propriété 1

$$(r^{(p)} \mid d_j) = 0 \qquad j = 0, \dots, p-1$$

$$r^{(p)} = r^{(0)} - \sum_{i=0}^{p-1} \alpha_i A d_i$$

relation entre $x^{(p)}$ et $x^{(0)}$

$$\left(b - Ax^{(p)} = b - Ax^{(0)} - \sum_{i=0}^{p-1} \alpha_i A d_i \right)$$

$$(r^{(p)} \mid d_j) = (r^{(0)} \mid d_j) - \alpha_j (A d_j \mid d_j)$$

$$\alpha_j = \frac{(d_j \mid r^{(j)})}{(d_j \mid A d_j)}$$

$$= (r^{(0)} \mid d_j) - (d_j \mid r^{(j)})$$

$$= 0 \quad \text{(propriété 0)}$$

Propriété 2

u_j colonne de A

$$(r^{(p)} \mid u_j) = 0 \qquad j = 0, \dots, p-1$$

$$(r^{(p)} \mid u_j) = \left(r^{(p)} \mid d_j - \sum_{i=0}^{j-1} \beta_{ji} d_i\right)$$

relation entre d_j et u_j de la A-ortho de GS

$$= \underbrace{(r^{(p)} \mid d_j)}_{= 0 \text{ (propriété 1)}} - \underbrace{\left(r^{(p)} \mid \sum_{i=0}^{j-1} \beta_{ji} d_i\right)}_{= 0} \qquad i < p$$

$$= 0$$

Propriété 3

$$\left(r^{(p)} \mid d_p\right) = \left(r^{p} \mid u_p\right) + \underbrace{\left(r^{(p)} \,\Big|\, \sum_{j=0}^{p-1} \beta_{pj}\, d_j\right)}_{=0 \ (\text{propriété } 1)}$$