deux directions de descente successives $r^{(p+1)}$ et $r^{(p)}$ sont orthogonales

$$\left(r^{(p+1)} \mid r^{(p)}\right) = \left(\underbrace{r^{(p)} - \lambda_p A r^{(p)}}_{\text{astuce transparent 18}} \mid r^{(p)}\right)$$

$$= \left(r^{(p)} \mid r^{(p)}\right) - \lambda_p \left(A r^{(p)} \mid r^{(p)}\right)$$

or $\lambda_p = \dfrac{\left(r^{(p)} \mid r^{(p)}\right)}{\left(A r^{(p)} \mid r^{(p)}\right)}$

$$\left(r^{(p+1)} \mid r^{(p)}\right) = 0$$

Soit u vecteur propre de A, λ valeur propre associée

$$x^{(0)} = x^* + \beta u \qquad \text{avec } \beta \neq 0$$

$\uparrow$ solution

on converge en 1 itération

$$\lambda_0 = \frac{(r^{(0)} | r^{(0)})}{(r^{(0)} | A r^{(0)})} = \frac{\| b - A x^{(0)} \|^2}{(b - A x^{(0)} | A (b - A x^{(0)}))}$$

$$x^{(1)} = x^{(0)} + \lambda_0 r^{(0)}$$

$$r^{(0)} = b - A x^{(0)} = b - A(x^* + \beta u) = 0 - \beta A u = -\beta \lambda u$$

$$x^{(1)} = x^{(0)} + \frac{\| -\beta \lambda u \|^2 \, (-\beta \lambda u)}{(-\beta \lambda u \,|\, A(-\beta \lambda u))}$$

$$= x^{(0)} + \frac{\beta^2 \lambda^2 \|u\|^2 (-\beta \lambda u)}{(-\beta \lambda u \,|\, -\beta \lambda^2 u)}$$

$$= x^{(0)} + \frac{-\beta^3 \lambda^3 \|u\|^2 \, u}{\beta^2 \lambda^3 \|u\|^2}$$

$$= x^{(0)} - \beta u$$

$$= x^*$$

A a toutes ses valeurs propres égales

A est multiple de l'identité $A = \mu I$

$$\lambda_0 = \frac{\|b - Ax_0\|^2}{(b - Ax_0 | A(b - Ax_0))} = \frac{1}{\mu} \frac{\|b - Ax_0\|^2}{\|b - Ax_0\|^2} = \frac{1}{\mu}$$

$$x_1 = x_0 + \frac{1}{\mu}(b - Ax_0)$$

$$x_1 = x_0 + \frac{1}{\mu}(b - \mu x_0)$$

$$x_1 = \frac{1}{\mu} b = A^{-1} b$$