

Assignment 4: Digital circuit

Attention: Recommend using L^AT_EX to complete your work. You can use any tool, such as Logisim, Visio, Draw.io, PowerPoint, etc., to create diagrams. However, handwritten or hand-drawn content is not acceptable.

1 Combinational logic

Analyze the circuit shown in Fig. 1 and answer the following questions:

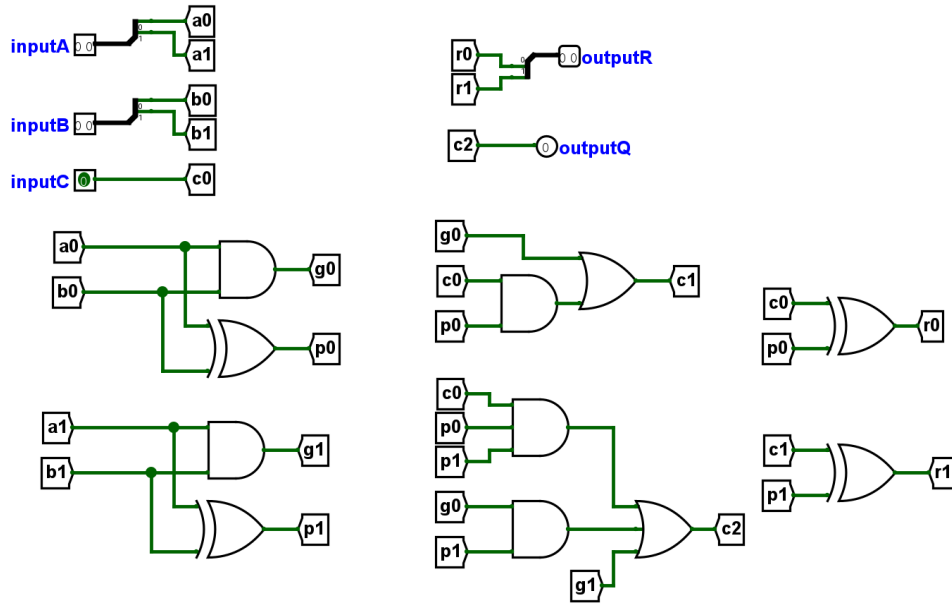


Figure 1: A 2-bit arithmetic circuit

- Draw the truth table of this circuit. [10 pt]
- Which kind of arithmetic operation (addition, subtraction, multiplication, division, shift, or comparison) is performed by this circuit? What are the advantages and disadvantages of the circuit in Fig. 1 compared to the corresponding arithmetic circuit mentioned in Digital circuits I? [10 pt]
- Assume that all 2-input logic gates have 1 ns delay, all 3-input logic gates have 2 ns delay, and other delays are not considered. Calculate the max delay of this circuit. [10 pt]

Answer to Question 1

(a)

A_0	A_1	B_0	B_1	c_0	g_0	g_1	p_0	p_1	c_1	c_2	r_0	r_1
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	1	0	0	0	1
0	0	0	1	1	0	0	0	1	0	0	1	1
0	0	1	0	0	0	0	1	0	0	0	1	0
0	0	1	0	1	0	0	1	0	1	0	0	1
0	0	1	1	0	0	0	1	1	0	0	1	1
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0	1	0	0	1	0	0	0	1	0	0	1	1
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0	1	0	1	1	0	1	0	0	0	1	1	0
0	1	1	0	0	0	0	1	1	0	0	1	1
0	1	1	0	1	0	0	1	1	1	1	0	0
0	1	1	1	0	0	1	1	0	0	1	1	0
0	1	1	1	1	0	1	1	0	1	1	0	1
1	0	0	0	0	0	0	1	0	0	0	1	0
1	0	0	0	1	0	0	1	0	1	0	0	1
1	0	0	1	0	0	0	1	1	0	0	1	1
1	0	0	1	1	0	0	1	1	1	1	0	0
1	0	1	0	0	1	0	0	0	1	0	0	1
1	0	1	0	1	1	0	0	0	1	0	1	1
1	0	1	1	0	1	0	0	1	1	1	0	0
1	0	1	1	1	1	0	0	1	1	1	1	0
1	1	0	0	0	0	0	1	1	0	0	1	1
1	1	0	0	1	0	0	1	1	1	1	0	0
1	1	0	1	0	0	1	1	0	0	1	1	0
1	1	0	1	1	0	1	1	0	1	1	0	1
1	1	1	0	0	1	0	0	1	1	1	0	0
1	1	1	0	1	1	0	0	1	1	1	1	0
1	1	1	1	0	1	1	0	0	1	1	0	1
1	1	1	1	1	1	1	0	0	1	1	1	1

(b) it is a addition operation.

Advantages: Shorter delay, capable of parallel computation across multiple bits;

Disadvantages: Requires more logic gates.

(c)

the longest circuit path is $c_2 \rightarrow 3\text{-or} \rightarrow 3\text{-and} \rightarrow p_0/p_1 \rightarrow 2\text{-xor}$

then the max delay is $2 \times 2 + 1 \times 1 = 5$

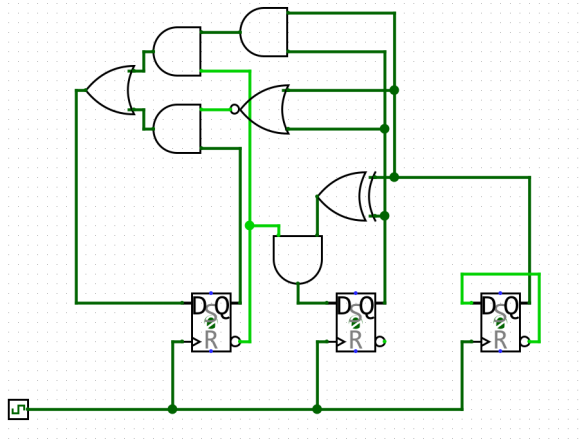
2 SDS

Draw a counter that counts from 0 to 5 using three D flip-flops (each flip-flop represents one output bit) and some 2-input logic gates (AND, OR, NOT). Please use the method taught in class to build a Moore FSM that implements the circular counter. Complete the state transition logic and output logic. [35 pt]

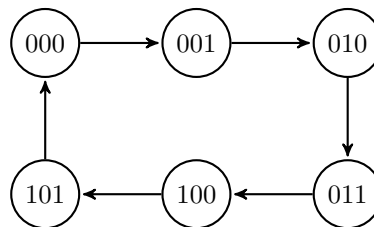


Figure 2: The counter cycles through the process of counting from 0 to 5.

Answer to Question 2



the FSM is:



a_{2t-1}	a_{1t-1}	a_{0t-1}	a_{2t}	a_{1t}	a_{0t}
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	0	0	0

⇒ after simplify, the output is:

$$a_0 = \overline{a_0}$$

$$a_1 = (a_0 \oplus a_1) \overline{a_2}$$

$$a_2 = a_0 a_1 \overline{a_2} + \overline{a_0 + a_1} a_2$$

3 Finite state machine

The function of a vending machine which sells bottles of soda is described below:

- Each bottle costs \$1.50.
- The machine only accepts \$0.50 and \$1 coins. If a customer inserts enough coins, the machine will dispense a bottle of soda (FSM will output “1”, otherwise “0”) and returns change if needed , e.g., the output of DISPENSE states may be “1 \$0.5”, other states’ output may be “0 \$0”.
- The process happens one coin at a time, and there is no simultaneous insertion of multiple coins or shipping of multiple bottles. After each transaction, the vending machine enters the IDLE state.
- We don’t need to account for a scenario where a customer inserts coins but decides not to make a purchase.

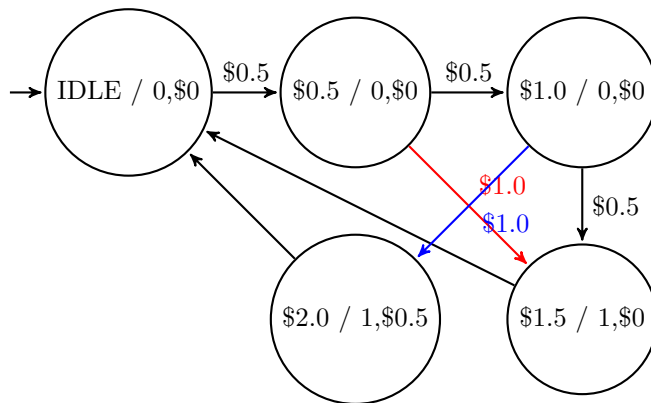
(a) Draw the FSM (Moore machine) for this vending machine.[15 pt]

(b) Draw the FSM (Mealy machine) for this vending machine.[10 pt]

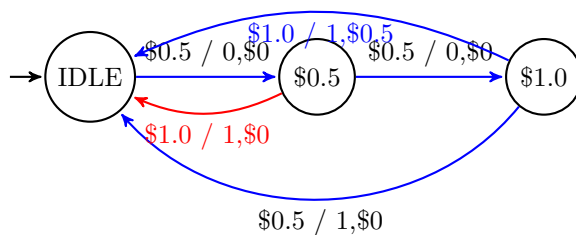
(c) Could Moore machines and Mealy machines be converted into each other to implement the same function? Compare their difference.[10 pt]

Answer to Question 3

(a)



(b)



(c)

They can be converted into each other.

The differences are: 1. Mealy Machines may have fewer nodes. 2. The outputs of a Mealy Machine depends on the current state and current input, while the outputs of a Moore Machine only depends the current state.