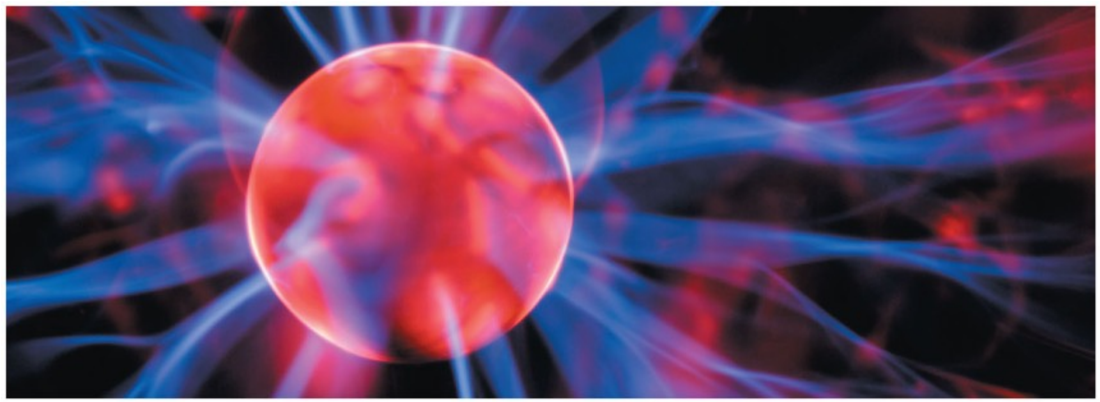
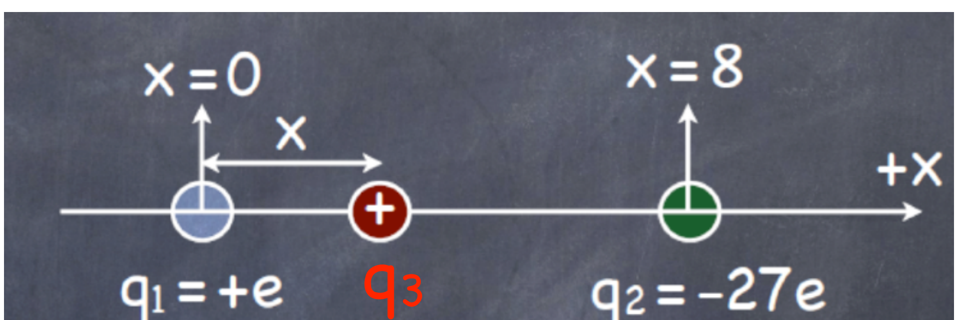
PHYS 2B: Announcements 10/02/25

"Equations are the devil's sentences!" -- Stephen Colbert

- Remember: HW1 due Sunday 10/05/25 by 11:59 PM



Last class example, minimize force on q_3



Superposition & minimization

$$F_{net} = k\frac{|q_1\, q_3|}{x^2} + k\frac{|q_2\, q_3|}{(8\text{ cm} - x)^2}$$

$$\frac{dF_{net}}{dx} = 0$$

note Please read the quiz instructions stop following 372 views

Actions

Quiz instructions

IMPORTANT: Quiz instructions

1. Students may need to identify themselves upon giving their completed quiz materials with their Student ID (No ID when requested, drop quiz). Your identity may be checked further against the roster pictures. Pictures may be taken during the quiz to compare against roster pictures later.
2. The quiz is closed book. No aid materials are allowed, no notecards, no cheat sheets.
3. **Students must seat at their assigned seat**, which will be posted by noon on the day of the quiz on Canvas.
4. Students may leave the room briefly to the bathroom but must leave their exam material, written materials, and all electronic devices in the classroom. Student will sign in and out of the room.
5. Students are required to bring their "Physics" scantron form and fill it out as required in the instructions for the scantron and exam. See class syllabus https://barreirolab.com/ucsd-phys2b
6. There are four quiz forms: A, B, C, and D. Students are responsible for having a quiz form different from their neighbors: students to your left and right. Failure to follow this may lead to a dropped quiz.
7. Students are **only allowed to use simple scientific calculators** TI-30 style, such as TI-30X or the TI-30XII-S, or an equivalent Casio or Sharp calculator (the TI30XS multiview Pro is NOT allowed, or any with integration, derivatives). These are non-graphing non-matrix calculators and are equivalent to the calculators used in the chem series. Not a comprehensive list, but these are some pretty common, cheap **simple** calculators.
8. Students are not allowed to use smartphones, tablets, laptops, **smartwatches, earphones,** or any electronic device other than your scientific calculator.
9. One more thing that all students have asked to implement: if your cellphone rings during the quiz, you have to drop that quiz.

Guidance: http://academicintegrity.ucsd.edu/take-action/prevent-cheating/faculty/exam-rules.html

Academic Integrity, take II

https://academicintegrity.org/**fundamental-values**/

Academic integrity is fundamental to the University and the work we are doing together in this classroom, I am professionally and ethically **responsible for reporting any violations** to the Academic Integrity Office.

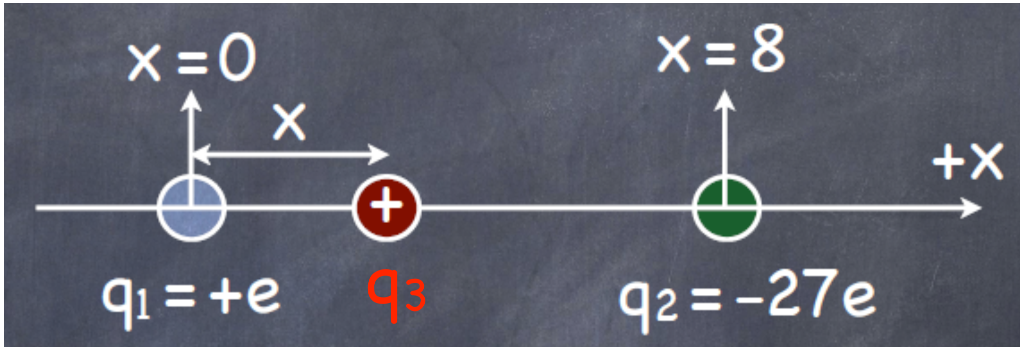
In addition, I will give at least a 0 on the assignment/exam in question, but generally pursue an F in the course, and the University will issue other consequences.

For more information on the consequences of cheating:

https://academicintegrity.ucsd.edu/process/consequences/**sanctioning-guidelines**.html

Coulomb's Law: superposition

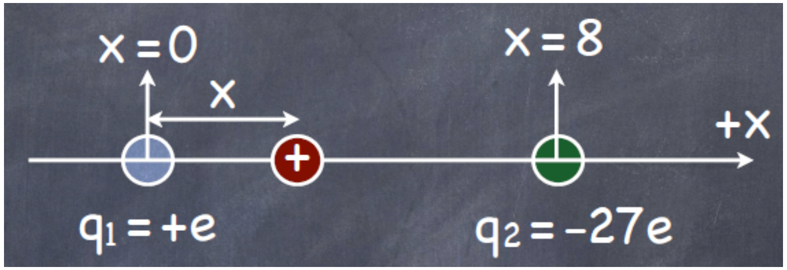
- Last time we were asked to find the position of charge q_3 (red charge below) where the force due to charges q_1 and q_2 would be a minimum.



- Note that the force there from both charges is to the right, so the minimum cannot be zero!
- Where would it be zero?
- Somewhere to the right or the left?

Let’s quickly plot in google

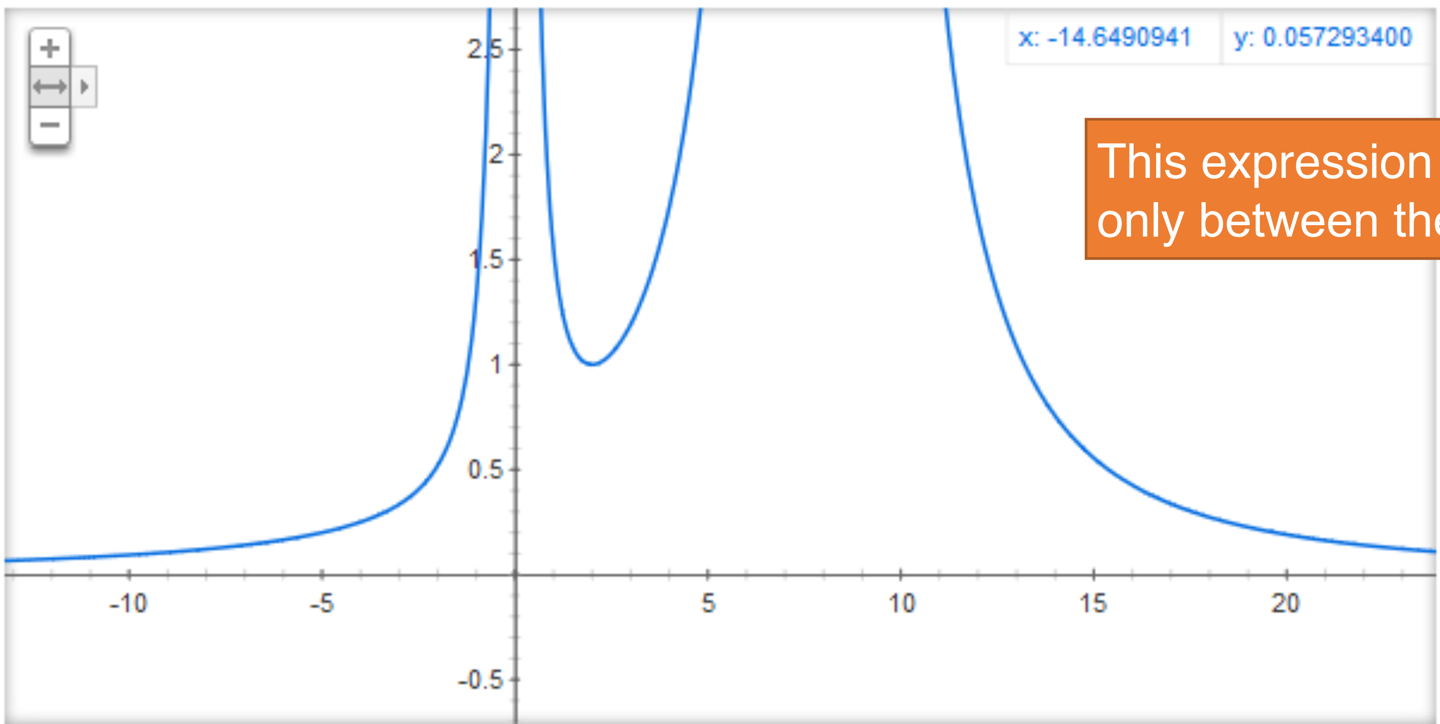
$$F_{net} = k\frac{|q_1\, q_3|}{x^2} + k\frac{|q_2\, q_3|}{(8\text{ cm} - x)^2}$$



- Note that k and q_3 are common factors

Google plot(1/x^2 +27/(8-x)^2)

Graph for 1/x^2+27/(8-x)^2

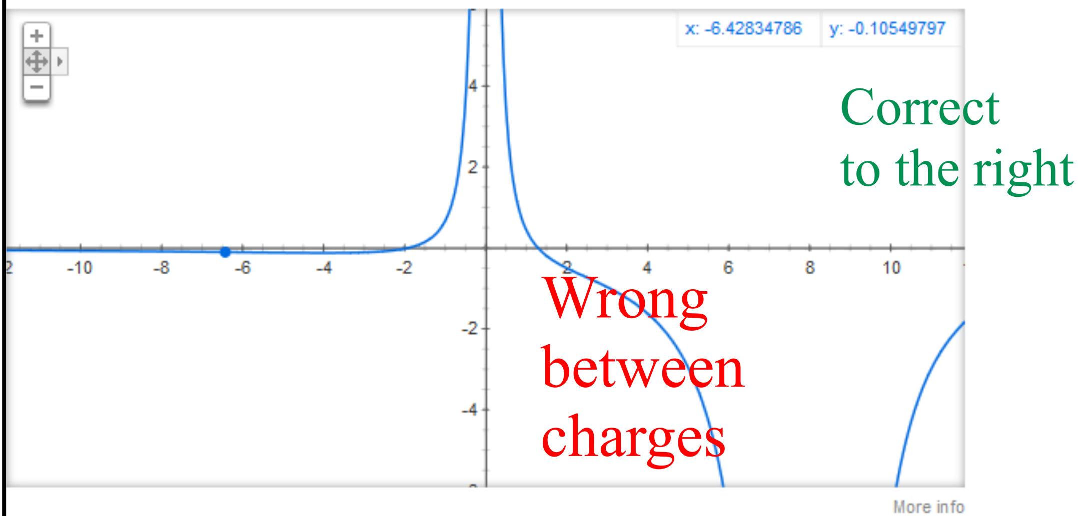


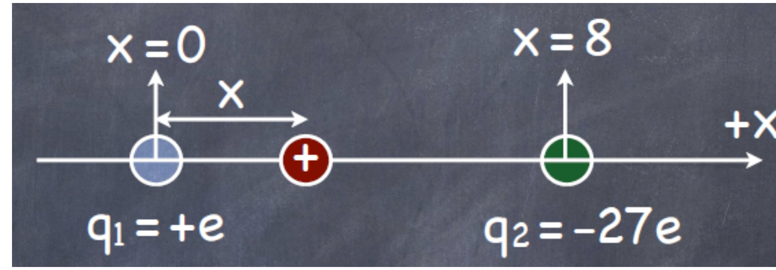
This expression is valid only between the charges!

Wait, how about writing the charges' signs

$$F_{net} = k\frac{|q_1\ q_3|}{x^2} - k\frac{|q_2\ q_3|}{(8\text{ cm} - x)^2}$$

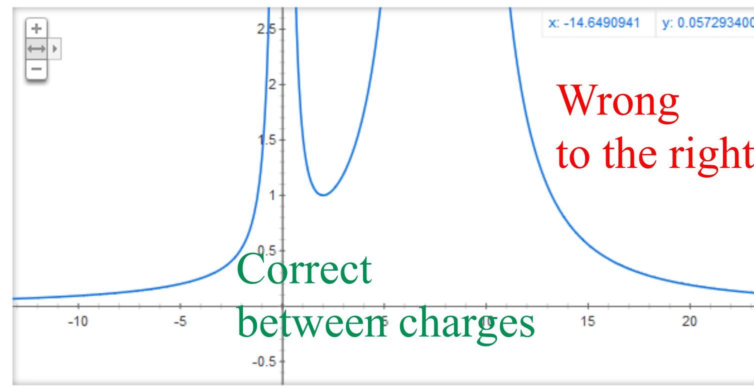
Graph for 1/x^2-27/(8-x)^2





$$F_{net} = k\frac{|q_1\ q_3|}{x^2} + k\frac{|q_2\ q_3|}{(8\text{ cm} - x)^2}$$

Graph for 1/x^2+27/(8-x)^2



$|\mathbf{F}| = k_e\,|q_1q_2|\,/\,r^2$ only tells us about the magnitude,
we do the "direction by hand", we'll see a more useful expression

iClicker question 3-1:

The direction of the force on charge $-q$ is

A. Up.
B. Down.
C. Right.
D. Left.
E. The force on $-q$ is zero.

Review: Another way of charging by induction

Start with neutral conductors

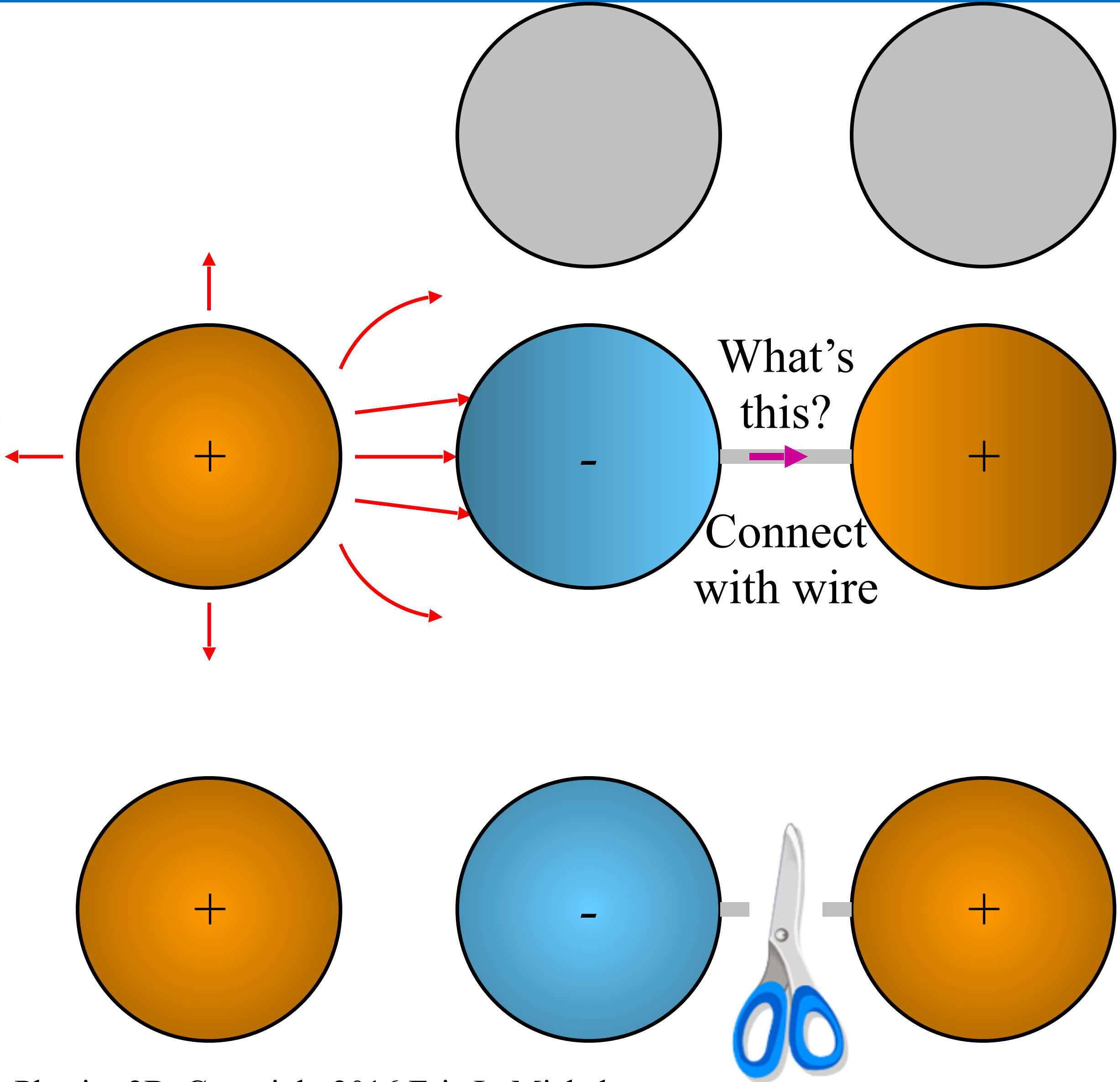
Electrically connect them

Induce

- Connect spheres by wire or touching

Separate

- (E-field not shown)



Gravity vs. Electricity

attractive

$$\mathbf{F}_{10} = -G\frac{m_1 m_0}{r^2}\hat{\mathbf{r}}_{10}$$

$\mathbf{g}(\mathbf{r}) \equiv$ force per unit mass

$$\mathbf{F}(\mathbf{r}) = m_0\mathbf{g}(\mathbf{r})$$

$$\Rightarrow \mathbf{g}(\mathbf{r}) = -G\frac{m_1}{r^2}\hat{\mathbf{r}}_{10}$$

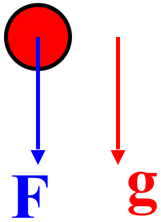
repulsive

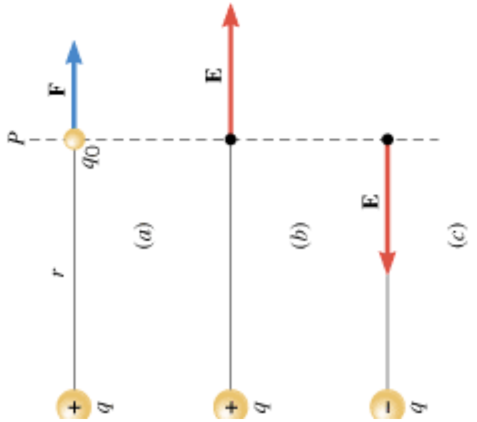
$$\mathbf{F}_{10} = k_e\frac{q_1 q_0}{r^2}\hat{\mathbf{r}}_{10}$$

$\mathbf{E}(\mathbf{r}) \equiv$ force per unit charge

$$\mathbf{F}(\mathbf{r}) = q_0\mathbf{E}(\mathbf{r})$$

$$\Rightarrow \mathbf{E}(\mathbf{r}) = k\frac{q_1}{r^2}\hat{\mathbf{r}}_{10}$$

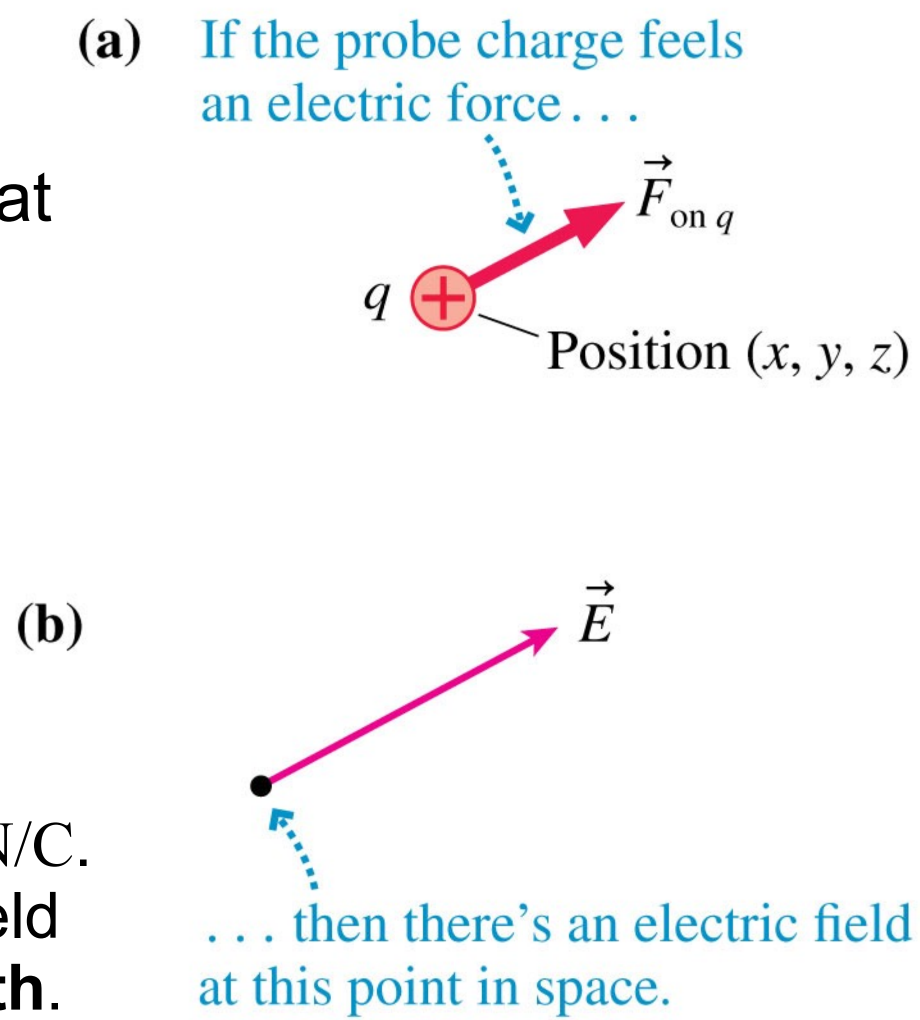




The Electric Field

- If a probe charge q experiences an electric force at a point in space, we say that there is an electric field at that point causing the force.

$$\vec{E}(x, y, z) = \frac{\vec{F}_{\text{on } q} \text{ at } (x, y, z)}{q}$$



The units of the electric field are N/C. The magnitude E of the electric field is called the **electric field strength**.

Field Model

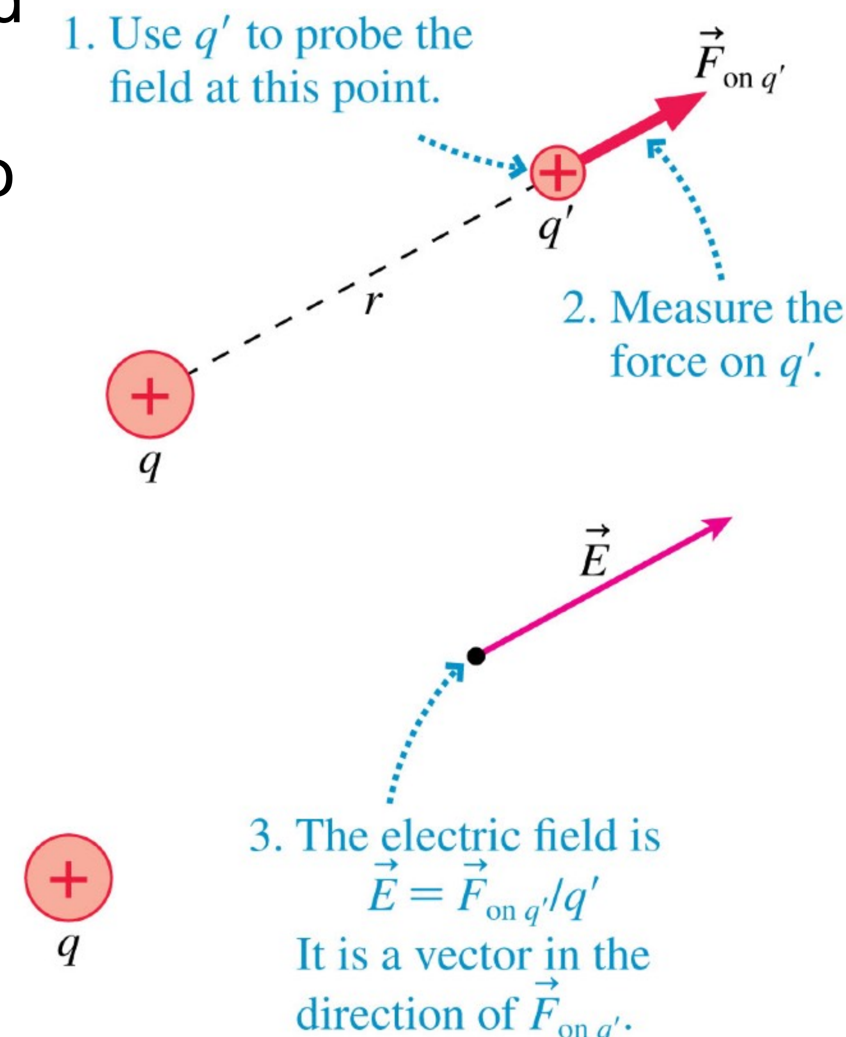
Source q $\boxed{\text{creates}}$ $\vec{E}$ field

$\vec{E}$ field $\boxed{\text{exerts } \vec{F} \text{ on}}$ test $q_\circ$

The electric field of a point charge

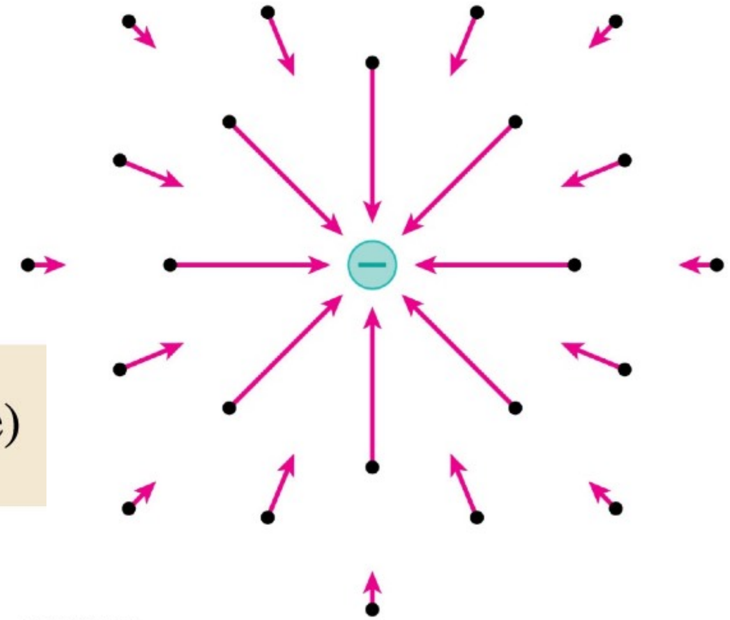
- The figure shows a charge q, and a point in space where we would like to know the electric field.
- We need a second charge, q', to serve as a probe for the electric field.
- The electric field is given by

$$\vec{E} = \frac{\vec{F}_{\text{on } q'}}{q'} = \left(\frac{1}{4\pi\epsilon_0} \frac{q}{r^2}, \text{ away from } q \right)$$



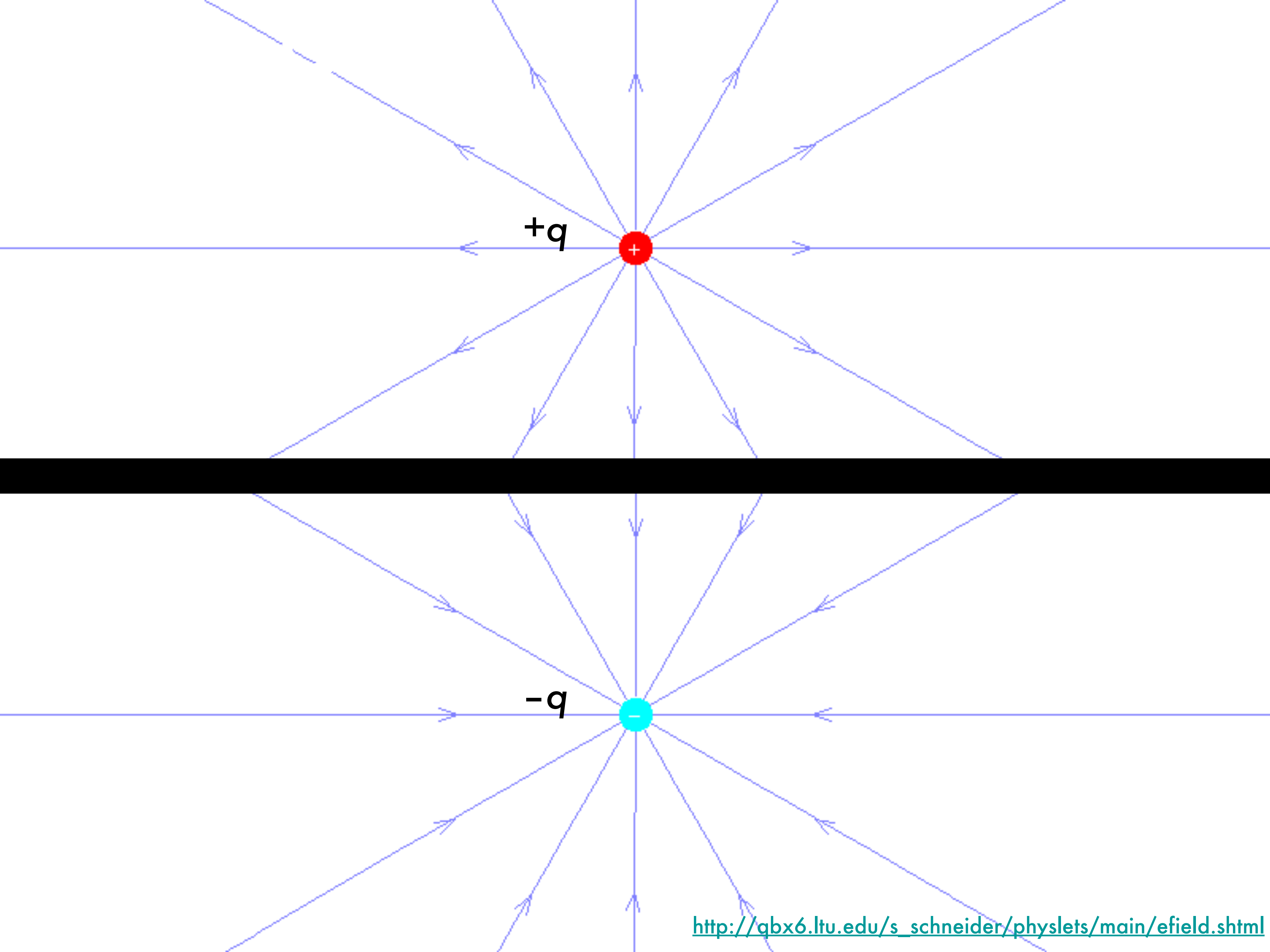
The Electric Field of a Point Charge

- Using unit vector notation, the electric field at a distance r from a point charge q is



$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \qquad \text{(electric field of a point charge)}$$

- A charge will never feel a force from its own field (Newton's 3rd law).
- A negative sign in front of a vector simply reverses its direction.
- The figure shows the electric field of a negative point charge.



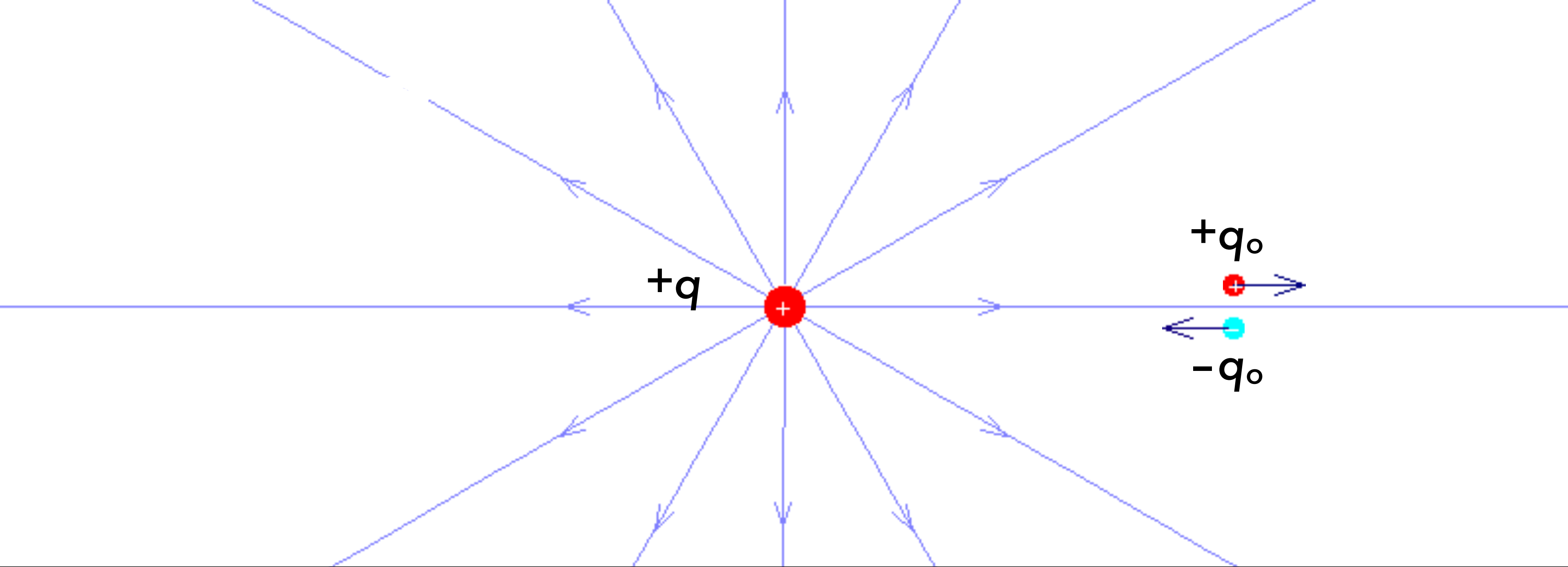
http://qbx6.ltu.edu/s_schneider/physlets/main/efield.shtml

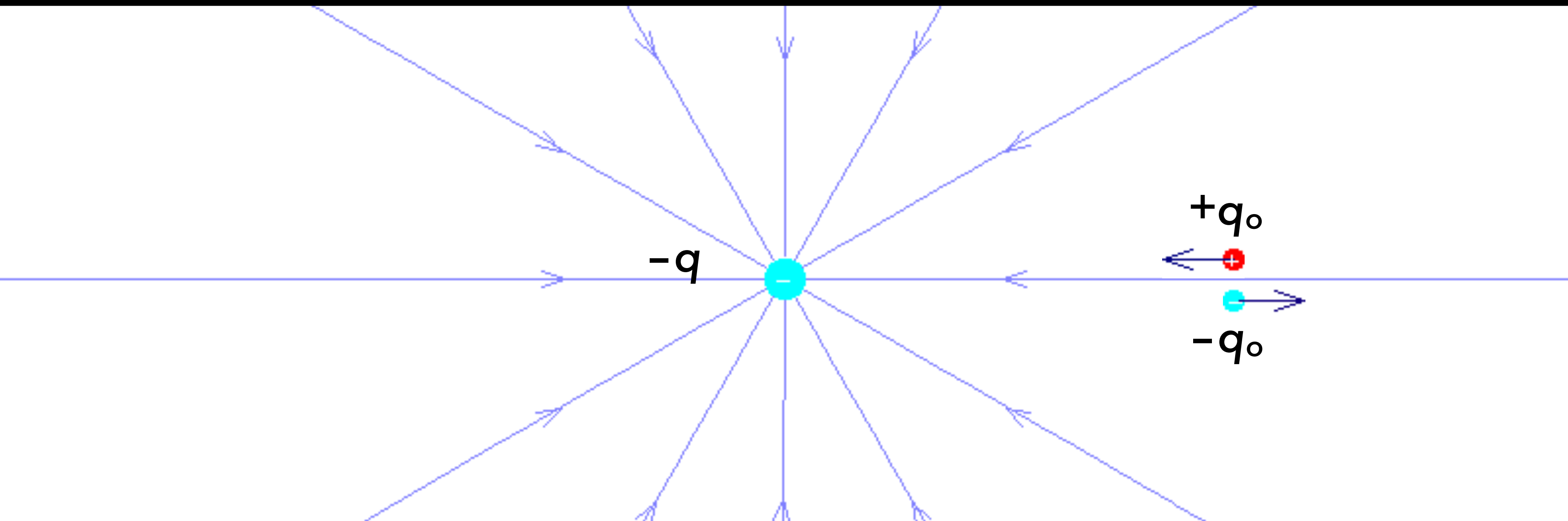
Electric Fields

- We relate the electric force and the electric field by the following equation:

$$\vec{F}_{elec} = q_o\vec{E}$$

- This equation is always valid (not just for point charges).
- Note that the electric field is a vector (it has both magnitude and direction).
- If q_o is positive then the electric force and the electric field point in the same direction.
- If q_o is negative then the electric force and the electric field point in opposite directions.

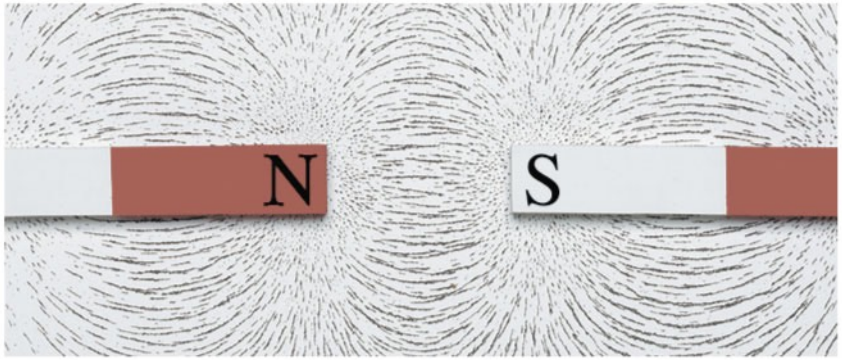




http://qbx6.ltu.edu/s_schneider/physlets/main/efield.shtml

Revisit: The field model

- The photo show the patterns that iron filings make when sprinkled around a magnet.



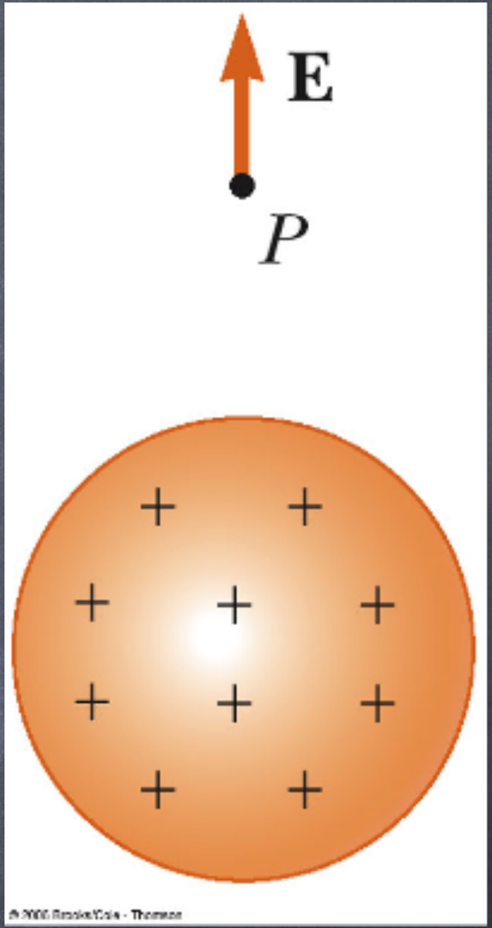
- These patterns suggest that space itself around the magnet is filled with magnetic influence.

- This is called the magnetic field.

- The concept of such a “field” was first introduced by Michael Faraday in 1821.

Electric Fields

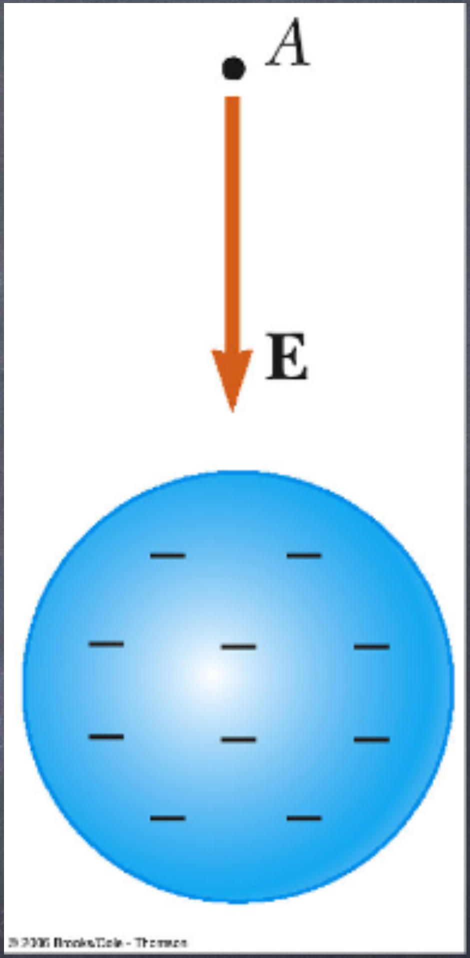
- For a positive point charge we get the following electric field at point P:
  - The electric field produced by a positive charge is directed away from the source charge.
  - A positive test charge would be repelled from the positive source charge.



- The electric field is essentially the direction the electric force would act if a positive **test** charge of magnitude 1 Coulomb were placed there.

Electric Fields

- For a negative point charge we get the following electric field at point A:
  - The electric field produced by a negative charge is directed toward the source charge.
  - A positive test charge would be attracted to the negative source charge.



- Please note that the electric field exists whether or not there is a test charge present.

iClicker question 4-1 (15 seconds)

What are the units of $\hat{\mathbf{r}}_{12} \equiv \dfrac{\mathbf{r}_{12}}{r}$?

A dimensionless

B m

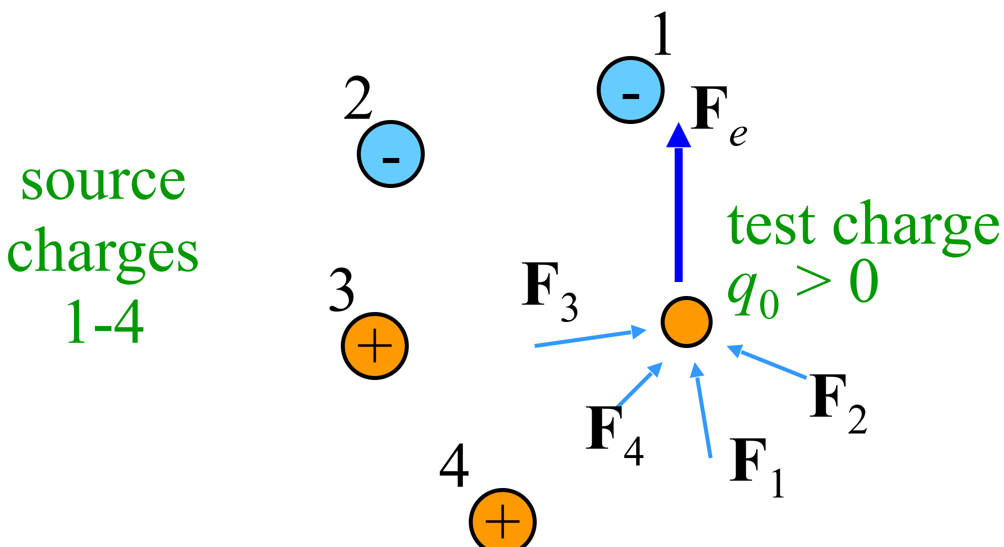
C m^2

D m^{-1}

E m^{-2}

Source charges and test charge

- Source charges usually considered fixed in space
- Test charge moves around
- No physical difference between "source" and "test"
- Coulomb's Law obeys superposition



$$\mathbf{F}_e = \sum_{i=1}^{4} k_e \frac{q_i q_0}{r_i^{\,2}} \hat{\mathbf{r}}_{i0}$$

$$= q_0 k_e \sum_{i=1}^{4} \frac{q_i}{r_i^{\,2}} \hat{\mathbf{r}}_{i0}$$

Electric Fields

There's no physical difference between source and test charges

- Consider a distribution of **source charges**
- Coulomb's law says the force on a **test charge** at any point is proportional to the test charge:

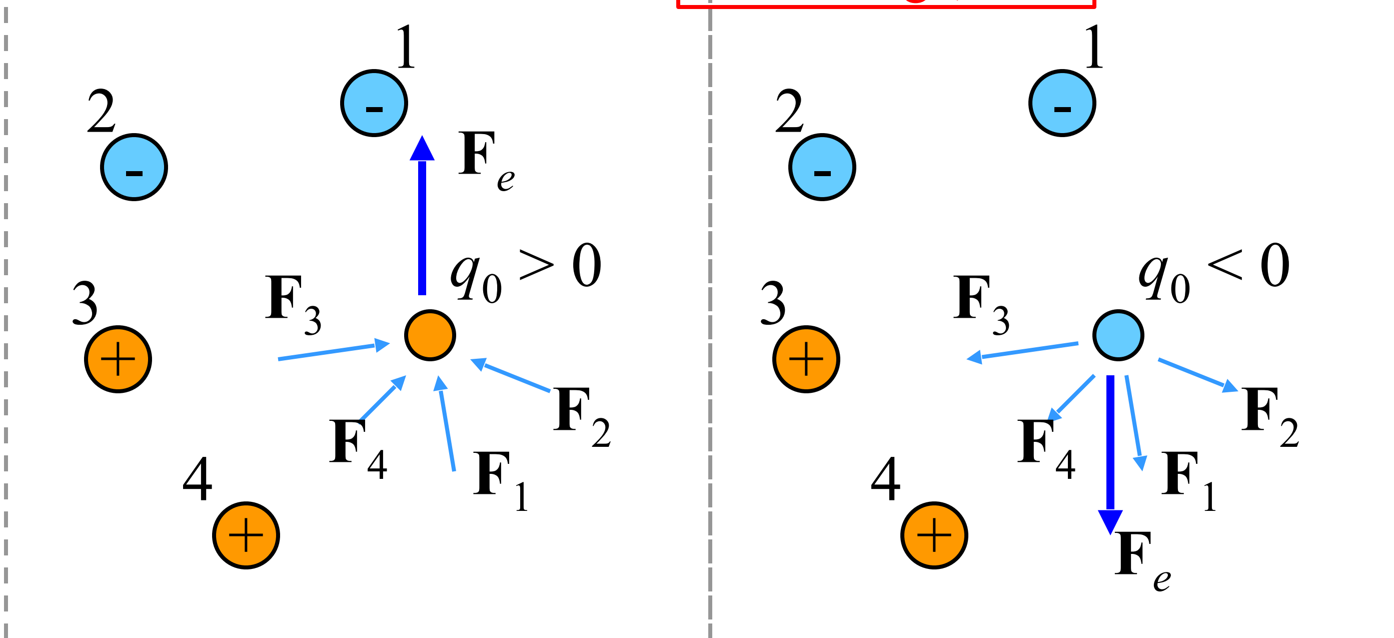
At a point: $\mathbf{F}_e \propto q_0 \qquad \Rightarrow \qquad \mathbf{F}_e = \mathbf{E} q_0 \qquad$ *where* $\quad \mathbf{E} \equiv$ constant vector

Extending to any point in space: $\quad \mathbf{F}_e(\mathbf{r}) = q_0 \mathbf{E}(\mathbf{r})$

$\mathbf{E}(\mathbf{r}) \equiv$ force per unit charge, at $\mathbf{r}$

Units of **E**?

Direction of **E**?

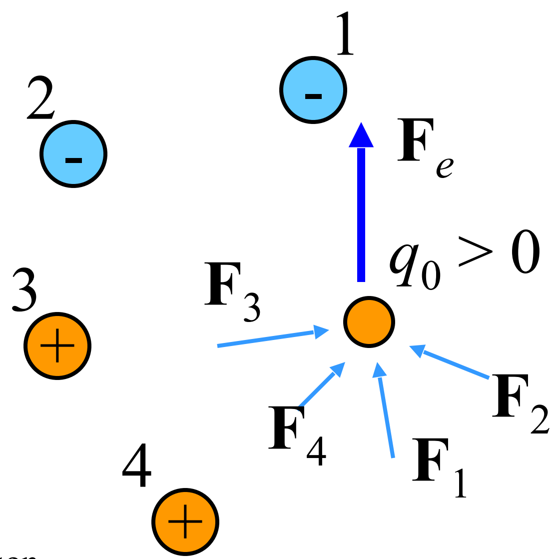


Electric field from charges

- Charges create an E-field
- When considering a single (**test**) charge, only the field from *other* charges acts on it
  - The book makes it sound like some charges create E-fields, and others don't

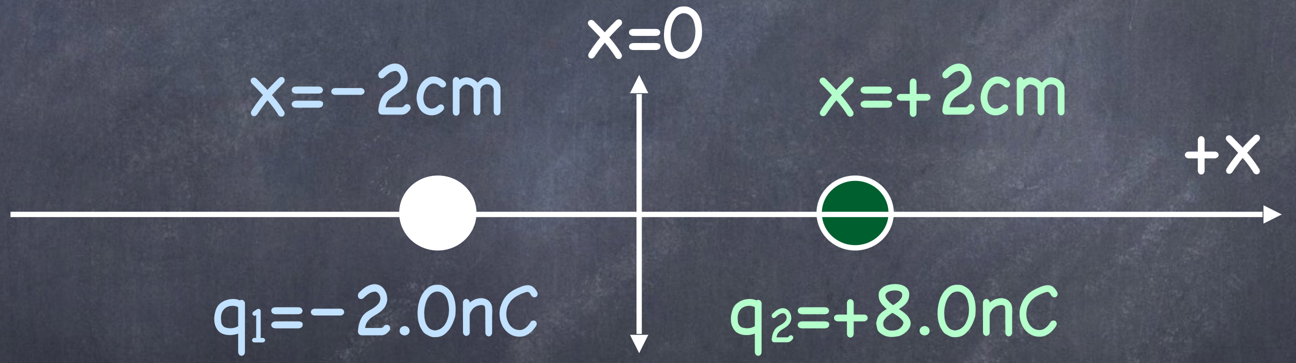
Recall: $\mathbf{F}_e = q_0 k_e \sum_{i=1}^{4} \frac{q_i}{r_i^2} \hat{\mathbf{r}}_{i0}, \quad \textit{and} \quad \mathbf{F}_e = q_0 \mathbf{E}$

$$\Rightarrow \qquad \mathbf{E}(\mathbf{r}) = k_e \sum_{i=1}^{4} \frac{q_i}{r_i^2} \hat{\mathbf{r}}_{i0}$$



Electric Fields

- Example
- Charged particles of −2.0nC and +8.0nC are located at x = −2.0cm and x = +2.0cm, respectively. At what point on the x-axis is the electric field equal to zero?

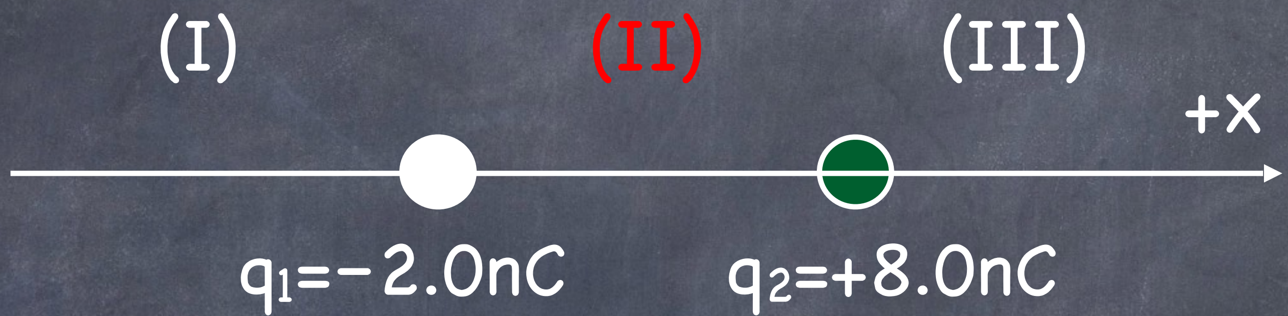


- Answer
- The coordinate system is already defined for you.

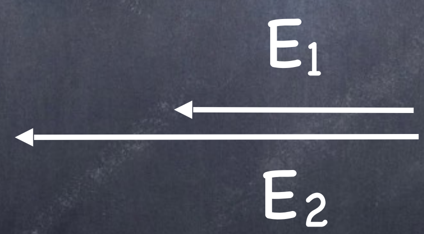
Electric Fields

- Answer

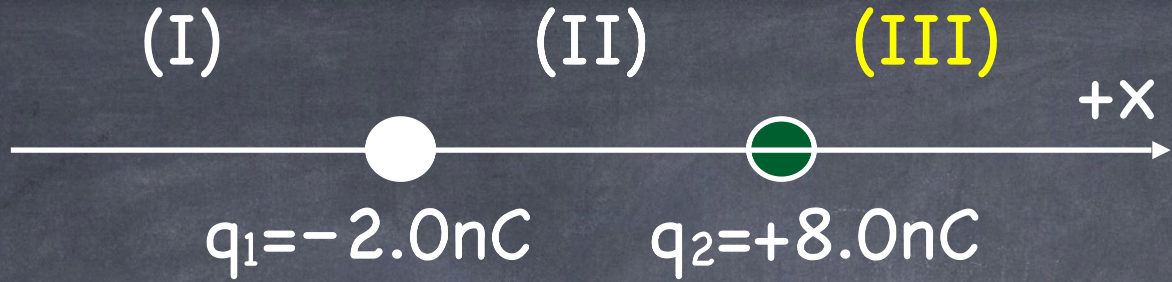
Let's divide the diagram into three distinct regions.



- In region (II), what direction will E_1 and E_2 be?



- Can E_1 and E_2 ever add to zero in region (II)?
- No, so the point cannot be located there.

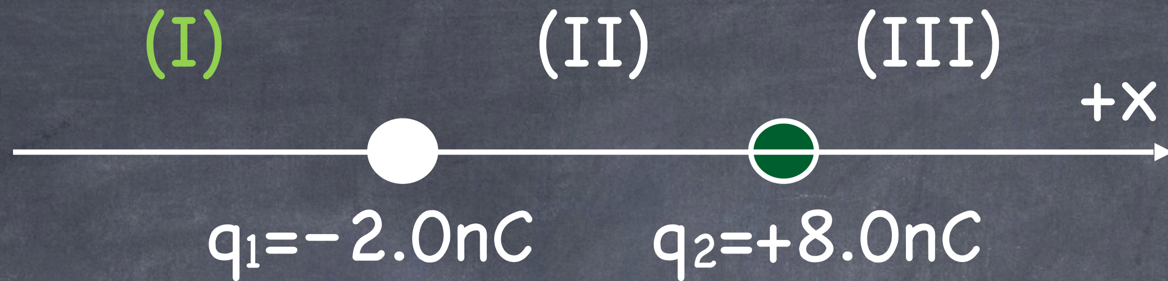


- In region (III), what direction will E_1 and E_2 be?

- In region (III), can E_1 and E_2 ever be equal in magnitude?

$$E_{\text{point charge}} = k_e \frac{q}{r^2}$$

  - E_2 will always be greater in magnitude since it has a larger charge ($q_2 > q_1$) and will always have a smaller separation distance ($r_2 < r_1$).
  - Can E_1 and E_2 ever add to zero in region (III)?
  - No, so the point cannot be located there.



- In region (I), what direction will E_1 and E_2 be?

- In region (I), can E_1 and E_2 ever be equal in magnitude?

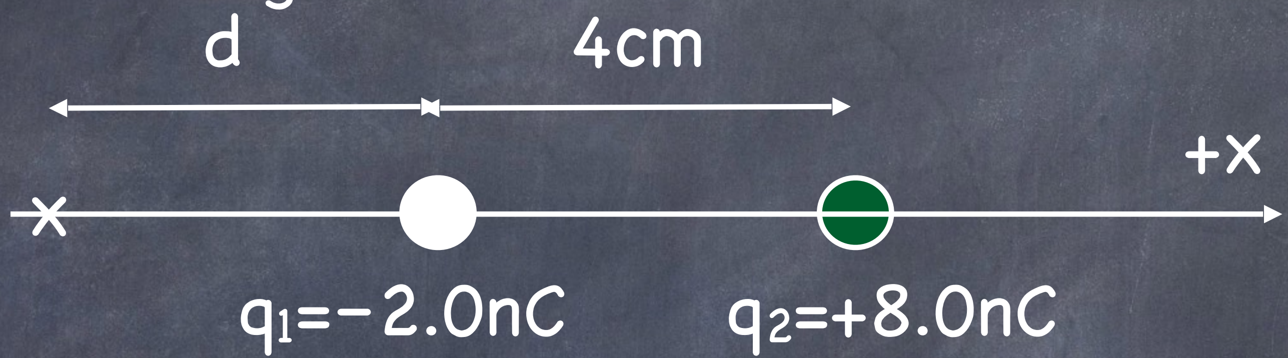
$$E_{\text{point charge}} = k_e \frac{q}{r^2}$$

- E_2 will have a larger charge ($q_2 > q_1$) but E_1 will have smaller separation distance ($r_2 > r_1$).
- Can E_1 and E_2 ever add to zero in region (I)?
- Yes, so the point can be located there.

Electric Fields

- Answer

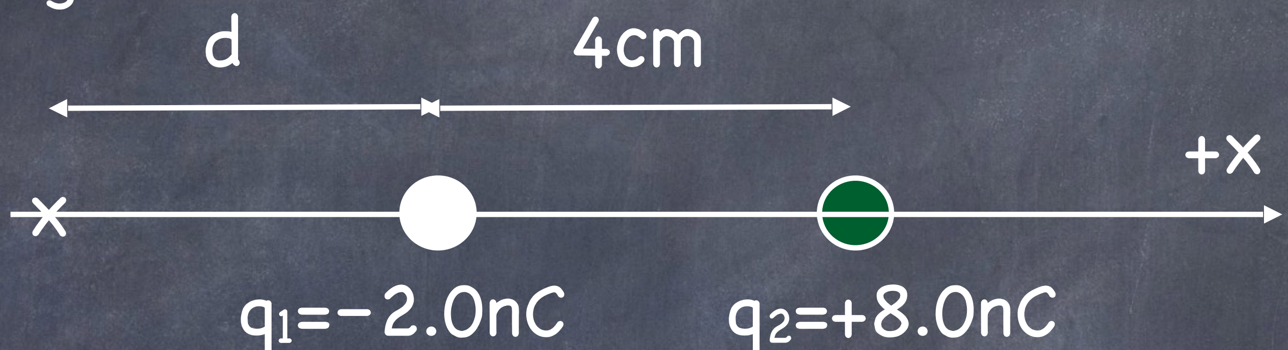
So, if the point exists where $E_{tot} = 0$, it must be to the left of the charges.



Electric Fields

- Answer

The point exists where $E_{tot} = 0$, it must be to the left of the charges.



Next, let's list the quantities that we know:

$q_1 = -2.0\text{x}10^{-9}$Coul

$q_2 = +8.0\text{x}10^{-9}$Coul

$r_1 = d$

$r_2 = d + 0.04\text{m}$

Note: we could not have completed this step without knowing what region we were in.

Electric Fields

- Answer

At this point the electric field vectors must have equal magnitude.

$$E_1 = E_2$$

$$k_e \frac{|q_1|}{r_1^2} = k_e \frac{|q_2|}{r_2^2}$$

$$\frac{|q_1|}{r_1^2} = \frac{|q_2|}{r_2^2}$$

$$\frac{|-2.0 \times 10^{-9}\mathrm{C}|}{d^2} = \frac{|+8.0 \times 10^{-9}\mathrm{C}|}{(d + 0.04\mathrm{m})^2}$$

$$(d + 0.04\mathrm{m})^2 = 4d^2$$

Electric Fields

- Answer

$$(d + 0.04\text{m}) = 2d$$

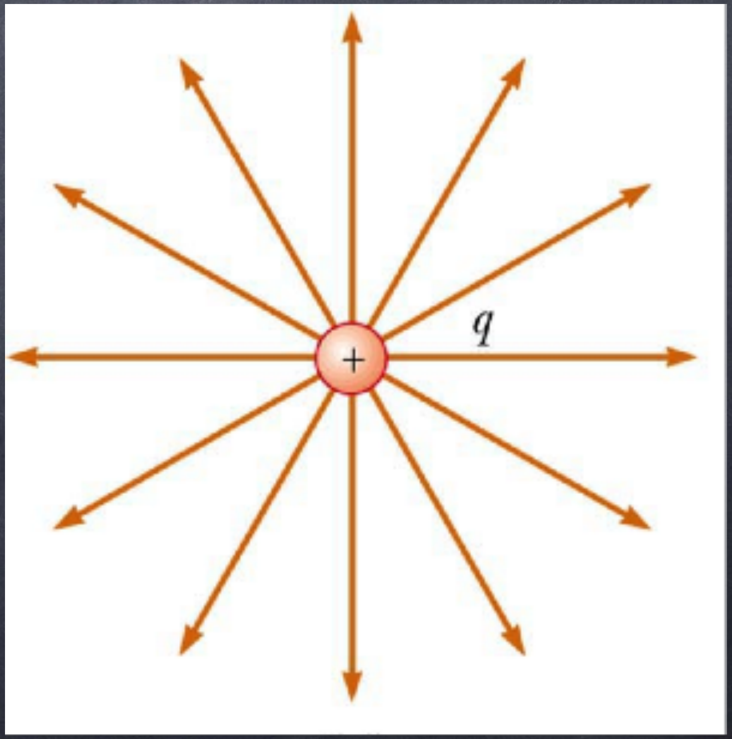
$$d = 0.04\text{m}$$

Since the −2.0nC charge is at x = −2.0cm, this means that the distance d is taken from that location.

The position on the x-axis will correspond to:
x = -0.060m or -6.0cm

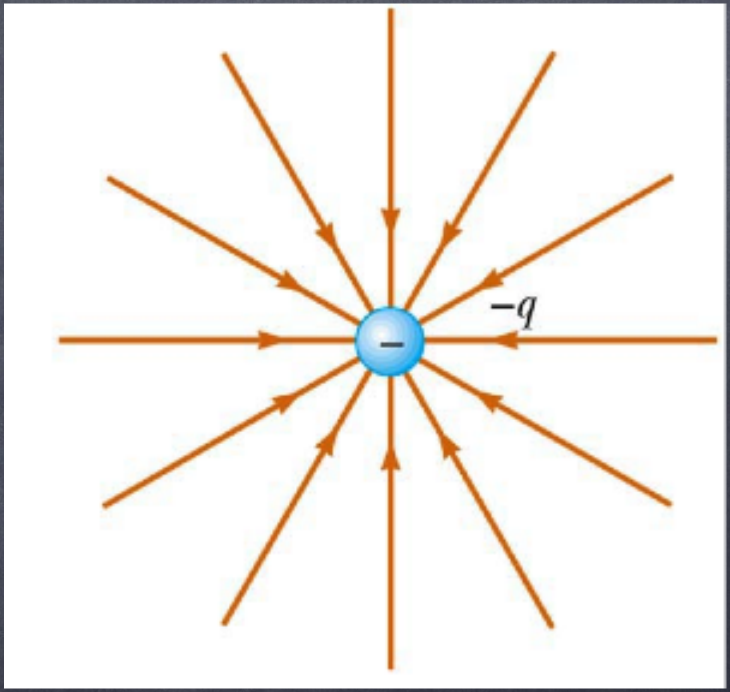
Electric Field Lines

- A convenient aid for visualizing electric field patterns is to draw lines pointing in the direction of the field vector at any point.
- These lines are called electric field lines.
- For a lone positive charge the field lines would radiate outward symmetrically.
- At any point, the electric field vector will point in the direction of the electric field lines.



Electric Field Lines

- For a lone negative charge the field lines would radiate inward symmetrically.
- The number of lines per unit area is proportional to the strength of the electric field in the given region.



- Know the electric field lines well for the lone positive and lone negative charges.
- All other charge configurations will be combinations of lone positive and lone negative charges.

Electric Field Lines

- There are some rules for electric field lines:
- 1) Electric field lines can **only begin and terminate on charges**.
- 2) No two field lines can cross.
- 3) The **number of field lines** drawn from a positive charge or to a negative charge is **proportional** to the **magnitude** of the charge.

Birth, life, and death of field lines

No matter how close we get to a positive charge, field lines point away from it

- Field lines *originate* on positive charges

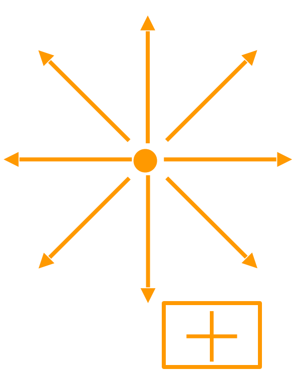
No matter how close we get to a negative charge, field lines point toward it

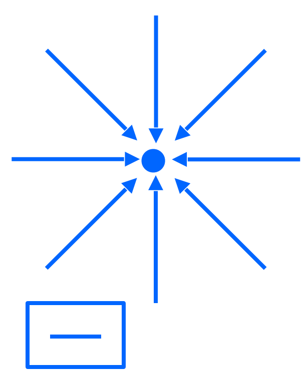
- Field lines *terminate* on negative charges

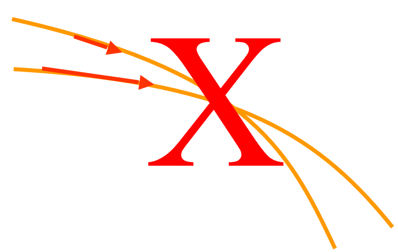
Field lines can't cross or split

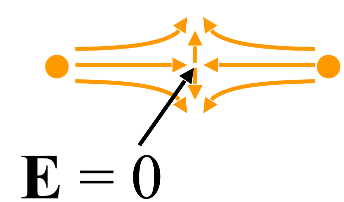
- ... because they follow the E-field, which can only point one way from every point

But they *can* turn sharply at a point of zero field









iClicker question 3-3 (1 minute)

At P, which way does the E-field point?

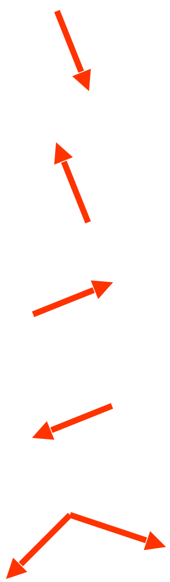
A

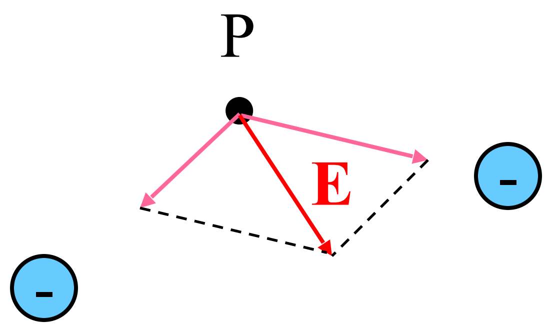
B

C

D

E





Summary of chapter 22, part I

Coulomb's Law

The forces between two charged particles q_1 and q_2 separated by distance r are

$$F_{1 \text{ on } 2} = F_{2 \text{ on } 1} = \frac{K|q_1||q_2|}{r^2}$$

The forces are repulsive for two like charges, attractive for two opposite charges.

To solve electrostatic force problems:

MODEL Model objects as point charges.

VISUALIZE Draw a picture showing charges and force vectors.

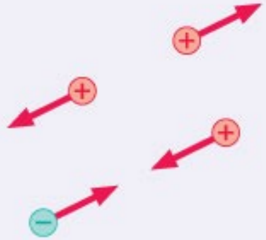
The Charge Model

There are two kinds of charge, positive and negative.

- Fundamental charges are protons and electrons, with charge $\pm e$ where $e = 1.60 \times 10^{-19}$ C.
- Objects are charged by adding or removing electrons.
- The amount of charge is $q = (N_p - N_e)e$.
- An object with an equal number of protons and electrons is **neutral**, meaning no *net* charge.

Charged objects exert electric forces on each other.

- Like charges repel, opposite charges attract.
- The force increases as the charge increases.
- The force decreases as the distance increases.



Summary Ch. 22, part II, questions?

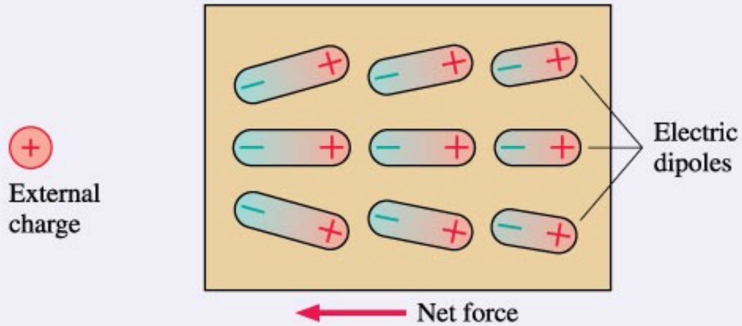
The Charge Model

There are two types of material, insulators and conductors.

- Charge remains fixed in or on an insulator.
- Charge moves easily through or along conductors.
- Charge is transferred by contact between objects.

Charged objects attract neutral objects.

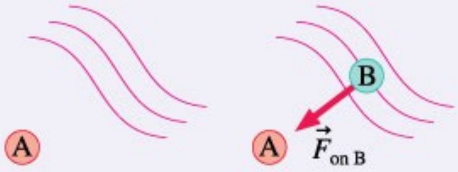
- Charge polarizes metal by shifting the electron sea.
- Charge polarizes atoms, creating electric dipoles.
- The **polarization** force is always an attractive force.



The Field Model

Charges interact with each other via the **electric field** $\vec{E}$.

- Charge A alters the space around it by creating an electric field.



- The field is the agent that exerts a force. The force on charge q_B is $\vec{F}_{\text{on B}} = q_B\vec{E}$.

An electric field is identified and measured in terms of the force on a **probe charge** q:

$$\vec{E} = \vec{F}_{\text{on } q}/q$$

- The electric field exists at all points in space.
- An electric field vector shows the field only at one point, the point at the tail of the vector.

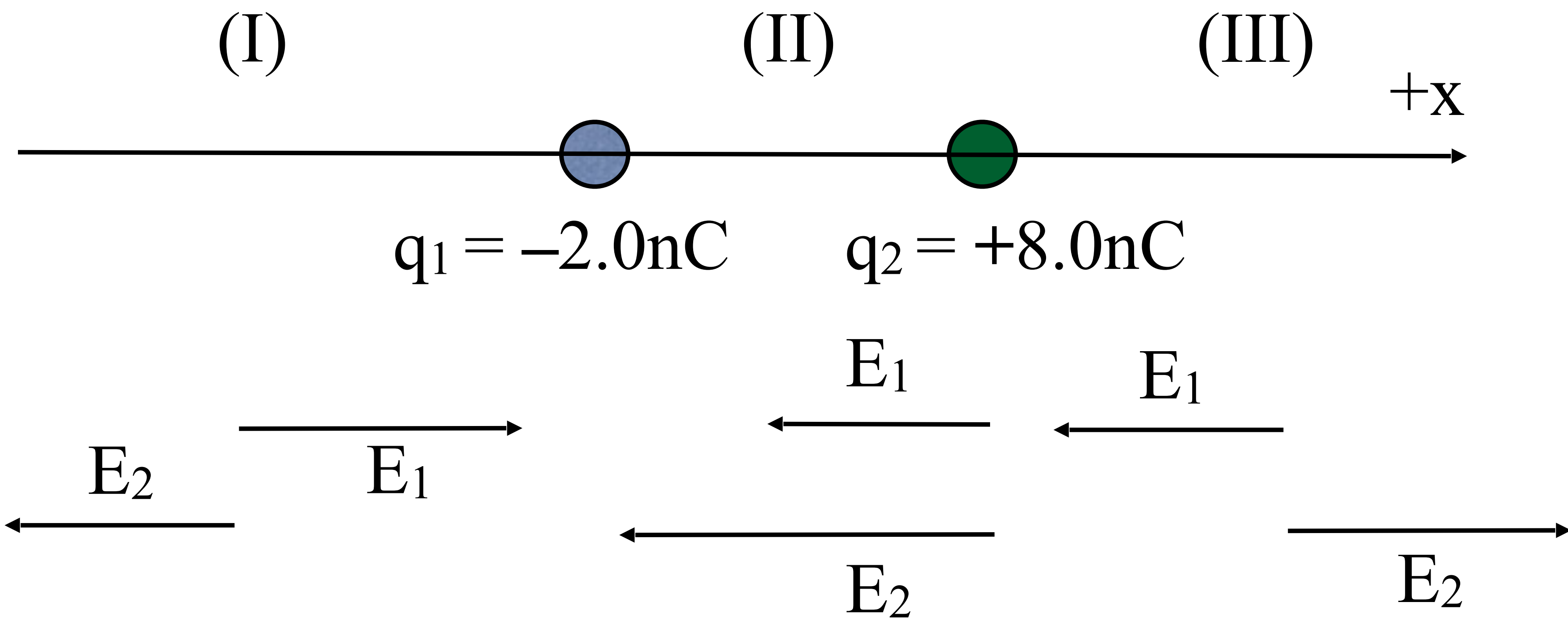
The electric field of a **point charge** is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

Unit vector $\hat{r}$ indicates "away from q."

Electric field is the sum of all contributions

- Example we saw before, three distinct regions, 1D:



Electric Field from multiple charges: superposition!

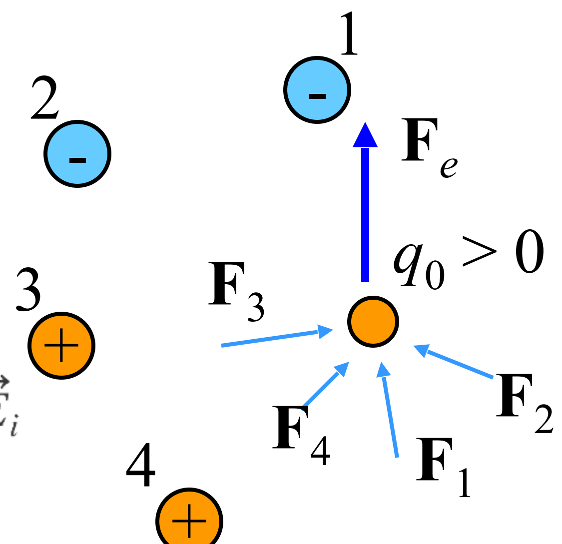
- Suppose the source of an electric field is a group of point charges q_1, q_2, …
- The net electric field $\vec{E}_{\text{net}}$ is the vector sum of the electric fields due to each charge.
- In other words, electric fields obey the *principle of superposition*.

$$\text{Recall:}\quad \mathbf{F}_e = q_0 k_e \sum_{i=1}^{4} \frac{q_i}{r_i^2}\hat{\mathbf{r}}_{i0}, \quad \textit{and} \quad \mathbf{F}_e = q_0\mathbf{E}$$

$$\Rightarrow \qquad \mathbf{E}(\mathbf{r}) = k_e \sum_{i=1}^{4} \frac{q_i}{r_i^2}\hat{\mathbf{r}}_{i0}$$

$q = q_0$

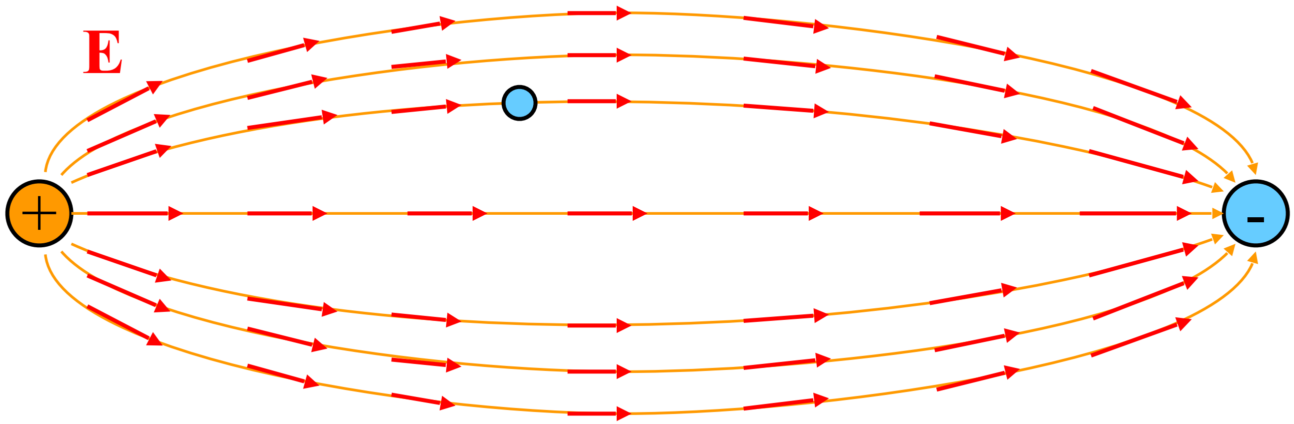
$$\vec{E}_{\text{net}} = \frac{\vec{F}_{\text{on } q}}{q} = \frac{\vec{F}_{1 \text{ on } q}}{q} + \frac{\vec{F}_{2 \text{ on } q}}{q} + \cdots = \vec{E}_1 + \vec{E}_2 + \cdots = \sum_i \vec{E}_i$$



Electric field lines

~~Lines of force~~

- Field of vectors in space: the **E**-field
- **Electric field lines** follow the arrows
  - E-field vectors at each point are tangent to the field line



iClicker question 3-4 (1 minute)

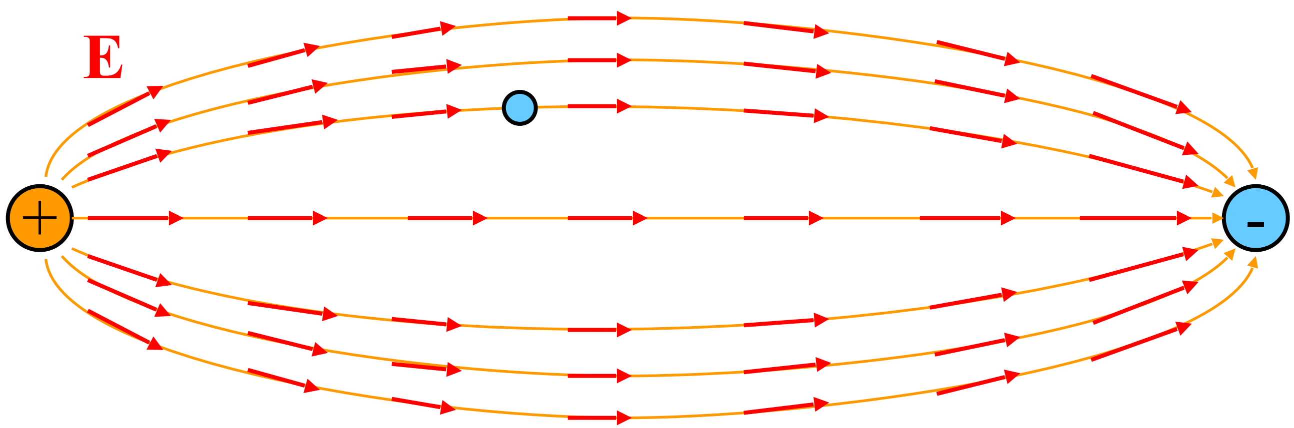
How does field-line density relate to E-field strength?

A There is no relationship

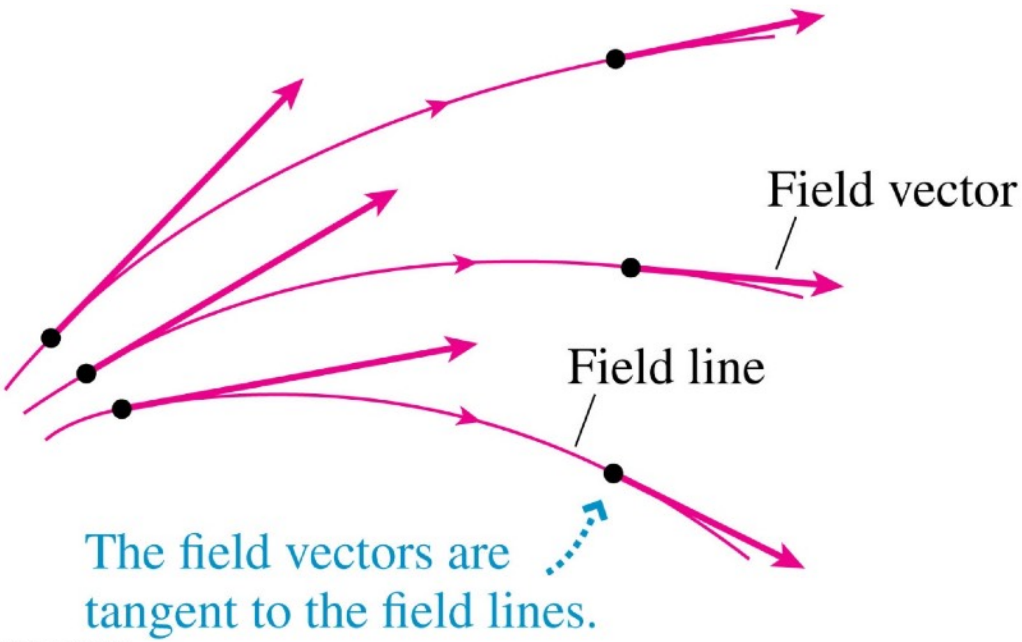
B Stronger fields have higher density

C Stronger fields have lower density

D Stronger fields have darker-colored lines



Electric Field Lines

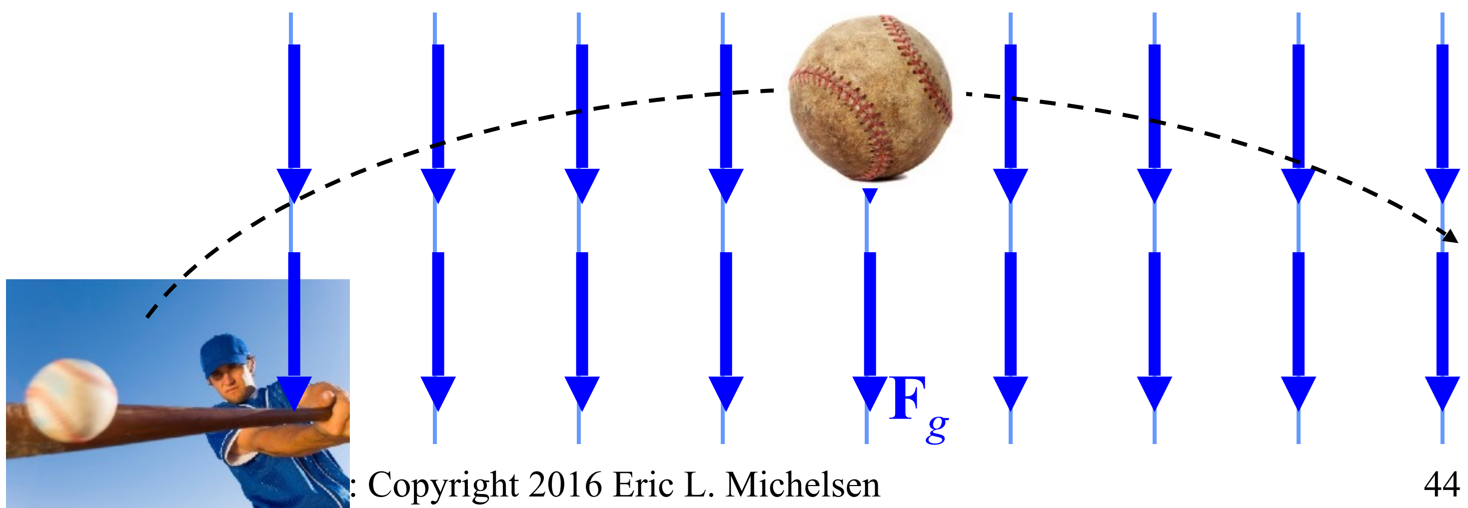


- Electric field lines are *continuous* curves tangent to the electric field vectors.
- Closely spaced field lines indicate a greater field strength.
- Electric field lines start on positive charges and end on negative charges.
- Electric field lines never cross.

Do objects always move in the direction of the force applied to them?

- How do bodies move in a gravitational field (g-field)?
- No. Objects bend toward the direction of applied force, but they generally don't move parallel to it

- Electric field lines are lines of *acceleration*
  - *Not* trajectories of motion
  - (Magnetic field lines are *not* lines of acceleration)



Gravity vs. Electricity

attractive

$$\mathbf{F}_{10} = -G\frac{m_1 m_0}{r^2}\hat{\mathbf{r}}_{10}$$

$\mathbf{g}(\mathbf{r}) \equiv$ force per unit mass

$$\mathbf{F}(\mathbf{r}) = m_0\mathbf{g}(\mathbf{r})$$

$$\Rightarrow \mathbf{g}(\mathbf{r}) = -G\frac{m_1}{r^2}\hat{\mathbf{r}}_{10}$$

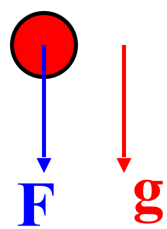
repulsive

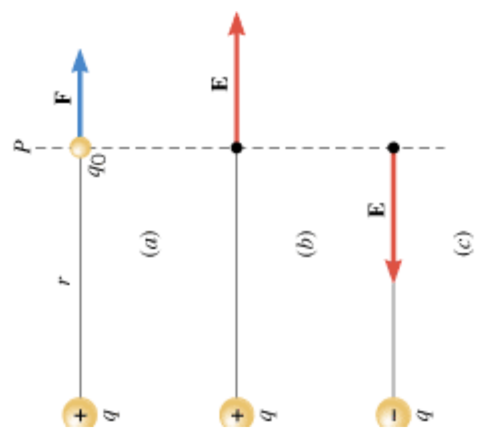
$$\mathbf{F}_{10} = k_e\frac{q_1 q_0}{r^2}\hat{\mathbf{r}}_{10}$$

$\mathbf{E}(\mathbf{r}) \equiv$ force per unit charge

$$\mathbf{F}(\mathbf{r}) = q_0\mathbf{E}(\mathbf{r})$$

$$\Rightarrow \mathbf{E}(\mathbf{r}) = k\frac{q_1}{r^2}\hat{\mathbf{r}}_{10}$$





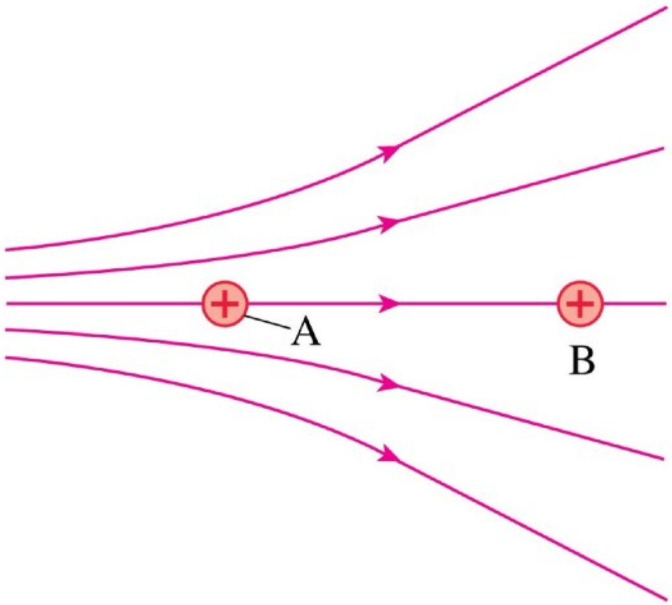
Take home iclicker question ;)

Two protons, A and B, are in an electric field. Which proton has the larger acceleration?

A. Proton A

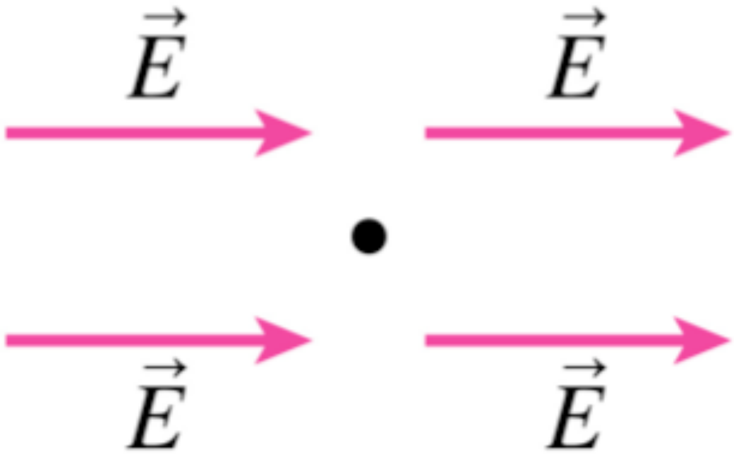
B. Proton B

C. Both have the same acceleration.



iClicker question 3-5 (2 minutes)

An electron is placed in a uniform electric field at the position marked by the dot. The force on the *electron* is:



A) To the right.

B) To the left.

C) Up.

D) Down.

E) Zero.

Take home iclicker question ;)

What are the directions of electric field $\boldsymbol{E}$ and force $\boldsymbol{F}$ on charge q_2 due to q_1?

$E, F?$

A) $\boldsymbol{E}$ $\longrightarrow$ $\boldsymbol{F}$ $\longrightarrow$

B) $\boldsymbol{E}$ $\longrightarrow$ $\boldsymbol{F}$ $\longleftarrow$

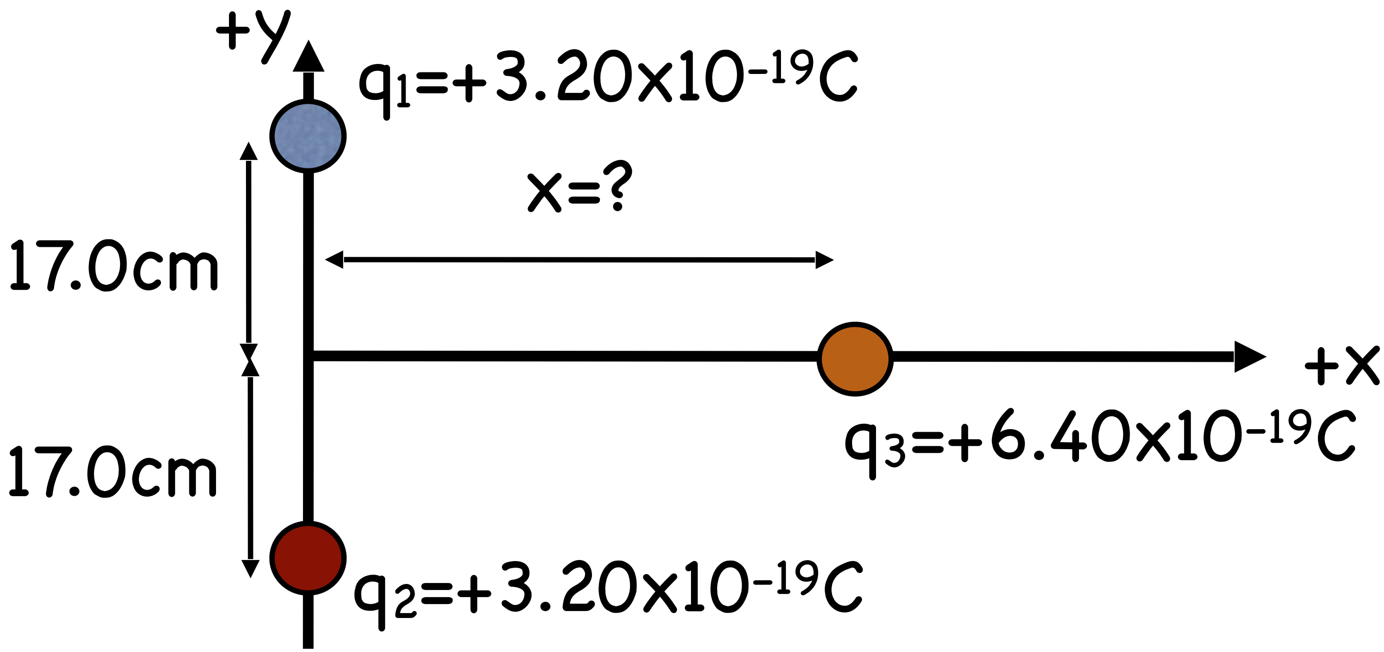
C) $\boldsymbol{E}$ $\longleftarrow$ $\boldsymbol{F}$ $\longrightarrow$

D) $\boldsymbol{E}$ $\longleftarrow$ $\boldsymbol{F}$ $\longleftarrow$

2-D Superposition

Example

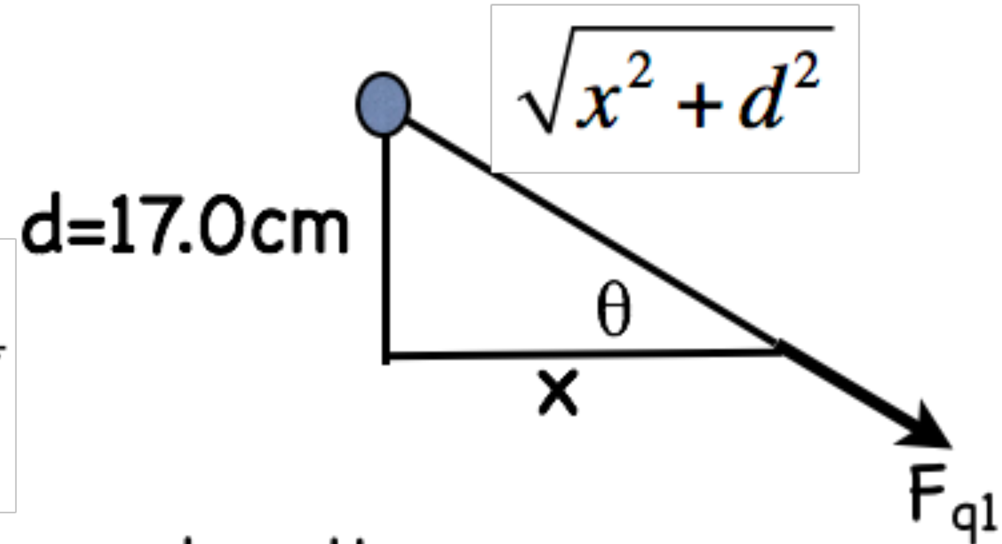
In the figure below, q_1 and q_2 are on a y-axis at a distance of 17.0cm from the origin. q_3 is moved gradually along the x-axis from x = 0m to x = +5.0m. At what values of x will the magnitude of the electrostatic force on q_3 [due to both q_1 and q_2] be (a) a minimum and (b) a maximum.



2-D Superposition

$$\cos\theta = \frac{x}{\sqrt{x^2 + d^2}}$$

Using Coulomb's Law, we find that:



$$F_{q1} = k_e \frac{q_1 q_3}{r^2} \cos\theta$$

$$F_{q1} = k_e \frac{(2e)(4e)}{\left(\sqrt{x^2 + d^2}\right)^2} \frac{x}{\sqrt{x^2 + d^2}}$$

$$F_{q1} = \frac{8 k_e e^2 x}{\left(x^2 + d^2\right)^{3/2}}$$

Since q_2 makes the same contribution, we can say that:

$$F_{net} = 2 F_{q1}$$

$$F_{net} = \frac{16 k_e e^2 x}{\left(x^2 + d^2\right)^{3/2}}$$

Answer

In order to minimize the force, you need to take the derivative (wrt to x)

Using the quotient rule, you get:

$$\frac{df}{dx}=\frac{d}{dx}\left(\frac{16k_e e^2 x}{\left(x^2+d^2\right)^{3/2}}\right)$$

$$\frac{df}{dx}=16k_e e^2\left(\frac{\sqrt{x^2+d^2}\left(2x^2-d^2\right)}{x^6+3d^2x^4+3d^4x^2+d^6}\right)$$

Setting this equal to zero, to find the extremum values we get:

$$\frac{df}{dx}=0$$

$$\sqrt{x^2+d^2}\left(2x^2-d^2\right)=0$$

$$\left(2x^2-d^2\right)=0$$

$$x=\frac{d}{\sqrt{2}}=\frac{17.0\text{cm}}{\sqrt{2}}=12.0\text{cm}$$

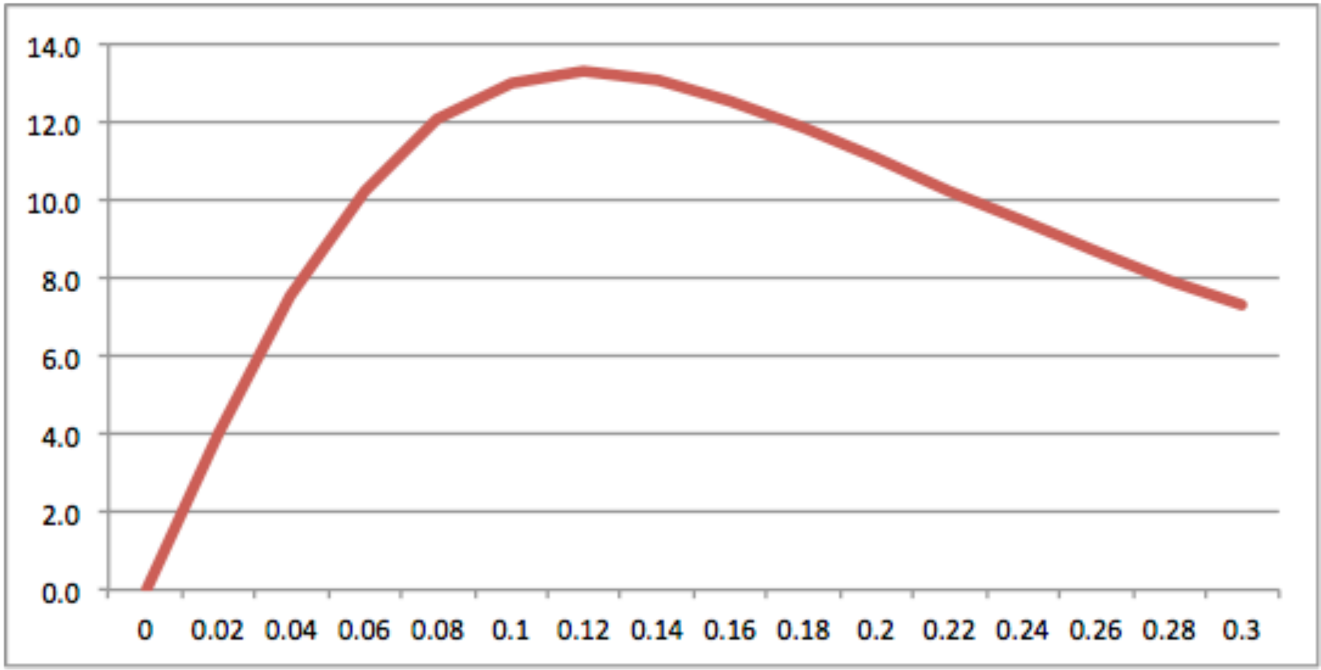
We find only one extremum. Is it a maximum or a minimum?

Answer

Looking back at the original equation, we find that putting in x=0 yields a net force of zero (this is the lowest value we can get for magnitude of the force).

$$F_{net} = \frac{16k_e e^2 x}{\left(x^2 + d^2\right)^{3/2}}$$

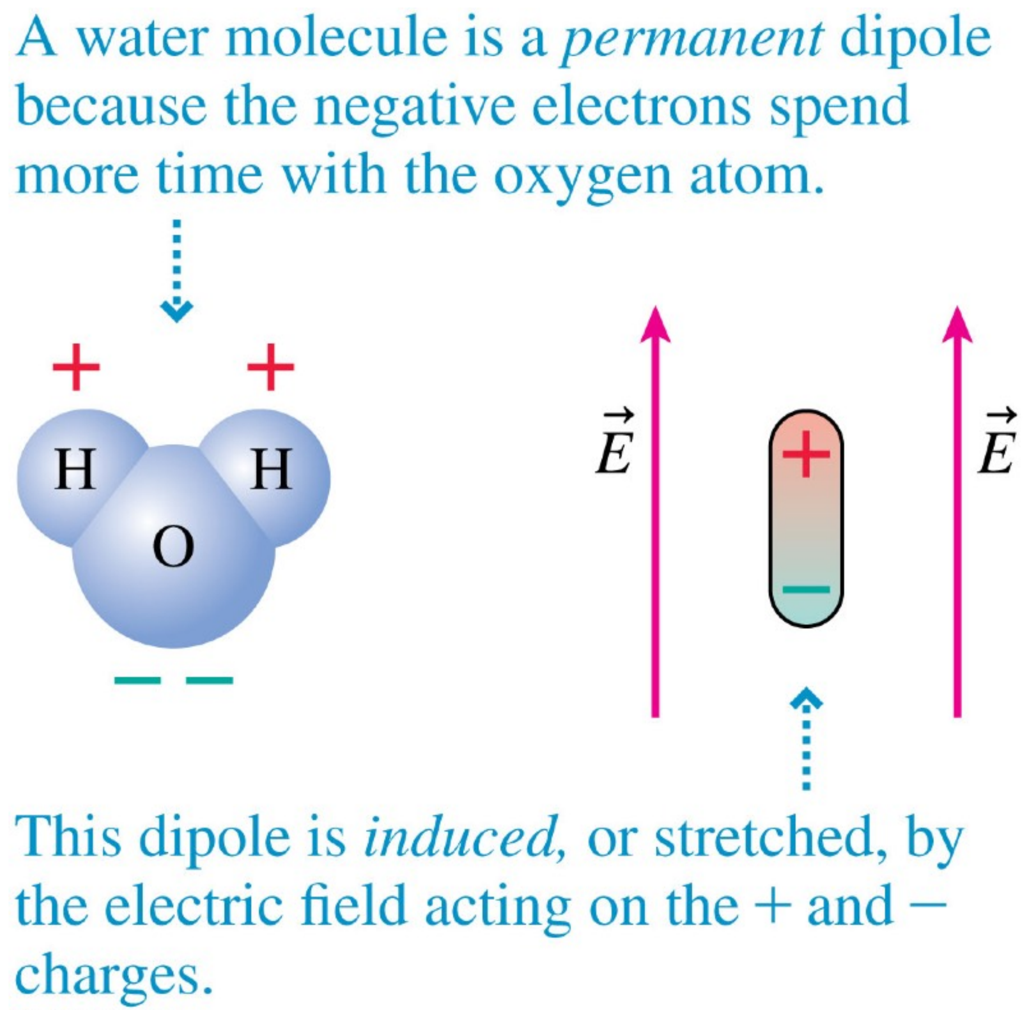
A quick graph confirms this result.



This means that the original value (x = 12.0cm) we calculated was the maximum and that x = 0 is the minimum.

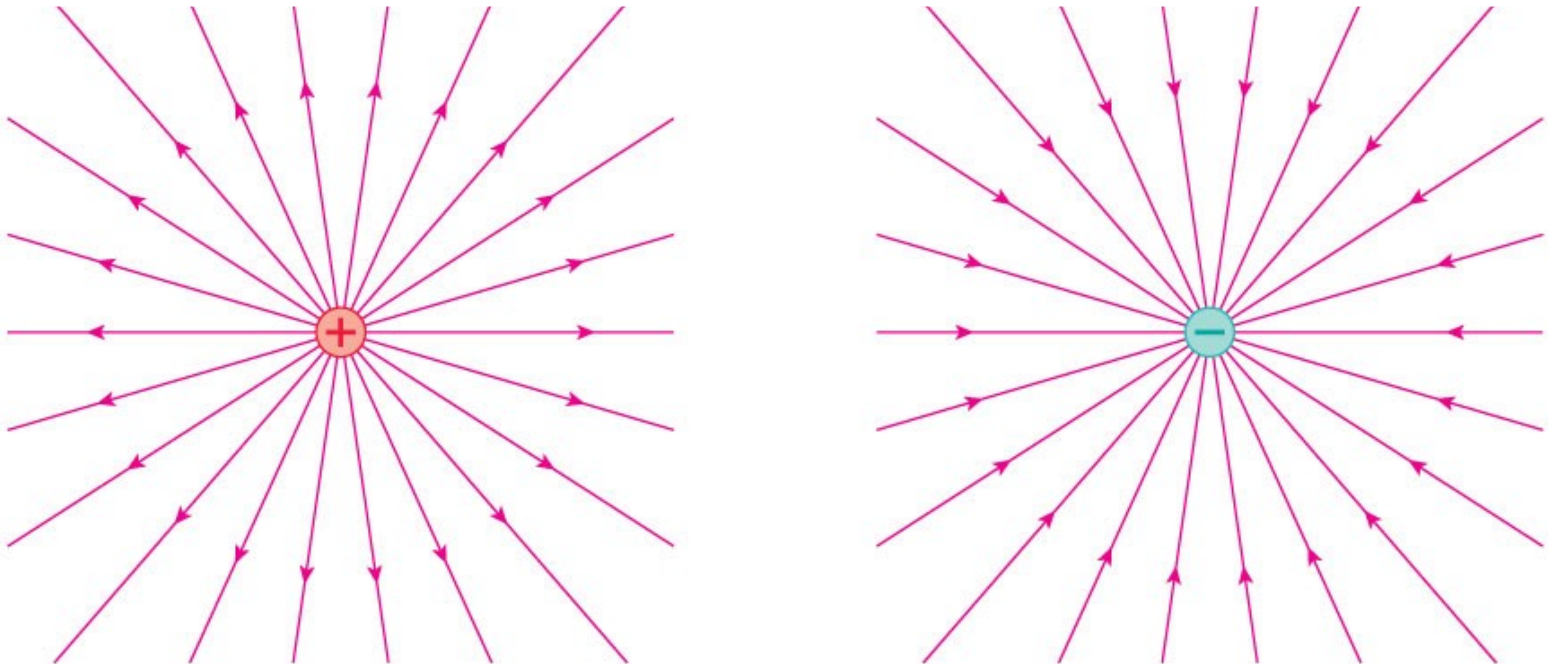
Electric Dipoles

- After describing lone charges, the next step is:
- Two equal but opposite charges separated by a small **fixed** distance form an *electric dipole*.
- The figure shows two examples.

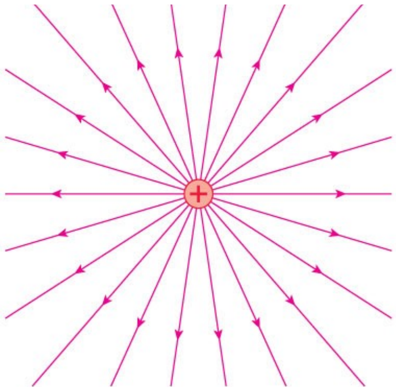


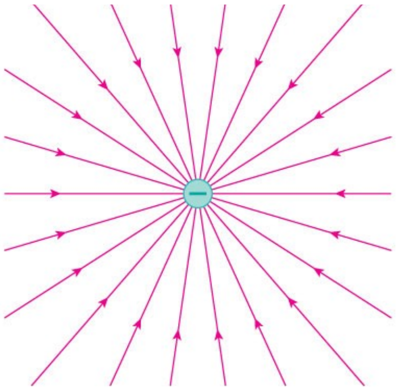
Remember: Electric Field Lines of a Point Charge

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \qquad \text{(electric field of a point charge)}$$

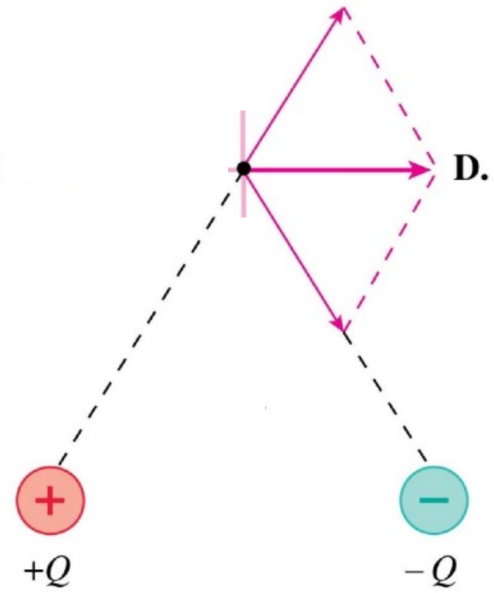


Electric Field Lines of two point charges

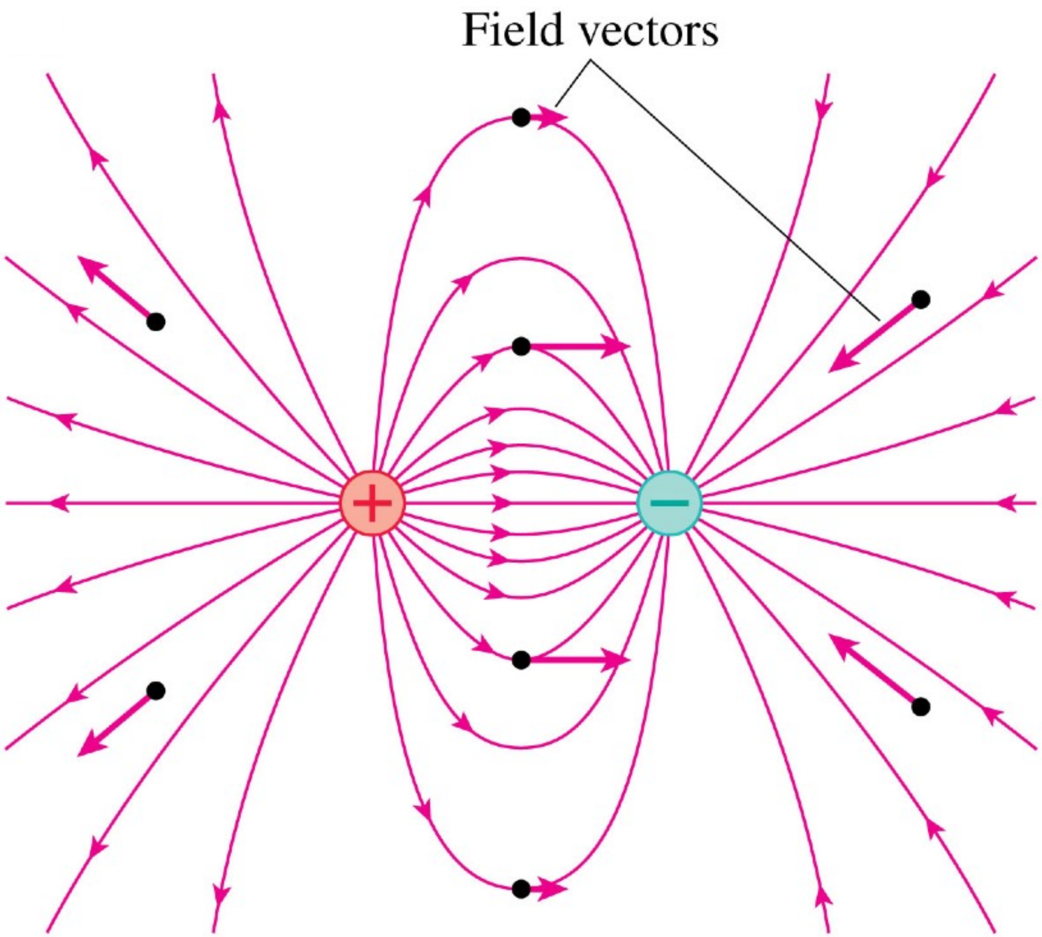


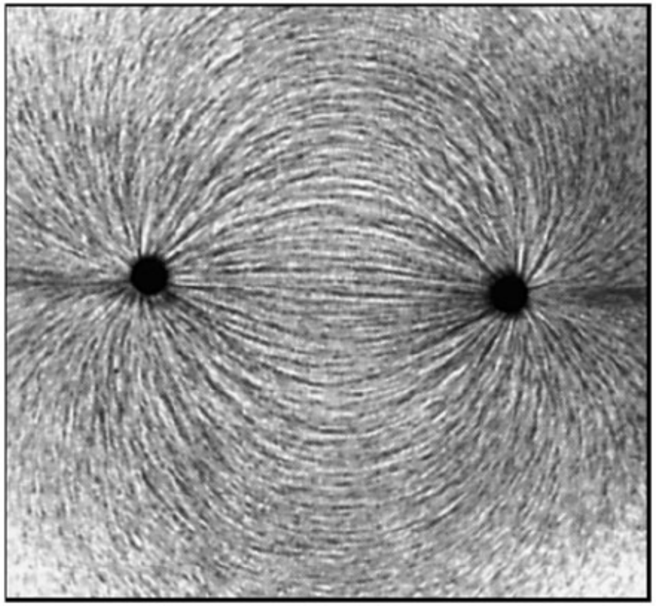


Direction of the electric field at the dot below?



The Electric Field of a Dipole

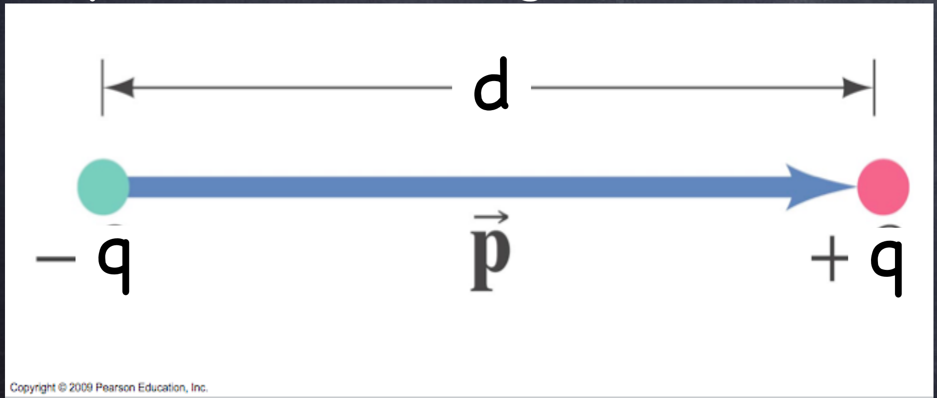




- This figure represents the electric field of a dipole using electric field lines.

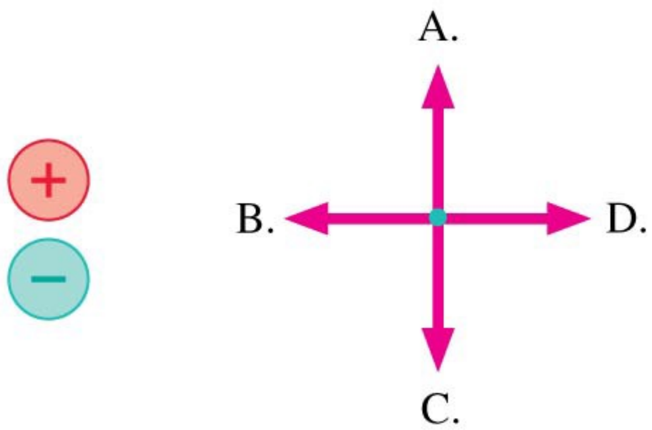
Electric Dipole

- The two charges in the electric dipole are separated by a distance, d.
- We define the magnitude of the electric dipole moment, p, as: $|\vec{p}| = qd$
- The dipole moment vector points from the negative charge to the positive charge.



iClicker question 4-1

An electron is in the plane that bisects a dipole. What is the direction of the electric **force** on the electron?



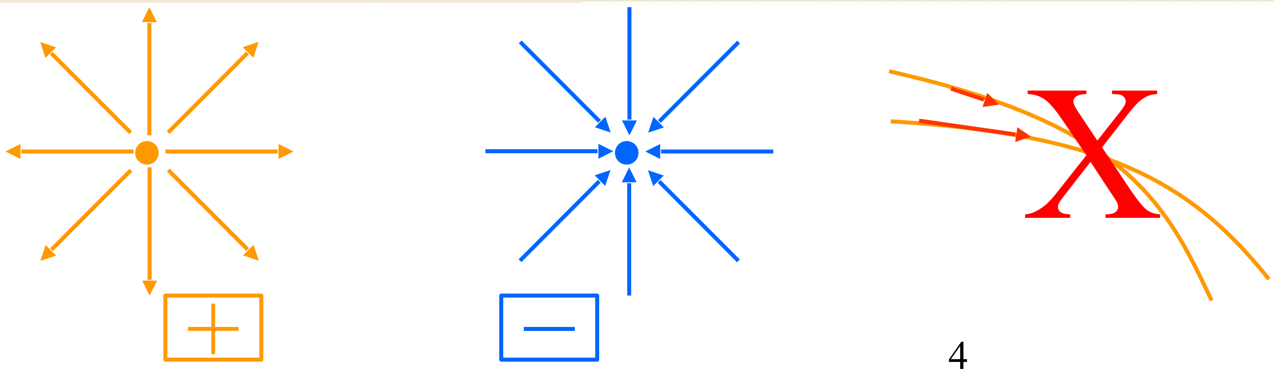
E. The force is zero.

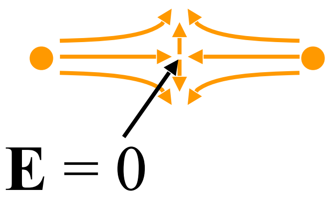
PHYS 2B: Announcements 10/07/25

- Quiz tomorrow 6:00 - 6:50 PM

Last class: Electric fields, properties, superposition.

$$\vec{E}(x,y,z) = \frac{\vec{F}_{\text{on } q} \text{ at } (x,y,z)}{q} \qquad \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \quad \text{(electric field of a point charge)}$$





Principle of superposition: $\mathbf{F}_e = q_0 k_e \sum_{i=1}^{4} \frac{q_i}{r_i^2}\hat{\mathbf{r}}_{i0}$, *and* $\mathbf{F}_e = q_0\mathbf{E}$

$$\Rightarrow \qquad \mathbf{E}(\mathbf{r}) = k_e \sum_{i=1}^{4} \frac{q_i}{r_i^2}\hat{\mathbf{r}}_{i0}$$

Today: Continuous charge distributions.

Electric Fields

- Which shapes you should know how to calculate the electric fields for:

1) point charge/sphere

$$E_{\text{point charge}} = k_e \frac{q}{r^2}$$

2) electric dipole

$$\left|\vec{E}_{dipole}\right| = \frac{1}{2\pi\varepsilon_o}\frac{p}{z^3}$$

3) infinite line of charge

$$E_y = \frac{1}{2\pi\varepsilon_o}\frac{\lambda}{z}$$

4) annular ring of charge

$$E_{ring} = \frac{1}{4\pi\varepsilon_o}\frac{qz}{\left(z^2+R^2\right)^{3/2}}$$

5) circular disk of charge

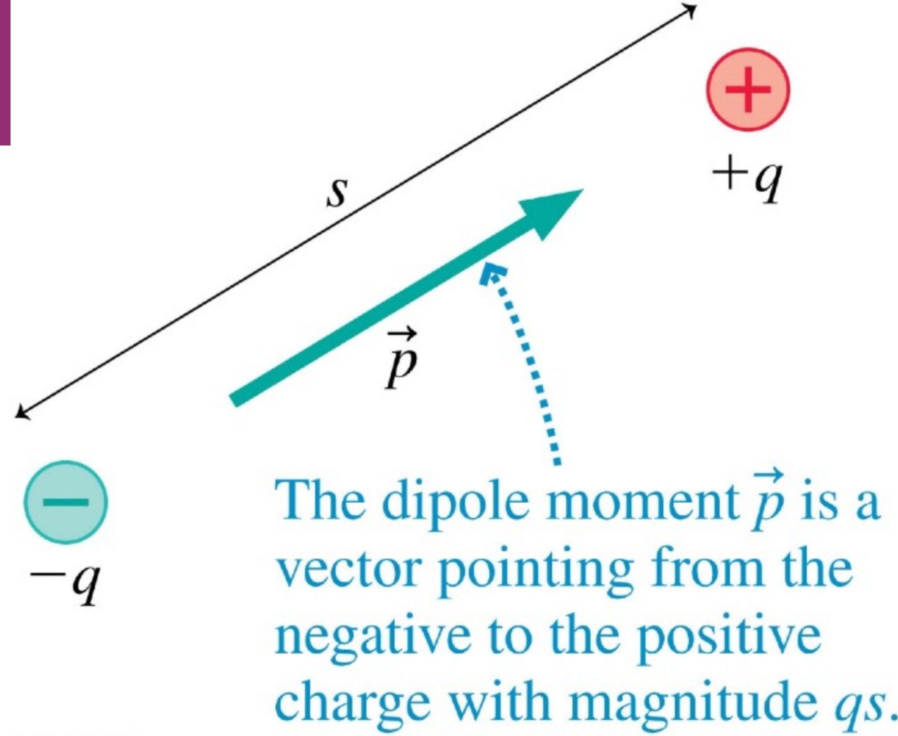
$$E_{disk} = \frac{\sigma}{2\varepsilon_o}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right)$$

6) infinite sheet of charge

$$E_{plane} = \frac{\sigma}{2\varepsilon_o}$$

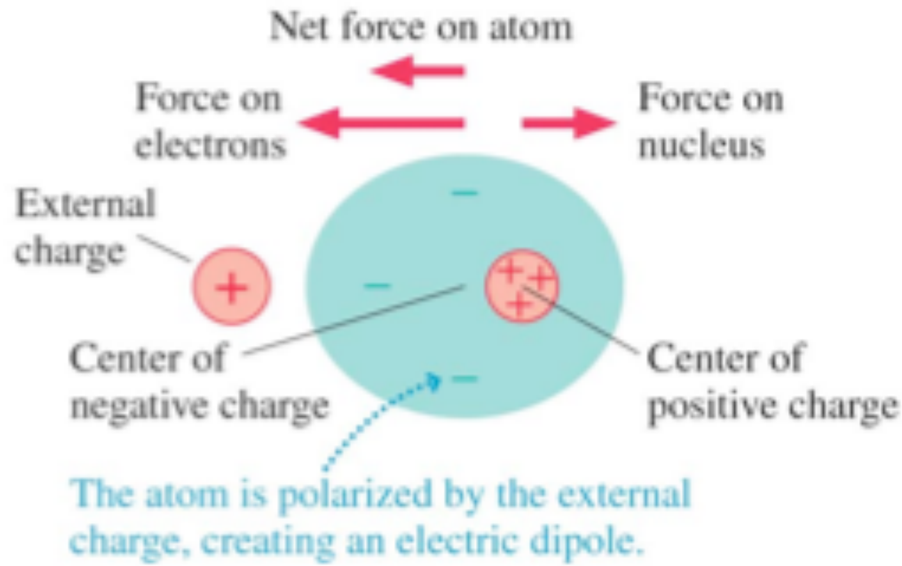
The Electric Dipole Moment

- It is useful to define the dipole moment $\vec{p}$, shown in the figure, as the vector:

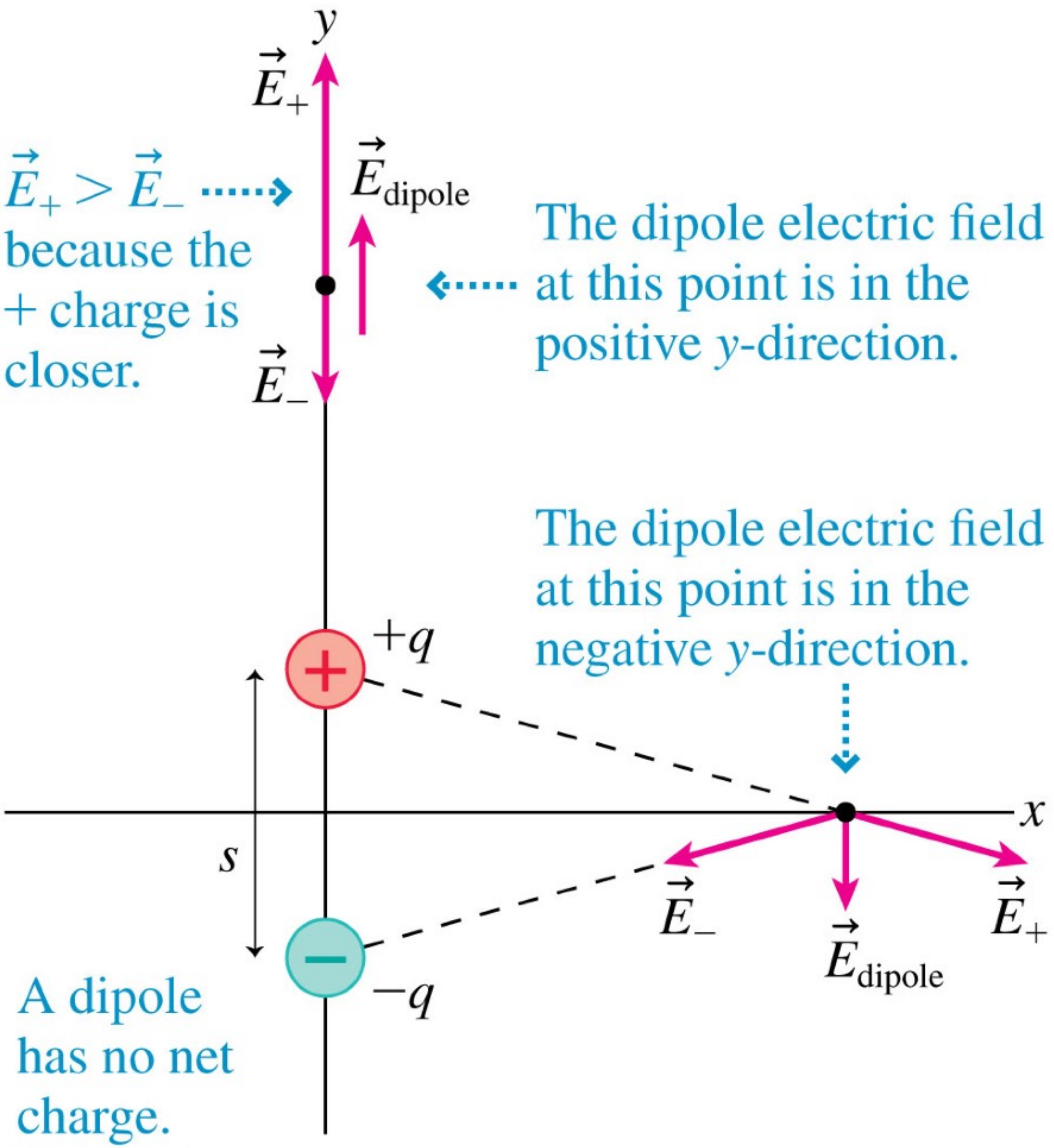


$$\vec{p} = (qs, \text{from the negative to the positive charge})$$

- The SI units of the dipole moment are C m.



The Dipole Electric Field at Two Points



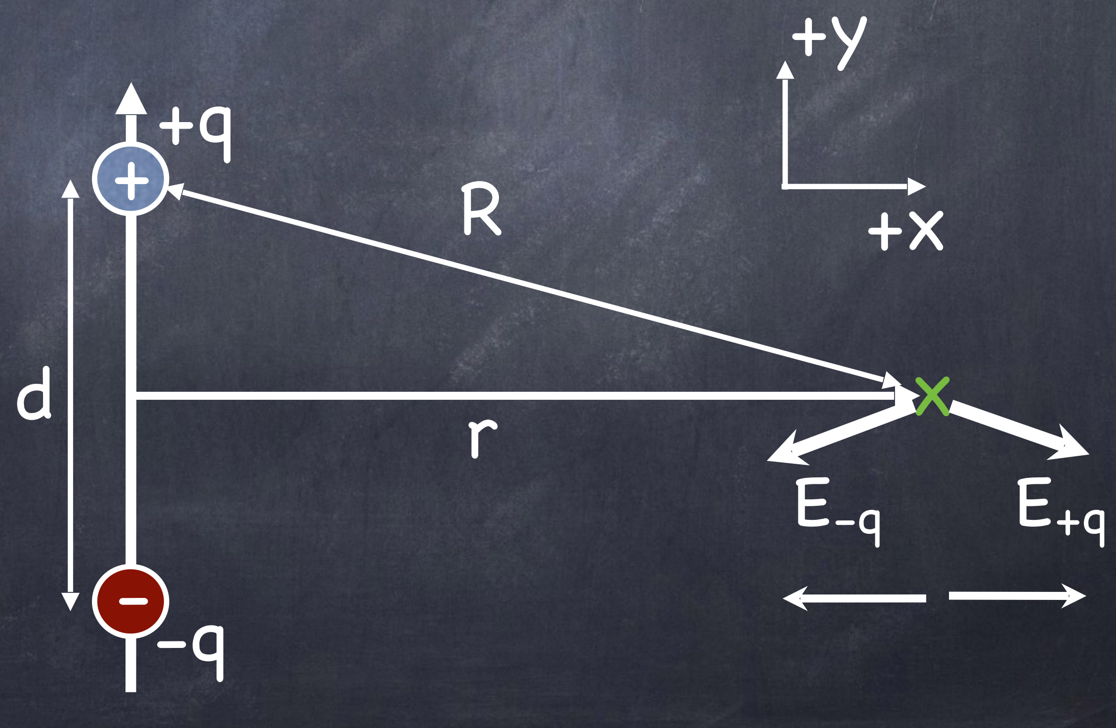
Electric Dipole

- We are going to attempt to calculate the electric field from an electric dipole on a perpendicular bisector of d (at point x below).
- Since we have two charges, we should calculate each magnitude separately and use superposition.

Due to symmetry we observe that the x-components of both forces will cancel out.

$$\left|\vec{E}_{+}\right| = \left|\vec{E}_{-}\right| = k_e \frac{\left|\pm q\right|}{R^2}$$

$$\left|\vec{p}\right| = qd$$



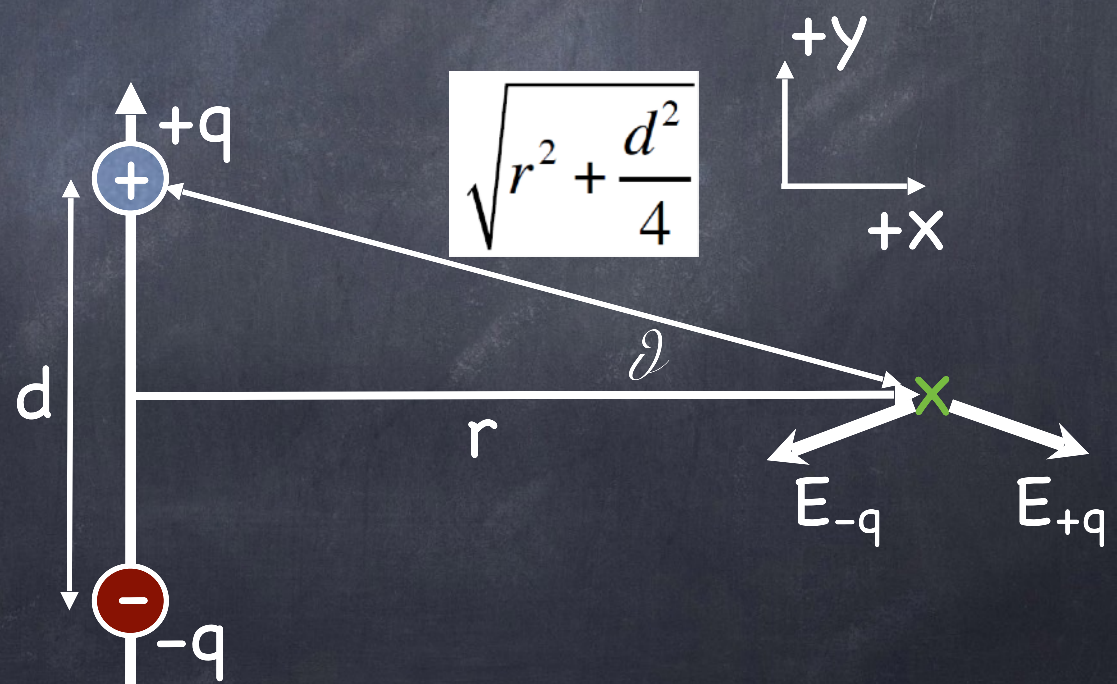
Electric Dipole

- The only contribution is from the y-component, which turns out to be the same magnitude for +q and -q.

$$\left|\vec{E}_{+,y}\right| = \left|\vec{E}_{+}\right| \sin\theta$$

$$\sin\theta = \frac{d/2}{\sqrt{r^2 + \frac{d^2}{4}}}$$

$$\left|\vec{E}_{+}\right| = k_e \frac{q}{r^2 + \frac{d^2}{4}}$$



- Combining these equations gives us:

$$\left|\vec{E}_{+,y}\right| = k_e \frac{q}{r^2 + \frac{d^2}{4}} \frac{d/2}{\sqrt{r^2 + \frac{d^2}{4}}}$$

Electric Dipole

$$\left|\vec{E}_{+,y}\right| = \frac{k_e \, qd/2}{\left(r^2 + \frac{d^2}{4}\right)^{3/2}}$$

- Combining E_+ and E_- yields:

$$\sum\left|\vec{E}\right| = 2\left|\vec{E}_+\right|\sin\theta$$

$$\sum\left|\vec{E}\right| = \frac{k_e qd}{\left(r^2 + \frac{d^2}{4}\right)^{3/2}}$$

- Since qd is the dipole moment, we can finally write:

$$\sum\left|\vec{E}\right| = \frac{k_e p}{\left(r^2 + \frac{d^2}{4}\right)^{3/2}}$$

Electric Dipole

- If we assume that we are very far away from the dipole, this means that r>>d and that $r^2 >>>> d^2$.

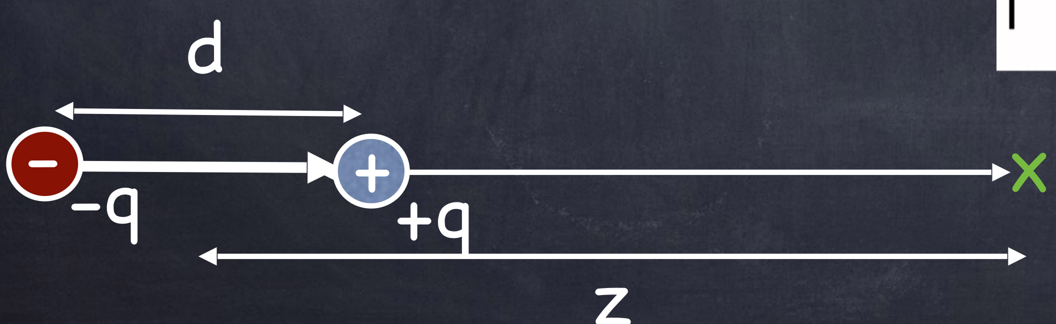
$$\sum\left|\vec{E}\right| = \frac{k_e p}{\left(r^2 + \frac{d^2}{4}\right)^{3/2}}$$

$$\left(r^2 + \frac{d^2}{4}\right)^{3/2} \approx \left(r^2\right)^{3/2} = r^3$$

$$\sum\left|\vec{E}\right| = \frac{k_e p}{r^3} = \frac{1}{4\pi\varepsilon_o}\frac{p}{r^3}$$

- You can choose a point along the line that joins the dipole to perform it's dipole calculation and get:

$$\left|\vec{E}_{dipole}\right| = \frac{1}{2\pi\varepsilon_o}\frac{p}{z^3}$$



Use this for calculations.

The Electric Field of a Dipole, in words

- The electric field at a point on the axis of a dipole is

$$\vec{E}_{\text{dipole}} \approx \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3} \qquad \text{(on the axis of an electric dipole)}$$

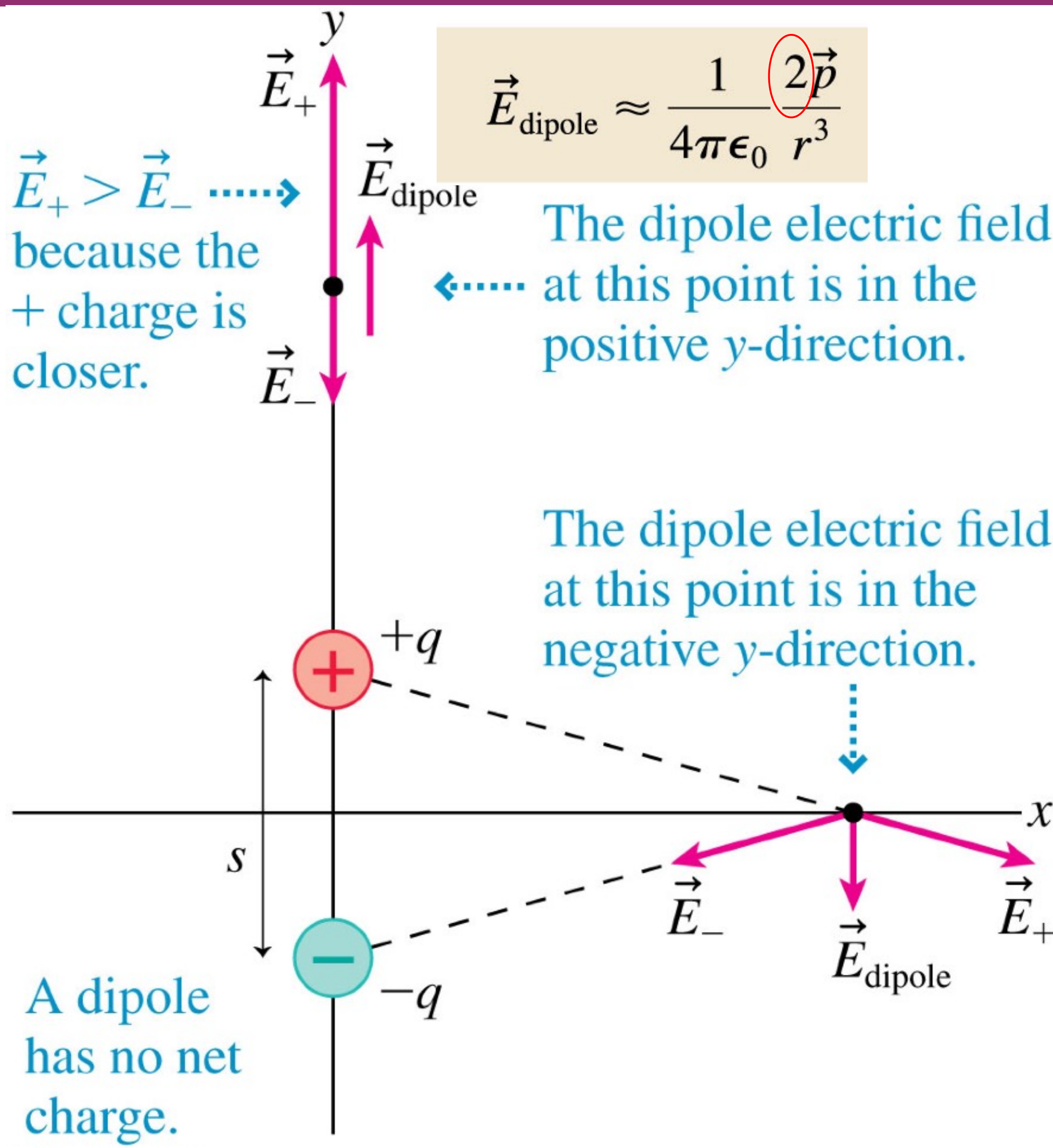
where r is the distance measured from the *center* of the dipole.

- The electric field in the plane that bisects and is perpendicular to the dipole is

$$\vec{E}_{\text{dipole}} \approx -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3} \qquad \text{(bisecting plane)}$$

- This field is opposite to the dipole direction, and it is only *half the strength* of the on-axis field at the same distance.

The Electric Field of a Dipole, in a picture



$$\vec{E}_{\text{dipole}} \approx \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3}$$

$$\vec{E}_{\text{dipole}} \approx -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3}$$

Electric Dipole far away...

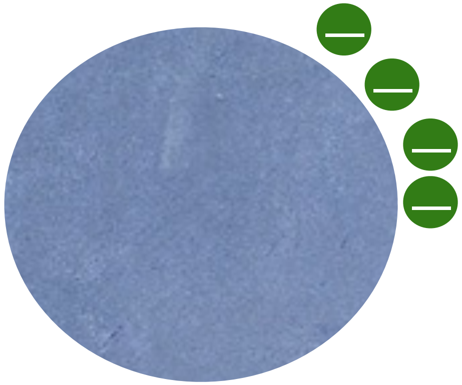
- No matter which equation you use, recall that the important thing is that the electric field exhibits a $1/r^3$ dependence.

$$\left|\vec{E}_{dipole}\right| = \frac{1}{2\pi\varepsilon_o}\frac{p}{z^3}$$

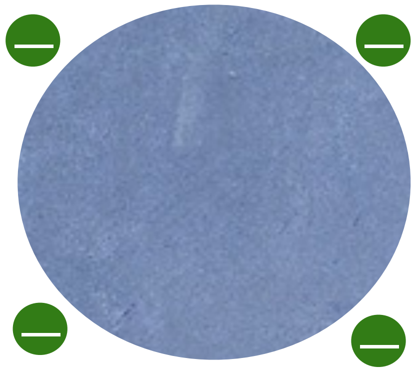
  - This field decreases faster than a single point charge (which exhibits $1/r^2$ dependence).
  - The reason for the more rapid decrease in magnitude is that, at large distances, electric dipoles look like two point charges that are right on top of one another.
  - They don't quite cancel each other out.

Back to **conductors**

- What would happen to a conductor if I put excess charged particles on one part of it?



- The charges would be repelled by one another and initially spread out along the edges of the conductor.
  - Once the charges have stopped moving the conductor is considered to be in electrostatic equilibrium.
  - At this point the charges won't move since there is not net force on them.



Properties of **conductors**

- An isolated conductor that is in static equilibrium will have some interesting properties:

1. Any excess charge will reside entirely on its surface.

2. The electric field is zero everywhere inside the conducting material.
   If this part were not true, there would still be charges moving around until it became true.

3. On an irregularly shaped conductor, charge accumulates on sharp points.

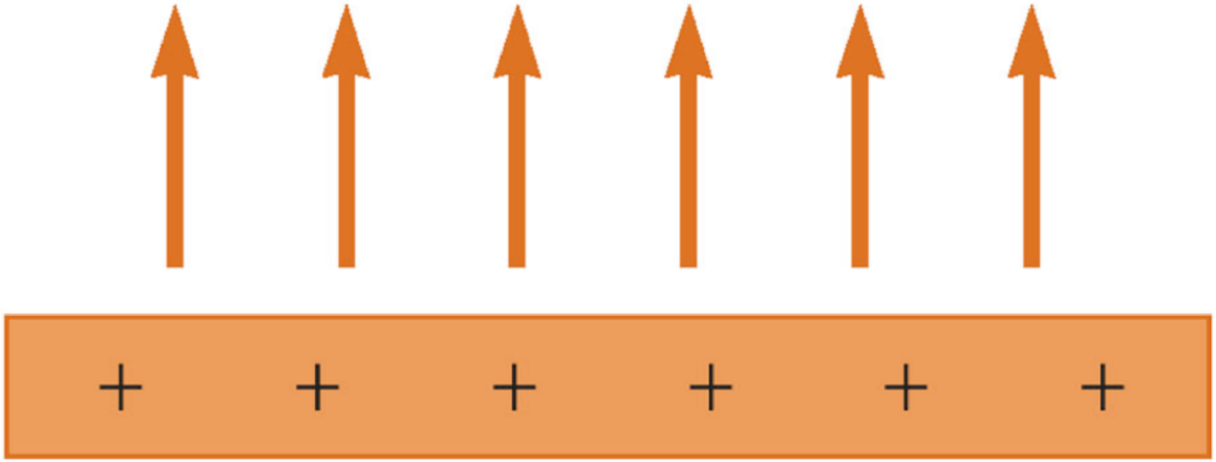
Properties of **conductors**

And finally:

4. The electric field just outside of a charged conductor is perpendicular to the conductor's surface.

- What would the electric field look like for a large positively charged conducting plane (like a floor)?

- Straight up, perpendicular to the surface.



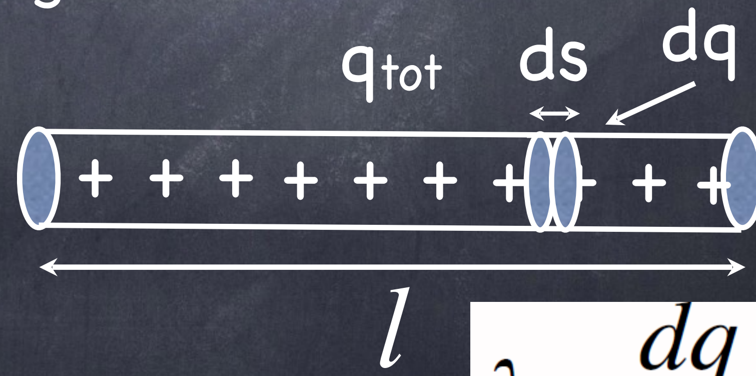
Continuous charge distributions

- If you have a discrete charge distribution (1, 2, 3... charges) then you can add up their individual electric fields by superposition.

- But if you have a continuous charge distribution, then you need to divide the distribution into very small (differential) elements of charge.

- It becomes useful to define charges in terms of densities.

- For a line of charge, we have linear charge density, d.



$$\lambda = \frac{q_{tot}}{\ell}$$

$$\lambda = \frac{dq}{ds}$$

An infinite line of charge

$$dE = k_e \frac{dq}{r^2}$$

- At point P, the differential electric field contribution will be:

$$dE = k_e \frac{\lambda dx}{\left(z^2 + x^2\right)}$$

- dE will have dE_x and dE_y components.

$$dE_x = dE \sin\theta \qquad dE_y = dE \cos\theta$$

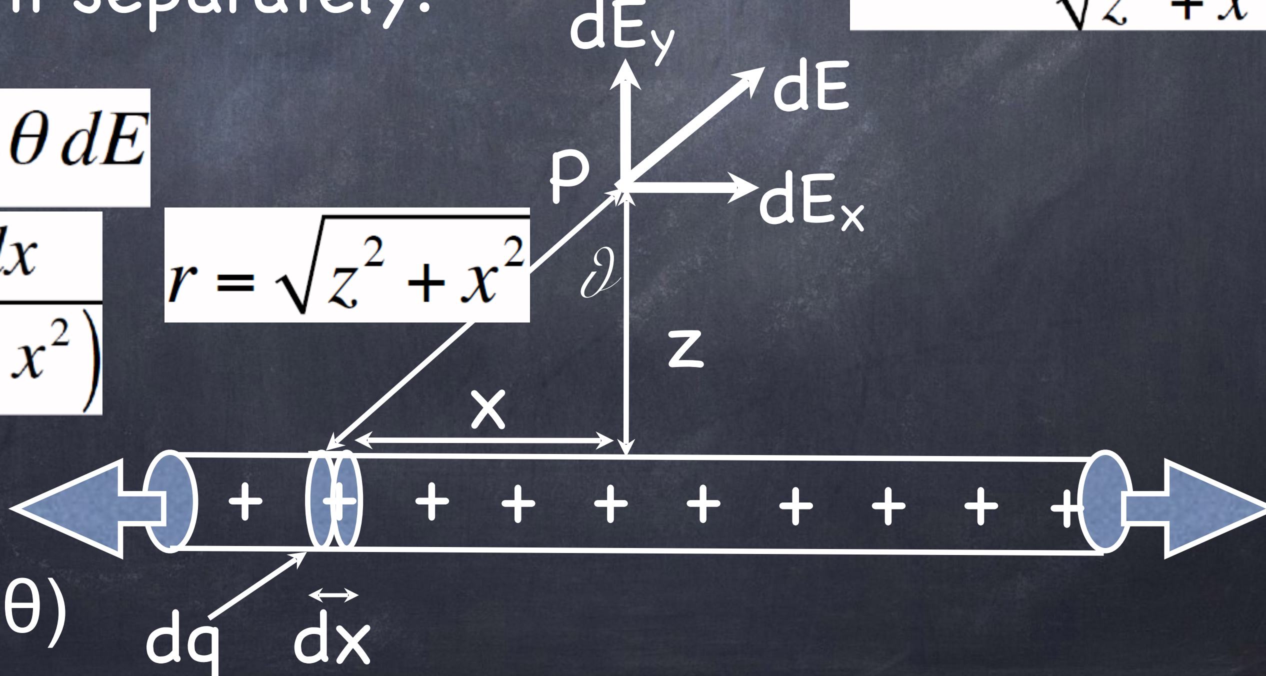
$$\cos\theta = \frac{z}{\sqrt{z^2 + x^2}}$$

- Let's handle them separately:

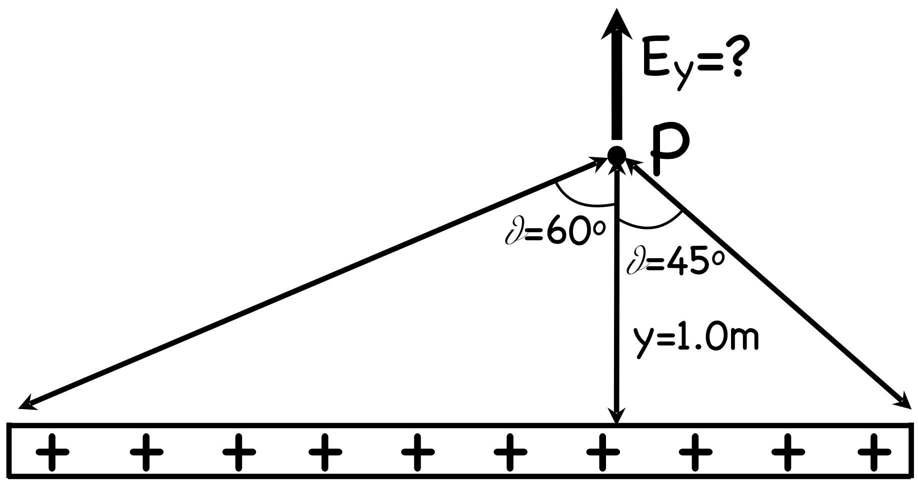
$$E_y = \int dE_y = \int \cos\theta \, dE$$

$$E_y = \int_{-\infty}^{+\infty} (\cos\theta) k_e \frac{\lambda dx}{\left(z^2 + x^2\right)}$$

$$r = \sqrt{z^2 + x^2}$$



- We have two variables (x and θ)



Ey=?
P
∂=60°
∂=45°
y=1.0m
+ + + + + + + + + +

An infinite line of charge

- Let's switch from dx to dθ in our formula:

$$\tan\theta = \frac{x}{z}$$

$$x = z\ \tan\theta$$

$$dx = z\ d\left(\tan\theta\right) = z\left(\sec^2\theta\right)d\theta$$

$$dx = \frac{z}{\cos^2\theta}d\theta$$

- And we have:

$$\cos\theta = \frac{z}{\sqrt{z^2+x^2}}$$

$$\frac{\cos^2\theta}{z^2} = \frac{1}{\left(z^2+x^2\right)}$$

- Giving us:

$$E_y = \lambda k_e \int_{-\infty}^{+\infty} (\cos\theta)\frac{dx}{\left(z^2+x^2\right)}$$

$$E_y = \lambda k_e \int_{-\pi/2}^{+\pi/2} (\cos\theta)\frac{\cos^2\theta}{z^2}\frac{z}{\cos^2\theta}d\theta$$

$$E_y = \frac{\lambda k_e}{z}\int_{-\pi/2}^{+\pi/2} (\cos\theta)d\theta$$

$$E_y = \frac{2\lambda k_e}{z}$$

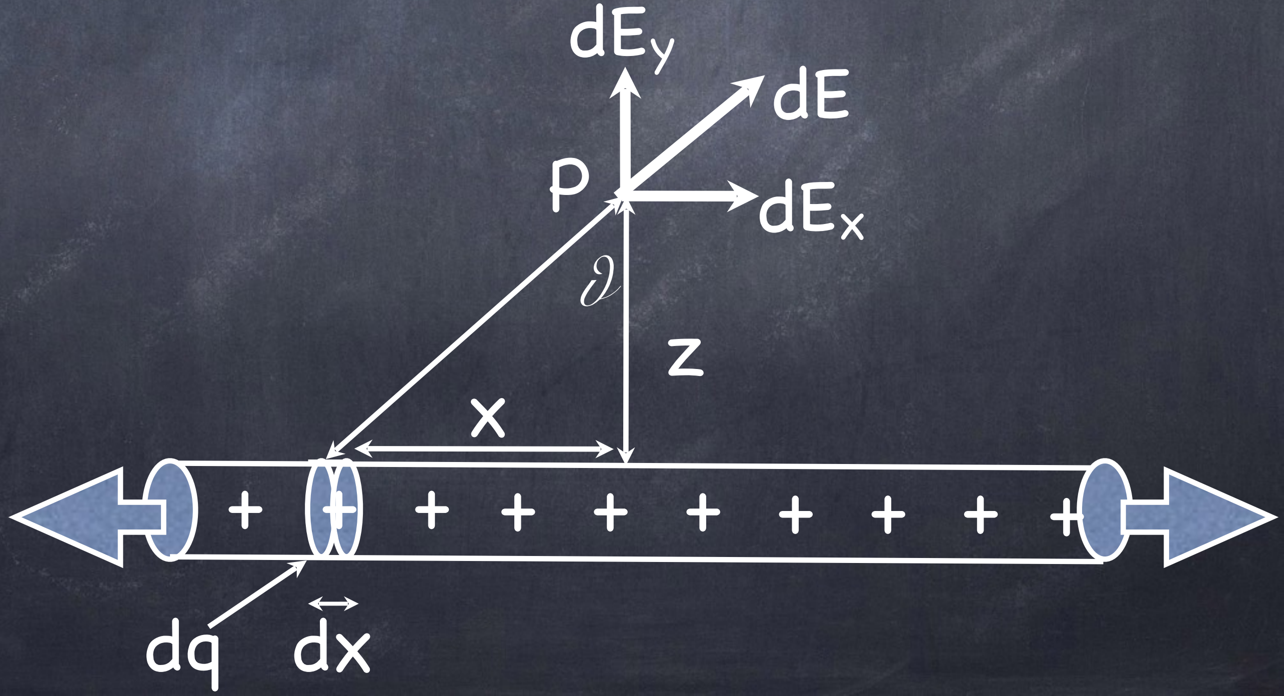
$$E_y = \frac{1}{2\pi\varepsilon_o}\frac{\lambda}{z}$$

An infinite line of charge

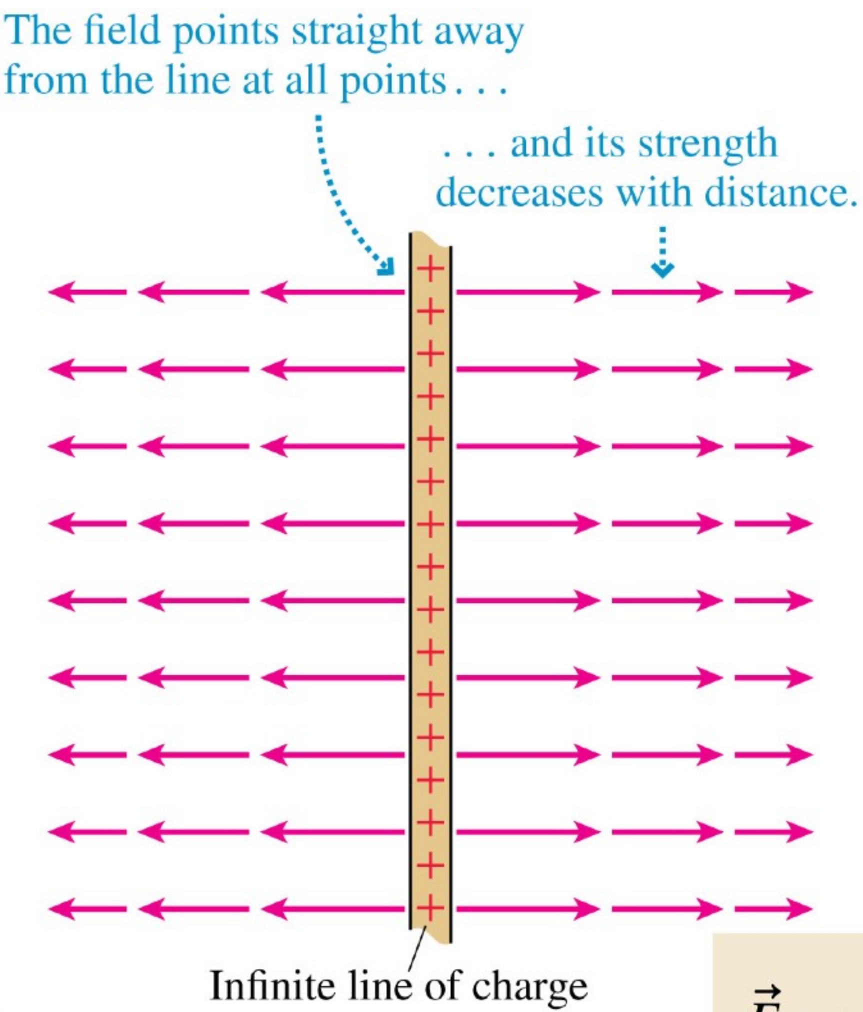
- The differential contributions for dE_x can easily be shown to cancel out via symmetry as you sum over the entire wire (you get sinθ instead of cosθ).
- Thus, the only component left is the dE_y.
- Our solution to this problem is:

$$\vec{E} = \frac{1}{2\pi\varepsilon_o}\frac{\lambda}{z}\hat{j}$$

- Note the linear dependence.



An Infinite Line of Charge (connecting w/book version)

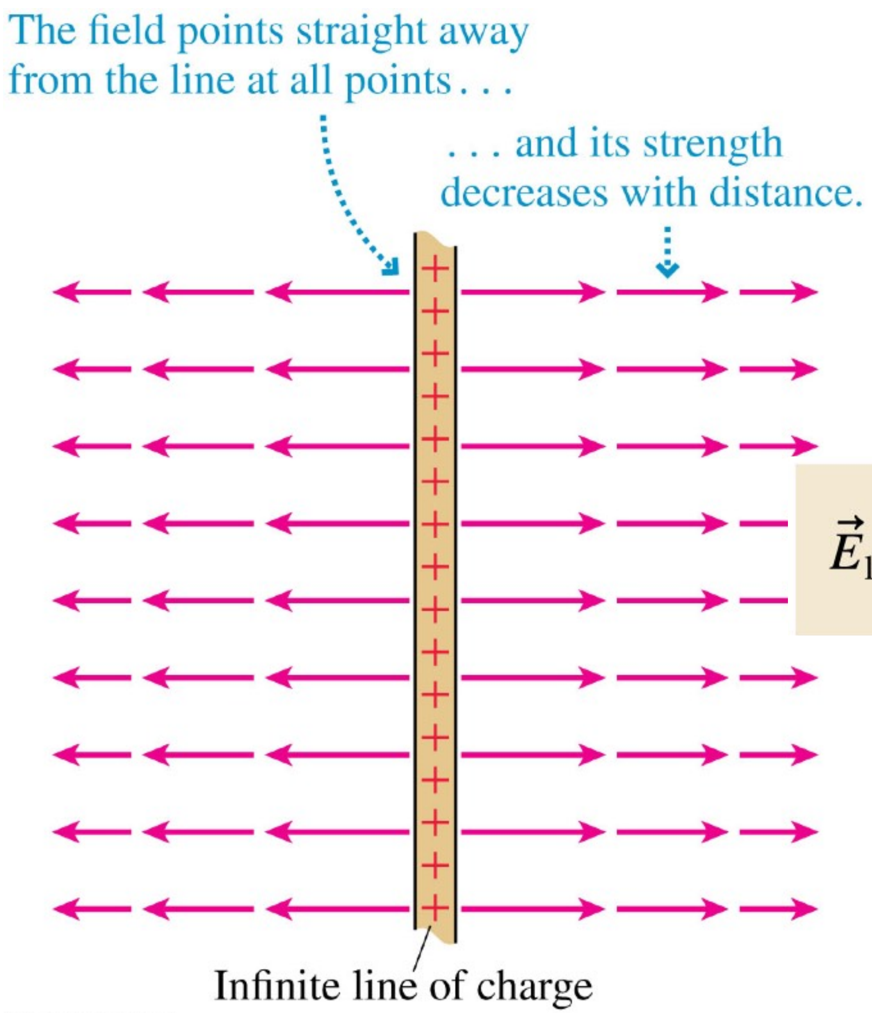


- Book version: The electric field of a thin, uniformly charged rod may be written

$$E_{\text{rod}} = \frac{1}{4\pi\epsilon_0}\frac{2|\lambda|}{r}\frac{1}{\sqrt{1+4r^2/L^2}}$$

- If we now let $L \to \infty$, the last term becomes simply 1 and we're left with

$$\vec{E}_{\text{line}} = \left(\frac{1}{4\pi\epsilon_0}\frac{2|\lambda|}{r}, \begin{cases}\text{away from line if charge } + \\ \text{toward line if charge } - \end{cases}\right)$$



$$E_{\text{rod}} = \frac{1}{4\pi\epsilon_0}\frac{2|\lambda|}{r}\frac{1}{\sqrt{1 + 4r^2/L^2}}$$

- If we now let $L \rightarrow \infty$, the last term becomes simply 1 and we're left with

$$\vec{E}_{\text{line}} = \left(\frac{1}{4\pi\epsilon_0}\frac{2|\lambda|}{r}, \begin{cases} \text{away from line if charge } + \\ \text{toward line if charge } - \end{cases}\right)$$

But, wait a minute!!!!

$\lambda = \frac{Q}{L}$ so λ *goes to zero?*

NO, Q *also increases so that the ratio* $\frac{Q}{L}$ *stays finite.*

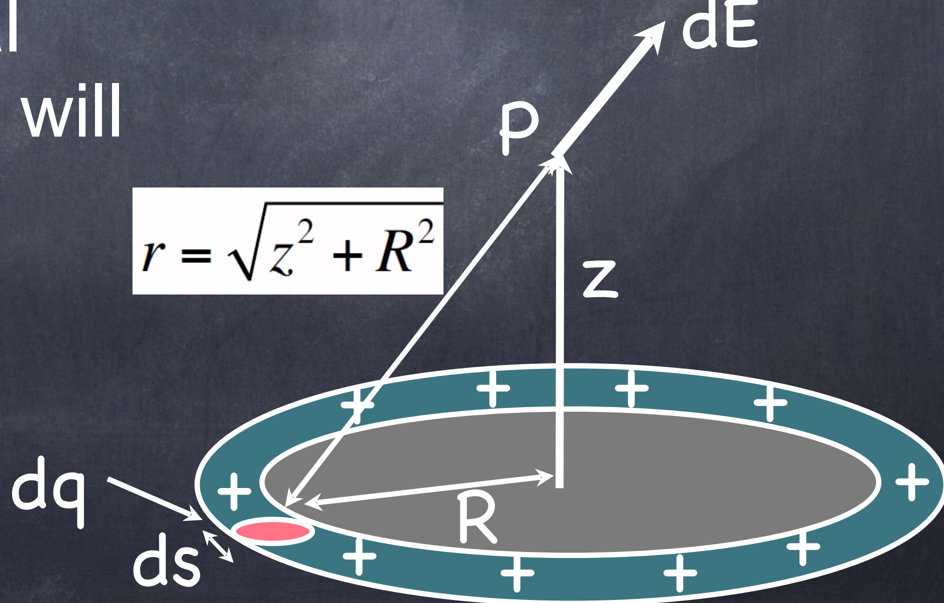
A ring of charge

- Next, let’s calculate the electric field at a point P which is a distance z along the central axis due to a charged ring of radius R and uniform linear charge density, λ (total charge q).
- Solution:
- A small segment of wire ds has a charge of $dq=\lambda\, ds$.
- At point P, the differential electric field contribution will be:

$$dE = k_e \frac{dq}{r^2}$$

$$dE = k_e \frac{\lambda ds}{\left(z^2 + R^2\right)}$$

$$r = \sqrt{z^2 + R^2}$$



A ring of charge

- The dE_x contributions from opposite sides of the ring will cancel, leaving only the dE_y contributions for finding dE.

$$dE_y = dE \sin\theta \qquad E_y = \int dE_y = \int \sin\theta \, dE$$

- From the diagram we see that:

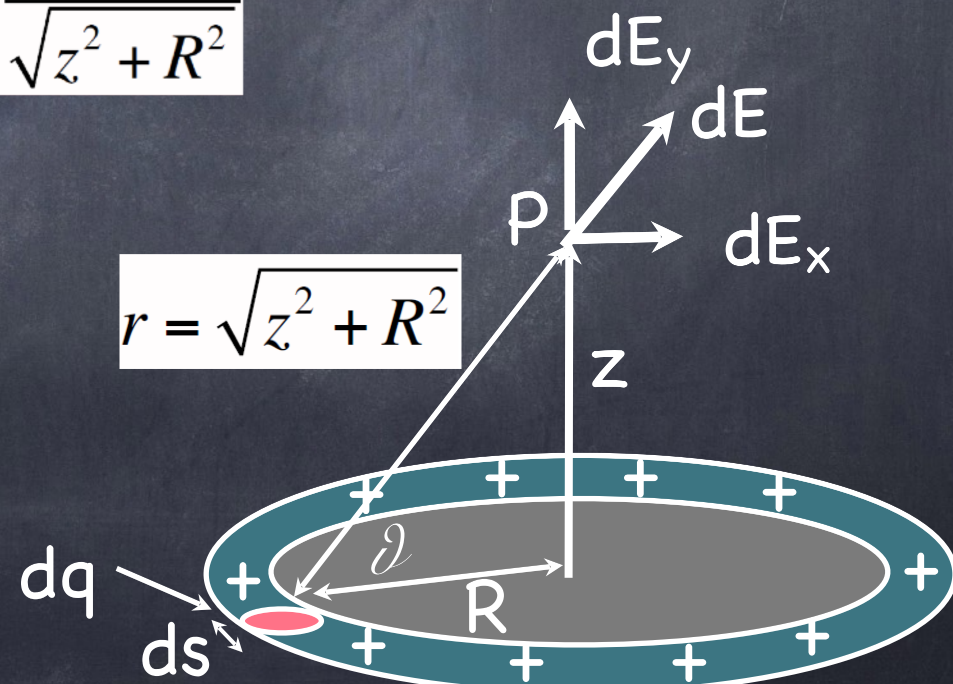
$$\sin\theta = \frac{z}{\sqrt{z^2 + R^2}}$$

- Giving us:

$$E_y = k_e \int \frac{z}{\sqrt{z^2+R^2}} \frac{\lambda ds}{\left(z^2+R^2\right)}$$

$$E_y = k_e \lambda \int_0^{2\pi R} \frac{z ds}{\left(z^2+R^2\right)^{3/2}}$$

$$r = \sqrt{z^2 + R^2}$$



A ring of charge

- Since, z and R are not variables, we can take them out of the integral:

$$E_y = k_e \lambda \frac{z}{\left(z^2 + R^2\right)^{3/2}} \int_0^{2\pi R} ds$$

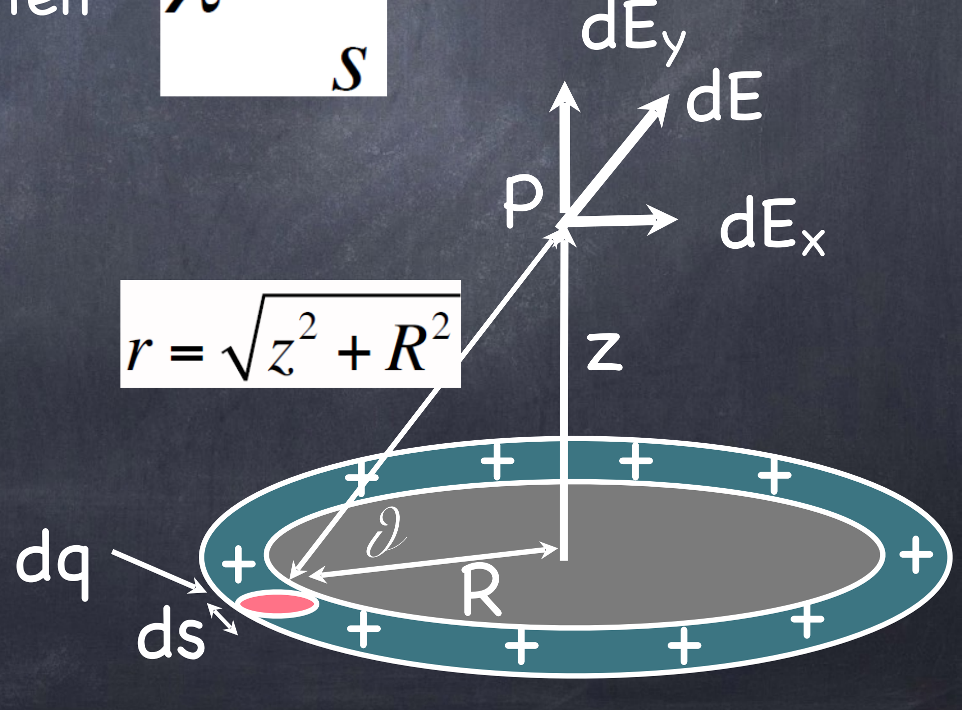
$$E_y = k_e \lambda \frac{z}{\left(z^2 + R^2\right)^{3/2}} (2\pi R)$$

- But we know that the linear charge density can be rewritten as:

$$\lambda = \frac{q}{s}$$

$$E_y = k_e \left(\frac{q}{2\pi R}\right) \frac{z}{\left(z^2 + R^2\right)^{3/2}} (2\pi R)$$

$$E_y = \frac{1}{4\pi\varepsilon_o} \frac{qz}{\left(z^2 + R^2\right)^{3/2}}$$



A ring of charge

- We say that the charge a distance z on the central axis due to a ring of radius R and total charge q is:

$$E_{ring} = \frac{1}{4\pi\varepsilon_o}\frac{qz}{\left(z^2 + R^2\right)^{3/2}}$$

- The direction will be either towards the ring or away from the ring (depending on the sign of the charge).

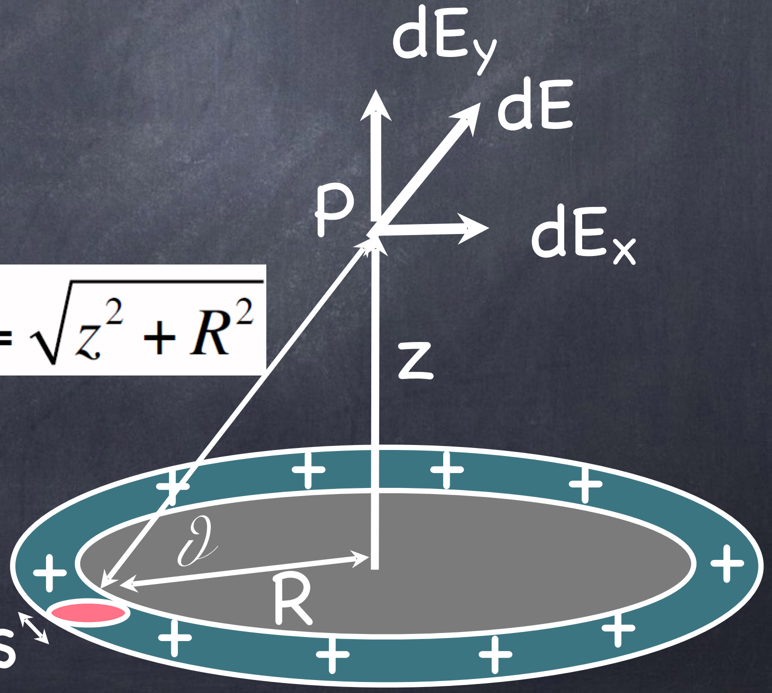
- If z>>R, then the equation reduces to:

$$E_{\text{point}} = \frac{1}{4\pi\varepsilon_o}\frac{qz}{\left(z^2\right)^{3/2}}$$

- Compare with Electric field for a point charge

$$E_{\text{point}} = \frac{1}{4\pi\varepsilon_o}\frac{q}{z^2}$$

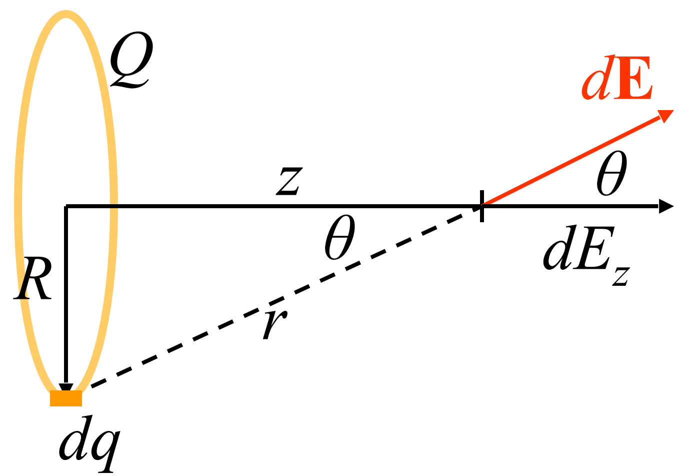
$$r = \sqrt{z^2 + R^2}$$



E-field of ring of charge:

(a continuous distribution of charge)

- What is E-field along the z-axis at distance z from center?
- Axial symmetry implies **E** parallel to axis



$$d\mathbf{E} = k_e \frac{dq}{r^2}\hat{\mathbf{r}} \qquad \Rightarrow \qquad dE_z = k_e \frac{dq}{r^2}\cos\theta$$

$$dE_z = k_e \frac{dq}{r^2}\frac{z}{r} = k_e \frac{z}{\left(R^2 + z^2\right)\sqrt{R^2 + z^2}}\,dq$$

$$\int_0^{E_z} dE_z = k_e \frac{z}{\left(R^2 + z^2\right)^{3/2}}\int_0^Q dq$$

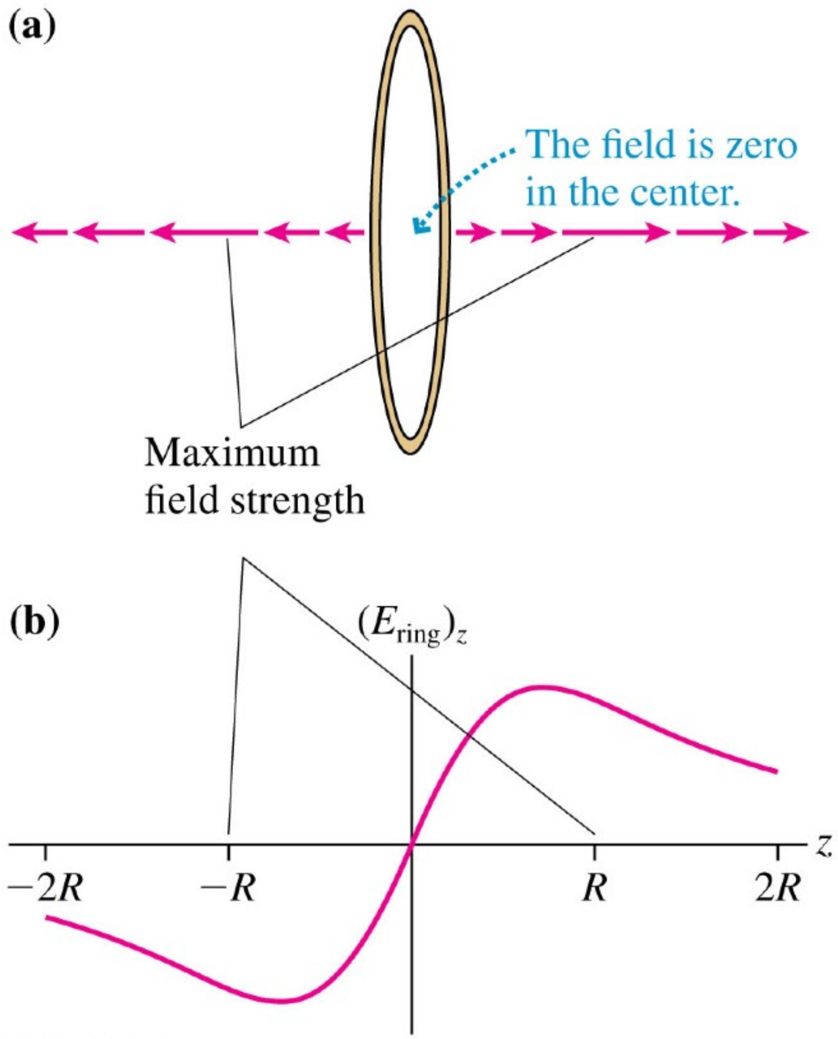
$$E_z \equiv E_z(z) = k_e \frac{z}{\left(R^2 + z^2\right)^{3/2}}Q, \qquad \mathbf{E}(z) = E_z(z)\hat{\mathbf{z}}$$

What happens when $z = 0$?

When $z = \infty$?

When z is large?

A Ring of Charge



- Consider the on-axis electric field of a positively charged ring of radius R.
- Define the z-axis to be the axis of the ring.
- The electric field on the z-axis points away from the center of the ring, increasing in strength until reaching a maximum when $|z| \approx R$, then decreasing:

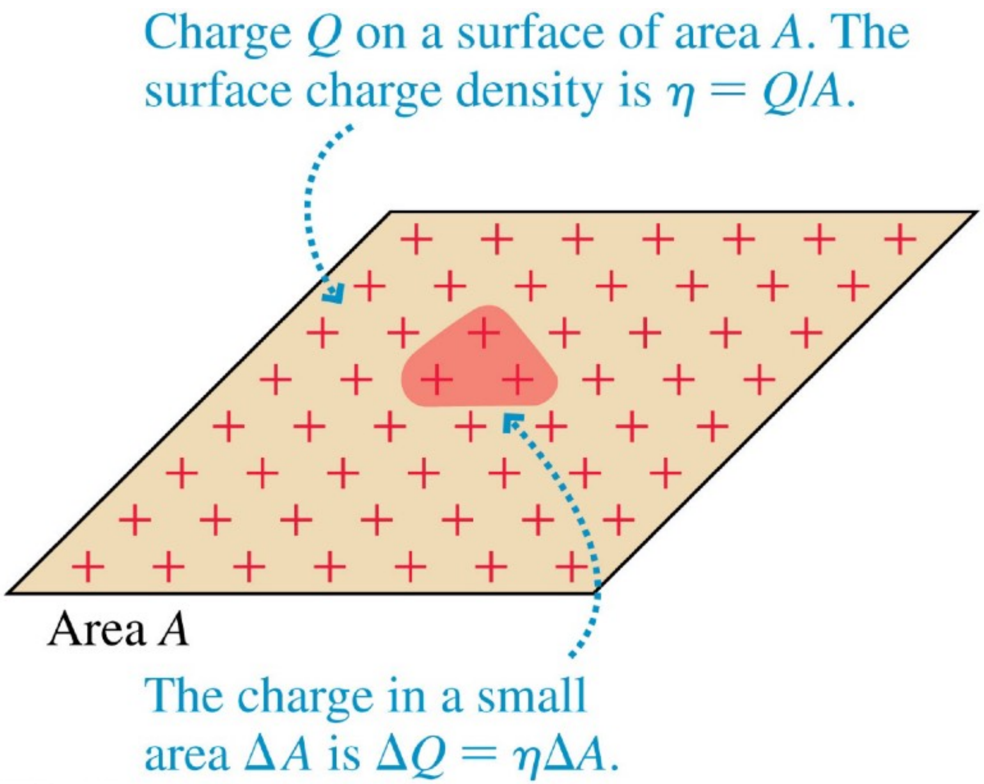
$$(E_{ring})_z = \frac{1}{4\pi\epsilon_0} \frac{zQ}{(z^2 + R^2)^{3/2}}$$

Continuous Charge Distributions

- The surface charge density of a two-dimensional distribution of charge across a surface of area is defined as

$$\eta = \frac{Q}{A}$$

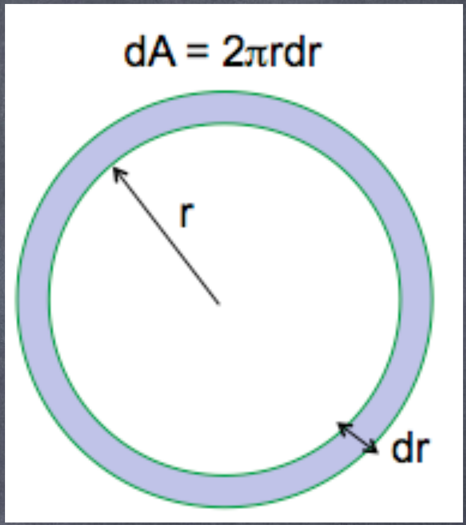
- Surface charge density, with units $\mathrm{C/m^2}$, is the amount of charge *per square meter*.



A disk of charge

- Next, let's calculate the electric field at a point P which is a distance z along the central axis due to a charged disk of radius R and uniform surface charge density, σ.

$$\sigma = \frac{q}{A}$$

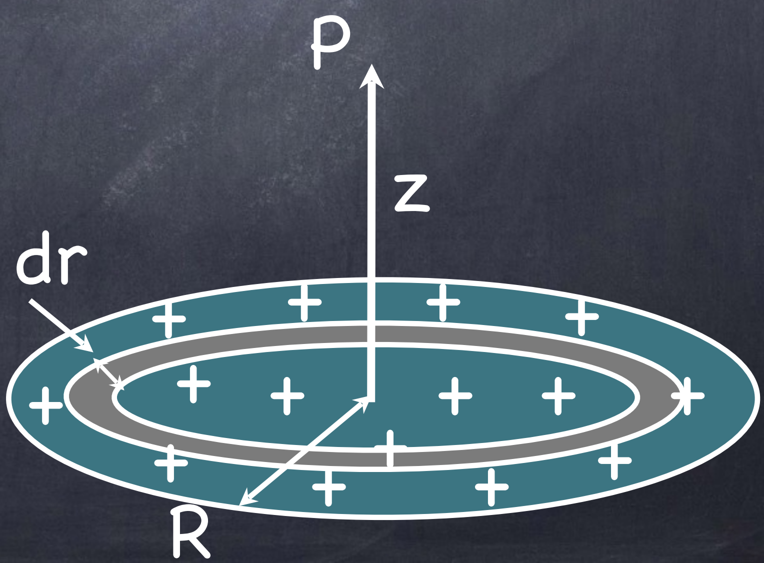


- Solution:
- We can say that the disk is composed of many differential rings each with area dA.

$$dA_{ring} = (2\pi r)\,dr$$

- Each ring will have a charge of dq = σ dA.

$$dq = \sigma(2\pi r)\,dr$$



A disk of charge

- From the previous example, we know that:

$$dE_{ring} = \frac{1}{4\pi\varepsilon_o}\frac{(z)dq}{(z^2+R^2)^{3/2}}$$

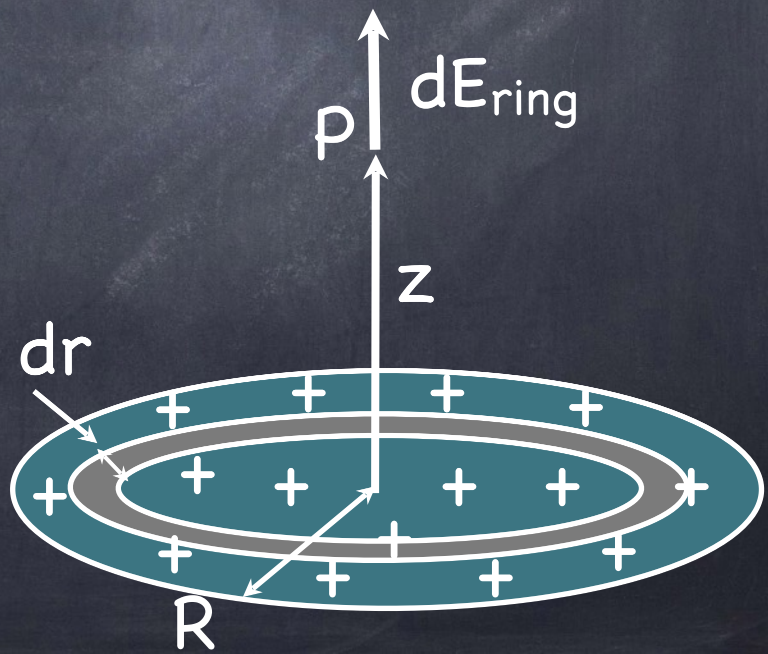
$$dE_{ring} = \frac{1}{4\pi\varepsilon_o}\frac{(z)\sigma(2\pi r)dr}{(z^2+r^2)^{3/2}}$$

- Integrating over rings from r = 0 to r = R yields:

$$dE_{ring} = \frac{\sigma}{4\varepsilon_o}\frac{(z)2rdr}{(z^2+r^2)^{3/2}}$$

$$E_{disk} = \int dE_{ring} = \frac{\sigma}{4\varepsilon_o}\int_0^R \frac{(z)2rdr}{(z^2+r^2)^{3/2}}$$

- In order to solve this integral we should perform a u-substitution.



A disk of charge

- Let's choose:

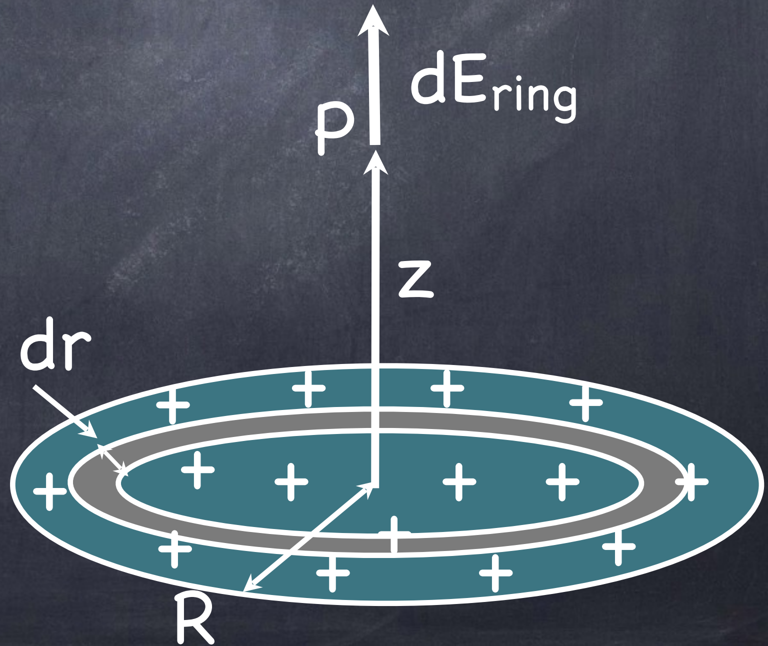
$$u = z^2 + r^2 \qquad du = 2r\,dr$$

- This changes our limits of integration for dr from (0 to R) to (z^2 to $z^2 + R^2$):

$$E_{disk} = \frac{\sigma}{4\varepsilon_o}\int_0^R \frac{(z)2r\,dr}{\left(z^2 + r^2\right)^{3/2}} \qquad E_{disk} = \frac{\sigma z}{4\varepsilon_o}\int_{z^2}^{z^2+R^2} \frac{du}{(u)^{3/2}}$$

$$E_{disk} = \frac{\sigma z}{4\varepsilon_o}\left(-2u^{-1/2}\right)_{z^2}^{z^2+R^2}$$

$$E_{disk} = \frac{-\sigma z}{2\varepsilon_o}\left(\frac{1}{\sqrt{z^2 + R^2}} - \frac{1}{\sqrt{z^2}}\right)$$



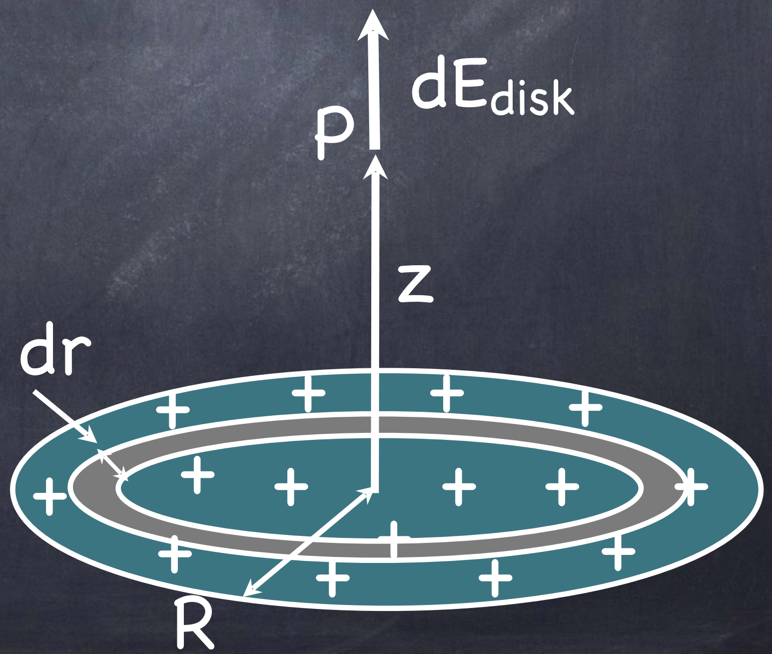
A disk of charge

- We can rewrite this is:

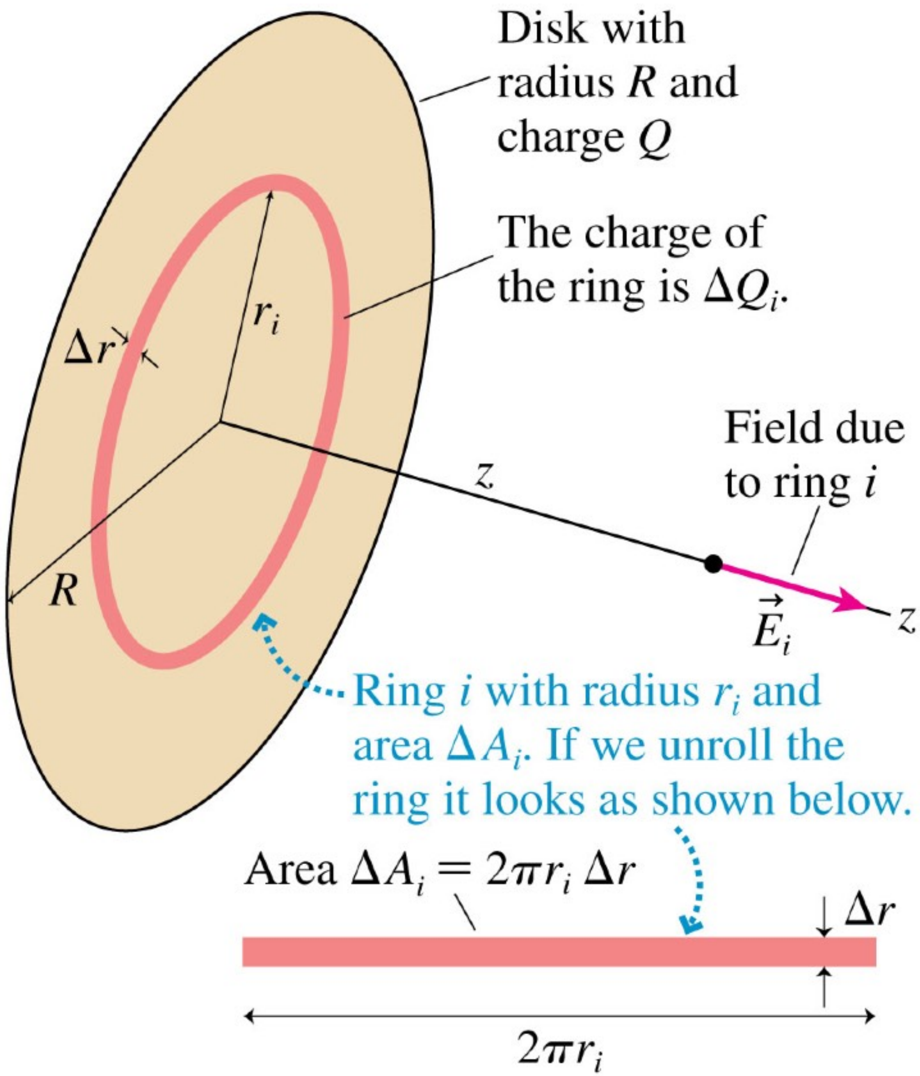
$$E_{disk} = \frac{\sigma}{2\varepsilon_o}\left(\frac{z}{z} - \frac{z}{\sqrt{z^2 + R^2}}\right) \qquad E_{disk} = \frac{\sigma}{2\varepsilon_o}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

- This is the electric field from a disk of charge at a point z on the central axis.

- The direction will be either towards the disk or away from the disk (depending on the sign of the charge).



A Disk of Charge



- Consider the on-axis electric field of a positively charged disk of radius R.
- Define the z-axis to be the axis of the disk.
- The electric field on the z-axis points away from the center of the disk, with magnitude:

$$(E_{\text{disk}})_z = \frac{\eta}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

A infinite sheet of charge

To now find the electric field from an infinite sheet of charge, just extend the radius of the disk to infinity (R -> ∞).

$$E_{disk} = \frac{\sigma}{2\varepsilon_o}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

$$E_{plane} = \frac{\sigma}{2\varepsilon_o}(1-0)$$

- This means that the electric field from an infinite sheet of charge at a point z above.

$$E_{plane} = \frac{\sigma}{2\varepsilon_o}$$

- Note that this electric field has **no dependence** on how far above the plane you are.

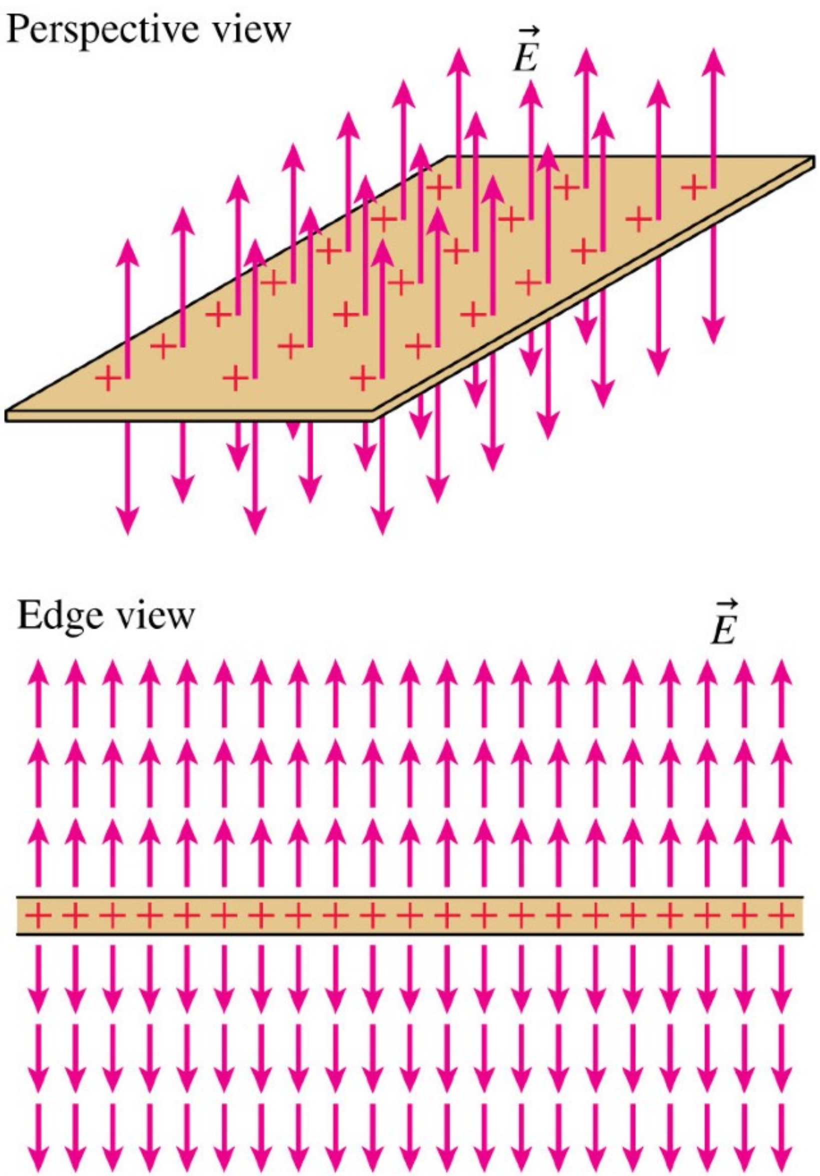
A Plane of Charge

- The electric field of a plane of charge is found from the on-axis field of a charged disk by letting the radius $R \to \infty$.
- The electric field of an infinite plane of charge with surface charge density η is

$$E_{\text{plane}} = \frac{\eta}{2\epsilon_0} = \text{constant}$$

- For a positively charged plane, with $\eta > 0$, the electric field points *away from* the plane on both sides of the plane.
- For a negatively charged plane, with $\eta < 0$, the electric field points *toward* the plane on both sides of the plane.

A Plane of Charge



$$\vec{E}_{\text{plane}} = \left(\frac{|\eta|}{2\epsilon_0}, \begin{cases} \text{away from plane if charge } + \\ \text{toward plane if charge } - \end{cases} \right)$$

Electric Fields

- Which shapes you should know how to calculate the electric fields f

1) point charge/sphere

$$E_{\text{point charge}} = k_e \frac{q}{r^2}$$

2) electric dipole

$$\left|\vec{E}_{dipole}\right| = \frac{1}{2\pi\varepsilon_o}\frac{p}{z^3}$$

3) infinite line of charge

$$E_y = \frac{1}{2\pi\varepsilon_o}\frac{\lambda}{z}$$

4) annular ring of charge

$$E_{ring} = \frac{1}{4\pi\varepsilon_o}\frac{qz}{\left(z^2+R^2\right)^{3/2}}$$

5) circular disk of charge

$$E_{disk} = \frac{\sigma}{2\varepsilon_o}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right)$$

6) infinite sheet of charge

$$E_{plane} = \frac{\sigma}{2\varepsilon_o}$$