

Progression : Recherche du point "le plus bas" dans la direction $r^{(p)}$

$$\text{Minimiser } f(\lambda) = F(x^{(p)} + \lambda r^{(p)}) = F(x^{(p)})$$

$$\begin{aligned} f(\lambda) &= \frac{1}{2} (x^{(p)} + \lambda r^{(p)})^T A (x^{(p)} + \lambda r^{(p)}) - (x^{(p)} + \lambda r^{(p)})^T b \\ &\quad - \frac{1}{2} x^{(p)T} A x^{(p)} + x^{(p)T} b \\ &= (\lambda r^{(p)})^T A x^{(p)} + \frac{1}{2} \lambda^2 r^{(p)T} A r^{(p)} - \lambda r^{(p)T} b \\ &= \frac{1}{2} \lambda^2 r^{(p)T} A r^{(p)} - \lambda r^{(p)T} \underbrace{(b - A x^{(p)})}_{= r^{(p)}} \\ &= \frac{1}{2} \lambda^2 r^{(p)T} A r^{(p)} - \lambda r^{(p)T} r^{(p)} \end{aligned}$$

$$f'(\lambda) = \lambda r^{(p)T} A r^{(p)} - r^{(p)T} r^{(p)}$$

$$f \text{ minimal en } \lambda_p \quad f'(\lambda_p) = 0$$

$$f'(\lambda_p) = 0 \Leftrightarrow \lambda_p = \frac{r^{(p)T} r^{(p)}}{r^{(p)T} A r^{(p)}}$$