Outline of the course

Learning Objectives

Learn to distinguish the different types of market microstructures

Learn the different types of market participants

Understand the process and outcomes of exchanging assets under explicit trading rules.

Learn how the informed traders exploit their information advantages

Understand the impact of irrational behavior on the stock prices

Explore the bubbles and crashes phenomena

Understand the debates on financial issues

Introduce the artificial intelligence

The importance of this course

Understanding the price setting mechanism is necessary to trade.

Generally speaking, investors require transparency and conversely they need opacity when they possess private information.

Traders must know the microstructure to maximize their expected profits (optimal strategies).

To price derivatives we need understanding new problems:

- Antiselection
- Liquidity
- High Frequency trading
- Irrational behavior

Prerequisites

Basics of Market Finance: Equities, Bonds, AAO, Derivatives, Demand/Supply, Present Value

Basics of Math: Valuation in discrete and continuous time, Probability, Optimization, Stochastic Calculus, Game Theory

Basics of Corporate Finance: Financing Policy, Cost of Capital, Discounting

Grading: Powerpoint Presentation on an article

Chapters

I Introduction

New economic paradigm

Since the mid 70 the economic has moved from industrial to financial

Agency theory arises Principal (Shareholders)/ Agent (Firm executive)

Markets : investment policies

II History

Silk Road

Champagne Fairs

Markets (Paris 1724)

Market locations

Market Organizations

The nature of Markets

Any trading mechanism can be viewed as a type of trading game in which players meet at some venue and act according to some rules.

The process of exchange occurs in many ways. Buyers and sellers can contact each other directly.

Traders can gather at a central setting or communicate through a computer screen.

A single intermediary can arrange every trade, or there can be numerous individuals who meet to set prices.

Order driven Markets

Order-driven market is a financial market where all buyers and sellers display the prices at which they wish to buy or sell a particular security, as well as the amounts of the security desired to be bought or sold. Moreover, all orders are collected in an order book.

Examples:

- EURONEXT,
- NEW YORK STOCK EXCHANGE (NYSE),
- Tokyo Stock Exchange
- Hong Kong Stock Exchange

Order driven Markets

A market order is an order to buy or sell a security immediately. This type of order guarantees that the order will be executed, but does not guarantee the execution price. A market order generally will execute at or near the current bid (for a sell order) or ask (for a buy order) price. However, it is important for investors to remember that the last-traded price is not necessarily the price at which a market order will be executed.

Investor can indicate **market order at any price**. In this case, the investor does not give any price indication and relies "at best" on the market. In other words, it is in the best interests of the market and not necessarily its interests. However, this type of order has priority over "in-the-money" orders.

Order driven Markets

A limit order is an order to buy or sell a security at a specific price or better. A buy limit order can only be executed at the limit price or lower, and a sell limit order can only be executed at the limit price or higher.

Example:

An investor wants to purchase shares of ABC stock for no more than $10. The investor could submit a limit order for this amount and this order will only execute if the price of ABC stock is $10 or lower.

Order driven Markets

A stop order, also referred to as a stop-loss order is an order to buy or sell a stock once the price of the stock reaches the specified price, known as the stop price. When the stop price is reached, a stop order becomes a market order.

A buy stop order is entered at a stop price above the current market price. Investors generally use a buy stop order to limit a loss or protect a profit on a stock that they have sold short. A sell stop order is entered at a stop price below the current market price. Investors generally use a sell stop order to limit a loss or protect a profit on a stock they own.

Quote driven Markets

Quote-driven market is a financial market where designated market makers have to set bid prices and ask prices at each time. These agents are knows as specialists when they only intervene for a specific security.

Examples:

- LONDON STOCK EXCHANGE (LES),
- NASDAQ (National Association of Securities Dealers Automated Quotation) began trading on February 8, 1971, it was the world's first electronic stock market.

Hybrid Markets

Hybrid market is a financial market where stockbroker can choose to trade either via an order-book or by dealing with a market maker. This mechanism is done to prevent the collusion between market makers (see Christie and Shultz 1994)

Example:

Frankfurt Stock Exchange

Hybrid Markets

The hybridization of the organization of financial markets is sometimes expressly requested by the supervisory authorities in the interest of clients.

Indeed, some risk-averse clients prefer to use an order book for the execution of small orders and use market makers for larger orders.

The execution risk on order-driven markets is that there is a time priority, the best offer has priority and finally that there is a risk for large quantities that they will not find a counterparty.

In order-driven markets, there are limit orders that submit an order up to a price. They provide liquidity and act as a kind of market maker.

Perfect Competition Market Model

An **auctioneer** set prices which are known by all agents

Traders choose the quantity of shares that maximizes their utility function

The price is set such to maximize the **trading volume**.

Perfect Competition Market Model

Pure or perfect competition is a theoretical market structure in which the following criteria are met:

- all firms sell an identical product (the product is a "commodity" or "homogeneous")
- all firms are price takers (they cannot influence the market price of their product); market share has no influence on price;
- buyers have complete or "perfect" information – in the past, present and future – about the product being sold and the prices charged by each firm
- resources such a labor are perfectly mobile
- and firms can enter or exit the market without cost

Market Participants

Market Participants

Traders or customers: they intervene in financial market for their own personal motives. He is an agent who sells or buys financial instruments (such that stocks, bonds, derivatives, commodities) in a financial market. According to his goals, he can be seen as hedger, arbitrageur or speculator.

Brokers: they transmit orders for customers, they don(t trade for their own account.

Market makers :is a liquidity provider. At each time he sets the bid (the price at which he accepts to buy the asset) and the ask (the price at which he accepts to sell the asset). His profits come from the bid-ask spread.

Market Participants

Market Makers:

On the NASDAQ, the number of market makers is very high. Some Dealers specialize in retail clients, others in institutional investors, others in equities or agents in a region.

Finally, Market Makers must be ready to buy or sell 100 shares at any time. The average number of market makers per asset is 13.3 in 2000 compared to 11.4 in 1999.

More precisely 58.8 for the most traded shares (top 1%) and 5.7 for the least traded shares (bottom 10%)

Market Participants

Insider is a informed trader who trade by exploiting a private information in order to realize a positive profit.

Liquidity or noise trader is a trader who submits order without possessing any specific information about the fundamental value of the risky asset. However, he participates for liquidity reason (sometimes for hedging motives)

Market Participants

There are intermediaries such as:

Block traders: they combine the brokerage and dealer functions with a broader search role.

Scalpers: they plays as a dealer but has no long-run market positions.

Saitori (order clerk): in Tokyo they clear the market by matching orders rather than by trading for their own account.

Bank dealers: in FX

Primary and Secondary Markets

Primary Market

When a company issues new stocks and bonds for the first time. This takes the form of an initial public offering (IPO).

When investors purchase securities on the primary capital market, the company offering the securities has already hired an underwriting firm to review the offering and create a prospectus outlining the price and other details of the securities to be issued.

Secondary Market

The secondary market is where securities are traded after the company has sold all the stocks and bonds offered on the primary market.

On the secondary market, small investors have a better chance of buying or selling securities, because they are no longer excluded from IPOs due to the small amount of money they represent.

Anyone can purchase securities on the secondary market as long as they are willing to pay the price for which the security is being traded.

Market efficiency Hypothesis

Definitions

The weak-form of the Efficient Market Hypothesis claims that prices on traded assets already reflect all past publicly available information.

The semi-strong form of the Efficient Market Hypothesis states both that prices reflect all past publicly available information and new public information released.

The strong-form of the Efficient Market Hypothesis claims that prices instantly reflect past and public information and hidden private information.

The weak-form of the Efficient Market Hypothesis

Weak-form efficiency. A market is weak efficient if prices reflect all past information.

In a weak-form efficient market, it is not possible to predict future returns by studying the history of the series of past returns.

The largest number of tests are conducted on the assumption of low efficiency, to see if returns are correlated. However, whatever the time horizon, the results obtained are very controversial. Some econometricians conclude in favor of efficiency. While others argue that nothing can be concluded (because, for example, co-integration sometimes leads to the presence of price predictability based on past dividends)

The semi-strong form of the Efficient Market Hypothesis

A market verifies semi-strong efficiency if prices reflect all past information and public information about the health of companies (announcements of annual results, issuance of new shares, etc...).

Thus, it is not possible to take advantage of public information to predict future returns in the semi-strong form of efficiency.

To test the semi-strong form, the event study methodology is used to detect possible abnormal returns compared to a normal theoretical return.

Theoretical models are market return, CAPM and APT.

The strong form of the Efficient Market Hypothesis

A market verifies strong efficiency if prices reflect all past information and public information about the health of companies (announcements of annual results, issuance of new shares, etc.) and all private information held by agents.

Thus, it is not possible to take advantage of private information to predict future returns in the highly efficient form.

To test the strong form, so-called insider trading models are used.

Institutional investors and market makers have a priori additional information. The last ones because they collect orders from all traders. The first because they invest a lot in information research

Grossman Stiglitz paradox

The paradox of Grossman Stiglitz (1980). They establish a model in which a fraction of traders possess expensive information and the other only follow prices to trade.

If the market is efficient, in the absence of noise, all private information is incorporated into prices. And from then on, the number of agents who invest in information collapses.

So a necessary condition for the market efficiency hypothesis is that the information must be free of charge!

Price Formation Process

The strong form of the Efficient Market Hypothesis

Bid-ask spread is the difference between the ask price and the bid price. It allows the market maker to overcome some costs.

- The order processing costs. (70% of the bid-ask spread)
- The inventory holding costs (above all when he is risk-averse) (20%)
- The adverse selection costs (when he faces with better-informed trader) (10%)

The order processing costs

It must make it possible to cover the costs of personnel and equipment necessary for the execution of orders.

the order processing costs would be about 70%. The variable part depends on the size of the orders (the larger, on average, an asset is traded, the lower the variable cost of processing it).

Finally, the market maker receives a market rent that allows him to raise this cost. According to Brock and Kleidon (1992), this rent is constant during the day but higher at the opening and closing.

Roll (1984) studied this component of the differential.

The inventory holding costs

It represents a remuneration to cover the risk taken by the market maker by maintaining a stock of risky stocks.

Depending on the market and the action, the inventory cost would explain up to 25% of the bid-ask spread, see for example (Huang Stoll (1997)).

Demsetz (1968), Stoll (1978), Amihud and Mendelson (1980), Ho and Stoll (1981) studied the cost of inventory.

The adverse selection costs

The adverse selection costs. It represents a remuneration to cover the risk taken by the market maker by trading with a trader who is more informed than he is.

Depending on the market and the action, the anti-selection cost would explain up to 8%-13% of the bid-ask spread, see for example (Huang Stoll (1997)).

Copeland and Galai (1983), Easley O'Hara (1987), Glosten and Milgrom (1985) studied the anti-selection cost that liquidity providers (whether market makers or limit orders) must pass on.

The optimal stock level

The market maker must have a stock of the most popular assets to meet customer demand.

If necessary, the market maker must be prepared to sell securities short, i.e. sell securities in the short term that it does not own by borrowing them, for example, from other holders for delivery (usually institutional investors).

It may happen that the market maker wishes to maintain a zero position (this is the case in options and foreign exchange markets where positions are closed at the end of the day).

The higher the volatility of a security, the greater the risk of the stock. Finally, the stock level of an asset depends on the total portfolio managed by the market maker (diversification).

The payment period

Financial market regulators recommend that delivery times be reduced as much as possible. This prevents market makers from speculating on their stock (or even trading long overdrawn). Thus the reduction to 7 days, then 3 days or even on the same day of the delivery time greatly reduces the counterparty risk.

In the Eurobond market, Clearstream and Euroclear are responsible for the circulation of securities. These are compensation funds.

Market makers have the possibility to trade with each other to reduce their position risk (when stocks move away from the desired position). In some markets, the exchange between content providers is more important than with customers. Inter dealer Broker or IDB system is reserved for market makers.

The adverse selection problem

The market maker must display his prices so he may have to trade against an agent:

- Who submits an order on the basis of public information before the market maker has had time to change his price (opportunity to react).
- Who orders an order on the basis of a personal informational advantage.

It can be seen that bad news has a stronger impact on prices than good news, but it is buyers who tend to be more informed than sellers (O'Hara 1987).

The free option

In order-driven markets, limit orders provide a free option in the sense that an informed agent can trade (or not) to their detriment. Thus Glosten (1994) show that principals record a gain on the price change resulting from a demand for immediate liquidity, but suffer a loss when this change comes from informed traders.

However, by dividing its orders into successive tranches, the limit order reduces the risk of anti-selection, but takes a risk of execution on the transaction.

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The liquidity

The liquidity concept has three components:

- The resiliency
- The market depth
- The floating part

Resiliency

Resilience. It measures price inertia. The liquidity parameter expresses this:

$$\lambda = \frac{\partial p}{\partial \omega}$$

Thus, there is strong resilience when the liquidity parameter is low, since a small change in volumes does not imply a significant change in prices

Market depth

Market depth. It is measured by the inverse of the liquidity parameter.

Thus, a market is deep if a small price change causes a large movement of trading in the stock.

The floating part

The width of the market. It is measured by the number of shares actually traded. This volume is all the greater when the float is high.

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Capital and Monetary Markets

Capital Markets

Capital markets are related to the stock and bond markets. As a result, the institutions operating in capital markets – stock exchanges, commercial banks and all types of corporations, including non-bank institutions such as insurance companies and mortgage banks –

The institutions operating in the capital markets access them to raise capital for long-term purposes, such as for a merger or acquisition, to expand a line of business or enter into a new business, or for other capital projects. Entities that are raising money for these long-term purposes come to one or more capital markets. In the bond market, companies may issue debt in the form of corporate bonds, while both local and federal governments may issue debt in the form of government bonds.

Monetary Markets

While investors are willing to take on more risk and have patience to invest in capital markets, money markets are a good place to "park" funds that are needed in a **shorter time period** – usually one year or less.

The instruments used in the money markets include deposits, collateral loans, acceptances and bills of exchange. Institutions operating in money markets are central banks, commercial banks and acceptance houses, among others.

Liquidity is often the main purpose for accessing money markets. When short-term debt is issued, it's often for the purpose of covering operating expenses or working capital for a company or government and not for capital improvements or large-scale projects.

The Market purposes

Financial markets can be classified in terms of whether the market is:

For new or existing claims (primary or secondary market)

For short-term or long term instruments (money or capital market)

For domestic or foreign securities

For immediate future or optional delivery (cash, future or option market)

Issuers

1) Issuers:

- Governments
- Corporations
- Banks
- Municipalities

Objectives

- To raise funds
- The hope to sell securities at the fair market price
- To provide liquidity to the shareholders
- To design and issue debt securities that best suit the needs of the firms and any shareholder

Overview

2) Financial intermediaries:

- Primary dealers
- Other dealers or brokers
- Investment banks
- Credit rating agencies
- Data providers
- Servicing companies

Objectives

- To make money by facilitating trades
- To provide information to the market
- To reduce transaction cost to market participants
- To provide a primary mechanism for issuers of securities
- To provide risk-management services

Overview

3) Investors:

- Governments
- Pension funds
- Mutual funds
- Insurance companies
- Commercial banks
- Hedge funds
- Individuals

Objectives

- To earn a rate of return for a given level of risk
- Diversification
- To speculate on interest rate movements
- To modify their interest rate risk exposure

Regulatory bodies

4) there are many authorities:

AMF (autorité des marchés financiers)

Banque de France

Ministère de l'économie

And so on

Objectives

- Insuring a fair, orderly and open market
- Implementing monetary/economic policy
- Assuring the safety and soundness of the banking system

Derivatives

Outline of this section

Chapter 1: Introduction

Outline of the topic

Definition and Useful Vocabulary

Uses

Some History

Pricing Principles

Definition and Useful Vocabulary

A derivative product is a financial security whose value depends on 1 or several UA

$$\pi_t = f\left(t, S_t^0, S_t^1, \ldots, S_t^N\right)$$

3 main product families:

Forward, Futures,

Options

Swaps

Definition and Useful Vocabulary

Where are derivatives are traded ?

Exchange-traded Markets : Futures and Options

Liffe, Eurex, CBOT, NYMEX

Standardized products

Trading floor or electronic platform (becoming the standard)

Over-The-Counter (OTC)

No-standard-product (customized)

- Phone/dealer
- Credit Risk

Definition and Useful Vocabulary

The trading of a financial asset involves at least four discrete steps

A buyer and a seller must locate one another and agree on a price

The trade must be cleared (the obligations of each party are specified)

The trade must be settled (the buyer and the seller must deliver the cash or securities to satisfy their obligations in the required period of time)

Ownership records are updated

Definition and Useful Vocabulary

There are at least four different measures of a market and its activity

Trading volume: this measures the number of financial claims that change hands

Market value: market value is the sum of the value of the claims that could be traded without regard to whether they have traded

Notional value: this measures the scale of a position

Open interest: it measures the total number of contracts for which counterparties have a future obligation to perform

Definition and Useful Vocabulary

Transaction costs and the bid ask spread

- Buying and selling a financial asset
- Brokers: commission (fees)
- Market Makers (bid-ask) : the order processing cost; the inventory holding cost; the adverse selection cost.

Short-selling when the price of an asset is expected to fall

- First: borrow and sell an asset (get $$)
- Then: buy back and return the asset (pay $)

→ If price fell in the mean-time: profit=$$-$

Uses

- Hedging (Risk Management)
- Speculation: Directional views on the future of the market
- To cash in an arbitrage profit
- To change the nature of the liability (fixed rate-floating rate, rate-share, etc.)
- To reduce the transaction costs
- To bypass regulations
- To create structured products (Financial Engineering)

Uses

- **Risk-management : irrationality :**

- **Risk aversion**
- **Feedback trading**
- **Herding**
- **Overconfidence**

Some History (1/2)

600 before JC : Thalès uses

580 before JC : Aristotle mentions a derivative strategy

1848: The Chicago Board of Trades opens. First standardized futures (1865).

1900: Louis Bachelier defends his doctoral thesis in mathematics at the Sorbonne. He paves the way to stock price modelling in continuous time which will be the milestone of the Black-Sholes Model

Some History (2/2)

1965: Paul Samuelson (option pricing model) but under historical probability measure

1973: The Chicago board Options Exchange opens

1973: Black, Scholes, Merton publish articles about options pricing which allow to throw off the weakness of previous models.

1983: Options on equity indices

1994: Options on default risk

2000: Credit derivatives

Pricing Principles

Hypotheses:

H1: Frictionless Market (no transaction costs, no bid-ask spread, no margin calls, no restrictions on short-selling, no taxes)

H2: no counterparty risk

H3: Competitive markets (traders are price takers)

H4: AAO

Chapter 2: Call/Put options

Derivative

Definition: a derivative is a contract that derives its value from the performance of an underlying entity (an asset, index, or interest rate, etc.)

3 types of derivatives : European, American, Bermudean

There are exotic (second generation) options more complex to price (asian, look-back, barrier, baskets, etc.) because they include characteristic such that:

- The value of the option depends on the historic of the UA' s value (path-dependent options such as look-back, asian barrier)
- The order of this option: the first rate options have a payment that only depends on the underlying itself. The options with a superior order have a payment contingent to one or several other options (option on option)

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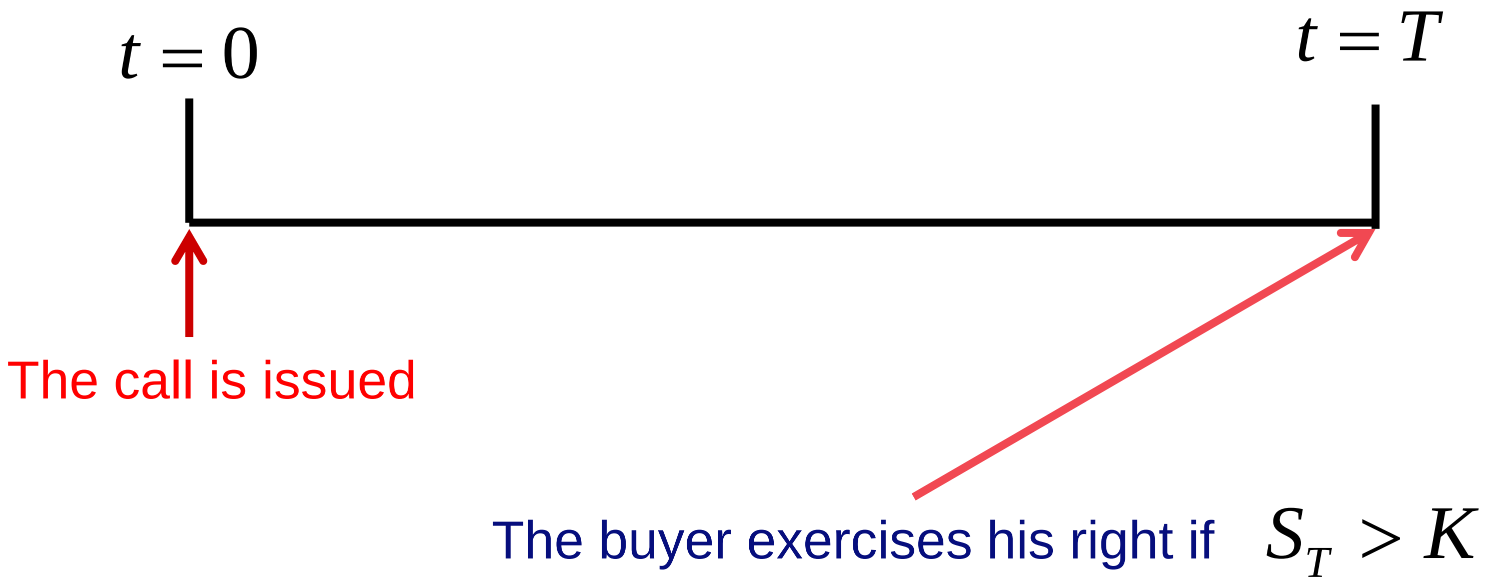
Call option (1/5)

Definition: a Call is the right but not the obligation to buy the UA at a specified future date T (expiry or maturity) and at a predetermined price K (strike price)

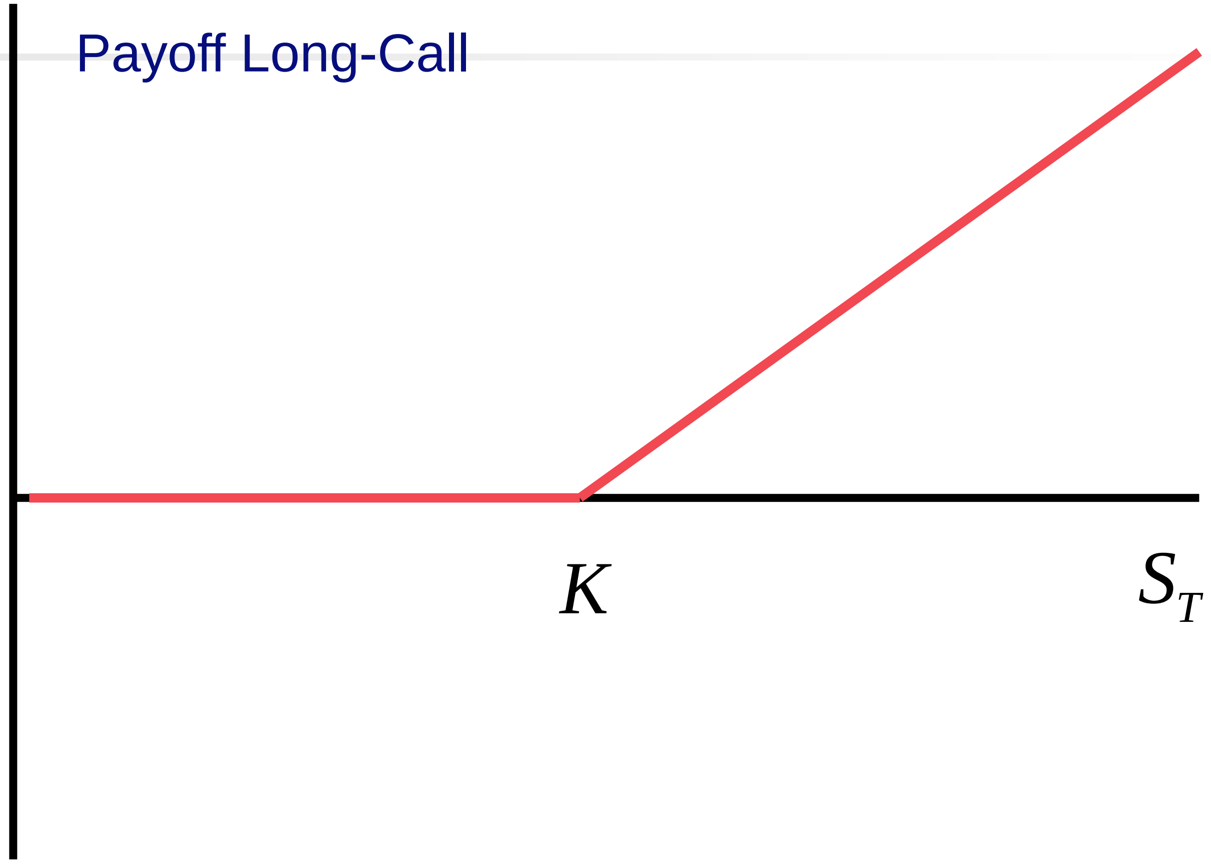
The seller has the corresponding obligation to fulfill the transaction—that is to sell —if the buyer (owner) "exercises" the option.

The buyer pays a premium to the seller for this right

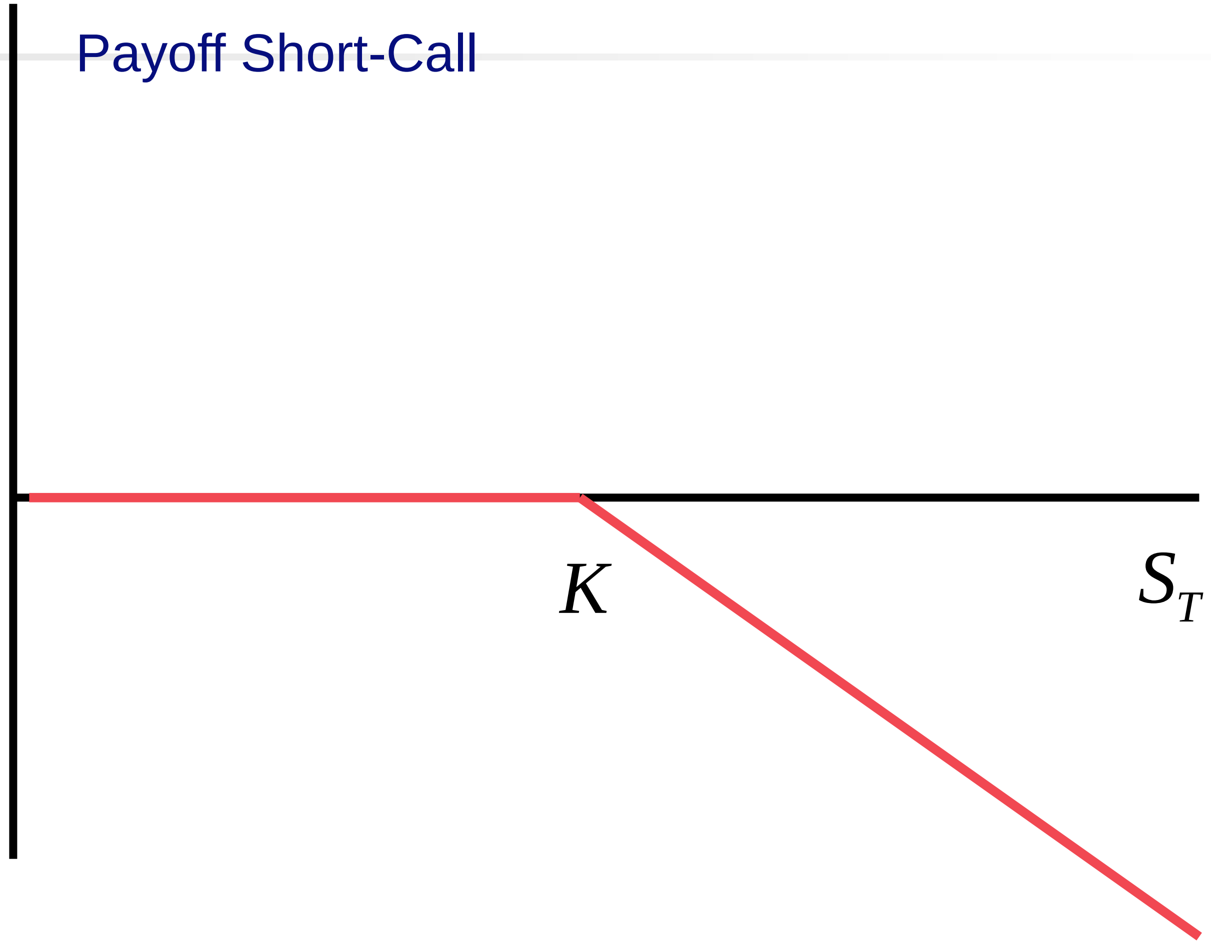
Call option (2/5)



Call option (3/5)



Call option (4/5)



Call option (5/5)

Exercise 1: At time $t = 0$

Mr Douglas owns 100$ he buys an underlying asset $S_0 = 100\$$

Sir Heath chooses to buy 10 calls, the value of each call is equal to $C = 10\$$

a) At time $t = T$, $S_T = 150\$$ calculate the different gains
b) At time $t = T$, $S_T = 99\$$ calculate the different gains
c) Make concluding remarks

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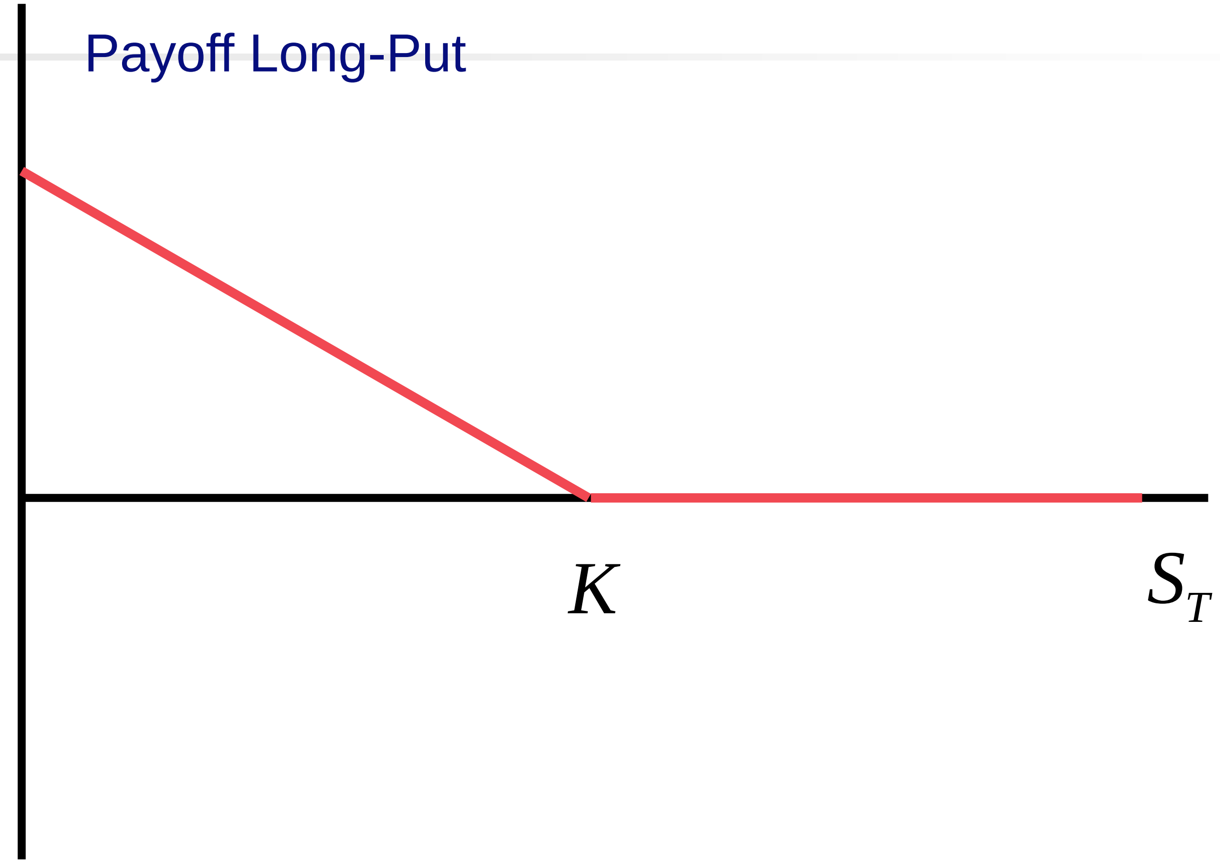
Put option (1/3)

Definition: a Put is the right but not the obligation to sell the UA at a specified future date T (expiry or maturity) and at a predetermined price K (strike price)

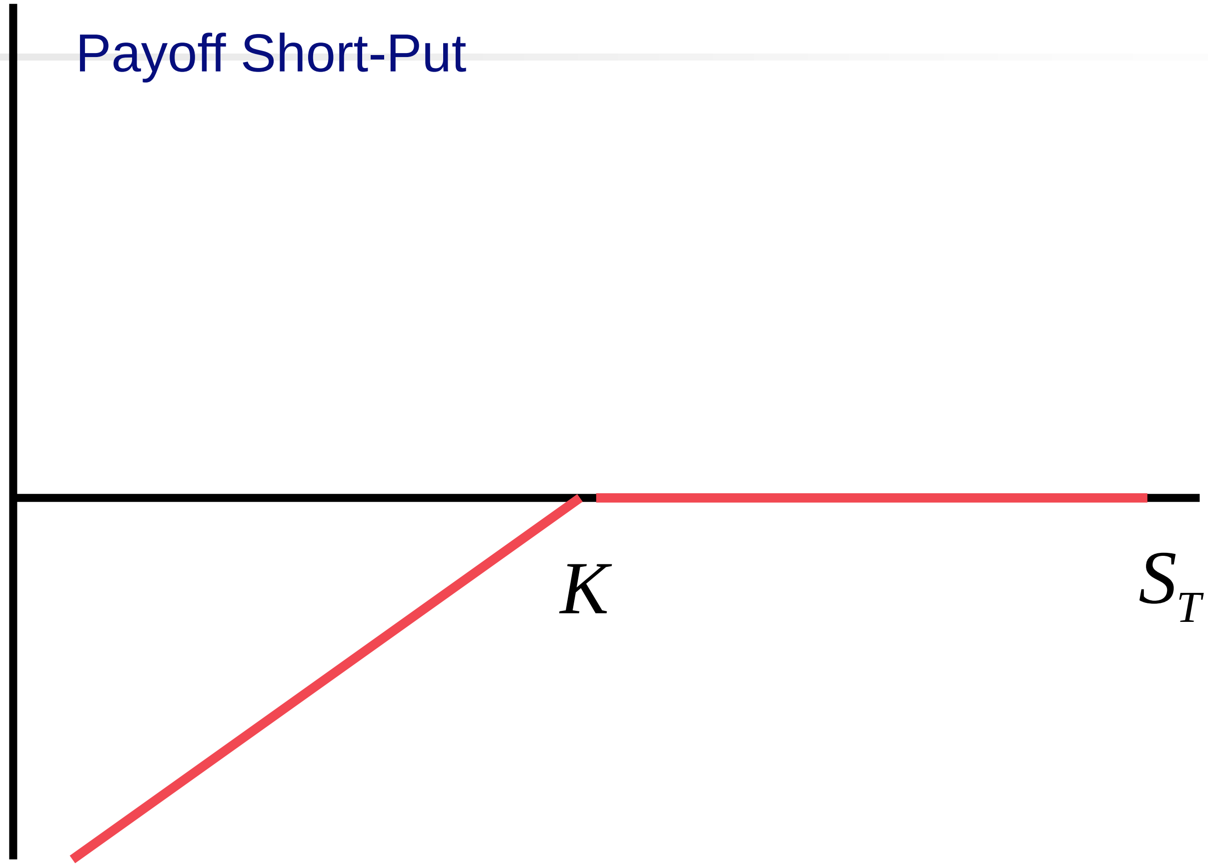
The seller has the corresponding obligation to fulfill the transaction—that is to buy —if the buyer (owner) "exercises" the option.

The buyer pays a premium to the seller for this right

Put option (2/3)



Put option (3/3)



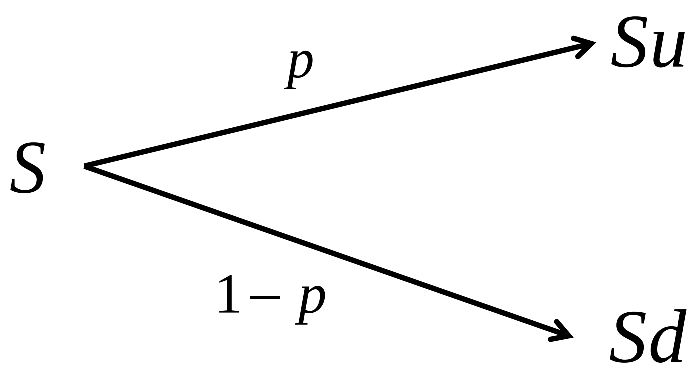
Chapter 3: Cox Ross Rubinstein Model

CRR (1/10)

Hypotheses: We consider a financial market, with only one auction and where two assets are traded

A risk-free asset with a return r

A risky asset yields a return $\tilde{R} = \dfrac{P_1}{P_0}$



CRR (2/10)

First property: show that the AAO implies that :

$$d < r < u$$

Deduce the existence of risk-neutral probability measure such that :

$$E_{P^*}\left[\tilde{R}\right] = r \qquad \textbf{with} \qquad P^*\left(\tilde{R} = u\right) = q$$

$$\begin{tikzcd}
 & Su \\
S \arrow[ur, "q"] \arrow[dr, "1-q"'] & \\
 & Sd
\end{tikzcd}$$

CRR (3/10)

By using the replicating portfolio find the value of the call with strike K and the maturity T=1.

At time t=0, the seller receives the premium C and buys Δ risky asset

At time t=1, we have:

$$\begin{cases} (C - \Delta S)r + \Delta S u = Su - K \\ (C - \Delta S)r + \Delta S d = 0 \end{cases}$$

CRR (4/10)

Proposition: The absence of arbitrage opportunities implies that there exists a risk- neutral probability measure.

Proposition: The price of a derivative is the present value of the expectation of the payoff of that derivative under the risk-neutral probability measure

CRR (5/10)

Definition: A complete market is a market in which the payoff of any derivative could be replicated by using the existing underlying assets.

Proposition: In a complete market with the absence of arbitrage opportunities there exists a unique risk-neutral probability measure.

CRR (6/10)

Exercise 2: We consider a static market model in which two assets are traded. A riskless asset guarantees a return $r = 1{,}05$

At the commencement of the trading game the value of the risky asset is $S_0 = 100\$$

At the end of the trading day the value of the risky asset is either $S_1 = 110\$$ or $S_1 = 90\$$

1) Calculate the return of the risky asset

CRR (7/10)

The historical probability measure gives

$$P\left(\tilde{R} = u\right) = P\left(\tilde{R} = d\right) = \frac{1}{2}$$

2) Calculate the expectation of the risky asset return.

3) Determine the risk-neutral probability measure.

CRR (8/10)

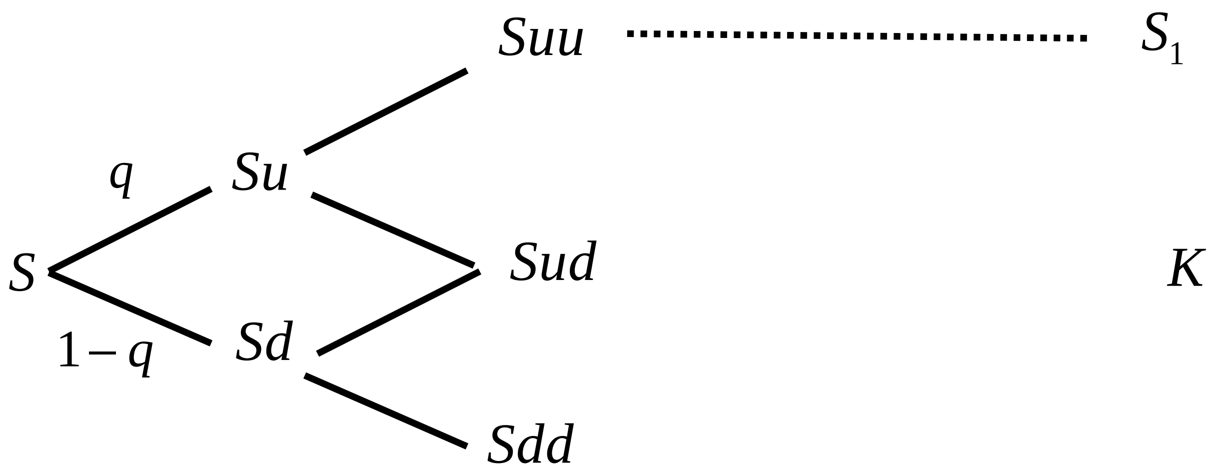
We consider at the opening of the market a call option on the risky asset with a maturity date $T = 1$ and a strike price $K = 100$

4) Determine the hedging strategy.

5) Determine the premium of that call.

CRR (9/10)

We consider a sequential-auction market with T periods



CRR (10/10)

The probability to have done *n* climbs:

$$\binom{T}{n} q^n (1-q)^{T-n}$$

And

$$n^* = \inf\{n \geq 1, S_1 \geq K\}$$

The premium

$$C = S\left\{\sum_{n=n^*}^{T} \frac{\binom{T}{n}(qu)^n\left((1-q)d\right)^{T-n}}{r^T}\right\} - \frac{K}{r^T}\left\{\sum_{n=n^*}^{T}\binom{T}{n} q^n (1-q)^{T-n}\right\}$$

Chapter 4: Black-Scholes model

Outline of the topic

Brownian Motion

Black-Scholes hypothesis

Black-Scholes formula

Put-call parity

Volatility smile

Brownian Motion

Brownian Motion: it has been used in physics to describe the motion of a particle that is subject to a large number of small molecular shocks.

Properties:

1) the change dB_t during a small period of time dt is

$dB_t \sim \sqrt{dt}\,N(0,1)$ where $N(0,1)$ is a standardized normal distribution

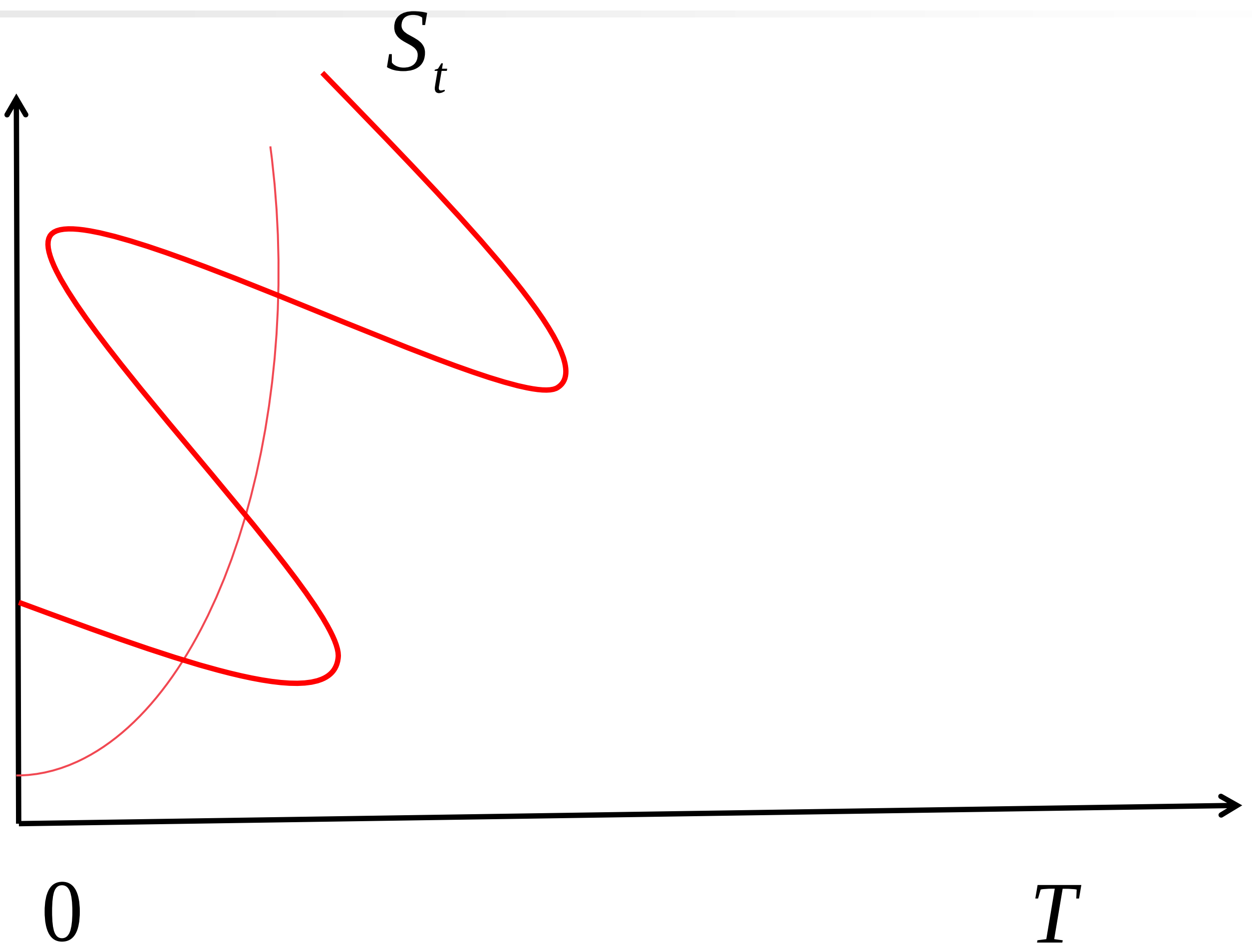
The changes for any two different short intervals of time are independent.

Black-Scholes Hypothesis

The dynamic of the underlying asset is given by

$$dS_t = \underbrace{rS_t dt}_{\text{The trend}} + \underbrace{\sigma S_t dB_t}_{\text{The random walk}}$$

Black-Scholes Hypothesis



Black-Scholes Hypothesis

If we match BS and CRR, we have to consider

$$u = e^{+\sigma\sqrt{dt}}$$

$$d = e^{-\sigma\sqrt{dt}}$$

$$q = \frac{e^{rdt} - d}{u - d}$$

Black Scholes Formula

The premium of the call is given by

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

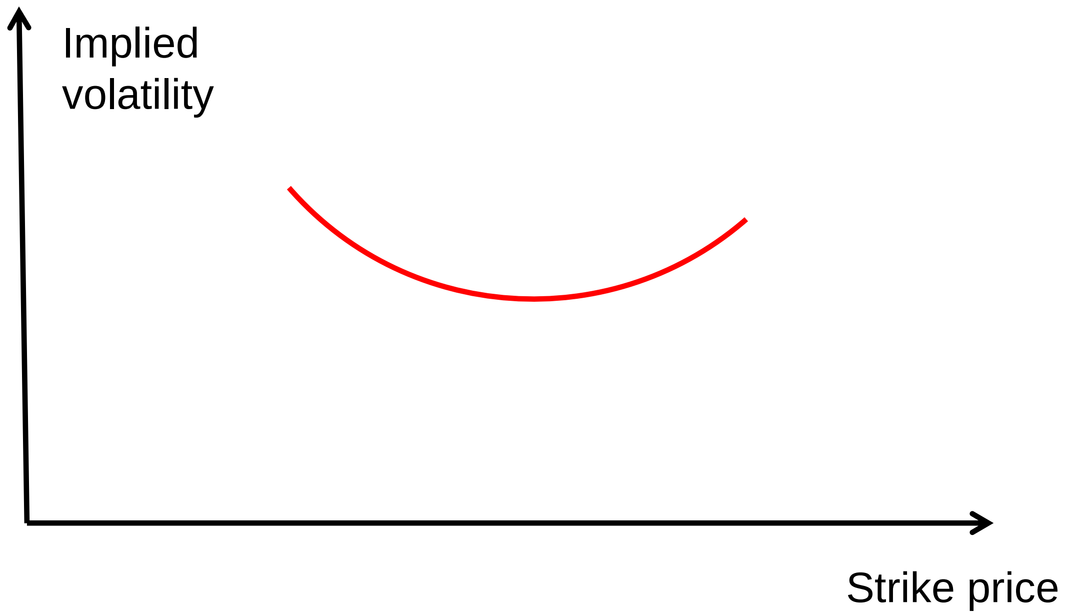
$$\begin{cases} d_1 = \dfrac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}} \\ d_2 = \dfrac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} \end{cases}$$

Put-call parity

$$P - C + S = Ke^{-rT}$$

Volatility smile

The volatility smile used by traders to price foreign currency options has the general form as follows:

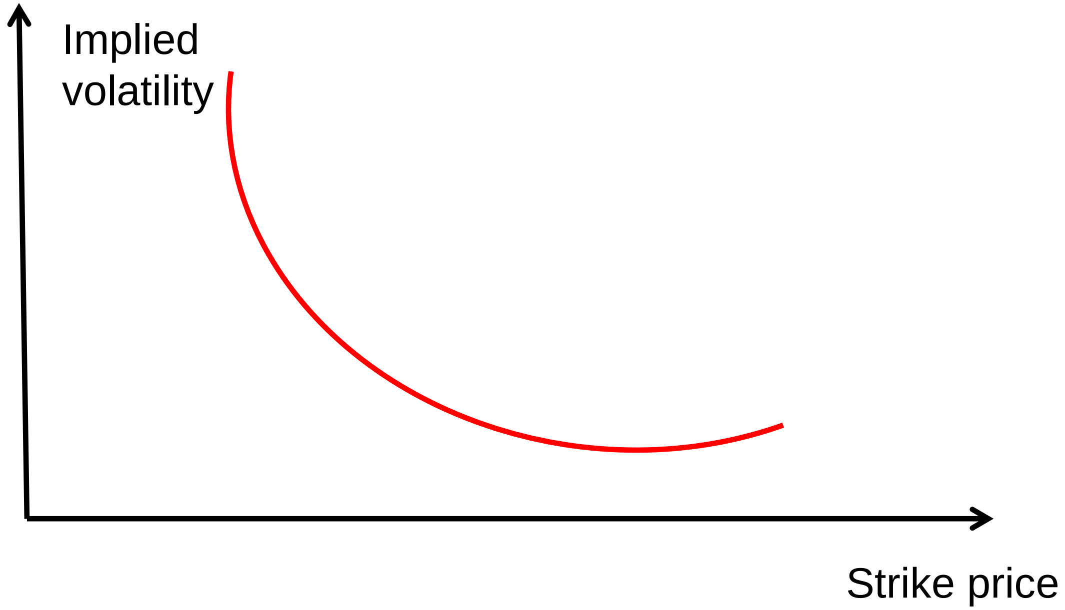


Volatility smile

The volatility is relatively low for at the money option and becomes progressively higher as an option moves either in the money or out of the money

Volatility smile

The volatility smile used by traders to price equity options has the general form as follows:



Volatility smile

The volatility decreases as the strike price increases. This sometimes referred as a volatility skew.

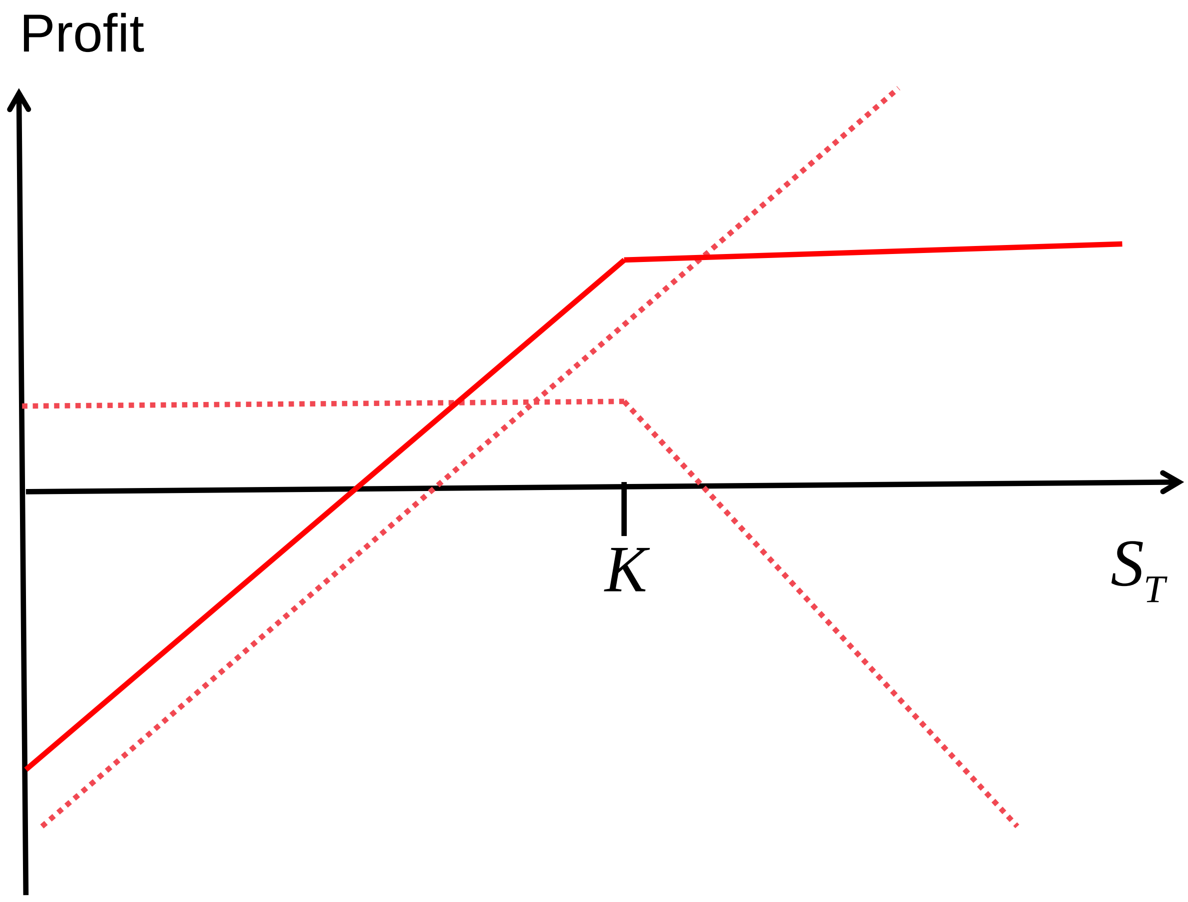
Chapter 5: Derivative Strategies

Writing a covered call (1/2)

The portfolio consists of a long position in a stock plus a short position in a call option.

The long stock position “covers” or protects the investor from the payoff on the short call that becomes necessary if there is a sharp rise in the stock price

Writing a covered call (2/2)

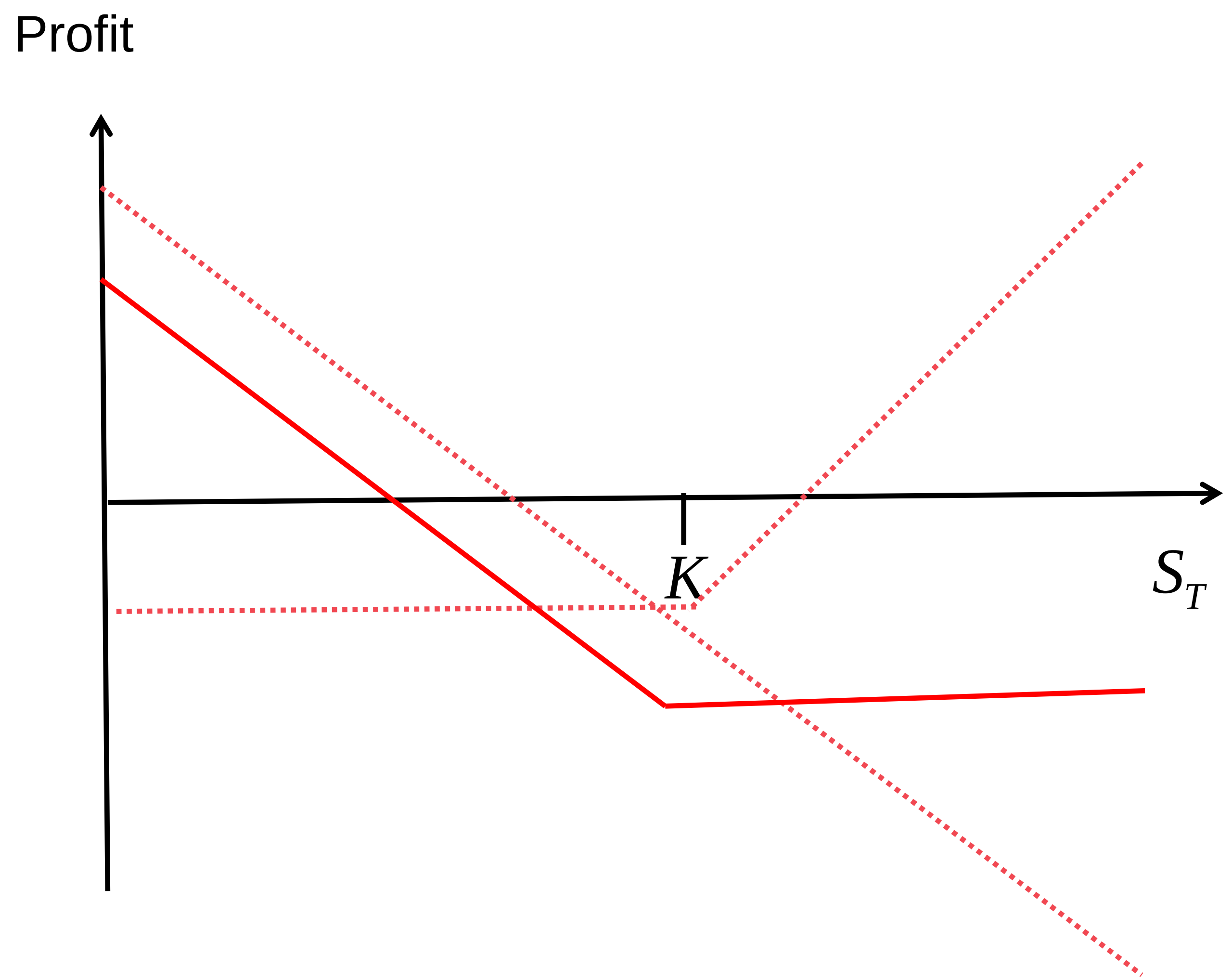


Other strategy 1

The portfolio consists of a short position in a stock plus a long position in a call option.

It is the reverse of writing a covered call

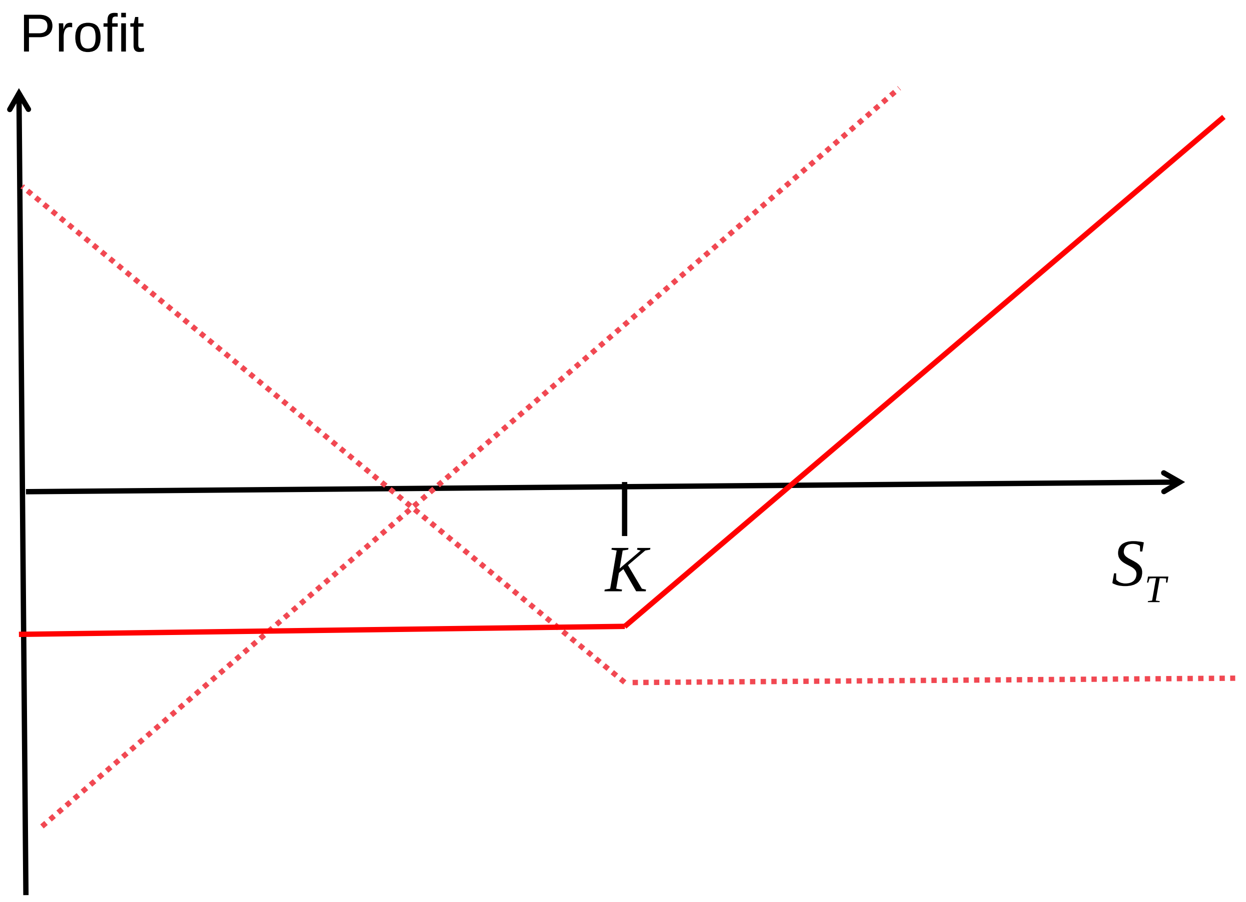
Other strategy 1



Protective put strategy (1/2)

The investment strategy involves buying a put option on a stock and the stock itself.

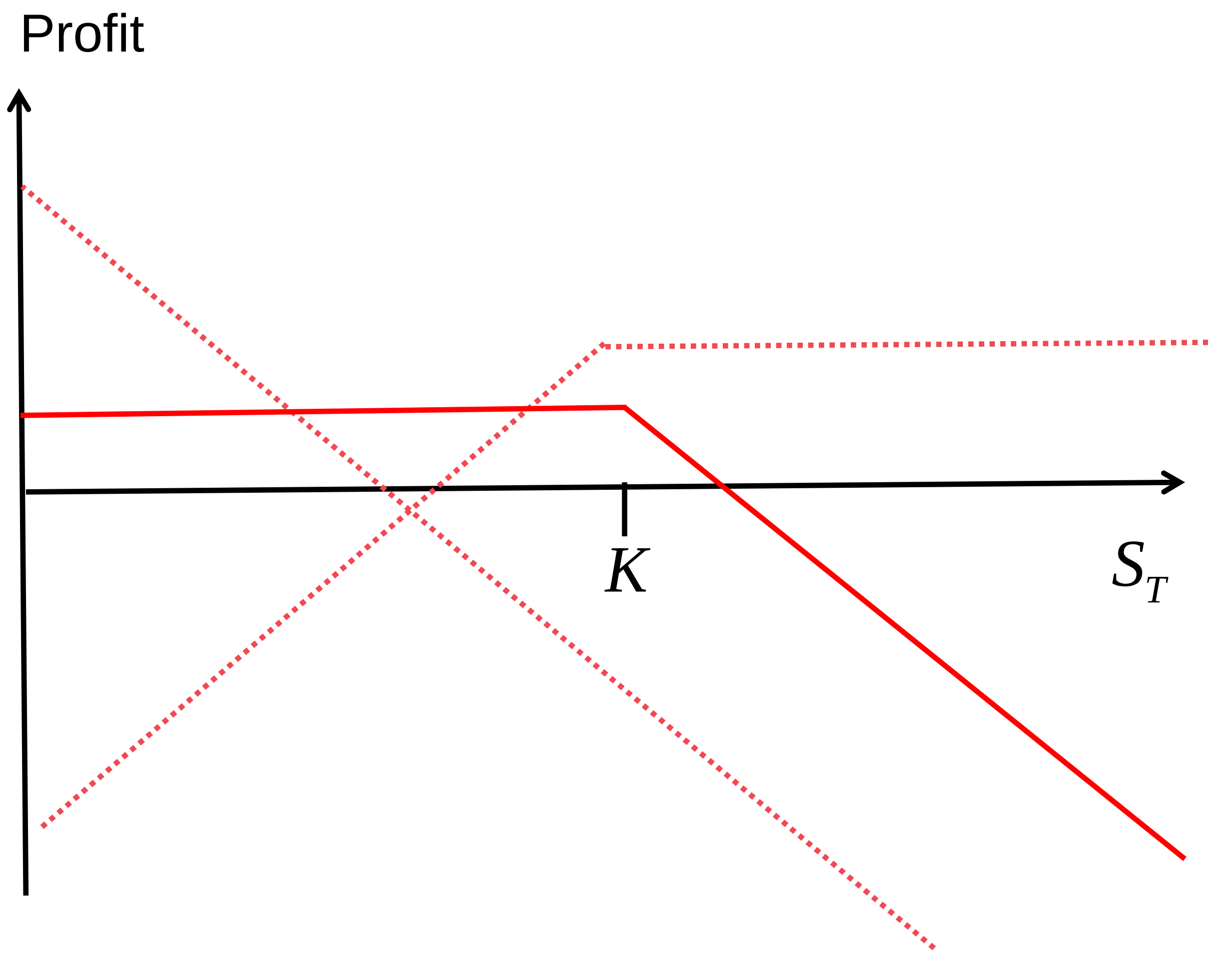
Protective put strategy (2/2)



Other strategy 2

We consider the reverse of a protective put strategy, that is a short position in a put option combined with a short position in the stock

Other strategy 2



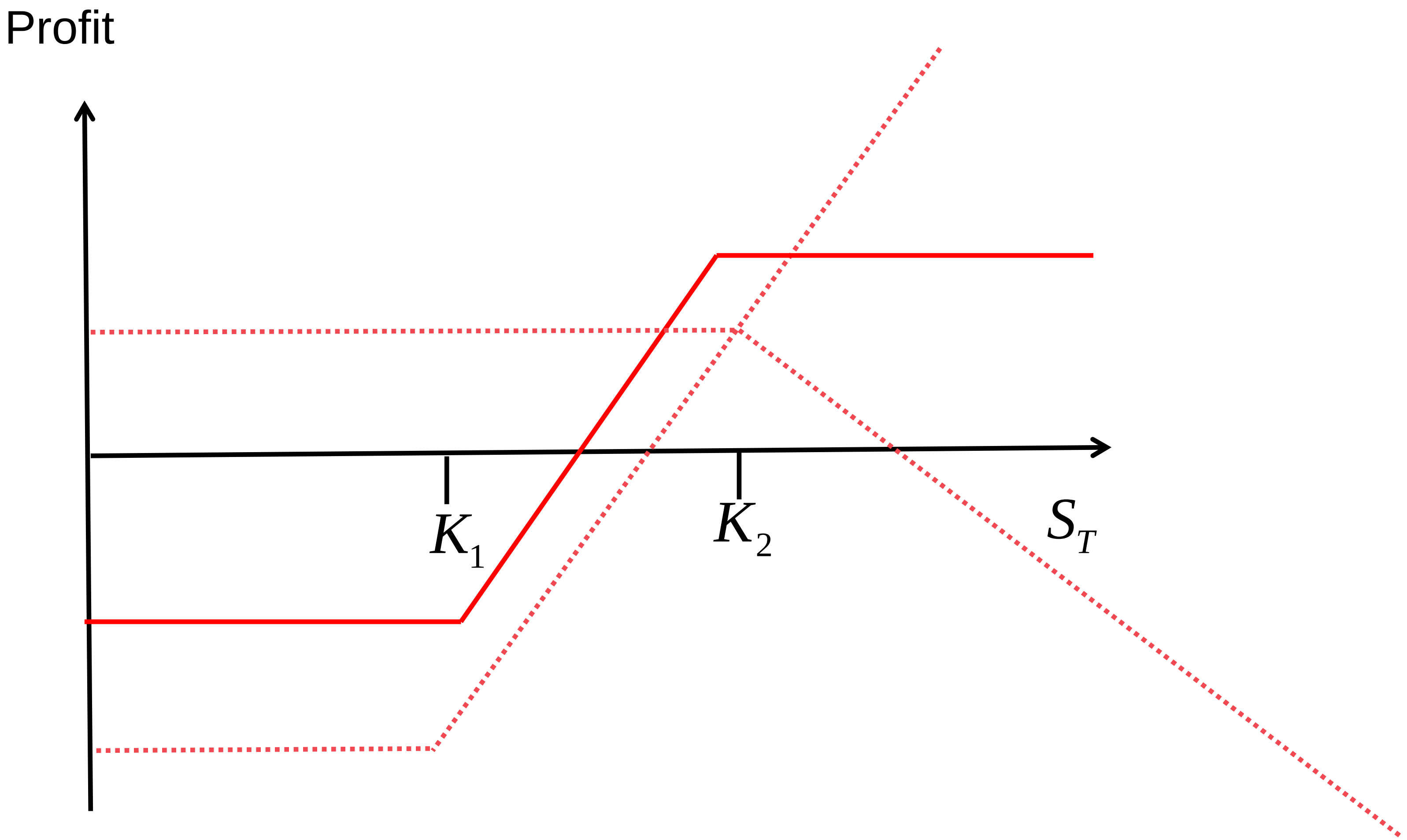
Bull spread strategy (1/3)

Generally speaking a bull spread strategy involves taking a position in two or more options of the same type (two or more calls; two or more puts).

Bull spread can be created by buying a call option on a stock with a certain strike and selling a call option on the same stock with a higher strike price. Both options have the same expiration date.

Because a call price always decreases as the strike price increases, the value of the option sold is always less than the value of the option bought. It requires an initial investment.

Bull spread strategy (1/3)



Bull spread strategy (3/3)

A bull spread strategy limits the investor's upside as well as downside risk.

The strategy can be described by saying that the investor has a call option with strike price equal to K_1 and has chosen to give up some upside potential by selling a call with strike price $K_2 > K_1$

Bear spread strategy (1/3)

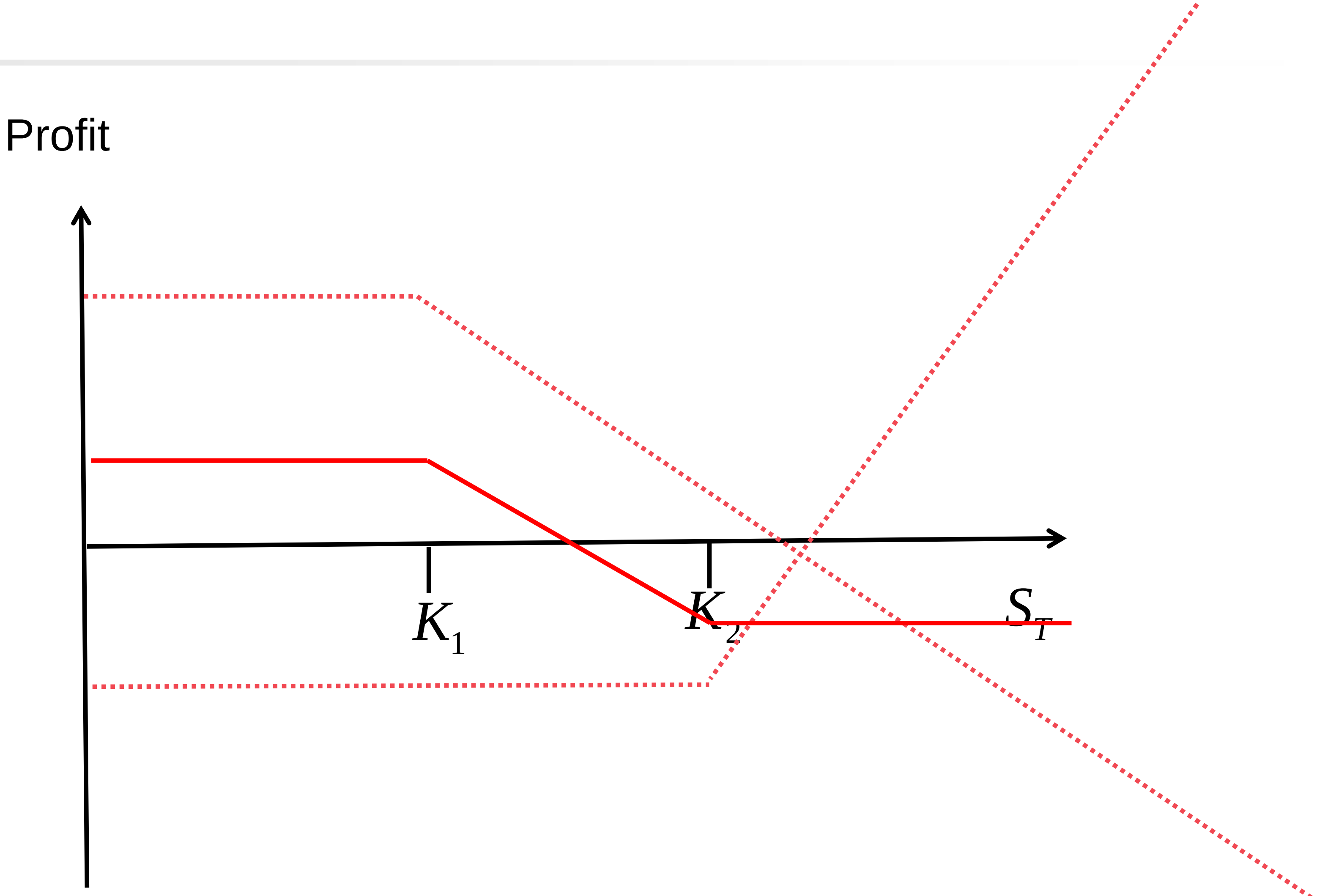
An investor who enters into a bull spread is hoping that the stock price will increase. By contrast, an investor who enters into a bear spread is hoping that the stock price will decline.

A bear spread can be created by selling a call option with a strike K_1 and by selling a call option with strike K_2

We have

$$K_2 > K_1$$

Bear spread strategy (1/3)



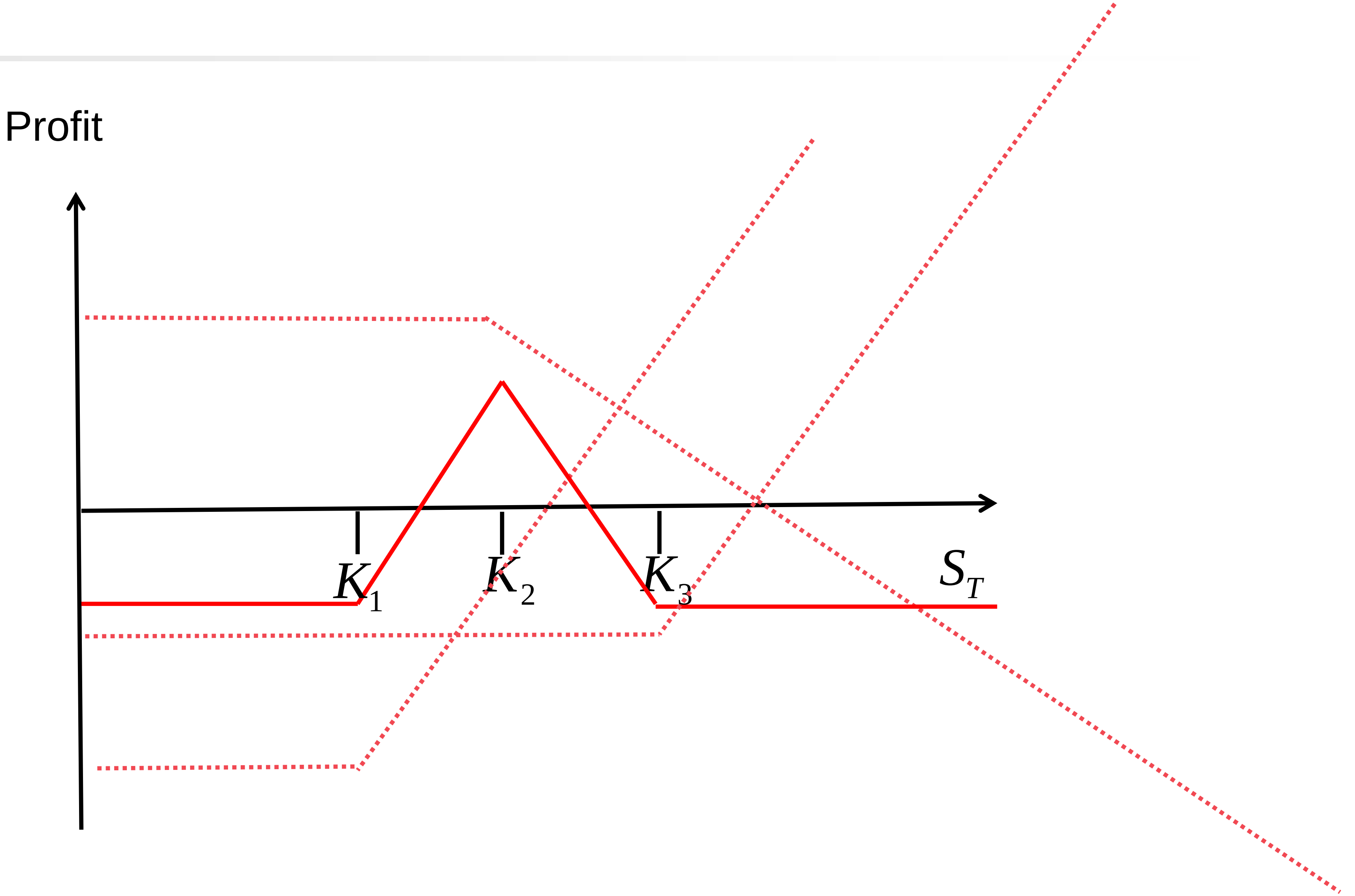
Bear spread strategy (3/3)

A bear spread strategy limits the investor's upside as well as downside risk.

Butterfly spread strategy (1/3)

A butterfly spread can be created by buying a call option with a relatively low strike price, K_1 ; buying a call option with a relatively high strike price K_3; and selling two call option with a strike price K_3 halfway between K_1 and K_2

Bear spread strategy (2/3)



Butterfly spread strategy (3/3)

Generally K_2 is close to the current stock price. A butterfly spread leads to a profit if the stock price stay close to K_2 but rises to a small loss if there is a significant stock price move in either direction.

Exercises

Exercise:

1) What is meant by a protective put?

2) When is it appropriate for an investor to purchase a butterfly spread ?

3)Describe an aggressive bear spread strategy.

Chapter 6: The greeks

Introduction

A financial institution that sells an option to a client in the over-the-counter markets is faced with the problem of managing its risk.

This section is devoted to the different dimension of the risk and each one is measured by a greek letter.

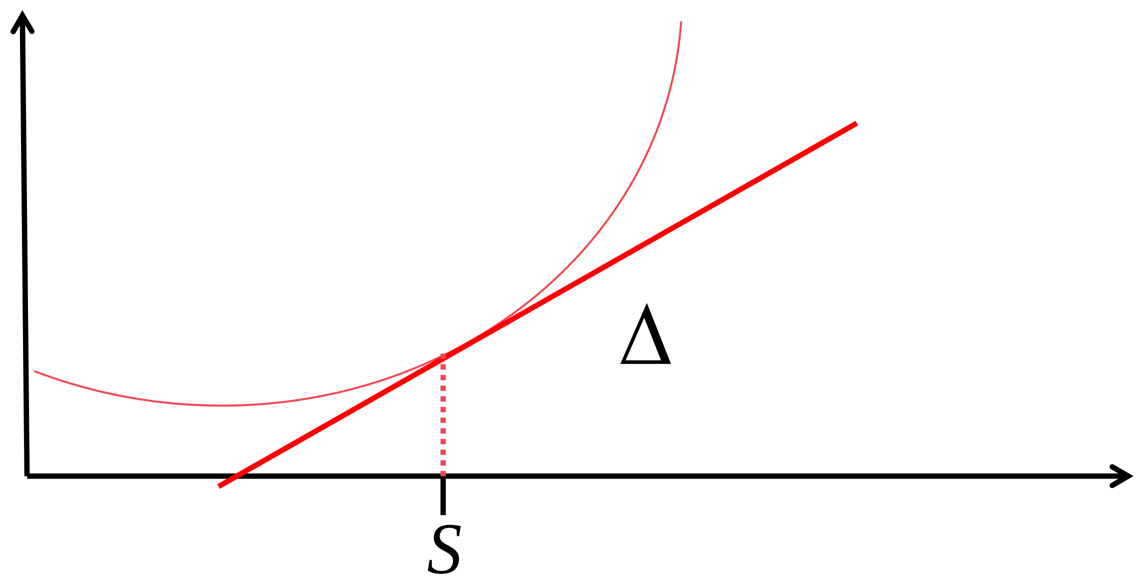
The Delta (1/4)

Definition: the Delta is defined as the rate of change of the option price with respect to the price of the underlying asset.

$$\Delta = \frac{\partial C}{\partial S}$$

The Delta (2/4)

Δ is the slope of the curve that relates the option price under to the underlying asset



The Delta (3/4)

Δ represents the hedging quantity

In the Black Scholes model

$$C = SN(d_1) - Ke^{-rT}N(d_2)$$

$$\Delta = N(d_1)$$

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The Delta (4/4)

Exercise 3: Calculate the option price change when the price rises by 1 unit:

$$S_0 = 18$$

$$C(18) = 3.3659$$

$$\Delta = 0.997$$

The Gamma (1/2)

Definition: the Gamma of a portfolio of options on an underlying asset, Γ, is the rate of change of the portfolio' s delta with respect to the price of the underlying

$$\Gamma = \frac{\partial^2 C}{\partial S^2}$$

The Theta

Definition: the Gamma of a portfolio of options on an underlying asset, Θ , is the rate of change of the portfolio' s delta with respect to the price of the underlying

$$\Theta = \frac{\partial^2 C}{\partial S^2}$$

Interest Rates

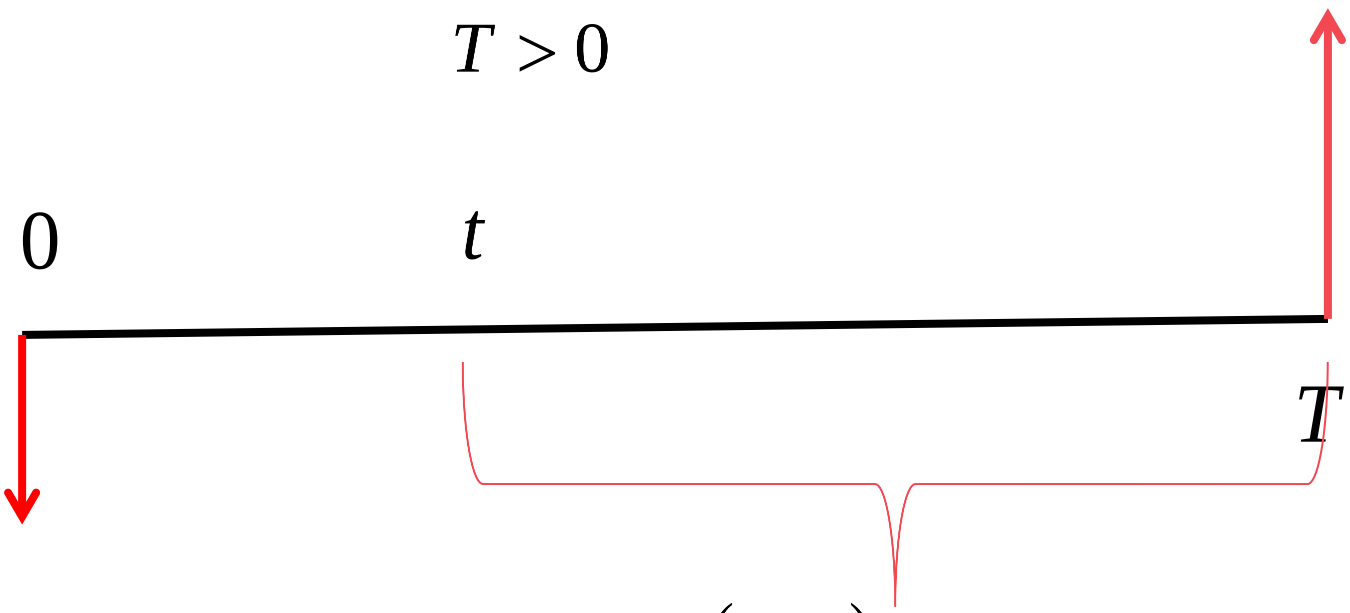
Outline of this section

Chapter 1: the day-count conventions

Time to maturity

<u>Definition :</u> The time to maturity $T-t$ is the amount of time (in years) from the present time t to the maturity time

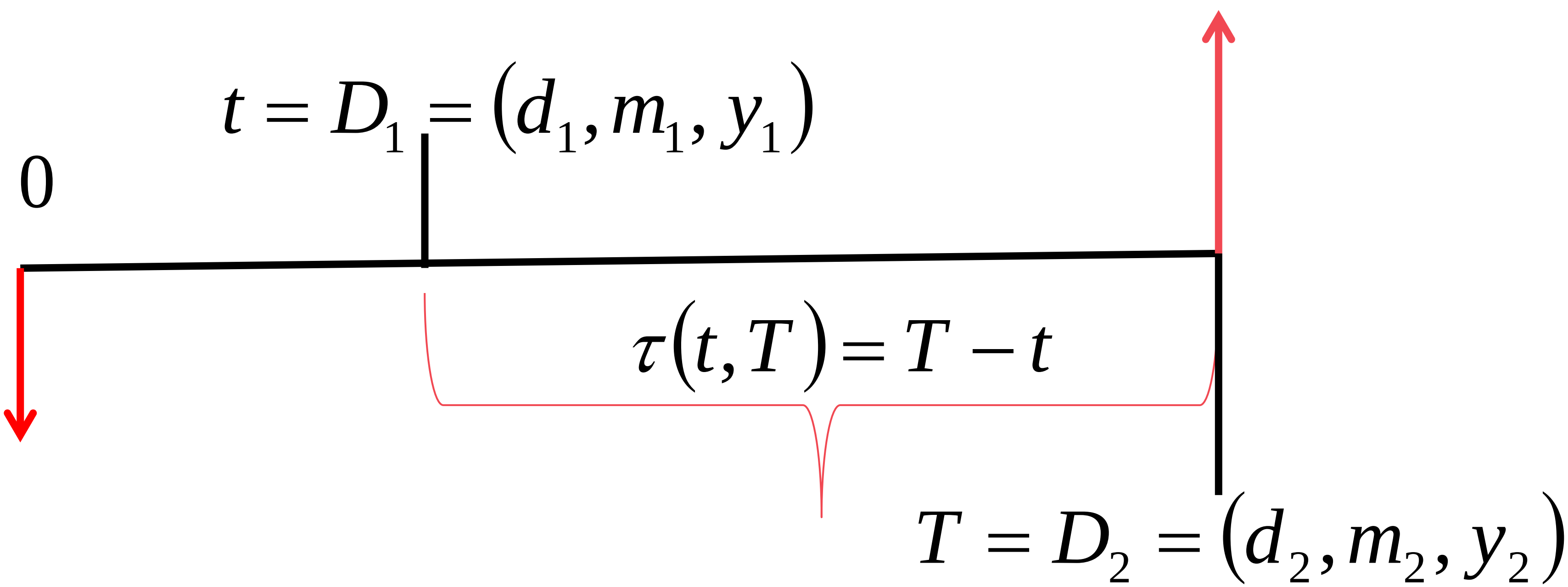
$$T>0$$



$$\tau(t,T)=T-t$$

Day-count convention

<u>Definition :</u> We denote by $\tau(t,T)$ the chosen time measure between *t* and *T*, which is usually referred to as year fraction between the dates *t* and *T*. *The choice to measure is known as the day-count convention*



Actual/365

In this case:

$$\tau(t,T) = \frac{D_2 - D_1}{365}$$

<u>Exercise 1</u>: Calculate the year fraction

$$D_1 = (15, February, 2018)$$

$$D_2 = (15, August, 2018)$$

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Actual/360

In this case:

$$\tau(t,T)=\frac{D_2-D_1}{360}$$

<u>Exercise 2</u>: Calculate the year fraction

$$D_1=(15,February,2018)$$

$$D_1=(15,February,2018)$$

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30/360

This day-count convention assume that each month has 30 days and the total number of days in the year is 360:

$$\tau(t,T) = \frac{\max(30-d_1,0)+\min(d_2,30)+360(y_2-y_1)+30(m_2-m_1-1)}{360}$$

<u>Exercise 3</u>: Calculate the year fraction

$$D_1 = (15, February, 2018)$$

$$D_2 = (15, August, 2018)$$

Conversion formulas

When you change the day-count convention the interest rate becomes as follows:

$$r_{360} = r_{365} \times \left(\frac{360}{365} \right)$$

<u>Exercise 4</u>: Calculate r_{360} **or** r_{365}

$$r_{365} = 10\%$$

$$r_{365} = 20\%$$

$$r_{360} = 20\%$$

Chapter 2: Zero-coupon Bonds

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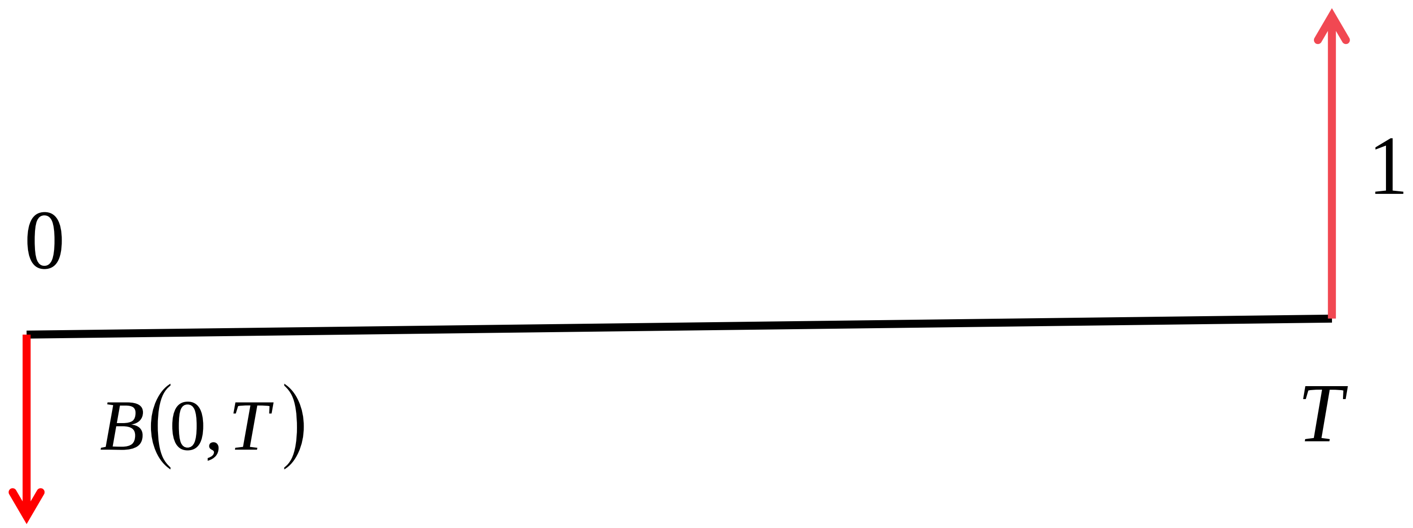
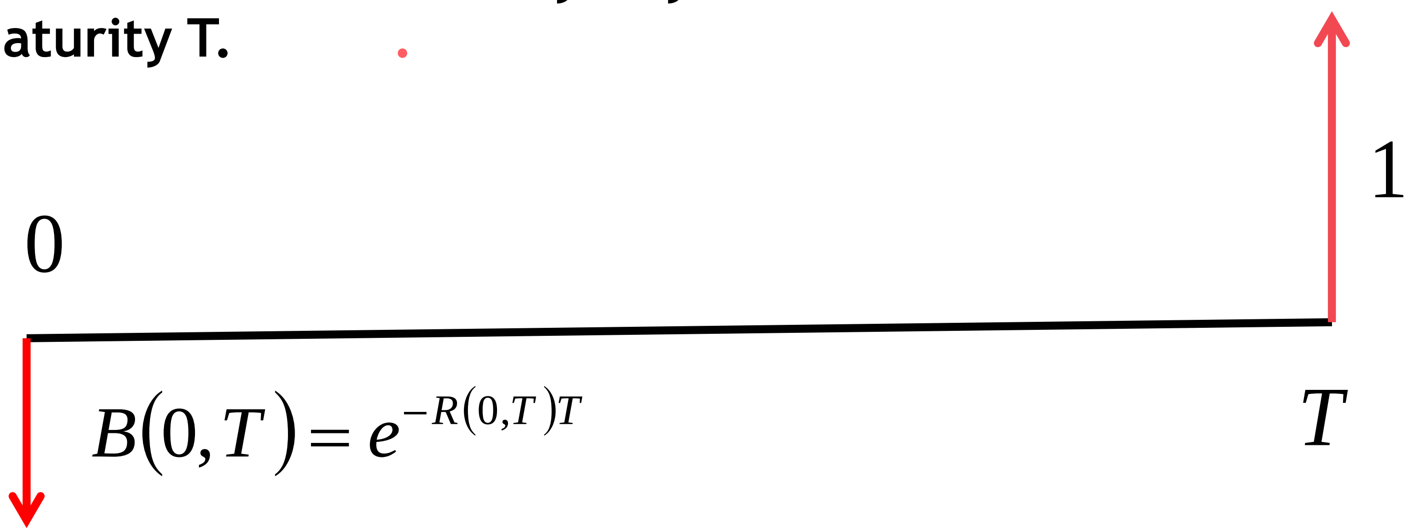
Definition

<u>Definition :</u> **A T-maturity zero-coupon bond is a contract that guarantees its holder the payment of one unit of currency at time T, with no intermediate payments.**



Zero-coupon rate

<u>Definition :</u> *The continuously-compounded spot interest rate* prevailing at time t=0, is denoted by $R(0,T)$ and is the constant rate at which an investment of $B(0,T)$ units of currency at time t=0 accrues continuously to yield a unit amount of currency at maturity T.



$$B(0,T) = e^{-R(0,T)T}$$

Zero-coupon rate

<u>Definition :</u> ***The simply-compounded spot interest rate*** **prevailing at time t=0, is denoted by $L(0,T)$ and is the constant rate at which an investment of $B(0,T)$ units of currency at time t=0 yields a unit amount of currency at maturity T. (Discrete time).**

$$B(0,T)=\frac{1}{1+L(0,T)\tau(0,T)}$$

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Zero-coupon rate

Exercise 5:

Assume that $B(0,T) = 0{,}5$ and $T = 0{,}5$

Calculate

$$R(0,T)$$

$$L(0,T)$$

Chapter 3: Term Structure of interest rates and the yield curve

Short and Long term interest rates

Definition: Short-term interest rate today for tomorrow $\tau(t, t+1day)$

Long-term interest rate when the year fraction is more than 1 day

We denote $r_t = \lim_{T \to t} R(t, T)$

We denote the long interest rate by $R(0, T)$

Fundamental Interest-Rate Curves

Arbitrage opportunity: It is defined in mathematical terms, as a self-financing strategy such that

$$V_0(\phi) = 0$$

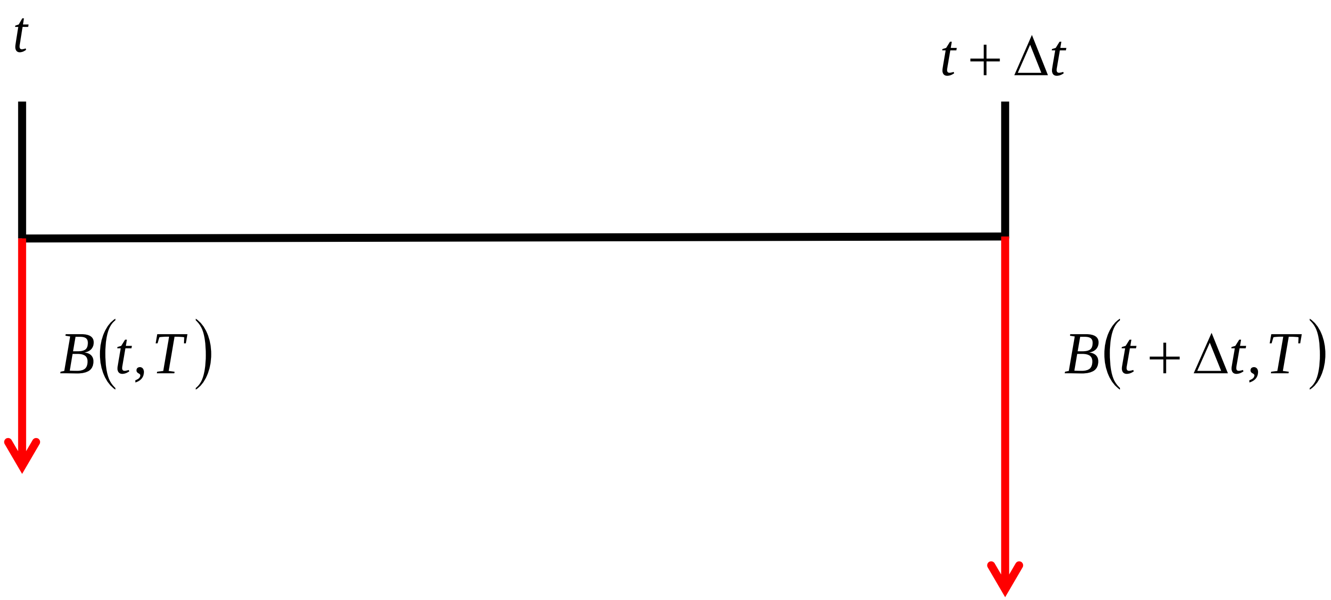
but

$$V_T(\phi) > 0$$

A strategy refers to the different amounts of assets which are purchased.

Fundamental Interest-Rate Curves

We consider an interest rate r_t

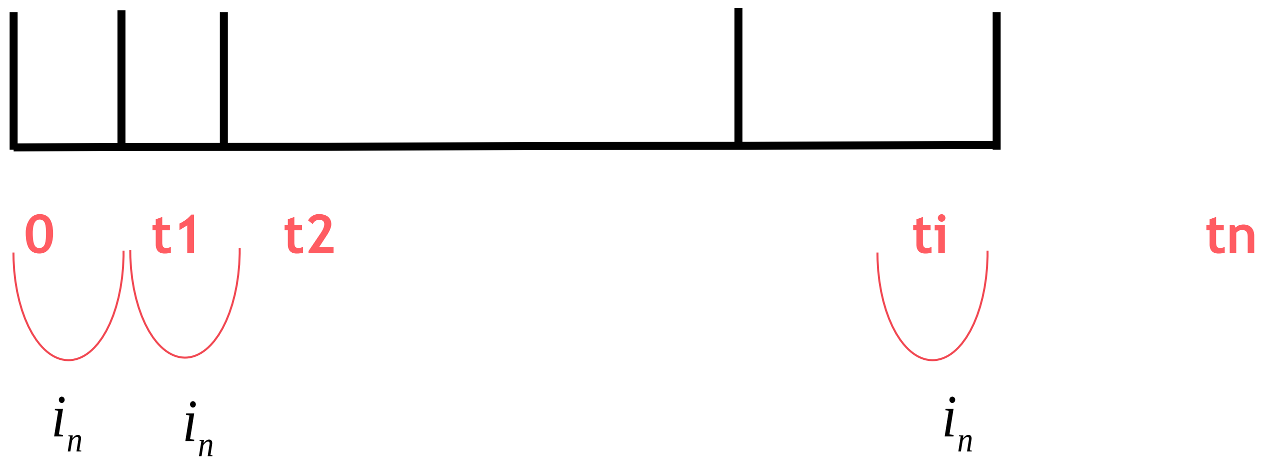


the zero-coupon bond grows at the rate: $B(t+\Delta t,T)=B(t,T)(1+r_t\Delta t)$

That results as $R(0,T)=\dfrac{1}{T}\int_{t=0}^{T} r_t\,dt$

Fundamental Interest-Rate Curves

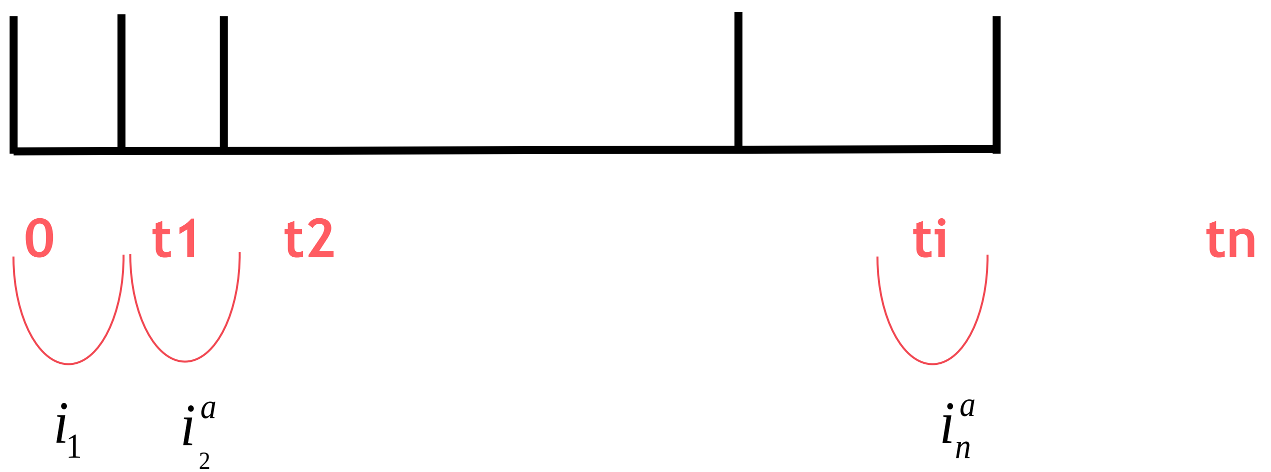
We consider long-term interest rate i_n



Therefore one euro invested at time t=0 yields $(1+i_n)^n$ at time t=tn

Fundamental Interest-Rate Curves

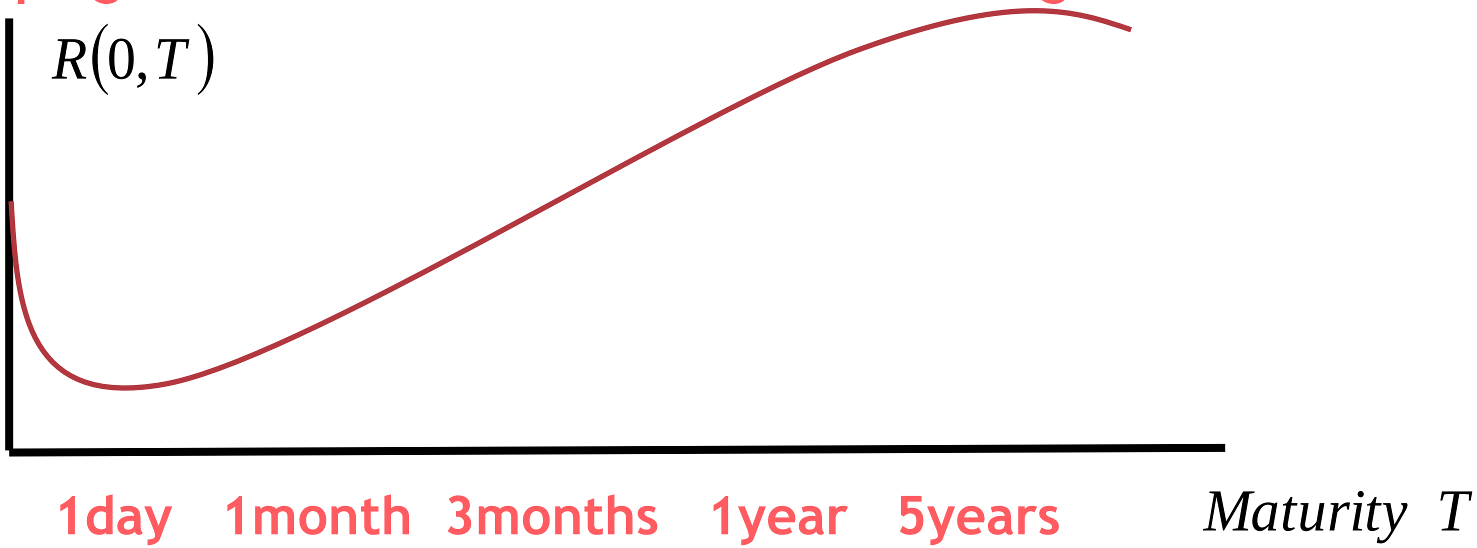
We consider short-term interest rates $i_1, i_2^a, \dots, i_n^a$



Therefore one euro invested at time t=0 yields $(1+i_1)(1+i_2^a)\dots(1+i_n^a)$ at time t=tn

Fundamental Interest-Rate Curves

The term structure of interest rates at time t=0 is the graph mapping the different interest rates according to the maturities



Exercises

Exercise 6: We assume that:

$$i_1 = 3\%,\ i_2^a = 4\%,\ i_3^a = 3{,}5\%,\ i_4^a = 3{,}9\%$$

Calculate the long-term interest rate

Exercise 7: We assume that:

$$\begin{cases} dr_t = a(b - r_t)dt \\ r(0) = r_0 \end{cases}$$

Calculate

$$r_t \quad and \quad R(0,T) \quad and \quad B(0,T)$$

Dynamics of Term Structure

The term structure interest rates changes in response to:

- Wide economic shocks
- Changes in economic agents expectations
- Market specific events
- Monetary policy

Practical Uses of Term Structure

The term structure is necessary to price bonds. Any fixed income security can be replicated by using bonds. Finally, its price is as follows:

$$P=\sum_{i=1}^{n}\frac{CF_i}{(1+R(0,t_i))}=\sum_{1}^{n}CF_iB(0,t_i)$$

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Practical Uses of Term Structure

<u>Exercise 8:</u> We assume that the cash flows grows at the rate of 10% per year and we have the following interest rates:

$$R(0,1) = 5\%$$
$$R(0,2) = 7\%$$
$$R(0,3) = 9\%$$

Finally, the cash flow at the end of the first year equals 100 euros.

By assuming the existence last 3 years calculate the price of this product.

What do we mean by Fixed Income securities ?

When we say “fixed” income, we really mean the market for debt and debt related instrument

In the old day of course, most debt truly had fixed rates

Obviously many bonds today have floating rates

In general, bondholders take priority over shareholders

Chapter 4: Financial markets

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Overview

Financial markets aim at financing the projects of corporations.

There are three types of financial markets:

Money markets

Equity markets

Debt markets

- Fixed income securities
- Derivatives

Overview

Financial markets can be classified in terms of whether the market is:

For new or existing claims (primary or secondary market)

For short-term or long term instruments (money or capital market)

For domestic or foreign securities

For immediate future or optional delivery (cash, future or option market)

Overview

Most fixed income securities trade in markets are not centrally organized, that is, there are no NYSE for bonds.

Instead they trade via an OTC (over the counter) system, which is essentially a series of computer and phone network among dealers and brokers.

Overview

Market participants are:

- **Issuers** of fixed income securities, i.e. users of the proceeds of fixed income issuance
- **Financial intermediaries** they make money by providing information to the market
- **Investors** those that supply funds to the market

Overview

1) Issuers:

- Governments
- Corporations
- Banks
- Municipalities

Objectives

- To raise funds
- The hope to sell securities at the fair market price
- To provide liquidity to the shareholders
- To design and issue debt securities that best suit the needs of the firms and any shareholder

Overview

2) Financial intermediaries:

- Primary dealers
- Other dealers or brokers
- Investment banks
- Credit rating agencies
- Data providers
- Servicing companies

Objectives

- To make money by facilitating trades
- To provide information to the market
- To reduce transaction cost to market participants
- To provide a primary mechanism for issuers of securities
- To provide risk-management services

Overview

3) Investors:

- Governments
- Pension funds
- Mutual funds
- Insurance companies
- Commercial banks
- Hedge funds
- Individuals

Objectives

- To earn a rate of return for a given level of risk
- Diversification
- To speculate on interest rate movements
- To modify their interest rate risk exposure

Regulatory bodies

4) there are many authorities:

AMF (autorité des marchés financiers)

Banque de France

Ministère de l'économie

And so on

Objectives

- Insuring a fair, orderly and open market
- Implementing monetary/economic policy
- Assuring the safety and soundness of the banking system

Bond markets

The characteristics of the bond are described in the indenture. Bond indenture is a legal document that defines the rights and the obligations of the borrower and the lender with respect to the bond issue. Basic component should be extracted:

- Coupon rate
- Number of payments per year
- Maturity
- Face value
- Different seniorities (Secured bonds, subordinated debentures, debendures)

General market participants

Informed Investors, they speculate on the basis of information

Liquidity traders, they trade for liquidity reasons

Market makers, they set a bid price and a ask price

Kyle Model

1 investor who knows the liquidation value of a risky asset

Liquidity traders

Several market makers who set the price.

The market is not strong efficient !!!

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Chapter 5: Interest Risk Management (Duration)

Sensitivity

The value of the different assets depends crucially on the variations of the interest rates.

First, we study the variation of these values with the interest rates

Second, we explore the different ways to hedge the interest rate risk exposure

<u>Definition:</u> the relative sensitivity is the derivative of the bond price with respect to the interest rate.

$$s = \frac{\partial P(r) / \partial r}{P(r)}$$

Duration (1/4)

Remark: The sensitivity is always a negative number since the bond price goes down when the interest rates goes up

Exercise 9: Calculate the sensitivity if the interest rate is constant and the bond price is as follows:

$$P(r) = \sum_{i=1}^{n} \frac{c_i}{(1+r)^{t_i}}$$

Duration (2/4)

<u>Duration :</u> **it is the weighted average maturity of the coupons (CF) received from the bond where weights are the relative values of these CF (Barycenter).**

$$-s = \frac{1}{(1+r)} D$$

$$D = \sum_{i=1}^{n} \alpha_i t_i$$

$$\alpha_i = \frac{c_i / (1+r)^{t_i}}{P(r)}$$

Duration (3/4)

Remark :

The Macauley duration of a zero-coupon bond equals its maturity

The Macauley duration of a fixed incomes security is always less than its maturity

Duration (4/4)

<u>Exercise 10:</u> We assume that the asset value of a bank depends on the interest rate. The expected interest rate is evaluated as $r^a = 5\%$

Considering this rate, we have found the following values for the bank TOTO:

The asset value $A(r^a) = 100$

The equities $E(r^a) = 10$

The debt $D(r^a) = 90$

The Macaulay duration of the assets is $D_A = 7\ years$

The Macaulay duration of the debt is $D_D = 1\ year$

Question 1. Calculate the equity value if the interest rate $\tilde{r} = 6\%$

Question 2. Calculate the equity value if the interest rate $\tilde{r} = 4\%$

Question 3. Comment these results.

Interest Rate Models (1/4)

<u>Vasicek Model:</u> The dynamic of the short-term interest rate is given by:

$$dr_t = a(b - r_t)dt + \sigma dN(0, dt)$$

This set-up implies that the interest rate can be negative with positive probability

We can calculate the interest rate according the different maturities

$$R(0,T) = R_\infty - (R_\infty - r_0)\alpha(T) + \frac{\sigma^2 T}{4a}\alpha^2(T)$$

$$\alpha(T) = \frac{1 - e^{-aT}}{aT}$$

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Interest Rate Models (2/4)

Cox Ingersoll and Ross (CIR) Model: The dynamic of the short-term interest rate is given by:

$$dr_t = a\left(b - r_t\right)dt + \sigma\sqrt{r_t}\,dN\left(0, dt\right)$$

This model has been a benchmark for many years because of its analytical tractability and the fact that contrary to the Vasicek model the instantaneous short rate is always positive.

Interest Rate Models (3/4)

Dothan Model: The dynamic of the short-term interest rate is given by:

$$dr_t = ar_t dt + \sigma r_t dN(0, dt)$$

The lognormal distribution that the short rate is always positive.

Interest Rate Models (4/4)

The two-additive-factor Gaussian Model G2++: The dynamic of the short-term interest rate is given by:

$$r_t = x_t + y_t + \varphi(t), \; r(0) = r_0$$

$$\begin{cases} dx_t = -a x_t dt + \sigma dB_t^1, \; x(0) = 0 \\ dy_t = -b y_t dt + \eta dB_t^2, \; y(0) = 0 \end{cases}$$

$$dB_t^1 dB_t^2 = \rho dt$$

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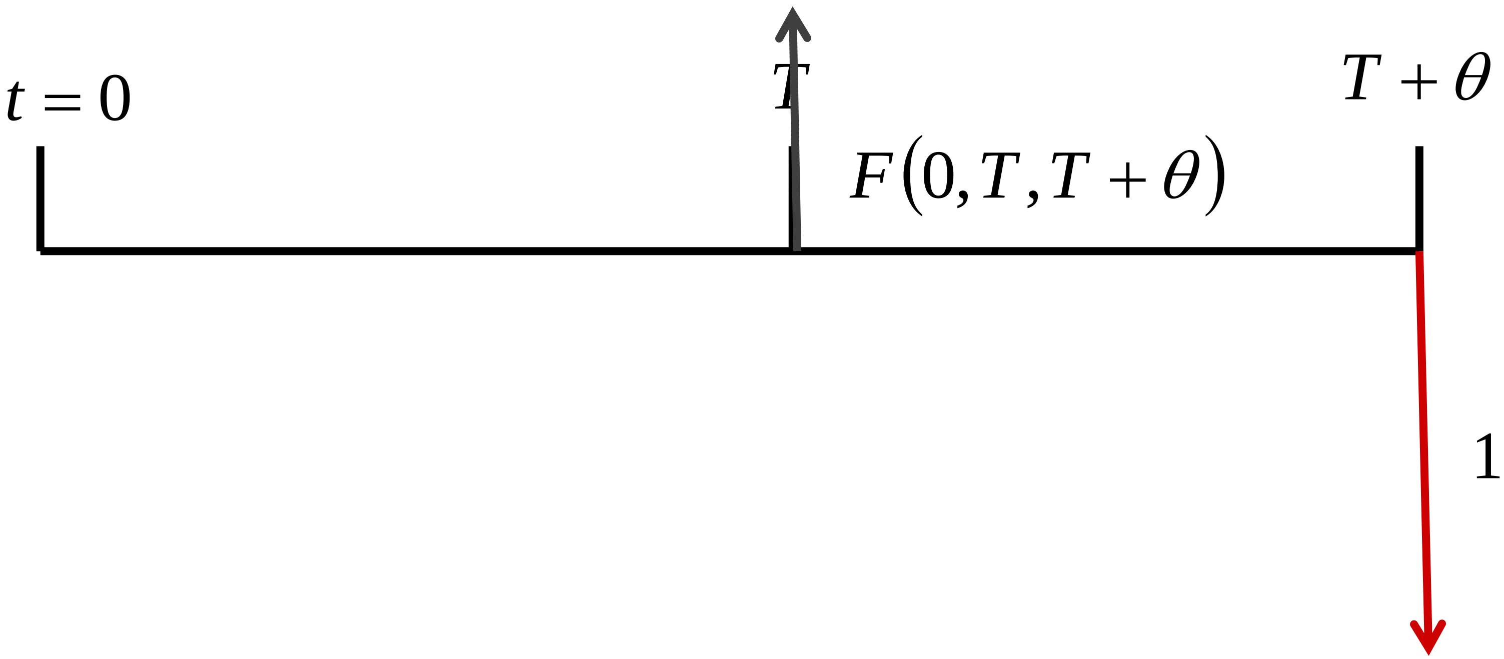
Chapter 7: Interest Rate derivatives

Forward Rate Agreement (1/4)

<u>Definition: </u>A forward Rate Agreement is a contract involving three time instants: The current time t=0, the expiry time T and the maturity time $T+\theta$



Forward Rate Agreement (2/4)

Proposition: One shows that:

$$F(0,T,T+\theta)=\frac{B(0,T+\theta)}{B(0,T)}$$

That results:

$$R_0(T,T+\theta)=\frac{(T+\theta)R(0,T+\theta)-TR(0,T)}{\theta}$$

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Forward Rate Agreement (3/4)

Exercise 11: We consider an interest-rate curve with :

$$R(0,1) = 3\%$$

$$R(0,2) = 5\%$$

Question 1. Calculate the 1-year-maturity zero-coupon $B(0,1)$

Question 2. Calculate the 2_years_maturity zero-coupon $B(0,2)$

Question 3. Calculate the FRA $R_0(1,2)$

Forward Rate Agreement (4/4)

<u>Exercise 12:</u> We consider an interest-rate curve resulting of the Vasicek dynamic :

$$dr_t = a(b - r_t)dt + \sigma dN(0, dt)$$

Question 1. Calculate $R(0,1)$

Question 2. Calculate $R(0,2)$

Question 3. Calculate the FRA $R_0(1,2)$

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Swap

<u>Interest Rate Swap (IRS):</u> **It is a contract that exchanges payments between two different indexed legs, starting from a future time instant. At every instant t_i in a prespecified set of dates $(t_1, \cdots, t_N)$ the fixed leg pays out the amount:**

$$N\tau_i K$$

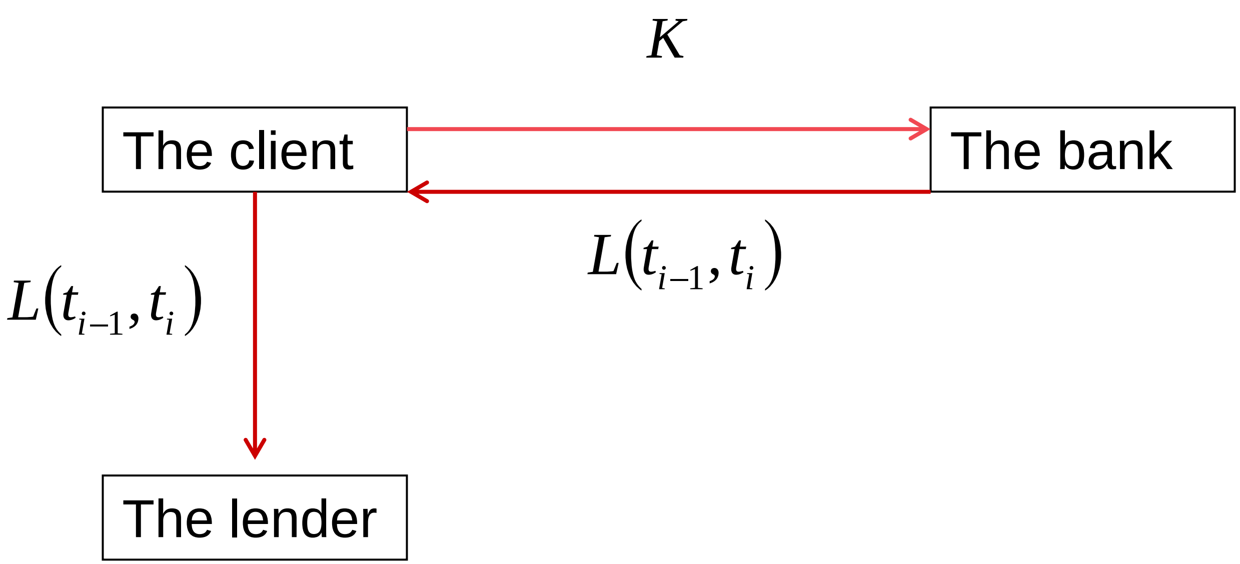
corresponding to a fixed interest rate *K*, a nominal value N and a year fraction τ_i between t_{i-1} and t_i, whereas the floating leg pays the amount:

$$N\tau_i L(t_{i-1}, t_i)$$

Corresponding to the interest rate $L(t_{i-1}, t_i)$ resetting at the t_i previous instant t_{i-1} for the maturity given by the current payment instant

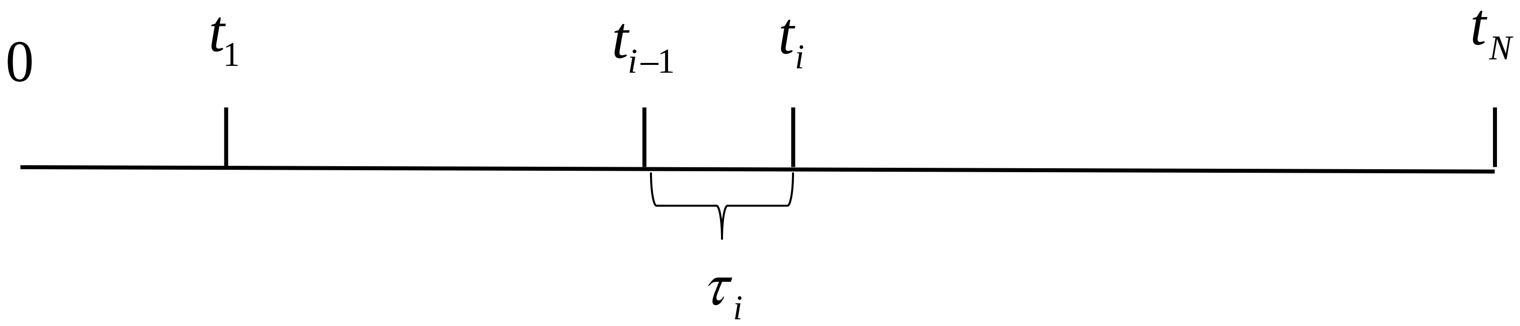
Swap

The client pays the fixed leg and receive the floating leg:



Cap

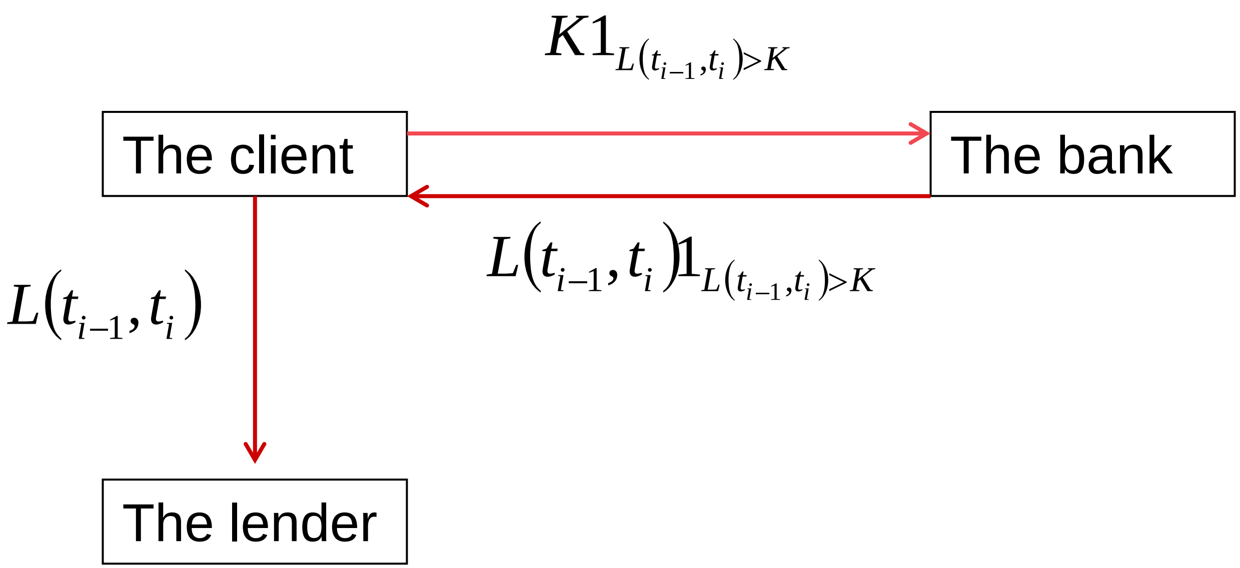
<u>Definition:</u> An interest rate cap is a derivative in which the buyer receives payments at the end of each period in which the interest rate exceeds the agreed strike price.



$$N\tau_i\left(L\left(t_{i-1},t_i\right)-K\right)1_{L\left(t_{i-1},t_i\right)>K}$$

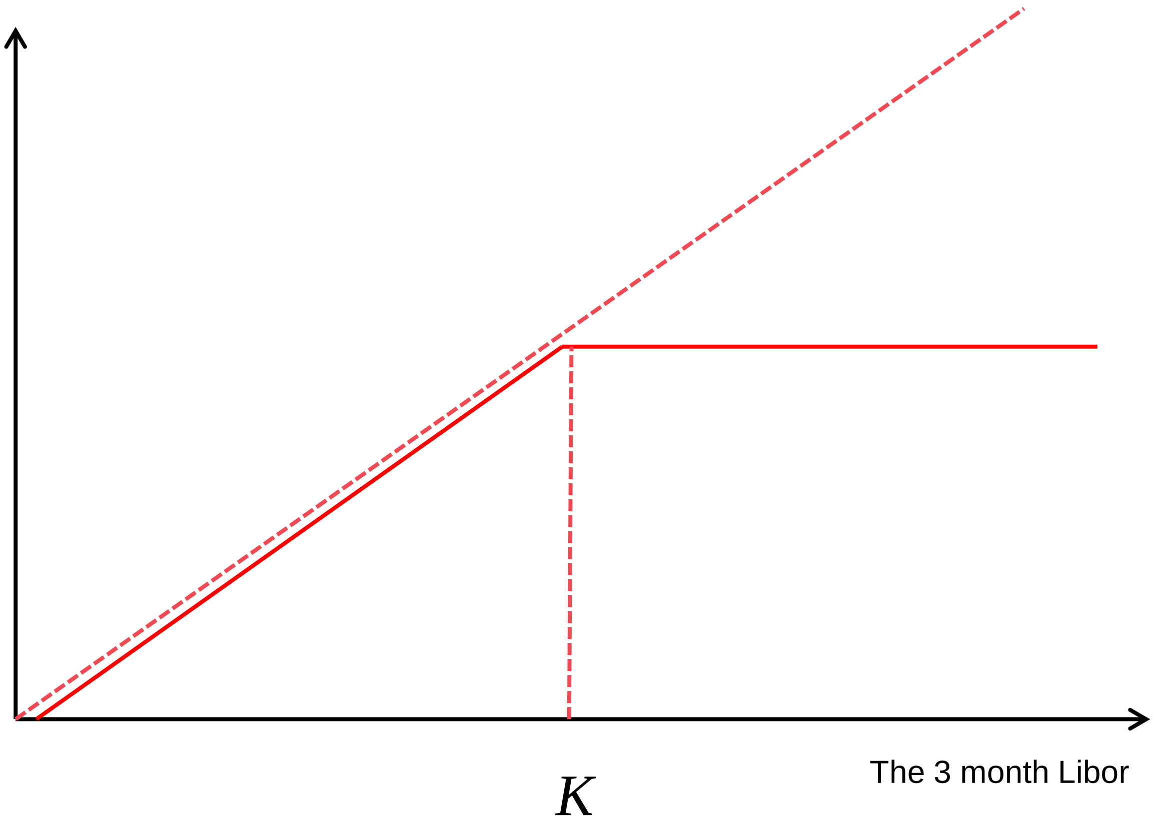
Cap

The client pays the fixed leg and receive the floating leg:



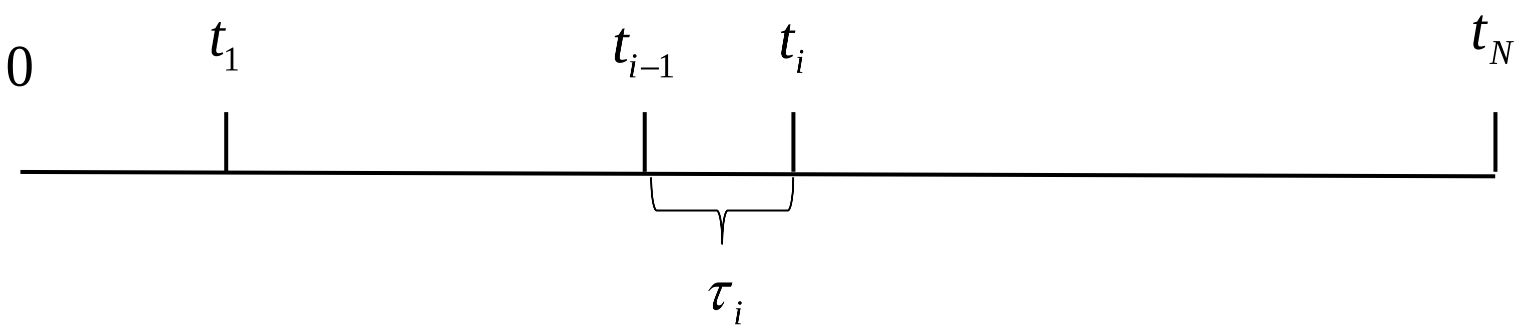
Cap

Payoff



Floor

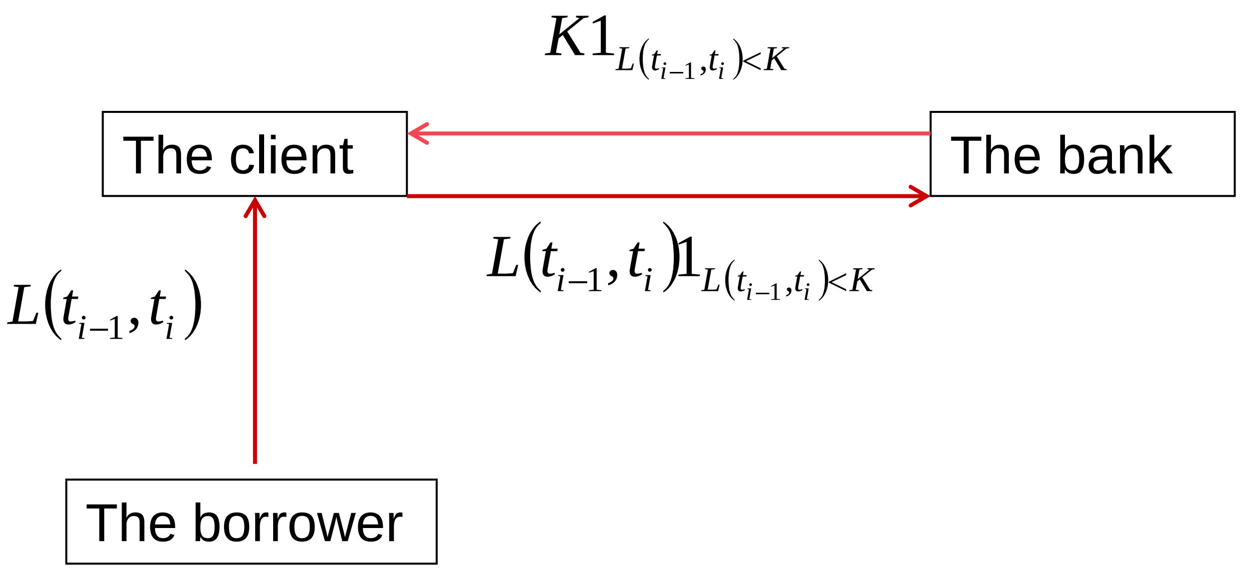
<u>Definition:</u> an interest rate floor is a derivative contract in which the buyer receives payments at the end of each period in which the interest rate is below the agreed strike price.



$$N\tau_i\left(K - L\left(t_{i-1}, t_i\right)\right)1_{L(t_{i-1}, t_i)<K}$$

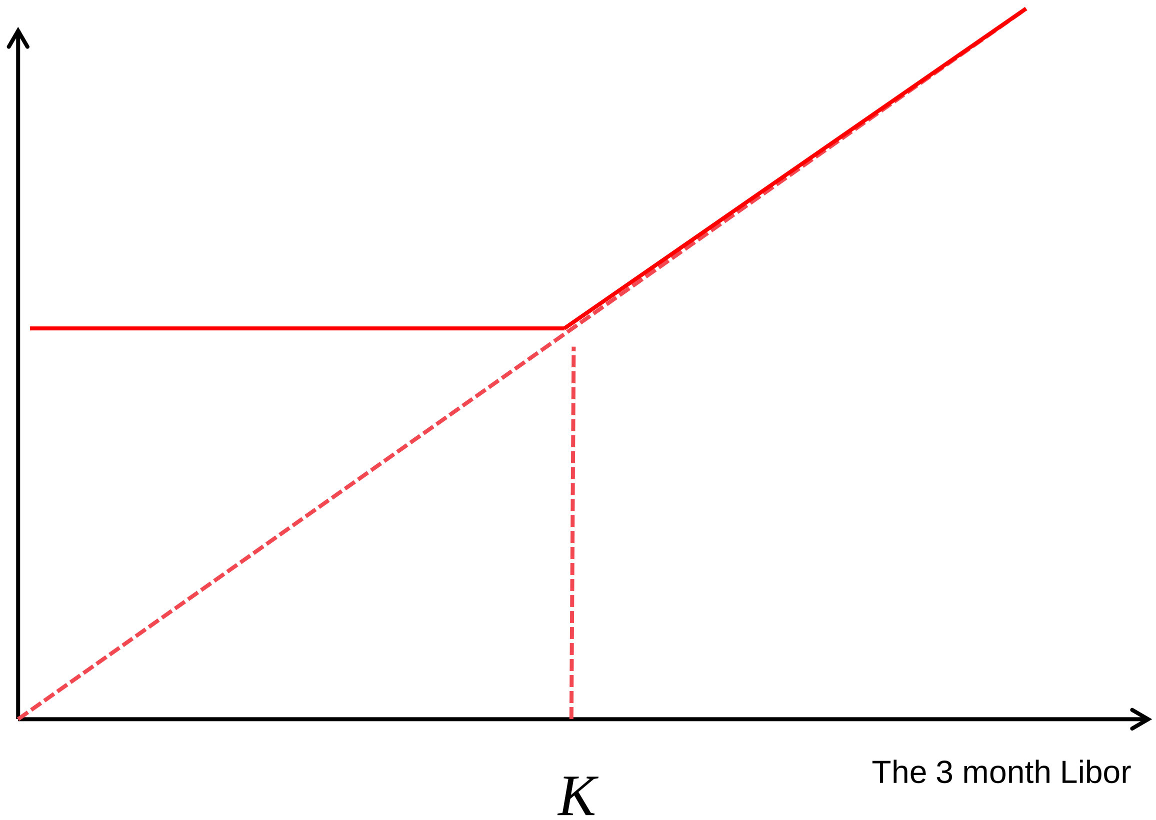
Floor

The client pays the fixed leg and receive the floating leg:



Floor

Payoff



Models

Kyle 1985 Model

Assumptions

We consider a financial market where a single risky asset is traded at time t=1. A the opening of the trading day (time t=0) N traders receive a Signal S about the liquidation value of the risky asset $\tilde{v} \sim N(0, \sigma_v^2)$

In this setup S=v

Moreover, noise traders submit an order $\tilde{u} \sim N(0, \sigma_u^2)$ independent of $\tilde{v}$

Market makers set the price by observing the aggregate order flow $\tilde{\omega} = \sum_{i=1}^{N} \tilde{x}_i + \tilde{u}$

At the end of the trading day the liquidation value is announced to the market.

Results : Equilibrium

<u>Proposition 1.</u>

There exist a unique linear equilibrium in which the reaction of each agent to his private information is $x = \beta\tilde{v}$

The price is given by $p = E\left[\tilde{v} \middle| \tilde{w}\right] = \lambda\tilde{w}$

Results: The reaction

The reaction is given by:

$$\beta = \frac{1}{\sqrt{N}} \frac{\sigma_u}{\sigma_v}$$

Therefore each insider reacts less aggressively to his private information when the competition is low (N, the number of insiders).

The reaction is higher when the insider can better dissimulate his private information (thanks to the noise σ_u)

Results: The liquidity

The liquidity parameter is given by:

$$\lambda = \frac{\sqrt{N}}{N+1} \frac{\sigma_v}{\sigma_u}$$

Therefore the liquidity parameter which measures the adverse selection increases with the noise stemming from the liquidity traders. Indeed, the insiders can hide their private information in this case. And the market makers fear to deal with better informed traders.

However, the competition leads the insiders to reveal their private information. And therefore the adverse selection problem drops with the competition

Results: Informativeness of price

The error variance of price is given by:

$$\Sigma = \operatorname{var}\left(\tilde{v} \middle| \tilde{w}\right) = \frac{1}{N+1}\sigma_v^2$$

When N=1, at time t=1 the single insider reveals the half of his private information. So the market is not strong-form efficient.

However, when the competition (N) leads the insiders to reveal almost all private information. The error variance of price tends to 0 when N tends to infinity.

Results: Profits

The individual and aggregate expected profits are given by:

$$\begin{cases} \pi_{individual} = \dfrac{1}{\sqrt{N}\left(N+1\right)}\sigma_u\sigma_v \\ \pi_{aggregate} = \dfrac{\sqrt{N}}{N+1}\sigma_u\sigma_v \end{cases}$$

When N=1, at time t=1 the single insider realizes a positive profits which increases with the level of noise stemming from the noise traders (σ_u) . So the market is not strong-form efficient.

However, when the competition (N) increases both individual and aggregate profits tend to zero

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Noise and Competition in Strategic Oligopoly

Noise and Competition in Strategic Oligopoly → Problématique (1/7)

The models à la Kyle (1985) point out two phenomena:

The informativeness of prices increases with the number of insiders.

The competition between the informed traders destroys their profits (Holden et Subramanyam (1992)).

To diminish the informativeness of prices and to prevent the profits from falling, we may introduce ex-ante a noise into the signals of the informed agents.

Several questions arise:

Is there an optimal level of noise on behalf of the insiders?

at this level of noise what is the amount of information released to the market?

Do the insiders keep a positive aggregate profit when their number tends to infinity?

Noise and Competition in Strategic Oligopoly → Model (2/7)

One considers a static market model where a risky asset is traded at t=1.

t=0 **t=1** $\tilde{v} \sim N(0, \sigma_v^2)$

There are *N* insiders who received at t=0 a signal about the fundamental value of the risky asset

$$\tilde{S}_i = \tilde{v} + \tilde{\varepsilon}_i \ \ pour \ \ i = 1, \ldots, N$$

.

Noise traders submit an order $\tilde{u} \sim N(0, \sigma_u^2)$ **independent of** $\tilde{v}$

Market makers set the price by observing the aggregate order flow

$$\tilde{\omega} = \sum_{i=1}^{N} \tilde{x}_i + \tilde{u}$$

Noise and Competition in Strategic Oligopoly → Symmetric case(3/7)

Proposition 1

There exists a unique linear equilibrium characterized by: the demand of insider i: .

$$x^* = \frac{\sigma_u}{\sigma_v} \frac{1}{\sqrt{N}\sqrt{1+\frac{\sigma_\varepsilon^2}{\sigma_v^2}}} S$$

The price set by the market makers:

$$p = \frac{\sigma_v}{\sigma_u} \frac{\sqrt{N}\sqrt{1+\frac{\sigma_\varepsilon^2}{\sigma_v^2}}}{N+1+2\frac{\sigma_\varepsilon^2}{\sigma_v^2}} \omega$$

Noise and Competition in Strategic Oligopoly → Symmetric case (4/7)

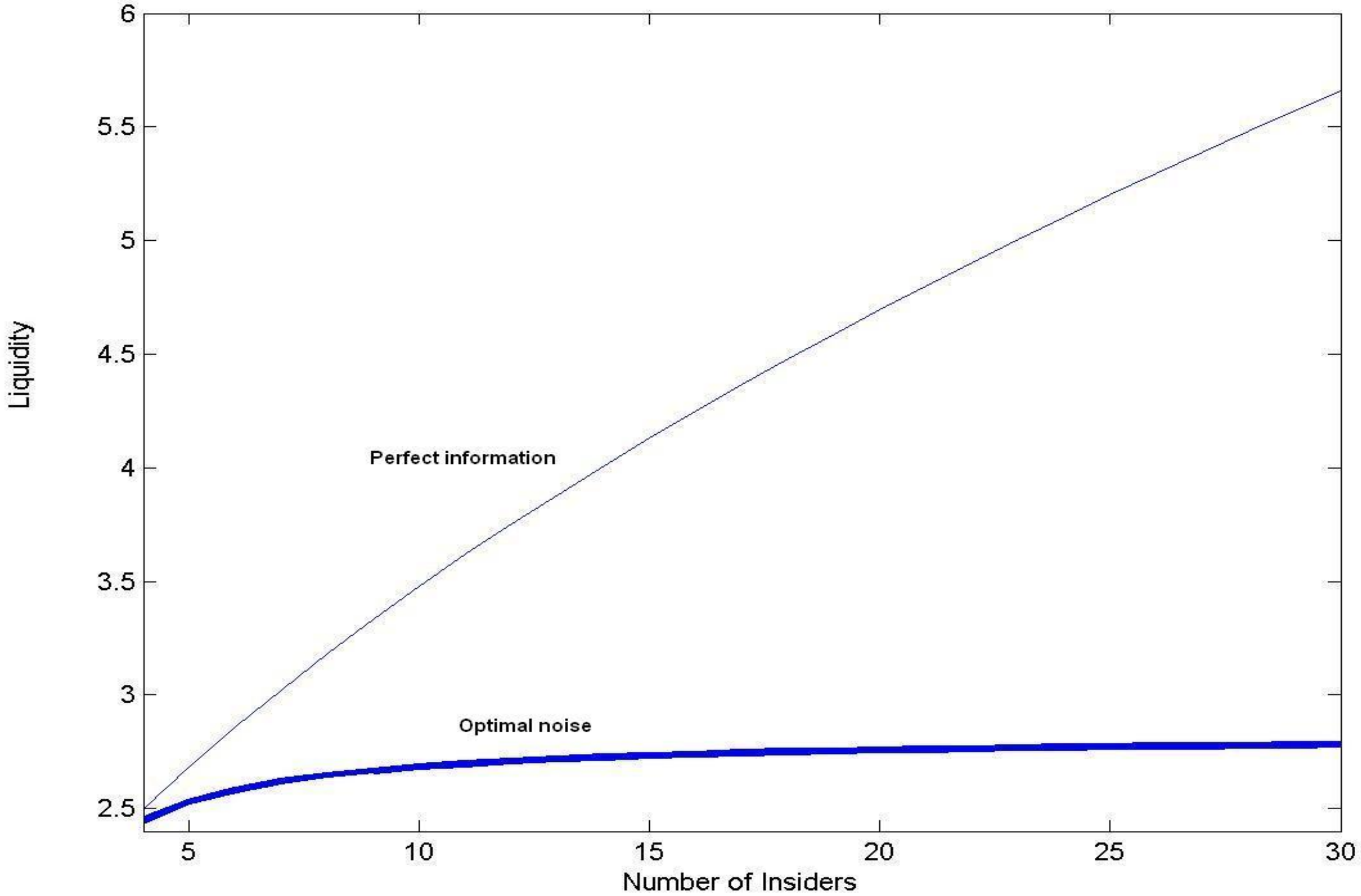
Liquidity **:**

At σ_ε fixed, the liquidity begins by descreasing and then by increasing with the number of insiders (*N*). If the signal is perfect then the liquidity always increases with *N*.

For $N < 4$, one observes that the liquidity increases with the level of noise in the signals.

For $N > 3$, the liquidity begins to decrease with the level of noise in the signals and after reaching a minimum it increases with the level of noise.

Noise and Competition in Strategic Oligopoly
→ Symmetric case(5/7)



Noise and Competition in Strategic Oligopoly → Symmetric case (6/7)

Proposition :

For N > 3 fixed, the individual profit increases with σ_ε until it reaches a maximum and then it decreases.

For N >3 fixed, there exists a range for the level noise in which the insiders' profits are higher than ones obtained with perfect information about the fundamental value of the risky asset.

At the optimal level of noise the aggregate profits if the insiders does not tend to zero when the number of insiders N tends to infinity.

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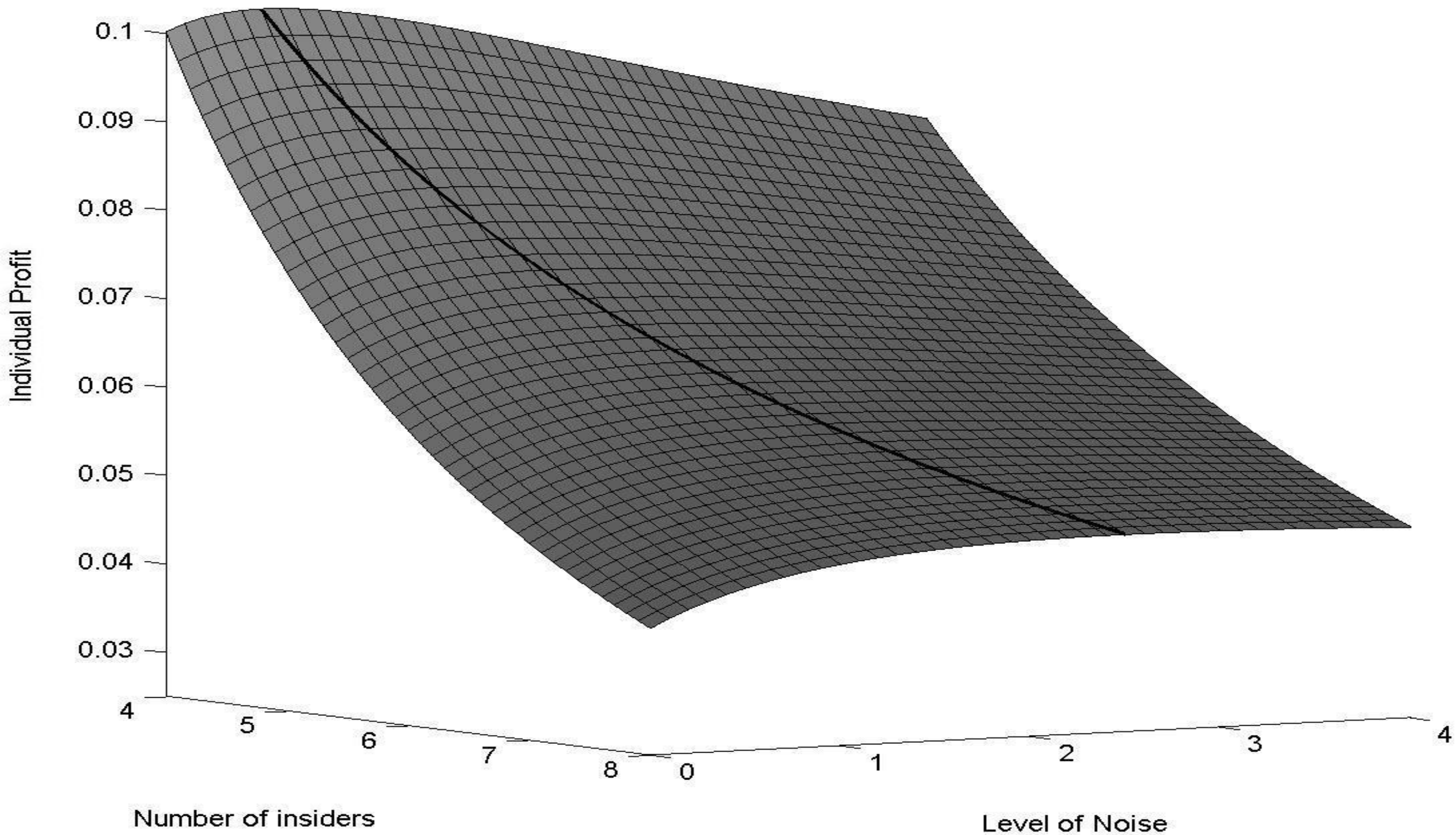
Noise and Competition in Strategic Oligopoly
→ Symmetric case (7/7)



The bold black line represents the optimal level of noise

Bullish Bearish Strategies of Trading

Journal of Financial and Quantitative Analysis (2004)

Bullish Bearish Strategies of Trading → Issues (1/5)

Informed agents receive a directional information (Bullish /Bearish).

This type of information differs from the ones à la Kyle (1985).

We study the impact of this new feature on the liquidity and the price setting

Are the insiders' profits higher in this model than the Kyle (1985) model?

Bullish Bearish Strategies of Trading → Model (2/5)

We consider a risky asset which is traded only at time t=1.

t=0 ——————————— t=1 $\tilde{v} \sim N(0, \sigma_v^2)$

There are N informed agents who receive a Bullish signal (S=+1) or Bearish signal (S=-1) about the risky asset at t=0.

Noise traders who submit an order $\tilde{u} \sim N(0, \sigma_u^2)$ **independent of** $\tilde{v}$

Market makers who set the price after observing the aggregate order flow

$$\tilde{\omega} = \sum_{i=1}^{N} \tilde{x}_i + \tilde{u}$$

Bullish Bearish Strategies of Trading → Equilibrium (3/5)

Proposition 1

There exists a unique linear equilibrium where the demand of the insider i and the price are respectively given by:

$$\tilde{x}_i = \gamma^*(N) = \frac{\sigma_u}{\sqrt{N}} \quad si \ S = 1$$

$$\tilde{x}_i = -\gamma^*(N) = -\frac{\sigma_u}{\sqrt{N}} \quad si \ S = -1$$

$$\tilde{p} = \frac{2\sigma_v}{\sqrt{2\pi}} \tanh\left(\frac{\omega\sqrt{N}}{\sigma_u}\right)$$

Proposition 2

Both the individual profit and the aggregate profit tend to zero when the number of informed traders tends to infinity.

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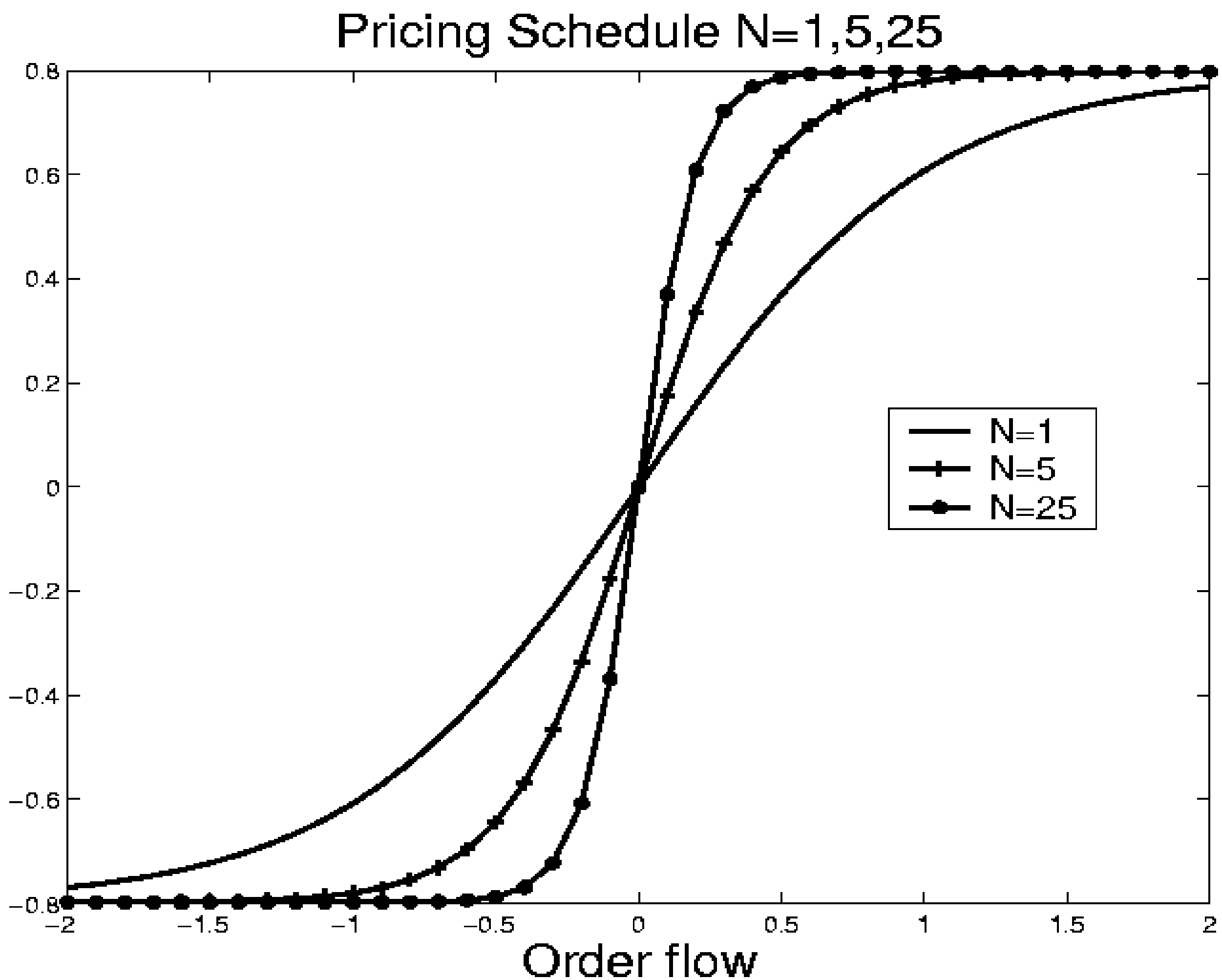
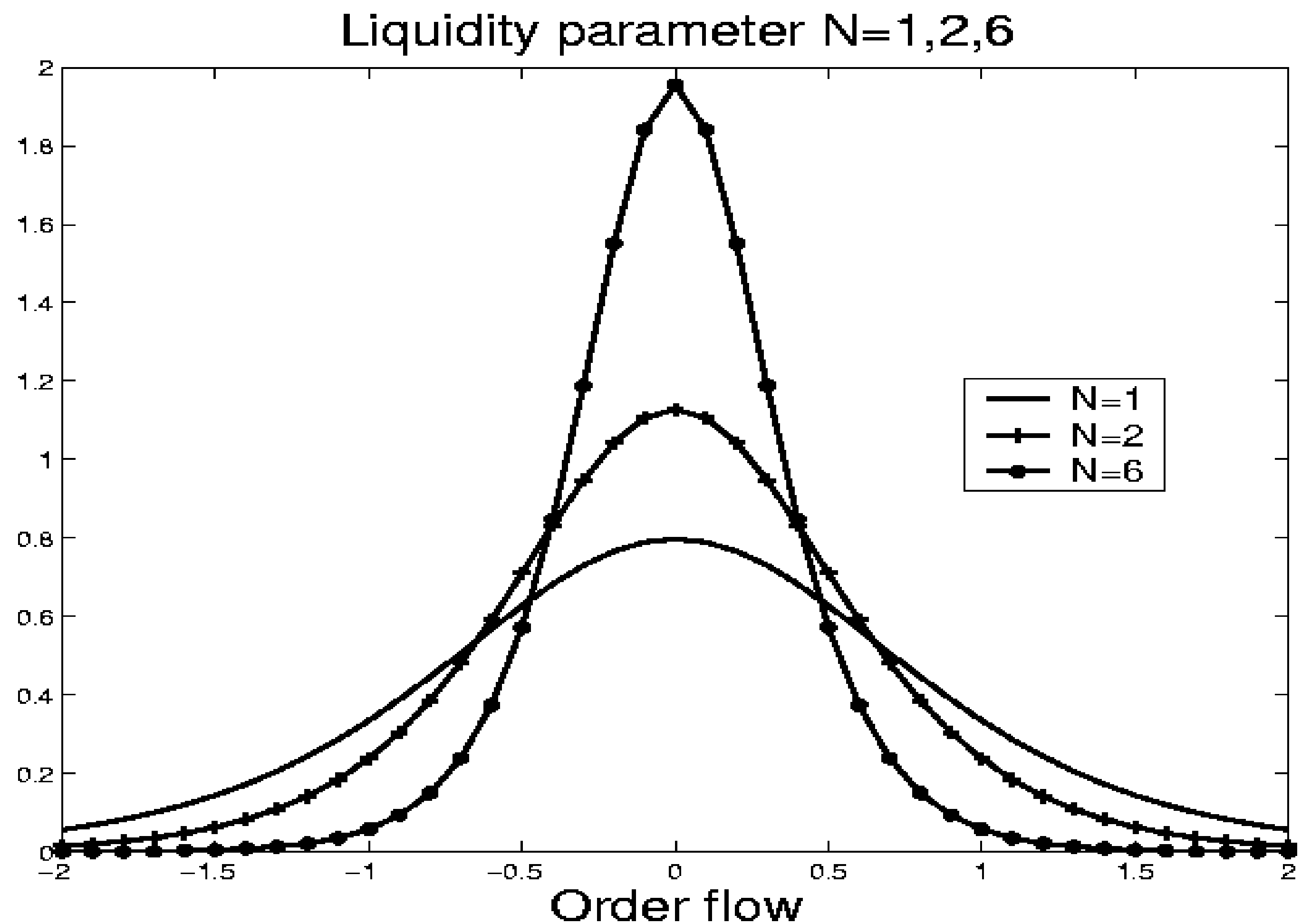
Bullish Bearish Strategies of Trading
→ Figures (4/5)



Bullish Bearish Strategies of Trading
→ Figures (5/5)



Heterogeneous Beliefs and Imperfect Competition in Sequential Auction Markets

Presentation Layout

- Introduction
- Literature Review
- Issues
- Model
- Equilibrium
- Informativess of prices
- Liquidity
- Competition
- Profits
- Conclusion

Introduction

Strategic trading models study the impact of the structure of information on:

- The efficiency of the markets,
- profits of the market participants

They explore mainly two cases :

- When the insiders know the liquidation value perfectly,
- When the informed traders' signals are initially correlated.

→ These setups cannot allow us to know the structure of information of all trades and to study the trade-off between noise and competition

Benchmark models

Kyle (1985); Holden Subramanyam (1992)

- In the monopolistic case the insider makes positive profits by limiting the size of his early trades. Information is gradually incorporated into prices.
- The competition is very fierce and leads the insiders to reveal almost all private information in the early auctions.

Foster and Vishwanathan (1996); Back Cao and Willard (2000)

- The results depend on the initial correlation. The insiders' profits are higher with positive but not perfect correlation than identical. The lower the signal correlation, the less informative is the price process. There are two phases : "rat race", "waiting game".

Our Setup

In contrast to the previous models with long-lived individual information, the sum of the signals are not equal to the fundamental value and we :

- Know the structure of each noisy signal
- Can study the trade-off between noise and competition
- Can study the phases of the competition owing to the level of noise (and/or) the correlation.

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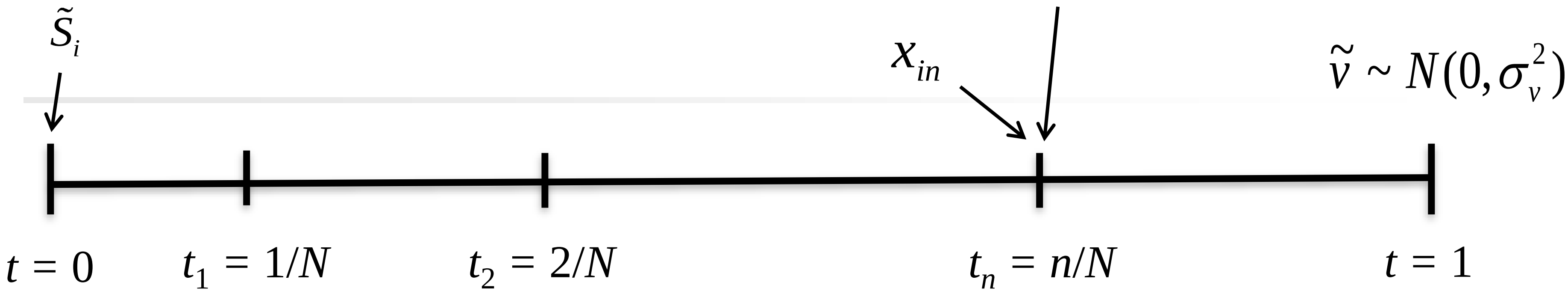
Issues

- What happens to insiders' profits when their private signals are noisy?
- What dynamic strategies should informed market participants use to maximize their profits?
- Is there an optimal level of noise that maximize traders' profits?
- How the liquidity and the informativeness of prices are affected by the two dimensions of the competition (temporal and spatial)?

Model



M risk-neutral informed traders who each receives a signal, at $t = 0$,

$$\tilde{S}_i = \tilde{v} + \tilde{\varepsilon}_i, \tilde{\varepsilon}_i \sim N(0, \sigma_\varepsilon^2), \forall i = 1, \ldots, M$$

Liquidity traders who submit an aggregate order at each auction n:

$$\Delta u_n \sim N(0, \sigma_u^2 \Delta t_n) \qquad \forall n = 1, \ldots, N$$

Competitive risk-neutral market makers who observe the aggregate orders, $\tilde{\omega}_n = \Delta \tilde{X}_n + \Delta \tilde{u}_n$ and set the price p_n, at each auction n.

Equilibrium

- **Proposition 1 If** $\frac{\sigma_\varepsilon^2}{M} < \Sigma_N$**, there exists a unique linear equilibrium in which:**
  - Insider *i* submits, at auction *n*, the order

$$\tilde{x}_{in} = \left(\alpha_n \tilde{S}_i + \beta_n p_{n-1}\right)\Delta t_n .$$

  - The price set by the market makers at auction *n* is given

$$p_n = p_{n-1} + \lambda_n \omega_n .$$

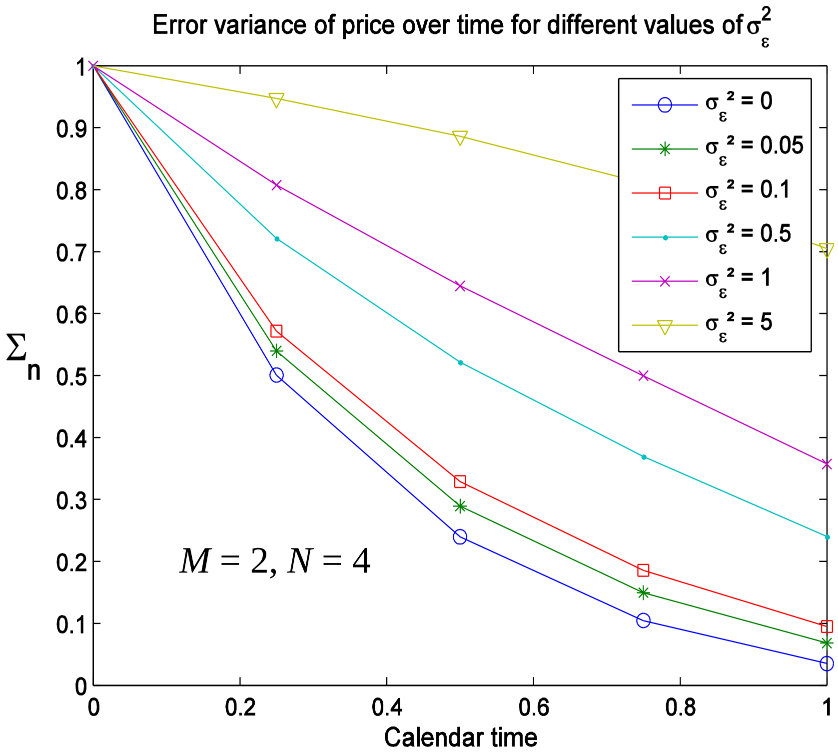
Interpretation

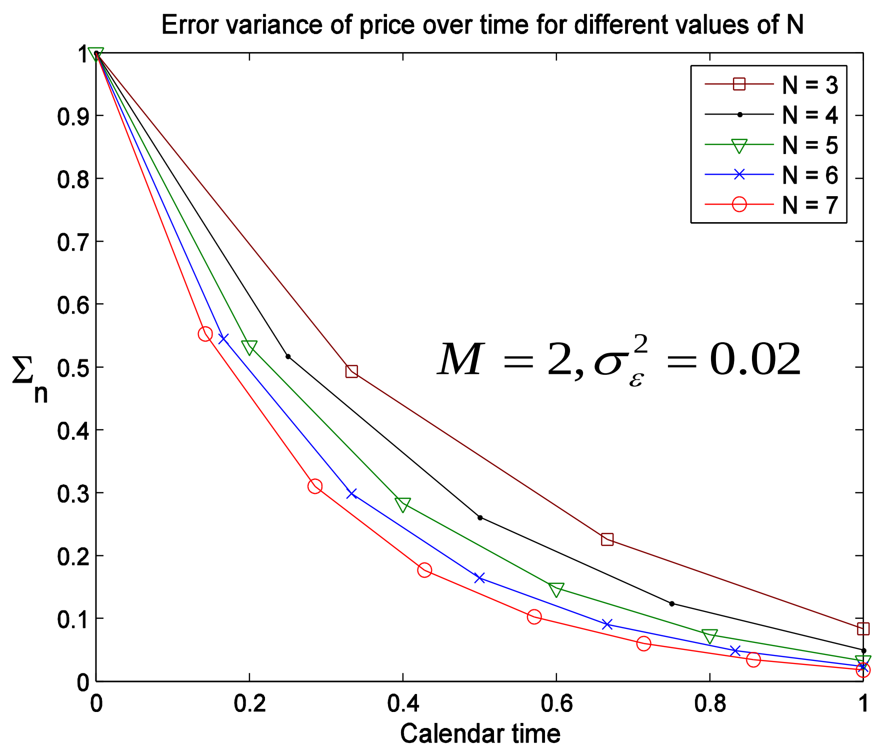
The trading process continues as long as informed agents still have some information that is not incorporated into prices.

The precision of the market maker’ s information is limited by the level of noise.

Price Informativeness

<u>Proposition 2</u> *When the level of noise in the private signals is low, prices incorporate quickly private information. When private signals are noisier, the decay of Σ_n becomes slower, and less information is acquired by the market maker during trading.*





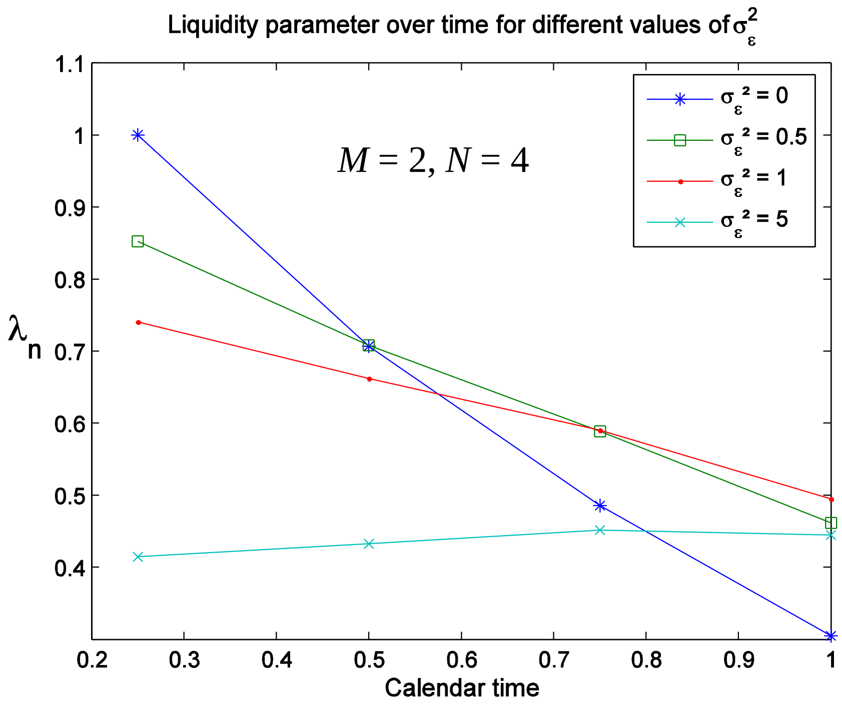
Interpretation

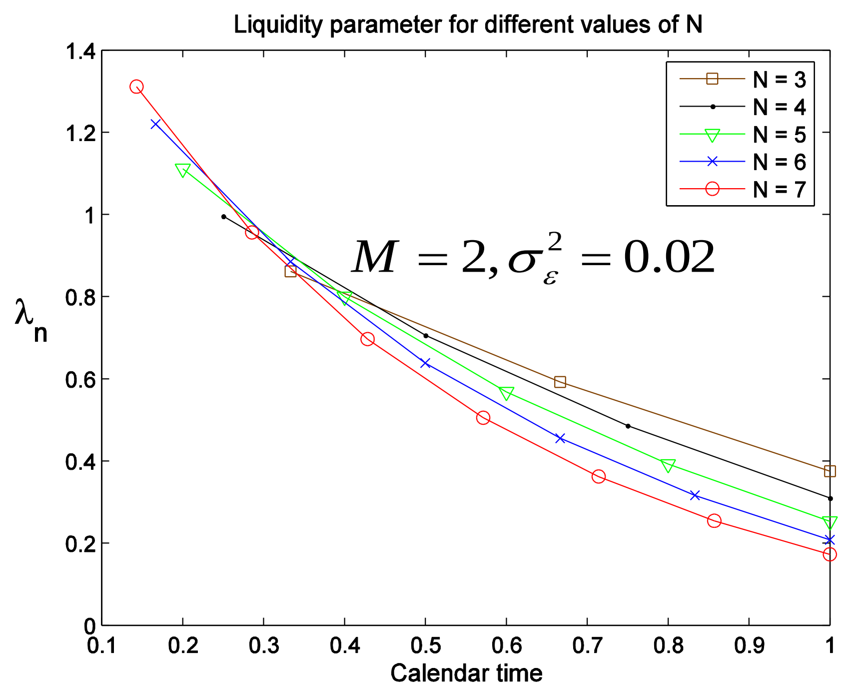
When the level of noise is low, insiders compete more aggressively on their private information and so release the largest part of their information in the first auctions.

The informativeness of prices increases with the number of auctions. (temporal component)

Liquidity

Proposition 3 *The liquidity parameter* λ_n *decreases over time when the level of noise is low. When the private information becomes noisier,* λ_n *declines more slowly. For large values of the noise* λ_n *increases over time.*





Interpretation

When the level of noise in the private signals is low, the competition is very fierce . The adverse selection problem drops at the first autions.

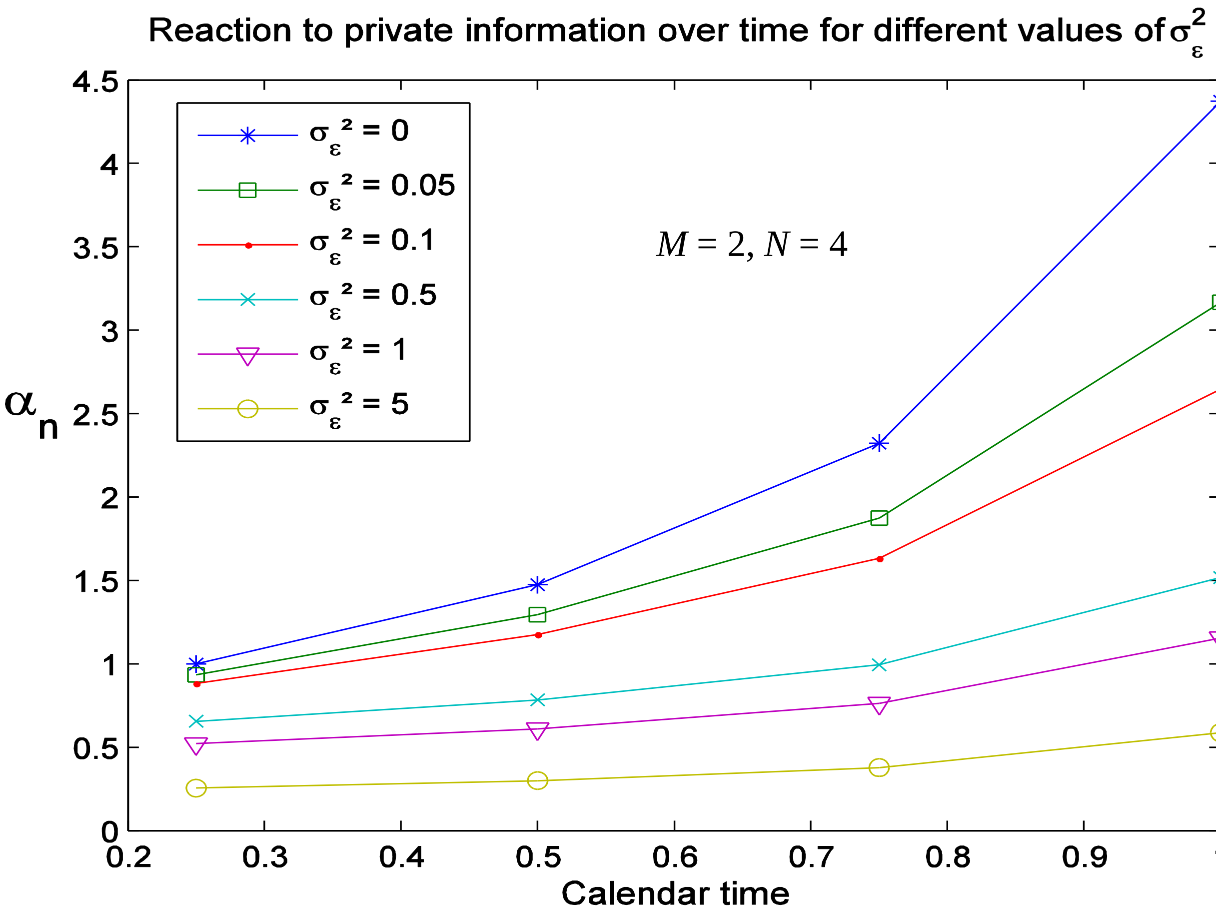
When there is a large level of noise, insiders delay their trades to the last auctions. Therefore, the market maker reacts more agressively to the order flow that appear in the last periods of trading.

When the number of auctions increases the adverse selection problem decreases

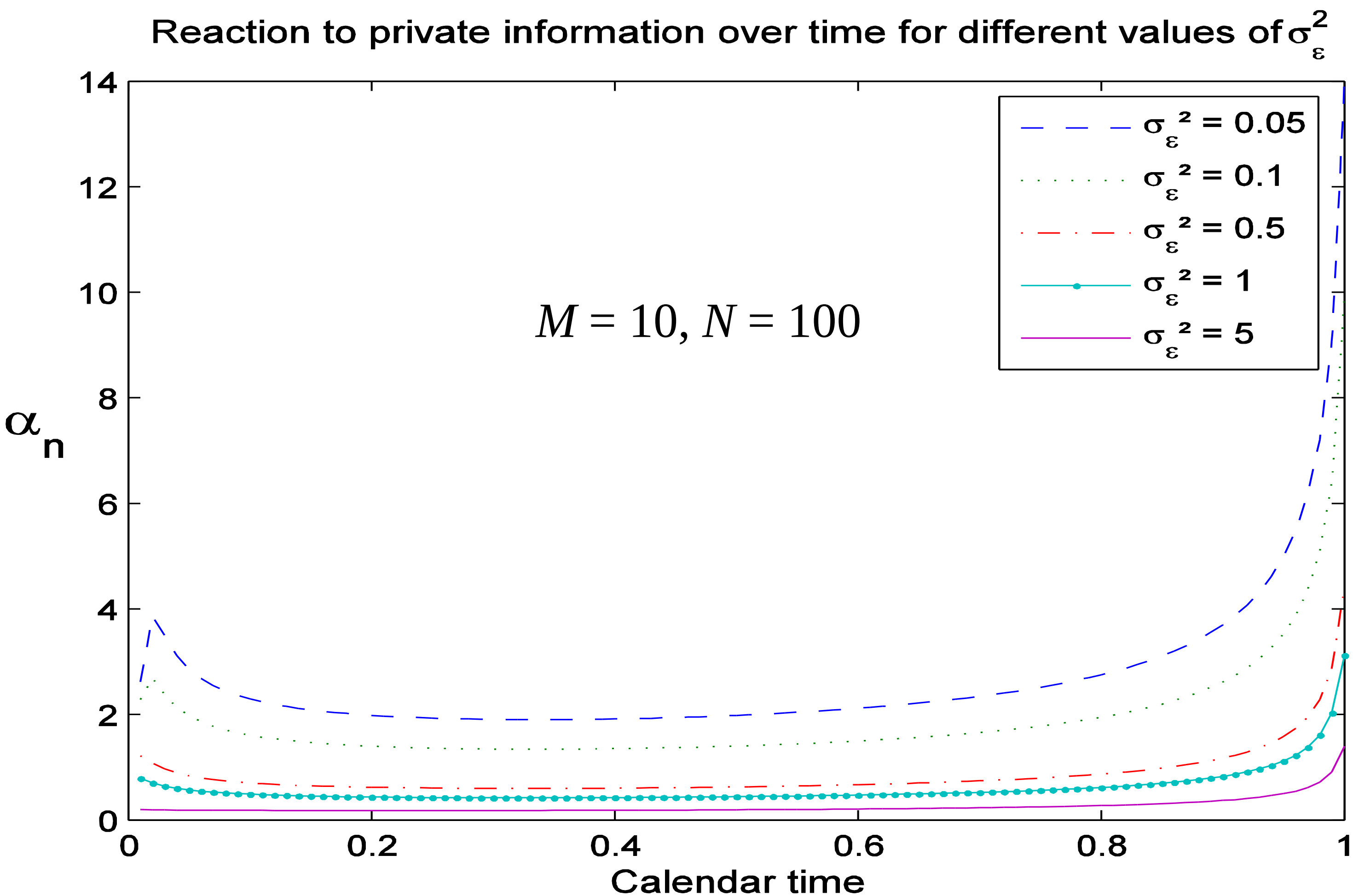
Competition →Reaction

Proposition 4   The informed market participants react more to both their private and public information at the end of the trading day.

Competition →Reaction



Competition →Reaction



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Interpretation

We find that the competition may have one of the following forms :

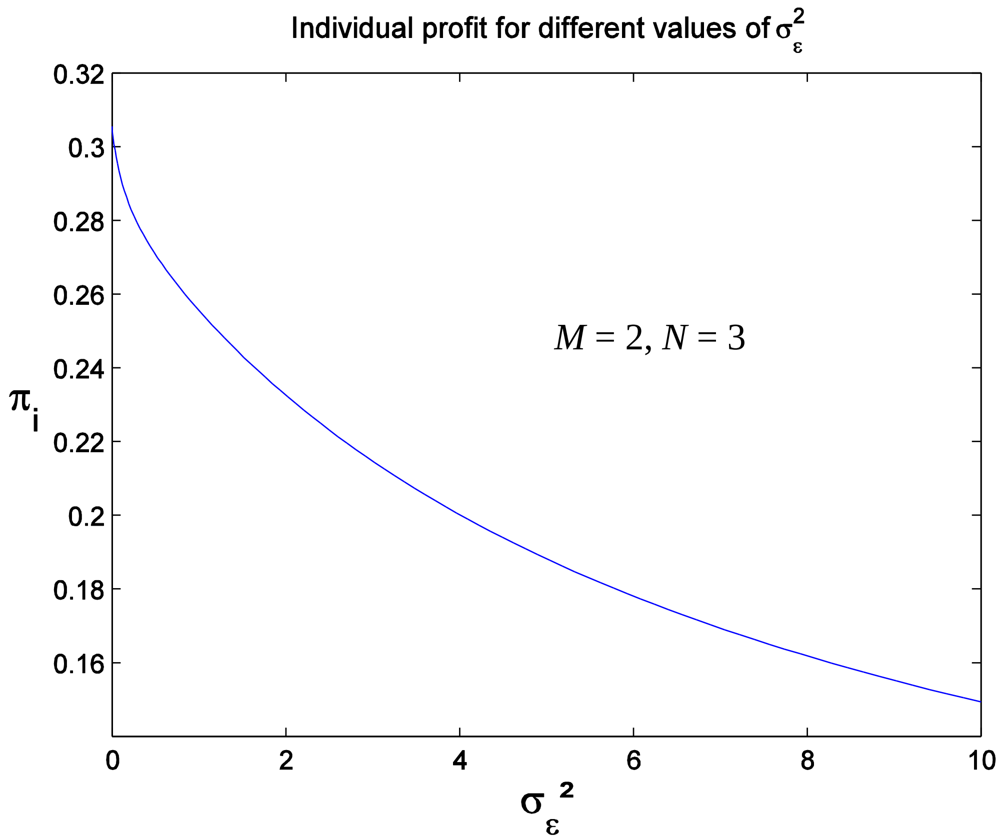
- A **"rat race"** phase when the level of noise in traders' signals is low. The intensity of this **"rate race"** increases with the number of auctions and insiders.

- A **"waiting game"** when the level of noise in traders' signals is high. This **"waiting game"** phase is followed by a **"rat race"** in the last periods of trading.

- A **"rate race"** that appears in the early periods of trading when the number of insiders is high. This early **"rate race"** becomes more pronounced when the level of noise drops and when the number of insiders / auctions increases.

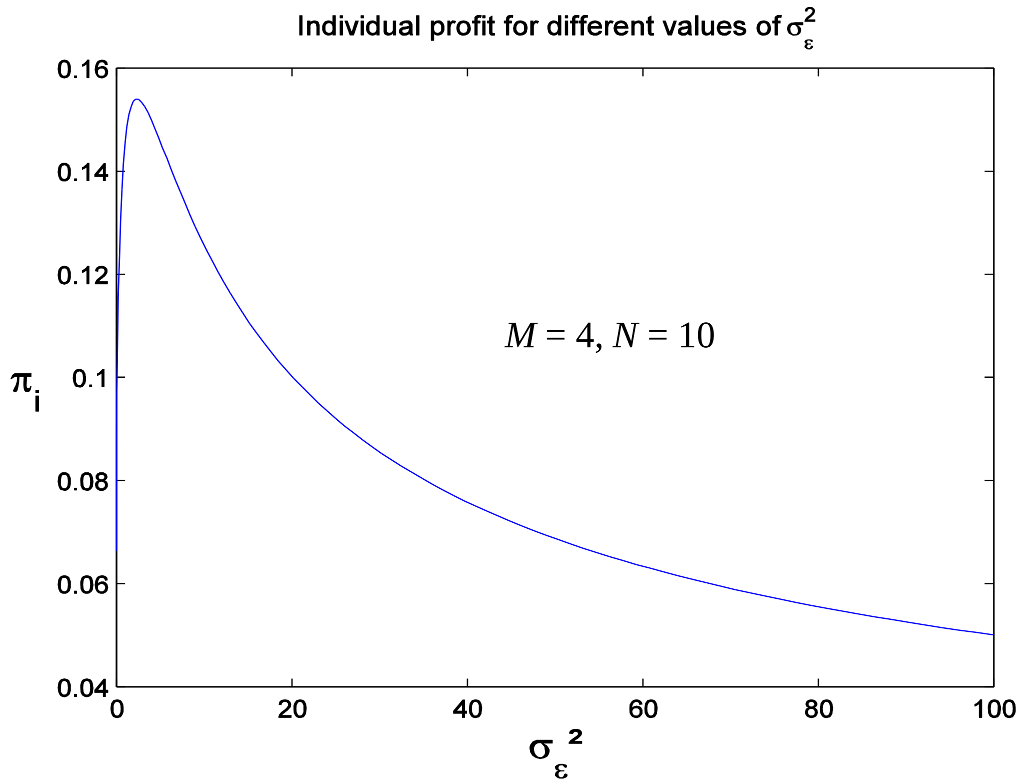
Profits

Proposition 4 ***When the competition is low (i.e. M = 2 and N < 7, M = 3 and N < 3, M = 4 and N = 1), the introduction of noise in the traders signals reduces their profits.***



Profits

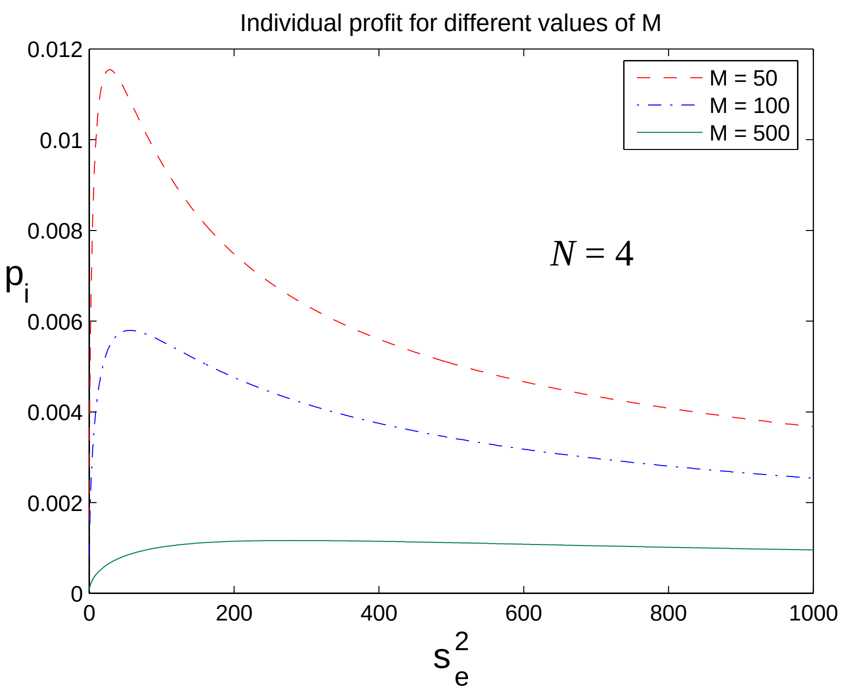
<u>Proposition 5</u>: *When the level of competition is high ($M = 2$ and $N \geq 7$, $M = 3$ and $N \geq 3$, $M = 4$ and $N \geq 2$, $M \geq 5$ and any N), the traders' profits are non-monotonic with the level of noise.*

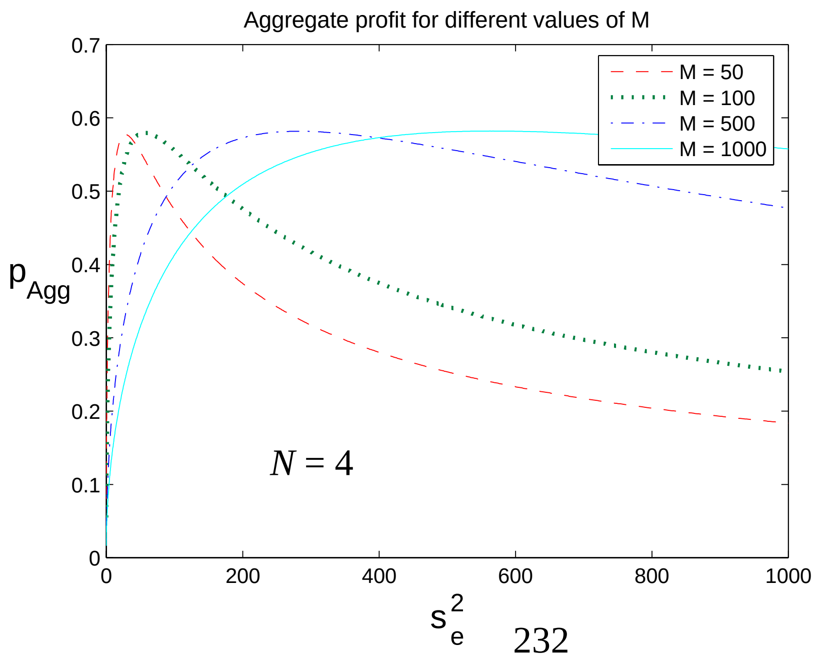


Profits at optimal noise

Proposition 6.

- *There exists an optimal level of noise which maximizes the expected profits of the traders when the number of insiders is relatively high.*
- *This level increases with the number of insiders.*
- *The optimal individual profit decreases with the number of insiders.*
- *The optimal aggregate profit converge to a finite positive value when M increases infinitely.*

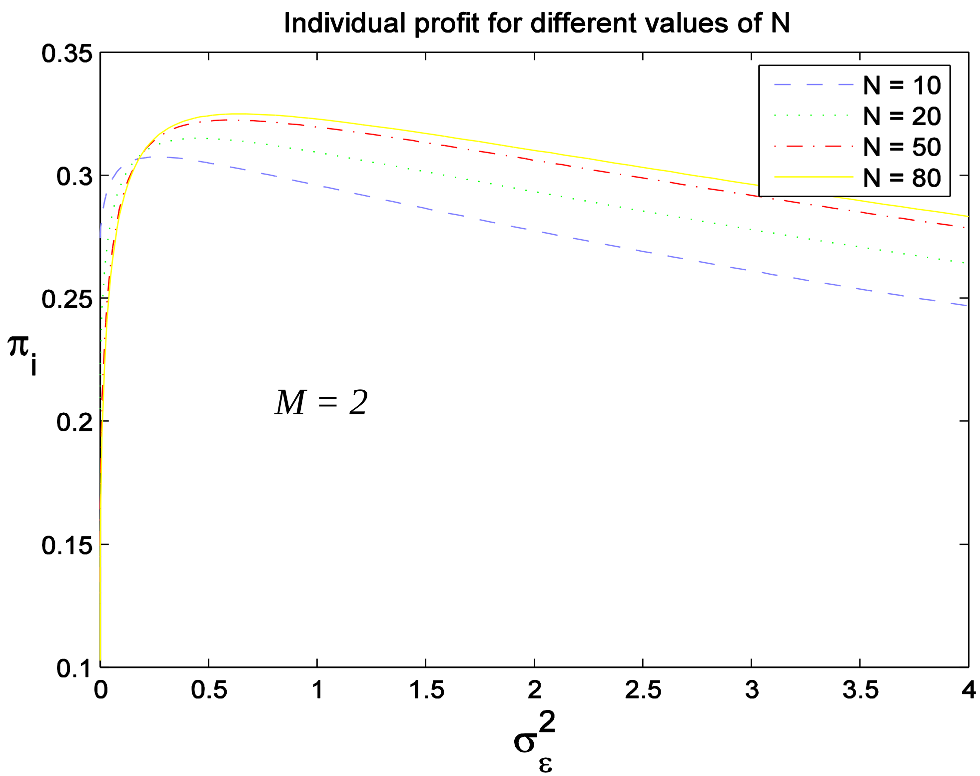




Profits at optimal noise

Proposition 7.

- *The optimal level of noise increases with the number of auctions.*
- *For a fixed level of noise, the individual profit decreases with the number of auctions.*
- *The optimal individual profit converges to a finite positive value when N increases infinitely.*



Interpretation

The trade-off between noise and competition has two dimensions: a spatial component, the number of traders, and one temporal component, the number of auctions.

When the competition is low, noise always destroys the profits

When the competition is high, noise can increase the profits

Conclusion

Our model allows us to explore the influence of noise on the insiders' expected profits and the price setting.

We exhibit the trade-off between noise and competition in a dynamic setting.

We show that there exists a unique linear equilibrium in terms of level of noise.

We show that it could be more profitable to possess noisy signals than to know the liquidation value of the risky asset perfectly.

Strategic Market Making and Risk Sharing

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Outline

Introduction

Literature Review

Issues

Model

Results

- Equilibrium
- Liquidity
- Profits

Conclusion

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Introduction

Common assumptions about price and liquidity in financial markets:

- Market makers behave competitively,
- Investors cannot split their orders among market makers,
- Market makers are risk-neutral.

→ **These assumptions imply that:**

- Market makers earn zero expected profit,
- Posted price equals expected value of asset given market makers' information and aggregate order flow,
- No inventory cost exploration.

Two benchmark models

Biais, Martimort and Rochet (2000):

- trader is risk averse
- market makers are risk-neutral

➢ when number of market makers is finite, they earn positive expected profit
➢ as the number of market makers tends to infinity, they earn zero expected profit

Bernhardt and Hughson (1997):

- all traders are risk neutral
- in equilibrium, market makers cannot earn zero expected profits

➢ the existence and the form of the equilibrium is shown only in the case of a duopoly

Our setup

In contrast to the previous models, we assume that:

K risk averse investors

each investor can split his orders

risk-averse market makers receive orders from several investors

Issues

How is the cost of trading affected by the level of the market markers' risk aversion?

How is that same cost influenced by the number of market makers?

How is the overall liquidity of an asset affected by both the number of market makers and their risk aversion?

What form of order do risk-averse traders choose in the face of:

- the number of market makers
- their private signals
- their hedging needs

Model (1/2)

We consider a market where a risky asset and a riskless asset are traded among K traders and N market makers.

The liquidation value of the risky asset is $\tilde{v} \sim N(0,\sigma_v^2)$. For convenience, the riskless asset has its rate normalized to zero.

All agents are risk averse and have a CARA utility function

$$\forall k = 1,\ldots,K \quad U(W_k) = -e^{-\rho W_k}$$

$$\forall n = 1,\ldots,N \quad U(W_n) = -e^{-\rho_m W_n}$$

Model (2/2)

$$\tilde{v} \sim N\left(0, \sigma_v^2\right)$$

t=0 **t=1**

K *traders receive:*

$$\tilde{S}_k = \tilde{v} + \tilde{\varepsilon}_k$$

$\tilde{w}_k$ *risky asset*

$\tilde{c}_k$ *riskless asset*

N *market makers set prices*

1

2

$\vdots$

N

Equilibrium (1/2)

Proposition 1

If $\rho\tau_w^{-1} > \tau_\varepsilon\left(1+\tau_v^{-1}\tau_\varepsilon\right)$, there exits a unique linear equilibrium in which the price set by the market maker n is as follows:

$$p_n = \lambda_n(N)y_n$$

where y_n is his anticipated aggregate order flow.

Each investor k submits to the different market makers a market order of the following form:

$$x(S_k, w_k) = a(N)\left(S_k - \rho\tau_\varepsilon^{-1}w_k\right) \forall k = 1,\dots,K$$

Equilibrium (2/2)

Interpretation

The condition for the existence of an equilibrium:

- hedging motives > speculation motives.

Trader splits his market order across different markets:

- cost of trading is equalized across markets.

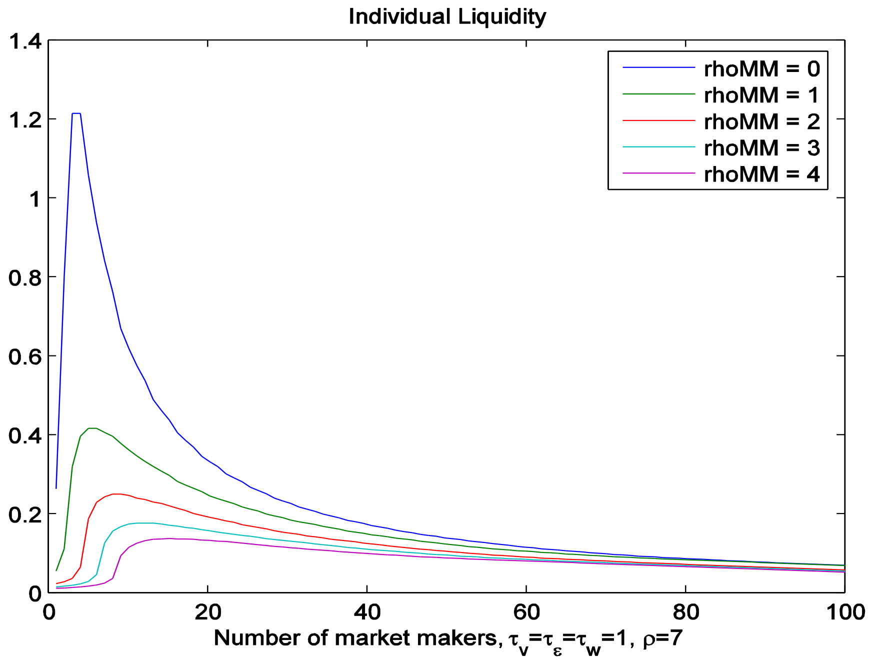
Properties of equilibrium
Liquidity (1/3)

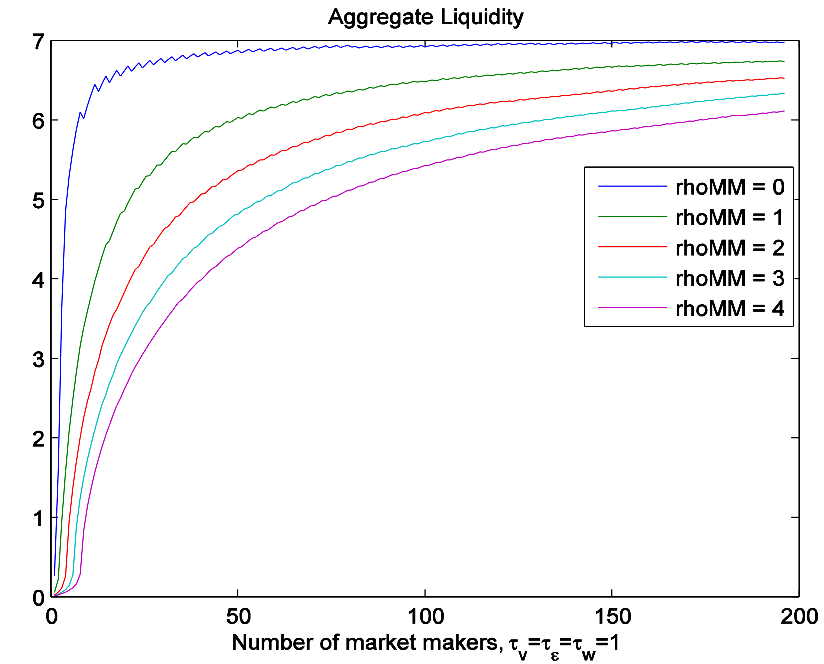
Proposition 2

Individual liquidity is a non-monotonic function of the number of market makers (*N*).

Aggregate liquidity increases with *N*.

Properties of equilibrium
Liquidity (2/3)





Properties of equilibrium
Liquidity (3/3)

Interpretation

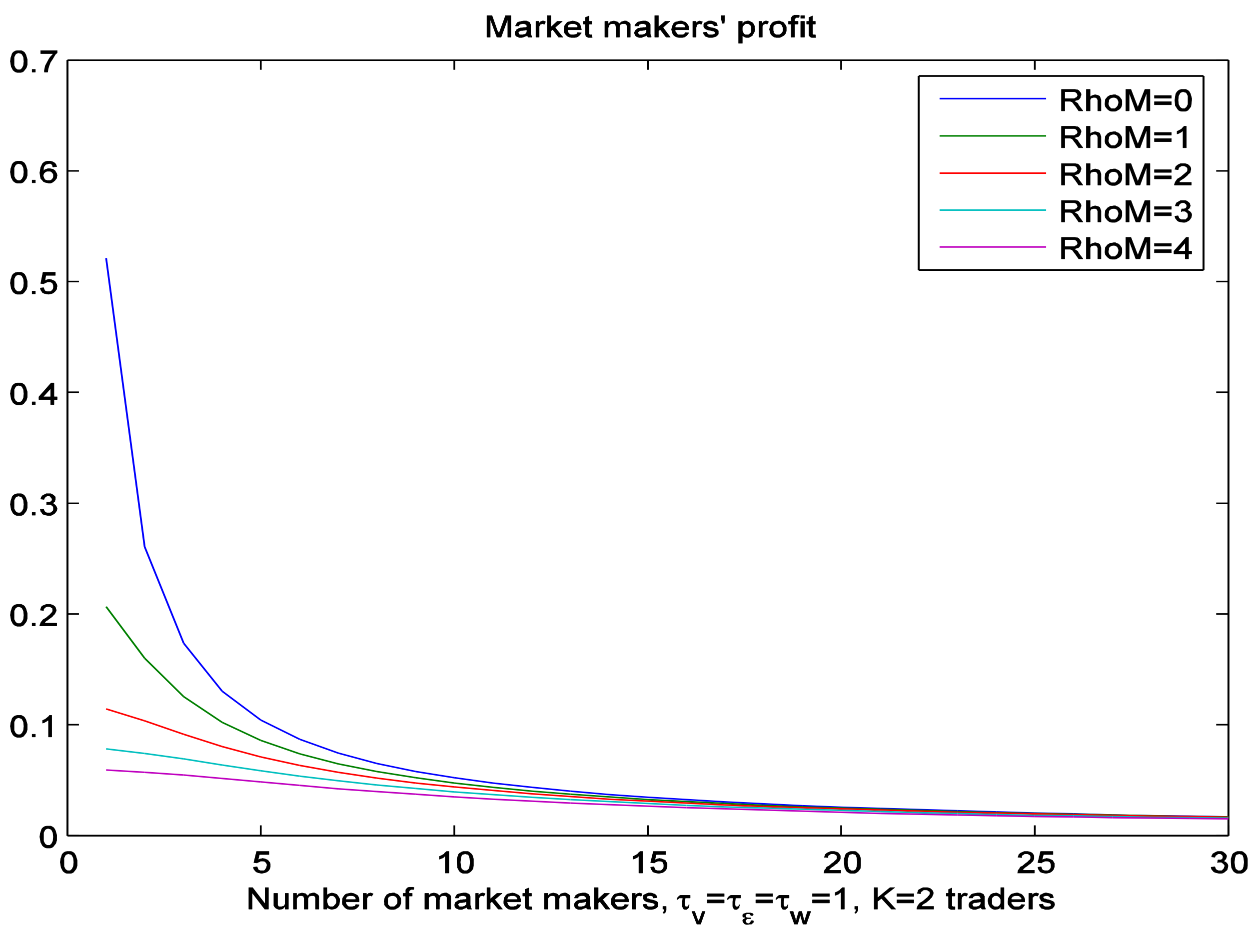
Market makers provide less and less *aggregate liquidity* as their risk aversion increases.

- In Bernhardt and Hughson (1997) (blue on the graph), the liquidity is the highest.

Individual liquidity

- decreases with the number of market makers
- for a low number of market makers, the increase is sharper for risk-neutral market makers than for risk-averse ones.

Results
Market makers’ expected utility (1/2)



Properties of equilibrium
Market makers' expected utility (2/2)

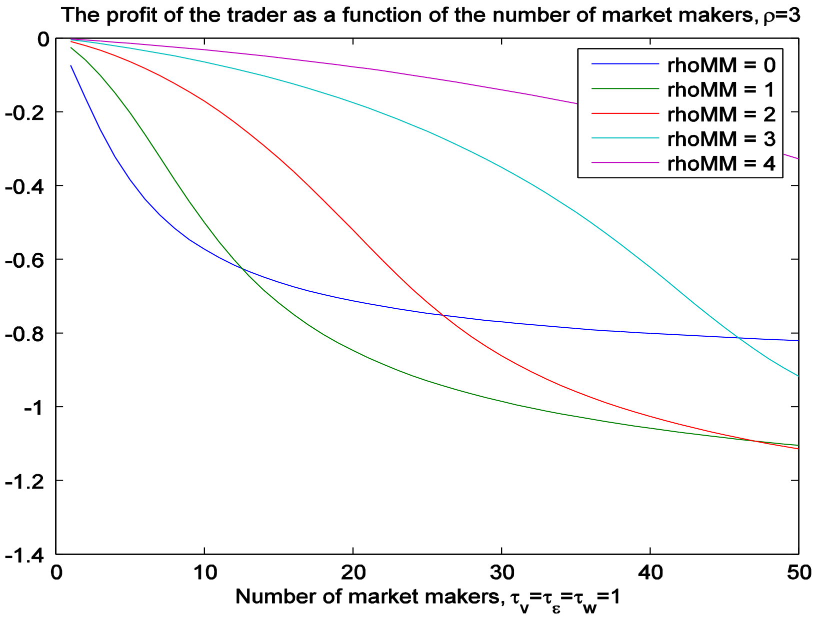
Interpretation

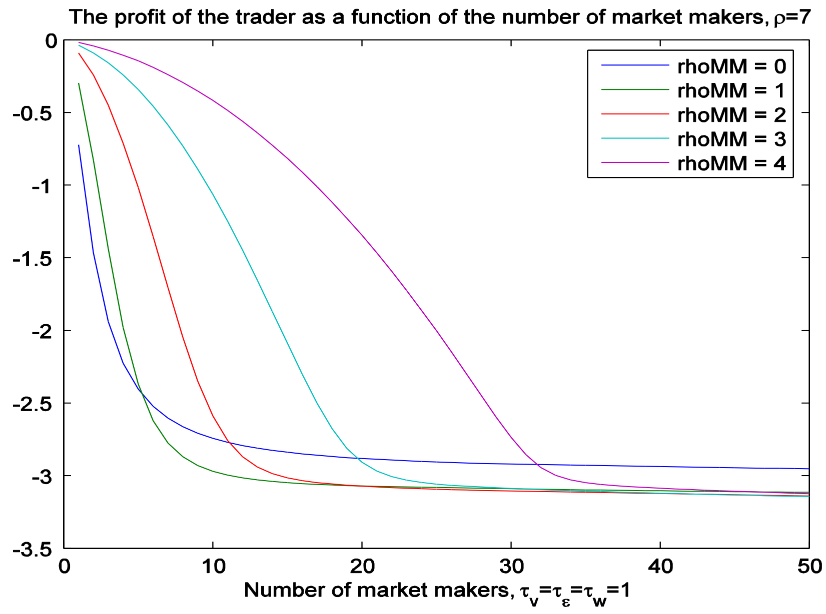
The market makers' aggregate expected utility of profit:

- always decreases with the number of market makers
- tends to zero when the number of market makers is infinite.

The higher the market makers' risk aversion, the smaller their aggregate expected utility.

Results
Cost of trading (1/2)





Properties of equilibrium
Cost of trading (2/2)

Interpretation

The cost of trading increases with traders' risk aversion.

Compared to Bernardt and Hughson (1997), in our model:

- for a low number of market makers the cost of trading is lower
- after a certain threshold, the cost of trading is higher.

Conclusion

We have studied a market in which risk-averse traders can split their orders among different risk-averse market makers.

Our key findings are:

- ➢The existence of a linear equilibrium.
- ➢Market makers' aggregate expected profits:
  - ➢ are positive when their number is low
  - ➢ decrease as their number increases
  - ➢ tend to zero as their number tends to infinity.

- ➢Investors' cost of trading increases with the number of market makers.
- ➢Liquidity increases with the number of market makers.

→Therefore liquidity is an inappropriate measure of investors' welfare.

When Overconfident Agents Meet Feedback Traders

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Introduction

Most financial studies assume that :

- Markets are efficient
- Market participants are rational

These assumptions are inconsistent with

- Booms and crashes,
- The under-reaction or over-reaction of agents (De Bondt and Thaler 1985, 1987, 1990)
- The excessive volume traded (Odean (1998)).

→ **To understand such phenomena, behavorial finance has emerged (Kahneman and Tversky (1979))**

Speculative Bubbles

Definition :

➢A speculative bubble exists when the price of an asset does not equal its market fundamentals for some period of time for reasons other than random shocks.

➢The presence of such bubbles can be explained by psychological traits of market participants:

Feedback trading (contrarian or momentum)
Overconfident trading
Herd trading (beauty contest)

Two benchmark models

Overconfidence: Odean (1998) examines the influence of the overconfidence on different types of markets.

➢He finds that overconfidence increases the trading volume and the market depth and decreases the expected utility of overconfident traders.

Feedback trading: De Long, Shleifer, Summers, Waldmann (1990) consider a market with three types of traders :

- Rational informed speculators, passive investors.
- Positive feedback investors

➢Their main result is: the rational speculators cannot stabilize asset prices.

Model

We analyze a model with four periods

$\tilde{y}_{3i} = \tilde{v} + \tilde{\varepsilon}_{3m} \quad M \text{ different signals}$

t=0 t=1 t=2 t=3 t=4

$$\tilde{v} \sim N\left(\bar{v}, h_v^{-1}\right)$$

$\tilde{y}_{2i} = \tilde{v} + \tilde{\varepsilon}_{2m} \quad M \text{ different signals}$

The average private signals at time t

$$\bar{Y}_t = \sum_{i=1}^{M} \frac{\tilde{y}_{ti}}{M}$$

- There are informed traders: N_1 overconfident agents and N_2 rational speculators.
- *P* positive feedback traders (resp. negative feedback traders).

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Equilibrium

Proposition 1. There exists a unique linear equilibrium

$$\begin{cases} P_3 = \alpha_{31} + \alpha_{32}\bar{Y}_2 + \alpha_{33}\bar{Y}_3 \\ P_2 = \alpha_{21} + \alpha_{22}\bar{Y}_2 \end{cases}$$

$$\alpha_{31} = \frac{(N_1\eta + N_2)h_v\bar{v} + N_1\eta h_v b - a(N_1 + N_2 + P)\bar{x}}{(N_1\eta + N_2)h_v + 2(N_1(\kappa + \gamma M - \gamma) + N_2 M)h_\varepsilon} + \frac{aP\beta}{(N_1\eta + N_2)h_v + 2(N_1(\kappa + \gamma M - \gamma) + N_2 M)h_\varepsilon}(\alpha_{21} - P_1)$$

$$\alpha_{32} = \alpha_{33} + \frac{aP\beta}{(N_1\eta + N_2)h_v + 2(N_1(\kappa + \gamma M - \gamma) + N_2 M)h_\varepsilon}\alpha_{22}$$

$$\alpha_{33} = \frac{N_1(\kappa + \gamma M - \gamma)h_\varepsilon + N_2 M h_\varepsilon}{(N_1\eta + N_2)h_v + 2(N_1(\kappa + \gamma M - \gamma) + N_2 M)h_\varepsilon}$$

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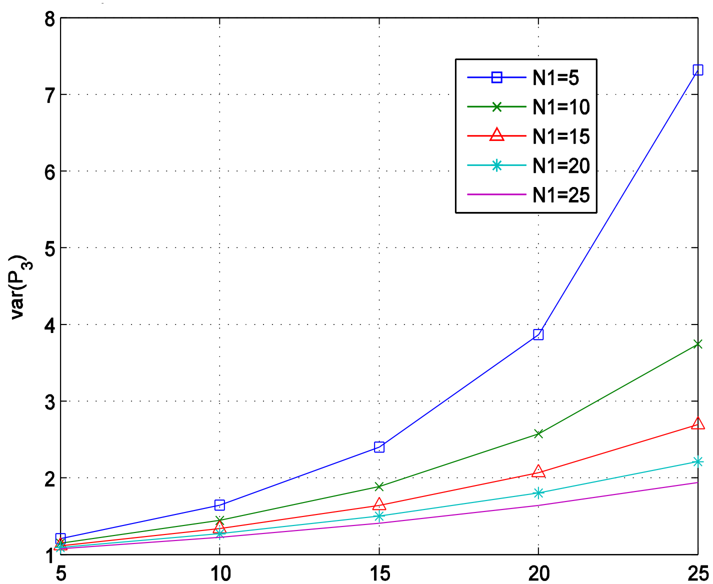
Properties
→ Volatility of prices (1/2)

Proposition 2. The variance of prices increases with the number of positive feedback traders.

When trend-chasing traders are present, the overconfidence can diminish the volatility of prices.

Property
→ Volatility of prices (2/2)

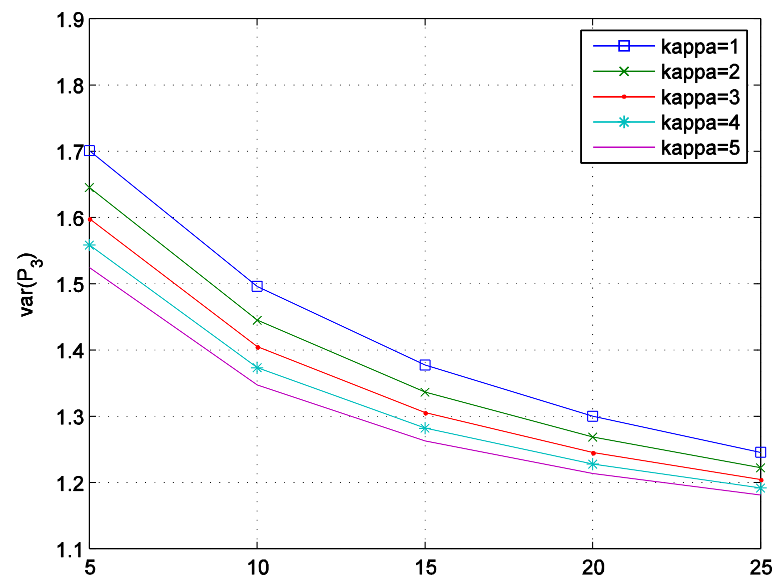
$$\mathrm{var}(P_3) = f(P)$$



***P*, the number of feedback traders**

$\kappa = 2\ ;\ \eta = \gamma = 0.5$

$$\mathrm{var}(P_3) = f(N_1)$$



***N1*, the number of overconfident agents**

$P = 10\ ;\ \eta = \gamma = 0.5$

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Properties
→ Volatility of prices (1/2)

Overconfidence has two opposite effects:

No feedback trading leads the market to be more volatile

With feedback traders, overconfidence stabilizes the prices

- indeed both rational and overconfident are informed and their trading release information.
- the risk bearing capacity of the market increases.

Increasing the number of overconfident traders destabilize the prices since more traders anticipate the trend-chasing behavior.

Property
→ serial correlation of prices (1/3)

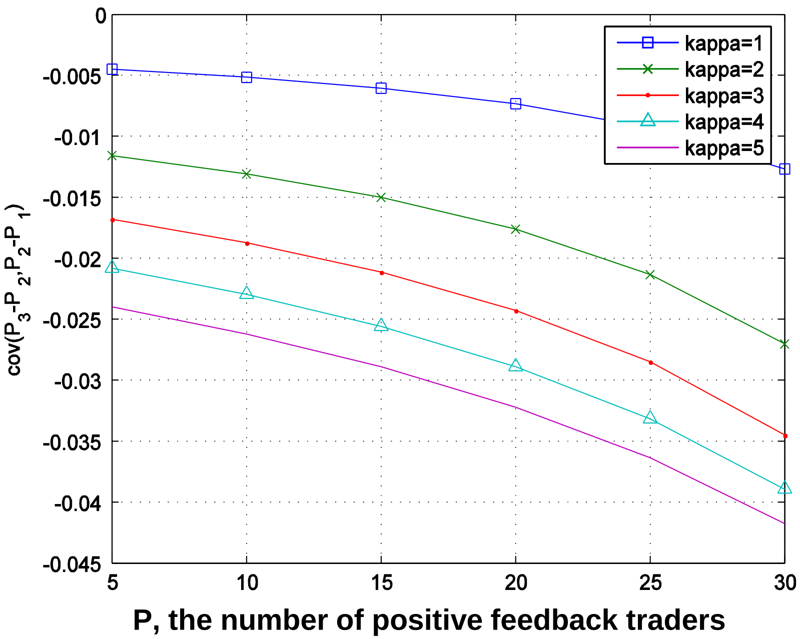
Result 1. The price changes are more important when the number of positive feedback traders is large.

$\mathrm{cov}(P_3 - P_2, P_2 - P_1)$ describes the correction phase made by the informed agents.

the higher the feedback trading → the greater the size of the departure of prices from fundamentals caused by the earlier informed trades → the more large price changes.

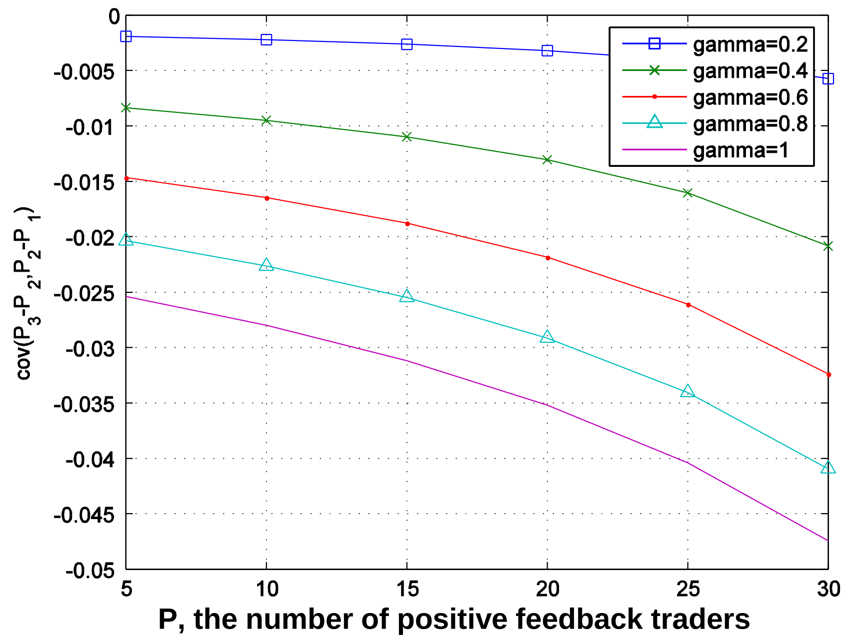
Property
→ serial correlation of prices (2/3)

$$\mathrm{cov}\left(P_3 - P_2, P_2 - P_1\right) = f\left(P\right)$$



$N_1 = N_2 = 10\ ;\ \eta = \gamma = 0.5$

$$\mathrm{cov}\left(P_3 - P_2, P_2 - P_1\right) = f\left(P\right)$$



$N_1 = N_2 = 10\ ;\ \eta = 0.5\ ;\ \kappa = 2$

Comments
→ serial correlation of prices (3/3)

The returns reversal increases with the number of feedback traders

The price change is higher when each overconfident agent overestimates the precision of his signal (measured by kappa)

The price change is less steep when each overconfident trader underestimates the precision of the other specific market participants' signals.

Property
→ Profits (1/3)

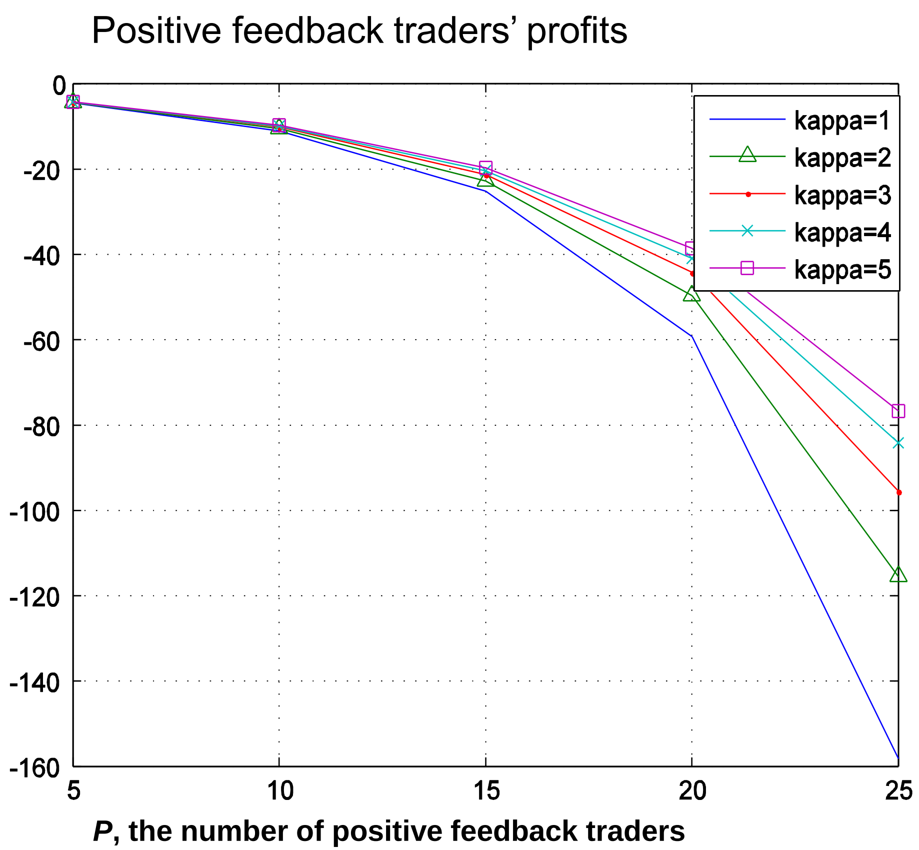
Result 2.

Positive feedback traders always earn negative profits.

If there are enough positive feedback traders, both overconfident and rational traders can earn positive profits.

When the number of trend chasers is large, overconfident traders earn more than rational speculators.

Property
→ Profits (2/3)



$N_1 = N_2 = 10$; $\eta = \gamma = 0.5$

Comments
→ Profits (3/3)

The positive feedback traders' losses are less important when their overconfident traders overestimate the precision of their signals (measured by kappa).

Property
→ Contrarian trading

__Result 3.__ Negative feedback traders:

Diminish the volatility of prices,

Worsen the quality of prices,

Can earn positive profits for a low number of contrarian traders.

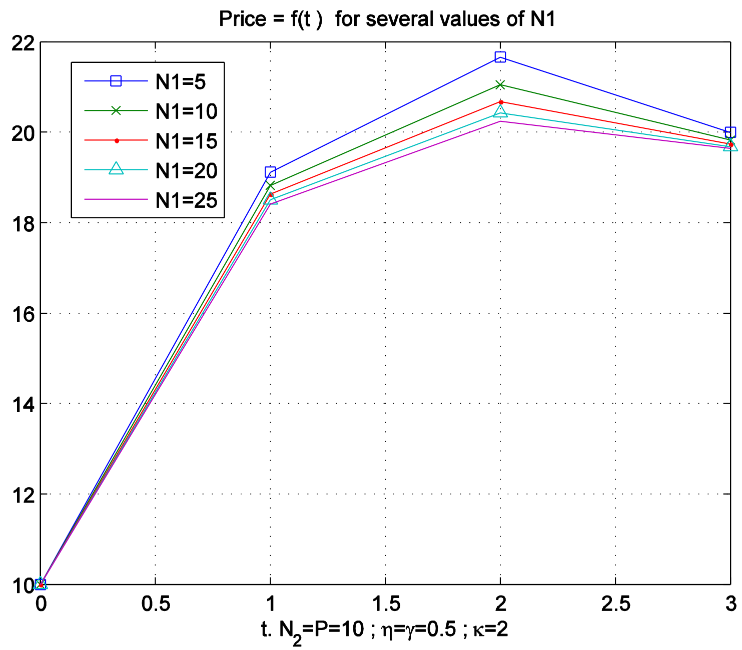
Conclusion

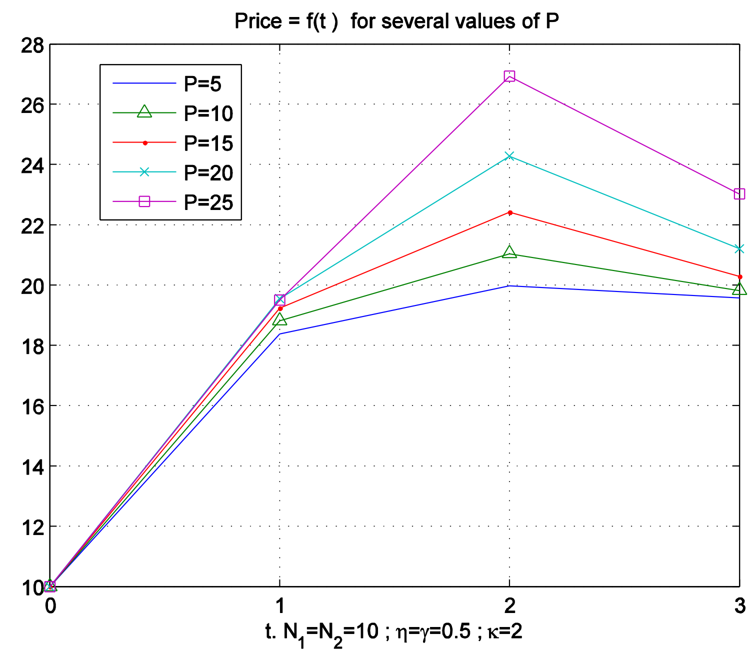
We have analyzed the effects of overconfidence and trend-chasing behavior on financial markets. Our key results are:

- Positive feedback traders always realize negative profits.
- Contrarian agents can earn positive profits for a low number.
- Depending on the size of feedback trading, overconfident agents can earn more than their rational counterparts.

➢ **A speculative bubble stems mainly from the feedback trading.**

Conclusion


Price = f(t) for several values of N1
N1=5
N1=10
N1=15
N1=20
N1=25
22
20
18
16
14
12
10
0
0.5
1
1.5
2
2.5
3
t. N_2=P=10 ; η=γ=0.5 ; κ=2


Price = f(t) for several values of P
P=5
P=10
P=15
P=20
P=25
28
26
24
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20
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16
14
12
10
0
1
2
3
t. N_1=N_2=10 ; η=γ=0.5 ; κ=2

High Frequency Trading: Strategic Competition Between Slow And Fast Traders

Laurent Germain (Toulouse Business School); Hervé Boco (Toulouse Business school) and Fabrice Rousseau (Maynooth University)

Presentation Layout

Introduction

Literature Review

Issues

Model and equilibrium

Numerical Results

Conclusion

Introduction

Introduction

Algorithmic Trading (AT)

Use of computers and advanced mathematical models to make decision about the timing, price and quantity of an order. Trades can be made without human intervention using information received electronically

High Frequency Trading (HFT), also called High-Speed Trading (HST) is a form of AT

Introduction Estimates

In 2010, it has been estimated that HFT represented 56% of all the trading in US equity markets, 38% in Europe, 10-30% in Asia-Pacific. However estimates go as high as 73% (TABB Group) and 85% (Brogaard et a. (2012)) for US stock exchange.

The different estimates lies in the difficulty to measure it (see Biais and Foucault (2014))

This phenomenon initially taking place almost exclusively on equity markets has developed to foreign exchange (FX) markets

In 2010, it is estimated that HFT represented 24-30% of spot turnover

It has also expanded to other markets and asset classes such as Futures and Fixed-Income markets

Introduction – HFT Features

HFT has the following features:

Very short holding periods

Submission of a large number of small-size,

Cancel most of orders after submission (quote stuffing),

Neutral position at the end of a trading day

Use of colocation (renting out space in close proximity to the trading servers) and proximity services to minimize latency.

They operate in low latency (delay between the transmission of information from a source and the reception of the information at a destination)

As such HFT firms require a liquid underlying market

Introduction – Effects

The most "famous" ones

2010 Flash Crash (large-order execution algorithm operating in an unexpected way)

2012 Facebook IPO (exchange system problem)

2012 Knight Capital incident (malfunction of an order routing system)

Literature Review

Literature empirical

Empirical literature suggests that AT and HFT can either increase or decrease volatility:

+ Boehmer et al. (2012), Zhang (2010)

- Chaboud et al. (2009), Hasbrouck and Saar (2012), Brogaard (2011)

Findings on liquidity are somehow mixed:

Hendershott et al. (2011) and Boehmer et al. (2012), Hendershott, Jones, and Menkveld (2011), Malinova, Parks, and Riordan (2012) find that HFT has improved liquidity.

However, Boehmer et al. (2012) show that Algorithmic liquidity is strategic and tends to decrease when volatility increases

Gai, Yao, and Ye (2012) find that a very high fraction (> 90%) of orders are cancelled on Nasdaq stocks

Literature
Theoretical

Most of the literature on HFT models the advantages that some traders have relative to others in terms of speed by introducing a form of asymmetric information such as Biais et al. (2012), Aït-sahalia and Saglam (2013), Foucault et al. (2013), Biais et al. (2015) to name but a few.

Biais et al. (2015) analyze the welfare consequences of adverse selection due to unequal speed of access to information.Two countervailing effects:

HFT increases the likelihood that investors can find a mutually profitable trading opportunity.

HFT generates a negative externality by increasing the risk of adverse selection, which raises the cost of trading (price impacts, i.e., market illiquidity) for all traders.

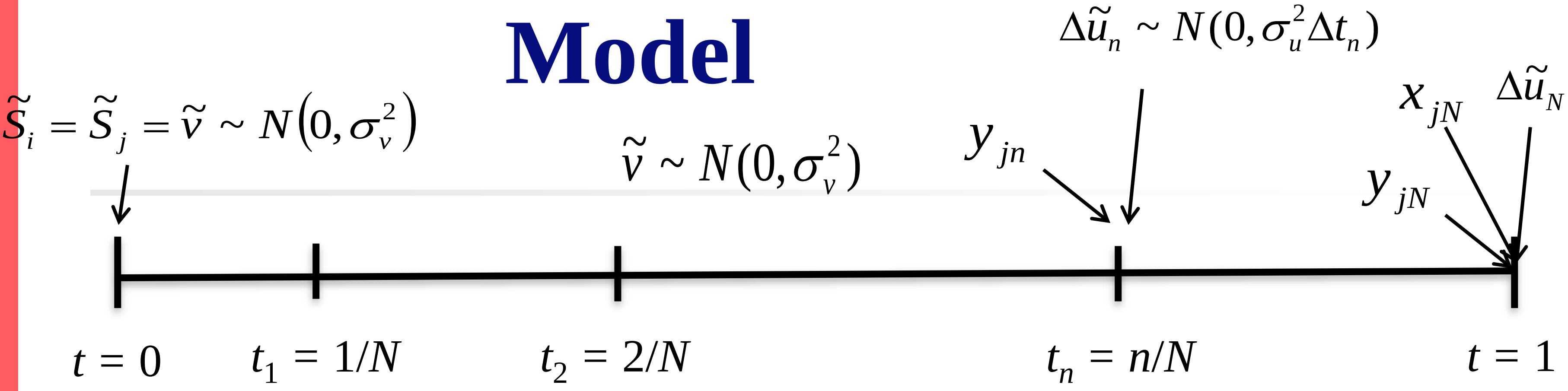
Foucault et al. (2013) show that as HFT trade on new short-lived information it may lead traders to trade less aggressively on their long-lived information. This effect could have a negative effect on price discovery.

Issues

Issues

- What happens to fast traders' profits when they face with a large number of slow insiders?
- What dynamic strategies should the different informed market participants use to maximize their profits?
- How the liquidity and the informativeness of prices are affected by the number of auctions and the number of the two types of informed agents?

Model



***M*$_1$ Fast risk-neutral informed traders** who each receives a signal, at $t = 0$,

$\tilde{S}_j = \tilde{v} \;\; for \;\; j = 1, \ldots, M_1$ they trade at each auction from *1* to *N*

***M*$_2$ Slow risk-neutral informed traders** who each receives a signal, at $t = 0$,

$\tilde{S}_i = \tilde{v} \;\; for \;\; i = 1, \ldots, M_2$ they trade only at the last auction *N*

Liquidity traders who submit an aggregate order at each auction *n*:

$$\Delta u_n \sim N(0, \sigma_u^2 \Delta t_n) \qquad \forall n = 1, \ldots, N$$

Competitive risk-neutral market makers who observe the aggregate orders, $\tilde{w}_n = \Delta X_n + \Delta Y_n + \Delta\tilde{u}_n$ and set the price p_n, at each auction *n*.

Equilibrium

- Proposition 1 There exists a unique linear equilibrium characterized by :
  - Each slow trader submits at *n*, the order

$$\begin{cases} x_n = 0 \;\; for\; n < N \\ x_N = \beta_N^X \left(\tilde{v} - p_0\right) \end{cases}$$

  - Each fast trader submits at n, the order

$$y_n = \beta_n^Y \left(\tilde{v} - p_{n-1}\right)\Delta t_n$$

  - The price set by the market makers at auction *n* is given

$$p_n = p_{n-1} + \lambda_n \omega_n .$$

Interpretation

The fast insiders determine their order to maximize their profits from the current auction *n* to the last one. Their reaction is linear to their long-lived private information $\tilde{v}$ and to the previous public price $\tilde{p}_{n-1}$. Obviously it is proportional to the time interval $\Delta t_n = t_n - t_{n-1}$

At the last auction both the fast traders and the slow traders participate. However, the slow insiders rely only on the data obtained at the beginning of the sequential auctions game.

Numerical Results

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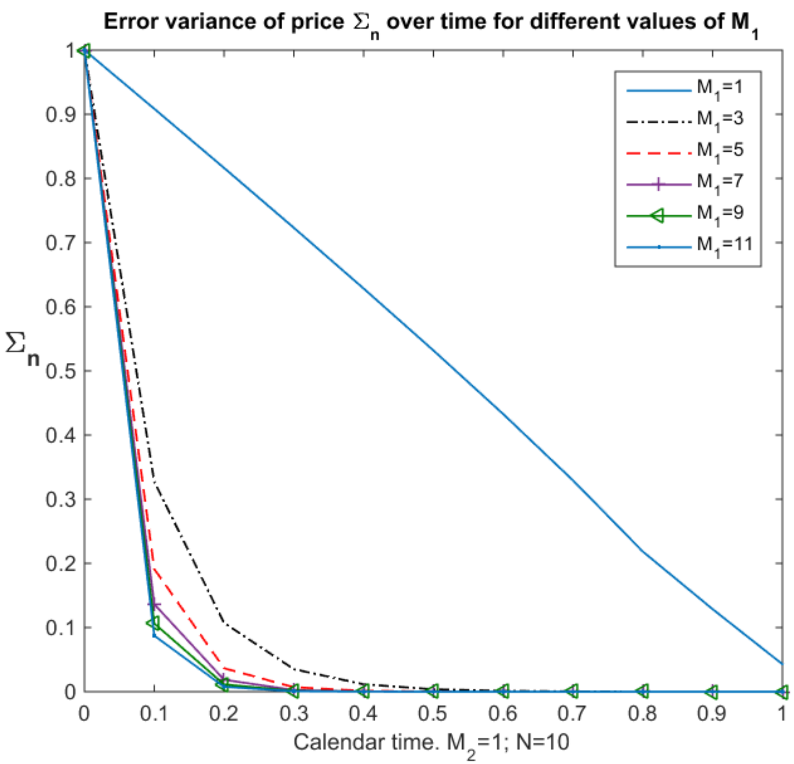
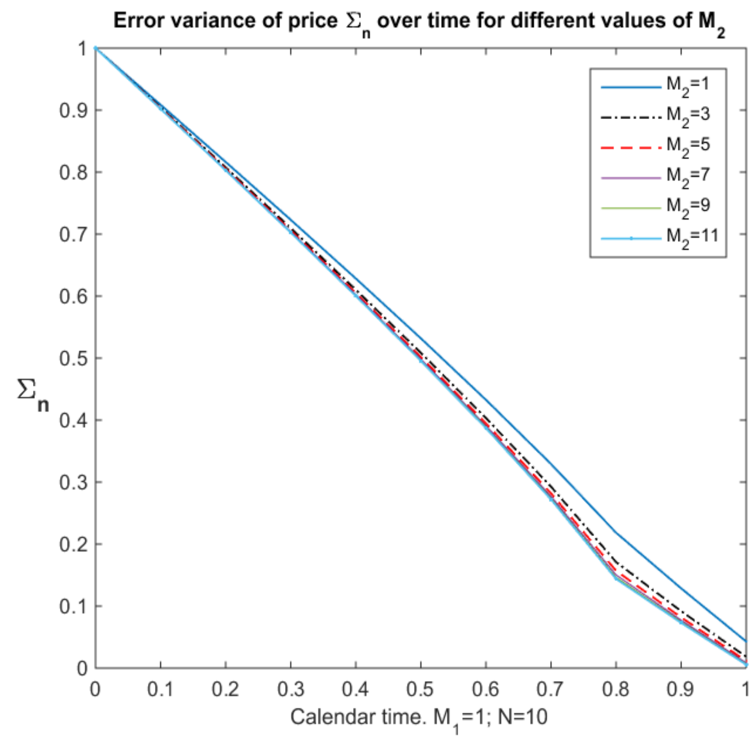
Price Informativeness(1/2)

Result 1 *When the number of fast traders increases, the price informativeness increases abruptly. However, the price informativeness increases slower when it is the number of slow insiders that increases.*





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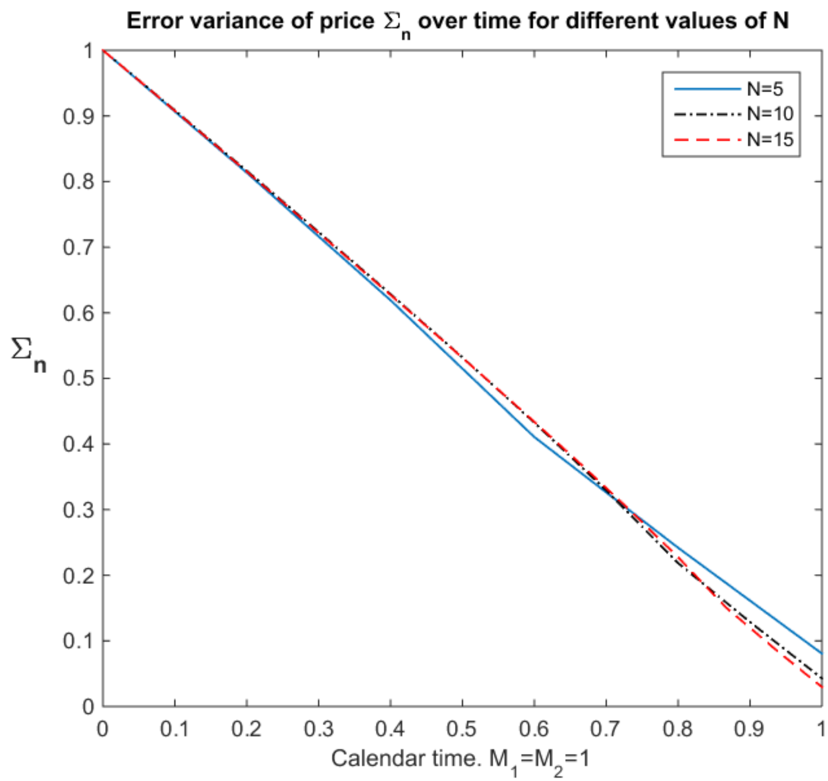
Interpretation

The noncooperative setting results in a fierce competition at the earlier auctions since the fast traders have the same information (they know the liquidation value perfectly). Therefore, they reveal their private information quickly. This result is consistent with HS and BGR.

Since the slow traders participate only at the last auction their orders mainly influence the price informativeness as one approaches the end of the trading day. Finally, the number of slow traders does not lead the price informativeness to drop significantly at the last auction.

Price Informativeness (2/2)

<u>Result 2</u> *When the number of auctions increases the price informativeness increases at the earlier auctions. However at the last auction the higher the number of auctions the lower the price informativeness is.*



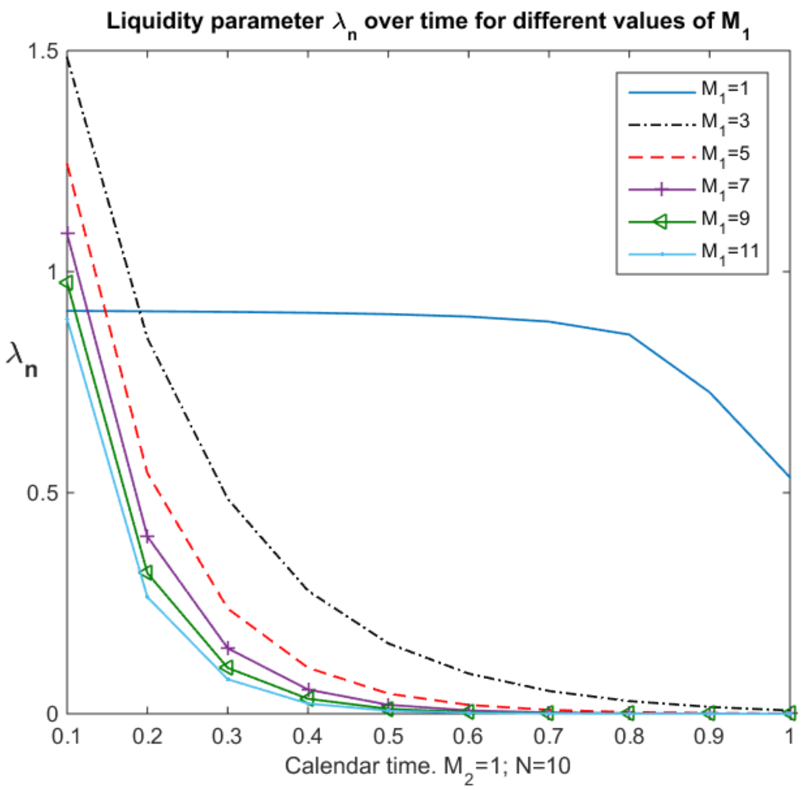
Interpretation

As the number of auction increases, the error variance of price declines slowly since the monopolistic fast trader can dissimulate his private information.

However, since the rate of information which is released to the market is constant over time the higher the number of auctions the higher the price informativeness is.

Liquidity (1/2)

<u>Result 3</u> *The liquidity parameter decreases over time. This parameter drops more quickly as the number of fast traders increases.*



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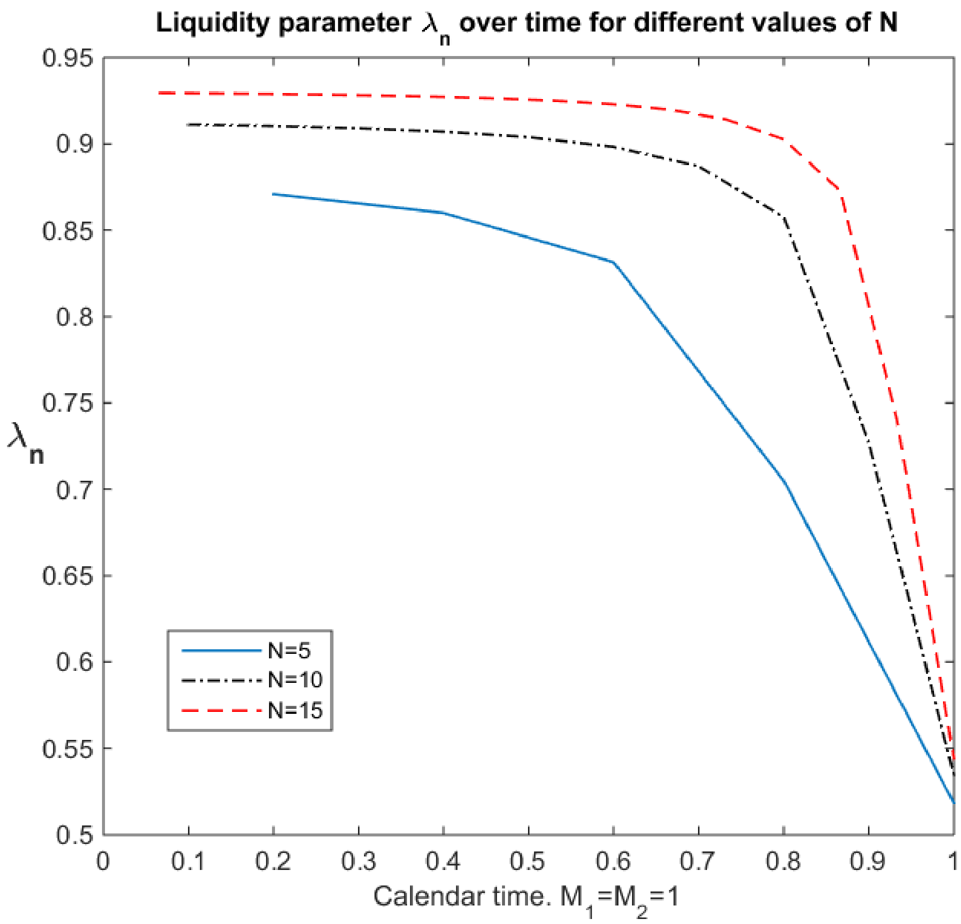
Interpretation

The liquidity parameter measures the adverse selection problem, in other words, the liquidity parameter is the market makers' sensitivity to the order flow. Since the competition is fierce between fast insiders possessing the same information, at the end of the trading day the insiders have already revealed almost all their private information. Thus, the information content is negligible at the last auction. Finally, the greater the number of fast insiders the weaker the liquidity parameter is.

Liquidity (2/2)

Result 4 *The liquidity parameter increases with the number of auctions.*

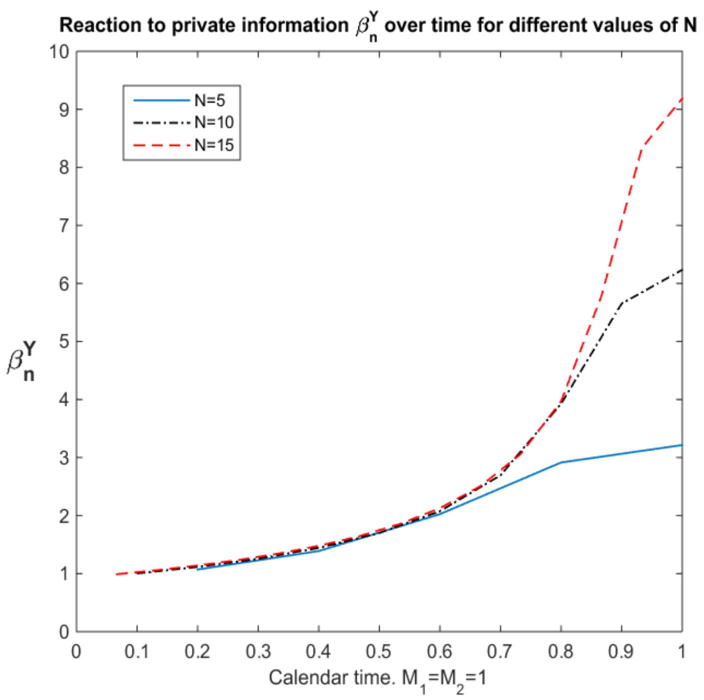


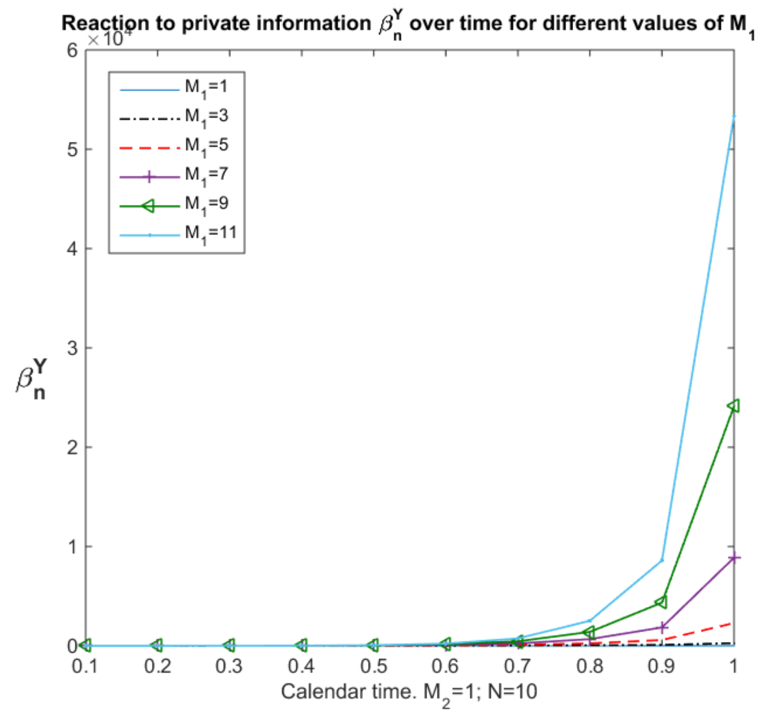
Interpretation

This result is consistent with HS and BGR. Indeed, the information content of the order flow is low at the first auctions since the monopolistic fast trader exploits his informational advantage slowly. Therefore when the number of auctions increases, the speed with the liquidity parameter drops, becomes weak.

Reaction

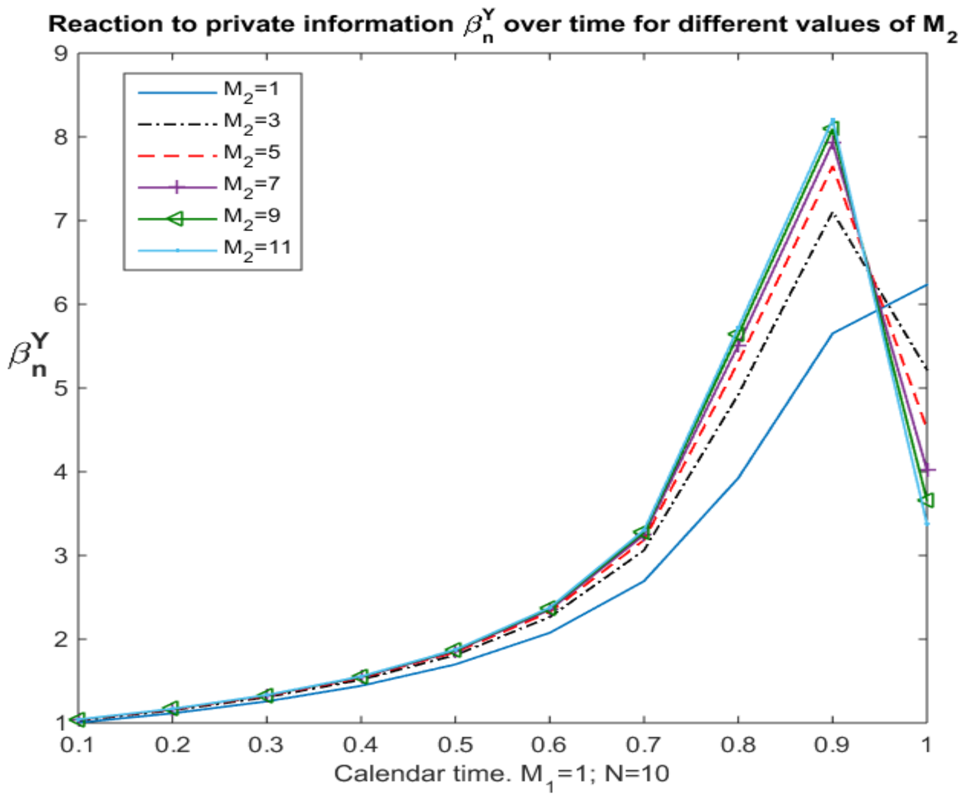
<u>Result 5</u> *the fast traders' reaction to private information increases with the number of fast traders and with the number auctions.*





Reaction

<u>Result 6</u> *the fast traders' reaction to private information increases with the number of slow traders at the earlier auctions. However, at the last auction this reaction decreases with the number of slow traders.*



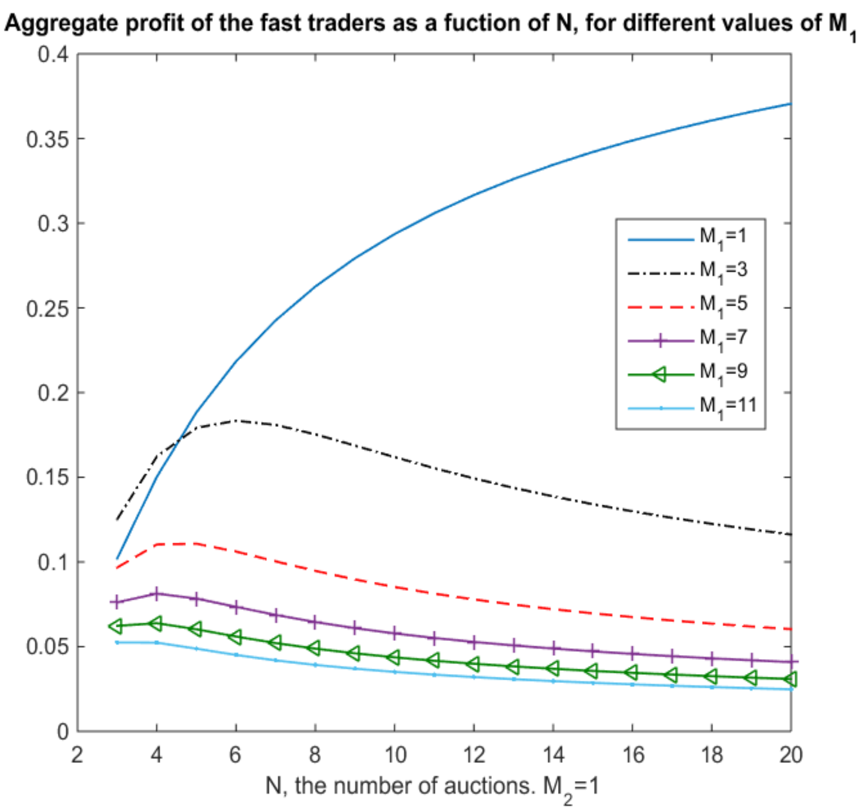
Interpretation

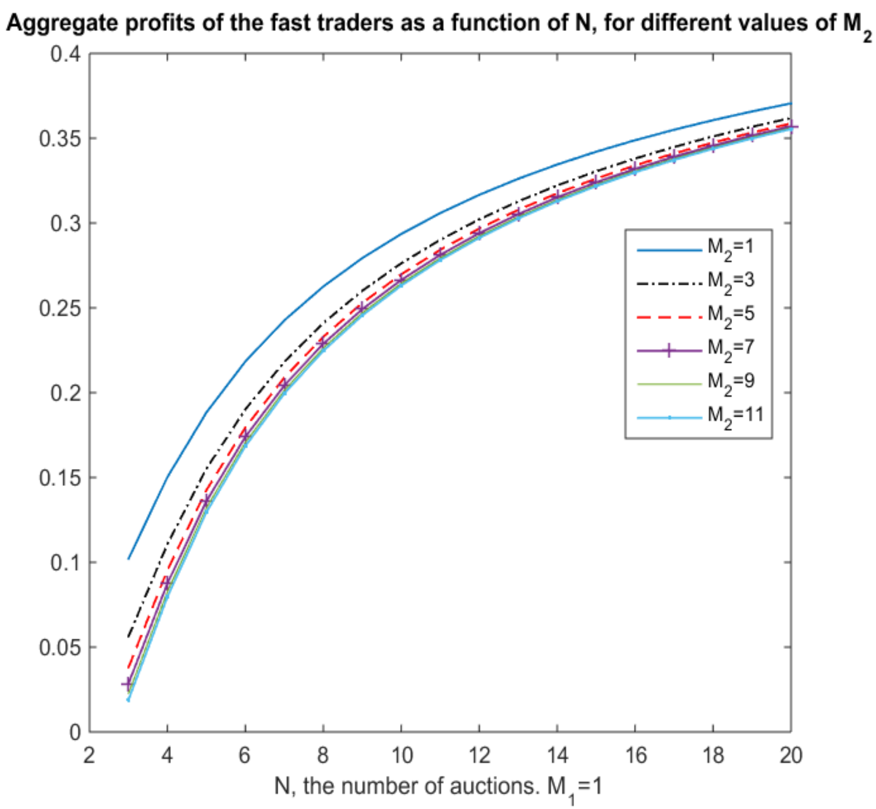
To soften the competition the fast insiders react to their private information less aggressively. However, at the last auction the higher the number of fast traders the higher their reaction. Indeed, they compete to share the remaining profits and their previous trades have released almost all their private information.

Due to the competition of the slow traders, the fast traders diminish their reaction at the last auction. Thus the greater the number of slow insiders, the less important the reaction **is.**

Aggregate Profits of the Fast traders

<u>Result 7</u> *the fast traders' profits decrease with both the number of fast traders and the slow traders. However, the aggregate profit of the fast traders is almost not affected by the number of slow traders.*





Interpretation

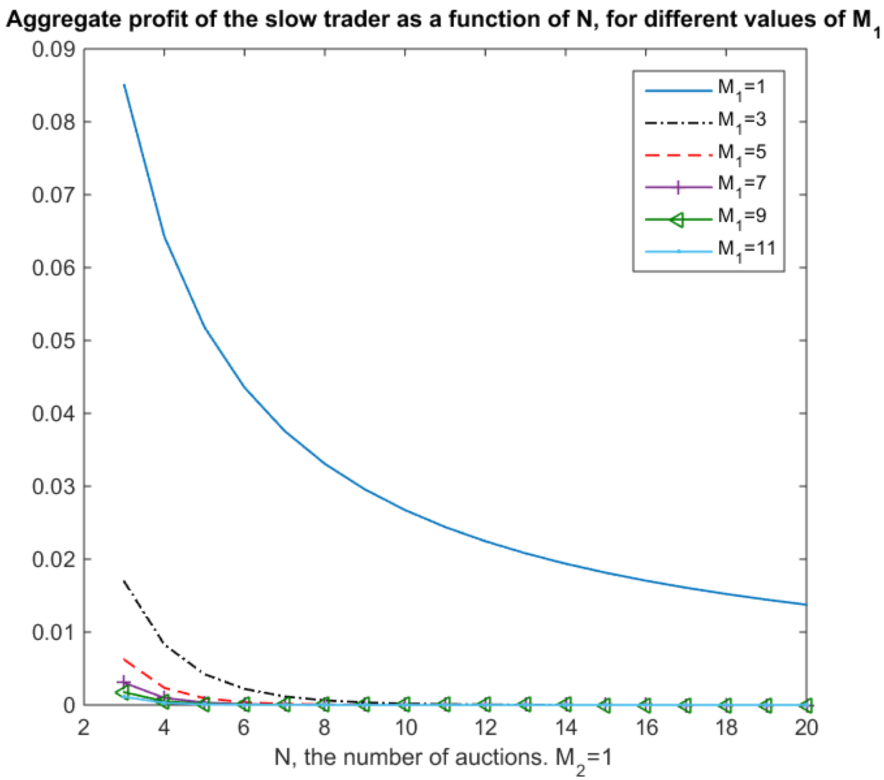
In our model, traders have only the "trading value" component of the profits (Foucault et al.), fast traders thus exploit their temporal advantage (long-run term).

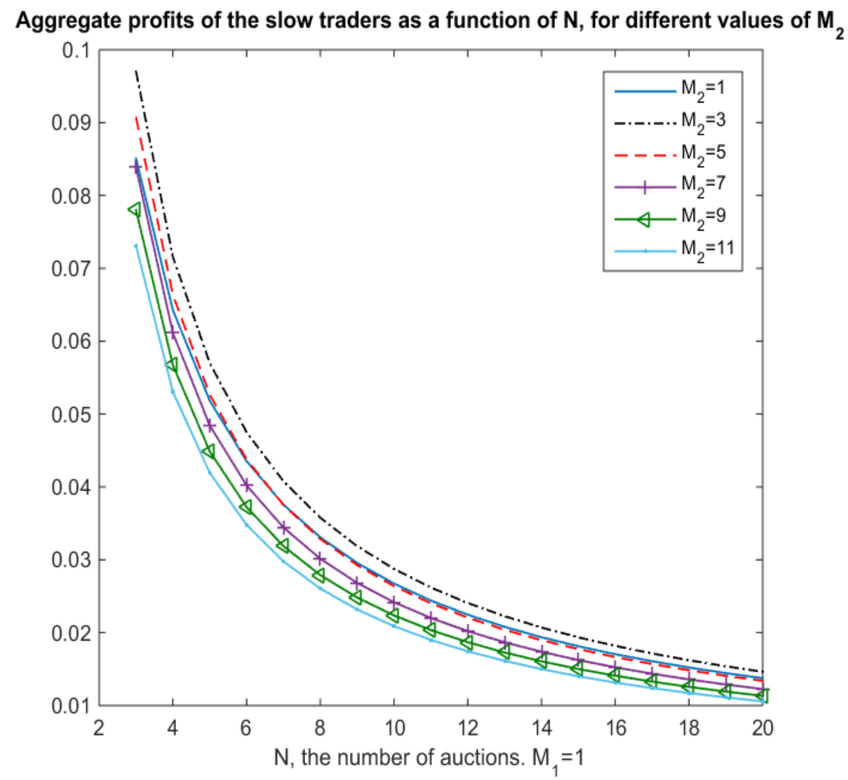
At the last auction, the slow traders share only a remaining profits which are not realized at the previous auctions by the fast speculators. Therefore, the aggregate profits of the fast traders are almost not affected when the number of slow traders increases

The competition is fierce, that leads the profits to go to zero when the number of fast traders increases.

Aggregate Profits of the Slow traders

<u>Result 8</u> *the fast traders' profits decrease with the number of fast traders. However, the aggregate profit of the fast traders is almost not affected by the number of slow traders.*





Interpretation

The slow insiders share the remaining profits at the last auction. Therefore, when the number of fast insiders increases the slow insiders' aggregate profit decreases since the competition is fierce at the earlier auctions between the fast traders.

The higher number of slow traders the lower the slow insiders aggregate profit is. Indeed, the competition is then fierce between all informed agents.

Conclusion

Our model allows us to explore the strategy of High Frequency Traders who trade ahead of their counterpart slow insiders.

We show that there exists a unique linear equilibrium.

We show that the fast traders realize almost all the profits at the earlier auctions at the expense of the slow traders who reach a part of them very small.

The presence of slow traders enhances the market efficiency at the end of the trading day.