执行测验: Homework 4

测试信息

描述

我们将在本次作业中允许多次尝试，不限制提交次数。请注意：

- 作业将使用最后一次尝试的成绩作为最终成绩；
- 未提交的尝试将被记为0分；
- 当开始新的尝试时，所填入的答案将被完全清除。

因此，当决定提交作业时，请在其他设备上妥善保存已经完成的答案；否则，请保存答案但不要提交。在截止日期之前，请确保作业的最后一次尝试已经提交。在截止日期之后，如果发现作业成绩有任何问题，可以随时联系助教处理。

FAQ

1. 作业有grace day吗?

BB作业没有grace day，Autolab编程作业有5个grace day。

2. 我忘记提交作业了，可以请助教帮忙提交吗?

在同时满足以下条件时，你可以联系助教在ddl之后为你提交作业：
a. 你的当前作业没有成绩，没有提交记录；
b. 你的作业完成记录显示你的所有操作在ddl之前完成。

注意，BB会记录助教的所有操作，这些操作也都将需要归档。

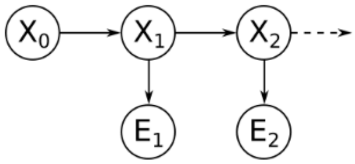
说明	注意：本作业不会自动提交。请在完成作业检查无误后，单击右下角"保存并提交"按钮提交作业。逾期未提交的作业不会被保存或计分。
多次尝试	此测试允许进行多次尝试。
强制完成	本测试可保存并稍后继续。

问题完成状态：

问题 1 30 分 已保存

单击"保存并提交"以保存并提交。单击"保存所有答案"以保存所有答案。

Consider the HMM shown below.



The prior probability $P(X_0)$, dynamics model $P(X_{t+1}|X_t)$, and sensor model $P(E_t|X_t)$ are as follows:

X_0	$P(X_0)$
0	0.35
1	0.65

X_{t+1}	X_t	$P(X_{t+1}\|X_t)$
0	0	0.05
1	0	0.95
0	1	0.05
1	1	0.95

E_t	X_t	$P(E_t\|X_t)$
a	0	0.15
b	0	0.3
c	0	0.55
a	1	0.7
b	1	0.05
c	1	0.25

We perform a first dynamics update, and fill in the resulting belief distribution $B'(X_1)$.

X_1	$B'(X_1)$
0	0.05
1	0.95

We incorporate the evidence $E_1 = c$.We fill in the evidence-weighted distribution $P(E_1 = c|X_1)B'(X_1)$, and the (normalized) belief distribution $B(X_1)$.

X_1	$P(E_1 = c\|X_1)B'(X_1)$
0	0.0275
1	0.2375

X_1	$B(X_1)$
0	0.103773584906
1	0.896226415094

You get to perform the second dynamics update. Fill in the resulting belief distribution $B'(X_2)$.

Please round your answers to 4 decimal places if necessary.

X_2	$B'(X_2)$
0	0.05
1	0.95

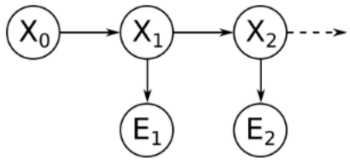
Now incorporate the evidence $E_2 = b$. Fill in the evidence-weighted distribution $P(E_2 = b|X_2)B'(X_2)$, and the (normalized) belief distribution $B(X_2)$.

X subscript 2	$P(E_2 = b\|X_2)B'(X_2)$
0	0.015
1	0.0475

X subscript 2	$B(X_2)$
0	0.24
1	0.76

单击“保存并提交”以保存并提交。单击“保存所有答案”以保存所有答案。

Consider the same HMM.



The prior probability $P(X_0)$, dynamics model $P(X_{t+1}|X_t)$, and sensor model $P(E_t|X_t)$ are as follows:

X_0	$P(X_0)$
0	0.5
1	0.5

X_{t+1}	X_t	$P(X_{t+1}\|X_t)$
0	0	0.5
1	0	0.5
0	1	0.2
1	1	0.8

E_t	X_t	$P(X_{t+1}\|X_t)$
a	0	0.5
b	0	0.3
c	0	0.2
a	1	0.8
b	1	0.1
c	1	0.1

In this question we'll assume the sensor is broken and we get no more evidence readings. We are forced to rely on dynamics updates only going forward. In the limit as $t \to \infty$, our belief about X_t should converge to a stationary distribution $\tilde{B}(X_\infty)$ defined as follows:

$$\tilde{B}(X_\infty) := \lim_{t\to\infty} P(X_t|E_1, E_2)$$

Recall that the stationary distribution satisfies the equation

$$\tilde{B}(X_\infty) = \sum_{X_\infty} P(X_{t+1}|X_t)\tilde{B}(X_\infty)$$

for all values in the domain of X.

In the case of this problem, we can write these relations as a set of linear equations of the form

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \tilde{B}(X_\infty = 0) \\ \tilde{B}(X_\infty = 1) \end{bmatrix} = \begin{bmatrix} \tilde{B}(X_\infty = 0) \\ \tilde{B}(X_\infty = 1) \end{bmatrix}$$

In the spaces below, fill in the coefficients of the linear system. The system you have written has many solutions (consider (0,0), for example), but to get a probability distribution we want the solution that sums to one. Fill in your solution in the table below. (Hint: to check your answer, you can also write some code and run till convergence.)

Your answers will be evaluated to 4 decimal places.

coefficient	value
a	0.5
b	0.2
c	0.5
d	0.8

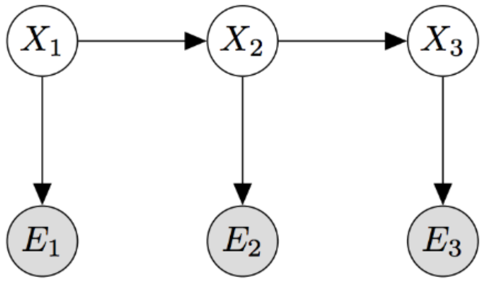
X_∞	$\tilde{B}(X_\infty)$

单击“保存并提交”以保存并提交。单击“保存所有答案”以保存所有答案。

问题 3

10 分 已保存

Consider the HMM graph structure shown below.



Recall the Forward algorithm is a two step iterative algorithm used to approximate the probability distribution

$P(X_t|e_1, \ldots, e_t)$.

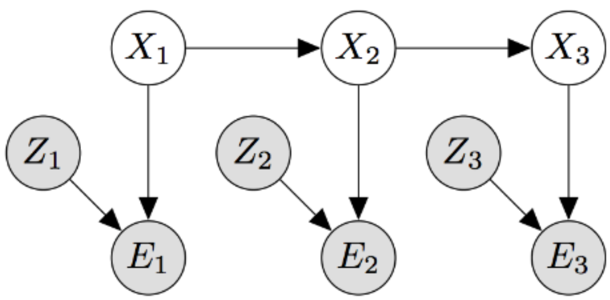
The two steps of the algorithm are as follows:

Elapse Time: $P(X_t|e_{1\ldots t-1}) = \sum_{x_{t-1}} P(X_t|x_{t-1})P(x_{t-1}|e_{1\ldots t-1})$

Observe Time: $P(X_t|e_{1\ldots t}) = \frac{P(e_t|X_t)P(X_t|e_{1\ldots t-1})}{\sum_{x_t} P(e_t|x_t)P(x_t|e_{1\ldots t-1})}$

For this problem we will consider modifying the forward algorithm as the HMM graph structure changes. Our goal will continue to be to create an iterative algorithm which is able to compute the distribution of states, X_t, given all available evidence from time 0 to time *t*.

Consider the graph below where new observed variables, Z_i, are introduced and influence the evidence.



What will the modified elapse time update be?

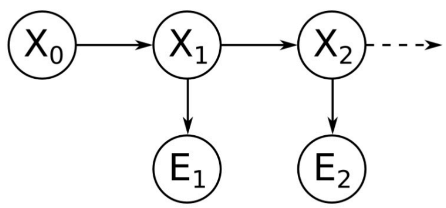
$P(X_t|e_{1\ldots t-1}, z_{1\ldots t-1})$ =

- A. $\sum_{x_{t-1}} P(X_t|z_{1\ldots t-1})P(x_{t-1}|e_{1\ldots t-1}, z_{1\ldots t-1})$
- B. $\sum_{x_{t-1}} P(X_t|e_{1\ldots t-1}, z_{1\ldots t-1})P(x_{t-1}|x_{t-1}, z_{1\ldots t-1})$
- C. $\sum_{x_{t-1}} P(X_t|x_{t-1})P(x_{t-1}|e_{1\ldots t-1})$
- D. $\sum_{x_{t-1}} P(X_t|x_{t-1})P(x_{t-1}|e_{1\ldots t-1}, z_{1\ldots t-1})$

问题 4

30 分 已保存

单击“保存并提交”以保存并提交。单击“保存所有答案”以保存所有答案。



The prior probability $$P(X_0)$$, dynamics model $$P(X_{t+1} | X_t)$$, and sensor model $$P(E_t|X_t)$$ are as follows:

X_0	$P(X_0)$
0	0.5
1	0.5

X_{t+1}	X_t	$P(X_{t+1}\|X_t)$
0	0	0.5
1	0	0.5
0	1	0.2
1	1	0.8

E_t	X_t	$P(E_t\|X_t)$
a	0	0.5
b	0	0.3
c	0	0.2
a	1	0.8
b	1	0.1
c	1	0.1

If the $E_1 = a, E_2 = b, E_3 = c$, what is the most likely explanation $X^*_{1:3} = argmax P(X_{1:3} | E_{1:3})$

X^*_1 = 1

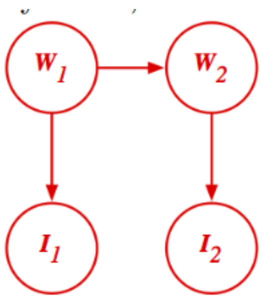
X^*_2 = 1

X^*_3 = 1

问题 5

30 分 已保存

Consider the following graph, where **W₁** and **W₂** can be either be **R** or **S**, and **I₁** and **I₂** can be either be **T** or **F**:



单击“保存并提交”以保存并提交。单击“保存所有答案”以保存所有答案。

$W_1 = \mathtt{S}$	$W_1 = \mathtt{R}$
.6	.4

(a) $P(W_1)$

	$W_2 = \mathtt{S}$	$W_2 = \mathtt{R}$
$W_1 = \mathtt{S}$	.7	.3
$W_1 = \mathtt{R}$	.5	.5

(b) $P(W_2 \mid W_1)$

	$I = \mathtt{T}$	$I = \mathtt{F}$
$W = \mathtt{S}$	.9	.1
$W = \mathtt{R}$	.2	.8

(c) $P(I \mid W)$

Now we want to try approximate inference through sampling. Applying likelihood weighting, suppose we generate the following six samples given the evidence I_1 = **T** and I_2 = **F**:

$$(W_1, I_1, W_2, I_2) = \left\{ \langle \mathtt{S}, \mathtt{T}, \mathtt{R}, \mathtt{F} \rangle, \langle \mathtt{R}, \mathtt{T}, \mathtt{R}, \mathtt{F} \rangle, \langle \mathtt{S}, \mathtt{T}, \mathtt{R}, \mathtt{F} \rangle, \langle \mathtt{S}, \mathtt{T}, \mathtt{S}, \mathtt{F} \rangle, \langle \mathtt{S}, \mathtt{T}, \mathtt{S}, \mathtt{F} \rangle, \langle \mathtt{R}, \mathtt{T}, \mathtt{S}, \mathtt{F} \rangle \right\}$$

Then the weight of the first sample **(S, T, R, F)** is 0.72 .

The result from likelihood weighting is:

$\widehat{P}(W_2 = R \mid I_1 = T, I_2 = F)$	0.8889
$\widehat{P}(W_2 = S \mid I_1 = T, I_2 = F)$	0.1111

问题 6

20 分 已保存

After observing step of particle filtering, the particles and its weight are as follow:

Particles	Weight
A	0.3
B	0.4
C	0.9
D	0.5
A	0.3
C	0.9
A	0.3
D	0.5
D	0.5
A	0.3

Fill in the weighted sample distribution $P'(X)$ you used in the resampling step. Your answers will be evaluated to 4 decimal places.

P'(A) = 0.2449

P'(B) = 0.0816

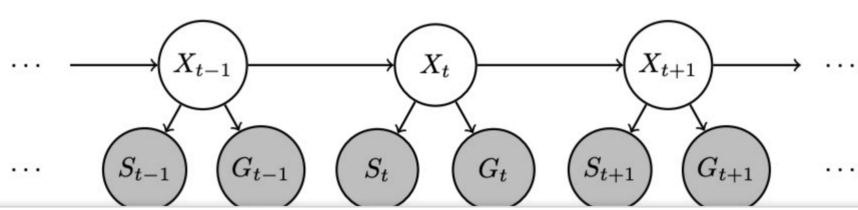
P'(C) = 0.3673

P'(D) = 0.3061

问题 7

10 分 已保存

Transportation researchers are trying to improve traffic in the city but, in order to do that, they first need to estimate the location of each of the cars in the city. They need our help to model this problem as an inference problem of an HMM. For this question, assume that only one car is being modeled. The structure of this modified HMM is given below, which includes X, the location of the car; S, the noisy location of the car from the signal strength at a nearby cell phone tower; and G, the noisy location of the car from GPS.

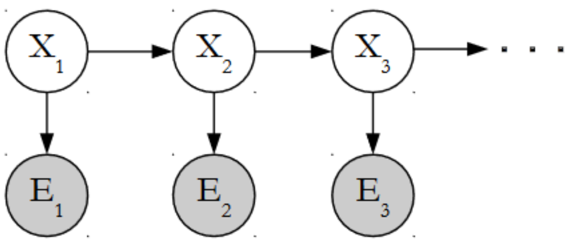


单击“保存并提交”以保存并提交。单击“保存所有答案”以保存所有答案。

all past and current observations. The dynamics update expression $P(x_t \mid s_{1:t-1}, g_{1:t-1})$ should be:

- ( ) A. None of the above
- ( ) B. $\sum_{x_{t-1}} P(x_t \mid x_{t-1})\, P(x_{t-1}, x_t \mid s_{1:t-1})\, P(x_{t-1}, x_t \mid g_{1:t-1})$
- ( ) C. $\sum_{x_{t-1}} P(x_t \mid x_{t-1})\, P(x_t \mid s_{1:t-1}, g_{1:t-1})$
- (●) D. $\sum_{x_{t-1}} p(x_t \mid x_{t-1})\, p(x_{t-1} \mid s_{1:t-1}, g_{1:t-1})$

问题 8 20 分 已保存



The Viterbi algorithm finds the most probable sequence of hidden states $X_{1:T}$, given a sequence of observations $e_{1:T}$. For the HMM structure above, which of the following probabilities are maximized by the sequence of states returned by the Viterbi algorithm? Select all correct option(s).

- [ ] A. $P(X_{1:T})$
- [ ] B. $P(X_T \mid e_T)$
- [x] C. $P(X_{1:T} \mid e_{1:T})$
- [x] D. $P(X_{1:T}, e_{1:T})$
- [x] E. $P(X_1)P(e_1 \mid X_1)\prod_{t=2}^{T} P(e_t \mid X_t)P(X_t \mid X_{t-1})$
- [ ] F. $P(X_1)\prod_{t=2}^{T} P(X_t \mid X_{t-1})$
- [ ] G. None of above

问题 9 20 分 已保存

- [x] $P(X_t \mid e_{1:t}, z_{1:t}) = \dfrac{P(e_t, z_t \mid X_t)P(X_t \mid e_{1:t-1}, z_{1:t-1})}{\sum_{x_t} P(e_t, z_t \mid x_t)P(x_t \mid e_{1:t-1}, z_{1:t-1})}$
- [ ] $P(X_t \mid e_{1:t}, z_{1:t}) = \dfrac{P(e_t \mid X_t)P(X_t \mid e_{1:t-1}, z_{1:t-1})}{\sum_{x_t} P(e_t \mid x_t)P(x_t \mid e_{1:t-1}, z_{1:t-1})}$
- [x] $P(X_t \mid e_{1:t}, z_{1:t}) = \dfrac{P(e_t \mid X_t, z_t)P(z_t)P(X_t \mid e_{1:t-1}, z_{1:t-1})}{\sum_{x_t} P(e_t \mid x_t, z_t)P(z_t)P(x_t \mid e_{1:t-1}, z_{1:t-1})}$
- [ ] $P(X_t \mid e_{1:t}, z_{1:t}) = \dfrac{P(e_t \mid X_t)P(e_t \mid z_t)P(X_t \mid e_{1:t-1}, z_{1:t-1})}{\sum_{x_t} P(e_t \mid x_t)P(e_t \mid z_t)P(x_t \mid e_{1:t-1}, z_{1:t-1})}$

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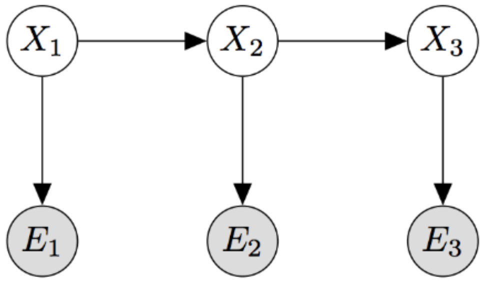

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Observe Time: $P(X_t|e_{1\ldots t})$ = $\frac{P(e_t|X_t)P(X_t|e_{1\ldots t-1})}{\sum_{x_t} P(e_t|x_t)P(x_t|e_{1\ldots t-1})}$

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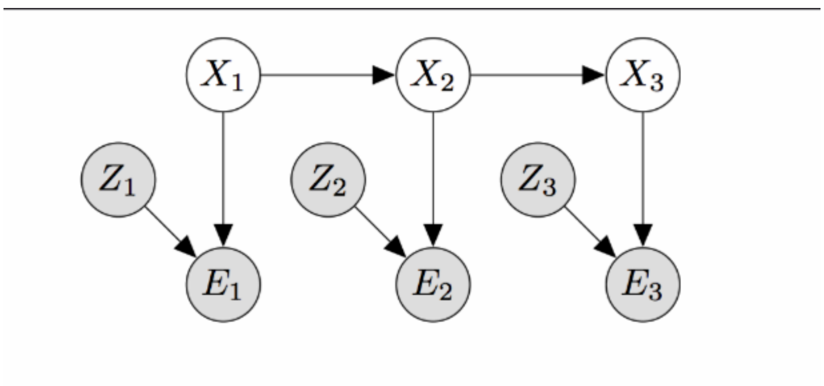

At the observe time $P(X_t | e_{1\ldots t}, z_{t\ldots t})$ equals to:

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