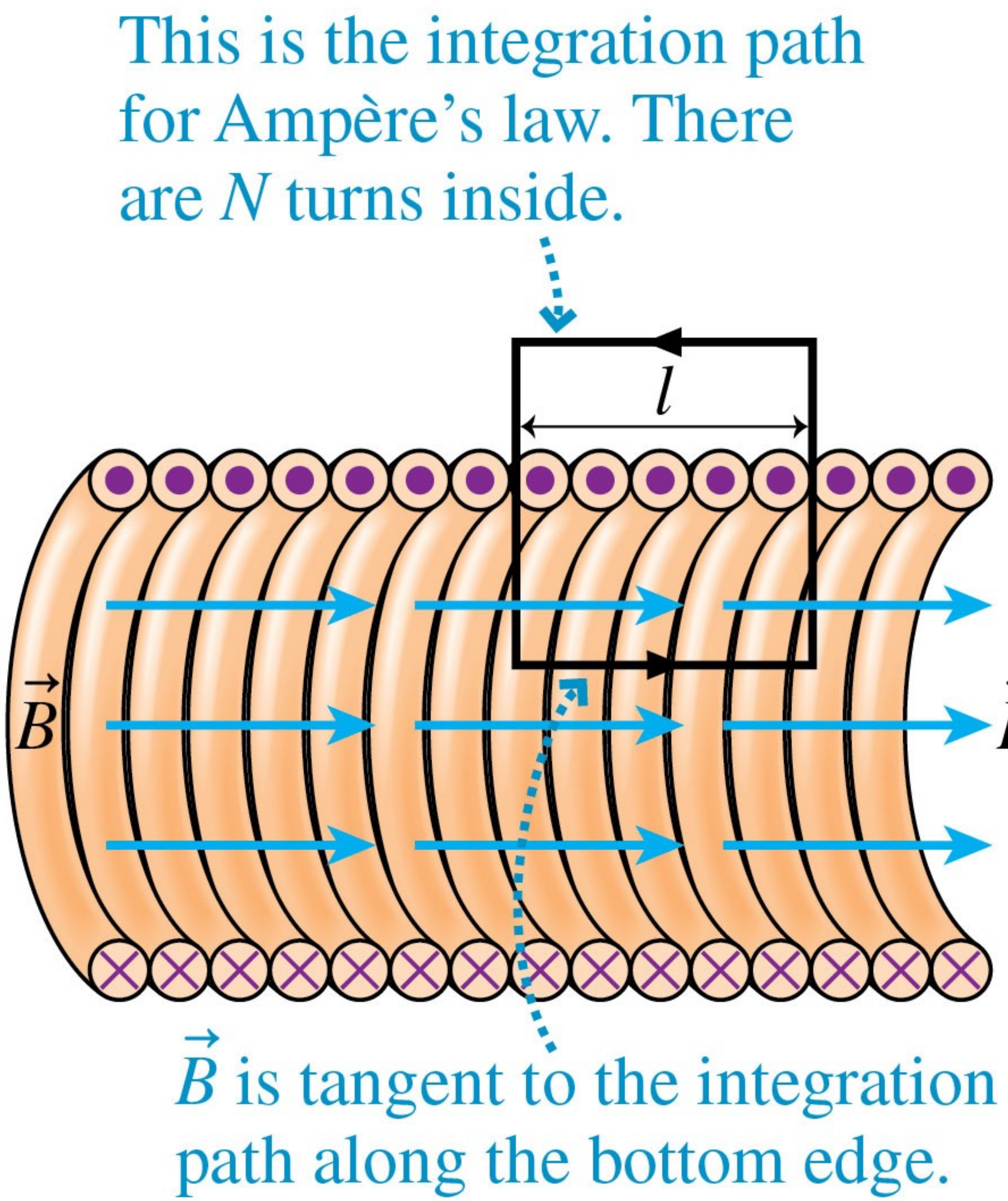
The Magnetic Field of a Solenoid



- The figure shows a cross section through an infinitely long solenoid.
- The integration path that we'll use is a rectangle.
- The current passing through this rectangle is $I_{\text{through}} = NI$.
- Ampère's Law is thus

$$\oint \vec{B} \cdot d\vec{s} = Bl = \mu_0 NI$$

The Magnetic Field of a Solenoid



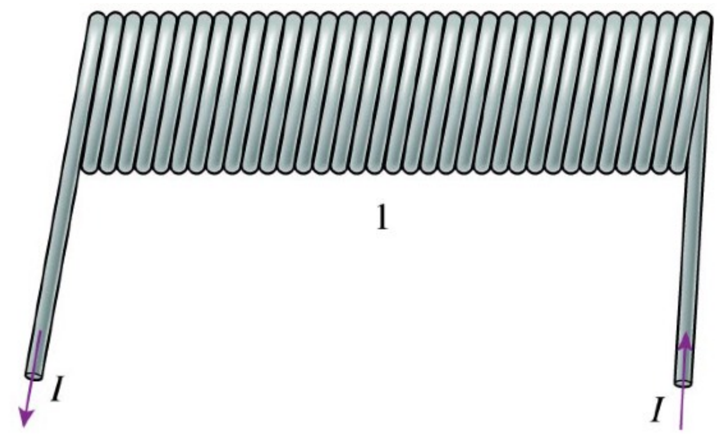
- Along the top, the line integral is zero since $B = 0$ outside the solenoid.
- Along the sides, the line integral is zero since the field is perpendicular to the path.
- Along the bottom, the line integral is simply Bl.
- Solving for B inside the solenoid:

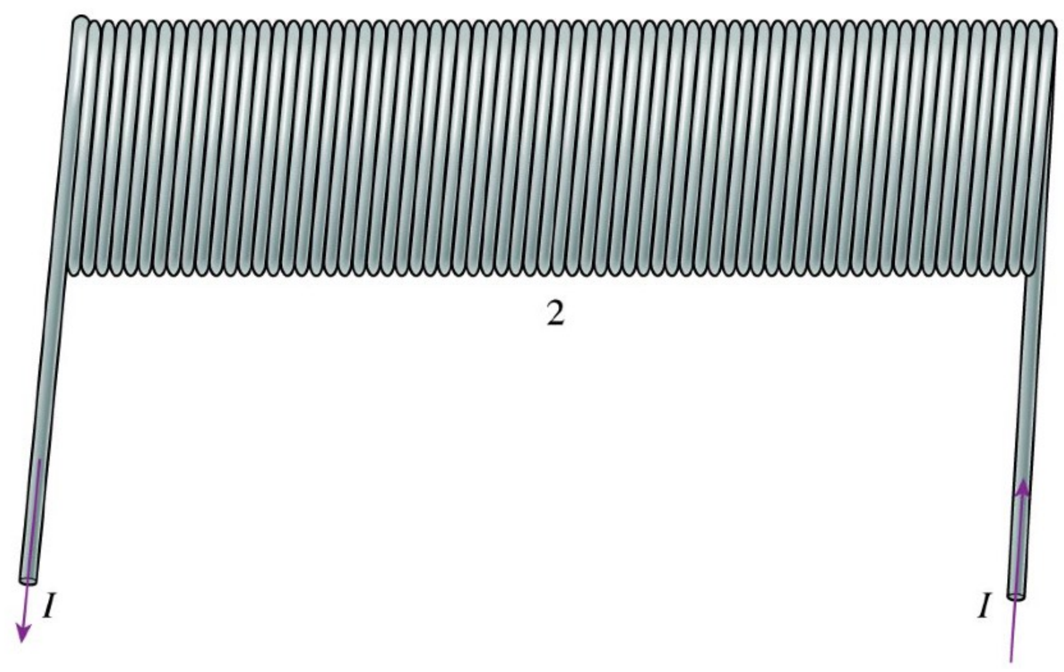
$$B_{\text{solenoid}} = \frac{\mu_0 N I}{l} = \mu_0 n I \qquad \text{(solenoid)}$$

where $n = N/l$ is the number of turns per unit length.

iClicker question #14-5

Solenoid 2 has twice the diameter, twice the length, and twice as many turns as solenoid 1. How does the field B_2 at the center of solenoid 2 compare to B_1 at the center of solenoid 1?





A. $B_2 = B_1/4$

B. $B_2 = B_1/2$

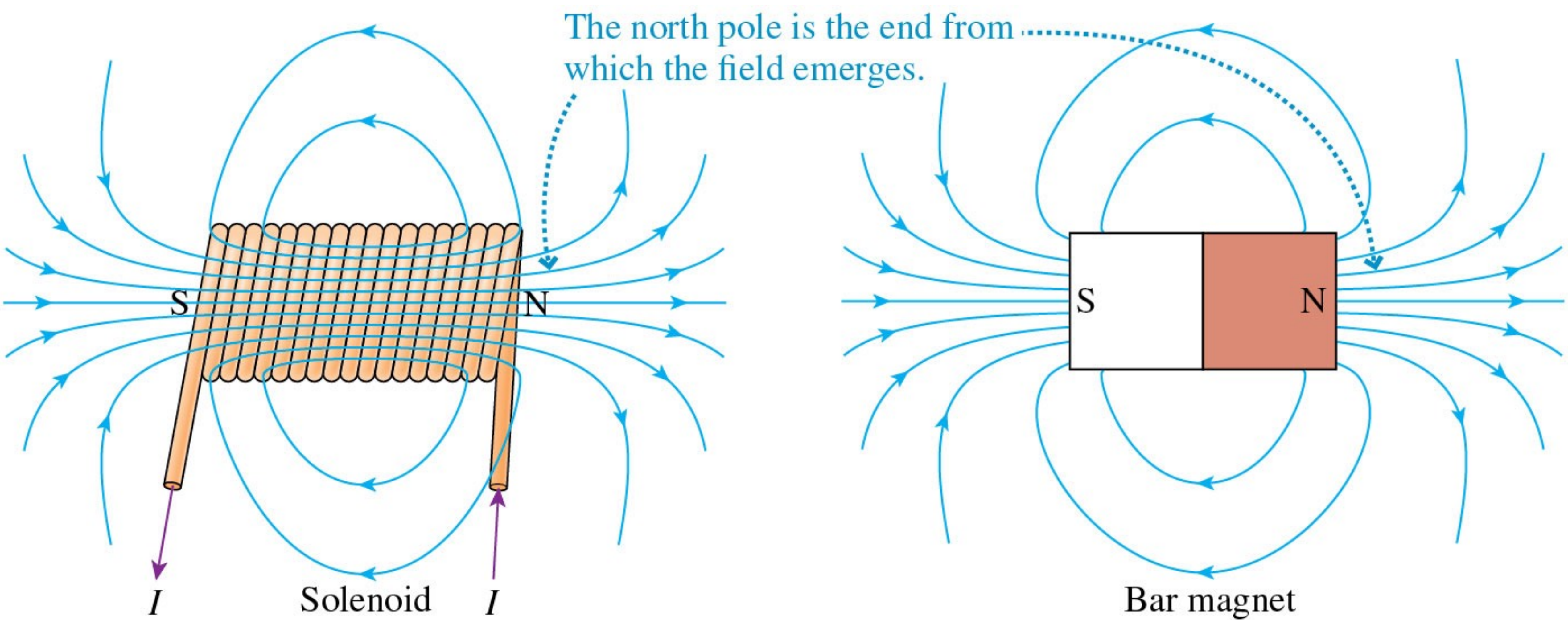
C. $B_2 = B_1$

D. $B_2 = 2B_1$

E. $B_2 = 4B_1$

The Magnetic Field Outside a Solenoid

- The magnetic field *outside* a solenoid looks like that of a bar magnet.
- Thus a solenoid is an electromagnet, and you can use the right-hand rule to identify the north-pole end.



Electromagnets, Breaking Bad