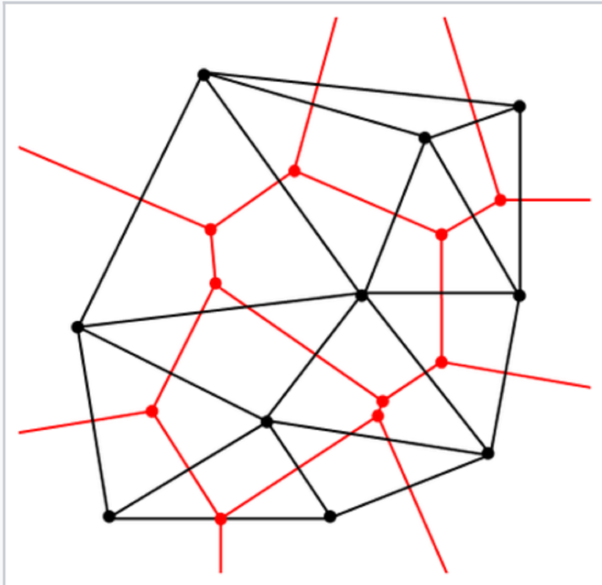
Un diagramme de Voronoï



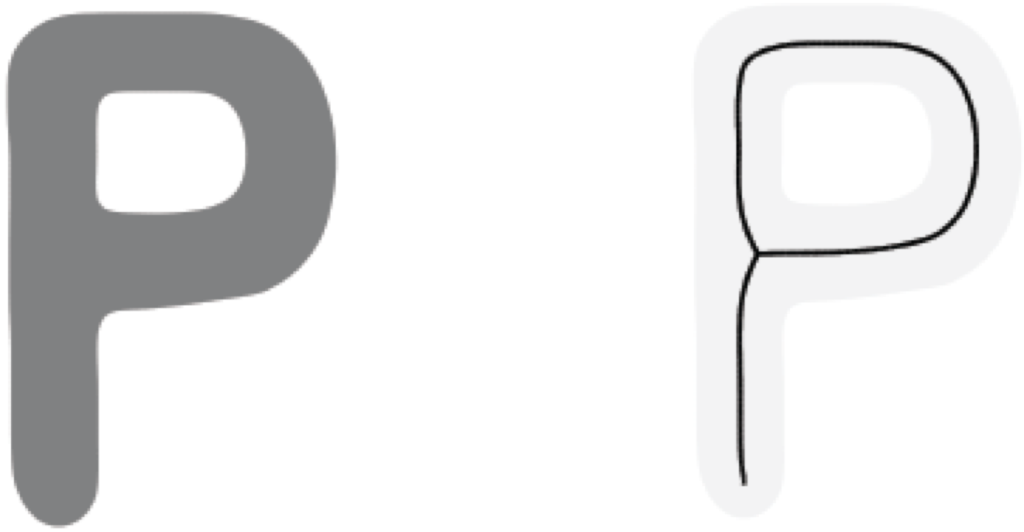
Superposition d'un diagramme de Voronoï (en rouge) et de sa triangulation de Delaunay (en noir).

Source : Wikipédia

Skeleton and medial axis

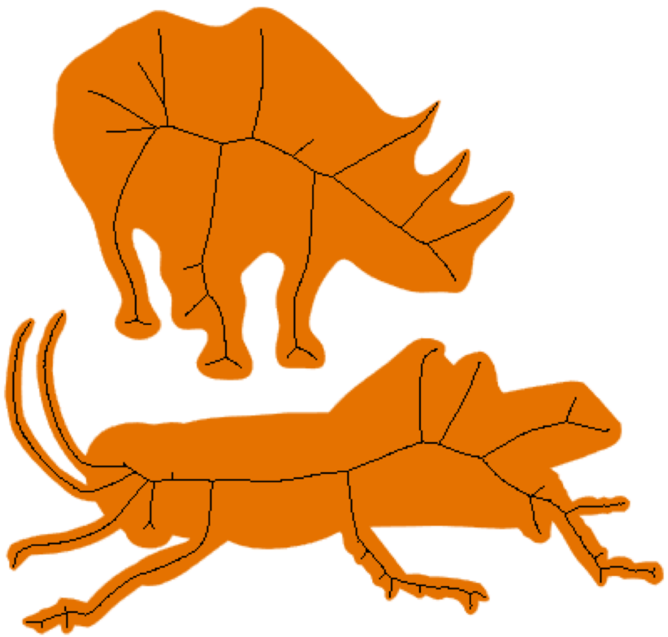
Medial representations of a shape use

- points "in the middle" of the shape
- local "thickness"

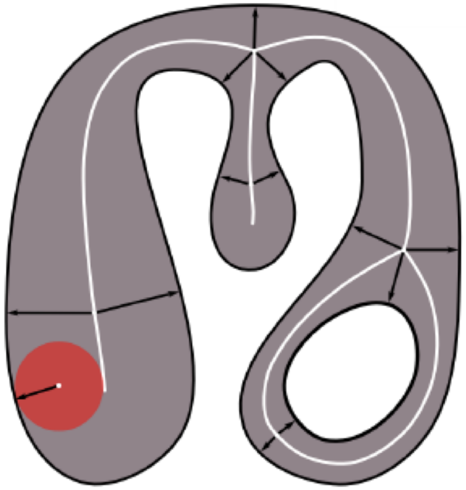


Medial Axes (MA)

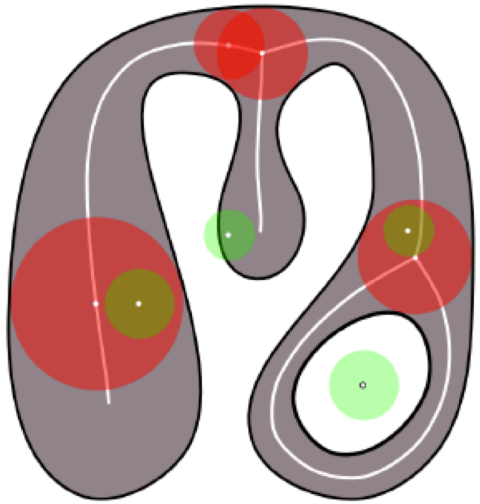
- Properties
  - ✓ Preserves object's shape
    - The object can be reconstructed from MA and its distances to the boundary



Skeleton and medial axis definitions



- Set of all points with at least two closest boundary points + radius of the associated ball



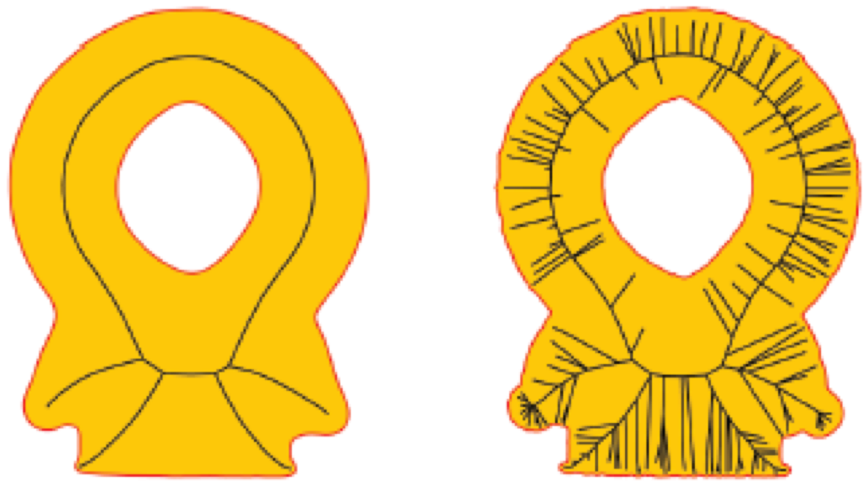
- Set of maximal balls' centers (Blum 67)
  - A maximally inscribed ball cannot be included in an other ball inner to the shape
  - Shape as set of these balls

Skeleton and medial axis definitions

Instability of the medial axis

- Little perturbations on the boundary of the shape induce many long additional branches

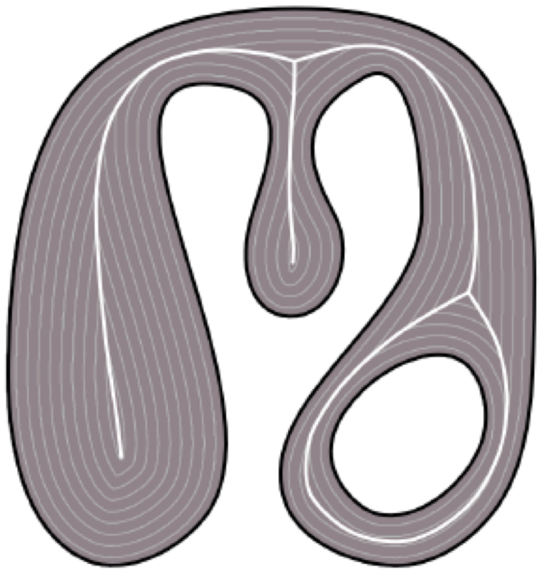
- Small medial balls = small surface features & noisy medial branches
- Large medial balls =



Skeleton and medial axis definitions

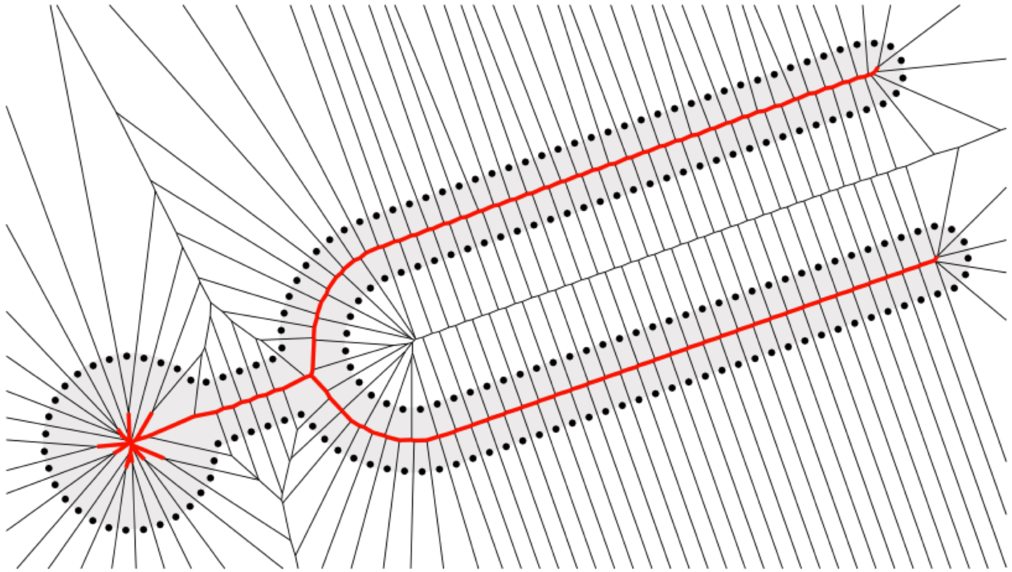
Shock graph of grass-fire evolution

- Fire propagates from boundaries to interior with uniform speed
- Skeleton = locations where two or more burning fronts meet

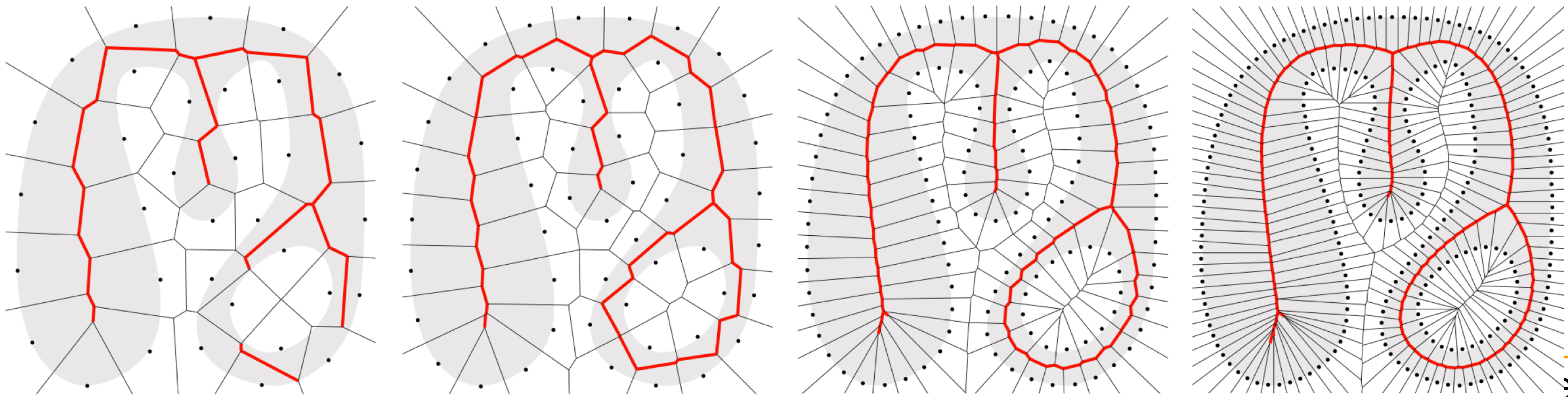


Medial axis / skeleton approximation from points sampled on shapes

- Input shape represented by a set of points sampled over its boundary
- Replace maximal balls by *Voronoï balls*

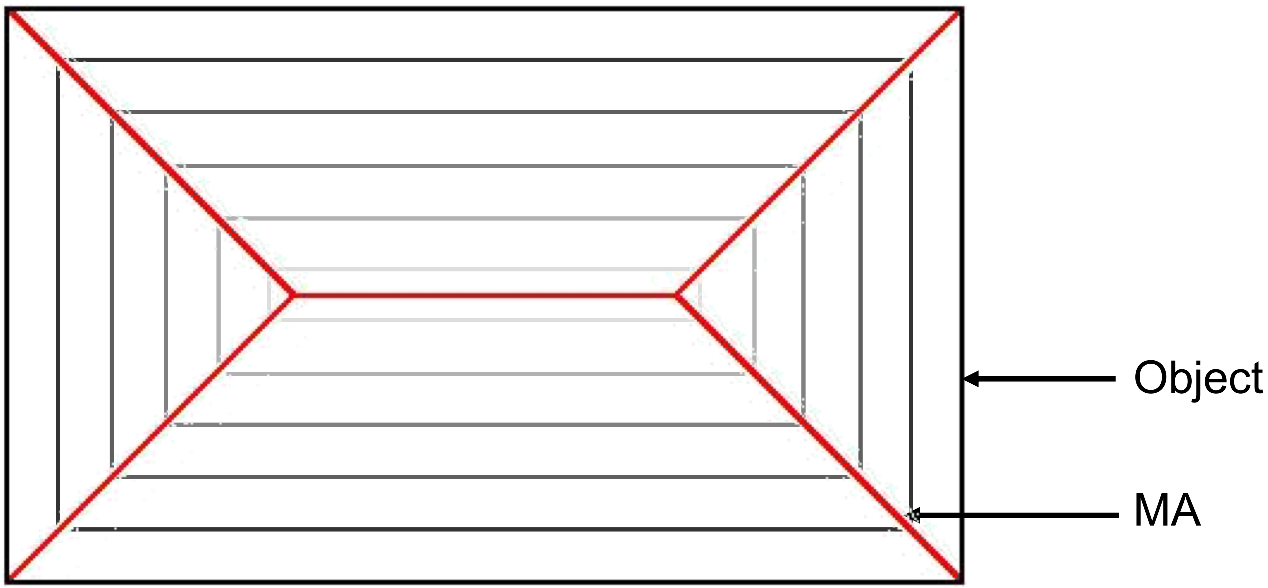


Medial axis / skeleton approximation from points sampled on shapes

- **Medial axis is included in the limit Voronoï graph, as density of the sampling increases** (Amenta et al 98)
- Voronoi graph : boundaries of the Voronoi cells
  - Voronoi vertices : on the boundaries of at least 3 regions in 2D
  - Voronoi edges : on the boundaries of 2 regions in 2D
- 

Another way to compute MA

- Grassfire analogy:
  - Let the object represent a field of grass. A fire starts at the field boundary, and burns across the field at uniform speed.
  - MA are where the fire fronts meet.



2D Thinning

- Iterative process that reduces a binary picture to a skeleton
  - Simulating the “grassfire burning” that defines MA

2D Thinning

- Iterative process that reduces a binary picture to a skeleton
  - Simulating the “grassfire burning” that defines MA

2D Thinning

- Thinning vs. morphological erosion

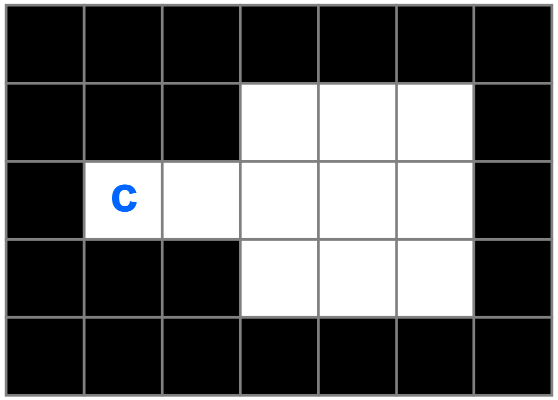
Thinning

Iterative erosion

  – Iterative erosion eventually eliminates the object, but thinning preserves key pixels (voxels) so that the shape and topology of the object is retained

2D Thinning

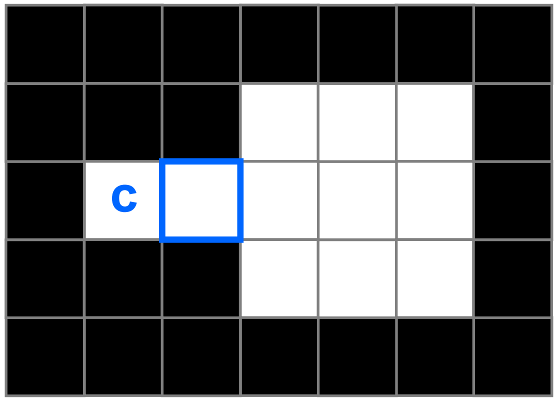
- Curve-end pixels
  - Object pixels lying at the ends of curves, whose removal would shrink the skeleton (and hence losing shape information).



Curve-end pixel

2D Thinning

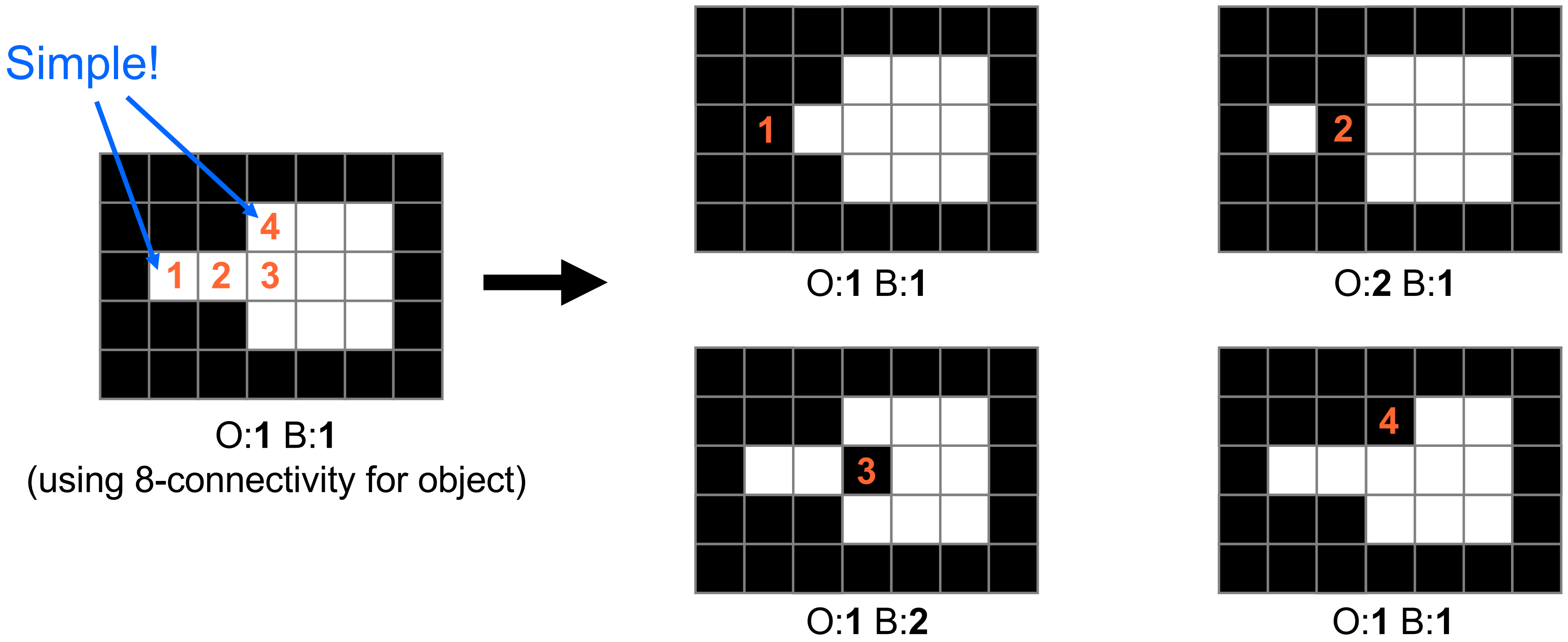
- Curve-end pixels criteria
  - Object pixel **c** is a curve-end pixel if and only if **c** has exactly one connected pixel in the object.



Curve-end pixel and its connected pixel

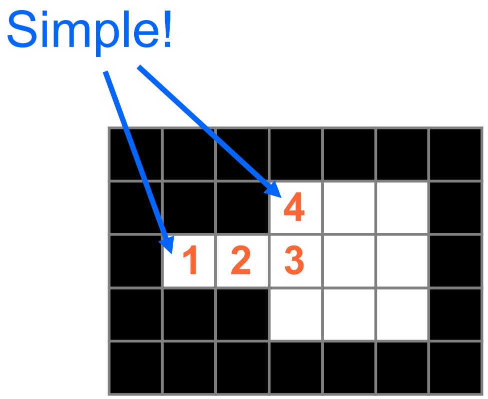
2D Thinning

- Simple pixels
  - Object pixels whose removal from the object does not change topology (i.e., # of components of object and background)

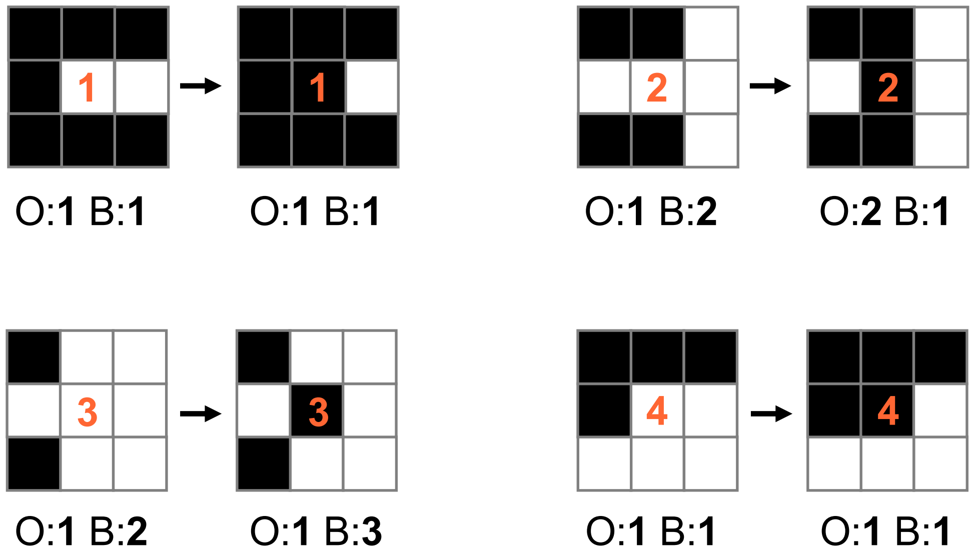


2D Thinning

- Simple pixels criteria
  - Object pixel **p** is simple if and only if setting **p** to background does not change the number of connected components of either the object or background in the 3x3 neighborhood of **p**.

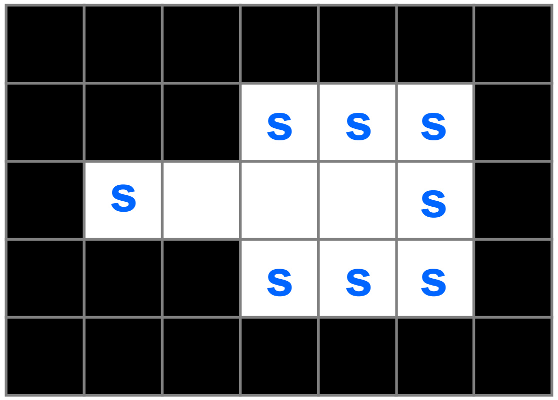


(using 8-connectivity for object)



2D Thinning

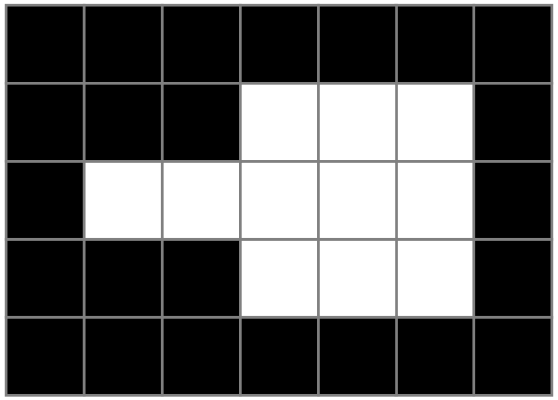
- Simple pixels criteria
  - Object pixel **p** is simple if and only if setting **p** to background does not change the number of connected components of either the object or background in the 3x3 neighborhood of **p**.



All simple pixels

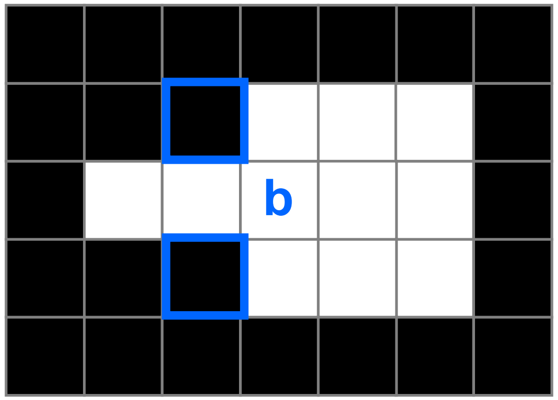
2D Thinning

- Border pixels
  - Pixels lying on the border of the object
  - To be considered for removal at each thinning iteration



2D Thinning

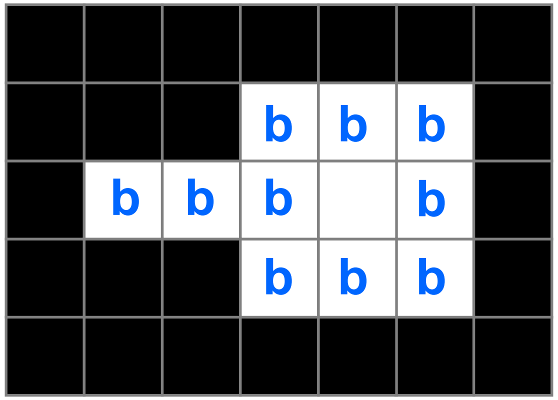
- Border pixels criteria
  - Object pixel **p** is on the border if and only if **p** is connected to some background pixel



A border pixel and its 8-connected background pixels

2D Thinning

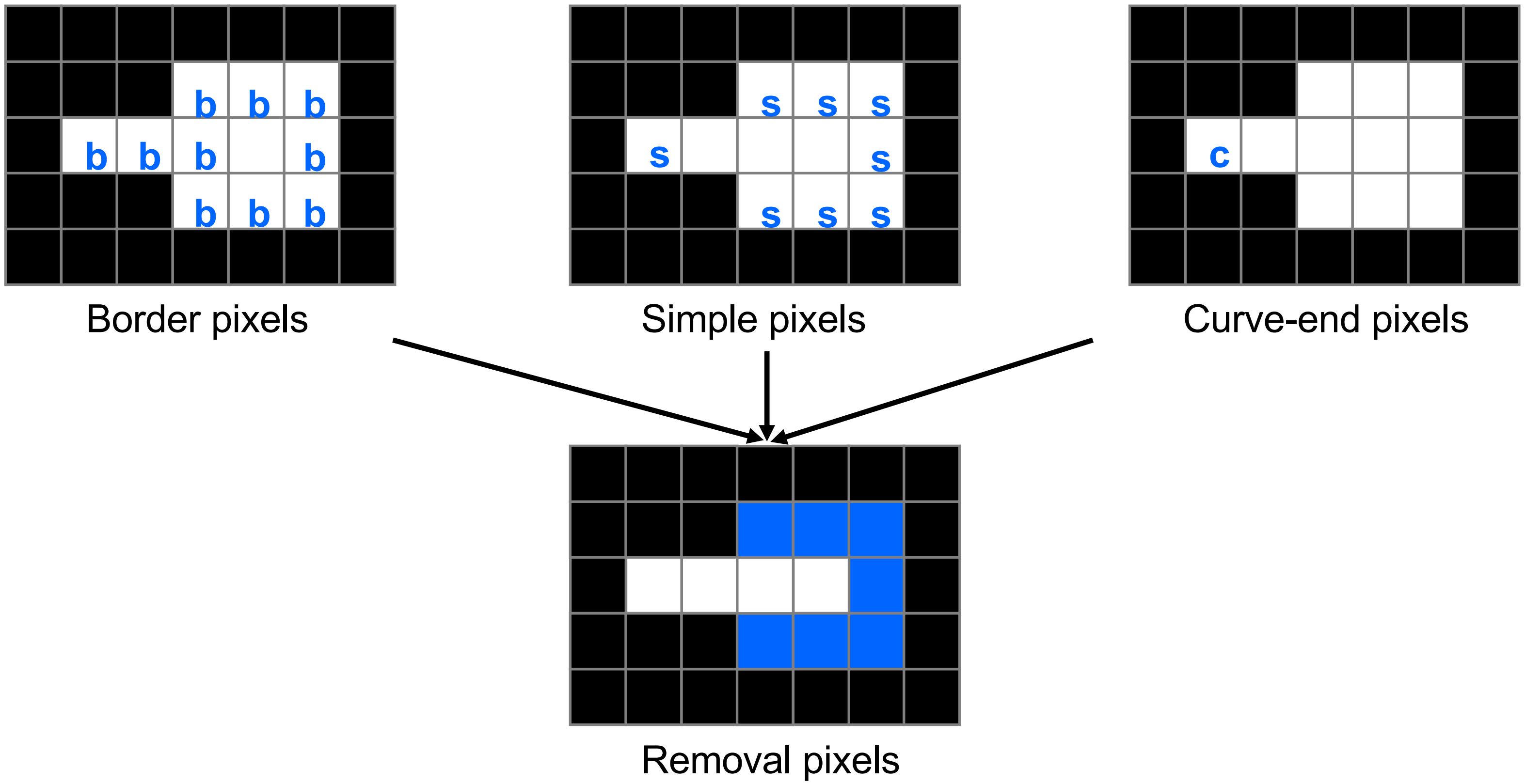
- Border pixels criteria
  – Object pixel **p** is on the border if and only if **p** is connected to some background pixel



Border pixels for 8-connectivity

2D Thinning

- Putting together: Removable pixels
  - Border pixels that are simple and not curve-end



2D Thinning

- Algorithm (attempt) 1
  - Simultaneous removal of all removable points (“Parallel thinning”)

```
// Parallel thinning on a binary image I
1. Repeat:
     1. Find all removable object pixels S in I
     2. If S is empty, Break.
     3. Set all pixels in S to be background in I
2. Output I
```

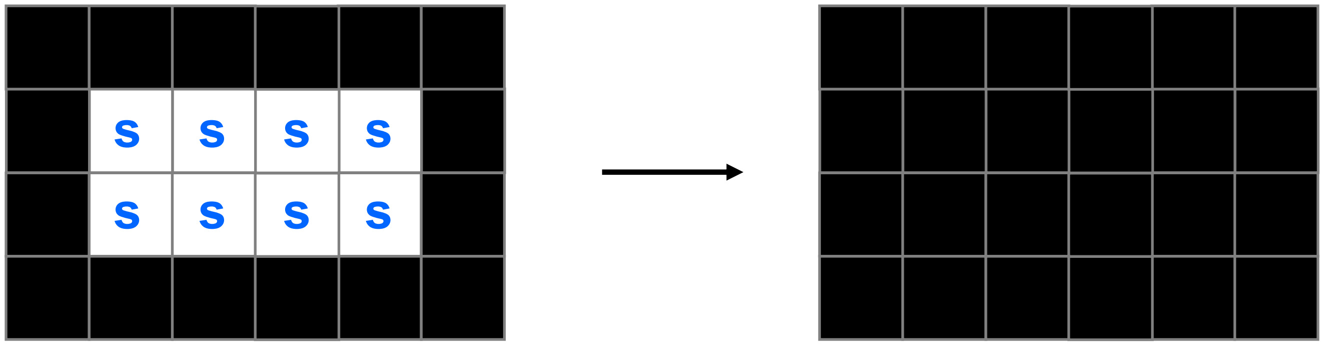
2D Thinning

- Algorithm (attempt) 1
  - Simultaneous removal of all removable points (“Parallel thinning”)
  - Does not work: the object topology is changed. How does this happen?

Naïve parallel thinning

2D Thinning

- A closer look at simple pixels
  - No longer "simple" when removed together



A simple example of how removing multiple simple pixels changes the object connectivity

  - Naïve parallel thinning is not topology-preserving; more sophisticated strategies are needed
    - see *Further Readings* slide

2D Thinning

- Algorithm 2
  - Sequentially visit each removable pixel and check its simple-ness before removing the pixel. ("Serial Thinning")

```
// Serial thinning on a binary image I
1. Repeat:
     1. Find all removable object pixels S in I
     2. If S is empty, Break.
     3. Repeat for each pixel x in S in some order:
          1. If x is simple, set x to be background in I
2. Output I
```

2D Thinning

- Algorithm 2
  - Sequentially visit each removable pixel and check its simple-ness before removing the pixel. (“Serial Thinning”)

Serial thinning

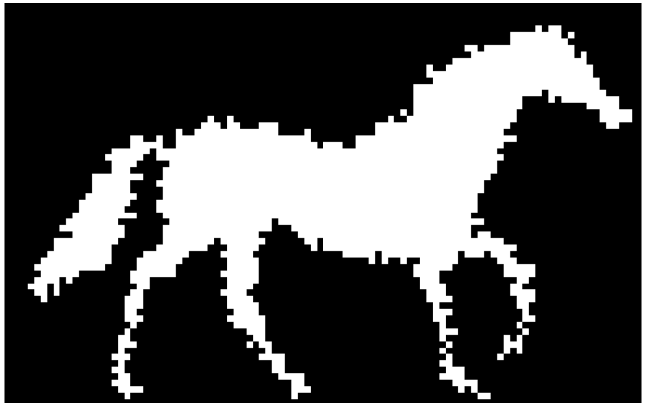
2D Thinning

- Algorithm 2
  - Sequentially visit each removable pixel and check its simplicity before removing the pixel. ("Serial Thinning")
  - Result is affected by the visiting "sequence"

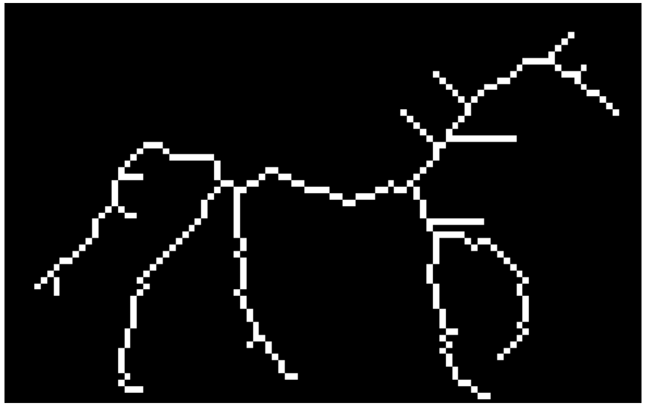
Serial thinning with two different visiting sequences of removable pixels

Skeleton Pruning

- Thinning is sensitive to boundary noise
  - Due to the instability of medial axes
- Skeleton pruning
  - During thinning
    - E.g., using more selective criteria for end pixels (voxels)
  - After thinning
    - E.g., based on branch length
  - See *Further Readings*



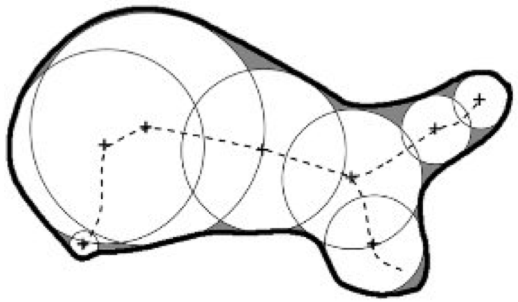
Object with boundary noise



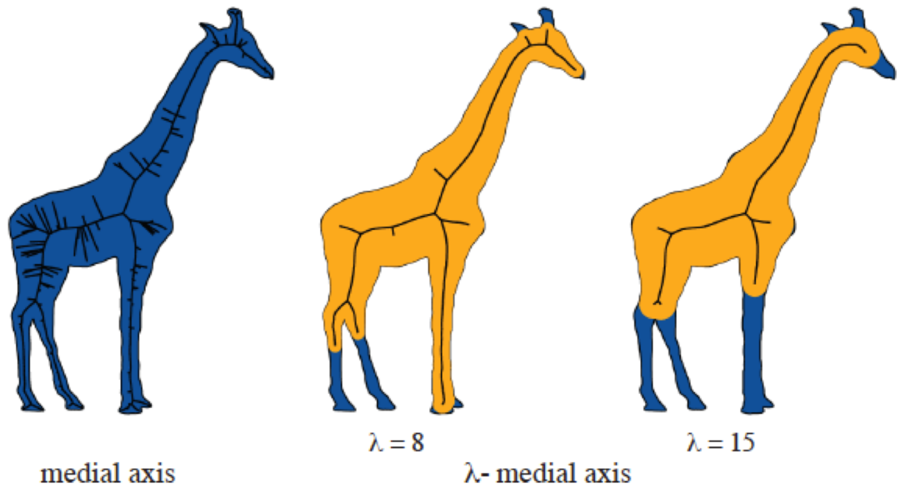
Resulting skeleton

Filtering / recovering of details

Medial axis branches pruning: based on local criteria on the maximal balls and corresponding pairs of contact points on the surface



- Filtering based on the angles and or radius (Attali and Montanvert 1996)
  - May disconnect the remaining parts
- Filtering based on the circumradius of contact points : Lambda medial axis (Chazal and Lieutier 2004)
  - Unable to capture the shape's multi-scale structure



Filtering / recovering of details

- How to obtain a better global consistency on the topological structure of the filtered medial axis?

- Scale Axis Transform (Giesen et al 2009)

  - **Scale** the medial balls of the shape **by a factor**

    - Small medial balls undergo a growth smaller than **nearby** larger medial balls

    - Whenever one of these scaled balls is not maximally inscribed anymore, it is **removed** from the set.

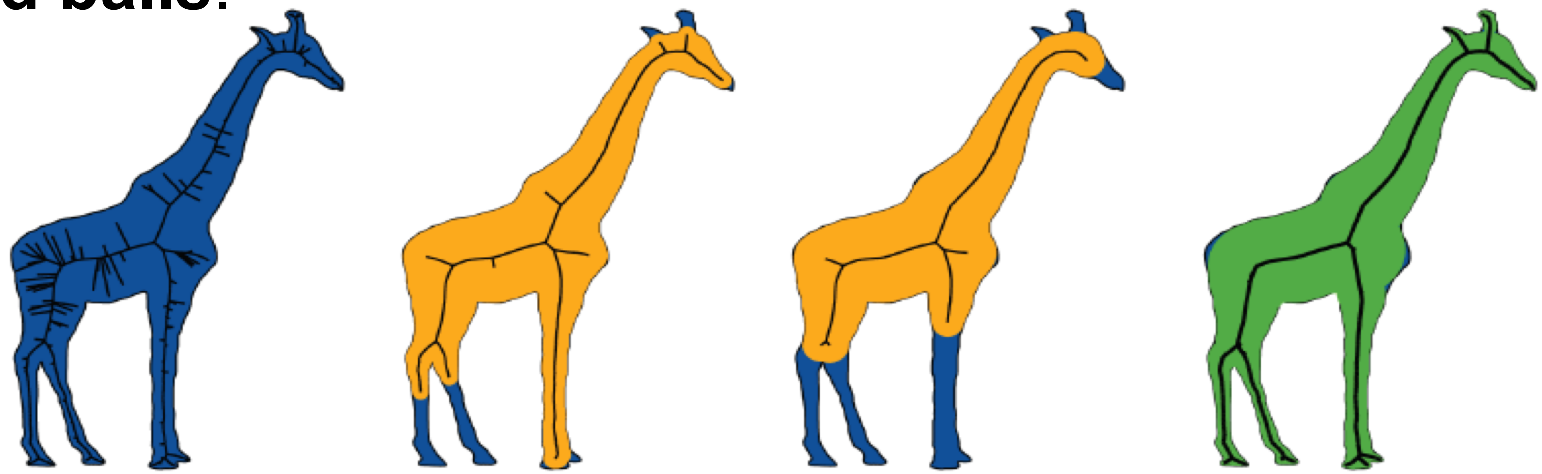
  - Filtering = Exact medial axis of **remaining set of de-scaled balls**.

Filtering / recovering of details

Scale Axis Transform (Giesen et al 2009)

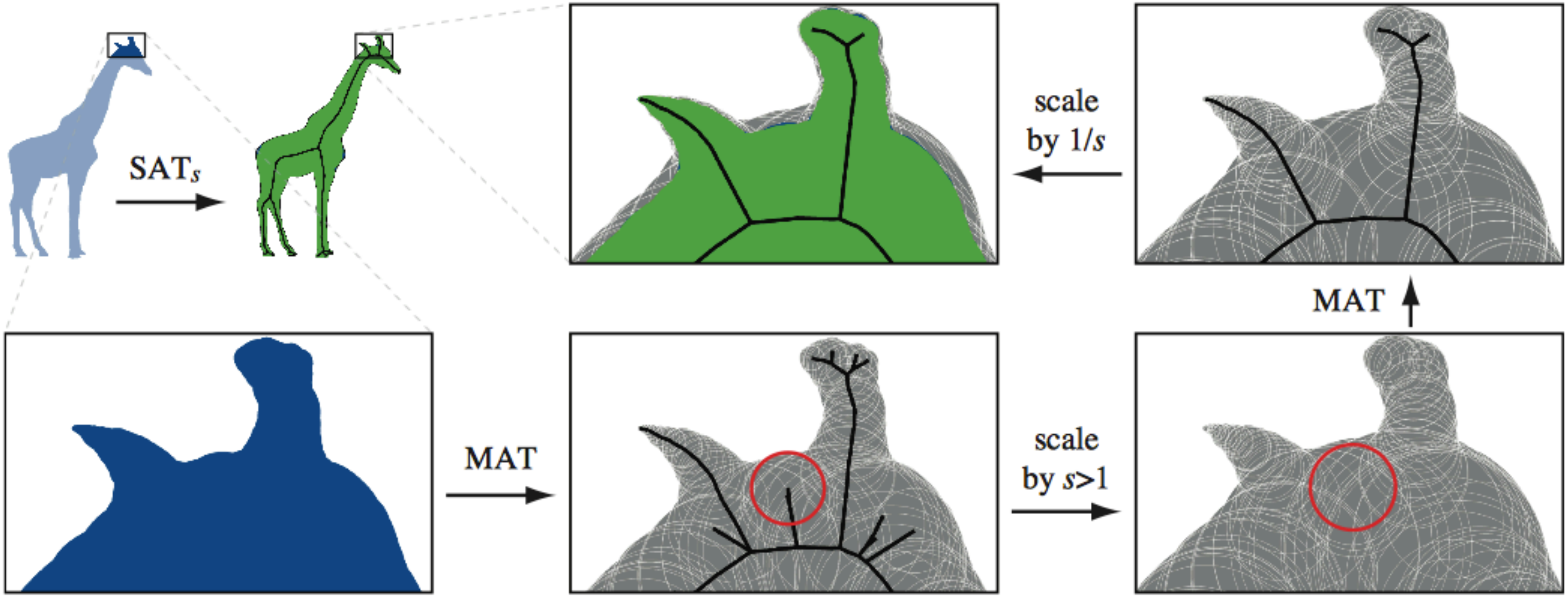
- Observe which balls remain maximal under **non uniform dilatation**
- Balls for details and noise covered by balls for rougher structures

Filtering = Exact medial axis of the **remaining set of de-scaled balls**.

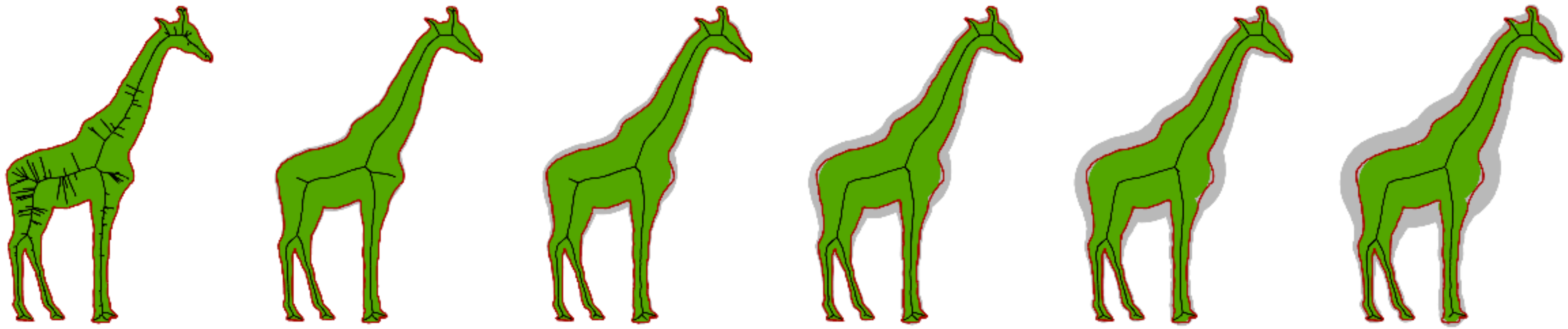


medial axis | $\lambda = 8$ | λ- medial axis | $\lambda = 15$ | $s = 1.34$ scale axis

Scale axis transform



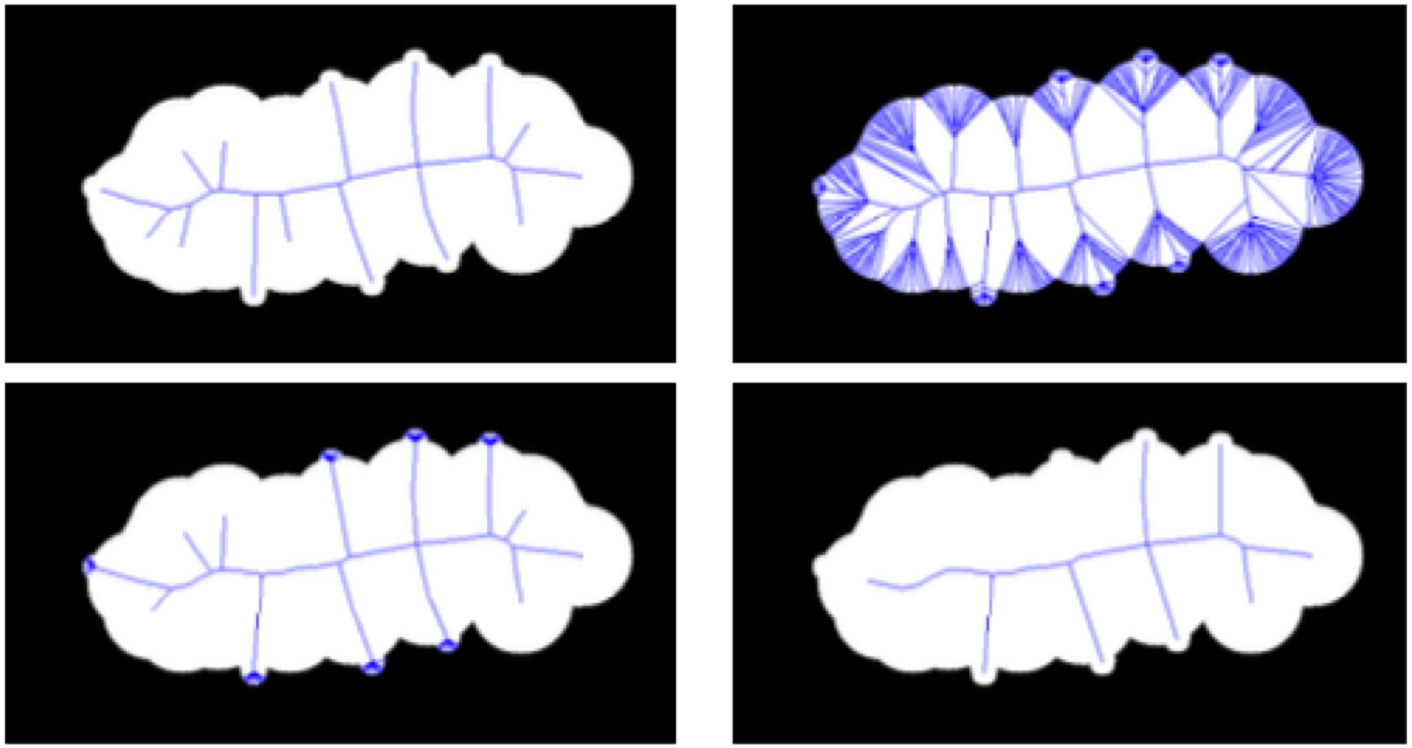
Scale axis transform



Multi-resolution representation

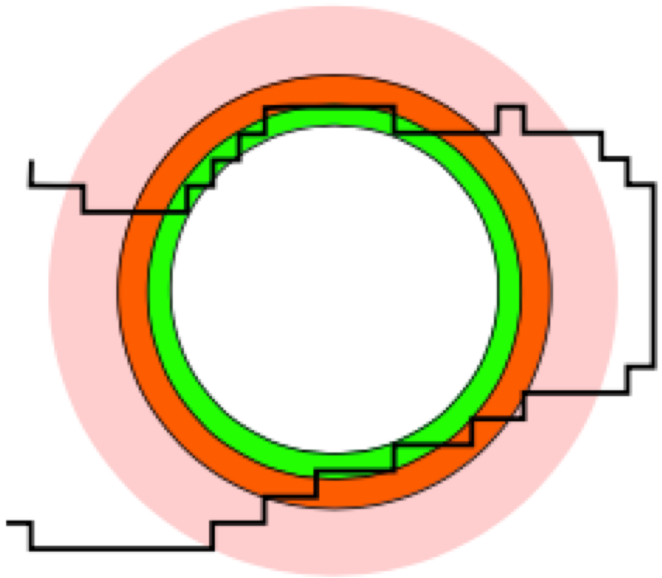
Limitation de SAT

Propagation Voronoï

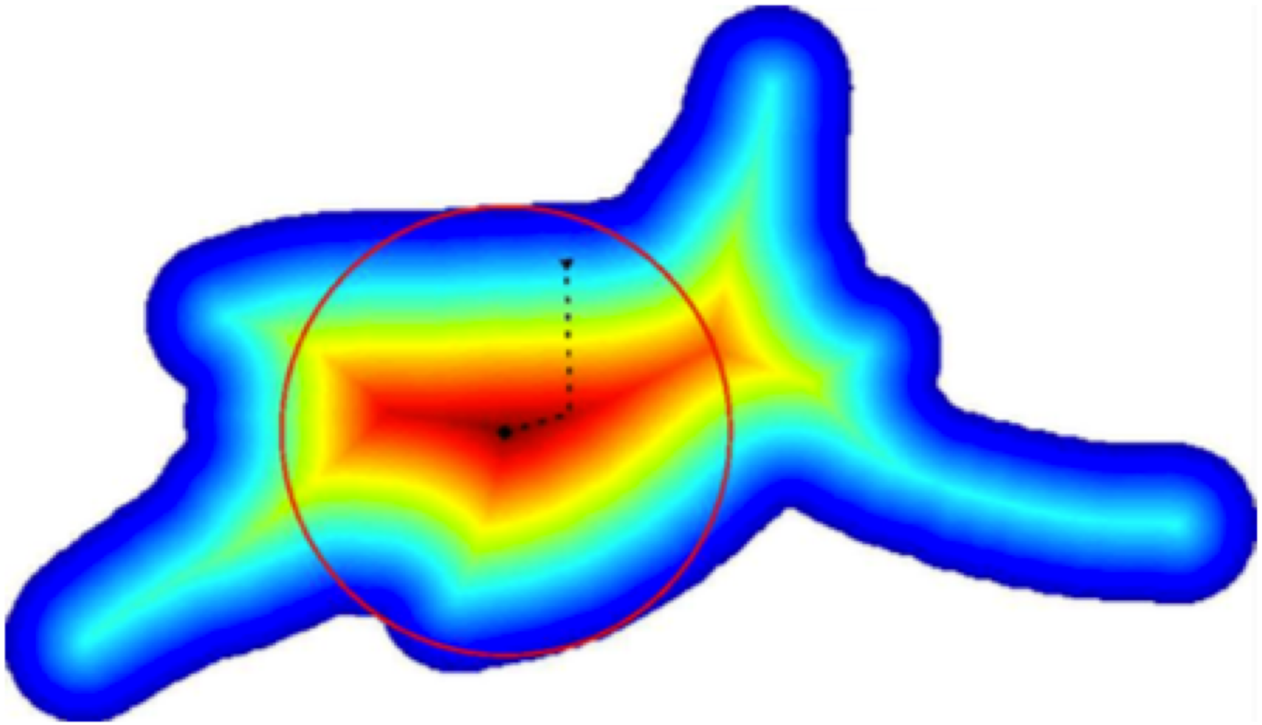


SAT avec $s = 1.1$ SAT avec $s = 1.2$

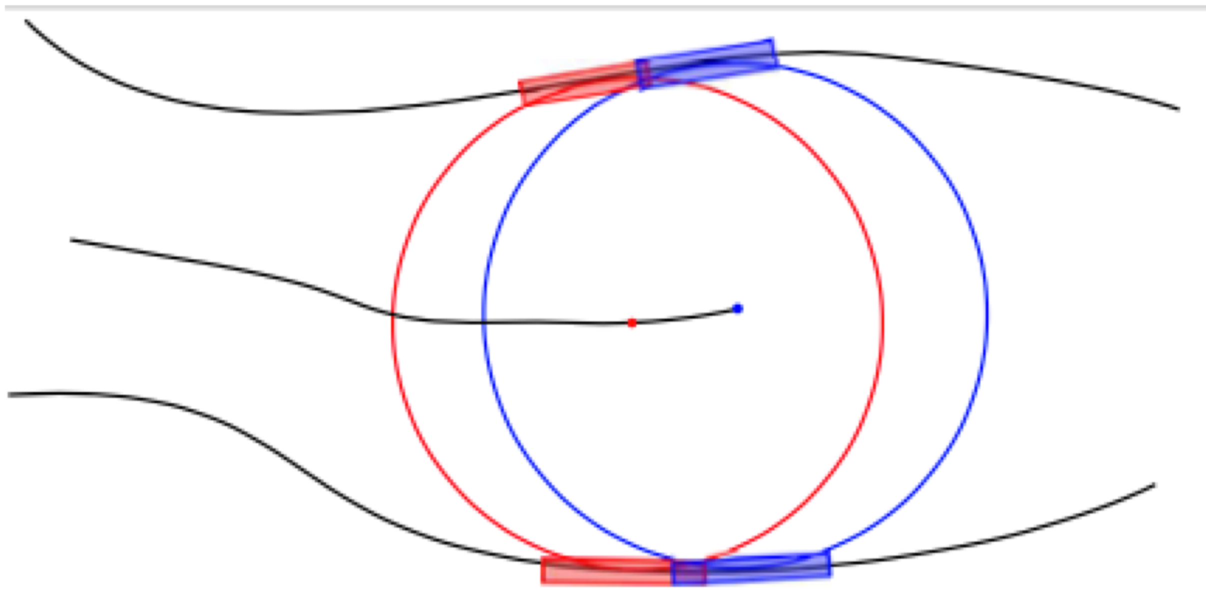
Cercle max à epsilon près



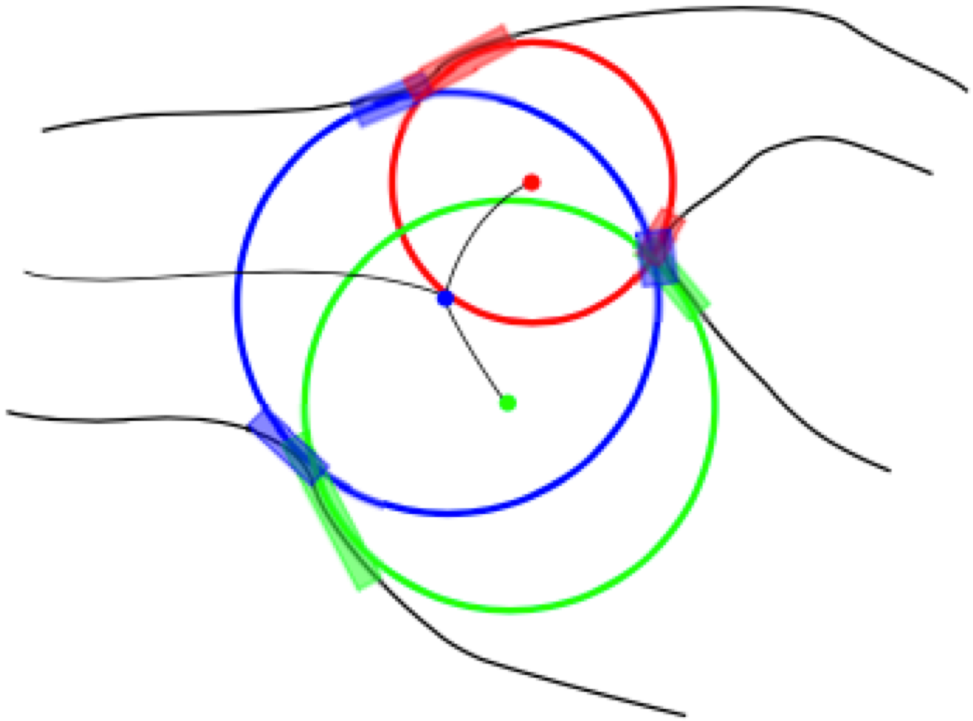
Initialisation



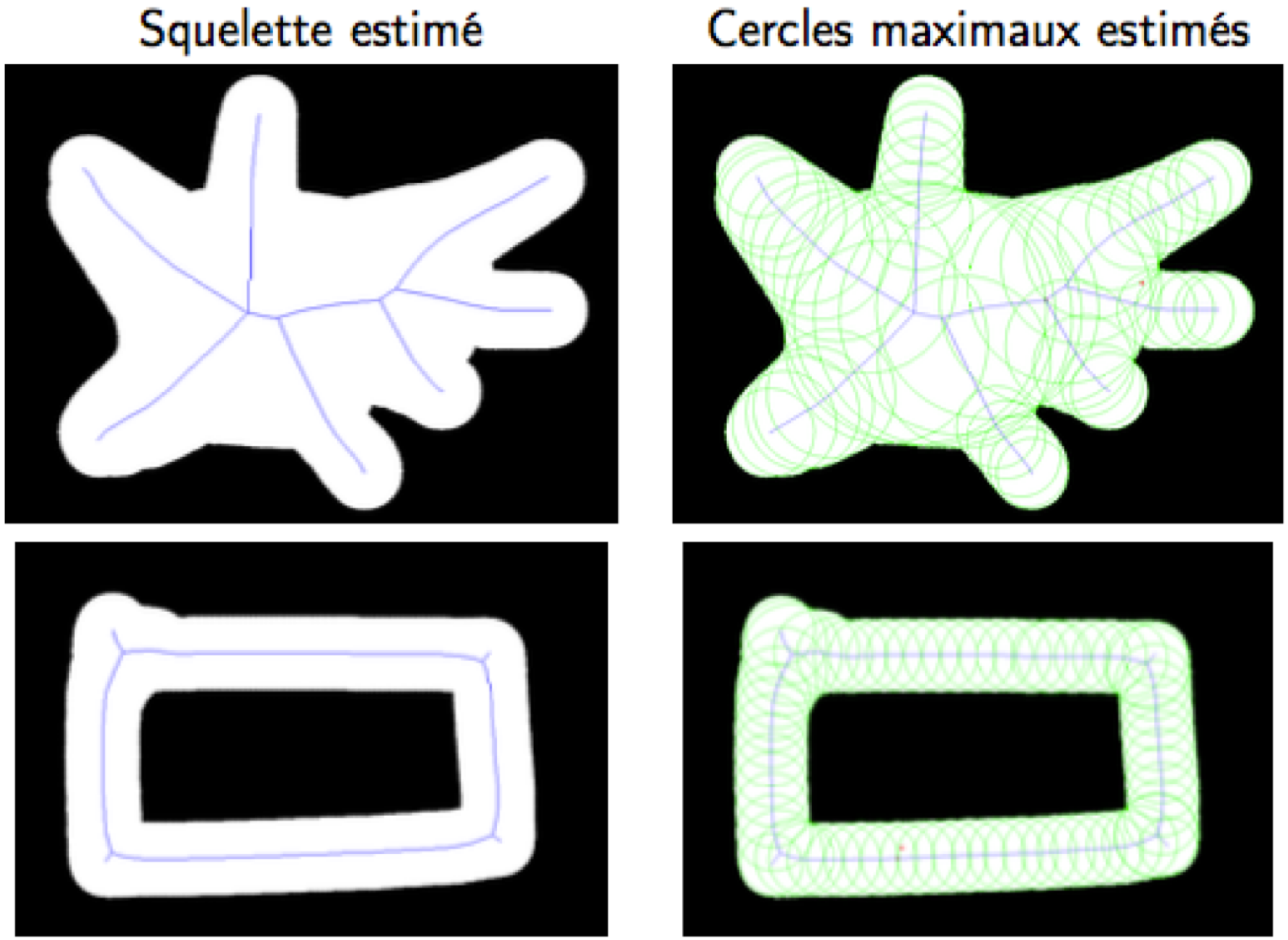
Propagation



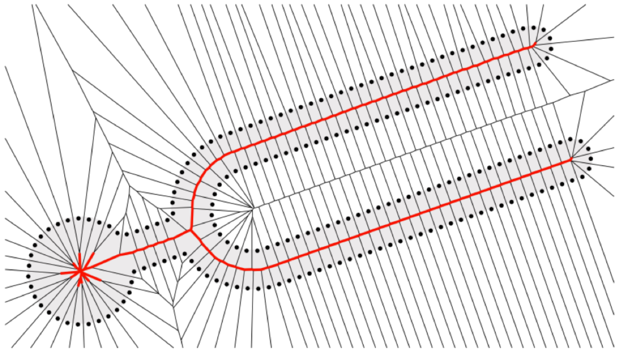
Intersection



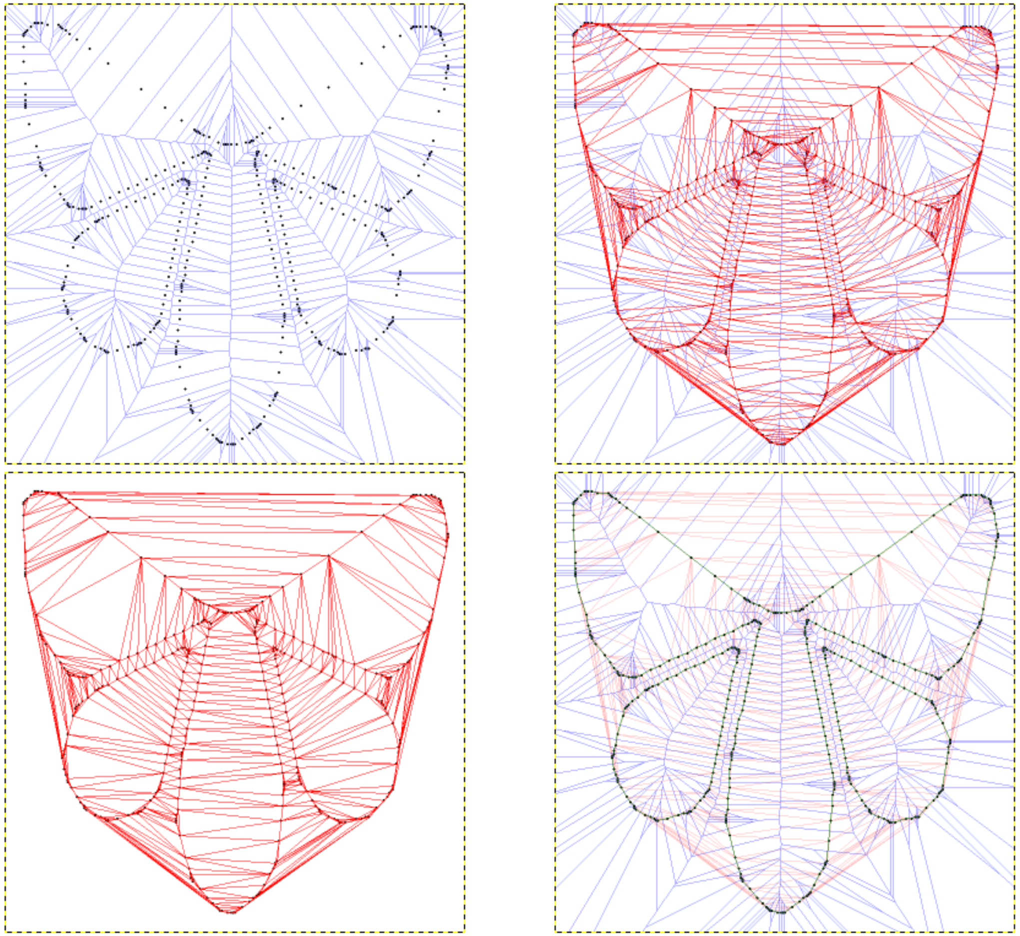
Squelette estimé



Application of medial axis to shape reconstruction

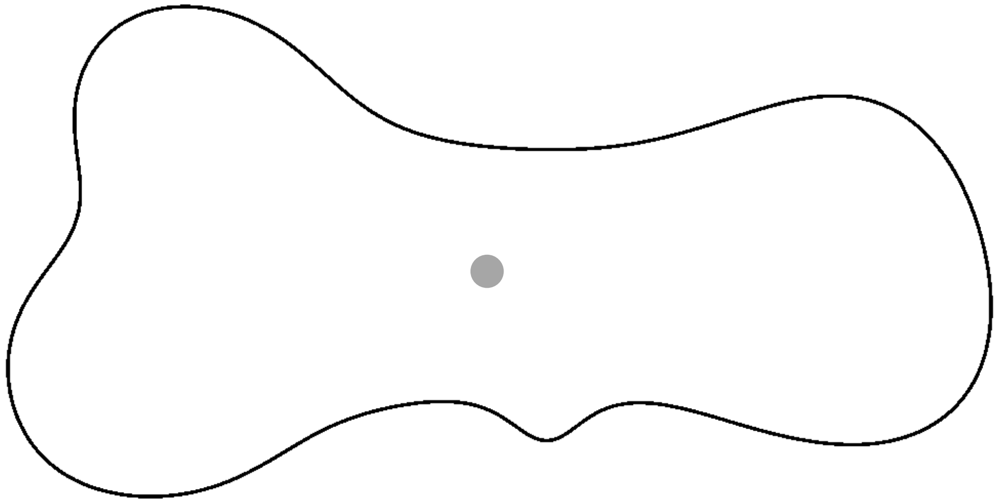


From Voronoï



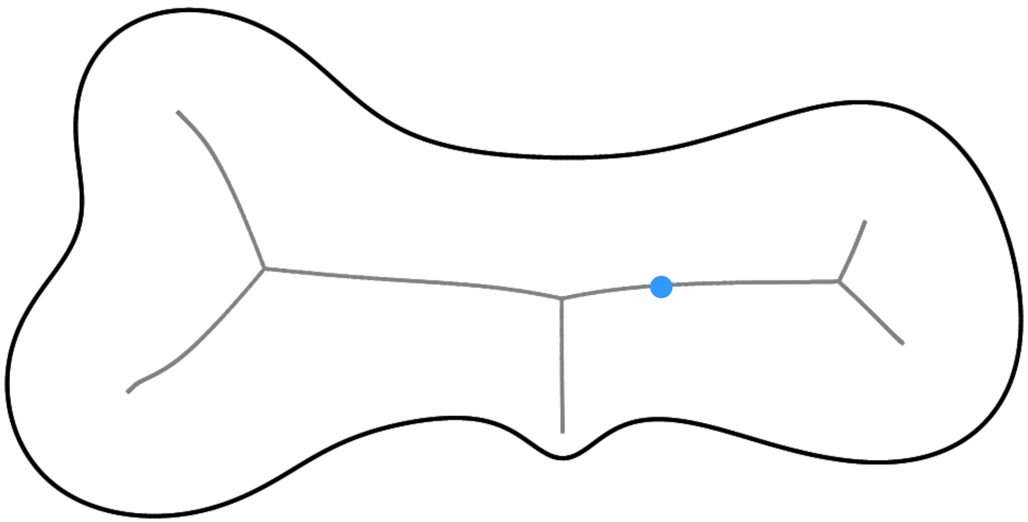
Shape center

- A center point is needed in various applications
  - Shape alignment
  - Motion tracking
  - Map annotation



Intuition

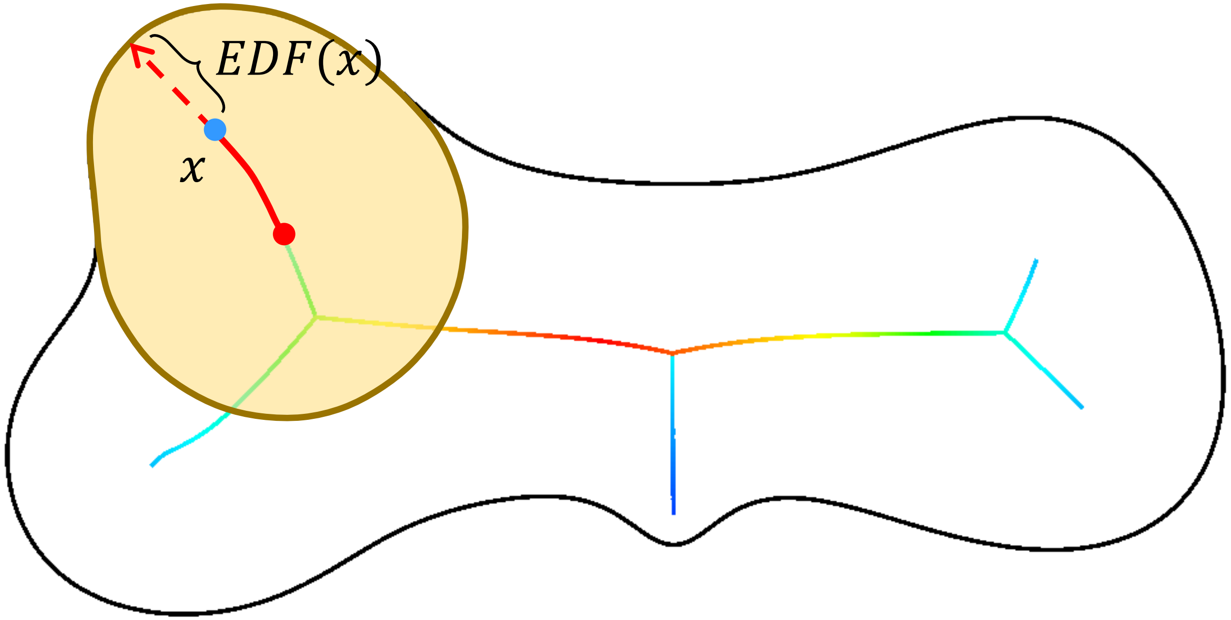
- Measure the shape elongation around a medial axis point
  - By the length of the longest “tube” that fits inside the shape and is centered at that point



EDF

- Extended Distance Function (EDF): radius of the longest tube

$$\boxed{EDF(x) = sup_t\, r_t(x)}$$

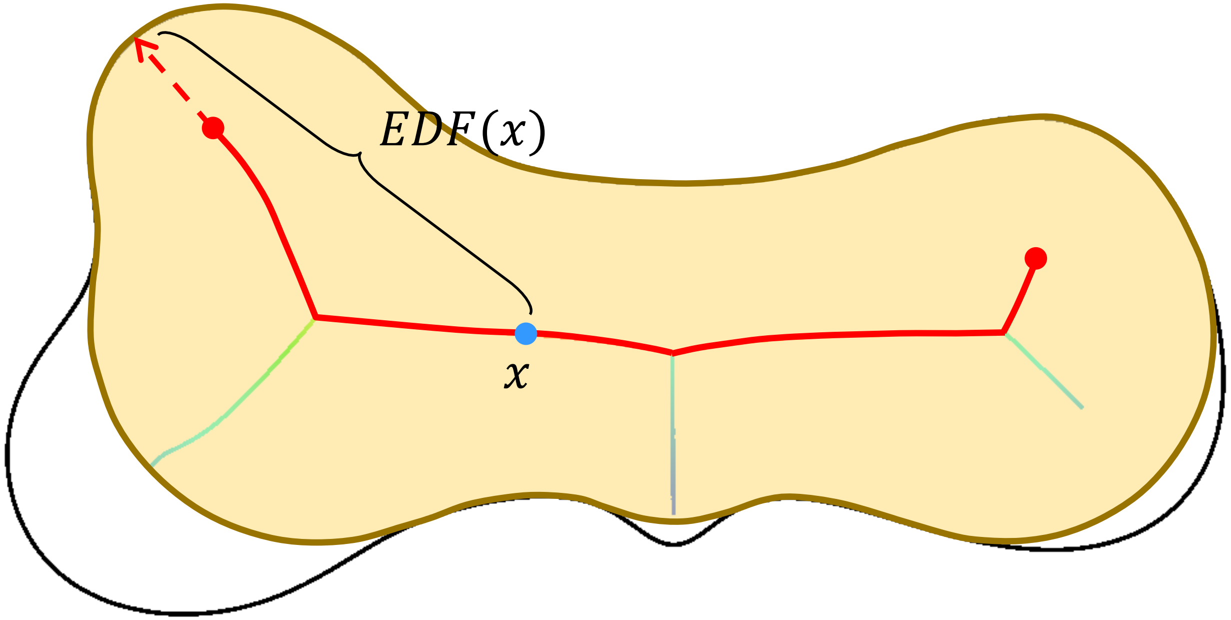


Simply connected shape

EDF

- Extended Distance Function (EDF): radius of the longest tube

$$EDF(x) = sup_t\, r_t(x)$$



Simply connected shape

EDF

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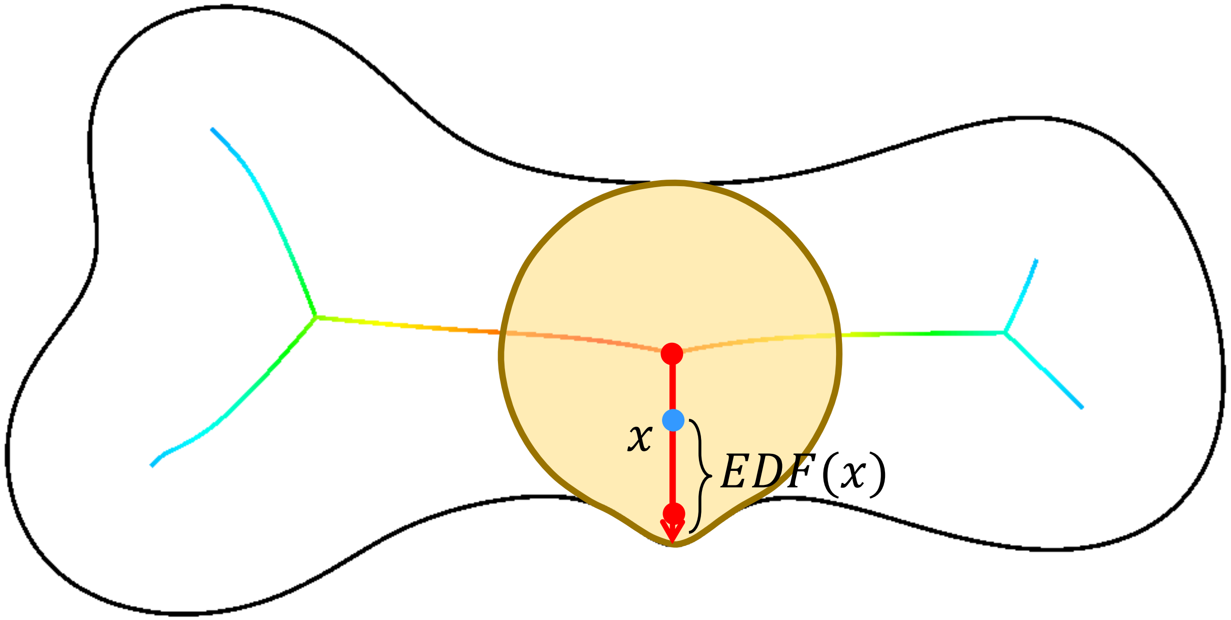
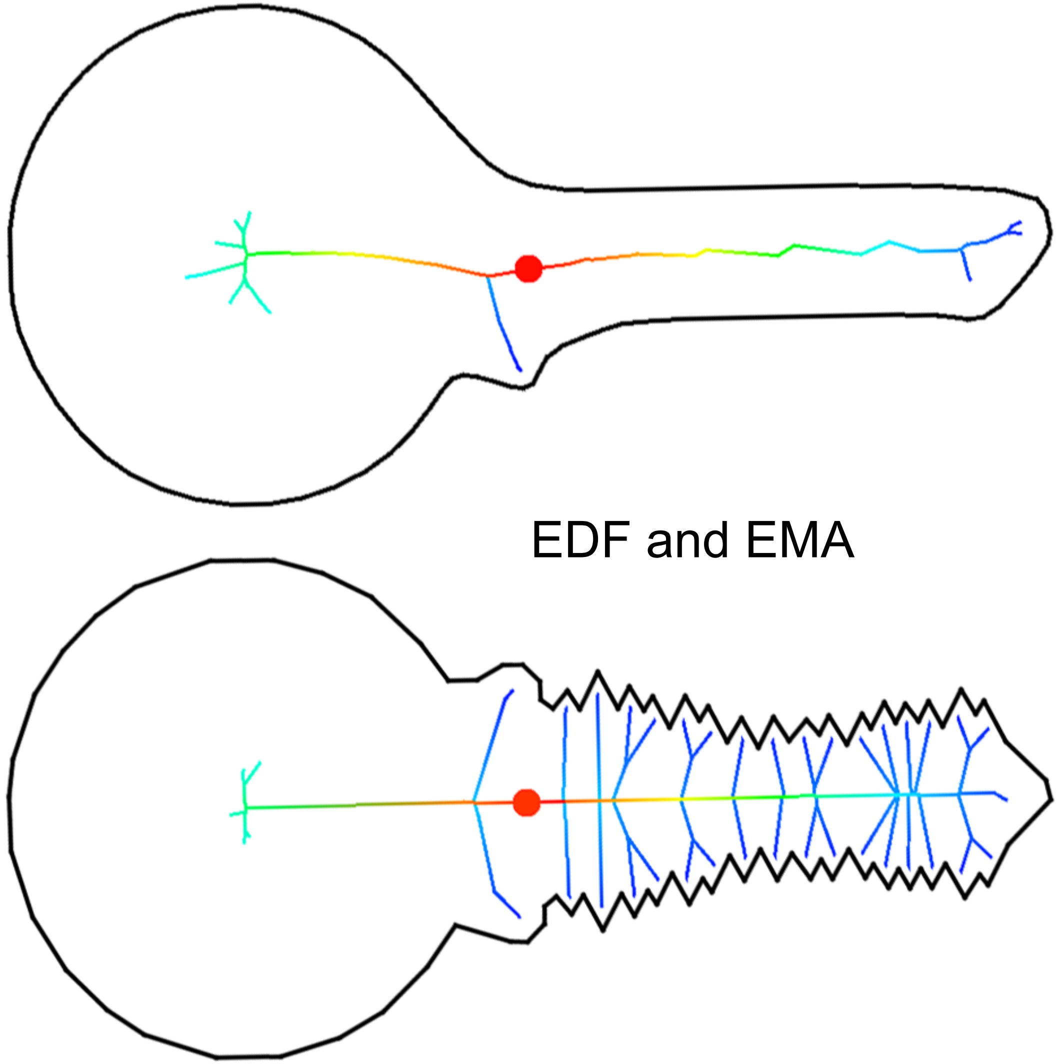


Simply connected shape

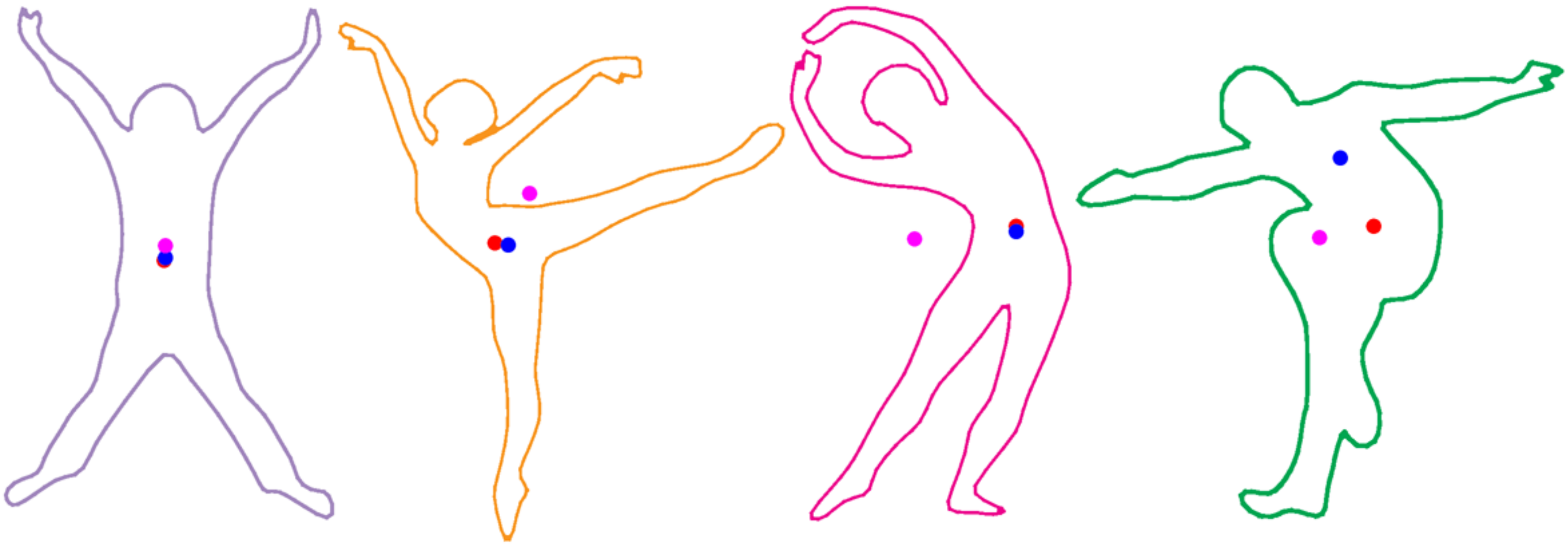
Comparison with PR/MGF

- EDF and EMA are more stable under boundary perturbation



Application: Shape alignment

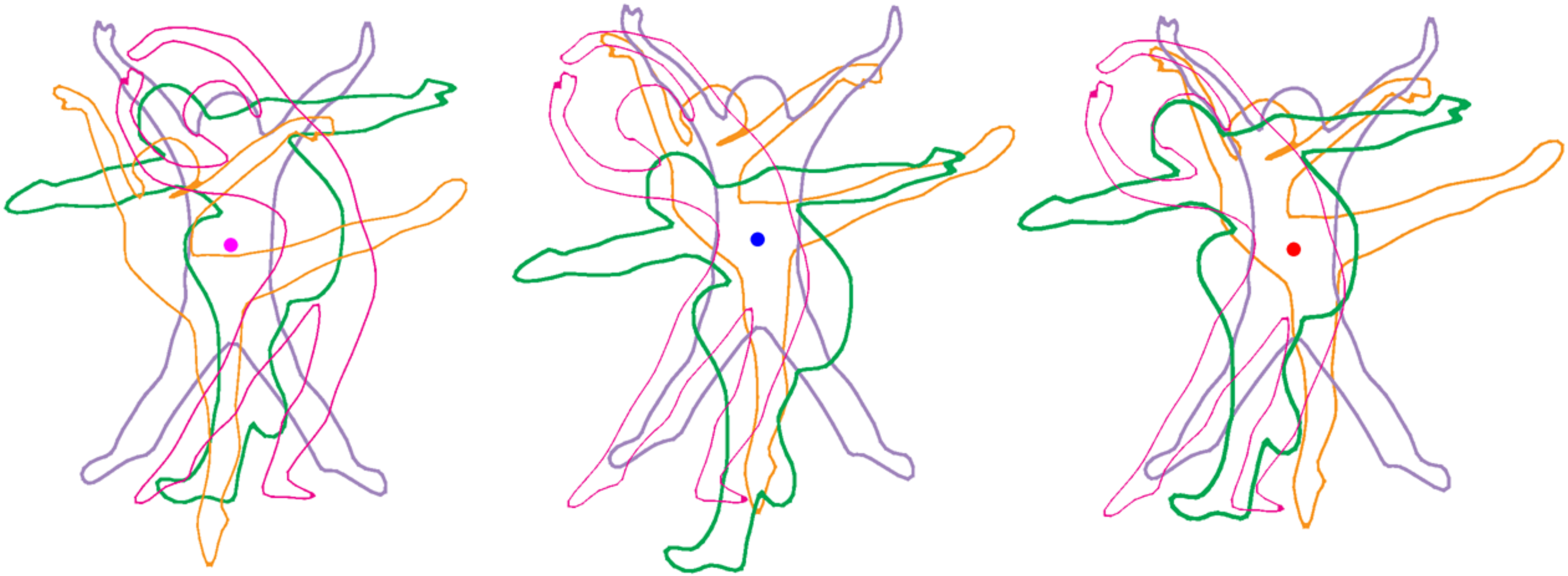
- Stable shape centers for alignment



● Centroid ● Maxima of PR ● EMA

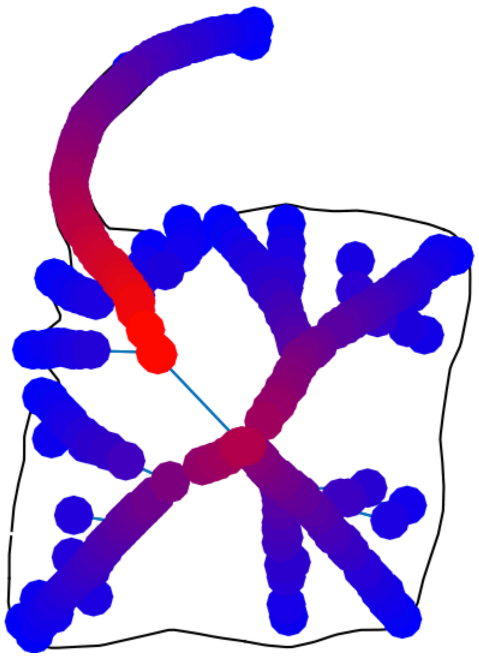
Application: Shape alignment

- Stable shape centers for alignment

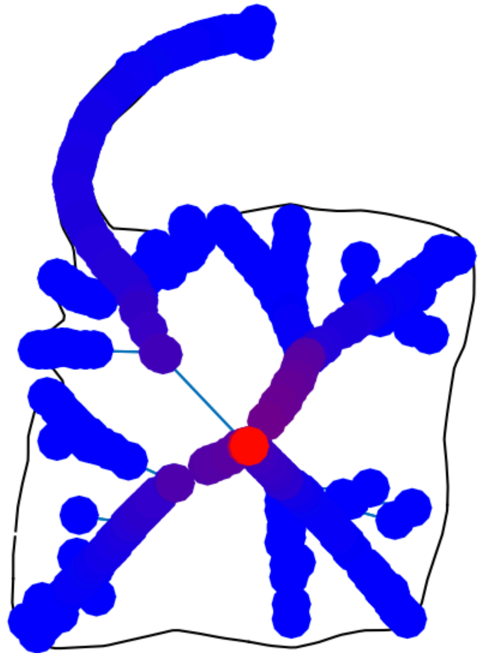


Centroid Maxima of PR EMA

A weigthed version of EDF: WEDF



EDF

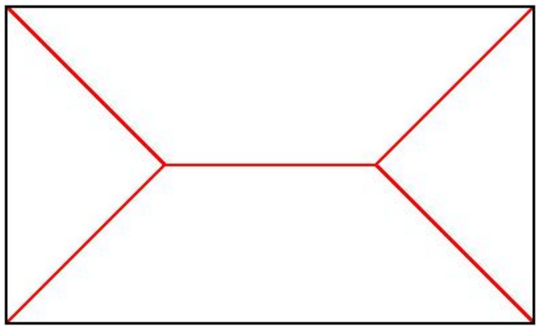


WEDF

EDF is perturbed by long but small features

Medial Axes in 3D

- MA are curves (1D) in a 2D object,
- and surfaces (2D) in a 3D object.

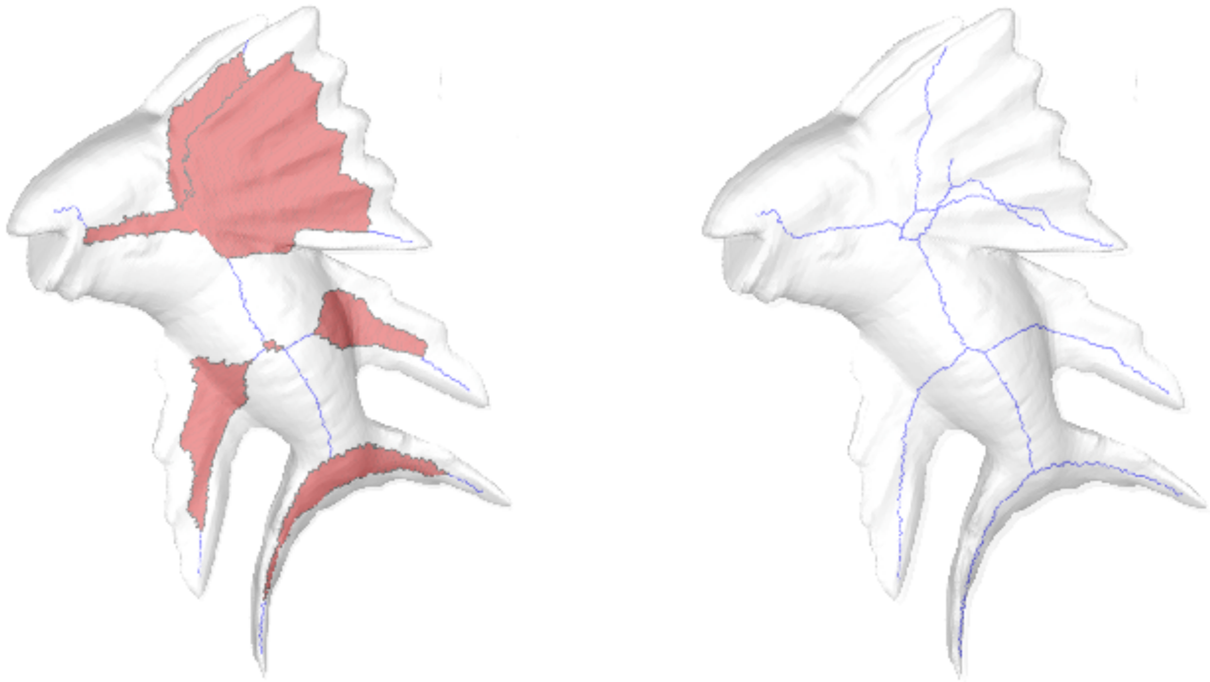


2D MA

3D MA

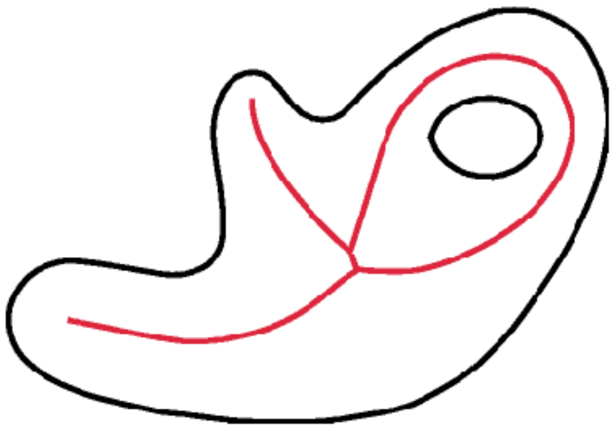
Medial Axes in 3D

– A mix of curves and surfaces, or ‘just’ curves

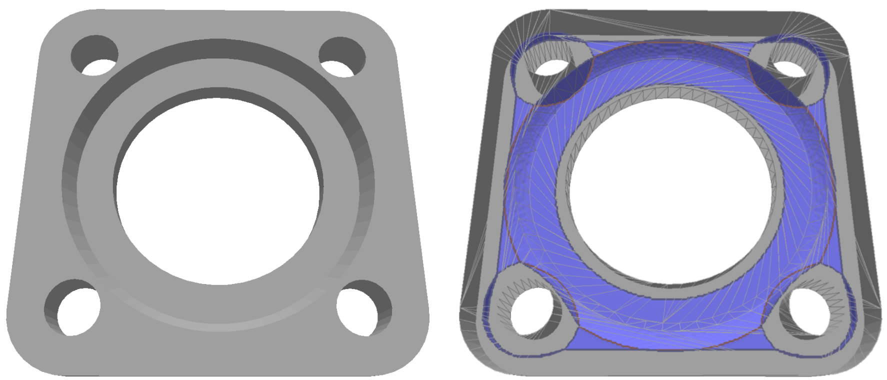


Medial Axes in 3D

- Properties
  - ✓ Preserves object's topology
    - 2D: # of connected components of object and background
    - 3D: # of connected components of object and background, and # of tunnels



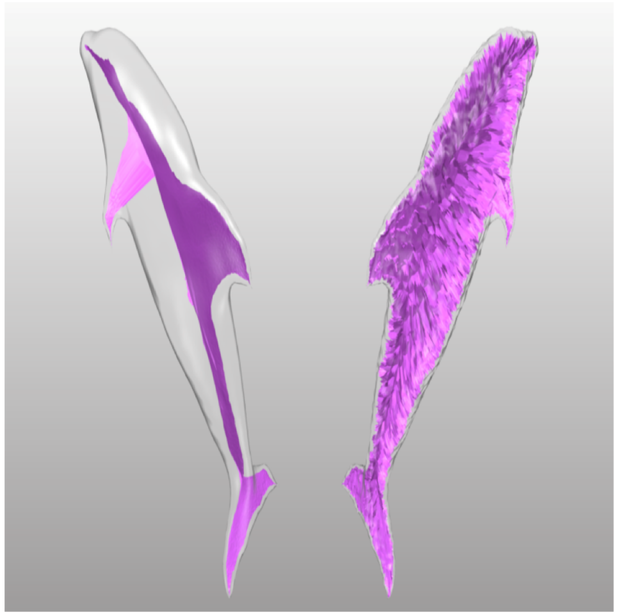
A 2D shape with 1 object component and 2 background components



A 3D shape with 5 tunnels

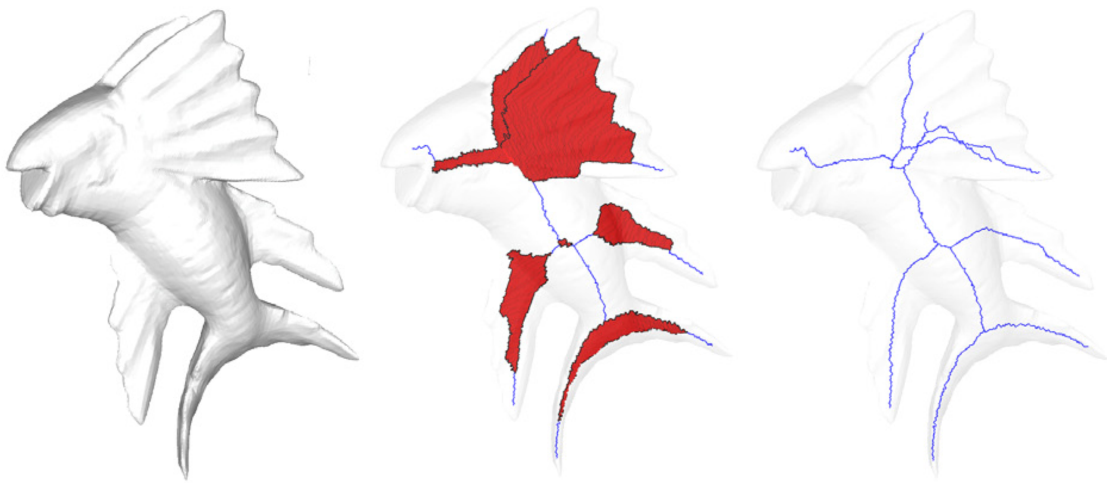
Medial Axes in 3D: still not robust!

- Change with noise!



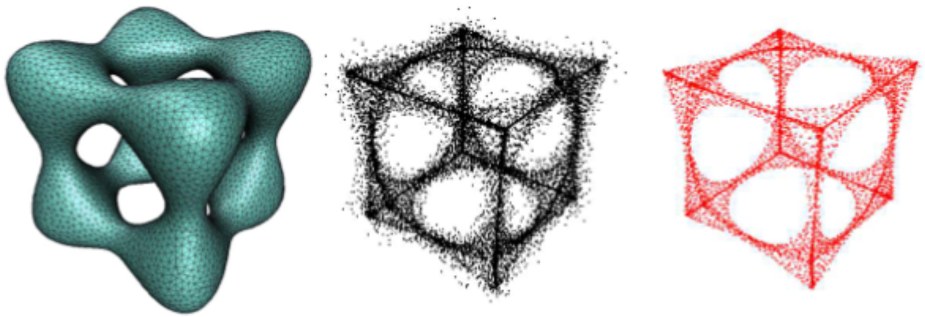
Thinning algorithms in 3D

- Work on cell complexes



Medial axis / skeleton approximation from points sampled on surfaces

- Problems with sliver in 3D …
  - Replace Voronoï balls by polar balls
  - Each sample point promotes two poles : the furthest vertex on its Voronoï cell and the furthest vertex on the opposite direction



- **Inner poles converge to the medial axis (MA), as the density of the sampling increases** (Amenta et al. 01)

Application to Surface reconstruction

Determination of polar balls from the point set (the MA could then be approximated by a subset of the Voronoï graph, with **polygonal faces**)

- Shape ≈ set of inner polar balls
- Power diagram of poles weighted by their polar balls radii
  - Surface ≈ set of inner cells in the power diagram
  - MA ≈ set of inner cells in the **regular triangulation** which is dual to the power diagram

For the lab

- Reconstruction in 3D
- Use 2D algorithm
- Additional knowledge with the image set