3 - OPTIMAL CAPITAL STRUCTURE IN IMPERFECT CAPITAL MARKETS

Part A - Taxes

Dr Laurent GERMAIN
Professor of FINANCE

LEVERAGE DOES MATTER

- Debt levels change over time
- Debt levels vary across industries
- Debt levels are similar within industry
- An increase in firm's debt level is perceived as good news for the firm's stock holders => **SIGNALLING EFFECT**

Determining firm's capital structure is an exercice in exploiting market imperfections

Dr Laurent GERMAIN
Professor of FINANCE

CAPITAL STRUCTURE AND CORPORATE TAXES

- Let t_c to be the corporate tax rate on profits

- Assume that individual investors do not pay taxes on their personal income

Value of an Unlevered firm = $$V_u = \underbrace{\frac{E(NOI)\overbrace{(1-t_c)}^{\text{Reduction of the value of the firm in proportion to the tax rate } t_c}}{r_A}}_{\text{Present value of the after tax expected net operating income}}$$

Dr Laurent GERMAIN
Professor of FINANCE

MM PROPOSITION 1 WITH TAXES

Tax code

- Interests = expense deductible $\Rightarrow$ tax shields
- Dividend ≠ expense

Annual interest rate payments*t_c = tax savings

- PV tax shields = $\Sigma[t_c *(r_D *D) / (1+r_D)^t]$
- $= (t_c *r_D D)*\Sigma [1 / (1+r_D)^t]$
- $= (t_c *r_D *D) / r_D$

PV tax shields = t_c*D

⇒ The value of the levered firm equals the value of the unlevered firm + PV of the tax savings on interests payments

$$\boxed{V_L = V_U + t_c*D}$$

Dr Laurent GERMAIN
Professor of FINANCE

PROOF OF PROPOSITION 1 WITH TAXES

Consider 2 firms – same distribution of NOI- Different capital structure (We will use the AOA method)

$$V_u = E_u$$

$$V_L = E_L + D$$

Strategies

1) Buy a proportion α of U

2) Buy a proportion α of E_L and a proportion $(1 - t_c)\,\alpha$ of D_L

Strategy 1 $\longrightarrow$ $\alpha NOI\,(1 - t_c)$

Strategy2 $\longrightarrow$ $\alpha(NOI - r_D D)\,(1 - t_c) + \alpha\, D\,(1 - t_c)\, r_D$

- Both generate an identical annual cash-flow : $\alpha NOI\ (1 - t_c)$
- $\alpha\ (E_L + (1 - t_c)\ D) = \alpha\ Vu$ by the law of one price (AOA)
- $V_L = V_u + t_cD$

Dr Laurent GERMAIN
Professor of FINANCE

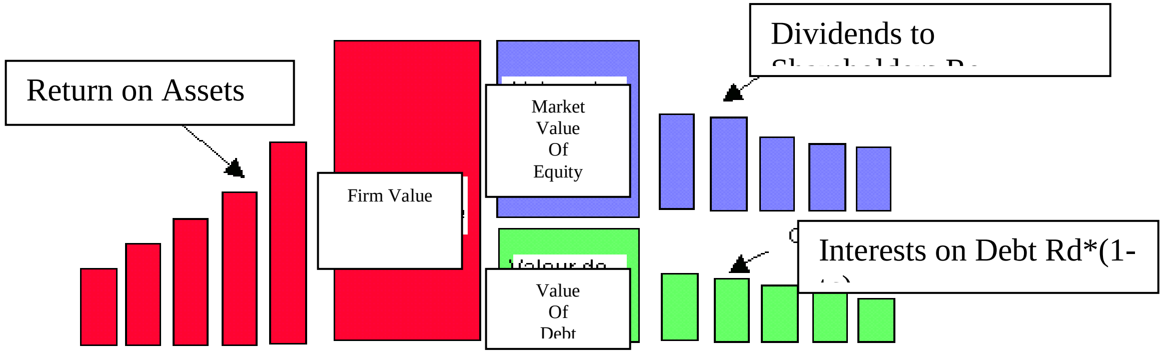
MM PROPOSITION 2 WITH TAXES

$$r_E(\text{levered}) = r_A(0) + (1 - t_c)*(r_A(0) - r_D)*(D/E_L)\ \textbf{[HAMADA]}$$

(see proof in appendix)

As in Proposition 1.0 but with taxes, the expected rate of return on levered assets is :

$$r_A(\text{levered}) = r_E(\text{levered})*(E_L/V_L) + r_D*(1 - t_c)*(D/V_L)$$



- Return on invested funds r_A=f(leveraged) on E_L + D
- Cost of Capital r_E = f(leveraged) on the equity E_LR
- Real cost of Debt is $r_D*(1 - t_c)$ thanks to annual tax savings of $t_{C*}r_D*D$ (Tax Shield)

$$\text{WACC} = r_E*(E_L/V_L) + r_D*(1 - t_c)*(D/V_L) = k$$

The same relation holds for the betas : Proposition 2 implies that the β of the equity increases with debt in the following way :

$$\beta_E = \beta_A + (\beta_A - \beta_D)(1 - t_c)\frac{D_L}{E_L} \quad \textbf{[HAMADA for Betas]}$$

Note : with $\beta_A = (\beta_E(0))$

- Inverting this formula, we obtain

$$\beta_A = \beta_D (1 - t_c)\frac{D_L}{V_u} + \beta_E \frac{E_L}{V_u}$$

Note that this formula is not a weighted average of β_D and β_E, because $D_L + E_L$ is larger than V_u.

MINIMUM OF WACC

If we identify r_E (levered) with HAMADA in WACC formula we find:

$$
\begin{aligned}
r_A(\text{levered}) &= (E_L/V_L)*[\ r_A(0) + (D/\ E_L)*(1 - t_c)*(\ r_A(0)\ - r_D)\ ] + r_D*(1 - t_c)*(D/V_L) \\
&= (E_L/V_L)*r_A(0) + (D/\ V_L)*(1 - t_c)*(\ r_A(0)\ - r_D)\ ] + r_D*(1 - t_c)*(D/V_L) \\
&= (E_L/V_L)*r_A(0) + (D/\ V_L)*(1 - t_c)*r_A(0) \\
&= r_A(0)*[\ E_L/V_L + D/\ V_L - t_c*(D/\ V_L)\ ] \\
&= r_A(0)*[\ 1 - t_c*(D/\ V_L)\ ]
\end{aligned}
$$

So :

$$\boxed{r_A(\text{levered}) = r_A(0)*[\ 1 - t_c*(D/\ V_L)\ ]}$$

Note in HAMADA we can replace $r_A(0)$ by $r_E(0)$ and obtain :

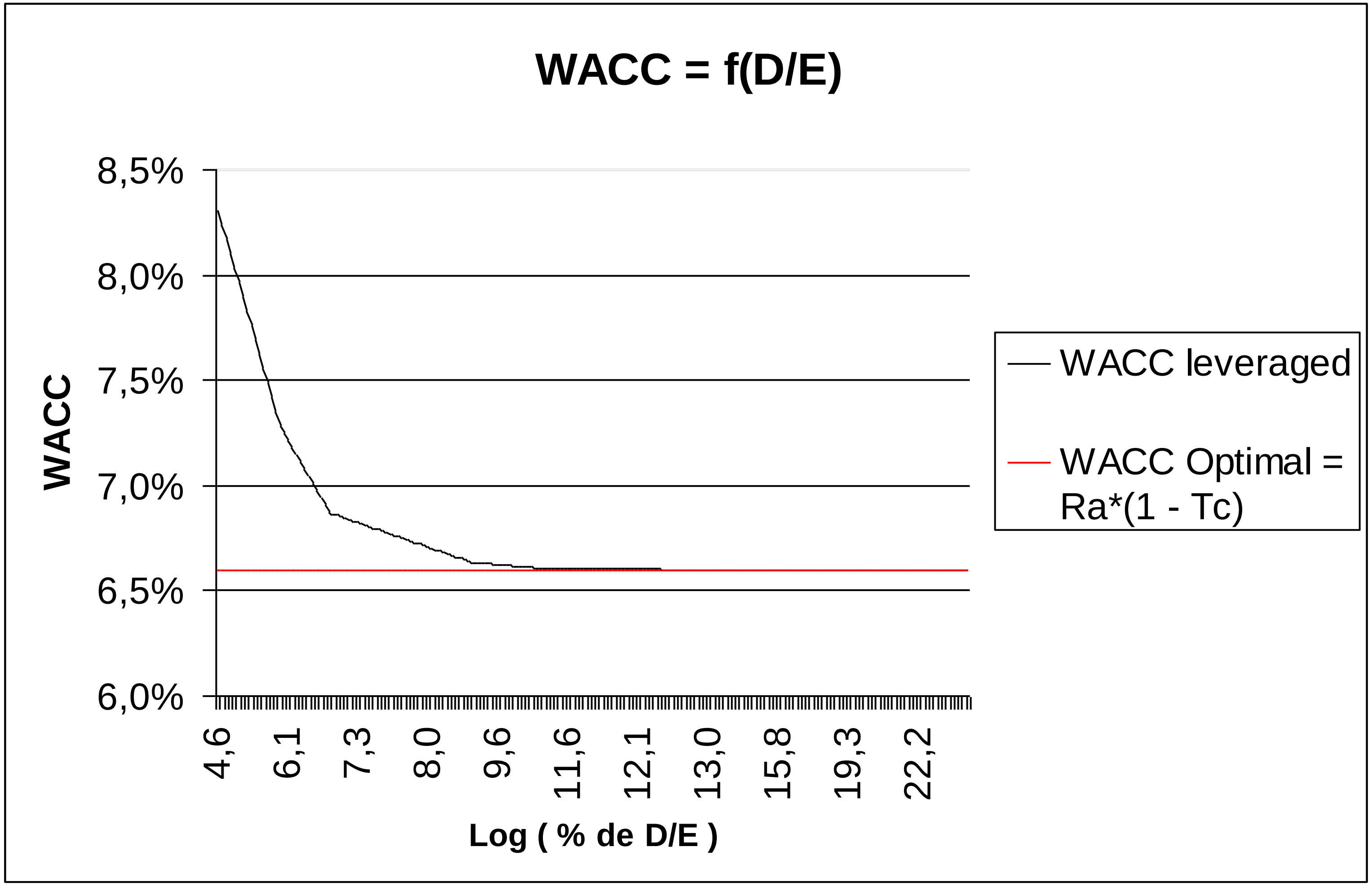
$$\boxed{r_E(\text{levered}) = r_E(0) + (1 - t_c)*(r_E(0) - r_D)*(D/E_L)}$$

- We have noticed that : WACC = r_A(levered) = $r_A(0)*[\ 1 - t_c*(D/\ V_L)\ ]$
$= [1-t_c*(D/E/(1+D/E))]*r_A(0)$

- Let us compute the value of WACC in function of D/E with initial data:

Items	Firm data
Tc	34%
Ra	10%
Rd	5%
Debt	40 000,0
Equity leveraged	10 000,0
V leveraged	50 000,0
D/E	400,0%
E/V	20,0%
D/V	80,0%
Re	23,2%
WACC leveraged	7,3%
WACC Optimal = Ra*(1 - Tc)	6,6%

  - We can notice the limit of WACC when D/E increases is $[1-t_c*]*r_A(0)$


WACC = f(D/E)
8,5%
8,0%
7,5%
7,0%
6,5%
6,0%
WACC
4,6
6,1
7,3
8,0
9,6
11,6
12,1
13,0
15,8
19,3
22,2
Log (% de D/E)
WACC leveraged
WACC Optimal = Ra*(1 - Tc)

IMPLICATIONS

0. The consistency of WACC : it is the only rate that gives the correct Fair Market Value (FMV) of the Equity but it also needs the balance between Debt and the FMV of the Equity to be computed !

1. WACC decreases with leverage thank to the tax shield

2. Min_D WACC = $r_A(0)*(1 – t_c)$

3. A marginal negative NPV project can become attractive if it is partially debt financed because of the tax subsidy.

4. WACC should include all sources of financing such as Preferred Stock and Convertibles (which should be respectively treated as Equity and Debt). Note that the tax effect should also be computed to all kinds of debt.

5. Beware of the “Opportunity Cost of Capital” = D/V* r_D + (1-D/V)* r_E (leveraged) (named r below) and which gives information about the hurdle rate of return on an investment a company should reach when undertaking a project. As a result, we shall incorporate capital structure into capital budgeting procedure.

IMPORTANT FORMULAS

- **Discount Rates**

i. **WACC = r_E*(E_L/V_L) + r_D*(1 – t_c)*(D/V_L) =k**
ii. **r_E(levered) = r_E(0) + (1 – t_c)*(r_E(0) - r_D)*(D/E_L) [HAMADA]**
iii. **r_A(levered) = r_A(0)*[1 – λ*t_c / (1 + λ)] where λ = D/E_L**

- **Values**

  - **V_L = CF / k = V_U + t_c*D** in a stationary world
  - **E_L = div_L / r_E** (Where div_L = CF – r_D*D called Cash to Equity Holders or dividends)

  - **V_L = CF / (k-g)**

model with growth g **[GORDON SHAPIRO]**

  - **E_L = div_L / (r_E –g)**

Dr Laurent GERMAIN
Professor of FINANCE

- $V_L = CF_1 / (1+k) + CF_2 / (1+k)^2 + ... + + CF_T / (1+k)^T + V_T / (1+k)^T$

  - **Where V_T =**
    - CF_T / k in a stationary world
    - $CF_T / (k-g)$ [GORDON SHAPIRO]
    - **Accounting Value of the Balance Sheet** (Historicalpoint of view)

- $E_L = div_1 / (1+r_E) + div_2 / (1+r_E)^2 + ... + + div_T / (1+r_E)^T + E_T / (1+r_E)^T$

  (Where $div_L = CF – r_D*D$ called Cash to Equity Holders or dividends)

  - **Where E_T =**
    - div_T / r_E in a stationary world
    - $div_T / (r_E-g)$ [GORDON SHAPIRO]
    - **Cumulated Accounting Value of the Equity in Balance Sheet**

CAPITAL STRUCTURE AND STOCK PRICE WITH TAXES

With corporate taxes, stock price increases with leverage

Proceeds from debt

- Dividends
- repurshase shares at a fair price (P_L anticipated price)

$$P_L = P_u + \frac{t_c D}{N} = \frac{P_u}{1 - t_c \frac{M}{N}}$$

P_u : stock price unlevered firm
P_L : stock price levered firm
N : number of outstanding shares of U
M : number of shares repurshased at price P_L with D

(See the demonstration in appendix)

Notes : the shareholder value decreased but :

- T_c D added to the value of the Company
- stock price Increases by $\dfrac{t_C * M * P_U}{N}$
- ROE has also increased like EPS

(See demonstration in appendix)

Dr Laurent GERMAIN
Professor of FINANCE

EXAMPLE Mojihto

- Consider an all-equity firm with E(NOI) of \$ 20 000, an expected return on assets of r_A= 10 % and a tax rate of t_c= 34%

Thus, $V_u = E(NOI)*(1- t_c) / r_A$ = \$ 132 000

- The firm's CFO has decided to isue \$ 80 000 perpetual debt with r_D = 5 % and use it to retire equity. Suppose you want to find the followings.

V_L = 132 000+ 80 000 * 34% = $ 159 200

$\mathbf{V_L = V_U + t_c{*}D}$

E_L = 159 200 – 80 000 = 79 200

$\mathbf{E_L = V_L - D}$

r_E=10%+(1–34%)*(10%-5%)*(80 000/79 200)
r_E = 13,3%

$\mathbf{r_E = r_A + (1 - t_c){*}(r_A - r_D){*}(D/E_L)}$

WACC= 13,3%*(79 200/159 200)
+5%*(1–34%)*(80 000/159 200)
WACC = 8,29%

$\mathbf{WACC = r_E{*}(E_L/V_L) + r_D{*}(1 - t_c){*}(D/V_L)}$

Check :

WACC = 20 000*(1-34%) / 159 200
= 8,29 %

$\mathbf{WACC = E(NOI){*}(1- t_c) / V_L}$

r_E = [20 000 –5%*80 000]*(1-34%) / 79 200
= 13,3%

$\mathbf{r_E = [E(NOI)-D{*} r_D]{*}(1- t_c) / E_L}$

Compute on Excel...

1	A	B	C	D
2	**Items**	**Figures**	**Literate Formulae**	**Excel Formulae**
3	Tc	34%	**DATA**	**DATA**
4	E(NOI)	20 000,0	**DATA**	**DATA**
5	E(NOI) after Tax	13 200,0	**= E(NOI)* (1-Tc)**	**=B4*(1-B3)**
6	Ra	10%	**DATA**	**DATA**
7	Equity unleveraged	132 000,0	**= E(NOI) * (1-Tc) / Ra**	**=B5/B6**
8	Debt	80 000,0	**DATA**	**DATA**
9	V leveraged	159 200,0	**= Equity Unleveraged + Tc * Debt**	**=B7+B8*B3**
10	Equity leveraged	79 200,0	**= V Leveraged - Equity Unleveraged**	**=B9-B8**
11	**WACC COMPUTATION**			
12	Ra	10%	**DATA**	**DATA**
13	Rd	5%	**DATA**	**DATA**
14	Debt	80 000,0	**DATA**	**DATA**
15	Equity leveraged	79 200,0	**= Equity leveraged**	**= B8**
16	V leveraged	159 200,0	**= V leveraged**	**= B9**
17	D/E	101,0%	**= Debt / Equity leveraged**	**=B14/B15**
18	E/V	49,7%	**= Equity leveraged / V leveraged**	**=B15/B16**
19	D/V	50,3%	**= Debt / V leveraged**	**=B14/B16**
20	Re	13,3%	**= Ra + (Ra - Rd)*(1-Tc)*D/E**	**=B12+(B12-B13)*(1-B3)*B17**
21	WACC	8,3%	**= Rd*(1-Tc)*D/V + Re*E/V**	**=B19*(1-B3)*B13+B18*B20**
22	Check WACC = (E(NOI) after Tax) / V leveraged	0,0%	**= WACC - (E(NOI) after Tax) / V leveraged**	**= B21 - B5/B9**

Dr Laurent GERMAIN
Professor of FINANCE

Capital structure with personal taxes

- Investors are interested In after corporate and Personal tax cash flow

Tax advantage of debt in the corporate level

Tax disadvantage of debt in the Personal level

- tpb : personal tax rate on interest income
- tps : personal tax rate on stock income

tpb > tps $\Rightarrow$ investors must be paid higher before tax rates of return on debt income than on stock income ⇨offsets some of the tax benefits of debt in the corporate level

- The basic comparison is as follows.

  Considere a firm with \$ 1 operating income.. two possible avenues :

  (a) **Pay as interest**

  Corporate tax = 0
  Personal tax = t_{pb}
  Total tax = t_{pb}
  Net cash flow = $1 – t_{pb}$

(b) **<u>Pay as equity income (dividend / capital gain)</u>**

Corporate tax $= t_c$
Personal tax $= (1 - t_c)t_{ps}$
Total tax $= t_c + (1 - t_c)t_{ps}$
Net cash flow $= (1 - t_c)\,(1 - t_{ps})$

- **<u>Conclusion</u>** : debt is preferred to equity if

$(1 - t_{pb}) > (1 - t_c)\,(1 - t_{ps})$
and equity is preferred otherwise

- Using strategies similar to those we used to prove M & M Proposition 1 (AOA), one can show

$$V_L = V_u + \left(1 - \frac{(1-t_c)(1-t_{ps})}{1-t_{pb}}\right) \times D_L$$

- <u>Special cases</u>

a) $T_{pb} = t_{ps} = t_c = 0 \Rightarrow V_L = Vu$
b) $T_{pb} = t_{ps} \Rightarrow V_L = V_u + t_c D_L$
c) $(1 - t_{pb}) = (1 - t_c)(1 - t_{ps}) \Rightarrow V_L = V_u$

(Corporate tax advantage of debt is wiped out by the personal tax disadvantage of debt)

- If t_{pb} and t_{ps} were the same across all investors and t_c was the same across all firms, then all firms would be either all equity financed or all debt financed.

EXERCISE Let consider a company with lambda=1.2, beta=1, rm=7, rf=5, rd=5.5. The CFO changes the capital structure to lambda=1.5. Compute E if D=100. Then, compute k by two different formulas.

Dr Laurent GERMAIN
Professor of FINANCE