Computing Essential of Some systems

- DES
- AES
- RSA
- DH
- ECC

Computing Essential of DES

- Feistel networks
  - Permutation operations (IP, Sapping left and right halves, PC)
  - XOR operations
  - Shift operations
  - Expansion operations (32 bits→48 bits)
  - S-box nonlinear substitution operations
    - Compression functions (48 bits→32 bits)
    - Boolean function f: $\{0,1\}^6 \rightarrow \{0,1\}^4$,
    - Bent function
    - Polynomial function: f: $GF(2^6) \rightarrow GF(2^4)$

AES

- Advanced Encryption Standard
- Initiated on April 15, 1997
- Rijndael was selected as the AES in Oct-2000
- issued as FIPS PUB 197 standard in Nov-2001(November 26, 2001)

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2. AES- Rijndael
3. Representation
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Origins

- clear a replacement for DES was needed
  - have theoretical attacks that can break it
  - have demonstrated exhaustive key search attacks
- can use Triple-DES – but slow with small blocks
- US NIST issued call for ciphers in 1997
- 15 candidates accepted in Jun 98
- 5 were shortlisted in Aug-99
- Rijndael was selected as the AES in Oct-2000
- issued as FIPS PUB 197 standard in Nov-2001

AES Requirements

- private key symmetric block cipher
- 128-bit data, 128/192/256-bit keys
- stronger & faster than Triple-DES
- active life of 20-30 years (+ archival use)
- provide full specification & design details
- both C & Java implementations
- NIST have released all submissions & unclassified analyses

AES Evaluation Criteria

- initial criteria:
  - security – effort to practically cryptanalyse
  - cost – computational
  - algorithm & implementation characteristics
- final criteria
  - general security
  - software & hardware implementation ease
  - implementation attacks
  - flexibility (in en/decrypt, keying, other factors)

AES Shortlist

- after testing and evaluation, shortlist in Aug-99:
  - MARS (IBM) - complex, fast, high security margin
  - RC6 (USA) - v. simple, v. fast, low security margin
  - Rijndael (Belgium) - clean, fast, good security margin
  - Serpent (Euro) - slow, clean, v. high security margin
  - Twofish (USA) - complex, v. fast, high security margin
- then subject to further analysis & comment
- saw contrast between algorithms with
  - few complex rounds verses many simple rounds
  - which refined existing ciphers verses new proposals

2. AES- Rijndael

The AES Cipher - Rijndael

- designed by Rijmen-Daemen in Belgium
- has 128/192/256 bit keys,
- 128 bit data
- an **iterative** rather than **feistel** cipher
  - treats data in 4 groups of 4 bytes
  - operates an entire block in every round
- designed to be:
  - resistant against known attacks
  - speed and code compactness on many CPUs
  - design simplicity

3. Representations

AES

Key length = 128 bits, $0 \le n < 16$;

Key length = 192 bits, $0 \le n < 24$;

Key length = 256 bits, $0 \le n < 32$.

Block length = 128 bits, $0 \le n < 16$;

AES

All byte values in the AES algorithm will be presented as the concatenation of its individual bit values (0 or 1) between braces in the order $\{b_7, b_6, b_5, b_4, b_3, b_2, b_1, b_0\}$. These bytes are interpreted as finite field elements using a polynomial representation:

$$b_7x^7 + b_6x^6 + b_5x^5 + b_4x^4 + b_3x^3 + b_2x^2 + b_1x + b_0 = \sum_{i=0}^{7} b_i x^i . \qquad (3.1)$$

For example, {01100011} identifies the specific finite field element $x^6 + x^5 + x + 1$.

AES

It is also convenient to denote byte values using hexadecimal notation with each of two groups of four bits being denoted by a single character as in Fig. 1.

Bit Pattern	Character
0000	0
0001	1
0010	2
0011	3

Bit Pattern	Character
0100	4
0101	5
0110	6
0111	7

Bit Pattern	Character
1000	8
1001	9
1010	a
1011	b

Bit Pattern	Character
1100	c
1101	d
1110	e
1111	f

Figure 1. Hexadecimal representation of bit patterns.

Hence the element {01100011} can be represented as {63}, where the character denoting the four-bit group containing the higher numbered bits is again to the left.

Some finite field operations involve one additional bit (b_8) to the left of an 8-bit byte. Where this extra bit is present, it will appear as '{01}' immediately preceding the 8-bit byte; for example, a 9-bit sequence will be presented as {01}{1b}.

Array of bytes

Arrays of bytes will be represented in the following form:

$$a_0 a_1 a_2 \ldots a_{15}$$

The bytes and the bit ordering within bytes are derived from the 128-bit input sequence

$$input_0\ input_1\ input_2 \ldots\ input_{126}\ input_{127}$$

as follows:

$$a_0 = \{input_0, input_1, \ldots, input_7\};$$

$$a_1 = \{input_8, input_9, \ldots, input_{15}\};$$

$$\vdots$$

$$a_{15} = \{input_{120}, input_{121}, \ldots, input_{127}\}.$$

The pattern can be extended to longer sequences (i.e., for 192- and 256-bit keys), so that, in general,

$$a_n = \{input_{8n}, input_{8n+1}, \ldots, input_{8n+7}\}. \qquad (3.2)$$

AES bytes and bits

Input bit sequence	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	...
Byte number	0								1								2								...
Bit numbers in byte	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	...

Figure 2. Indices for Bytes and Bits.

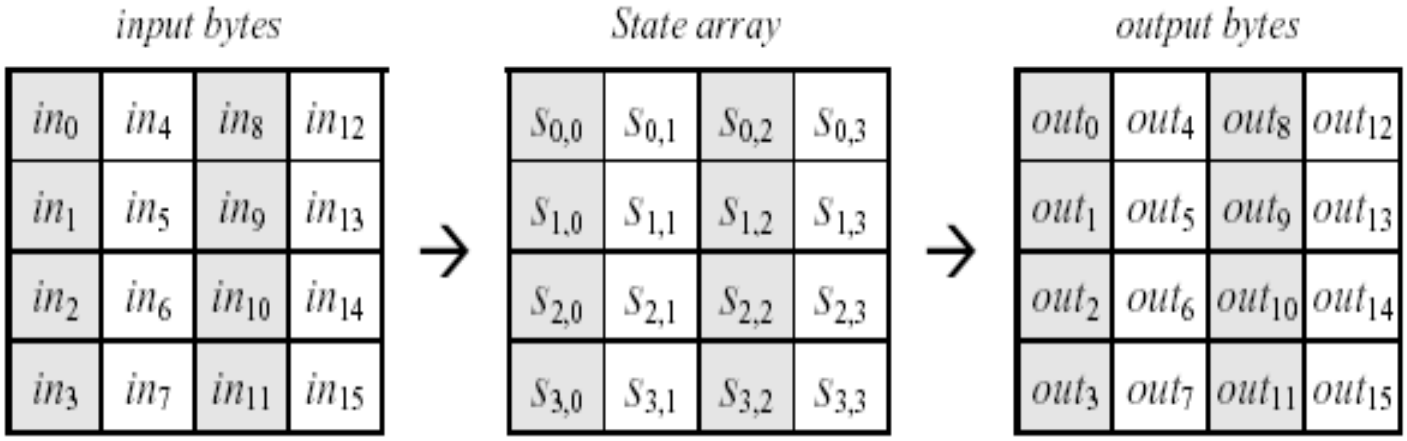


Figure 3. State array input and output.

Hence, at the beginning of the Cipher or Inverse Cipher, the input array, *in*, is copied to the State array according to the scheme:

$$s[r, c] = in[r + 4c] \qquad \text{for } 0 \le r < 4 \text{ and } 0 \le c < Nb, \qquad (3.3)$$

and at the end of the Cipher and Inverse Cipher, the State is copied to the output array *out* as follows:

$$out[r + 4c] = s[r, c] \qquad \text{for } 0 \le r < 4 \text{ and } 0 \le c < Nb. \qquad (3.4)$$

The State as an Array of Columns

The four bytes in each column of the State array form 32-bit **words**, where the row number r provides an index for the four bytes within each word. The state can hence be interpreted as a one-dimensional array of 32 bit words (columns), $w_0 \ldots w_3$, where the column number c provides an index into this array. Hence, for the example in Fig. 3, the State can be considered as an array of four words, as follows:

$$w_0 = s_{0,0}\, s_{1,0}\, s_{2,0}\, s_{3,0} \qquad w_2 = s_{0,2}\, s_{1,2}\, s_{2,2}\, s_{3,2}$$

$$w_1 = s_{0,1}\, s_{1,1}\, s_{2,1}\, s_{3,1} \qquad w_3 = s_{0,3}\, s_{1,3}\, s_{2,3}\, s_{3,3}\,. \qquad (3.5)$$

4. Mathematical operations

- Finite field

AES

- processes data as 4 groups of 4 bytes (state)
- has 9/11/13 rounds in which state undergoes:
  - byte substitution (1 S-box used on every byte)
  - shift rows (permute bytes between groups/columns)
  - mix columns (subs using matrix multiply of groups)
  - add round key (XOR state with key material)
- initial XOR key material & incomplete last round
- all operations can be combined into XOR and table lookups - hence very fast & efficient

4.1 Addition

The addition of two elements in a finite field is achieved by "adding" the coefficients for the corresponding powers in the polynomials for the two elements. The addition is performed with the XOR operation (denoted by $\oplus$) - i.e., modulo 2 - so that $1 \oplus 1 = 0$, $1 \oplus 0 = 1$, and $0 \oplus 0 = 0$. Consequently, subtraction of polynomials is identical to addition of polynomials.

Alternatively, addition of finite field elements can be described as the modulo 2 addition of corresponding bits in the byte. For two bytes $\{a_7 a_6 a_5 a_4 a_3 a_2 a_1 a_0\}$ and $\{b_7 b_6 b_5 b_4 b_2 b_2 b_1 b_0\}$, the sum is $\{c_7 c_6 c_5 c_4 c_3 c_2 c_1 c_0\}$, where each $c_i = a_i \oplus b_i$ (i.e., $c_7 = a_7 \oplus b_7$, $c_6 = a_6 \oplus b_6$, ... $c_0 = a_0 \oplus b_0$).

For example, the following expressions are equivalent to one another:

$$(x^6 + x^4 + x^2 + x + 1) + (x^7 + x + 1) = x^7 + x^6 + x^4 + x^2 \qquad \text{(polynomial notation);}$$

$$\{01010111\} \oplus \{10000011\} = \{11010100\} \qquad \text{(binary notation);}$$

$$\{57\} \oplus \{83\} = \{d4\} \qquad \text{(hexadecimal notation).}$$

4.2 Multiplication

In the polynomial representation, multiplication in GF(2^8) (denoted by •) corresponds with the multiplication of polynomials modulo an **irreducible polynomial** of degree 8. A polynomial is irreducible if its only divisors are one and itself. **For the AES algorithm, this irreducible polynomial is**

$$m(x) = x^8 + x^4 + x^3 + x + 1, \tag{4.1}$$

or {01}{1b} in hexadecimal notation.

For example, {57} • {83} = {c1}, because

$$\begin{aligned}(x^6 + x^4 + x^2 + x + 1)\,(x^7 + x + 1) &= x^{13} + x^{11} + x^9 + x^8 + x^7 + \\ &\quad x^7 + x^5 + x^3 + x^2 + x + \\ &\quad x^6 + x^4 + x^2 + x + 1 \\ &= x^{13} + x^{11} + x^9 + x^8 + x^6 + x^5 + x^4 + x^3 + 1\end{aligned}$$

and

$$\begin{aligned}x^{13} + x^{11} + x^9 + x^8 + x^6 + x^5 + x^4 + x^3 + 1 \ \text{modulo}\ (x^8 + x^4 + x^3 + x + 1) \\ = x^7 + x^6 + 1.\end{aligned}$$

The modular reduction by $m(x)$ ensures that the result will be a binary polynomial of degree less than 8, and thus can be represented by a byte. Unlike addition, there is no simple operation at the byte level that corresponds to this multiplication.

The multiplication defined above is associative, and the element {01} is the multiplicative identity. For any non-zero binary polynomial $b(x)$ of degree less than 8, the multiplicative inverse of $b(x)$, denoted $b^{-1}(x)$, can be found as follows: the extended Euclidean algorithm [7] is used to compute polynomials $a(x)$ and $c(x)$ such that

$$b(x)a(x) + m(x)c(x) = 1. \tag{4.2}$$

Hence, $a(x) \bullet b(x) \bmod m(x) = 1$, which means

$$b^{-1}(x) = a(x) \bmod m(x). \tag{4.3}$$

Moreover, for any $a(x)$, $b(x)$ and $c(x)$ in the field, it holds that

$$a(x) \bullet (b(x) + c(x)) = a(x) \bullet b(x) + a(x) \bullet c(x).$$

It follows that the set of 256 possible byte values, with XOR used as addition and the multiplication defined as above, has the structure of the finite field GF(2^8).

4.2.1 Multiplication by x

Multiplying the binary polynomial defined in equation (3.1) with the polynomial x results in

$$b_7x^8 + b_6x^7 + b_5x^6 + b_4x^5 + b_3x^4 + b_2x^3 + b_1x^2 + b_0x\,. \qquad (4.4)$$

The result $x \bullet b(x)$ is obtained by reducing the above result modulo $m(x)$, as defined in equation (4.1). If $b_7 = 0$, the result is already in reduced form. If $b_7 = 1$, the reduction is accomplished by subtracting (i.e., XORing) the polynomial $m(x)$. It follows that multiplication by x (i.e., {00000010} or {02}) can be implemented at the byte level as a left shift and a subsequent conditional bitwise XOR with {1b}. This operation on bytes is denoted by `xtime()`. Multiplication by higher powers of x can be implemented by repeated application of `xtime()`. By adding intermediate results, multiplication by any constant can be implemented.

For example, {57} • {13} = {fe} because

$$\{57\} \bullet \{02\} = \text{xtime}(\{57\}) = \{ae\}$$

$$\{57\} \bullet \{04\} = \text{xtime}(\{ae\}) = \{47\}$$

$$\{57\} \bullet \{08\} = \text{xtime}(\{47\}) = \{8e\}$$

$$\{57\} \bullet \{10\} = \text{xtime}(\{8e\}) = \{07\},$$

thus,

$$\begin{aligned}\{57\} \bullet \{13\} &= \{57\} \bullet (\{01\} \oplus \{02\} \oplus \{10\}) \\ &= \{57\} \oplus \{ae\} \oplus \{07\} \\ &= \{fe\}.\end{aligned}$$

4.3 Polynomials with Coefficients in GF(2^8)

Four-term polynomials can be defined - with coefficients that are finite field elements - as:

$$a(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0 \qquad (4.5)$$

which will be denoted as a word in the form $[a_0, a_1, a_2, a_3]$. Note that the polynomials in this section behave somewhat differently than the polynomials used in the definition of finite field elements, even though both types of polynomials use the same indeterminate, x. The coefficients in this section are themselves finite field elements, i.e., bytes, instead of bits; also, the multiplication of four-term polynomials uses a different reduction polynomial, defined below. The distinction should always be clear from the context.

To illustrate the addition and multiplication operations, let

$$b(x) = b_3 x^3 + b_2 x^2 + b_1 x + b_0 \qquad (4.6)$$

define a second four-term polynomial. Addition is performed by adding the finite field coefficients of like powers of x. This addition corresponds to an XOR operation between the corresponding bytes in each of the words – in other words, the XOR of the complete word values.

Thus, using the equations of (4.5) and (4.6),

$$a(x) + b(x) = (a_3 \oplus b_3)x^3 + (a_2 \oplus b_2)x^2 + (a_1 \oplus b_1)x + (a_0 \oplus b_0) \qquad (4.7)$$

Multiplication is achieved in two steps. In the first step, the polynomial product $c(x) = a(x) \bullet b(x)$ is algebraically expanded, and like powers are collected to give

$$c(x) = c_6 x^6 + c_5 x^5 + c_4 x^4 + c_3 x^3 + c_2 x^2 + c_1 x + c_0 \tag{4.8}$$

where

$$\begin{aligned} c_0 &= a_0 \bullet b_0 & c_4 &= a_3 \bullet b_1 \oplus a_2 \bullet b_2 \oplus a_1 \bullet b_3 \\ c_1 &= a_1 \bullet b_0 \oplus a_0 \bullet b_1 & c_5 &= a_3 \bullet b_2 \oplus a_2 \bullet b_3 \\ c_2 &= a_2 \bullet b_0 \oplus a_1 \bullet b_1 \oplus a_0 \bullet b_2 & c_6 &= a_3 \bullet b_3 \end{aligned} \tag{4.9}$$

$$c_3 = a_3 \bullet b_0 \oplus a_2 \bullet b_1 \oplus a_1 \bullet b_2 \oplus a_0 \bullet b_3 .$$

The result, $c(x)$, does not represent a four-byte word. Therefore, the second step of the multiplication is to reduce $c(x)$ modulo a polynomial of degree 4; the result can be reduced to a polynomial of degree less than 4. **For the AES algorithm, this is accomplished with the polynomial $x^4 + 1$**, so that

$$x^j \bmod (x^4 + 1) = x^{j \bmod 4} . \tag{4.10}$$

The modular product of $a(x)$ and $b(x)$, denoted by $a(x) \otimes b(x)$, is given by the four-term polynomial $d(x)$, defined as follows:

$$d(x) = d_3 x^3 + d_2 x^2 + d_1 x + d_0 \tag{4.11}$$

with

$$\begin{aligned} d_0 &= (a_0 \bullet b_0) \oplus (a_3 \bullet b_1) \oplus (a_2 \bullet b_2) \oplus (a_1 \bullet b_3) \\ d_1 &= (a_1 \bullet b_0) \oplus (a_0 \bullet b_1) \oplus (a_3 \bullet b_2) \oplus (a_2 \bullet b_3) \\ d_2 &= (a_2 \bullet b_0) \oplus (a_1 \bullet b_1) \oplus (a_0 \bullet b_2) \oplus (a_3 \bullet b_3) \\ d_3 &= (a_3 \bullet b_0) \oplus (a_2 \bullet b_1) \oplus (a_1 \bullet b_2) \oplus (a_0 \bullet b_3) \end{aligned} \tag{4.12}$$

When $a(x)$ is a fixed polynomial, the operation defined in equation (4.11) can be written in matrix form as:

$$\begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} a_0 & a_3 & a_2 & a_1 \\ a_1 & a_0 & a_3 & a_2 \\ a_2 & a_1 & a_0 & a_3 \\ a_3 & a_2 & a_1 & a_0 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{4.13}$$

Because $x^4 + 1$ is not an irreducible polynomial over GF(2^8), multiplication by a fixed four-term polynomial is not necessarily invertible. However, the AES algorithm specifies a fixed four-term polynomial that *does* have an inverse (see Sec. 5.1.3 and Sec. 5.3.3):

$$a(x) = \{03\}x^3 + \{01\}x^2 + \{01\}x + \{02\} \tag{4.14}$$

$$a^{-1}(x) = \{0b\}x^3 + \{0d\}x^2 + \{09\}x + \{0e\}. \tag{4.15}$$

Another polynomial used in the AES algorithm (see the `RotWord()` function in Sec. 5.2) has $a_0 = a_1 = a_2 = \{00\}$ and $a_3 = \{01\}$, which is the polynomial x^3. Inspection of equation (4.13) above will show that its effect is to form the output word by rotating bytes in the input word. This means that $[b_0, b_1, b_2, b_3]$ is transformed into $[b_1, b_2, b_3, b_0]$.

5. Algorithm specification

5. Algorithm Specification

For the AES algorithm, **the length of the input block, the output block and the State is 128 bits.** This is represented by $Nb = 4$, which reflects the number of 32-bit words (number of columns) in the State.

For the AES algorithm, **the length of the Cipher Key, *K*, is 128, 192, or 256 bits.** The key length is represented by $Nk = 4, 6, \text{or } 8$, which reflects the number of 32-bit words (number of columns) in the Cipher Key.

For the AES algorithm, the number of rounds to be performed during the execution of the algorithm is dependent on the key size. The number of rounds is represented by Nr, where $Nr = 10$ when $Nk = 4$, $Nr = 12$ when $Nk = 6$, and $Nr = 14$ when $Nk = 8$.

The only Key-Block-Round combinations that conform to this standard are given in Fig. 4. For implementation issues relating to the key length, block size and number of rounds, see Sec. 6.3.

	Key Length *(Nk words)*	**Block Size** *(Nb words)*	**Number of Rounds** *(Nr)*
AES-128	4	4	10
AES-192	6	4	12
AES-256	8	4	14

Figure 4. Key-Block-Round Combinations.

For both its Cipher and Inverse Cipher, the AES algorithm uses a round function that is composed of four different byte-oriented transformations: 1) byte substitution using a substitution table (S-box), 2) shifting rows of the State array by different offsets, 3) mixing the data within each column of the State array, and 4) adding a Round Key to the State. These transformations (and their inverses) are described in Sec. 5.1.1-5.1.4 and 5.3.1-5.3.4.

The Cipher and Inverse Cipher are described in Sec. 5.1 and Sec. 5.3, respectively, while the Key Schedule is described in Sec. 5.2.

5.1 Cipher

At the start of the Cipher, the input is copied to the State array using the conventions described in Sec. 3.4. After an initial Round Key addition, the State array is transformed by implementing a round function 10, 12, or 14 times (depending on the key length), with the final round differing slightly from the first $Nr - 1$ rounds. The final State is then copied to the output as described in Sec. 3.4.

The round function is parameterized using a key schedule that consists of a one-dimensional array of four-byte words derived using the Key Expansion routine described in Sec. 5.2.

The Cipher is described in the pseudo code in Fig. 5. The individual transformations - `SubBytes()`, `ShiftRows()`, `MixColumns()`, and `AddRoundKey()` – process the State and are described in the following subsections. In Fig. 5, the array `w[]` contains the key schedule, which is described in Sec. 5.2.

As shown in Fig. 5, all *Nr* rounds are identical with the exception of the final round, which does not include the `MixColumns()` transformation.

```
Cipher(byte in[4*Nb], byte out[4*Nb], word w[Nb*(Nr+1)])
begin
   byte  state[4,Nb]

   state = in

   AddRoundKey(state, w[0, Nb-1])                    // See Sec. 5.1.4

   for round = 1 step 1 to Nr-1
      SubBytes(state)                                // See Sec. 5.1.1
      ShiftRows(state)                               // See Sec. 5.1.2
      MixColumns(state)                              // See Sec. 5.1.3
      AddRoundKey(state, w[round*Nb, (round+1)*Nb-1])
   end for

   SubBytes(state)
   ShiftRows(state)
   AddRoundKey(state, w[Nr*Nb, (Nr+1)*Nb-1])

   out = state
end
```

Figure 5. Pseudo Code for the Cipher.[1]

5.1.1 `SubBytes()` Transformation

The `SubBytes()` transformation is a non-linear byte substitution that operates independently on each byte of the State using a substitution table (S-box). This S-box (Fig. 7), which is invertible, is constructed by composing two transformations:

1. Take the multiplicative inverse in the finite field GF(2^8), described in Sec. 4.2; the element {00} is mapped to itself.
2. Apply the following affine transformation (over GF(2)):

$$b_i' = b_i \oplus b_{(i+4) \bmod 8} \oplus b_{(i+5) \bmod 8} \oplus b_{(i+6) \bmod 8} \oplus b_{(i+7) \bmod 8} \oplus c_i \qquad (5.1)$$

for $0 \le i < 8$, where b_i is the i^{th} bit of the byte, and c_i is the i^{th} bit of a byte c with the value {63} or {01100011}. Here and elsewhere, a prime on a variable (e.g., b') indicates that the variable is to be updated with the value on the right.

In matrix form, the affine transformation element of the S-box can be expressed as:

$$\begin{bmatrix} b_0' \\ b_1' \\ b_2' \\ b_3' \\ b_4' \\ b_5' \\ b_6' \\ b_7' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}. \qquad (5.2)$$

Figure 6 illustrates the effect of the `SubBytes()` transformation on the State.

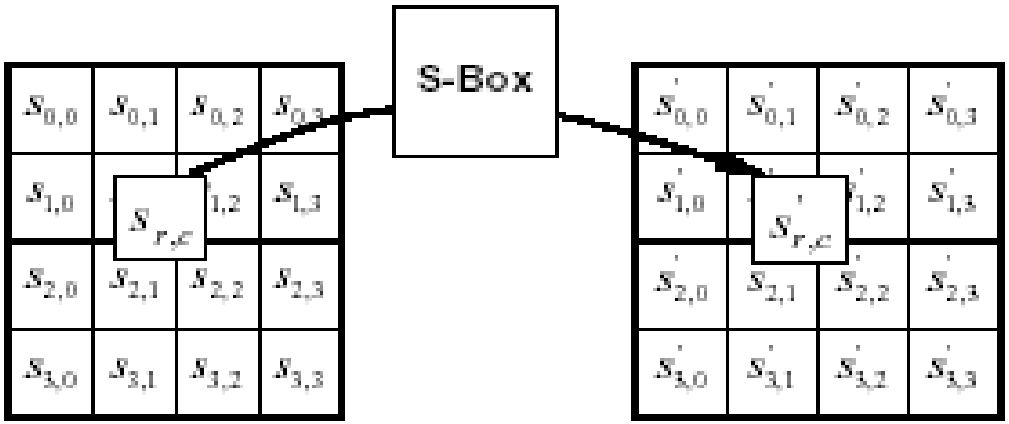


Figure 6. `SubBytes()` applies the S-box to each byte of the State.

The S-box used in the `SubBytes()` transformation is presented in hexadecimal form in Fig. 7. For example, if $s_{1,1} = \{53\}$, then the substitution value would be determined by the intersection of the row with index '5' and the column with index '3' in Fig. 7. This would result in $s'_{1,1}$ having a value of $\{ed\}$.

		y															
		0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
x	0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
	1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
	2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
	3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
	4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
	5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
	6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
	7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
	8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
	9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
	a	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
	b	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
	c	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
	d	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
	e	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
	f	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

Figure 7. S-box: substitution values for the byte xy (in hexadecimal format).

5.1.2 `ShiftRows()` Transformation

In the `ShiftRows()` transformation, the bytes in the last three rows of the State are cyclically shifted over different numbers of bytes (offsets). The first row, $r = 0$, is not shifted.

Specifically, the `ShiftRows()` transformation proceeds as follows:

$$s'_{r,c} = s_{r,(c+shift(r,Nb)) \bmod Nb} \quad \text{for } 0 < r < 4 \quad \text{and} \quad 0 \le c < Nb, \tag{5.3}$$

where the shift value $shift(r,Nb)$ depends on the row number, r, as follows (recall that $Nb = 4$):

$$shift(1,4) = 1; \quad shift(2,4) = 2; \quad shift(3,4) = 3. \tag{5.4}$$

This has the effect of moving bytes to "lower" positions in the row (i.e., lower values of c in a given row), while the "lowest" bytes wrap around into the "top" of the row (i.e., higher values of c in a given row).

Figure 8 illustrates the `ShiftRows()` transformation.

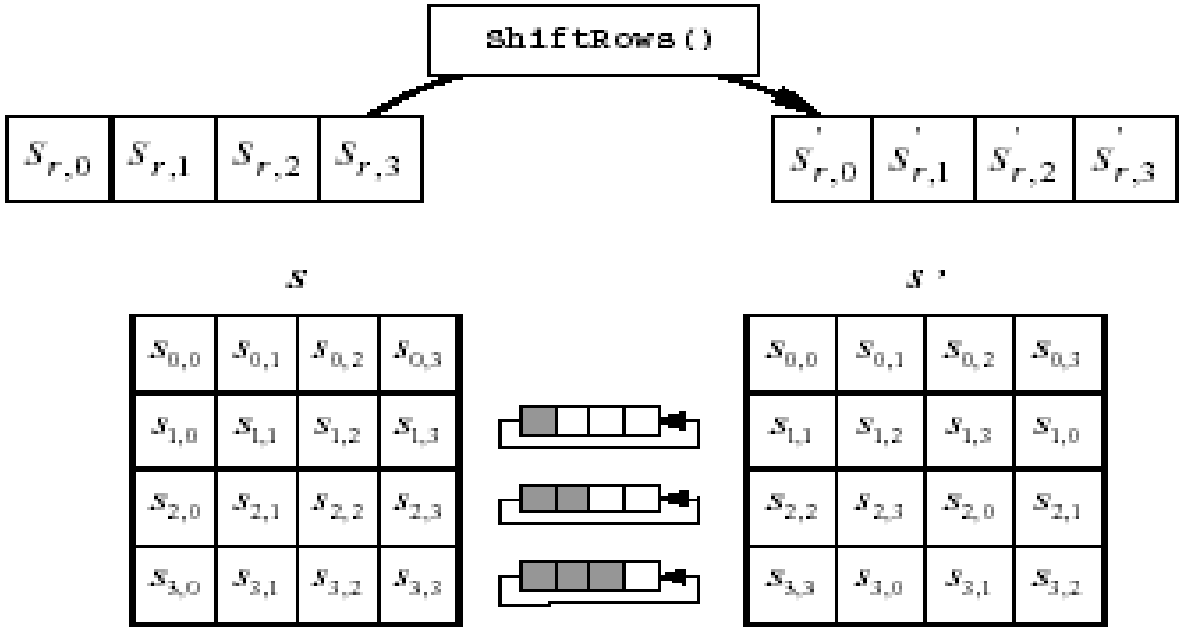


Figure 8. `ShiftRows()` cyclically shifts the last three rows in the State.

5.1.3 `MixColumns()` Transformation

The `MixColumns()` transformation operates on the State column-by-column, treating each column as a four-term polynomial as described in Sec. 4.3. The columns are considered as polynomials over GF(2^8) and multiplied modulo $x^4 + 1$ with a fixed polynomial $a(x)$, given by

$$a(x) = \{03\}x^3 + \{01\}x^2 + \{01\}x + \{02\}. \qquad (5.5)$$

As described in Sec. 4.3, this can be written as a matrix multiplication. Let

$s'(x) = a(x) \otimes s(x)$:

$$\begin{bmatrix} s'_{0,c} \\ s'_{1,c} \\ s'_{2,c} \\ s'_{3,c} \end{bmatrix} = \begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,c} \\ s_{1,c} \\ s_{2,c} \\ s_{3,c} \end{bmatrix} \quad \text{for } 0 \le c < Nb. \qquad (5.6)$$

As a result of this multiplication, the four bytes in a column are replaced by the following:

$$s'_{0,c} = (\{02\} \bullet s_{0,c}) \oplus (\{03\} \bullet s_{1,c}) \oplus s_{2,c} \oplus s_{3,c}$$

$$s'_{1,c} = s_{0,c} \oplus (\{02\} \bullet s_{1,c}) \oplus (\{03\} \bullet s_{2,c}) \oplus s_{3,c}$$

$$s'_{2,c} = s_{0,c} \oplus s_{1,c} \oplus (\{02\} \bullet s_{2,c}) \oplus (\{03\} \bullet s_{3,c})$$

$$s'_{3,c} = (\{03\} \bullet s_{0,c}) \oplus s_{1,c} \oplus s_{2,c} \oplus (\{02\} \bullet s_{3,c}).$$

Figure 9 illustrates the `MixColumns()` transformation.

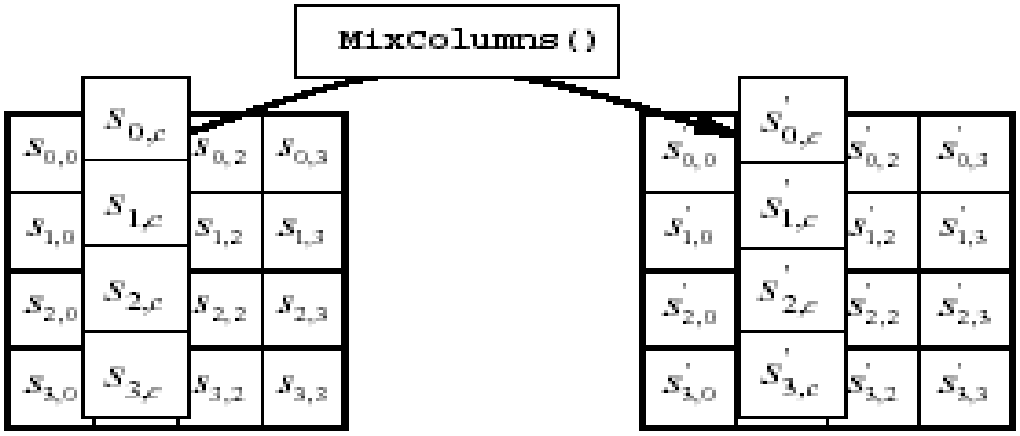


Figure 9. `MixColumns()` operates on the State column-by-column.

5.1.4 AddRoundKey() Transformation

In the AddRoundKey() transformation, a Round Key is added to the State by a simple bitwise XOR operation. Each Round Key consists of *Nb* words from the key schedule (described in Sec. 5.2). Those *Nb* words are each added into the columns of the State, such that

$$[s'_{0,c}, s'_{1,c}, s'_{2,c}, s'_{3,c}] = [s_{0,c}, s_{1,c}, s_{2,c}, s_{3,c}] \oplus [w_{round*Nb+c}] \quad \text{for } 0 \le c < Nb, \qquad (5.7)$$

where $[w_i]$ are the key schedule words described in Sec. 5.2, and *round* is a value in the range $0 \le round \le Nr$. In the Cipher, the initial Round Key addition occurs when $round = 0$, prior to the first application of the round function (see Fig. 5). The application of the AddRoundKey() transformation to the *Nr* rounds of the Cipher occurs when $1 \le round \le Nr$.

The action of this transformation is illustrated in Fig. 10, where $l = round * Nb$. The byte address within words of the key schedule was described in Sec. 3.1.

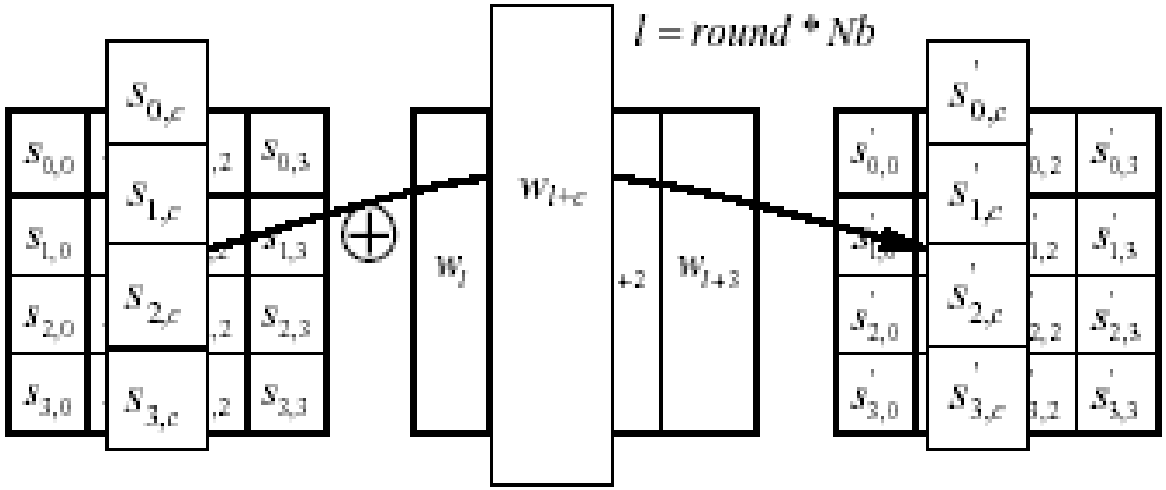


Figure 10. `AddRoundKey()` XORs each column of the State with a word from the key schedule.

5.2 Key Expansion

The AES algorithm takes the Cipher Key, K, and performs a Key Expansion routine to generate a key schedule. The Key Expansion generates a total of $Nb\ (Nr + 1)$ words: the algorithm requires an initial set of Nb words, and each of the Nr rounds requires Nb words of key data. The resulting key schedule consists of a linear array of 4-byte words, denoted $[w_i]$, with i in the range $0 \le i < Nb(Nr + 1)$.

The expansion of the input key into the key schedule proceeds according to the pseudo code in Fig. 11.

`SubWord()` is a function that takes a four-byte input word and applies the S-box (Sec. 5.1.1, Fig. 7) to each of the four bytes to produce an output word. The function `RotWord()` takes a word $[a_0,a_1,a_2,a_3]$ as input, performs a cyclic permutation, and returns the word $[a_1,a_2,a_3,a_0]$. The round constant word array, `Rcon[i]`, contains the values given by $[x^{i-1},\{00\},\{00\},\{00\}]$, with x^{i-1} being powers of x (x is denoted as $\{02\}$) in the field GF(2^8), as discussed in Sec. 4.2 (note that i starts at 1, not 0).

From Fig. 11, it can be seen that the first Nk words of the expanded key are filled with the Cipher Key. Every following word, `w[i]`, is equal to the XOR of the previous word, `w[i-1]`, and the word Nk positions earlier, `w[i-Nk]`. For words in positions that are a multiple of Nk, a transformation is applied to `w[i-1]` prior to the XOR, followed by an XOR with a round constant, `Rcon[i]`. This transformation consists of a cyclic shift of the bytes in a word (`RotWord()`), followed by the application of a table lookup to all four bytes of the word (`SubWord()`).

It is important to note that the Key Expansion routine for 256-bit Cipher Keys ($Nk = 8$) is slightly different than for 128- and 192-bit Cipher Keys. If $Nk = 8$ and `i-4` is a multiple of Nk, then `SubWord()` is applied to `w[i-1]` prior to the XOR.

```
KeyExpansion(byte key[4*Nk], word w[Nb*(Nr+1)], Nk)
begin
    word  temp

    i = 0

    while (i < Nk)
        w[i] = word(key[4*i], key[4*i+1], key[4*i+2], key[4*i+3])
        i = i+1
    end while

    i = Nk

    while (i < Nb * (Nr+1)]
        temp = w[i-1]
        if (i mod Nk = 0)
            temp = SubWord(RotWord(temp)) xor Rcon[i/Nk]
        else if (Nk > 6 and i mod Nk = 4)
            temp = SubWord(temp)
        end if
        w[i] = w[i-Nk] xor temp
        i = i + 1
    end while
end

Note that Nk=4, 6, and 8 do not all have to be implemented;
they are all included in the conditional statement above for
conciseness.   Specific implementation requirements for the
Cipher Key are presented in Sec. 6.1.
```

Figure 11. Pseudo Code for Key Expansion.[2]

[2] The functions `SubWord()` and `RotWord()` return a result that is a transformation of the function input, whereas the transformations in the Cipher and Inverse Cipher (e.g., `ShiftRows()`, `SubBytes()`, etc.) transform the State array that is addressed by the '`state`' pointer.

5.3 Inverse Cipher

The Cipher transformations in Sec. 5.1 can be inverted and then implemented in reverse order to produce a straightforward Inverse Cipher for the AES algorithm. The individual transformations used in the Inverse Cipher - `InvShiftRows()`, `InvSubBytes()`,`InvMixColumns()`, and `AddRoundKey()` – process the State and are described in the following subsections.

The Inverse Cipher is described in the pseudo code in Fig. 12. In Fig. 12, the array `w[]` contains the key schedule, which was described previously in Sec. 5.2.

```
InvCipher(byte in[4*Nb], byte out[4*Nb], word w[Nb*(Nr+1)])
begin
   byte  state[4,Nb]

   state = in

   AddRoundKey(state, w[Nr*Nb, (Nr+1)*Nb-1]) // See Sec. 5.1.4

   for round = Nr-1 step -1 downto 1
      InvShiftRows(state)                    // See Sec. 5.3.1
      InvSubBytes(state)                     // See Sec. 5.3.2
      AddRoundKey(state, w[round*Nb, (round+1)*Nb-1])
      InvMixColumns(state)                   // See Sec. 5.3.3
   end for

   InvShiftRows(state)
   InvSubBytes(state)
   AddRoundKey(state, w[0, Nb-1])

   out = state
end
```

Figure 12. Pseudo Code for the Inverse Cipher.[3]

---

[3] The various transformations (e.g., `InvSubBytes()`, `InvShiftRows()`, etc.) act upon the State array that is addressed by the '`state`' pointer. `AddRoundKey()` uses an additional pointer to address the Round Key.

5.3.1 `InvShiftRows()` Transformation

`InvShiftRows()` is the inverse of the `ShiftRows()` transformation. The bytes in the last three rows of the State are cyclically shifted over different numbers of bytes (offsets). The first row, $r = 0$, is not shifted. The bottom three rows are cyclically shifted by $Nb - shift(r, Nb)$ bytes, where the shift value *shift(r,Nb)* depends on the row number, and is given in equation (5.4) (see Sec. 5.1.2).

Specifically, the `InvShiftRows()` transformation proceeds as follows:

$$s'_{r,(c+shift(r,Nb)) \bmod Nb} = s_{r,c} \quad \text{for } 0 < r < 4 \quad \text{and} \quad 0 \le c < Nb \tag{5.8}$$

Figure 13 illustrates the `InvShiftRows()` transformation.

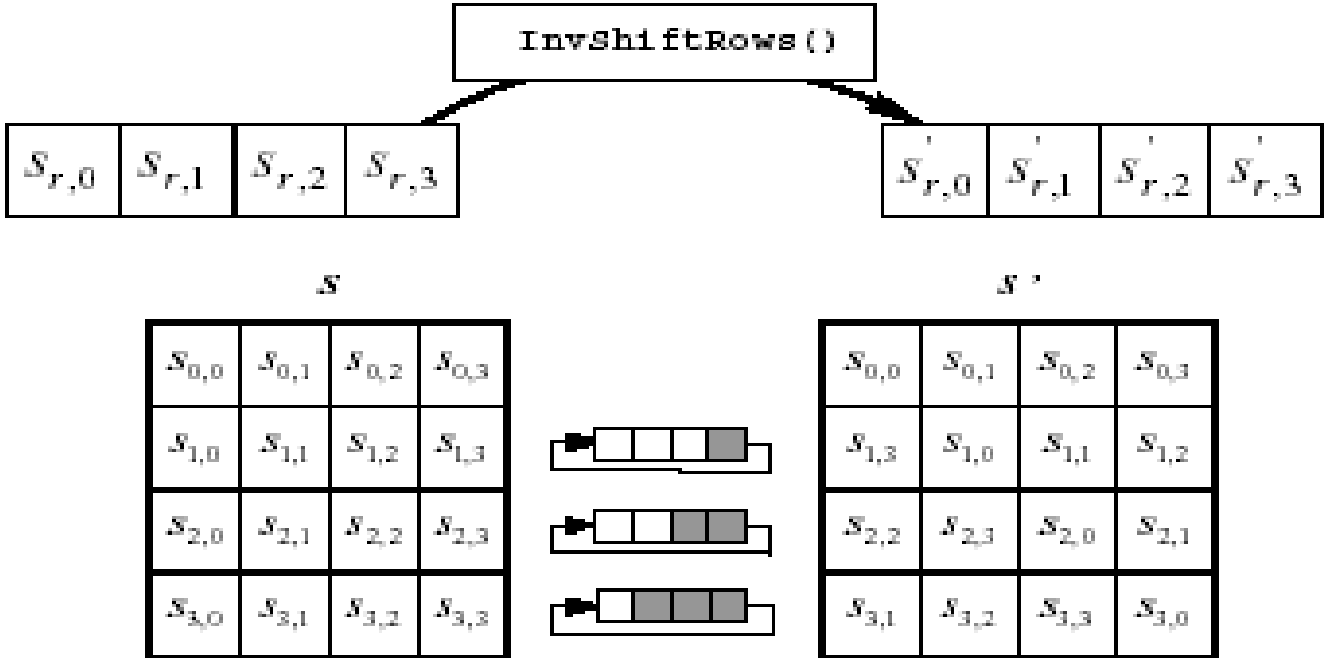


Figure 13. `InvShiftRows()` cyclically shifts the last three rows in the State.

5.3.2 `InvSubBytes()` Transformation

`InvSubBytes()` is the inverse of the byte substitution transformation, in which the inverse S-box is applied to each byte of the State. This is obtained by applying the inverse of the affine transformation (5.1) followed by taking the multiplicative inverse in $GF(2^8)$.

The inverse S-box used in the `InvSubBytes()` transformation is presented in Fig. 14:

		y															
		0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
x	0	52	09	6a	d5	30	36	a5	38	bf	40	a3	9e	81	f3	d7	fb
	1	7c	e3	39	82	9b	2f	ff	87	34	8e	43	44	c4	de	e9	cb
	2	54	7b	94	32	a6	c2	23	3d	ee	4c	95	0b	42	fa	c3	4e
	3	08	2e	a1	66	28	d9	24	b2	76	5b	a2	49	6d	8b	d1	25
	4	72	f8	f6	64	86	68	98	16	d4	a4	5c	cc	5d	65	b6	92
	5	6c	70	48	50	fd	ed	b9	da	5e	15	46	57	a7	8d	9d	84
	6	90	d8	ab	00	8c	bc	d3	0a	f7	e4	58	05	b8	b3	45	06
	7	d0	2c	1e	8f	ca	3f	0f	02	c1	af	bd	03	01	13	8a	6b
	8	3a	91	11	41	4f	67	dc	ea	97	f2	cf	ce	f0	b4	e6	73
	9	96	ac	74	22	e7	ad	35	85	e2	f9	37	e8	1c	75	df	6e
	a	47	f1	1a	71	1d	29	c5	89	6f	b7	62	0e	aa	18	be	1b
	b	fc	56	3e	4b	c6	d2	79	20	9a	db	c0	fe	78	cd	5a	f4
	c	1f	dd	a8	33	88	07	c7	31	b1	12	10	59	27	80	ec	5f
	d	60	51	7f	a9	19	b5	4a	0d	2d	e5	7a	9f	93	c9	9c	ef
	e	a0	e0	3b	4d	ae	2a	f5	b0	c8	eb	bb	3c	83	53	99	61
	f	17	2b	04	7e	ba	77	d6	26	e1	69	14	63	55	21	0c	7d

Figure 14. Inverse S-box: substitution values for the byte xy (in hexadecimal format).

5.3.3 `InvMixColumns()` Transformation

`InvMixColumns()` is the inverse of the `MixColumns()` transformation. `InvMixColumns()` operates on the State column-by-column, treating each column as a four-term polynomial as described in Sec. 4.3. The columns are considered as polynomials over GF(2^8) and multiplied modulo $x^4 + 1$ with a fixed polynomial $a^{-1}(x)$, given by

$$a^{-1}(x) = \{0b\}x^3 + \{0d\}x^2 + \{09\}x + \{0e\}. \tag{5.9}$$

As described in Sec. 4.3, this can be written as a matrix multiplication. Let

$s'(x) = a^{-1}(x) \otimes s(x)$:

$$\begin{bmatrix} s'_{0,c} \\ s'_{1,c} \\ s'_{2,c} \\ s'_{3,c} \end{bmatrix} = \begin{bmatrix} 0e & 0b & 0d & 09 \\ 09 & 0e & 0b & 0d \\ 0d & 09 & 0e & 0b \\ 0b & 0d & 09 & 0e \end{bmatrix} \begin{bmatrix} s_{0,c} \\ s_{1,c} \\ s_{2,c} \\ s_{3,c} \end{bmatrix} \quad \text{for } 0 \le c < Nb. \tag{5.10}$$

As a result of this multiplication, the four bytes in a column are replaced by the following:

$$s'_{0,c} = (\{0e\} \bullet s_{0,c}) \oplus (\{0b\} \bullet s_{1,c}) \oplus (\{0d\} \bullet s_{2,c}) \oplus (\{09\} \bullet s_{3,c})$$

$$s'_{1,c} = (\{09\} \bullet s_{0,c}) \oplus (\{0e\} \bullet s_{1,c}) \oplus (\{0b\} \bullet s_{2,c}) \oplus (\{0d\} \bullet s_{3,c})$$

$$s'_{2,c} = (\{0d\} \bullet s_{0,c}) \oplus (\{09\} \bullet s_{1,c}) \oplus (\{0e\} \bullet s_{2,c}) \oplus (\{0b\} \bullet s_{3,c})$$

$$s'_{3,c} = (\{0b\} \bullet s_{0,c}) \oplus (\{0d\} \bullet s_{1,c}) \oplus (\{09\} \bullet s_{2,c}) \oplus (\{0e\} \bullet s_{3,c})$$

5.3.4 Inverse of the `AddRoundKey()` Transformation

`AddRoundKey()`, which was described in Sec. 5.1.4, is its own inverse, since it only involves an application of the XOR operation.

5.3.5 Equivalent Inverse Cipher

In the straightforward Inverse Cipher presented in Sec. 5.3 and Fig. 12, the sequence of the transformations differs from that of the Cipher, while the form of the key schedules for encryption and decryption remains the same. However, several properties of the AES algorithm allow for an Equivalent Inverse Cipher that has the same sequence of transformations as the Cipher (with the transformations replaced by their inverses). This is accomplished with a change in the key schedule.

The two properties that allow for this Equivalent Inverse Cipher are as follows:

1. The `SubBytes()` and `ShiftRows()` transformations commute; that is, a `SubBytes()` transformation immediately followed by a `ShiftRows()` transformation is equivalent to a `ShiftRows()` transformation immediately followed buy a `SubBytes()` transformation. The same is true for their inverses, `InvSubBytes()` and `InvShiftRows`.

2. The column mixing operations - `MixColumns()` and `InvMixColumns()` - are linear with respect to the column input, which means

```
InvMixColumns(state XOR Round Key) =
                InvMixColumns(state) XOR InvMixColumns(Round Key).
```

These properties allow the order of `InvSubBytes()` and `InvShiftRows()` transformations to be reversed. The order of the `AddRoundKey()` and `InvMixColumns()` transformations can also be reversed, provided that the columns (words) of the decryption key schedule are modified using the `InvMixColumns()` transformation.

The equivalent inverse cipher is defined by reversing the order of the `InvSubBytes()` and `InvShiftRows()` transformations shown in Fig. 12, and by reversing the order of the `AddRoundKey()` and `InvMixColumns()` transformations used in the "round loop" after first modifying the decryption key schedule for *round* = 1 to *Nr*-1 using the `InvMixColumns()` transformation. The first and last *Nb* words of the decryption key schedule shall *not* be modified in this manner.

Given these changes, the resulting Equivalent Inverse Cipher offers a more efficient structure than the Inverse Cipher described in Sec. 5.3 and Fig. 12. Pseudo code for the Equivalent Inverse Cipher appears in Fig. 15. (The word array `dw[]` contains the modified decryption key schedule. The modification to the Key Expansion routine is also provided in Fig. 15.)

```
EqInvCipher(byte in[4*Nb], byte out[4*Nb], word dw[Nb*(Nr+1)])
begin
   byte  state[4,Nb]

   state = in

   AddRoundKey(state, dw[Nr*Nb, (Nr+1)*Nb-1])

   for round = Nr-1 step -1 downto 1
      InvSubBytes(state)
      InvShiftRows(state)
      InvMixColumns(state)
      AddRoundKey(state, dw[round*Nb, (round+1)*Nb-1])
   end for

   InvSubBytes(state)
   InvShiftRows(state)
   AddRoundKey(state, dw[0, Nb-1])

   out = state
end
```

For the Equivalent Inverse Cipher, the following pseudo code is added at the end of the Key Expansion routine (Sec. 5.2):

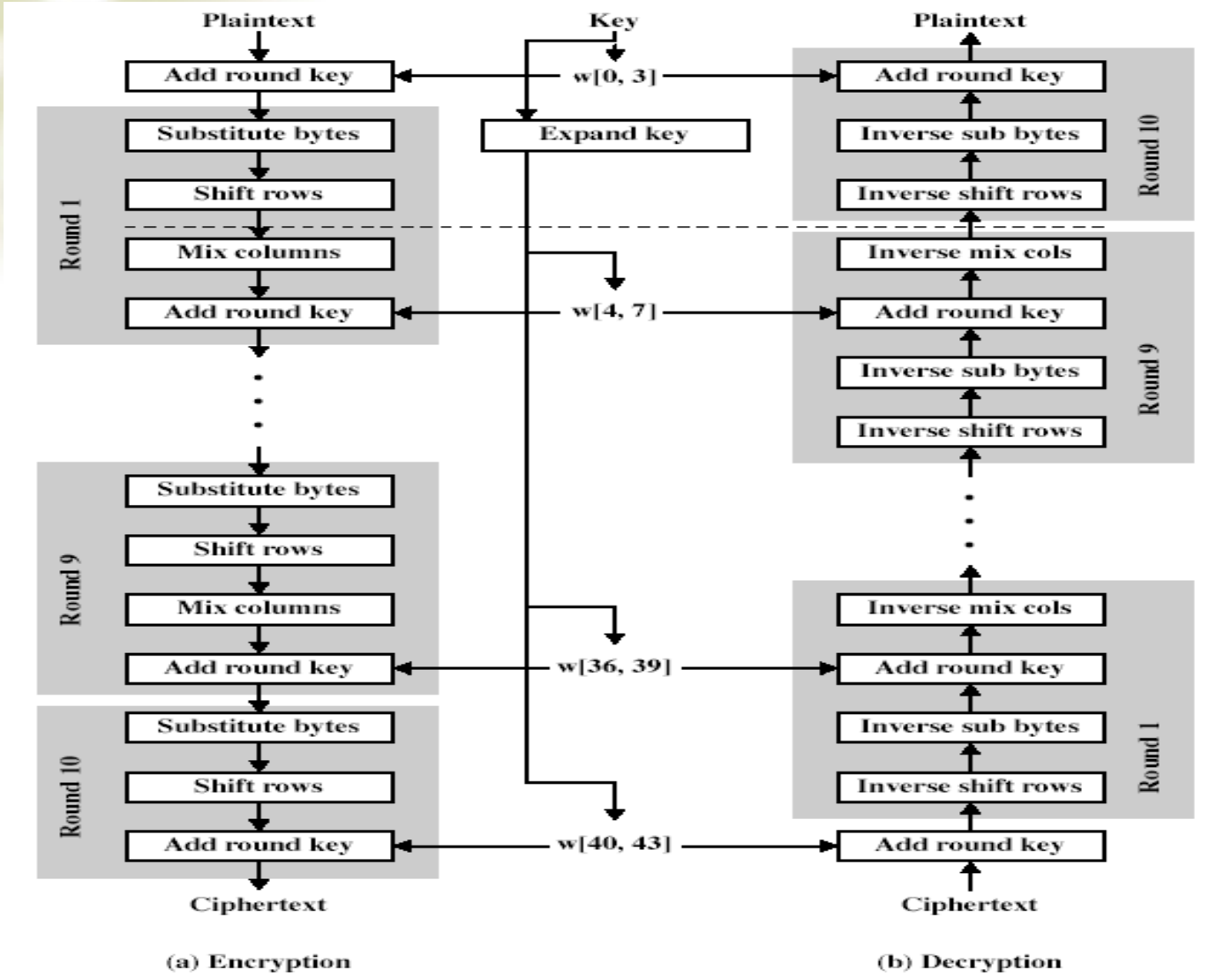
```
   for i = 0 step 1 to (Nr+1)*Nb-1
      dw[i] = w[i]
   end for


   for round = 1 step 1 to Nr-1
      InvMixColumns(dw[round*Nb, (round+1)*Nb-1])      //  note  change  of
type
   end for
```

Note that, since InvMixColumns operates on a two-dimensional array of bytes while the Round Keys are held in an array of words, the call to InvMixColumns in this code sequence involves a change of type (i.e. the input to InvMixColumns() is normally the State array, which is considered to be a two-dimensional array of bytes, whereas the input here is a Round Key computed as a one-dimensional array of words).

Figure 15. Pseudo Code for the Equivalent Inverse Cipher.

Rijndael


Plaintext
Key
Plaintext
Add round key
w[0, 3]
Add round key
Substitute bytes
Expand key
Inverse sub bytes
Shift rows
Inverse shift rows
Round 1
Round 10
Mix columns
Inverse mix cols
Add round key
w[4, 7]
Add round key
Inverse sub bytes
Round 9
Inverse shift rows
Substitute bytes
Shift rows
Inverse mix cols
Mix columns
Add round key
w[36, 39]
Add round key
Substitute bytes
Inverse sub bytes
Round 1
Shift rows
Inverse shift rows
Round 10
Add round key
w[40, 43]
Add round key
Ciphertext
Ciphertext
(a) Encryption
(b) Decryption

Byte Substitution

- a simple substitution of each byte
- uses one table of 16x16 bytes containing a permutation of all 256 8-bit values
- each byte of state is replaced by byte in row (left 4-bits) & column (right 4-bits)
  - eg. byte {95} is replaced by row 9 col 5 byte
  - which is the value {2A}
- S-box is constructed using a defined transformation of the values in GF(2^8)
- designed to be resistant to all known attacks

Shift Rows

- a circular byte shift in each each
  - 1st row is unchanged
  - 2nd row does 1 byte circular shift to left
  - 3rd row does 2 byte circular shift to left
  - 4th row does 3 byte circular shift to left
- decrypt does shifts to right
- since state is processed by columns, this step permutes bytes between the columns

Mix Columns

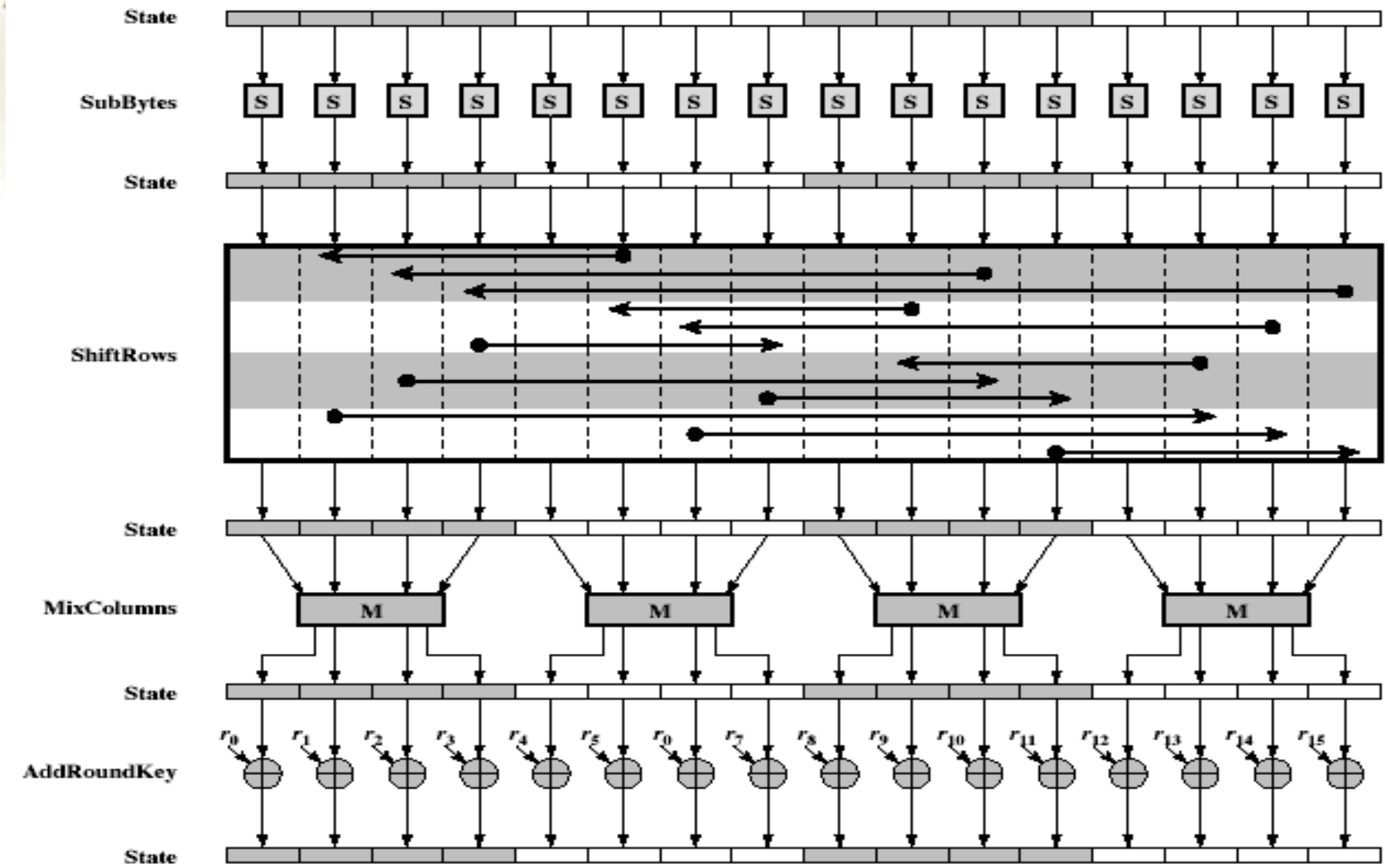
- each column is processed separately
- each byte is replaced by a value dependent on all 4 bytes in the column
- effectively a matrix multiplication in GF(2^8) using prime poly $m(x) = x^8+x^4+x^3+x+1$

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix}$$

Add Round Key

- XOR state with 128-bits of the round key
- again processed by column (though effectively a series of byte operations)
- inverse for decryption is identical since XOR is own inverse, just with correct round key
- designed to be as simple as possible

AES Round


State
SubBytes
S
State
ShiftRows
State
MixColumns
M
State
AddRoundKey
State

AES Key Expansion

- takes 128-bit (16-byte) key and expands into array of 44/52/60 32-bit words
- start by copying key into first 4 words
- then loop creating words that depend on values in previous & 4 places back
  - in 3 of 4 cases just XOR these together
  - every 4^{th} has S-box + rotate + XOR constant of previous before XOR together
- designed to resist known attacks

AES Decryption

- AES decryption is not identical to encryption since steps done in reverse
- but can define an equivalent inverse cipher with steps as for encryption
  - but using inverses of each step
  - with a different key schedule
- works since result is unchanged when
  - swap byte substitution & shift rows
  - swap mix columns & add (tweaked) round key

Implementation Aspects

- can efficiently implement on 8-bit CPU
  - byte substitution works on bytes using a table of 256 entries
  - shift rows is simple byte shifting
  - add round key works on byte XORs
  - mix columns requires matrix multiply in GF(2^8) which works on byte values, can be simplified to use a table lookup

Implementation Aspects

- can efficiently implement on 32-bit CPU
  - redefine steps to use 32-bit words
  - can precompute 4 tables of 256-words
  - then each column in each round can be computed using 4 table lookups + 4 XORs
  - at a cost of 16Kb to store tables
- designers believe this very efficient implementation was a key factor in its selection as the AES cipher

Summary

- have considered:
  - the AES selection process
  - the details of Rijndael – the AES cipher
  - looked at the steps in each round
  - the key expansion
  - implementation aspects