Part II

Risk and Return

...in a Perfect Market under Risk Aversion

We are now moving on to the next step in complexity. We shall still (cowardly) maintain that financial markets are perfect: no information differences, no transaction costs, no taxes, and many buyers and sellers. But we are now abandoning the assumption that investors are risk-neutral—that they are indifferent between receiving \$1 million for sure, and receiving \$500,000 or \$1,500,000 with equal probability. An investor who is risk-averse prefers the safe \$1 million.

We now introduce a complication that adds not only complexity but also realism: risk aversion. Under risk aversion, projects can influence one another from an "overall risk" perspective. If one project's return is always high (say, +20%) when the other project's return is low (say, –20%), and vice-versa, then it can even be possible that the overall risk cancels out completely! This simple insight means that determining the best investment choices, selected from the large universe of available investment projects, becomes a much more difficult task for corporate investors and consequently, for their corporations' managers. Projects are no longer self-contained islands.

As a corporate manager, it now becomes a question of how your corporate projects work together with your other projects (for internal corporate risk management) or even with your investors' projects elsewhere. This also means that you need to first understand your investors' problems before you can answer what projects they would like you to undertake. So, who are your investors, what do they like and dislike, and how should you evaluate your project relative to what you believe your investors' alternatives are? What exactly *are* your investors' alternatives? How do your projects interact with your investors' other projects? This is a wide and deep subject, which is why we require an unprecedented four chapters: It requires a larger expedition into the world of uncertainty.

Although the details of how to invest now become more difficult, fortunately, all the important questions and tasks still remain the same—and, fortunately, so do many of the answers. As a corporate executive, you must still understand how to work with rates of return and how to decide whether to accept or reject investment projects. You can still use the net present value method. You still need knowledge of projects' expected cash flows, $\mathsf{E}(C)$, and of the cost of capital, $\mathsf{E}(r)$,

$$\text{NPV} = C_0 + \frac{\mathsf{E}(C_1)}{1+\mathsf{E}(r_1)} + \frac{\mathsf{E}(C_2)}{1+\mathsf{E}(r_2)} + \cdots$$

The novel complication arises in the denominator. Investors' risk aversion influences the NPV (only) through $\mathsf{E}(r)$. Still, it continues to be best to think of it as the opportunity cost of capital. As a manager, the difficulty is only that you must somehow calculate what it should be on behalf of your corporation's owners (investors). The cost of capital still measures the same thing: whether your investors have better alternatives elsewhere in the economy. If they do, you should return their capital to them and let them invest their money there. The opportunities elsewhere determine your corporation's cost of capital, which in turn determines what projects you should take.

What You Want to Learn in this Part

In sum, we now assume that investors are risk-averse—as they truly are in the real world. Then what is the correct $\mathsf{E}(r)$, the opportunity cost of capital, in the NPV formula? As in earlier chapters, great opportunities elsewhere in the economy still manifest themselves as a high cost of capital $\mathsf{E}(r)$ that you should apply to your projects. But in this part of the book, you must judge all opportunities not only by their rewards, but also by their risks.

- Chapter 7 gives you a short tour of historical rates of return on various asset classes to whet your appetite. Its appendix explains some of the (ever-changing) current institutional setups of U.S. equity markets.

  Typical questions: Did stocks, bonds, or cash perform better over the last 30 years? How safe were stocks compared to bonds or cash? What are the roles of brokers and exchanges? How do stocks appear and disappear?

- Chapter 8 considers choices if investors like more reward and less risk. It takes the perspective of an investor. It explains how you should measure risk and reward, and how diversification reduces risk. It draws a strong distinction between a security's own risk and a security's contribution to an investor's overall portfolio risk.

  Typical questions: What is the standard deviation of the rate of return on my portfolio? What is Intel's market beta, and what does it mean for my portfolio? What is Intel's own risk, and should I care? What is the average market beta of my portfolio?

- Chapter 9 looks at two key quantities: The price of time (i.e., the risk-free rate) and the price of risk (i.e., the expected rate of the stock market above an equivalent risk-free rate). As a corporate CFO, you can benchmark your cost of capital to these quantities. If you offer no-risk securities, it is enough for your projects to meet the risk-free rate of return. If you offer projects about as risky as the stock market, they should offer expected rates of return just like those of the stock market. These are the opportunity costs of capital for projects of different types for your investors.

  Typical questions: What should a short-term safe investment offer? What should a long-term safe investment offer? What should a risky investment offer?

- Chapter 10 takes this perspective one step further. It explains how to determine the degree to which projects are like bonds and stocks through the market-beta. An extreme version thereof is the "capital asset pricing model" (CAPM), which even states an exact relation between a project's expected rates of return and its market-beta. Alas, it holds only under very special circumstances.

  Typical questions: What characteristics should influence the appropriate expected rate of return that your investors care about? What should be the appropriate expected rate of return for any one particular project? Can you trust the CAPM?

Looking ahead, Part III will explain what happens when financial markets or decision rules are not perfect.

5

Time-Varying Rates of Return and the Yield Curve

When Rates of Return are Different

In this chapter, we will make the world a little more complex and a lot more realistic, although we are still assuming perfect foresight and perfect markets. The first assumption that we will abandon is that rates of return are the same no matter what the investment time horizon is. In the previous chapters, the interest rate was the same every period—if a 30-year bond offered an interest rate of 5% per annum, so did a 1-year bond. But this is not the case in the real world. Rates of return usually vary with the length of time an investment requires.

The U.S. Treasury Department Resource Center informs you of this fact every day. For example, on Dec 31, 2015, a U.S. Treasury paid 0.65% *per annum* for a payment to be delivered in one year (Dec 31, 2016), but 3.01% *per annum* for a payment to be delivered in 30 years (Dec 31, 2045):

	In Percent Per Annum										
	1m	3m	6m	1y	2y	3y	5y	7y	10y	20y	30y
12/31/2015	0.14	0.16	0.49	0.65	1.06	1.31	1.76	2.09	2.27	2.67	3.01

Is this stuff that just bond traders need to know? Not at all. In fact, this stuff matters to you, too. Have you ever wondered why your bank's one-month CD offered only 0.21% *per annum*, while their five-year CD offered 0.8% *per annum*? (These were the average national rates on Dec 31, 2015.) Which should you choose? Why? And does inflation matter?

CEOs must also know how to compare what they should have to pay for investors willing to give them money for a return promise in a 1-year project vs. a 30-year project. If the U.S. Treasury has to offer higher rates for lengthier investment return periods, surely so will firms!

In this chapter, you will learn how to work with time-dependent rates of return and inflation. In addition, this chapter contains an (optional) section that explains the U.S. Treasury yield curve.

5.1 Working with Time-Varying Rates of Return

Interest rates can differ based on the length of the commitment.

In the real world, rates of return usually differ depending on when the payments are made. For example, the interest rate next year could be higher or lower than it is this year. Moreover, it is often the case that long-term bonds offer different interest rates than short-term bonds. You must be able to work in such an environment, so let me give you the tools.

Compounding Different Rates of Return

A compounding example with time-dependent rates of return.

Fortunately, when working with time-varying interest rates, all the tools you have learned in previous chapters remain applicable (as promised). In particular, compounding still works exactly the same way. For example, what is the two-year holding rate of return if the rate of return is 20% in the first year and 30% in the second year? (The latter is sometimes called the **reinvestment rate**.) You can determine the two-year holding rate of return from the two 1-year rates of return using the same compounding formula as before:

$$\begin{aligned}(1 + r_{0,2}) &= (1 + 20\%) \cdot (1 + 30\%) = (1 + 56\%) \\ & \quad\; (1 + r_{0,1}) \cdot (1 + r_{1,2}) \;=\; (1 + r_{0,2})\end{aligned}$$

Subtract 1, and the answer is a total two-year holding rate of return of 56%. If you prefer it shorter,

$$r_{0,2} = 1.20 \cdot 1.30 - 1 = 1.56 - 1 = 56\%$$

The calculation is not conceptually more difficult, but the notation is. You have to subscript not just the interest rates that begin now, but also the interest rates that begin in the future. Therefore, most of the examples in this chapter must use two subscripts: one for the time when the money is deposited, and one for the time when the money is returned. Thus, $r_{1,2}$ describes an interest rate from time 1 to time 2. Aside from this extra notation, the compounding formula is still the very same multiplicative "one-plus formula" for each interest rate (subtracting 1 at the end).

The general formula for compounding over many periods.

You can also compound to determine holding rates of return in the future. For example, if the 1-year rate of return is 30% from year 1 to year 2, 40% from year 2 to year 3, and 50% from year 3 to year 4, then what is your holding rate of return for investing beginning next year for three years? It is

$$\begin{aligned}\text{Given:}\quad & r_{1,2} = 30\% \quad r_{2,3} = 40\% \quad r_{3,4} = 50\% \\ (1 + r_{1,4}) &= (1 + 30\%) \cdot (1 + 40\%) \cdot (1 + 50\%) = (1 + 173\%) \\ & \quad\; (1 + r_{1,2}) \cdot (1 + r_{2,3}) \cdot (1 + r_{3,4}) \;=\; (1 + r_{1,4})\end{aligned}$$

Subtracting 1, you see that the three-year holding rate of return for an investment that takes money *next* year (not today!) and returns money in 4 years (appropriately called $r_{1,4}$) is 173%. Let's be clear about the timing. For example, say it was midnight of December 31, 2016, right now. This would be time 0. Time 1 would be midnight December 31, 2017, and this is when you would invest your \$1. Three years later, on midnight December 31, 2020 (time 4), you would receive your original dollar plus an additional \$1.73, for a total return of \$2.73. Interest rates that begin right now—where the first subscript would be 0—are usually called **spot rates**. Interest rates that begin in the future are usually called **forward rates**.

Q 5.1. If the first-year interest rate is 2% and the second year interest is 3%, what is the two-year total interest rate?

Q 5.2. Although a two-year project had returned 22% in its first year, overall it lost half of its value. What was the project's rate of return after the first year?

Q 5.3. From the closing of December 31, 2009 to December 31, 2015, Vanguard's S&P 500 fund (which received and paid dividends on the underlying constituent stocks to its fund investors, but charged administration fees) returned the following annual rates of return:

2010	2011	2012	2013	2014	2015
15.0%	2.1%	16.0%	32.3%	13.7%	1.3%

What was the rate of return over the first 3 years, and what was it over the second 3 years? What was the rate of return over the whole 6 years? Was the *realized* rate of return time-varying?

Q 5.4. A project lost one-third of its value the first year, then gained fifty percent of its value, then lost two-thirds of its value, and finally doubled in value. What was the average rate of return? What was the investment's overall four-year rate of return? If one is positive, is the other, too?

Annualized Rates of Return

Per-unit standard measures are statistics that are conceptual aids.

Time-varying rates of return create a new complication that is best explained by an analogy. Is a car that travels 163,680 yards in 93 minutes fast or slow? It is not easy to say, because you are used to thinking in "miles per sixty minutes," not in "yards per ninety-three minutes." It makes sense to translate speeds into miles per hour for the purpose of comparing them. You can even do this for sprinters, who run for only 10 seconds. Speeds are just a standard measure of the rate of accumulation of distance per unit of time.

A per-unit standard for rates of return: annualized rates.

The same issue applies to rates of return: A rate of return of 58.6% over 8.32 years is not as easy to compare to other rates of return as a rate of return per year. Therefore, most rates of return are quoted as **annualized rates**. The average annualized rate of return is just a convenient unit of measurement for the rate at which money accumulates—a "sort-of-average" measure of performance. Of course, when you compute such an annualized rate of return, you do not mean that the investment earned the same annualized rate of return of, say, 5.7% each year—just as the car need not have traveled at 60 mph (163,680 yards in 93 minutes) each instant.

Return to our example: You want to annualize our three-year total holding rate of return.

If you were earning a total three-year holding rate of return of 173% over the three-year period, what would your *annualized* rate of return be? The answer is not the **average rate of return** of $173\%/3 \approx 57.7\%$, because if you earned 57.7% per year, you would have ended up with $1.577^3 - 1 \approx 292\%$, not 173%. This incorrect answer of 57.7% ignores the *compounded interest on the interest* that you would earn after the first and second years. Instead, to compute the annualized rate of return, you need to find a single hypothetical annual rate of return that, if you received it each and every year, would give you a three-year holding rate of return of 173%.

To find the t-year annualized interest rate, take the t-th root of the total return (t is number of years).

How can you compute this? Call this hypothetical annual rate that you would have to earn each year for three years $r_{\overline{3}}$ (note the bar above the 3 to denote *annualized*) in order to end up with a holding rate of return of 173%. To find $r_{\overline{3}}$, solve the equation

$$(1 + r_{\overline{3}}) \cdot (1 + r_{\overline{3}}) \cdot (1 + r_{\overline{3}}) = (1 + 173\%)$$

$$(1 + r_{\overline{3}}) \cdot (1 + r_{\overline{3}}) \cdot (1 + r_{\overline{3}}) = (1 + r_{0,3})$$

or, for short,

$$(1 + r_{\overline{3}})^3 = (1 + 173\%)$$
$$(1 + r_{\overline{t}})^t = (1 + r_{0,t}) \quad (5.1)$$

In our example, the holding rate of return $r_{0,3}$ is known (173%) and the annualized rate of return $r_{\overline{3}}$ is unknown. Earning the same rate ($r_{\overline{3}}$) three years in a row should result in a holding rate of return of 173%. It is a "smoothed-out" rate of return of the three years' rates of return. Think of it as a hypothetical, single, constant-speed rate at which your money would have ended up as quickly at 173% as it did with the 30%, 40%, and 50% individual annual rates of return. The correct solution for $r_{\overline{3}}$ is obtained by computing the third root of 1 plus the total holding rate of return:

➤ Math Background, Chapter A, Pg.621.

$$(1 + r_{\overline{3}}) = (1 + 173\%)^{(1/3)} = \sqrt[3]{1 + 173\%} \approx 1 + 39.76\%$$
$$(1 + r_{0,t})^{(1/t)} = \sqrt[t]{1 + r_{0,t}} = (1 + r_{\overline{t}})$$

Confirm with your calculator that $r_{\overline{3}} \approx 39.76\%$,

$$1.3976 \cdot 1.3976 \cdot 1.3976 \approx (1 + 173\%)$$
$$(1 + r_{\overline{3}}) \cdot (1 + r_{\overline{3}}) \cdot (1 + r_{\overline{3}}) = (1 + r_{0,3})$$

In sum, if you invested money at a rate of 39.76% per annum for 3 years, you would end up with a total three-year holding rate of return of 173%. As is the case here, for very long periods, the order of magnitude of the annualized rate will often be so different from the holding rate that you will intuitively immediately register whether the quantity $r_{0,3}$ or $r_{\overline{3}}$ is meant. In the real world, very few rates of return, especially over long horizons, are quoted as holding rates of return. Almost all rates are quoted in annualized terms instead.

IMPORTANT

The total holding rate of return over t years, called $r_{0,t}$, is translated into an annualized rate of return, called $r_{\overline{t}}$, by taking the t-th root:

$$(1 + r_{\overline{t}}) = \sqrt[t]{1 + r_{0,t}} = (1 + r_{0,t})^{1/t}$$

Compounding the annualized rate of return over t years yields the total holding rate of return.

Translating long-term dollar returns into annualized rates of return.

You also will often need to compute annualized rates of return from payoffs yourself. For example, what annualized rate of return would you expect from a \$100 investment today that promises a return of \$240 in 30 years? The first step is computing the total holding rate of return. Take the ending value (\$240) minus your beginning value (\$100), and divide by the beginning value. Thus, the total 30-year holding rate of return is

$$r_{0,30} = \frac{\$240 - \$100}{\$100} = 140\%$$
$$r_{0,30} = \frac{C_{30} - C_0}{C_0}$$

The annualized rate of return is the rate $r_{\overline{30}}$, which, if compounded for 30 years, offers a 140% rate of return,

$$(1 + r_{\overline{30}})^{30} = (1 + 140\%)$$
$$(1 + r_{\overline{t}})^t = (1 + r_{0,t})$$

Solve this by taking the 30th root,

$$(1 + r_{\overline{30}}) = (1 + 140\%)^{1/30} = \sqrt[30]{1 + 140\%} \approx 1 + 2.96\%$$
$$(1 + r_{\overline{t}}) = (1 + r_{0,t})^{1/t} = \sqrt[t]{1 + r_{0,t}}$$

Subtracting 1, you see that a return of \$240 in 30 years for an initial \$100 investment is equivalent to a 2.96% annualized rate of return.

Compounding ≈ adding. Annualizing ≈ averaging.

In the context of rates of return, compounding is similar to adding, while annualizing is similar to averaging. If you earn 1% twice, your compounded rate is 2.01%, similar to the rates themselves added (2%). Your annualized rate of return is 1%, similar to the average rate of return of 2.01%/2 = 1.005%. The difference is the interest on the interest.

Averaging can lead to surprising results—returns that are much higher than what you earned per year.

Now assume that you have an investment that doubles in value in the first year and then falls back to its original value. What would its average rate of return be? Doubling from, say, \$100 to \$200 is a rate of return of +100%. Falling back to \$100 is a rate of return of (\$100–\$200)/\$200 = –50%. Therefore, the average rate of return would be [+100% + (–50%)]/2 = +25%.

But you have not made any money! You started with \$100 and ended up with \$100. If you compound the returns, you get the answer of 0% that you were intuitively expecting:

Look how deceptive!

$$(1 + 100\%) \cdot (1 - 50\%) = 1 + 0\% \Rightarrow r_{0,2} = 0\%$$
$$(1 + r_{0,1}) \cdot (1 + r_{1,2}) = (1 + r_{0,2})$$

It follows that the annualized rate of return $r_{\overline{2}}$ is also 0%. Conversely, an investment that produces +20% followed by –20% has an average rate of return of 0% but leaves you with a loss:

$$(1 + 20\%) \cdot (1 - 20\%) = (1 - 4\%) \Rightarrow r_{0,2} = -4\%$$
$$(1 + r_{0,1}) \cdot (1 + r_{1,2}) = (1 + r_{0,2})$$

For every \$100 of your original investment, you now have only \$96. The average rate of return of 0% does not reflect this loss. Both the compounded and therefore the annualized rates of return do tell you that you had a loss:

$$1 + r_{\overline{2}} = \sqrt{(1 + r_{0,2})} = \sqrt{1 - 4\%} = 1 - 2.02\% \Rightarrow r_{\overline{2}} \approx -2.02\%$$

➤ More about how arithmetic returns are "too high" in normally-distributed stock returns, Pg.141.

If you were an investment advisor and quoting your historical performance, would you rather quote your average historical rate of return or your annualized rate of return? (Hint: The industry standard is to quote the average rate of return, not the annualized rate of return!)

Do it!

Make sure to solve the following questions to gain more experience with compounding and annualizing over different time horizons.

Q 5.5. If you earn a rate of return of 5% over 4 months, what is the annualized rate of return?

Q 5.6. Assume that the two-year holding rate of return is 40%. The average (arithmetic) rate of return is therefore 20% per year. What is the annualized (geometric) rate of return? Is the annualized rate the same as the average rate?

Q 5.7. Is the compounded rate of return higher or lower than the sum of the individual rates of return? Is the annualized rate of return higher or lower than the average of the individual rates of return? Why?

Q 5.8. Return to Question 5.3. What was the annualized rate of return on the S&P 500 over the six years in the table?

Q 5.9. If the total holding interest rate is 50% for a five-year investment, what is the annualized rate of return?

Q 5.10. If the per-year interest rate is 10% for each of the next 5 years, what is the annualized five-year rate of return?

Duration and Maturity

We sometimes need summary statistics of how long a bond or a project will last. Maturity is the date of the very last payment of a bond. However, you would probably not consider a bond that pays \$1 in one year and \$1 in 30 years a 30-year bond—it's more like a 15-year bond. The **duration** can be calculated as

$$\text{Duration}(C_1 = \$1, C_{30} = \$1) = \frac{1 \times \$1 + 2 \times \$0 + \ldots + 29 \times \$0 + 30 \times \$1}{\$1 + \$0 + \ldots + \$1} = 15.5 \text{ years}$$

$$\text{Duration}(\{C_t\}) = \sum_t t/W \times C_t/$$

where W is the sum of all payments, $W = \sum_t C_t$. Intuitively, this bond has about a 15-year duration. Also intuitively, a bond that pays \$100 in one year and \$1 in 30 years has a shorter duration,

$$\text{Duration}(C_1 = \$100, C_{30} = \$1) = \frac{1 \times \$100 + 30 \times \$1}{\$100 + \$1} \approx 1.287$$

A zero bond has a duration equal to its maturity. One important variation is the **Macauley Duration**, which essentially replaces the raw cash flow with its present value. The intent is to discount far-away payouts more, which thus further shortens duration. For example, using our December 2015 yield curve,

$$\text{Duration}(C_1 = \$1, C_{100} = \$1) = \frac{1 \times \$100/1.0065 + 99 \times \$1/1.0301^{30}}{\$100/1.0065 + \$1/1.0301^{30}} \approx 1.004$$

There are also other measures of duration, but they are all basically summary statistics of when your "average" cash flow is going to occur.

Present Values with Time-Varying Interest Rates

The PV formula still looks very similar.

Let's proceed now to net present value with time-varying interest rates. What do you need to learn about the role of time-varying interest rates when computing NPV? The answer is essentially nothing new. You already know everything you need to know here. The net present value formula is still

$$\begin{aligned}
\text{NPV} &= \text{PV}(C_0) + \text{PV}(C_1) + \text{PV}(C_2) + \text{PV}(C_3) + \cdots \\
&= C_0 + \frac{C_1}{1 + r_{0,1}} + \frac{C_2}{1 + r_{0,2}} + \frac{C_3}{1 + r_{0,3}} + \cdots \\
&= C_0 + \frac{C_1}{1 + \bar{r}_1} + \frac{C_2}{(1 + \bar{r}_2)^2} + \frac{C_3}{(1 + \bar{r}_3)^3} + \cdots \\
&= C_0 + \frac{C_1}{1 + r_{0,1}} + \frac{C_2}{(1 + r_{0,1}) \cdot (1 + r_{1,2})} + \frac{C_3}{(1 + r_{0,1}) \cdot (1 + r_{1,2}) \cdot (1 + r_{2,3})} + \cdots
\end{aligned}$$

The only novelty is that you need to be more careful with your subscripts. You cannot simply assume that the multiyear holding returns (e.g., $1 + r_{0,2}$) are the squared 1-year rates of return ($(1 + r_{0,1})^2$). Instead, you must work with time-dependent costs of capital (interest rates). That's it.

Present values are still alike and thus can be added, subtracted, compared, and so on.

For example, say you have a project with an initial investment of \$12 that pays \$10 in one year and \$8 in five years. Assume that the 1-year interest rate is 5% and the five-year annualized interest rate is 6% per annum. In this case,

$$\text{PV}(\$10 \text{ in } 1 \text{ year}) = \frac{\$10}{1.05} \approx \$9.52$$

$$\text{PV}(\$8 \text{ in } 5 \text{ years}) = \frac{\$8}{1.06^5} \approx \$5.98$$

It follows that the project's total value *today* (time 0) is \$15.50. If the project costs \$12, its net present value is

$$\text{NPV} = -\$12 + \frac{\$10}{1.05} + \frac{\$8}{1.06^5} \approx \$3.50$$

$$\text{NPV} = C_0 + \frac{C_1}{1+r_{0,1}} + \frac{C_5}{1+r_{0,5}} = \text{NPV}$$

You can also rework a more involved project, similar to that in Exhibit 2.3,. But to make it more interesting, let's now use a hypothetical current term structure of interest rates that is upward-sloping. Assume this project requires an appropriate annualized discount rate of 5% over one year, and 0.5% more for every subsequent year, so that the cost of capital reaches 7% annualized in the fifth year. The valuation method works the same way as it did in Exhibit 2.3—you only have to be a little more careful with the interest rate subscripts. The project's value is thus

Here is a typical NPV example.

➤ Hypothetical Project Cash Flows, Exhibit 2.3, Pg.28.

Time	Project Cash Flow	Annualized Rate	Com-pounded	Discount Factor	Value
t	C_t	$r_{\bar{t}}$	$r_{0,t}$	$\frac{1}{1+r_{0,t}}$	$\text{PV}(C_t)$
Today	–\$900	any		1.0000	–\$900.00
Y1	+\$200	5.0%	5.0%	0.9524	\$190.48
Y2	+\$200	5.5%	11.3%	0.8985	\$179.69
Y3	+\$400	6.0%	19.1%	0.8396	\$335.85
Y4	+\$400	6.5%	28.6%	0.7773	\$310.93
Y5	–\$100	7.0%	40.3%	0.7130	–\$71.30
			Net Present Value (Sum):		\$45.65

Q 5.11. A project costs \$200 and will provide cash flows of +\$100, +\$300, and +\$500 in consecutive years. The annualized interest rate is 3% per annum over one year, 4% per annum over two years, and 4.5% per annum over three years. What is this project's NPV?

5.2 Inflation

Inflation is the increase in the price of the same good.

Let's make our world even more realistic—and complex—by working out the effects of inflation. **Inflation** is the process by which the same good costs more in the future than it does today. With inflation, the price level is rising and thus money is losing its value. For example, if inflation is 100%, an apple that costs $0.50 today will cost $1 next year, a banana that costs $2 today will cost $4, and corporate finance textbooks that cost $200 today will cost $400.

Inflation matters when contracts are not written to adjust for it.

Inflation may or may not matter in a corporate context, depending on how contracts are written. If you ignore inflation and write a contract that promises to deliver bread for the price of $1 next year, it is said to be in **nominal terms**—and you may have made a big mistake. The money you will be paid will be worth only half as much. You will only be able to buy one apple for each loaf of bread that you had agreed to sell for $1, not the two apples that anyone else will enjoy. On the other hand, you could write your contract in **real terms** (or **inflation-indexed terms**) today, in which case the inflationary price change would not matter. That is, you could build into your promised banana delivery price the inflation rate from today to next year. An example would be a contract that promises to deliver bananas at the rate of four apples per banana. If a contract is indexed to inflation, then inflation does not matter. However, in the United States inflation often does matter, because most contracts are in nominal terms and not inflation-indexed. Therefore, you have to learn how to work with inflation. What effect, then, does inflation have on returns? On (net) present values? This is our next subject.

Measuring the Inflation Rate

The CPI is the most common inflation measure.

The first important question is how you should define the inflation rate. Is the rate of change of the price of apples the best measure of inflation? What if apples (the fruit) become more expensive, but Apples (the computers) become less expensive? Defining inflation is actually rather tricky. To solve this problem, economists have invented *baskets* or *bundles* of goods and services that are deemed to be representative. Economists then measure an average price change for these items. The official source of most inflation measures is the **Bureau of Labor Statistics** (**BLS**), which determines the compositions of a number of common bundles (indexes) and publishes the average total price of these bundles on a monthly basis. The most prominent such inflation measure is a hypothetical bundle of average household consumption, called the **Consumer Price Index** (**CPI**). (The CPI components are roughly: housing 40%, food 20%, transportation 15%, medical care 10%, clothing 5%, entertainment 5%, others 5%.) The BLS offers inflation data at http://www.bls.gov/cpi/, and the *Wall Street Journal* prints the percent change in the CPI at the end of its regular column "Money Rates."

From Dec 2014 to Dec 2015, the inflation rate was a remarkably low 0.7% per annum. (And there were some months with negative inflation rates, too—called **deflation**!)

Year	2010	2011	2012	2013	2014	2015
CPI	1.5	3.0	1.7	1.5	0.8	0.7

A number of other indexes are also commonly used as inflation measures, such as the **Producer Price Index** (**PPI**) or the broader **GDP Deflator**. They typically move very similarly to the CPI. Over short periods, one expects these rates to move fairly close together (on average, not every month); but over longer periods, they can diverge. There are also more specialized bundles, such as computer or flash memory chip inflation indexes (their prices usually decline), or price indexes for particular regions.

The CPI matters—even if it is calculated incorrectly.

The official inflation rate is not just a number that mirrors reality—it is important in itself, because many contracts are specifically indexed to a particular inflation definition. For example, even if actual true inflation is zero, if the officially reported CPI rate is positive, the government

The German Hyperinflation of 1922

Many economists now believe that a modest inflation rate between 1% and 3% per year is a healthy number. It's not so easy to maintain.

The Roman Emperor Gallienus is still infamous two-thousand years later for having changed the value of the Roman Denarius by reducing its silver content by a factor of 100.

The most famous episode of extreme inflation (**hyperinflation**) occurred in Germany from August 1922 to November 1923. Prices more than quadrupled every month. The price for goods was higher in the evening than in the morning! Stamps had to be overprinted by the day, and shoppers went out in the morning with bags of money that were worthless by the end of the day. By the time Germany printed 1,000 billion Mark Bank Notes, no one trusted Marks anymore. This hyperinflation was stopped only by a drastic currency and financial system reform. But high inflation is not just a historic artifact. In 2015, Venezuela had an inflation rate of 180%. Don't trust Venezuelan Bolivars!

Yet recent experience has humbled us economists (further) by proving that it can also be difficult to push *up* inflation. In the **Great Recession** (the financial crisis of 2008-11), the Fed tried to fuel inflation by pushing money into the hands of consumers. The idea was to get them to lose just a little trust in the currency and spend it, so as to raise the value of much underwater real estate. But consumers turned around and deposited the money back into their banks, which in turn redeposited the money with—you guessed it—the Fed.

must pay out more to Social Security recipients. The lower the official inflation rate, the less the government has to pay. You would therefore think that the government has the incentive to understate inflation. But, strangely, this has not been the case. On the contrary, there are strong political interest groups that hinder the BLS from even fixing mistakes that everyone knows overstate the CPI—that is, corrections that would result in *lower* official inflation numbers. In 1996, the Boskin Commission, consisting of a number of eminent economists, found that the CPI overstates inflation by about 74 basis points per annum—a huge difference. The main reasons were and continue to be that the BLS has been tardy in recognizing the growing importance of such factors as effective price declines in the computer and telecommunications industries and the role of superstores such as Wal-Mart and Target.

Before we get moving, a final warning:

IMPORTANT

The common statement "in today's dollars" can be ambiguous. Most people mean "inflation-adjusted." Some people mean present values (i.e., "compared to an investment in risk-free bonds"). When in doubt, ask!

Q 5.12. Read the Bureau of Labor Statistics' website descriptions of the CPI and the PPI. How does the CPI differ conceptually from the PPI? Are the two official rates different right now?

Real and Nominal Interest Rates

To work with inflation and to learn how you would properly index a contract for inflation, you first need to learn the difference between a **nominal return** and a **real return**. The nominal rate is what is usually quoted—a return that has not been adjusted for inflation. In contrast, the real rate of return "somehow takes out" inflation from the nominal rate in order to calculate a return "as if" there had been no price inflation to begin with. The real return reflects the fact that, in the presence of inflation, dollars in the future will have less purchasing power than dollars today.

Nominal returns are what is normally quoted. Real returns are adjusted for inflation. They are what you want to know if you want to consume.

It measures your trade-off between present and future consumption, taking into account the change in prices.

An extreme 100% inflation rate example: Prices double every year.

Start with a simple exaggerated scenario: Assume that the inflation rate is 100% per year and you can buy a bond that promises a *nominal* interest rate of 700%. What is your *real* rate of return? To find out, assume that $1 buys one apple today. With an inflation rate of 100%, you need $2 next year to buy the same apple. Your gross return will be $\$1 \cdot (1 + 700\%) = \$8$ for today's $1 of investment. But this $8 will apply to apples costing $2 each. Your $8 will buy 4 apples, not 8 apples. Your real rate of return (1 apple yields 4 apples) is therefore

$$r_{\text{real}} = \frac{(\text{4 Apples for \$8}) - (\text{1 Apple for \$2})}{(\text{1 Apple for \$2})} = 300\%$$

For each dollar invested today, you will be able to buy only 300% more apples next year (not 700% more) than you could buy today. This is because inflation will reduce the purchasing power of your dollar by more than one half.

Here is the correct conversion formula from nominal to real rates.

The correct formula to adjust for the inflation rate (π) is again a "one-plus" type formula. In our example, it is

$$\begin{array}{ccccc} (1 + 700\%) & = & (1 + 300\%) & \cdot & (1 + 100\%) \\ (1 + r_{\text{nominal}}) & = & (1 + r_{\text{real}}) & \cdot & (1 + \pi) \end{array}$$

Turning this formula around gives you the real rate of return,

$$(1 + r_{\text{real}}) = \frac{1 + 700\%}{1 + 100\%} = 1 + 300\%$$

$$(1 + r_{\text{real}}) = \frac{(1 + r_{\text{nominal}})}{(1 + \pi)}$$

In plain English, a nominal interest rate of 700% is the same as a real interest rate of 300%, given an inflation rate of 100%.

IMPORTANT

The relation between the nominal rate of return (r_{nominal}), the real rate of return (r_{real}), and the inflation rate (π) is

$$(1 + r_{\text{nominal}}) = (1 + r_{\text{real}}) \cdot (1 + \pi) \qquad (5.2)$$

For small rates, adding/subtracting is an okay approximation.

As with compounding, if the rates are small, the mistake of just subtracting the inflation rate from the nominal interest rate to obtain the real interest rate is not too grave. For example, with our (30-year) 3% Treasury, if inflation were to remain 0.7%, the correct real interest rate would be 2.28% and not 2.30%:

➤ Bills, notes, and bonds, Sect. 5.3, Pg.86.

$$\begin{array}{ccccccccccc} (1 + 3\%) & \approx & (1 + 2.28...\%) & \cdot & (1 + 0.7\%) & \approx & 1 + 2.28...\% & + & 0.7\% & + & 0.0002.... \\ (1 + r_{\text{nominal}}) & = & (1 + r_{\text{real}}) & \cdot & (1 + \pi) & = & 1 + r_{\text{real}} & + & \pi & + & \underbrace{r_{\text{real}} \cdot \pi}_{\text{cross-term}} \end{array}$$

The difference between the correct and the approximation, i.e., the cross-term, is trivial, and easily swamped by measurement noise in the current inflation rate and uncertainty about future inflation rates. However, when inflation and interest rates are high—as they were, for example, in the United States in the late 1970s—then the cross-term can be important.

➤ Adding or compounding interest rates, and the cross-term, Pg.20.

A positive time value of money—that the same amount of money today is worth more than tomorrow—is only true for nominal quantities, not for real quantities. Only nominal interest rates are (usually) not negative. In the presence of inflation, real interest rates not only *can* be negative, but often *are* negative. For example, in December 2015, the one-month Treasury paid 0.14% while the inflation rate was 0.7%, implying a real interest rate of about –0.56%. Every dollar you invested in such U.S. Treasuries would be worth *less* in real purchasing power one month later. You would have ended up with more cash—but also with *less* purchasing power.

Real interest rates can be negative.

Gold

Sometimes, the price of gold has been used as a measure of inflation. Although gold is not a great measure of purchasing power in general, it does make it easy to conduct long-run comparisons. A Roman legionary was paid the equivalent of 2.31 oz of gold a year. A U.S. Army private nowadays is paid the equivalent of 11.01 oz—about five times as much. A Roman centurion was paid 38.58 oz. A U.S. army captain is paid 27.8 oz, about a quarter less. Thus, as an asset class, an investment in gold would have earned a rate of return roughly in line with the income growth over two millenia. *Erb and Harvey (2013)*

Q 5.13. From memory, write down the relationship between nominal rates of return ($r_{nominal}$), real rates of return (r_{real}), and the inflation rate (π).

Q 5.14. The nominal interest rate is 20%. Inflation is 5%. What is the real interest rate?

Inflation in Net Present Values

When it comes to inflation and net present value, there is a simple rule: Never mix apples and oranges. The beauty of NPV is that every project's cash flows are translated into the same units: today's dollars. Keep everything in the same units in the presence of inflation, so that this NPV advantage is not lost. When you use the NPV formula, always discount nominal cash flows with nominal discount rates, and real (inflation-adjusted) cash flows with real (inflation-adjusted) discount rates.

The most fundamental rule: never mix apples and oranges. Nominal cash flows must be discounted with nominal interest rates.

Let's return to our "apple" example. With 700% nominal interest rates and 100% inflation, the real interest rate is $(1 + 700\%)/(1 + 100\%) - 1 = 300\%$. What is the value of a project that gives 12 apples next year, given that apples cost \$1 each today and \$2 each next year?

Our example discounted both in real and nominal terms.

There are two methods you can use:

1. Discount the nominal cash flow of 12 apples next year ($\$2 \cdot 12 = \$24$) with the nominal interest rate. Thus, the 12 future apples are worth

$$\frac{\text{Nominal Cash Flow}}{1 + \text{Nominal Rate}} = \frac{\$24}{1 + 700\%} = \$3$$

Discount nominal cash flows with nominal rates. Discount real cash flows with real rates.

2. Discount the real cash flows of 12 apples next year with the real interest rate. Thus, the 12 future apples are worth

$$\frac{\text{Real Cash Flow}}{1 + \text{Real Rate}} = \frac{12 \text{ Apples}}{1 + 300\%} = 3 \text{ Apples}$$

in today's apples. Because an apple costs \$1 today, the 12 apples next year are worth \$3 today.

Both the real and the nominal methods arrive at the same NPV result. The opportunity cost of capital is that if you invest one apple today, you can quadruple your apple holdings by next year. Thus, a 12-apple harvest next year is worth 3 apples to you today. The higher nominal interest rates already reflect the fact that nominal cash flows next year are worth less than they are this year. As simple as this may sound, I have seen corporations first work out the real value of their goods in the future, and then discount this with standard nominal interest rates. Just don't!

IMPORTANT

- Discount nominal cash flows with nominal interest rates.
- Discount real cash flows with real interest rates.

Either works. Never discount nominal cash flows with real interest rates, or vice-versa.

Usually, it is best to work only with nominal quantities.

If you want to see this in algebra, the reason that the two methods come to the same result is that the inflation rate cancels out,

$$\begin{aligned} \text{PV} &= \frac{\$24}{1 + 700\%} = \frac{12\text{A}}{1 + 300\%} = \frac{12\text{A} \cdot (1 + 100\%)}{(1 + 300\%) \cdot (1 + 100\%)} \\ &= \frac{\text{N}}{1 + r_{\text{nominal}}} = \frac{\text{R}}{1 + r_{\text{real}}} = \frac{\text{R} \cdot (1 + \pi)}{(1 + r_{\text{real}}) \cdot (1 + \pi)} \end{aligned}$$

where N is the nominal cash flow, r is the real cash flow, and π is the inflation rate. Most of the time, it is easier to work in nominal quantities. Nominal interest rates are far more common than real interest rates, and you can simply use published inflation rates to adjust the future price of goods to obtain future expected nominal cash flows.

Q 5.15. If the real interest is 3% per annum and the inflation rate is 8% per annum, then what is the present value of a \$500,000 nominal payment next year?

5.3 U.S. Treasuries and the Yield Curve

The simplest and most important benchmark bonds nowadays are Treasuries. They have known and certain payouts.

It is now time to talk in more detail about the most important financial market in the world today: the market for bonds issued by the U.S. government. These bonds are called Treasuries and are perhaps the simplest projects around. This is because, in theory, Treasuries cannot fail to pay. They promise to pay U.S. dollars, and the U.S.-controlled Federal Reserve has the right to print more U.S. dollars if it were ever to run out. Thus, there is no uncertainty about repayment for Treasuries. (In contrast, some European countries or U.S. states that borrow in currencies that they cannot create may well not have the money to pay and therefore default.)

U.S. Treasury bills, notes, and bonds have different maturities.

The shorthand "Treasury" comes from the fact that the debt itself is issued by the U.S. Treasury Department. There are three main types:

1. **Treasury bills** (often abbreviated as **T-bills**) have maturities of up to one year.
2. **Treasury notes** have maturities between one and ten years.
3. **Treasury bonds** have maturities greater than ten years.

The 30-year bond is often called the **long bond**. Together, the three are usually just called **Treasuries**. Conceptually, there is really no difference among them. All are really just obligations issued by the U.S. Treasury. Indeed, there can be Treasury notes today that are due in 3 months—such as a 9-year Treasury note that was issued 8 years and 9 months ago. This is really the same obligation as a 3-month Treasury bill that was just issued. Thus, we shall be (very) casual with name distinctions.

Magnitudes

In late 2015, the United States Federal Government owed over $18 trillion in Treasury obligations (on a GDP of about $17.5 trillion). With a population of 322 million, this debt translated to over $55,000 per person. With 125 million households and 121 million full-time workers, it represented about $150,000 per household or worker. (A worse problem, however, is that the United States has already promised benefits to future retirees that far exceed this number.) But the United States also has assets. It owns more than $100 trillion in land, infrastructure, and mineral rights under the land.

The Treasuries market is one of the most important financial markets in the world.

After Treasuries are sold by the government, they are then actively traded in what is one of the most important financial markets in the world today. It would not be uncommon for a dedicated bond trader to buy $100 million of a Treasury note originally issued 10 years ago that has 5 years remaining, and 10 seconds later sell $120 million of a three-year Treasury note issued 6 years ago. Large buyers and sellers of Treasuries are easily found, and transaction costs are very low. Trading volume is huge: Around 2015, it was about $500 billion per trading day. Therefore, the annual trading volume in U.S. Treasuries—about $252 \cdot \$500$ billion $\approx \$130$ trillion, where 252 is the approximate number of trading days per year—was about an order of magnitude larger than the annual U.S. gross domestic product (GDP).

Who owns them?

Who owns them? About $6 trillion is owed to foreigners, with the Chinese (our largest creditor) holding about $1.2 trillion of our bonds. about $12 trillion is owed to ourselves. The U.S. Federal Reserve estimated that domestically about 20% were held by individuals, 25% by banks and mutual funds, 15% by public and private pension funds, 15% by state and local governments, and 25% by other investors.

The yield curve shows the annualized interest rate as a function of bond maturity.

Interest rates on Treasuries change every moment, depending on their maturity terms. Fortunately, you already know how to handle time-varying rates of return, so we can now put your knowledge to the test. The principal tool for working with Treasury bonds is the **yield curve** (or **term structure of interest rates** or just the **term structure**). It is a graphical representation, where the time to maturity is on the x-axis and the annualized interest rates are on the y-axis. There are also yield curves on non-Treasury bonds, but the Treasury yield curve is so prominent that unless clarified further, the yield curve should be assumed to mean investments in U.S. Treasuries. (A more precise name would be the "U.S. Treasuries yield curve.") This yield curve is so important that most other debts in the financial markets, like mortgage rates or bank lending rates, are "benchmarked" relative to the Treasury yield curve. For example, if your firm wants to issue a five-year bond, your creditors will want to compare your interest rate to that offered by equivalent Treasuries, and often will even describe your bond as offering "x basis points above the equivalent Treasury."

Q 5.16. What are the three types of Treasuries? How do they differ?

Yield Curve Shapes

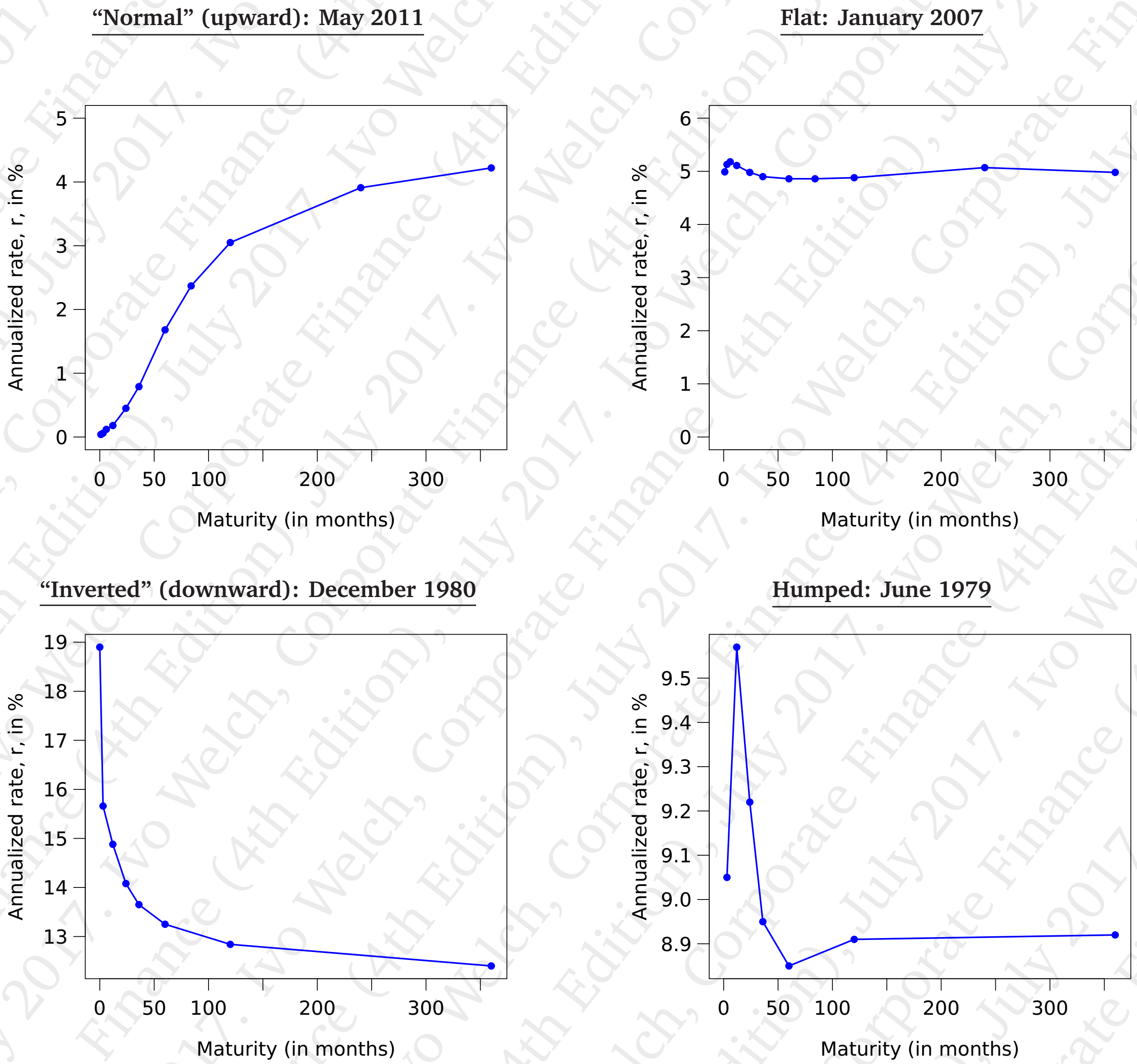


Exhibit 5.1: *Various Historical Yield-Curve Shapes.* The upward slope is so common that it is considered the "normal" shape. In 2016, the yield curve was normal, but not as steep as that in May 2011. A downward slope is sometimes called "inverted."

Exhibit 5.1 shows some historical yield curves. They are commonly classified into four basic shapes:

Yield curves are often, but not always, upward-sloping.

1. **Flat:** There is little or no difference between annualized short-term and long-term rates. A flat yield curve is basically the scenario that was the subject of the previous chapter. It means you can simplify $(1+r_{0,t}) \approx (1+r)^t$.
2. **Upward-sloping ("normal"):** Short-term rates are lower than long-term rates. This is the most common shape. It means that longer-term interest rates are higher than shorter-term interest rates. Since 1934, the steepest yield curve (the biggest difference between the long-term and the short-term Treasury rates) occurred in October 1992, when the long-term interest rate was 7.3% and the short-term interest rate was 2.9%—just as the economy pulled out of the recession of 1991. As of 2016, the yield curve has been upward-sloping since the Great Recession.
3. **Downward-sloping ("inverted"):** Short-term rates are higher than long-term rates.
4. **Humped:** Short-term rates and long-term rates are lower than medium-term rates.

Inverted and humped yield curves are relatively rare.

Macroeconomic Implications of Different Yield Curve Shapes

Economists and pundits have long wondered what they can learn from the shape of the yield curve about the future of the economy. It appears that the yield curve shape is a useful—though unreliable and noisy—signal of where the economy is heading. Steep yield curves often signal emergence from a recession, as you will see in Exhibit 5.4 on Page 96. Inverted yield curves often signal an impending recession. But can't the Federal Reserve Bank control the yield curve and thereby control the economy? It is true that the Fed can influence the yield curve—and since 2008, it has worked on influencing it like never before. But ultimately the Fed does not control it—instead, it is the broader demand and supply for savings and credit in the economy that determines it. Economic research has shown that the Fed typically has a good deal of influence on the short end of the Treasury curve—by expanding and contracting the supply of money and short-term loans in the economy—but not much influence on the long end of the Treasury curve, especially in the long-run. And even in the financial crisis of 2008, the Fed's influence on the short end was ultimately limited, too—the nominal rate already stood at 0% and there was little the Fed could do to drop it further. (Large negative nominal rates are not possible.) In fact, by flooding the economy with cheap money, the Fed was trying to push banks to lend and people to spend money—but people instead just deposited the cash right back into the banks!

If you want to undertake your own research, you can find historical interest rates at the St. Louis Federal Reserve Bank at http://research.stlouisfed.org/fred. There are also the Treasury Management Pages at http://www.tmpages.com/. Or you can look at SmartMoney.com for historical yield curves. PiperJaffray.com has the current yield curve—as do many other financial sites and newspapers. Finance.yahoo.com/bonds provides not only the Treasury yield curve but also yield curves for many other types of bonds.

Common data sources for interest rates.

An Example: The Yield Curve on December 31, 2015

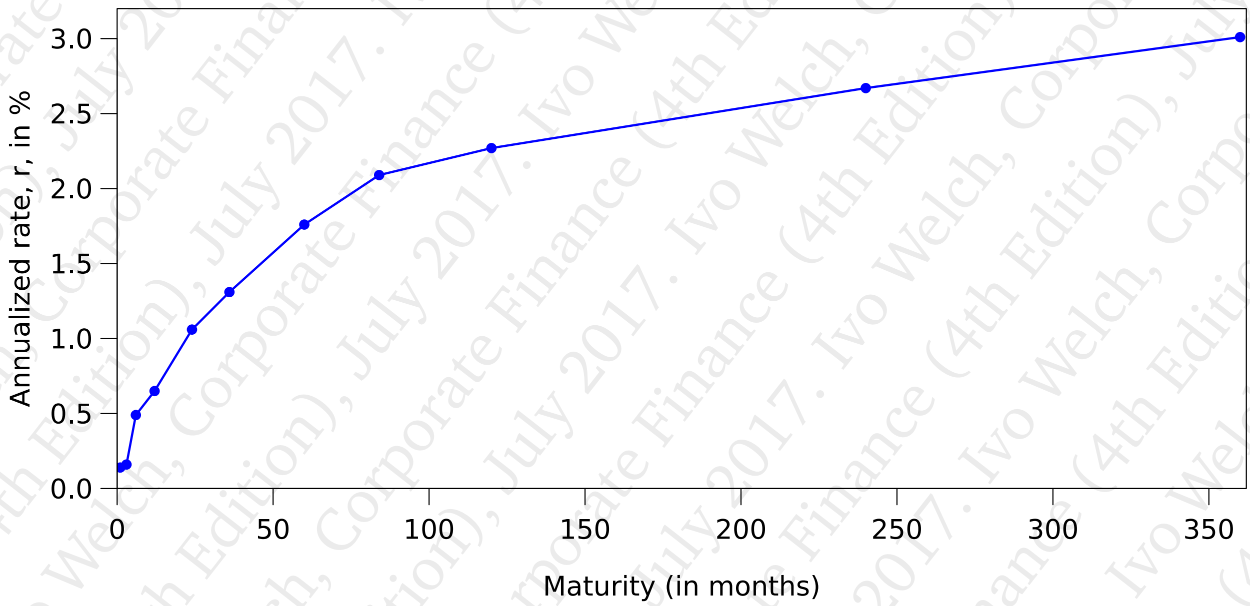
Let's focus on working with one particular yield curve. Exhibit 5.2 shows the Treasury yields on December 31, 2015. This yield curve had the most common upward slope. The curve tells you that if you had bought a 3-month Treasury at the end of the day on December 31, 2015, your annualized interest rate would have been 0.16% per annum. (A \$100 investment would turn into $\$100 \cdot (1+0.16\%)^{1/4} \approx \$100 \cdot 1.0003998 \approx \$100.04$ on March 31, 2016.) If you had

We will analyze the Treasury yield curve at the end of December 2015.

bought a thiry-year bond, your annualized interest rate would have been 3.01% per annum, or \$243.43 on December 2045 for a \$100 investment.

You can interpolate annualized interest rates on the yield curve.

Sometimes it is necessary to determine an interest rate for a bond that is not listed. This is usually done by interpolation. For example, if you had wanted to find the yield for a 4-year bond, a reasonable guess would have been an interest rate halfway between the 3-year bond and the 5-year bond. In December 2015, this would have been an annualized yield of $(1.31\%+1.76\%)/2 \approx 1.54\%$. (This is *not* exact, as you can guess by noticing that the yield curve looks more concave.)



	⟵ Months			Years ⟶							
	1	3	6	1	2	3	5	7	10	20	30
12/31/2015	0.14	0.16	0.49	0.65	1.06	1.31	1.76	2.09	2.27	2.67	3.01

Exhibit 5.2: *The Treasury Yield Curve on December 31, 2015.* These rates are annualized yields to maturity (internal rates of return) calculated from Treasury prices. If they were truly Treasury zero-bonds, they would just be the standard discount rates computed from the final payment and today's price, but we ignore the details here. Such yield curves can be found on many websites. The yield curve changes every day—although day-to-day changes are usually small. Our example works primarily with this particular yield curve. Source: Federal Reserve, www.federalreserve.gov/releases/h15/data.htm.

The December 2015 yield curve was upward-sloping: Annualized interest rates were higher for longer maturities.

As notation for the annualized horizon-dependent interest rates, we continue using our earlier method. We call the two-year annualized interest rate $r_{\overline{2}}$ (here, 1.06%), the three-year

Deeper: There are some small inaccuracies in my description of yield curve computations. My main simplification is that U.S. yield curves are based on semi-annually-compounded coupon bonds in real life, whereas our textbook pretends that the yield is quoted on a zero bond. In corporate finance, the yield difference between annual compounding and semi-annual compounding is almost always inconsequential. However, if you want to become a fixed-income trader, you cannot take this approximation literally. Consult a dedicated fixed-income text instead.

annualized interest rate $r_{\overline{3}}$ (here, 1.31%), and so on. It is always these overlined-subscript yields that are graphed in yield curves. Let's work with this particular yield curve, assuming it is based exclusively on zero-bonds, so you don't have to worry about interim payments.

Computing the holding rate of return for 2-year and 30-year Treasuries.

Holding rates of return First, let's figure out how much money you will have at maturity. That is, how much does an investment of \$500,000 in U.S. two-year notes (i.e., a loan to the U.S. government of \$500,000) on December 31, 2015, return on December 31, 2017? Use the data in Exhibit 5.2. Because the yield curve prints annualized rates of return, the total two-year holding rate of return (as in Formula 5.1) is the twice compounded annualized rate of return,

➤ Annualizing, Formula 5.1, Pg.78.

$$r_{0,2} = 1.0106 \cdot 1.0106 - 1 \approx 2.13\%$$
$$r_{0,2} = (1 + r_{\overline{2}}) \cdot (1 + r_{\overline{2}}) - 1$$

so your \$500,000 will turn into

$$C_2 \approx (1 + 2.13\%) \cdot \$500{,}000 \approx \$510{,}656$$
$$C_2 = (1 + r_{0,2}) \cdot C_0$$

on December 31, 2017. (In the real world, you might have to pay a commission to arrange this transaction, so you would end up with a little less.)

What if you invest \$500,000 into 30-year Treasuries? Your 30-year holding rate of return is

$$r_{0,30} = 1.0301^{30} - 1 \approx 2.434 - 1 \approx 143.4\%$$
$$r_{0,30} = (1 + r_{\overline{30}})^{30} - 1$$

Thus, an investment of $C_0 = \$500{,}000$ in December 2015 turns into a return of $C_{30} \approx \$1.2$ million by December 2045.

Let's work out one forward rate implied by the December 2015 yield curve.

Forward rates of return Second, let's figure out what the yield curve in December 2015 implied about the 1-year interest rate from December *2016* to December 2017. This would be best named $r_{1,2}$. It is an interest rate that begins in one year and ends in two years. As already mentioned, this is called the *forward rate*.

The 1-year annualized interest rate is $r_{\overline{1}} = 0.65\%$. The two-year annualized rate of return is $r_{\overline{2}} = 1.06\%$. You already know that you can work out the two holding rates of return, $r_{0,1} = 0.65\%$ and $r_{0,2} = (1 + r_{\overline{2}})^2 - 1 \approx 1.0213\%$. You only need to use the compounding formula to determine $r_{1,2}$:

$$(1 + 2.13\%) = (1 + 0.65\%) \cdot (1 + r_{1,2}) \Rightarrow r_{1,2} \approx 1.47\%$$
$$(1 + r_{0,2}) = (1 + r_{0,1}) \cdot (1 + r_{1,2})$$

Note that this forward rate $r_{1,2}$ is higher than both $r_{\overline{1}}$ and $r_{\overline{2}}$ from which you computed it.

Is the proliferation of subscripts torture or necessity?

Exhibit 5.3 summarizes our two-year calculations, and extends them by another year. (This helps you to check your results in an exercise below.) One question you should ask yourself is whether I use so many subscripts in the notation just because I enjoy torturing you. The answer is an emphatic no: The subscripts are there for good reason. When you look at Exhibit 5.3, for example, you have to distinguish between the following:

- the three holding rates of return, $r_{0,t}$ (0.65%, 2.13%, and 3.98%)
- the three annualized rates of return, $r_{\overline{t}}$ (0.65%, 1.06%, and 1.31%)

Maturity	Total Holding	Rates of Return: Annualized	Compounded Rates
1 Year	$(1+0.65\%)$ $(1+r_{0,1})$	$= (1+0.65\%)^1$ $= (1+\overline{r_1})^1$	$= (1+0.65\%)$ $= (1+r_{0,1})$
2 Years	$(1+2.13\%)$ $(1+r_{0,2})$	$\approx (1+1.06\%)^2$ $= (1+\overline{r_2})^2$	$\approx (1+0.65\%)\cdot(1+1.47\%)$ $= (1+r_{0,1})\cdot(1+r_{1,2})$
3 Years	$(1+3.98\%)$ $(1+r_{0,3})$	$\approx (1+1.31\%)^3$ $= (1+\overline{r_3})^3$	$\approx (1+0.65\%)\cdot(1+1.47\%)\cdot(1+1.81\%)$ $= (1+r_{0,1})\cdot(1+r_{1,2})\cdot(1+r_{2,3})$

Exhibit 5.3: *Relation between Holding Returns, Annualized Returns, and Year-by-Year Returns on December 31, 2015, by Formula.* The individually compounded rates are the future interest rates. They are implied by the annualized rates quoted in the middle column. The text worked out the two-year case. You will work out the three-year case in Question 5.17.

- the three individual annual rates of return $r_{t-1,t}$ (0.65%, 1.47%, and 1.81%), where the second and third forward rates begin at different moments in the future.

In real life, you have not just three yearly Treasuries, but many Treasuries between 1 day and 30 years. Anyone dealing with Treasuries (or CDs or any other fixed-income investment) that can have different maturities or start in the future must be prepared to suffer double subscripts.

Yes, corporate projects have double subscripts, too!

If Treasuries offer different annualized rates of return over different horizons, do corporate projects have to do so, too? Almost surely yes. If nothing else, they compete with Treasury bonds for investors' money. And just like Treasury bonds, many corporate projects do not begin immediately, but may take a year or more to prepare. Such project rates of return are essentially forward rates of return. Double subscripts—yikes, but sometimes there is no way out of painful notation in the real world!

Q 5.17. Compute the three-year holding rate of return on December 31, 2015. Then, using the two-year holding rate of return on December 31, 2015, and your calculated three-year holding rate of return, compute the forward interest rate for a 1-year investment beginning on December 31, *2017*, and ending on December 31, 2018. Are these the numbers in Exhibit 5.3?

Q 5.18. Repeat the calculation with the five-year annualized rate of return of 1.76%. That is, what is the five-year holding rate of return, and how can you compute the *annualized* forward interest rate for a *two*-year investment beginning on December 31, *2018*, and ending on December 31, 2020?

Bond Payoffs and Your Investment Horizon

Your investment horizon has no link to the time patterns of bond payoffs you invest in. You can always sell long-term bonds to get money quickly, if need be.

Should there be a link between your personal investment horizon and the kinds of bonds you may be holding? Let's say that you want to buy a three-year zero-coupon bond because it offers 1.31%, which is more than the 0.65% that a 1-year zero-coupon bond offers—but you also want to consume in one year. Can you still buy the longer-term bond? There is good news and bad news. The good news is that the answer is yes: There is no link whatsoever between your desire to get your money back and consume, and the point in time when your three-year bond pays off. You can always buy a three-year bond today, and sell it before maturity, such as next year when it will have become a two-year bond. The bad news is that in our perfect and certain market, this investment strategy will still only get you the 0.65% that the 1-year bond offers. If you buy \$100 of the three-year bond for $P = \$100/1.0131^3 \approx \$96.17$ today, next year it will be a two-year bond with an interest rate of 1.47% in the first year and 1.81% in the second year (both worked out in Exhibit 5.3). You can sell this bond next year for a price of

$$\frac{\$100}{1+r_{1,3}} = \frac{\$100}{(1+r_{1,2})\cdot(1+r_{2,3})} = \frac{\$100}{1.0147\cdot 1.0181} \approx \$96.80$$

Your 1-year holding rate of return would therefore be only $(\$96.80-\$96.17)/\ \$96.17 \approx 0.65\%$—the same rate of return you would have received if you had bought a 1-year bond to begin with. There is no free lunch here.

The Effect of Interest Rate Changes on Short-Term and Long-Term Bonds

Treasuries pay what they promise. They have no default risk. They do have the risk of interim interest rate changes.

Are long-term bonds riskier than short-term bonds? Of course, recall that repayment is no less certain with long-term Treasury bonds than short-term Treasury notes. (This would be an issue of concern if you were to evaluate corporate projects that can go bankrupt. Long-term corporate bonds are often riskier than short-term corporate bonds—most firms are unlikely to go bankrupt this week, but more likely to go bankrupt over a multidecade time horizon.) So, for Treasury bonds, as long as Congress does not go crazy, there should be no uncertainty as far as payment uncertainty is concerned. But there may still be some interim risk of a different kind; and even though we have not yet fully covered it, you can still intuitively figure out why this is so. Ask yourself how economy-wide bond prices (interest rates) can change in the interim (before maturity). What are the effects of sudden interest rate changes before maturity on bond values? It turns out that an equal-sized interest rate movement can be much more dramatic for long-term bonds than for short-term bonds. Let me try to illustrate why.

First, the effect of a 10 bp yield change on the price of a 30-year bond.

The 30-year bond: Work out the value of a \$1,000 30-year zero-bond at the 3.01% interest rate prevailing. It costs $\$1{,}000/1.0301^{30} \approx \$410.79$. You already know that when prevailing interest rates go up, the prices of outstanding bonds drop and you will lose money. For example, if interest rates increase by 10 basis points to 3.11%, the bond value decreases to $\$1{,}000/1.0311^{30} \approx \$399.00$. If interest rates decrease by 10 basis points to 2.91%, the bond value increases to $\$1{,}000/1.0291^{30} \approx \$422.93$. Thus, the effect of a 10-basis-point change in the prevailing 30-year yield induces an immediate percent change (an instant rate of return) in the value V of your bond of

$$\text{Up 10 bp:}\quad r = \frac{V(r_{\overline{30}} = 3.11\%) - V(r_{\overline{30}} = 3.01\%)}{V(r_{\overline{30}} = 3.01\%)} \approx \frac{\$399.00-\$410.79}{\$410.79} \approx -2.87\%$$

$$\text{Down 10 bp:}\quad r = \frac{V(r_{\overline{30}} = 2.91\%) - V(r_{\overline{30}} = 3.01\%)}{V(r_{\overline{30}} = 3.01\%)} \approx \frac{\$422.93-\$410.79}{\$410.79} \approx +2.96\%$$

So for every \$1 million you invest in 30-year bonds, you expose yourself to about \$30,000 in instant risk for every 10-basis-point yield change in the economy.

Second, the effect of a 10 bp point change on the price of a 1-year note.

The 1-year note: To keep the example identical, let's now assume that the 1-year note also has an interest rate of 3.01% and consider the same 10-basis-point change in the prevailing interest rate. In this case, the equivalent computations for the value of a 1-year note are \$970.78 at 3.01%, \$971.72 at 2.91%, and \$969.84 at 3.11%. Therefore, the equivalent instant rates of return are

$$\text{Up 10 bp:}\quad r = \frac{V(r_{\bar{1}} = 3.11\%) - V(r_{\bar{1}} = 3.01\%)}{V(r_{\bar{1}} = 3.01\%)} \approx \frac{\$971.72 - \$970.78}{\$970.78} \approx -0.097\%$$

$$\text{Down 10 bp:}\quad r = \frac{V(r_{\bar{1}} = 3.11\%) - V(r_{\bar{1}} = 3.01\%)}{V(r_{\bar{1}} = 3.01\%)} \approx \frac{\$969.84 - \$970.78}{\$970.78} \approx +0.097\%$$

For every \$1 million you invest in 1-year notes, you expose yourself to about \$1,000 risk in instant risk for every 10-basis-point yield change in the economy.

An equal interest rate move affects longer-term bonds more strongly.

It follows that the value effect of an *equal-sized* change in prevailing interest rates is more severe for longer-term bonds. In turn, it follows that if the bond is due tomorrow, interest rate changes can usually wreak very little havoc. You will be able to reinvest tomorrow at whatever the new rate will be. A long-term bond, on the other hand, may lose (or gain) a lot of value.

Again, in the interim, T-bonds are not risk-free!

In sum, you should always remember that Treasury bonds are risk-free in the sense that they cannot default (fail to return the promised payments), but they are risky in the sense that interim interest changes can alter their values. Only the most short-term Treasury bills (say, due overnight) can truly be considered risk-free—virtually everything else suffers interest-rate change risk.

IMPORTANT

Though "fixed income," even Treasuries do not guarantee a "fixed rate of return" over horizons shorter than their maturities. Day to day, long-term Treasury bonds are generally riskier investments than short-term Treasury bills, because interest-rate changes have more impact on them.

For illustration, I have ignored volatility of changes and earned interest.

Confession time: I have pulled two cheap tricks on you. First, in the real world, what if short-term, economy-wide interest rates typically experienced yield shifts of plus or minus 100 basis points, while long-term, economy-wide interest rates never budged? If this were true, long-term bonds could even be safer than short-term bonds. However, the empirical evidence from 1990 to 2016 suggests that day-to-day changes of both were of similar magnitude—about plus or minus 5 basis points a day. (One-month yields changed by about 7 basis points a day.) Second, I ignored that between today and tomorrow, you would also earn 1 day of interest. On a \$1,000,000 investment in 1-years, this would be about \$25; in 30-years, about \$120. Thus about \$100 should be added to the long-term bond investment strategy—but \$100 on a \$30,000 risk exposure was small enough to keep ignorance bliss.

Q 5.19. A ten-year and a 1-year zero-bond both offer an interest rate of 8% per annum.

1. How does an increase of 1 basis point in the prevailing interest rate change the value of the 1-year bond? (Use 5 decimals in your calculation.)
2. How does an increase of 1 basis point in the prevailing interest rate change the value of the ten-year bond?
3. What is the ratio of the value change over the interest change? (In calculus, this would be called the derivative of the value with respect to interest rate changes.) Which derivative is larger?

A Tragic Error: "Paper Losses"?!

"Only" a paper loss: A cardinal error!

If you really need cash from a bond investment in 20 years, doesn't a prevailing interest rate increase cause only an interim **paper loss**? This is a capital logical error many investors commit. Say that a 10-basis-point increase happened overnight, and you had invested $1 million yesterday. You would have lost $30,000 of your net worth in 1 day! Put differently, waiting 1 day would have saved you $30,000 or allowed you to buy the same item for $30,000 less. Paper money is actual wealth. Thinking paper losses are any different from actual losses is a common but capital error. (The only exception to this rule is that realized gains and losses have different tax implications than unrealized gains and losses.) Avoiding this conceptual mistake is more important than learning any formulas in this book.

➤ Tax treatment of realized and unrealized capital gains, Sect. 11.4, Pg.257.

IMPORTANT

"Paper losses" are no less real than realized losses.

5.4 Why Does the Yield Curve Usually Slope Up?

But why? Why? Why?

Aren't you already wondering *why* the yield curve is not usually flat? Take our example yield curve from December 2015. Why did the 30-year Treasury bonds in December 2015 pay 3.01% per year, while the one-month Treasury bills paid only 0.14% per year? And why is the upward slope the most common shape?

Historical Yield Curves

First, let's look at the historical data. We cannot easily visualize the entire historical yield curve in a two-dimensional graph, but we can plot the historical yields on the short-term 3-month Treasury and, say, the 20-year Treasury. Exhibit 5.4 shows that the spread between them ranged from negative (though usually only briefly) to about 300 bp per year. (Humped shapes are rare, so if the long-term rate is below the short-term rate, the yield curve is most likely inverted.) The plot shows that inverted shapes occurred often just before a recession. Since the Great Recession of 2008, the Fed has kept short-term rates close to zero, which has resulted in unusually large spreads to the long-term Treasury. Long-term rates have been coming down.

The two possible explanations are (1) higher future interest rates and/or (2) compensation for risk.

To understand these dynamics better, let's work with a simpler two-year example. Let's say that the yield curve tells you that the 1-year rate is $r_{\bar{1}} = 5\%$ and the two-year rate is $r_{\bar{2}} = 10\%$. You can work out that the 1-year forward rate is then $r_{1,2} \approx 15.24\%$. There are really only two possible explanations:

1. The 1-year interest rate next year will be higher than the 5% that it is today. Indeed, maybe next year's 1-year interest rate will be the 15.24% that it would be in a perfect world with perfect certainty. In equation form, is it the case that $(1+r_{1,2})\cdot(1+E(r_{2,3})) = (1+r_{1,3})$, so $E(r_{2,3}) = (1+r_{1,3})/(1+r_{1,2})-1$? (I am using "E" as an abbreviation for "expected"—which you will learn about in the next chapter.)
2. Investors tend to earn higher rates of return holding long-term bonds than they do holding short-term bonds. For example, if the yield curve were to remain at exactly the same shape next year, then a $100 investment in consecutive 1-year bonds would give you interest of only about $10.25, while the same investment in two-year bonds would give you (on average) $21. Is $E(r_{2,3}) < (1+r_{1,3})/(1+r_{1,2})-1$?

In other words, the question is whether higher long-term interest rates today predict higher interest rates in the future, or whether they offer extra compensation for investors willing to hold longer-term bonds. Let's consider two possible variants of each of these two possibilities.

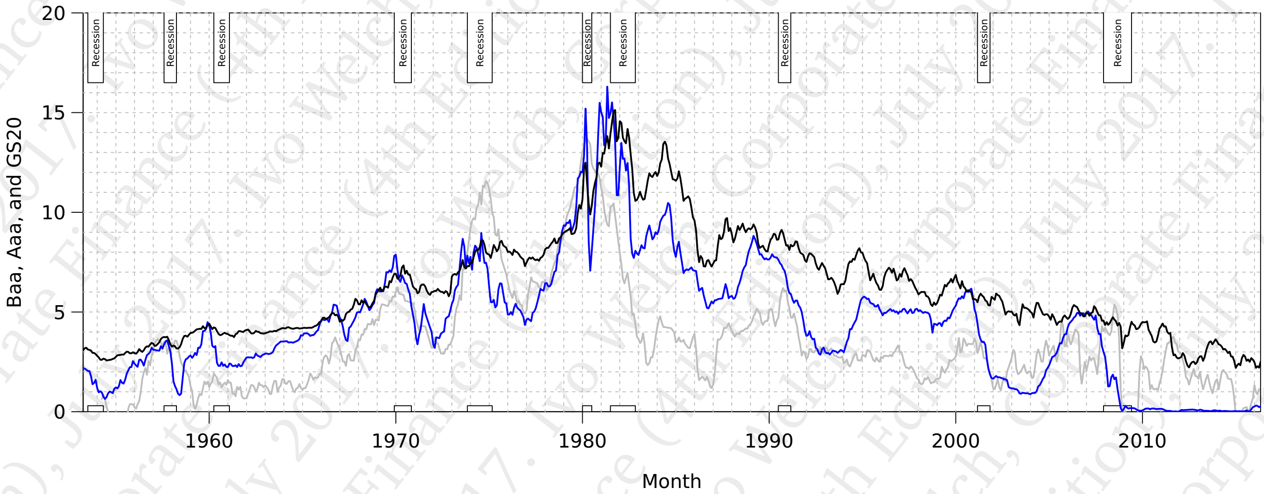


Exhibit 5.4: *3-Month and 20-year Treasury Yields.* The blue line is the yield on 3-month Treasury bills; the black line is the yield on 20-year Treasury bonds. When the blue line was above the black line, the yield curve was inverted. The gray line is the 12-month moving average inflation rate. Recessions are marked at the top (and compressed at the bottom). The data comes from the Federal Reserve FRED database.

Does the Yield Curve Predict Higher Future Inflation?

If inflation is high, investors (typically) demand higher interest rates.

In general, when inflation is higher, you would expect investors to demand higher nominal interest rates. Consequently, you would expect nominal rates to go up when inflation rate expectations are going up. Similarly, you would expect nominal rates to go down when inflation rate expectations are going down. Of course, demand and supply do not mean that real rates of return need to be positive—indeed, the real rate of return is often negative, but the alternative of storing money under the mattress is even worse.

Are higher future inflation rates the cause of higher future interest rates?

Therefore, our first potential explanation for an upward-sloping yield curve is that investors believe that cash will be worth progressively less in the more distant future. That is, even though you will be able to earn higher interest rates over the long run, you may also believe that the inflation rate will increase from today's rate. Because inflation erodes the value of higher interest rates, interest rates should then be higher in the future just to compensate you for the lesser value of money in the future. Of course, this argument would apply only to a yield curve computed from Treasury debt that pays off in nominal terms. It should not apply to any bond payoffs that are inflation-indexed.

TIPS are inflation-indexed Treasury bonds. They are not affected by inflation.

Fortunately, since 1997 the Treasury has been selling bonds that are inflation-indexed. These bond contracts are written so that they pay out the promised interest rate plus the CPI inflation rate. They are called Treasury Inflation Protected Securities (**TIPS**), or sometimes just **CPI bonds**. By definition, they should not be affected by inflation in a perfect market. If the nominal yield curve is upward-sloping because of higher future inflation rates, then a TIPS-based real yield curve should not be upward-sloping.

Inflation-Neutral Bonds

As it turns out, inflation-adjusted bonds had already been invented once before! The world's first known inflation-indexed bonds were issued by the Commonwealth of Massachusetts in 1780 during the Revolutionary War. These bonds were invented to deal with severe wartime inflation and discontent among soldiers in the U.S. Army with the decline in the purchasing power of their pay. Although the bonds were successful, the concept of indexed bonds was abandoned after the immediate extreme inflationary environment passed, and largely forgotten. In 1780, the bonds were viewed as an irregular expedient, because there was no formulated economic theory to justify indexation.

Robert Shiller, "The Invention of Inflation-Indexed Bonds in Early America," October 2003

Conveniently, the Treasury website also shows a TIPS-based yield curve. They were

Inflation-adjusted bond prices suggest expectations of inflation were not the main driver of the upward-sloping yield curve.

➤ Inflation Adjusting, Formula 5.2, Pg.84.

Maturity	5-year	7-year	10-year	20-year	30-year
TIPS Interest Rate	0.45%	0.59%	0.73%	1.07%	1.28%

Recall that for small figures, the difference between the nominal and the real rate is about the inflation rate.

Maturity	5-year	7-year	10-year	20-year	30-year
Ordinary Treasury Bonds	1.76%	2.09%	2.27%	2.67%	3.01%
TIPS Interest Rate	0.45%	0.59%	0.73%	1.07%	1.28%
Implied Inflation	1.31%	1.50%	1.54%	1.60%	1.73%

Without going into more details, the implied inflation rate contains a little bit of risk compensation, too, and more for longer-term projects. Thus, differences in inflation expectations can explain *at most* an 0.4% difference between 5-year and 30-year nominal interest rates, and likely quite a bit less. Say 0.25% is a good guestimate. This 0.25% is only about one-fifth part of the 1.25% spread in nominal interest rates. Trust me that these numbers have also been reasonably representative for the last few decades of U.S. history. We can conclude that, even if increasing inflation expectations can play a minority role, they were not the main reason for the common upward nominal yield curve slope.

Q 5.20. In June, 2016, an inflation-adjusted 30-year Treasury bond offered a real yield of about 0.7% per year. The equivalent non-inflation-adjusted bond offered 2.25% per year. In what inflation scenario would you be better off buying one or the other? (The most recent historical inflation rate was 1% per year.)

Does The Yield Curve Predict Higher Future Interest Rates?

Does a high forward interest rate predict a high future interest rate?

A closely related possibility is that the yield curve is typically upward-sloping because short-term interest rates will be higher in the future. This is more generic than the previous explanation—higher future interest rates need not be caused by higher future inflation expectations. Maybe the 30-year yield of 3.01% was much higher than the 1-year yield of 0.65% because investors expected the 1-year interest rate in 2044 to be much much higher than 3% (the forward rate, $r_{29,30}$). This does not tell you *why* investors would expect interest rates to be so much higher in 2044 than in 2015—maybe they expect that capital will be more scarce then and investment opportunities will be better—but we can speculate about this even if we do not know the precise reason.

Alas, the historical data tells us "probably not much."

Unfortunately, we do not have a direct estimate of future interest rates the way we had a direct estimate of future inflation rates (from TIPS). Therefore, investigating this hypothesis requires looking at many years of evidence to learn whether future interest rates were well predicted by prevailing forward rates. The details are beyond our scope. However, I can tell you the punchline: Expectations of higher future rates of return are not the reason why the yield curve is typically upward-sloping (except maybe at the very short end of the yield curve, say, for interest rates that are for cash investments for less than 1 month).

Does The Yield Curve Identify Bargains?

It must be either higher future interest rates or higher compensation for long-term bond investors.

If it is not the case that future interest rates are higher when forward rates are higher, it means that we are dealing with the second possible reason: On average, investors must have earned more in long-term bonds than in rolled-over short-term bonds. The empirical data confirms that you would have ended up historically with more money if you had bought 30-year bonds than if you had bought one-month bonds every month for 30 years.

Different Preferences?

One reason why this may have been the case is the habitat theory: Different investors may only like different types of bonds, to the point that different-maturity bond markets are segmented. The fact that there are habitats may well be true, but it is not so clear why long-term bond investors would then be so scarce that they require more compensation, while short-term bond investors would be so abundant that bills can be sold for higher prices. And if this were the only reason, then why would borrowers not always prefer to sell short-term bonds instead? And why would other investors not try to step in and get rich (by "arbitraging" the difference)?

Free money? Not in a perfect market.

So why were long-term bonds better investments than short-term bonds? Maybe the yield curve was upward-sloping because investors were stupid. In this case, you might conclude that the 30-year bond offering 3.01% was a much better deal than the 1-year bond offering 0.65%. Alas, investor stupidity seems highly unlikely as a good explanation. The market for Treasury bond investments is close to perfect in the sense that we have used the definition. It is very competitive. If there was a great deal to be had, thousands of traders would have immediately jumped on it. More likely, the interest rate differential does not overthrow the old tried-and-true axiom, *You get what you pay for*. It is just a fact of life that investments for which the payments are tied down to occur in 30 years must offer higher interest rates now in order to entice investors—for some good reason yet to be identified. Again, it is important that you keep in mind that your cash and consumption are *not* tied down if you invest in a 30-year bond, because you can, of course, sell your 30-year bond tomorrow to another investor if you so desire.

Does It Compensate Investors For Risk?

The answer is probably compensation for risk.

➤ Bond Risk, Sect. 5.3, Pg.93.

If it isn't market stupidity that allows you to earn more money in long-term bonds than in rolled-over short-term bonds, then what else could it be? The empirical evidence suggests that it is most likely the phenomenon explained in Section 5.3: Interim changes in prevailing interest rates have much more impact on long-term bonds than they have on short-term bonds. Recall that rolling over short-term bonds insulates you from the risk that interest rates will change in the future. If you hold a one-day bond and interest rates double by tomorrow, you can just purchase more bonds tomorrow that will offer you twice the interest rate. In contrast, if you hold a long-term bond, you could lose your shirt if interest rates go up in the future. With long-term bonds being riskier than short-term bonds, investors only seem to want to buy them if they get some extra rate of return. Otherwise, they prefer rolling short bonds. Thus, long-term bonds need to offer investors more return on average than short-term bonds.

5.5 Corporate Time-Varying Costs of Capital

Now that you understand that the yield curve is usually upward-sloping for a good reason, you should recognize the family resemblance: Corporate projects are offering cash flows, just like Treasury bonds. Thus, it should not surprise you that longer-term projects usually have to offer higher rates of return than shorter-term projects. And just because a longer-term project offers a higher expected rate of return does not necessarily mean that it has a higher NPV. Conversely, just because shorter-term borrowing allows firms to pay a lower expected rate of return does not necessarily mean that this creates value. (Neither firms nor the U.S. Treasury rely exclusively on short-term borrowing.) Paying a higher expected rate of return for longer-term obligations is (usually) a fact of life.

Extend this insight to corporations: Longer-term projects, even if they are not more likely to default, often face a higher cost of capital, and therefore should have to deliver higher returns.

IMPORTANT

Even in a perfect market without uncertainty:

- The appropriate cost of capital (rate of return) should usually depend on how long-term the project is.
- The term structure is usually upward-sloping. Short-term corporate projects usually have lower costs of capital than long-term projects.
- Conversely, corporations usually face lower costs of capital (expected rates of return offered to creditors) if they borrow short term rather than long term.

The difference between long-term and short-term rates is called the **Term Premium**.

Let me give you a short preview now. In Chapter 10, you will learn about the CAPM. The CAPM is the most common model used to discount future cash flows in NPV applications. It is a model that relates your project's required expected rate of return to its risk. In practice, the CAPM allows you to use higher (risk-free) rates of return for cash flows farther in the future. Thus, the CAPM has one term that is (more or less reflective of) the term-premium and one term for the risk-premium. In this sense, it can be viewed as a generalization of the point of this chapter that longer-term projects usually require higher (opportunity) costs of capital. If the second term is zero, then all you have left is the term premium.

Time-Varying Expected Rates of Return vs. Time- and Risk-Varying Expected Rates of Return.

It turns out that the first term (the term premium) has worked much better than the second (the risk premium). Thus, in real life, it is often more important for you to understand that you usually need to increase your cost-of-capital estimate for longer-term cash flows, than it is for you to understand the much more complex second CAPM term.

Basic Usage and Reasonable guesstimate.

Alas, there is one second-order complication: the term-premium for corporate cash flows can be different from the term-premium for Treasuries. Although this is true, the first-order approximation is usually that the two are similar. That is, if a 20-year Treasury offers an expected rate of return that is 2% higher than that of a 1-year Treasury, you should probably guess that a corporate cash flow in 20 years should offer an expected rate of return that is also about 2% higher than that of its 1-year counterpart. (The second-order corrections depend on much deeper, more difficult, and harder-to-prove reasoning and will most likely gross up or shade down the Treasury spread only by a little bit.)

Summary

This chapter covered the following major points:

- Different horizon investments can offer different rates of return. This phenomenon is often called time-varying rates of return.
- The general formula for compounding works just as well for time-varying rates of return as it does for time-constant rates of return. You only lose the ability to exponentiate (one plus the 1-year rate of return) when you want to compute multiyear rates of return.
- A holding rate of return can be annualized for easier interpretation.
- The graph of annualized interest rates as a function of maturity is called the "term structure of interest rates" or the "yield curve."
- The yield curve is usually upward-sloping. However, no law of finance is violated if it is downward-sloping (inverted), humped, or flat.
- Net present value also works just as well for time-varying interest rates. You merely need to use the appropriate rate of return as the opportunity cost of capital in the denominator.
- An important side observation: "Paper losses" are no different from real losses.
- Inflation is the process by which money will buy fewer goods in the future than it does today. If contracts are inflation-indexed in a perfect market, inflation is irrelevant. This is rarely the case.
- The relationship between nominal interest rates, real interest rates, and inflation rates is

$$(1 + r_{\text{nominal}}) = (1 + r_{\text{real}}) \cdot (1 + \text{Inflation Rate})$$

- Unlike nominal interest rates, real interest rates can—and often have been—negative.
- In NPV, you can either discount real cash flows with real interest rates, or discount nominal cash flows with nominal interest rates. The latter is usually more convenient.
- TIPS are Treasury bonds that protect against future inflation. Short-term bond buyers are also less exposed to inflation rate changes than long-term bond buyers.
- Higher long-term interest rates could be due either to expectations of higher future interest rates or to extra required compensation for investors willing to hold longer-term bonds. The empirical evidence suggests that historically the latter has been the more important factor.
- Corporations should realize that corporate project cash flows need to be discounted with specific costs of capital that may depend on when the cash flows will come due. It is not unusual that cash flows in the more distant future require higher discount rates.

Preview of the Chapter Appendix in the Companion

The appendix to this chapter explains

- how you can translate the yield curve (in Exhibit 5.3) step by step into forward and total holding rates of return.
- how shorting works in the real world, and how you can lock in a future interest today with clever bond transactions today.
- (again) how the "duration" of bonds helps you measure when you receive your cash flows on average.
- continuous compounding, which is a different way of quoting interest rates.
- that Treasury notes and bonds are not really zero bonds (as we pretended in this chapter) but coupon bonds, and why this rarely matters in a corporate context. True Treasury zero bonds are called STRIPS.

Keywords

Answers

Q 5.1 $r_{0,2} = (1+r_{0,1}) \cdot (1+r_{1,2}) - 1 = 1.02 \cdot 1.03 - 1 = 5.06\%$

Q 5.2 Solve $(1+x) \cdot (1+22\%) = (1-50\%)$, so the project had a rate of return of –59.00%.

Q 5.3 The first three-year compounded rate of return was $r_{2010,2012} \approx (1+0.150) \cdot (1+0.021) \cdot (1+0.16) - 1 \approx +36.2\%$. (The notation is a bit ambiguous when month and day are omitted, because the first return is from the end of 2009 to the end of 2010.) The second three-year rate was $r_{2013,2015} \approx +52.42\%$. The full six-year compounded rate of return was thus $r_{2010,2015} \approx (1+.362\%) \cdot (1+.524\%) - 1 \approx +107.6\%$. Although these were very fat years for stock investors. the realized rate was indeed time-varying.

Q 5.4 The returns were $(-33\%, +50\%, -67\%, +100\%)$. Thus the average rate was 12.5% and the overall rate of return was –33.33%. It is always true that the compound rate of return is always less than the average rate of return. The example shows that the two can differ in sign.

Q 5.5 $1.05^{12/4} \approx 15.76\%$

Q 5.6 The annualized rate of return is $\sqrt{1.4} - 1 \approx 18.32\%$. It is therefore lower than the 20% average rate of return.

Q 5.7 The compounded rate of return is always higher than the sum, because you earn interest on interest. The annualized rate of return is lower than the average rate of return, again because you earn interest on the interest. For example, an investment of \$100 that turns into an investment of \$200 in two years has a total holding rate of return of 100%—which is an average rate of return of $100\%/2 = 50\%$ and an annualized rate of return of $\sqrt{(1+100\%)} - 1 \approx 41.42\%$. Investing \$100 at 41% per annum would yield \$200, which is lower than 50% per annum.

Q 5.8 The six-year holding rate of return was 107.6%. Thus, the annualized rate of return was $r_{\overline{6}} = \sqrt[6]{1+107.6\%} - 1 \approx 12.9\%$.

Q 5.9 $r_{0,5} = 50\%$ $(1+r_{\overline{5}})^5 = 1.50 \implies r_{\overline{5}} = 1.50^{1/5} - 1 \approx 8.45\%$.

Q 5.10 The annualized five-year rate of return is the same 10%.

Q 5.11 This project is worth

$$-\$200 + \frac{\$100}{1.03} + \frac{\$300}{1.04^2} + \frac{\$500}{1.045^3} \approx \$612.60$$

Q 5.12 The CPI is the average price change to the consumer for a specific basket of goods. The PPI measures the price that producers are paying. Taxes, distribution costs, government subsidies, and basket composition drive a wedge between these two inflation measures.

Q 5.13 $(1+r_{\text{nominal}}) = (1+r_{\text{real}}) \cdot (1+\pi)$

Q 5.14 $1.20/1.05 \approx 1.1429$. The real interest rate is 14.29%.

Q 5.15 The nominal interest rate is $1.03 \cdot 1.08 - 1 = 11.24\%$. Therefore, the cash flow is worth about $\$500{,}000/1.1124 \approx \$449{,}479$.

Q 5.16 Bills, notes, and bonds. T-bills have maturities of less than 1 year. T-notes have maturities from 1 to 10 years. T-bonds have maturities greater than 10 years.

Q 5.17 Yes. The answers are right in the table. The three-year rate of return is $1.0131^3 - 1 \approx 3.98\%$. The forward rate is $1.0398/(1.0065 \cdot 1.0147) - 1 \approx 1.81\%$.

Q 5.18 $r_{0,5} = 1.0176^5 - 1 \approx 9.12\%$. Therefore, $1 + r_{3,5} = 1.0176^5/1.0131^3 - 1 \approx 4.94\%$. This is a two-year forward holding rate of return. Thus, it is $1.0494^{(}1/2) - 1 \approx 2.44\%$ in annualized terms.

Q 5.19 (1) For the 1-year bond, the value of a \$100 bond changes from $\$100/1.0800 \approx \$92.59259$ to $\$100/1.0801 \approx \$92.58402$. This is about a –0.009% change. (2) For the ten-year bond, the value of a \$100 bond changes from $\$100/1.08^{10} \approx \$46.31935$ to $\$100/1.0801^{10} \approx \$46.27648$. This is a –0.09% change—ten times that of the 1-year bond. (3) The derivative of the 1-year bond is $-0.009/0.01 = -0.9 \approx -1$. The derivative of the ten-year bond is $-0.09/0.01 \approx -9$. The derivative of the ten-year bond is about nine times more negative.

Q 5.20 If the inflation rate will increase to more than $1.0225/1.007 - 1 \approx 1.5\%$ per year, the inflation-adjusted bond will be better. Otherwise, the non-inflation adjusted bond will be better.

End of Chapter Problems

Q 5.21. Are you better off if a project first returns –10% followed by +30%, or if it first returns +30% followed by –10%?

Q 5.22. Compare two stocks. Both have earned 8% per year on average. However, stock A has oscillated between 6% and 10%. Stock B has oscillated between 3% and 13%. (For simplicity, say that they alternated.) If you had bought $500 in each stock, how much would you have had 10 years later?

Q 5.23. (Strange) Stock A always alternated between +20% and –10% in the past. Stock B earned 4.5% per annum.

1. What was the average rate of return for stock A?
2. What was the average rate of return for stock B?
3. On a 1-year basis, would a risk-neutral investor prefer +20% or –10% with equal probability, or 4.5% for sure?
4. How much would each dollar invested 10 years ago in stock A have earned?
5. How much would each dollar invested 10 years ago in stock B have earned?
6. What is going on here?

Q 5.24. Return to Question 5.3. What was the annualized geometric rate of return, and what was the average rate of return on the S&P 500? Would stock brokers prefer to tell their clients the former or the latter?

Q 5.25. On June 23, 2016, the Brits voted to exit the EU. The following were the daily values of an investment (in a fund called SPY):

June	27	28
Dollars	199.60	203.20

If returns were to accumulate at the same rate over an entire year (252 trading days), what would a $100 investment turn into?

Q 5.26. If the annualized five-year rate of return is 10%, what is the total five-year holding rate of return?

Q 5.27. If the annualized five-year rate of return is 10%, and if the first year's rate of return is 15%, and if the returns in all other years are equal, what are they?

Q 5.28. The annual interest rate from year t to year $t+1$ is $r_{t,t+1} = 5\% + 0.3\% \cdot t$ (e.g., the rate of return from year 5 to year 6 is $5\% + 0.3\% \cdot 5 = 6.5\%$).

1. What is the holding rate of return of a ten-year investment today?
2. What is the annualized interest rate of this investment?

Q 5.29. A project has cash flows of +$100 (now at time 0), and –$100, +$100, and –$100 at the end of consecutive years. The interest rate is 6% per annum.

1. What is the project's NPV?
2. How does the value change if all cash flows will occur one year later?
3. Repeat these two questions, but assume that the 1-year (annualized) interest rate is 5%, the two-year is 6%, the three-year is 7%, the four-year is 8%, and so on.

Q 5.30. What is the current inflation rate?

Q 5.31. What is the annualized current nominal interest rate on 30-day U.S. Treasury bills?

Q 5.32. Using the information from Questions 5.30 and 5.31, compute the annualized current real interest rate on 30-day Treasuries.

Q 5.33. If the nominal interest rate is 7% per year and the inflation rate is 2% per year, what is the exact real rate of return?

Q 5.34. The inflation rate is 1.5% per year. The real rate of return is 2.0% per year. A perpetuity project that paid $100 this year will provide income that grows by the inflation rate. Show what this project is truly worth. Do this in both nominal and real terms. (Be clear on what *never* to do.)

Q 5.35. If the annualized rate of return on insured tax-exempt municipal bonds will be 3% per annum and the inflation rate remains at 2% per annum, then what will be their real rate of return over 30 years?

Q 5.36. If the real interest rate is –1% per annum and the inflation rate is 3% per annum, then what is the present value of a $1,000,000 nominal payment next year?

Q 5.37. Inflation is 2% per year; the interest rate is 8% per year. Your perpetuity project has cash flows that grow at 1% faster than inflation forever, starting with $20 next year.

1. What is the real interest rate, both accurate (the "1+" version) and approximate (the subtraction version)?
2. What is the correct project PV?
3. What would you get if you grew a perpetuity project of $20 by the real growth rate of 1%, and then discounted it at the nominal cost of capital?
4. What would you get if you grew a perpetuity project of $20 by the nominal growth rate of 3%, and then discounted it at the real cost of capital?

Performing either of the latter two calculations is not an uncommon mistake in practice.

Q 5.38. You must value a perpetual lease. It will cost $100,000 each year *in real terms*—that is, its proceeds will not grow in real terms, but just contractually keep pace with inflation. The prevailing interest rate is 8% per year, and the inflation rate is 2% per year forever. The first cash flow of your project *next year* is $100,000 *quoted in today's real dollars*. What is the PV of the project? (Warning: Watch the timing and amount of your first payment.)

Q 5.39. If the real rate of return has been about 1% per month for long-term bonds, what would be the value of an investment that costs $100 today and returned $200 in 10 years?

Q 5.40. At your own personal bank, what is the prevailing savings account interest rate?

Q 5.41. Look up today's yield curve on a financial website. What is the 1-year rate of return on a risk-free Treasury? What is the ten-year rate of return on a risk-free Treasury? What is the 30-year rate of return on a risk-free Treasury?

Q 5.42. The 1-year forward interest rates are

Y1	Y2	Y3	Y4	Y5	Y6
3%	4%	5%	6%	6%	6%
Y7	**Y8**	**Y9**	**Y10**	**Y11**	**Y12**
7%	7%	7%	6%	5%	4%

1. Compute the 12 n-year compounded holding rates of return from now to year n.
2. Compute the 12 annualized rates of return.
3. Draw the yield curve.
4. Is there anything wrong in this example?

Q 5.43. The *annualized* interest rates are

Y1	Y2	Y3	Y4	Y5	Y6
3%	4%	5%	6%	6%	6%
Y7	**Y8**	**Y9**	**Y10**	**Y11**	**Y12**
7%	7%	7%	6%	5%	4%

1. Draw the yield curve.
2. Compute the 12 n-year compounded holding rates of return from now to year n.
3. Compute the 12 1-year forward rates of return.
4. Is there anything wrong in this example?

Q 5.44. At today's prevailing Treasury rates, how much money would you receive from an investment of $100 in 1 year, 10 years, and 30 years? What are their annualized rates of return? What are their total holding rates of return?

Q 5.45. Do long-term bonds pay more than short-term bonds because you only get money after a long time—money that you could need earlier?

Q 5.46. A five-year, zero-coupon bond offers an interest rate of 8% per annum.

1. How does a 1-basis-point increase in the prevailing interest rate change the value of this bond in relative terms?
2. What is the ratio of the relative bond value change over the interest change? (This is the derivative of the value with respect to interest rate changes.)
3. How does the derivative of wealth with respect to the interest rate vary with the length of the bond?

Q 5.47. Look at this week's interest rate on ordinary T-bonds and on TIPS. (You should be able to find this information, e.g., in the *Wall Street Journal* or through a fund on the Vanguard website.) What is the implied inflation rate at various time horizons?

Q 5.48. The yield curve is usually upward-sloping. Assess whether this means that the following statements are true or false:

1. Investors earn a higher annualized rate of return from long-term T-bonds than short-term T-bills.
2. Long-term T-bonds are better investments than short-term T-bills.
3. Investors are expecting higher inflation in the future than they are today.
4. Investors who are willing to take the risk of investing in long-term bonds on average earn a higher rate of return because they are taking more risk (that in the interim bond prices fall / interest rates rise).

Q 5.49. Evaluate and Discuss: Does the evidence suggest that long-term bonds tend to earn higher average rates of return than short-term bonds? If yes, why is this the case? If no, why is this not possible?

6

Uncertainty, Default, and Risk

Risk-neutral Promised versus Expected Returns; and Debt versus Equity

You are now entering the world of uncertainty and abandoning the pleasant idea that you have perfect foresight. We shall still pretend, however, that you live in a perfect market with no taxes, no transaction costs, no differences of opinion, and infinitely many investors and firms. But you will learn in this chapter that the presence of uncertainty adds quite a bit of additional complexity and realism.

Net present value still rules supreme, but you will now have to face the sad fact that it is no longer easy to use. It is *not* the NPV concept that is difficult. Instead, it is the *inputs* that are difficult—the expected cash flows and appropriate costs of capital that you now have to guesstimate.

In a world of uncertainty, there are scenarios in which you will get more cash than you expected and scenarios in which you will get less. The single most important insight under uncertainty is that you must always draw a sharp distinction between *promised* (or *quoted* or *stated*) returns and *expected* returns. Because firms can default on payments or go bankrupt in the future, expected returns are lower than promised returns.

After some necessary statistical background, this chapter will cover two important finance topics: First, you must learn how much lenders should charge borrowers if there is the possibility of default. Second, you must learn how to work with the two building blocks of financing—namely, debt and equity.

6.1 An Introduction to Statistics

Statistics is about characterizing an uncertain world.

Statistics has the reputation of being the most painful of the foundation sciences for finance—but you absolutely need to understand it to describe an uncertain future. Yes, it can be a difficult subject, but if you have ever placed a bet in the past, chances are that you already have a good intuitive grasp of what you need. In fact, I had already sneaked the term "expected" into previous chapters, even though it is only now that this book covers what this precisely means.

Random Variables and Expected Values

The "expected value" is the average outcome if the random draw is repeated infinitely often. It need not be a possible realization.

The most important statistical concept is the **expected value**, which is the probability-weighted average of all possible outcomes. It is very similar to a **mean** or **average**. The difference is that the latter two names are used if you work with *past* outcomes, while the expected value applies if you work with *future* outcomes. For example, say you toss a coin, which can come up either heads or tails with equal probability. You receive \$1 if the coin comes up heads and \$2 if the coin comes up tails. Because you know that there is a 50% chance of \$1 and a 50% chance of

$2, the expected value of each coin toss is $1.50. If you repeated this infinitely often, and if you recorded the series of **realizations** (actual outcomes), the mean would converge to exactly $1.50. Of course, in any one throw, $1.50 can never come up—the expected value does not need to be a possible realization of a single coin toss.

IMPORTANT

The expected value is just the mean (a fancy word for average) if you could repeat an experiment (the random draws) infinitely often.

A random variable is a number whose realization is not yet known.

To make it easier to work with uncertainty, statisticians have invented the concept of the **random variable**. It is a variable whose outcome has not yet been determined. In the coin toss example, you can define a random variable named c (for "coin toss outcome") that takes the value $1 with 50% probability and the value $2 with 50% probability. (Random variables are often written with tildes over them, such as $\tilde{c}$, but we will dispense with this formality in our book.) The expected value of c is $1.50. To denote the expected value, we use the notation $\mathcal{E}$. In this bet,

$$\begin{array}{ccccccc} \mathcal{E}(c) & = & 50\% \cdot \$1 & + & 50\% \cdot \$2 & = & \$1.50 \\ \text{Expected Value(Coin Toss)} & = & \text{Prob(Heads)} \cdot \$1 & + & \text{Prob(Tails)} \cdot \$2 & & \end{array}$$

After the coin has been tossed, the actual outcome c could, for example, be c = $2. After the toss, this c is no longer a random variable. Also, if you are certain about the outcome, perhaps because there is only one possible outcome, then the actual realization and the expected value are the same. The random variable is then the same as an ordinary nonrandom variable. Is the expected outcome of the coin toss a random variable? No: You know the expected outcome is $1.50 even before the toss of the coin. The expected value is known; the uncertain outcome is not. The expected value is an ordinary nonrandom variable; the possible outcome is a random variable. Is the outcome of the coin throw *after* it has come down heads a random variable? No: It is an actual outcome and you know what it is (heads), so it is no longer a random variable.

A random variable is a statistical distribution.

A random variable is defined by the **probability distribution** of its possible outcomes. The coin throw distribution is simple: the value $1 with 50% probability and the value $2 with 50% probability. This is sometimes graphed in a **histogram**, which is a graph that has the possible outcomes on the x-axis and the frequency (or probability) on the y-axis. Exhibit 6.1 shows the histogram for the coin throw. In fact, you can think of a random variable as a placeholder for a histogram.

A final note—perfect markets.

One final reminder: In this chapter, we are eliminating our certainty assumption. But we are *not* (yet) eliminating our perfect market assumption. The assumption of no-disagreement means that we all must agree on the probabilities of all possible outcomes. An example of an imperfect market would be if you believed that there was a 51% probability of an outcome of $1, and I believed there was a 50% probability of $1.

Fair Bets

An example with three possible outcomes.

A **fair bet** is a bet that costs its expected value. If repeated infinitely often, both the person offering the bet and the person taking the bet would expect to end up even. For example, call D your payoff based on the following structure:

- There is a 1/4 chance that you will be paid $2;
- a 1/4 chance that you will be paid $10;
- and a 2/4 chance that you will be paid $8.

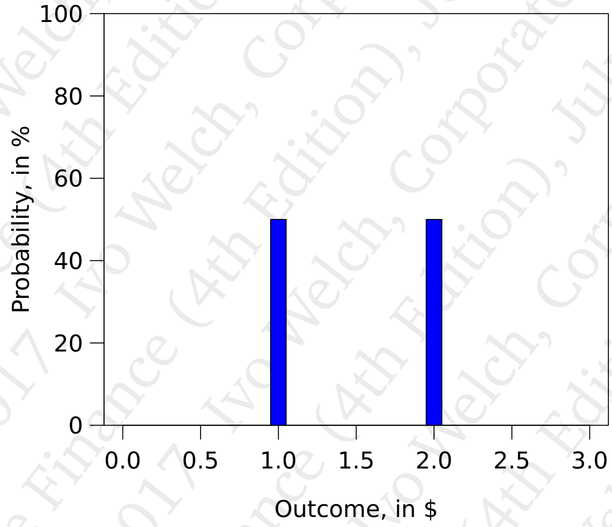


Exhibit 6.1: *A Histogram for a Random Variable with Two Equally Likely Outcomes, $1 and $2.*

You can simulate this payoff structure by drawing a card from a complete deck. If it is ♣, you get a value V of $2; if it is ♦, you get $10, and if it is ♥or ♠, you get $8. What would be a fair price for this card bet? The uncertain payoff is a random variable. Let's call it D. First, you must determine $\mathsf{E}(\mathrm{D})$. It is

$$\mathsf{E}(\mathrm{D}) = \tfrac{1}{4} \cdot \$2 + \tfrac{1}{4} \cdot \$10 + \tfrac{2}{4} \cdot \$8 = \$7$$

$$\mathsf{E}(\mathrm{D}) = \mathsf{Prob}(\clubsuit) \cdot \mathrm{V}\clubsuit + \mathsf{Prob}(\diamondsuit) \cdot \mathrm{V}\diamondsuit + \mathsf{Prob}(\heartsuit \text{ or } \spadesuit) \cdot \mathrm{V}\heartsuit \text{ or } \spadesuit$$

If you repeat this bet a zillion times, you would expect to earn $7 zillion. On average, each bet would earn $7, although some sampling variation in actual trials would make this a little more or a little less. If it costs $7 to buy each single bet, it would be fair.

Generally, the procedure to compute expected values is always the same: Multiply each outcome by its probability and add up all these products.

The expected value is the probability-weighted sum of all possible outcomes.

$$\begin{aligned} \mathsf{E}(\mathrm{X}) = \; & \mathsf{Prob}(\text{First Possible Outcome}) \cdot \text{Value of First Possible Outcome} \\ + \; & \mathsf{Prob}(\text{Second Possible Outcome}) \cdot \text{Value of Second Possible Outcome} \\ + \; & \qquad \vdots \\ + \; & \mathsf{Prob}(\text{Last Possible Outcome}) \cdot \text{Value of Last Possible Outcome} \end{aligned}$$

This is the formula that you used above,

$$\begin{aligned} \mathsf{E}(\mathrm{D}) &= \tfrac{1}{4} \cdot \$2 + \tfrac{1}{4} \cdot \$10 + \tfrac{2}{4} \cdot \$8 = \$7 \\ &= \text{Sum of } [\mathsf{Prob}(\text{Each Outcome}) \times \text{Value of Each Outcome}] \end{aligned}$$

Note that the formula is general. It works even with outcomes that are impossible. You would just assign probabilities of zero to them.

IMPORTANT

You must understand the following:

1. The difference between an ordinary variable and a random variable
2. The difference between a realization and an expectation
3. How to compute an expected value, given probabilities and outcomes
4. What a fair bet is

Q 6.1. Is the expected outcome (value) of a die throw a random variable?

Q 6.2. Could it be that the expected value of a bet is a random variable?

Q 6.3. For an ordinary die, assume that the random variable is the number on the die times two. Say the die throw came up with a "six" yesterday. What was its expected outcome before the throw? What was its realization?

Q 6.4.A stock that has the following probability distribution (outcome P_{+1}) costs \$50. Is an investment in this stock a fair bet?

Prob	P_{+1}	Prob	P_{+1}	Prob	P_{+1}	Prob	P_{+1}
5%	\$41	20%	\$45	20%	\$58	5%	\$75
10%	\$42	30%	\$48	10%	\$70		

Variance and Standard Deviation

We will measure the "reward" as the expected value. Looking ahead, the standard deviation is the most common measure of "risk" (spread).

In finance, we often need to measure the (average) **reward** that you expect to receive from making an investment. Ordinarily, we use the expected value of the investment as our measure of reward. We also often need to measure a second characteristic of an investment, its **risk**. Thus, we also need summary measures of how spread out the possible outcomes are. These two concepts will play starring roles in the next few chapters, where you will explore them in great detail. For now, if you are curious, think of risk as a measure of the variability of outcomes around your expected mean. The most common measure of risk is the standard deviation, which takes the square root of the sum of squared deviations from the mean—a mouthful. Let's just do it once for our card-draw problem. Recall our formula: the expectation are probability-weighted values. First, work out each squared deviation from the mean:

(Computing the variance can be a demeaning task.)

The first outcome is \$2. The mean is \$7, so the deviation from the mean is \$2 – \$7 = –\$5. You need the squared deviation from the mean, which is $(–\$5)^2 = +\$\$25$. The units are strange—dollars squared—and impossible to interpret intuitively. Don't even try.

The second outcome is \$10, so the deviation from the mean is \$10 – \$7 = +\$3. You need the squared deviation from the mean, which is $(+\$3)^2 = +\$\$9$.

The third outcome is \$8, so the deviation from the mean is \$8 – \$7 = +\$1. You need the squared deviation from the mean, which is $(\$1)^2 = +\$\$1$.

Together, in one table, this is

Probability	1/4	1/4	2/4
Outcome	\$2	\$10	\$8
Net of the Mean (\$7)	– \$5	+ \$3	\$1
Squared Net of Mean	\$\$25	\$\$9	\$\$1

Now compute the expected value of these squared deviations, which is called the **variance**:

$$\text{Var}\big(\text{Card Pay}\big) \;=\; 1/4 \cdot (\$\$25) + 1/4 \cdot (\$\$9) + 2/4 \cdot \$\$1 \;=\; \$\$9$$

The **standard deviation** is therefore

$$\text{Sdv}\big(\text{Card Pay}\big) \;=\; \sqrt{\$\$9} \approx \$3$$

There you have it—our mouthful: The standard deviation is the square root of the average squared deviation from the mean. Unlike the variance, the standard deviation has sensible units. Together, the mean and standard deviation allow you to characterize your bet. It is common phrasing, though a bit loose, to state that you expect to earn \$7 (the expected value) from a single card draw, plus or minus \$3 (the standard deviation).

Q 6.5. Reconsider the stock investment from Question 6.4. What is its risk—that is, what is the standard deviation of its outcome P_{+1}?

Risk Neutrality (and Preview of Risk Aversion)

Choosing investments only on the basis of expected values is assuming risk neutrality.

Fortunately, the expected value is all you need to learn about statistics for this chapter. This is because we are assuming—only for learning purposes—that everyone is **risk-neutral**. Essentially, this means that investors are willing to write or take any fair bet. For example, if you are risk-neutral, you would be indifferent between getting \$1 for sure and getting either \$0 or \$2, each with 50% probability. And you would be indifferent between earning 10% from a risk-free bond and earning either 0% or 20%, again with fifty-fifty probability, from a risky bond. You have no preference between investments with equal expected values, no matter how safe or uncertain these investments may be.

Risk aversion means you would prefer the safe project. Put differently, you would demand an extra "kicker" to take the riskier project instead.

If, instead, you are **risk-averse**, you would not want to invest in the more risky alternative if both the risky and safe alternatives offered the same expected rate of return. You would prefer the safe \$1 to the unsafe \$0 or \$2 investment. You would prefer a 10% risk-free bond to the unsafe corporate bond that would pay either 0% or 20%. In this case, if I wanted to sell you a risky project or a risky bond, I would have to offer you a higher expected rate of return as risk compensation. I might have to pay you, say, 5 cents to get you to be willing to accept the project that pays off \$0 or \$2 if you can instead earn \$1 elsewhere. Alternatively, I would have to lower the price of my corporate bond so that it offers you a higher expected rate of return, say, 1% or 21% instead of 0% or 20%.

For a given investor, bigger bets usually require more compensation for risk.

It is true that if you are risk-averse, you should not accept fair bets. (You can think of this as the definition of risk aversion.) But would you really worry about a bet for either +\$1 or –\$1? Probably not. For small bets, you are probably close to risk-neutral—I may not have to pay you even 1 cent extra to induce you to take this bet. But what about a bet for plus or minus \$100? Or for plus or minus \$10,000? My guess is that you would be fairly reluctant to accept the latter bet without getting extra compensation for risk bearing. If you are like most investors, you are more risk-averse when the bet is larger. To take the plus or minus \$10,000 bet, I would probably have to offer you several hundred dollars extra.

Financial markets can spread risk and thereby lower the aggregate risk aversion.

However, your own personal risk aversion is not what matters in financial markets. Instead, the financial markets set investments prices in line with the market's aggregate risk aversion. The reason is risk sharing. For example, if you could share the $10,000 bet with 10,000 other students in your class, your own part of the bet would be only plus or minus $1. And some of your colleagues may be willing to accept even more risk for relatively less extra risk compensation—they may have healthier bank accounts or wealthier parents. Therefore, when you can lay bets across many investors, the effective risk aversion of the group will be lower than that of any of its members. And this is exactly how financial markets work: Their aggregate risk absorption capabilities are considerably higher than those of their individual investors. In effect, the financial markets are less risk-averse than individual investors.

The tools you learn now will remain applicable under risk aversion.

You will study risk aversion in the next chapters. In this chapter, we will focus on pricing under risk neutrality. But, as always, all tools you learn in this simpler scenario will remain applicable in the more complex scenario in which investors are risk-averse. Moreover, in the real world, the differences between promised and expected returns that are discussed in this chapter are often more important (in terms of value) than the extra compensation for risk aversion that is ignored in this chapter.

Q 6.6. Are investors more risk-averse for small bets or for large bets? Should "small" be defined relative to investor wealth?

Q 6.7. Can the aggregate financial market be less risk-averse than each of its individual investors?

6.2 Interest Rates and Credit Risk (Default Risk)

Risk-free and risky lending.

Most loans in the real world are not risk-free, because the borrower may not fully pay back what was promised. We will assume that there is one exception, which is that U.S. Treasuries are risk-free loans in nominal terms. In principle, the United States can always tax more and print more dollars to satisfy all promised bond payments. Therefore, it is reasonable to assume the United States cannot default. (Intelligent people can disagree. Washington politics is so dysfunctional that the U.S. may actually default not for lack of dollars, but by choice.) So, how do you compute appropriate expected rates of return for risky bonds?

The Ruin of the First Financial System

The earliest known example of widespread financial default occurred in the year 1788 B.C.E., when King Rim-Sin of Uruk (Mesopotamia) repealed *all* loan repayments. The royal edict effectively destroyed a system of flourishing commerce and finance, which was already many thousands of years old! It is not known why Rim-Sin did so. Interest rates were modest, roughly 4% per annum for five-year loans.

William Goetzmann, Yale University

Risk-Neutral Investors Demand Higher Promised Rates

If my repayment is certain, you should charge me the same interest rate that the U.S. Treasury offers.

Now, put yourself into the position of a banker. Assume that a 1-year Treasury note offers a safe annual rate of return of 10%. Your immediate problem is that you are contemplating making a 1-year loan of $1 million to me. What interest rate should you charge me on the loan? If you are 100% certain that I will fully pay the agreed-upon amount, you can just charge me 10%. You earn just as much from me as from the Treasury note. Both will pay back $1,100,000.

If you quote me the same interest rate, you would expect to earn a lower interest rate if there is a chance of default.

However, in the real world, there are few borrowers for whom you can be 100% certain that they will fully repay a loan. For example, assume you believe there is only a 50% chance that I will pay back the principal plus interest. (If I do pay it back, I will be called **solvent**). There is also a 50% chance that I will **default** (fail to pay all that I have promised). This is often informally called bankruptcy. In this case, I may only be able to pay back \$750,000—all that I have left. If, as the bank, you were to charge me a 10% interest rate, your expected payout would be

$$\underbrace{50\%}_{\text{Prob(Default)}} \cdot \underbrace{\$750{,}000}_{\text{(Pay if Default)}} + \underbrace{50\%}_{\text{Prob(Solvent)}} \cdot \underbrace{\$1{,}100{,}000}_{\text{(Pay if Solvent)}} = \$925{,}000$$

Your *expected* return would not be \$1,100,000, but only \$925,000. Your *expected* rate of return would not be +10%, but only $\$925,000/\$1,000,000 - 1 = -7.5\%$. Extending such a loan would not be—pardon the pun—in your best interest: You can do better by investing your \$1,000,000 into government Treasury notes.

A Short History of Bankruptcy

The framers of the United States Constitution had the English bankruptcy system in mind when they included the power to enact "uniform laws on the subject of bankruptcies" in Article I (powers of the legislative branch). The first bankruptcy law, passed in 1800, virtually copied the existing English law. Our bankruptcy laws thus have their conceptual origins in English bankruptcy law prior to 1800. On both sides of the Atlantic, however, much has changed since then.

Early English law had a distinctly pro-creditor orientation and was noteworthy for its harsh treatment of defaulting debtors. Imprisonment for debt was the order of the day, from the time of the Statute of Merchants in 1285 until Charles Dickens's time in the mid-nineteenth century. (In fact, when Dickens was a child, his father spent time in debtor's prison.) The common law *Writs of Capias* authorized "body execution," that is, seizure of the body of the debtor, to be held until payment of the debt.

English law was not unique in its lack of solicitude for debtors. History's annals are replete with tales of harsh treatment of debtors. Punishments inflicted upon debtors included forfeiture of all property, relinquishment of the consortium of a spouse (think about this one!), imprisonment, and death. In Rome, creditors were apparently authorized to carve up the body of the debtor. However, scholars debate the extent to which the letter of that law was actually enforced.

Charles Jordan Tabb, 1995, "The History of the Bankruptcy Laws in the United States."

You must ask for a higher promised interest—received only in good times—in order to make up for my default risk.

You should conclude that you must demand a higher interest rate from risky borrowers as a banker, even if you just want to "break even" (i.e., expect to earn the same \$1,100,000 that you could earn in Treasury notes). If you solve

$$\underbrace{50\%}_{\text{Prob}} \cdot \underbrace{\$750{,}000}_{\text{(Payment if Default)}} + \underbrace{50\%}_{\text{Prob}} \cdot \underbrace{\text{(Promised Repayment)}}_{\text{(Payment if Solvent)}} = \underbrace{\$1{,}100{,}000}_{\text{Treasury Payment}}$$

for the desired promised repayment, you will find that you must ask me for \$1,450,000. The promised interest rate is therefore $\$1,450,000/\$1,000,000 - 1 = 45\%$. Of this 45%, 10% is the **time premium** that the Treasury pays. Therefore, you can call the remaining 35% the **default premium**—the difference between the promised rate and the expected rate that you, the lender, would have to demand just to break even. It is very important that you realize that the default premium is not extra compensation for your taking on more risk, say, relative to holding Treasuries. You don't receive any such extra compensation in a risk-neutral world. The default premium just fills the gap between the expected return and the promised return.

You are always quoted promised returns, and not expected returns. The risk is called "credit risk."

➤ IRR, YTM, Sect. 4.2, Pg.59.

You rarely observe expected rates of return directly. Newspaper and financial documents almost always provide only the **promised interest rate**, which is therefore also called the **quoted interest rate** or the **stated interest rate**. When you read a published yield-to-maturity, it is also usually only a promised rate, not an expected rate—that is, the published yield is an internal rate of return that is calculated from promised payments, not from expected payments. Of course, you should never make capital budgeting decisions based on promised IRRs. You almost always want to use an expected IRR (YTM). But you usually have easy access only to the promised rate, not the expected rate. On Wall Street, the default premium is often called the **credit premium**, and **default risk** is often called **credit risk**.

Q 6.8. For what kind of bonds are expected and promised interest rates the same?

A More Elaborate Example With Probability Ranges

Again, I sometimes may not be able to repay.

This distinction between expected and promised rates is so important that it is worthwhile to work another more involved example. Assume again that I ask you to lend me money. You believe that I will pay you what I promise with 98% probability; that I will repay half of what I borrowed with 1% probability; and that I will repay nothing with 1% probability. I want to borrow $200 from you, which you could alternatively invest into a government bond promising $210 (i.e., a 5% interest rate). What interest rate would you ask of me?

If you ask me to pay the risk-free interest rate, you will on average earn less than the risk-free interest rate.

If you ask me for a 5% interest rate, next year (time 1), your $200 investment today (time 0) will produce the following:

Payoff (C_1)	Rate of Return (r)	Frequency (Prob)
$210	+5.0%	98% of the time
$100	–50.0%	1% of the time
$0	–100.0%	1% of the time

Therefore, your expected payoff is

$$E(C_1) = 98\% \cdot \$210 + 1\% \cdot \$100 + 1\% \cdot \$0 = \$206.80$$
$$= \text{Prob} \cdot \text{Cash Flow} + \text{Prob} \cdot \text{Cash Flow} + \text{Prob} \cdot \text{Cash Flow}$$

Your expected return of $206.80 is less than the $210 that the government promises. Put differently, if I *promise* you a rate of return of 5%,

$$\text{Promised}(r) = \frac{\$210 - \$200}{\$200} = 5.00\%$$
$$\text{Promised}(r) = \frac{\text{Promised}(C_1) - C_0}{C_0}$$

then your expected rate of return would be only

$$E(r) = \frac{\$206.80 - \$200}{\$200} = 3.40\%$$
$$E(r) = \frac{E(C_1) - C_0}{C_0}$$

This is less than the 5% interest rate that Uncle Sam promises—and surely delivers.

You need to determine how much I have to promise you just to break even. You want to expect to end up with the same \$210 that you could receive from Uncle Sam. The expected loan payoff is the probability-weighted average payoff. You want this payoff to be not \$206.80 but the \$210 that you can earn if you invest your \$200 into government bonds. You need to solve for an amount x that you receive if I have money,

Let's determine how much more interest promise you need to break even.

$$\mathsf{E}(C_1) = 98\% \cdot x + 1\% \cdot \$100 + 1\% \cdot \$0 = \$210.00$$

The solution is that if I promise you $x \approx \$213.27$, you will expect to earn the same 5% interest rate that you can earn in Treasury notes. This \$213.27 for a cash investment of \$200 is a *promised* interest rate of

$$\mathsf{Promised}(r) \approx \frac{\$213.27 - \$200}{\$200} \approx 6.63\%$$

$$\mathsf{Promised}(r) = \frac{\mathsf{Promised}(C_1) - C_0}{C_0}$$

Such a promise provides the following:

Payoff (C_1)	Rate of Return (r)	Frequency (Prob)
\$213.27	+6.63%	98% of the time
\$100.00	–50.00%	1% of the time
\$0.00	–100.00%	1% of the time

This comes to an *expected* interest rate of

$$\mathsf{E}(r) \approx 98\% \cdot (+6.63\%) + 1\% \cdot (-50\%) + 1\% \cdot (-100\%) \approx 5\%$$

Q 6.9. Recompute the example from the text, but assume now that the probability of receiving full payment in one year on a \$200 investment of \$210 is only 95%, the probability of receiving \$100 is 1%, and the probability of receiving absolutely no payment is 4%.

1. At the promised interest rate of 5%, what is the expected interest rate?
2. What interest rate is required as a promise to ensure an expected interest rate of 5%?

Deconstructing Quoted Rates of Return—Time and Default Premiums

The difference of 1.63% between the promised (or quoted) interest rate of 6.63% and the expected interest rate of 5% is the default premium—it is the extra interest rate that is caused by the default risk. Of course, you only receive this 6.63% *if* everything goes perfectly. In our perfect market with risk-neutral investors,

The difference between the promised and expected interest rate in a risk-neutral perfect world is the default premium.

$$\begin{array}{ccccc} 6.63\% & = & 5\% & + & 1.63\% \\ \text{"Promised Interest Rate"} & = & \text{"Time Premium"} & + & \text{"Default Premium"} \end{array}$$

IMPORTANT

Except for 100%-safe bonds, the promised (or quoted) rate of return is higher than the expected rate of return. Never confuse the promised rate with the (lower) expected rate. If you only remember one thing from this book, this should be it!

Financial securities and information providers rarely, if ever, provide information about expected rates of return. They almost always provide only quoted rates of return.

In a perfect risk-neutral world, all securities have the same expected rate of return.

On average, the expected rate of return is the expected time premium plus the expected default premium. Because the *expected* default premium is zero *on average*,

$$\begin{aligned} E(\text{Rate of Return}) &= E(\text{Time Premium}) + 0 \\ &= E(\text{Time Premium}) + E(\text{Realized Default Premium}) \end{aligned}$$

If you want to work this out, you can compute the expected realized default premium as follows: You will receive $6.63\% - 5\% = 1.63\%$ in 98% of all cases; $-50\% - 5\% = -55\%$ in 1% of all cases (note that you lose the time premium); and $-100\% - 5\% = -105\%$ in the remaining 1% of all cases (i.e., you lose not only all your money, but also the time premium). Therefore,

$$E(\text{Realized Default Premium}) \approx 98\% \cdot (+1.63\%) + 1\% \cdot (-55\%) + 1\% \cdot (-105\%) \approx 0\%$$

Warning: Additional premiums will follow later.

In addition to the 5% time premium and the 1.63% default premium, in the real world, there are also other premiums that we have not yet covered:

Risk premiums that compensate you with (even) *higher* expected rates of return for your willingness to take on risk. They will be the subject of Chapter 10.

Imperfect market premiums (e.g., liquidity premiums) that compensate you for future difficulties in finding buyers for your bonds. They will be the subject of Chapter 11.

In normal times, these premiums are typically much lower than time premiums and default premiums in a bond context.

Q 6.10. Is the expected default premium positive?

Credit Ratings and Default Rates

Bond rating agencies: The most important corporate credit ratings are from Moody's and Standard & Poor's.

To make it easier for lenders to judge the probability of default, a number of data vendors for credit ratings have appeared. For individuals, Experian, Transunion, and Equifax provide credit ratings—you should request a free credit report for yourself from the Federal Trade Commission if you have never seen one. For small companies, Dun & Bradstreet provides similar credit scores. For corporations, the two biggest credit rating agencies are **Moody's** and **Standard&Poor's** (**S&P**). (There are also other less influential ones, like *Duff and Phelps* and *Fitch*.) For a fee, these agencies rate the probability that the issuer's bonds will default. This fee depends on a number of factors, such as the identity of the issuer, the desired detail in the agencies' investigations and descriptions, and the features of the bond (e.g., a bond that will pay off within one year is usually less likely to default before maturity than a bond that will pay off in thirty years; thus, the former is easier to grade).

Investment Grade

	Best									Barely
Moody's	Aaa	Aa1	Aa2	Aa3	A1	A2	A3	Baa1	Baa2	Baa3
Standard & Poor's	AAA	AA+	AA	AA–	A+	A	A–	BBB+	BBB	BBB–

Non-Investment Grade (Speculative or "Junk")

	Speculative							in Default
Moody's	Ba1	Ba2	Ba3	B1	B2	B3	Caa1, Caa2, Caa3, Ca , C	D
Standard & Poor's	BB+	BB	BB–	B+	B	B–	CCC	D

Exhibit 6.2: *Bond Rating Categories Used by Moody's and Standard & Poor's.*

The credit rating agencies ultimately do not provide a whole set of default probabilities (e.g., 1% chance of 100% loss, 1.2% chance of 99% loss, etc.), but just an overall rating grade. Exhibit 6.2 shows the categories for Moody's and Standard & Poor's. It is then up to the lender to translate the rating into an appropriate compensation for default risk. The top rating grades are called **investment grade**, while the bottom grades are called **speculative grade** (or **junk grade**).

The most important grade distinction is "junk" versus "investment grade."

Ratings have limited usefulness:

Conflicted Ratings (and the Great Recession).

1. They do not consider common risk, wherein many bonds would default at the same time. This will be an important concept in the next few chapters. See, most bond buyers should care more about the (small) risk of all their bonds blowing up at the same time and care less about one small individual bond in their many-bond portfolios defaulting. But common risk assessments are *not* what rating agencies provide.
2. Unlike most other financial market experts, rating agencies are not liable for their ratings or perspectives even if they deliberately deceive investors. (The 2010 Dodd-Frank Act repealed this exemption, but the SEC has granted indefinite "no-action" relief for most ratings.)
3. The strangest aspect, however, is how the rating agencies are paid. They collect fees for rating securities by the investment banks—how critical would you be of their bond products in this case? Not surprisingly, although they need to maintain some independence and reputation, the agencies have also often been good game when it comes to being manipulated—some would even call it bribed. A good part of the **Great Recession** (the financial crisis of 2009), falls squarely on the shoulders of the rating agencies, which earned billions providing optimistic ratings for issues explicitly engineered by investment banks to have high ratings. And although they have taken some steps to improve the situation, the basic conflicts of interest are still there. When public attention will have moved on to another "issue of the day," chances are that the ratings will return back to business as usual.
4. Ratings change over time: after their issue, bonds move up or down with probabilities of about 3-10% each per year.

Nevertheless, despite all their flaws, they are a useful source of information for potential bond buyers.

Empirical Evidence on Default

Here are historical probabilities of bond defaults by credit ratings.

Ed Altman and his coauthors (from New York University) collected corporate bond statistics from 1971 to 2015. Exhibit 6.3 gives you a sketch of how likely default was. (It is standard to define default as missing at least one coupon payment. It is *not* complete ultimate non-payment.) The average default rate was about 3.5% per year—but the left plot shows that it was much higher in recessions, where defaults typically shot up to over 10%. For example, in the Great Recession, about 11% of bonds failed to pay. (In 2008, about half of all defaults were Lehman Brothers' bankruptcy.) By 2010, the worst seemed to have passed. In retrospect, the Great Recession financial crisis ended up "not so bad" for most public corporations. (Lucky!) The right plot shows that corporate bonds originally rated A or better rarely defaulted, even 10 years after issue. However, about half of all CCC junk bonds would fail to pay at least one coupon within the first five years of issue.

➤ Senior and Junior Bonds, Chapter 16, Pg.425.

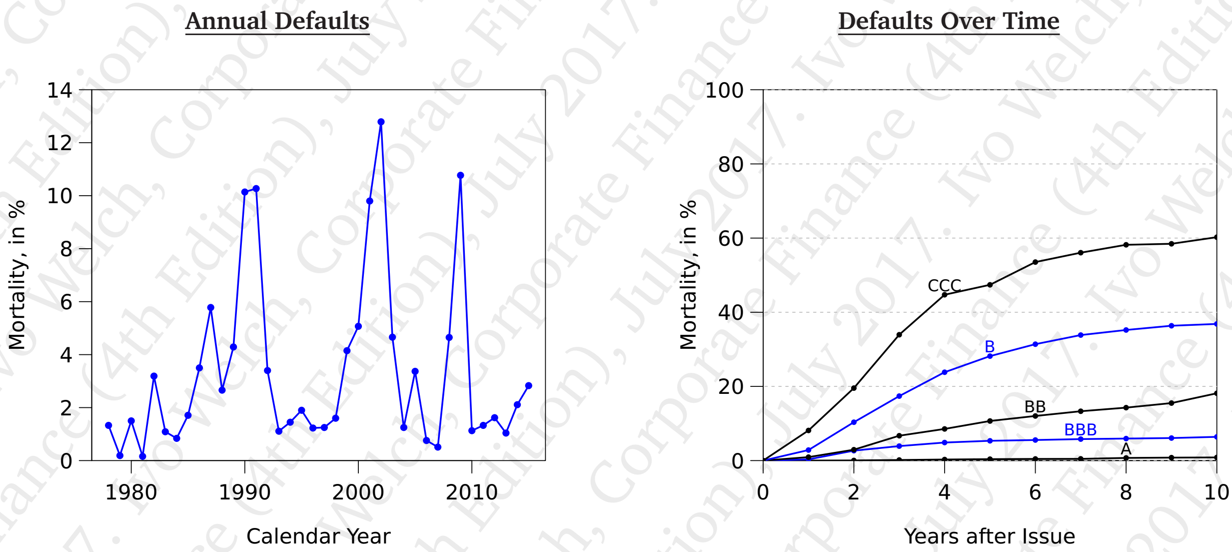


Exhibit 6.3: *Cumulative Historical Frequency of Default by Original Bond Rating, 1971–2015.* The left plot shows the rate at which bonds defaulted. For example, in 2009, about 11% of all corporate bonds failed to make at least one payment. The right plot shows the frequency of default within x years after issue, given the bond rating *at-issue* (not updated). For example, at some point during the first 7 years of their issue, about 1-in-3 bonds originally issued as B (poor) had not delivered on at least one promised bond payment. Corporate bonds originally rated A and better essentially did not default over their first 10 years. Source: Edward Altman and Brenda Kuehne, New York University, June 2016.

Moody's monthly *Default Report* lists recovery rates after bonds default. In 2005, they reported that from 1982 on, recovery rates were about 60% for senior secured bonds, 45% for senior unsecured bonds, and 30% for junior bonds. The typical recovery in a default was about 30-40 cents on the dollar, with 25 cents in recessions and 50 cents in booms. Low-rated bonds would pay less. These numbers seem to have remained similar in the 2010-2016 time period, too, but there is a lot of idiosyncratic variation across individual bonds.

Bond Contract Option Features

Before I show you how bonds are priced, I need to let you know that bonds in the real world differ from one another not just in credit risk. Most bonds have additional contract features that may also influence their quoted rates of return. For example, many corporate bonds allow the issuer to repay the loan early. (The same applies to almost all domestic mortgages.) If the interest rates in the future fall, this can be a good thing for the borrower and a bad thing for the lender. The borrower would pay off the loan and borrow more cheaply elsewhere. If the interest rates in the future rise, the borrower gets to pay just the earlier low interest rate. For example, assume that the interest rate is 10% today and you are lending me $90,909 in exchange for my promise to pay you $100,000 next year. One second after you extend the loan, one of two scenarios can happen:

Before I show you real-world quoted returns, I must explain that they can contain contract premiums.

1. The interest may fall to 5%. I would then simply repay your $90,909 loan and refinance at this lower interest rate elsewhere.
2. The interest rate may rise to 15%. In this case, I keep my $100,000 promise to pay next year—I received $90,909 for a loan that should have given me only $100,000/1.15 ≈ $86,957.

This would not be a good arrangement for you—unless you are appropriately compensated for giving me this option to prepay. Borrowers who want the right to repay without penalty therefore have to pay higher interest rates when they issue such bonds. Virtually all mortgage bonds in the United States allow prepayment and therefore carry higher interest rates than they would if they did not have a prepayment feature. Loosely speaking, you can classify these contract option features as default premiums, too, because on average they tend not to add or subtract from your expected rate of return. Sometimes they increase the amount paid, and sometimes they decrease the amount paid by the lender—just as a solvent bond would pay more to the lender and an insolvent bond would pay less to the lender.

Q 6.11. Does the historical evidence show that lower-grade borrowers default more often or that they pay less upon default?

Differences in Quoted Bond Returns in June 2016

So how do real-world credit risk and reflecting bond credit ratings translate into differences in promised (quoted) bond yields? Exhibit 6.4 lists the borrowing rates of various issuers on June 10, 2016 from Yahoo.

Historical rates of return: Riskier bonds indeed have higher stated rates of return.

The data looks broadly consistent with the theory—bonds that have higher default risk have to offer higher promised rates of return. Bonds with higher (better) credit ratings can find lenders at lower interest rates (higher bond prices).

Do lenders who extend loans to riskier creditors end up with higher average rates of return? This would be the case in a perfect market in which lenders and borrowers are risk-neutral. The evidence suggests that this is not exactly true, but it is also not as far from reality as naive readers would think. The majority of the investment-grade bond spread above the Treasury that you see in Exhibit 6.4 simply made borrowers and lenders come out about even. That is, the *expected* rates of return were much more similar to one another than the *promised* rates of return in the

Riskier bonds have to promise higher rates of return, but...

	Years			
Type	2	5	10	20
AAA	n/a	1.37	2.26	2.57
only Johnson&Johnson and Microsoft				
AA	0.78	1.52	2.43	3.58
e.g., Walmart, Apple, Intel				
U.S.	0.76	1.20	1.66	2.04

	Years			
Type	2	5	10	20
A	0.91	1.75	2.69	3.80
e.g., IBM, Morgan-Stanley, Target				
≈B		5% to 12%		
e.g., Tesla, Victoria's Secret.				
U.S.	0.76	1.20	1.66	2.04

Exhibit 6.4: *Promised Interest Rates and Treasury Bonds on June 9, 2016.* Source: Federal Reserve Economic Database, FRED, itself partially sourced from Moody's and Bank of America Merrill Lynch.

table suggest. If you put a gun to my head and asked me for an opinion, I would guess that about 80% of the promised spreads over Treasury are credit risk; about 10% are due to measurement or contract features (e.g., the timing of coupons or some option contingencies); and only about 10% are extra compensation that the creditors earn on average above and beyond what they would earn in equivalent U.S. Treasuries. (And, of course, all interest income is taxable.)

➤ De Jong-Driessen, Exhibit 11.1, Pg.266.

Historical Yields of Corporate Bonds

You already saw in Exhibit 6.3 that actual defaults by reasonably large corporations were cyclical and increased only briefly in the Great Recession. Similarly but even more extreme, the interest rates that lower-rated corporations paid (which are *promise*d rates that reflect expectations of non-payment over the full life of the bond) were cyclical and spiked in 2009. Exhibit 6.5 plots the historical yields of the 20-year Treasury bond, of Moody's-rated Aaa and Baa investment-grade bond portfolios, and non-investment grade (high-yield) bond portfolios. The typical investment-grade bond promised about 100-200 basis points above the Treasury, while the typical junk bond promised about 200-600 basis points above the Treasury. (Junk is relative—non-publicly traded corporations usually pay even higher rates on non-collateralized obligations.) Any non-investment grade corporation that had to borrow during the Great Recession was in trouble. Promised interest rates were more than 2,000 bp higher than Treasuries, as investors were fleeing to safety. While investment-grade bonds have pretty much returned to normal by 2016, non-investment grade issuers continue to have to promise relatively high spreads.

Dilbert on libor: 2012-10-04

Other Rates

We could discuss risk and reward for many other types of credit. Credit risks are not always similar. Mortgage investors in Arizona can face different interest rates than London banks. The latter is called the London Interbank Offered Rate (**Libor**). Incidentally, Libor plays an important role, because it is also a commonly used benchmark for about $350 trillion of derivatives. Even your mortgage interest rate may be tied to Libor.

The Libor Scandal of 2008

The dependence of many other financial contracts on the daily quoted Libor rate made its active manipulation by large banks extraordinarily profitable. This illegal manipulation seemed to have begun in about 1991 and lasted until a WSJ investigation discovered and wrote about it in 2008. As usual, the traders kept their bonuses, while the fairly innocent shareholders of the major banks had to pay many billions of dollars in penalties.

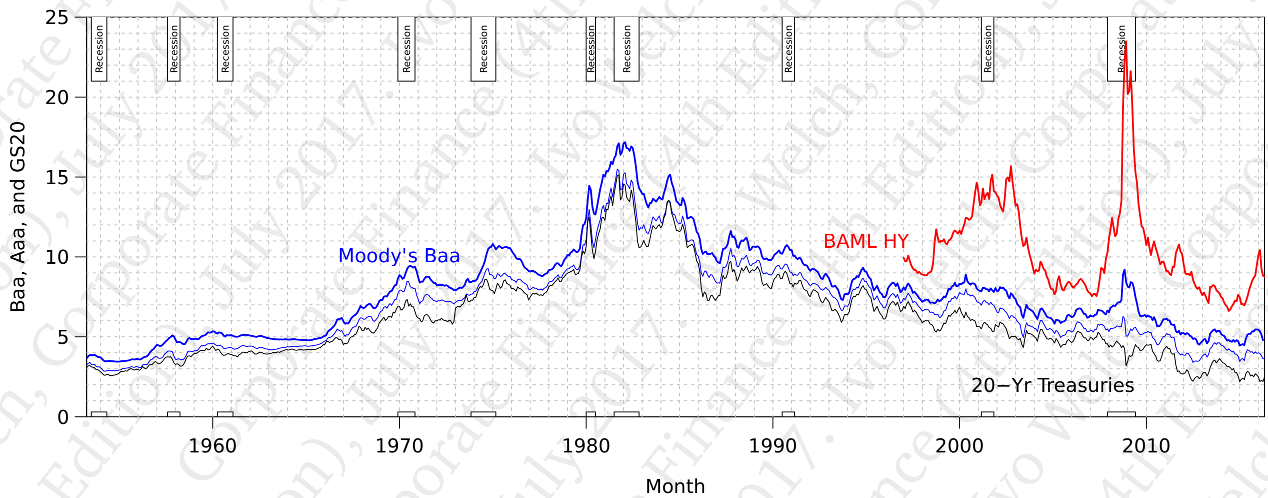


Exhibit 6.5: *Yields on 20-Year Treasuries (black) and Promised Yields on Corporate Bonds, 1955–2016.* The plot shows that the typical Baa corporate bond, in blue, spread over the 20-year Treasury was about 150 bp (ranging from about 40 to about 500 bp). Aaa bond yields were between those of the long-term Treasury and the Baas. In contrast, high-yield (BAML HY) corporate bonds (after option adjustments), in red, offered promised spreads of about 400-600 bp, higher in recessions, and a spike as large as 2,000 bp (!) in the Great Recession. Source: FRED (Moody's and Bank of America Merrill Lynch).

Credit Default Swaps

A large newish market: credit default swaps.

The financial world is always changing and innovating. The components of bond returns described above used to be primarily a conceptual curiosity—firms would borrow money from their lenders, paying one interest rate that just contained all premiums. But then, with the introduction of **credit default swaps** (often abbreviated **credit swaps** or **CDS**), some premium components suddenly became themselves tradable.

A CDS example: The swap seller insures the swap buyer.

Here is an example of a CDS: A large pension fund that owned a $15 million bond issued by *Hospital Corporation of America* (HCA) may have wanted to purchase a $10 million credit swap from a hedge fund that in turn wanted to bet that HCA would *not* go bankrupt. Upon HCA bankruptcy, the hedge fund would owe the pension fund $10 million. The *Wall Street Journal* reported that this CDS contract cost about $130,000 in June 2006, but rose to over $400,000 in July, because of a potential buyout deal that increased the risk of future default. And, in this case, purchasing the CDS in June would have been a lucky deal for the pension fund and an unlucky deal for the hedge fund, because HCA did indeed go bankrupt.

Conceptually...

The best way to think of such credit swaps is as insurance contracts, in which the swap sellers (the hedge funds) are the insurance providers. The seller of the swap thus takes on the credit risk from the buyer—just like an insurance company takes on some risk from the insured party—in exchange for a premium payment upfront. The insurance then usually pays out in case of a credit event (e.g., a failed payment or bankruptcy)—typically for one particular bond within a given number of years. The payment itself can be formula-determined, or it can be a guarantee by the CDS seller to buy the bond at a predetermined price. One way of thinking of the upfront cost (the $130,000 that increased to $400,000) is that it contains the bond's default premium.

In effect, credit swaps allow investors to hold different premium components.

Credit swaps allow different funds to hold different premiums of a bond. In our example, the pension fund decided to earn primarily the time premium component of HCA's bonds, divesting itself of the credit risk and other components. The hedge fund took over the credit premium. It decided to speculate that HCA would not go bankrupt, and it could do so without having to take a large cash position in HCA's bonds. Of course, hedge funds and other investors could also have speculated with CDS's that HCA would go bankrupt.

The CDS market size was huge.

➤ Over-the-counter, Sect. 7.2, Pg.157.

Credit swaps are typically traded in lots of $5 million and last for 5 years (but 3 to 10 years is not unusual, either). This market is **over-the-counter** (**OTC**)—that is, negotiated one-to-one between two parties. This market is also very big: in 2016, there was more than $17 trillion outstanding in single-name swaps.

The financial crisis and the CDS collapse. The shifting of risk everywhere.

The CDS market collapsed temporarily in the financial crisis of 2008. An important insurer, American International Group (AIG), had a financial arm that had sold too many CDSs and the traders booked them as outright profit. This worked for a while...until it did not. We, the taxpayers, had to bail out AIG, because the Treasury feared that too many bond buyers were relying on AIG's insurance, and would themselves have to default if this insurance had become worthless. Unfortunately, even after the market has risen again (after 2010), it still remains relatively "dark": no one really knows how big or small it is, who is trading, who has exposures, and so on—and this includes the Federal Reserve and the Treasury. In 2007, I wrote in the first edition of this book that no one knows who is really holding most of the credit risk in the economy nowadays. I gave as an example the German bank IKB, which had collapsed to everyone's surprise, because it had owned too many financial securities that were tied to U.S. mortgages. I was either prescient or just lucky. During the 2008 financial crisis (the Great Recession), investors did not want to trust even good corporations and banks any longer, simply because they did not know what their actual exposures were. The CDS market is large and competitive—but also opaque and rife with manipulation. Some corporations have even been pressured into defaulting for the sake of triggering CDS. And, in the believe-it-or-not category, there is actually a committee of (conflicted) major banks that can decide whether a bond is really in default or not when the creditor offers an exchange. However, CDSs are not intrinsically evil. Like most other financial instruments, they can be used to reduce or increase risk. The social problem, even today, is that (1) traders have incentives to speculate too much with them, because their risk is hard to measure, and banks and their shareholders intrinsically like risk (but in cases of a crisis, it is taxpayers that will be on the hook again!); and (2) no one really knows what is going on in this OTC market (and many other financial markets). If large institutions speculate too much with them (and pay non-recoverable bonuses based on estimated profitability) and then fail, we the people may have no choice but to bail them out *again*. Despite these problems, the CDS market has come back strongly. As of 2016, there is "only" about $500-$600 trillion in notional amounts outstanding, with $15-$20 trillion in gross market value. By any measure, these numbers are *huge*.

6.3 Uncertainty in Capital Budgeting

Next you learn about payoff diagrams, to characterize the main future contingencies.

Let's now return to the basic tasks of capital budgeting: selecting projects under uncertainty. Your task is to compute present values with imperfect knowledge about future outcomes. The principal tool in this task will be the **payoff table** (or **state table**), which assigns probabilities to the project value in each possible future-value-relevant scenario. For example, the value of a factory producing hard disks may depend on computer sales (say, low, medium, or high), whether hard disks have become obsolete (yes or no), whether the economy is in a recession or expansion, and what the oil price (a major transportation cost factor) turns out to be. It is the manager's task to create the appropriate "state" table, which specifies what variables and scenarios are most value-relevant and how the business will perform in each of them. Clearly, it is not an easy task even to understand what the key factors are, much less to determine the probabilities under which these factors will take on one or another value. Assessing how your own project will respond to them is an even harder task—but it is an inevitable one. If you want to understand the value of your project, you must understand what your project's key value drivers are and how your project will respond to these value drivers. Fortunately, for many projects, it is usually not necessary to describe all possible outcomes in the most minute detail—just a dozen or so scenarios are often enough to cover the most important possibilities.

Present Value with Outcome-Contingent Payoff Tables

Our example of this section: A building can end up with one of two possible future values.

We begin with the hypothetical purchase of a building for which the future value is uncertain. Next year, this investment will be worth either \$60 thousand (with 1/4 probability) or \$100 thousand (with 3/4 probability). (In case you are worried that real firms last longer than one year, you can think of these values as themselves reflecting further future outcomes for the firm.) To help you remember the two possible states, let's just call the bad outcome "Rain" and the good outcome "Sun." (If you are from California, be aware that rain is the bad outcome and sun is the good outcome.)

The Building's Expected Value

A payoff table.

If you own the full building, your payoff table, omitting thousands henceforth, is as follows:

Event	Probability	Value
Rain	1/4	\$60
Sun	3/4	\$100
⟹ **Expected Future Value**		\$90

To obtain the expected future cash value of the building, multiply each possible outcome by its probability.

The expected future building value of \$90 (thousand) was computed as

$$\mathsf{E}\big(\text{Value at Time 1}\big) = 1/4 \cdot \$60 + 3/4 \cdot \$100 = \$90$$
$$= \text{Prob} \cdot \text{Value Rain} + \text{Prob} \cdot \text{Value Sun}$$

This is *not* yet discounted. It is only your expectation of the future outcome.

Then discount back the expected cash value using the appropriate cost of capital.

Now assume that the appropriate expected rate of return for a project of type "building" with this type of riskiness and with 1-year maturity is 20%. (This 20% discount rate is provided by demand and supply in the financial markets, and it is assumed to be known by you, the manager.) Your goal is to determine the present value—the appropriate price—for the building *today*.

You can use NPV with expected (rather than actual, known) cash flows and expected (rather than actual, known) rates of return.

There are two methods to arrive at the present value of the building—and they are almost identical to what you have done earlier. You only need to replace the known value with the expected value, and the known future rate of return with an expected rate of return. The first PV method is to compute the expected value of the building next period and to discount it at the cost of capital, here 20%:

$$\text{PV} = \frac{\$90}{1+20\%} \approx \$75$$
$$= \frac{E(\text{Value at Time } 1)}{1+E(r)}$$

Taking expectations and discounting can be done in any order.

The second method is to compute the discounted state-contingent value of the building, and then take expected values. To do this, augment the earlier table:

Event	Probability	Value	Discount Factor	⇒	PV
Rain	1/4	\$60	1/(1+20%)	⇒	\$50
Sun	3/4	\$100	1/(1+20%)	⇒	\$83.33

If it rains, the present value is \$50. *If* the sun shines, the present value is \$83.33. Thus, the expected value of the building can also be computed as

$$E(\text{Value at Time } 1) = \underbrace{1/4}_{\text{Prob}} \cdot \underbrace{\$50}_{\text{Value if Rain}} + \underbrace{3/4}_{\text{Prob}} \cdot \underbrace{\$83.33}_{\text{Value if Sun}} = \$75$$

Both methods lead to the same result: You can either first compute the expected value of the investment next year ($1/4 \cdot \$60 + 3/4 \cdot \$100 = \$90$) and then discount this expected value of \$90 to \$75; or you can first discount all possible future outcomes (\$60 to \$50, and \$100 to \$83.33) and then compute the expected value of the discounted values ($1/4 \cdot \$50 + 3/4 \cdot \$83.33 = \$75$.)

IMPORTANT

Under uncertainty, in the NPV formula,

- known future cash flows are replaced by expected discounted cash flows, and
- known appropriate rates of return are replaced by appropriate expected rates of return.

You can first do the discounting and then take expectations, or vice-versa. The order does not matter.

The State-Contingent Rates of Return

The state-contingent rates of return can also be probability-weighted to arrive at the average (expected) rate of return.

What would the rates of return be in the two states, and what would your overall expected rate of return be? If you have bought the building for \$75 and it will be sunny, your actual rate of return will be

$$\text{If Sun:} \quad r \approx \frac{\$100 - \$75}{\$75} \approx +33\%$$

If it's rainy, your rate of return will be

$$\text{If Rain:} \quad r \approx \frac{\$60 - \$75}{\$75} \approx -20\%$$

Therefore, your expected rate of return is

$$E(r) \approx \underbrace{1/4}_{\text{Prob}} \cdot \underbrace{(-20\%)}_{\text{Rain Rate of Return}} + \underbrace{3/4}_{\text{Prob}} \cdot \underbrace{(+33\%)}_{\text{Sun Rate of Return}} \approx 20\%$$

The probability state-weighted rates of return add up to the expected overall rate of return. This is as it should be: After all, you derived the proper price of the building today using a 20% expected rate of return.

Q 6.12. What changes have to be made to the NPV formula to handle an uncertain future?

Q 6.13. A factory can be worth $500,000 or $1,000,000 in two years, depending on product demand, each with equal probability. The appropriate cost of capital is 6% per year. What is the present value of the factory?

Q 6.14. A new product may be a dud (20% probability), an average seller (70% probability), or dynamite (10% probability). If it is a dud, the payoff will be $20,000; if it is an average seller, the payoff will be $40,000; and if it is dynamite, the payoff will be $80,000.

1. What is the expected payoff of the project?
2. The appropriate expected rate of return for such payoffs is 8%. What is the PV of the payoff?
3. If the project is bought for the appropriate present value, what will be the rates of return in each of the three outcomes?
4. Confirm the expected rate of return when computed from the individual outcome-specific rates of return.

6.4 Splitting Uncertain Project Payoffs into Debt and Equity

Most projects are financed with a mix of debt and equity.

The most important reason for you to learn about state payoff tables is that they will help you understand cash flow rights. This leads to one of the most important concepts in finance: the difference between a **loan** (also called **debt** or **leverage**) and **levered ownership** (also called **levered equity** or simply **equity** or **stock**). Almost all companies and projects are financed with both debt and levered equity. You already know in principle what debt is. Levered equity is simply what accrues to the business owner *after* the debt is paid off. We leave it to later chapters to make a distinction between financial debt and other obligations—for example, tax obligations—and to cover the control rights that flow from securities—for example, how debt can force borrowers to pay up and how equity can replace poorly performing managers.

Other projects are financed the same way.

You probably already have an intuitive understanding about the distinction between debt and equity. If you own a house with a mortgage, you really own the house only after you have made all debt payments. If you have student loans, you *yourself* are the levered owner of your future income stream. That is, you get to consume “your” residual income only *after* your liabilities (including your nonfinancial debt) are paid back. But what will the levered owner and the lender get if the company’s projects fail, if the house collapses, or if your career takes a turn toward Rikers Island? What is the appropriate compensation for the lender and the levered owner? The split of net present value streams into loans (debt) and levered equity lies at the heart of finance.

Outcome (or “state”)-contingent claims have payoffs that depend on future states of nature.

You now know how to compute the present value of state-contingent payoffs—your building paid off differently in two different states of nature. Thus, your building was a state-contingent claim—its payoff depended on the outcome. But it is just one of many possible state-contingent claims. Another might promise to pay $1 if the sun shines and $25 if rain falls. Using payoff tables, you can work out the value of *any* state-contingent claim and, in particular, the value of our two most important state-contingent claims, debt and equity.

The Loan

Assume that the building is funded by (a) a mortgagor and (b) a residual (the levered building owner).

Let's assume you want to finance the building purchase of \$75 with a mortgage of \$70. In effect, the single project "building" is being turned into two different projects, each of which can be owned by a different party. The first project is "Mortgage Lending." The second project is "Residual Building Ownership," that is, ownership of the building but bundled with the obligation to repay the mortgage. The "Residual Building Ownership" investor will not receive a dime until *after* the debt has been satisfied. As already explained, such residual ownership is called levered equity, or just equity (or even stock) in the building. This avoids calling it "what's-left-over-after-the-loans-have-been-paid-off."

The first goal is to determine the appropriate promised interest rate on a "\$70 value today" mortgage loan on the building.

What sort of interest rate would the creditor demand? To answer this question, you need to know what will happen if the building were to be worth less than than the mortgage promise. Let's say that the value of the building will be \$60 next year if rain falls. (The roof is partly water-soluble.) We are assuming that the owner could walk away, and the creditor could repossess the building but not any of the borrower's other assets. Such a mortgage loan is called a **no-recourse loan**. There is no recourse other than taking possession of the asset itself. This arrangement is called **limited liability**. The building owner cannot lose more than the money that he originally puts in. Limited liability is the mainstay of many financial securities: For example, if you buy stock in a company in the stock market, you cannot be held liable for more than your investment, regardless of how badly the company performs.

Start with the payoff table, and write down payoffs to project "Mortgage Lending."

➤ Credit Risk, Sect. 6.2, Pg.110.

To compute the present value for the project "Mortgage Lending," return to the problem of setting an appropriate interest rate, given credit risk (from Section 6.2). Start with the following payoff table:

Event	Prob	Value	Discount Factor
Rain	1/4	\$60	1/1.20
Sun	3/4	Promised	1/1.20

Limited Liability

Limited liability was invented after the Renaissance, but it became common only in the nineteenth and twentieth centuries. Ultimately, it is this legal construction that allowed corporations to evolve into entities distinct from their owners. Thus, in 1911, the President of Columbia University wrote: "The limited liability corporation is the greatest single discovery of modern times.... Even steam and electricity are less important."

William Goetzmann, Yale University

The quoted (or promised) payoff.

The creditor receives the property worth \$60 if it rains, or the full promised amount (to be determined) if the sun shines. To break even, the creditor must solve for the payoff to be received if the sun shines in exchange for lending \$70 today. This is the "quoted" or "promised" payoff:

$$\$70 = 1/4 \cdot \left(\frac{\$60}{1+20\%}\right) + 3/4 \cdot \left(\frac{\text{Promise}}{1+20\%}\right)$$

$$\text{Loan Value}_0 = \text{Prob} \cdot \text{Rain Loan PV} + \text{Prob} \cdot \text{Sun Loan PV}$$

You can solve this for the necessary promise, which is

Nerdnote: Special liability and tax rules apply to private residences. Mortgages can have limited liability ("non recourse") or unlimited liability ("full recourse"). The latter can also have further nasty tax consequences, where a capital loss in the home can create a large ordinary income tax obligation, adding insult to injury. (If interested, google for "cancellation-of-debt income.") Moreover, as a home owner, you can deduct interest only on the first \$1 million in mortgage; and capital losses on the home do not create a tax credit, but large capital gains can create a tax obligation.

$$\text{Promise} = \frac{(1+20\%)\cdot \$70 - 1/4 \cdot \$60}{3/4} = \$92$$

$$= \frac{[1+\text{E}(r)]\cdot \text{Loan Value}_0 - \text{Prob}(\text{Rain}) \cdot \text{Rain Value}}{\text{Prob}(\text{Sun})}$$

in repayment, paid by the borrower only if the sun shines.

With this promised payoff of $92 (if the sun shines), the lender's rate of return will be the **promised rate of return**:

The state-contingent rates of return in the rainy ("default") state and in the sunny ("solvent") state can be probability-weighted to arrive at the expected rate of return.

$$\text{If Sun:} \quad r = \frac{\$92 - \$70}{\$70} \approx +31.4\%$$

The lender would not provide the mortgage at any lower promised interest rate. If it rains, the owner walks away, and the lender's rate of return will be

$$\text{If Rain:} \quad r = \frac{\$60 - \$70}{\$70} \approx -14.3\%$$

Therefore, the lender's *expected* rate of return is

$$\text{E}(r) = \underbrace{1/4}_{\text{Prob}} \cdot \underbrace{(-14.3\%)}_{\text{Rain Rate of Return}} + \underbrace{3/4}_{\text{Prob}} \cdot \underbrace{(+31.4\%)}_{\text{Sun Rate of Return}} \approx 20\%$$

The stated rate of return is 31.4% (and it is not an exorbitant rate!), but the expected rate of return is 20%. After all, in our risk-neutral perfect market, anyone investing for one year expects to earn an expected rate of return of 20%.

The Levered Equity

As the residual building owner, what rate of return would you expect as proper compensation? You already know the building is worth $75 today. Thus, after the loan of $70, you need to pay in $5—presumably from your personal savings. Of course, you must compensate your lender: To contribute the $70 to the building purchase today, you must promise to pay the lender $92 next year. If it rains, the lender will confiscate your house, and all your invested personal savings will be lost. However, if the sun shines, the building will be worth $100 minus the promised $92, or $8. Your payoff table as the levered equity building owner is as follows:

Now compute the payoffs of the post-mortgage (i.e., levered) ownership of the building. The method is exactly the same.

Event	Prob	Value	Discount Factor
Rain	1/4	$0	1/1.20
Sun	3/4	$8	1/1.20

This allows you to determine that the *expected* future levered building ownership payoff is $1/4 \cdot \$0 + 3/4 \cdot \$8 = \$6$. Therefore, the present value of levered building ownership is

$$\text{PV} = \underbrace{1/4}_{\text{Prob}} \cdot \underbrace{\left(\frac{\$0}{1+20\%}\right)}_{\text{Rain PV}} + \underbrace{3/4}_{\text{Prob}} \cdot \underbrace{\left(\frac{\$8}{1+20\%}\right)}_{\text{Sun PV}} \approx \$5$$

Your rates of return are

Again, knowing the state-contingent cash flows permits computation of state-contingent rates of return and the expected rate of return.

$$\text{If Sun:} \quad r \approx \frac{\$8 - \$5}{\$5} = +60\%$$

$$\text{If Rain:} \quad r \approx \frac{\$0 - \$5}{\$5} = -100.00\%$$

The expected rate of return of levered equity ownership, that is, the building with the bundled mortgage obligation, is

$$E(r) = \underbrace{1/4}_{\text{Prob}} \cdot \underbrace{(-100.00\%)}_{\text{Rain Rate of Return}} + \underbrace{3/4}_{\text{Prob}} \cdot \underbrace{(+60\%)}_{\text{Sun Rate of Return}} = 20\%$$

Reflections On The Example: Payoff Tables

Payoff tables are great conceptual tools.

Payoff tables are fundamental tools to help you think about projects and financial claims. Admittedly, they can sometimes be tedious, especially if there are many different possible states. (There may even be infinitely many states, as in a bell-shaped, normally-distributed project outcome—but you can usually approximate even the most continuous and complex outcomes fairly well with no more than 10 discrete possible outcomes.)

There are three possible investment opportunities here. The bank is just another investor, with particular payoff patterns.

Exhibit 6.6 shows how elegant such a table can be. It describes everything you need in a very concise manner: the state-contingent payoffs, expected payoffs, net present value, and expected rates of return for your house scenario. Because owning the mortgage and the levered equity is the same as owning the full building, the last two columns must add up to the values in the "Building Value" column. You could decide to be any kind of investor: a creditor (bank) who is loaning money in exchange for promised payment; a levered building owner who is taking a "piece left over after a loan"; or an unlevered building owner who is investing money into an unlevered project (i.e., the whole piece). All three investments are just state-contingent claims.

Event	Prob	Building Value	$92-Promise Mortgage	Levered Equity
Rain	1/4	$60	$60	$0
Sun	3/4	$100	$92	$8
Expected Value at Time 1		$90	$84	$6
Present Value at Time 0		$75	$70	$5
From Time 0 to Time 1, $E(r)$		20%	20%	20%

Exhibit 6.6: *Payoff Table and Overall Values and Returns.* In this example, the project is financed with $70 in mortgage promising $92 in payment.

IMPORTANT

Whenever possible, in the presence of uncertainty, write down a payoff table to describe the probabilities of each possible event ("state") with its state-contingent payoff.

Q 6.15. In the example, the building was worth $75, the mortgage was worth $70, and the equity was worth $5. The mortgage thus financed about 93.3% of the cost of the building, and the equity financed 6.7%. Is the arrangement identical to one in which two partners purchase the building together—one puts in $70 and owns 93.3% of the building, and the other puts in $5 and owns 6.7%?

Q 6.16. Buildings are frequently financed with mortgages that cover 80% of the purchase price, not 93.3% ($70 of $75). Produce a table similar to Exhibit 6.6 for this case.

Reflections on The Example: Debt and Equity Risk

We have only briefly mentioned risk. It was just not necessary to illustrate the main insights. In a risk-neutral world, all that matters is the expected rate of return, not the uncertainty about what you will receive. Of course, you can assess the risk even in our risk-neutral world where risk earns no extra compensation (a risk premium). So, which investment is riskiest: full ownership, loan ownership, or levered ownership?

Evaluate the risk of the three types of projects, even if riskier projects do not earn higher expected rates of return.

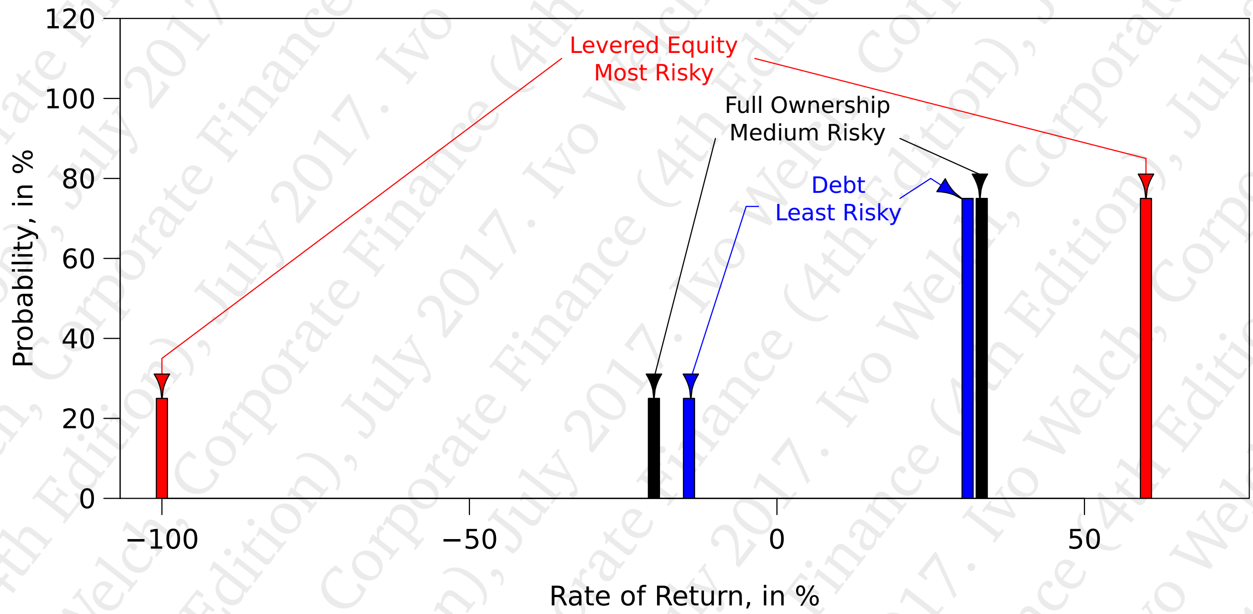


Exhibit 6.7: *Three Probability Histograms for Project Rates of Return.* The solid red bars are the payoffs to equity, the riskiest investment. The solid black bars are the payoffs to full ownership. The blue bars are the payoffs to debt, the least risky investment. You can judge risk by how spread out the two bars are.

Exhibit 6.7 plots the histograms of the rates of return for each of the three types of investments. The equity loses everything (–100%) with a 1/4 probability but earns 60% with 3/4 probability. The debt loses about 14.3% with 1/4 probability and gains 31.4% with 3/4 probability. The full ownership loses about –20% with 1/4 and gains 33.3% with 3/4 probabilities. As the visuals show, the loan is least risky, followed by the full ownership, followed by the levered ownership. There is an interesting intuition here. By taking the mortgage, the medium-risk project "building" has been split into one more risky project ("levered building") and one less risky project ("mortgage"). The combined "full building ownership" project therefore has an average risk.

Leveraging (mortgaging) a project splits it into a safer loan and a riskier levered ownership.

Of course, regardless of leverage, all investment projects in our risk-neutral world expect to earn a 20% rate of return. After all, 20% is the universal time premium here for investing money. (The default premium is a component only of promised interest rates, not of expected interest rates; see Section 6.2.) By assuming that investors are risk-neutral, we have assumed that the risk premium is zero. Investors are willing to take any investment that offers an expected rate of return of 20%, regardless of risk. (If investors were risk-averse, debt would offer a lower expected rate of return than the project, which would offer a lower expected rate of return than equity.)

If everyone is risk-neutral, everyone should expect to earn 20%.

Unrealistic, maybe! But ultimately, this is the basis for more realistic examples, and illustrative of the most important concepts.

Although our example was a little sterile because we assumed away risk preferences, it is nevertheless very useful. Almost all projects in the real world are financed with loans extended by one party and levered ownership held by another party. Understanding debt and equity is as important to corporations as it is to building owners. After all, stocks in corporations are basically levered ownership claims that provide money only *after* the corporation has paid back its liabilities. The building example has given you the skills to compute state-contingent, promised, and expected payoffs, as well as state-contingent, promised, and expected rates of return. These are the necessary tools to work with debt, equity, or any other state-contingent claim. And really, all that will happen later when we introduce risk aversion is that you will add a few extra basis points of required compensation—more to equity (the riskiest claim), fewer to the project (the medium-risk claim), and still fewer to debt (the safest claim).

Q 6.17. Compare a "junk" mortgage (with its requisite junk equity, receiving payments only if the junk mortgage is paid off) that promises to pay off $70 with a "solid" mortgage (with its requisite solid equity) that promises to pay off $60.

1. Does the junk mortgage seem riskier than the solid mortgage?
2. Does the junk equity seem riskier than the solid equity?
3. Does the building seem riskier if financed with a junk mortgage rather than with a solid mortgage?

What "Leverage" Really Means—Financial and Operational Leverage

Leverage "amplifies" the equity stake.

I have already mentioned that debt is often called leverage and equity is called "levered equity." Let me now explain why. A lever is a mechanical device that can amplify effects. In finance, a lever is something that allows a smaller equity investment to still control the firm and be more exposed to the underlying firm's gain or loss than unlevered ownership. That is, with leverage, a small change in the underlying project value translates into a larger change in value for levered equity, both up and down. You have seen this leverage mechanism in our house example above, and specifically in Exhibit 6.7. Ordinary ownership would have cost you $75. But with leverage, you could take control of the house with cash of only $5. In addition, it also meant that if the sun had shone, you would have earned ($8 – $5)/$5 = 60%, not just ($100 – $75)/$75 = 33%; but if it had rained, you would have earned –100% (lost *everything*), not just ($60 – $75)/$75 = –20%. Leverage amplified your stake.

The leverage concept can encompass more than just financial debt.

➤ Calculating leverage, Sect. 15.6, Pg.412.

Financial debt is a lever, but it is not the only one. Leverage can also be calculated using all corporate liabilities (which may include, e.g., accounts payable and pension obligations). More importantly, because leverage is a general concept rather than an accounting term, you should think of it in even broader terms. The idea of leverage is always that a smaller equity investment can control the firm and is more sensitive to firm value changes. Exhibit 6.8 illustrates some different types of levers. In this table, you can pay $475 for machine and labor, and receive either $200 or $1,000 in product revenues, plus $150 as resale value for the machine. In the first line of the exhibit, you can see that the bad state, you lose 26%; and in the good state, you earn 142%. The second line shows that financial leverage can magnify these rates of return into –100% or +540%. But instead of taking on financial debt, you could also lease the machine, which costs you $250 in leasing fees (with no residual ownership of the machine at the end), and pay for labor of $75. In this case, you have effectively levered up, increasing your risk to –38% and +208% but without taking on any financial leverage. It is the lease that has now become your leverage! And you can also combine real and financial leverage. Finally, there can even be differences in the degree to which the production technologies themselves are levered. The final example in the fourth line shows a different method of production, which is intrinsically more levered.

Example Assumptions:

- The machine costs $400 and can be resold for $150. The net operating cost is thus $250.
- In a hypothetical lease, the lessee would pay the lessor $250 for use of the machine, and the lessor would own the machine ($150) at the end.
- Labor costs are $75.
- Product produces $200 ("Bad") or $1,000 ("Good").
- Assume prevailing interest rate is 0.

Leverage	Investment	Out of Pocket	Dollars Bad	Dollars Good	Percent Bad	Percent Good	FLR
None	Pay for everything.	$475	$350	$1,150	–26%	+142%	0%
Financial	Borrow $350.	$125	$0	$800	–100%	+540%	74%
Real	Lease machine, Pay $250.	$325	$200	$1,000	–38%	+208%	0%
Real+Financial	Lease machine & Borrow $200.	$125	$0	$800	–100%	+540%	62%
Different Technology—Labor costs $40, different machine costs $400, has residual value of $115.							
Technology	Pay for everything.	$440	$315	$1,115	–28%	+153%	0%

Exhibit 6.8: *Financial and Real Leverage.* FLR is financial leverage, which is defined as the fraction of financial debt divided by the sum of debt and equity.

Working with More Than Two Possible Outcomes

Multiple outcomes will cause multiple breakpoints in the relation from promised to expected payoffs.

In the real world, possible outcomes can often range from 0 to infinity. Can you use the same method if you have more than two scenarios? For example, assume that the building could be worth $60, $70, $80, $90, or $100 with equal probability (for an expected value of $80) and that the appropriate expected interest rate is 20%. It follows that the building has a PV of $80/1.20 ≈ $67. If a loan promised to pay $60 at time 1, how much would it expect to receive? The full $60, of course, because the building is always worth at least this much:

$$\mathrm{E}\big(\text{ Payoff(\$0 } \le \text{ Loan Promise} = x \ \le \text{\$60) } \big) \;=\; 100\% \cdot x$$

But if a loan promised $61, how much would it expect to receive then? It would expect $60 for sure, plus the extra "marginal" $1 with 80% probability (because there is an 80% chance that $61 is covered; only if the outcome is $60, which happens 20% of the time, would it not receive the full $61). Thus, for the $61 loan promise, it would expect to receive $60.80. In fact, it would expect only 80 cents for each dollar promised between $60 and $70. So, if a loan promised x between $60 and $70, it would expect to receive

$$\mathrm{E}\big(\text{ Payoff(\$60 } \le \text{ Loan Promise} = x \ \le \text{\$70) } \big) \;=\; \$60 \;+\; 80\% \cdot (x - \$60)$$

If a loan promised $71, how much would it expect to receive? It would expect $60 for sure, plus $8 for the next promised $10, plus 60 cents on the dollar for anything above $70, i.e., $68.60,

$$\mathrm{E}\big(\text{ Payoff(\$70 } \le \text{ Loan Promise} = x \ \le \text{\$80) } \big) \;=\; \$60 \;+\; \$8 + 60\% \cdot (x - \$70)$$

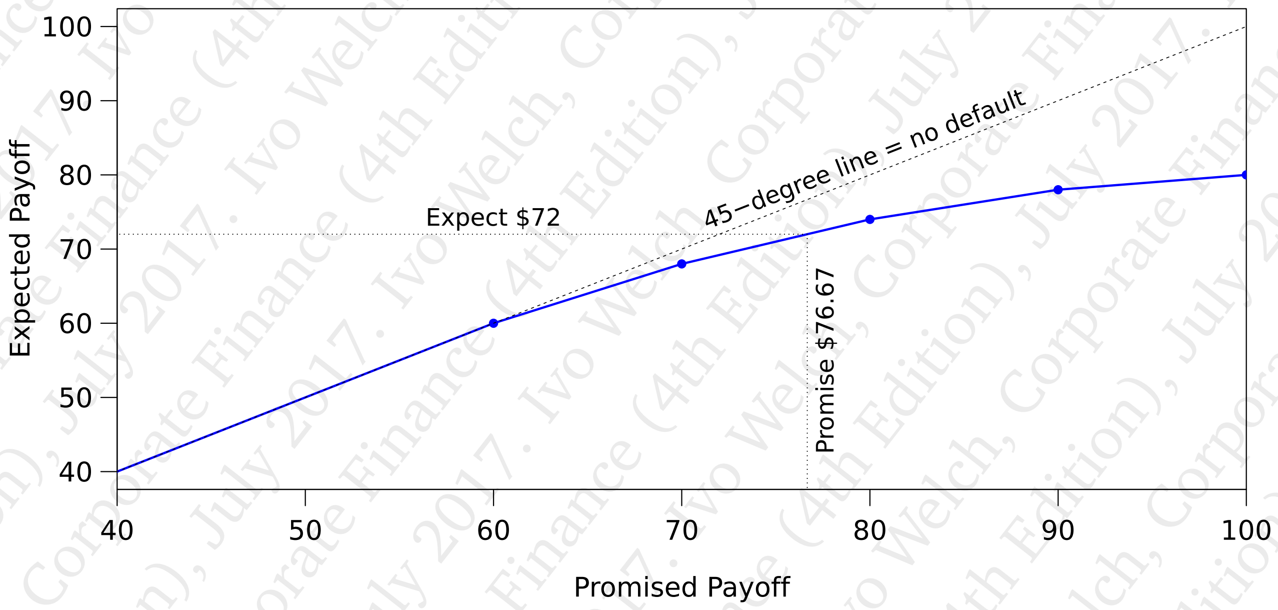


Exhibit 6.9: *Promised versus Expected Payoff for a Loan on the Project with Five Possible Payoffs.* The dotted line is one-to-one, where a promised dollar is an expected dollar (i.e., risk-free). The blue line shows the payoffs to the bond. The firm will be worth $60, $70, $80, $90, or $100, each with equal probability. To borrow $60 today, the bond must offer an expected payoff of $\$60 \cdot 1.2 = \$72$ next year. Following the arrow from the y-axis at $72 to the payoff function and then down to the x-axis shows that this high an expected payoff requires a promised payoff of $76.67.

You can now read off the appropriate promised value from the graph for any mortgage.

Exhibit 6.9 plots these expected payoffs as a function of the promised payoffs. With this figure, mortgage valuation becomes easy. For example, how much would the loan have to promise to provide $50 today? The expected payoff would have to be $(1+20\%) \cdot \$50 = \$60$. This is on the linear segment, so you would have to promise $60. Of course, you cannot offer an expected payoff of more than $80, so forget about borrowing more than $\$80/1.2 \approx \$66.67$ today.

The same approach would also work when possible value outcomes are "normally distributed" (i.e., following a bell-curve). The math is a more complex, but the method remains the same.

Q 6.18. What is the formula for a promised loan payoff between $80 and $90?

Q 6.19. What is the expected payoff if the promised payoff is $72?

Q 6.20. If you want to borrow $65, what do you have to promise?

Q 6.21. If there were infinitely many possible outcomes (e.g., if the building value followed a statistical normal distribution), what would the graph of expected payoffs of the loan as a function of promised payoffs look like?

Q 6.22. A new product may be a dud (20% probability), an average seller (70% probability), or dynamite (10% probability). If it is a dud, the payoff will be $20,000; if it is an average seller, the payoff will be $40,000; if it is dynamite, the payoff will be $80,000. The appropriate expected rate of return is 6% per year. If a loan promises to pay off $40,000, what are the promised and expected rates of return?

An Error: Discounting Promised Cash Flows with the Promised Cost of Capital

A common mistake is the attempt to avoid the need to estimate expected values by discounting promised cash flows with promised discount rates. After all, both numbers reflect default risk. The two default issues might cancel out one another, and you might end up with the correct inference. *Or they might not cancel out, in which case you will end up with an incorrect decision!*

To illustrate, say the appropriate expected rate of return is 20%. A newly available bond investment promises $25 for a $100 investment with fully insured principal but a 50% probability of default on the interest payment. Say that other risky bonds in the economy offer 11.4% (for a total interest rate of 31.4%). If you discounted the promised interest payment of $25 with the quoted interest rate on your benchmark bonds, you would get

$$\text{Bad NPV Calculation} = -\$100 + \frac{\$100}{1+20\%} + \frac{\$25}{1+31.4\%} \approx +\$2.36$$

Wrong! Instead, you must work with expected values:

$$\text{Correct NPV Calculation} = -\$100 + \frac{\$100}{1+20\%} + \frac{\$12.50}{1+20\%} = -\$6.25$$

This bond would be a bad investment.

Summary

This chapter covered the following major points:

- Uncertainty means that a project may not return its promised amount.
- A random variable is one whose outcome has not yet been determined. It is characterized by its distribution of possible future outcomes.
- The "expected value" is the probability-weighted sum of all possible outcomes. It is the "average" or "mean," but it is applied to the future instead of to a historical data series. It is a measure of "reward."
- Risk neutrality means indifference between a safe bet and a risky bet if their expected rates of return are the same.
- The possibility of future default causes promised (quoted) interest rates to be higher than expected interest rates. Default risk is also often called credit risk.
- Most of the difference between promised and expected interest rates is due to default. Extra compensation for bearing more risk—the risk premium—and other premiums are typically smaller than the default premium for bonds.
- Credit ratings can help judge the probability of potential losses in default. Moody's and S&P are the two most prominent vendors of ratings for corporate bonds.
- The key tool for thinking about uncertainty is the payoff table. Each row represents one possible outcome, which contains the probability that the state will come about, the total project value that can be distributed, and the allocation of this total project value to different state-contingent claims. The state-contingent claims "carve up" the possible project payoffs.
- Most real-world projects are financed with the two most common state-contingent claims—debt and equity. Their payoff rights are best thought of in terms of payoff tables.
- Debt and equity are methods to parcel out total firm risk into one component that is safer than the overall firm (debt) and one that is riskier than the overall firm (equity).
- The presence of debt "levers up" equity investments. That is, a smaller upfront cash investment becomes more exposed to swings in the value of the underlying firm. However, there are also other leverage mechanisms that firms can choose (e.g., leasing or technology).
- If debt promises to pay more than the project can deliver in the worst state of nature, then the debt is risky and requires a promised interest rate in excess of its expected interest rate.
- NPV is robust to modest errors in the expected interest rate (the discount rate) for near-term cash flows. However, NPV is not necessarily robust with respect to modest errors in either expected cash flows or discount rates for distant cash flows.
- NPV is about discounting *expected* cash flows with *expected* rates of return. You cannot discount *promised* cash flows with *promised* rates of return.

Keywords

Answers

Q 6.1 No! The expected outcome (value) is assumed to be known—at least for an untampered die throw. The following is almost philosophy and beyond what you are supposed to know or answer here: It might, however, be that the expected value of an investment is not really known. In this case, it, too, could be a random variable in one sense—although you are assumed to be able to form an expectation (opinion) over anything, so in this sense, it would not be a random variable, either.

Q 6.2 If you do not know the exact bet, you may not know the expected value, which means that even the expected value is unknown. This may be the case for stocks, where you are often forced to guess what the expected rate of return will be (unlike for a die, for which you know the underlying physical process, which assures an expected value of 3.5). However, almost all finance theories assume that you know the expected value. Fortunately, even if you do not know the expected value, finance theories hope you still often have a pretty good idea.

Q 6.3 If the random variable is the number of dots on the die times two, then the expected outcome is $1/6 \cdot (2) + 1/6 \cdot (4) + 1/6 \cdot (6) + 1/6 \cdot (8) + 1/6 \cdot (10) + 1/6 \cdot (12) = 7$. The realization was 12.

Q 6.4 The expected value of the stock investment is $5\% \cdot (\$41) + 10\% \cdot (\$42) + 20\% \cdot (\$45) + 30\% \cdot (\$48) + 20\% \cdot (\$58) + 10\% \cdot (\$70) + 5\% \cdot (\$75) = \$52$. Therefore, buying the stock at \$50 is not a fair bet, but it is a good bet.

Q 6.5 The variance of the P_{+1} stock investment is $\text{Var}(P_{+1}) = 5\% \cdot (\$41 - \$52)^2 + 10\% \cdot (\$42 - \$52)^2 + 20\% \cdot (\$45 - \$52)^2 + 30\% \cdot (\$48 - \$52)^2 + 20\% \cdot (\$58 - \$52)^2 + 10\% \cdot (\$70 - \$52)^2 + 5\% \cdot (\$75 - \$52)^2 = 5\% \cdot \$\$121 + 10\% \cdot \$\$100 + 20\% \cdot \$\$49 + 30\% \cdot \$\$16 + 20\% \cdot \$\$36 + 10\% \cdot \$\$324 + 5\% \cdot \$\$529 = \$\$96.70$. Therefore, the standard deviation (risk) is $\text{Sdv}(P_{+1}) = \sqrt{\$\$96.70} \approx \$9.83$.

Q 6.6 Investors are more risk-averse for large bets relative to their wealth.

Q 6.7 Yes, individual investors are typically more risk-averse than investors in the aggregate. This can even be the case for all investors.

Q 6.8 Expected and promised rates are the same only for risk-free (i.e., government) bonds. Most other bonds have some kind of default risk—though even the U.S. Treasury is now rated to have some credit risk.

Q 6.9 With the revised probabilities:

1. The expected payoff is now $95\% \cdot \$210 + 1\% \cdot \$100 + 4\% \cdot \$0 = \$200.50$. Therefore, the expected rate of return is $\$200.50/\$200 = 0.25\%$.
2. You require an expected payoff of \$210 to expect to end up with 5%. Therefore, you must solve for a promised payment $95\% \cdot P + 1\% \cdot \$100 + 4\% \cdot \$0 = \$210 \Rightarrow P = \$209/0.95 = \$220$. On a loan of \$200, this is a 10% promised interest rate.

Q 6.10 No, the expected default premium is zero by definition.

Q 6.11 Both. The historical evidence is that lower-grade borrowers both default more often and pay less upon default.

Q 6.12 The actual cash flow is replaced by the expected cash flow, and the actual rate of return is replaced by the expected rate of return.

Q 6.13 The factory's expected value is $\text{E}(\text{Value at Time 2}) = [0.5 \cdot \$500{,}000 + 0.5 \cdot \$1{,}000{,}000] = \$750{,}000$. Its present value is therefore $\$750{,}000/1.06^2 \approx \$667{,}497.33$.

Q 6.14 For the dynamite/dud project:

1. The expected payoff is $\text{E}(P) = 20\% \cdot \$20{,}000 + 70\% \cdot \$40{,}000 + 10\% \cdot \$80{,}000 = \$40{,}000$.
2. The present value of the expected payoff is $\$40{,}000/1.08 \approx \$37{,}037$.
3. The three rate of return outcomes are $\$20{,}000/\$37{,}037 - 1 \approx -46\%$, $\$40{,}000/\$37{,}037 - 1 \approx +8\%$, $\$80{,}000/\$37{,}037 - 1 \approx +116\%$.
4. The expected rate of return is $20\% \cdot (-46\%) + 70\% \cdot (+8\%) + 10\% \cdot (+116\%) \approx 8\%$.

Q 6.15 No! Partners would share payoffs proportionally, not according to "debt comes first." For example, if it rains, the 6.7% partner would still receive \$4, and not \$0 that the levered equity owner would receive.

Q 6.16 To finance 80% of a \$75 building, the mortgage has to provide \$60 today. Start with the payoff table that contains what you know:

Event	Prob	Building	80% Mortgage	Levered
Rain	1/4	\$60	\$60	\$0
Sun	3/4	\$100	x	\$100-x
$E(V)$, Time 1		\$90	y	\$90-y
PV, Time 0		\$75	\$60	\$15
$E(r_{0,1})$		20%	20%	20%

In this interest environment, a mortgage that has a value of \$60 today must have an expected value of $y = \$60 \cdot (1 + 20\%) = \$72$. \$60 next year are worth \$50 today. Thus, $1/4 \cdot \$50 + 3/4 \cdot x = \$60$, which tells you that the promise to pay must be $x = \$76$.

Q 6.17 The text worked out the rates of return in the case of the junk mortgage. The previous question worked out the rates of return in the case of the solid mortgage.

	Rain	Sun	Expected
Junk Mortgage (\$70)	–14.3%	+31.4%	20%
Junk Equity (\$70)	–100.0%	+60.0%	20%
Solid Mortgage (\$60)	0%	+26.7%	20%
Solid Equity (\$60)	–100%	+60.0%	20%

The junk mortgage is indeed riskier than the solid mortgage. The junk equity is no riskier than the solid equity (though in a more general example, it would be). The building is the same building, and thus its risk has not changed.

Q 6.18

$$E(\text{ Payoff}(\ \$80 \le \text{Loan Promise} = x \le \$90)\ ) = \$60 + \$8 + \$6 + 40\% \cdot (x - \$80)$$

Q 6.19 The relevant line segment (and numeric answer) are $E = \$68 + 60\% \cdot (\$72 - \$70) = \$69.20$.

Q 6.20 The \$65 today requires an expected payoff of $1.2 \cdot \$65 = \$78$. This is on the final line segment. The formula is

$$\begin{aligned} E(\text{ Payoff}(\ \$90 \le \text{Loan Promise} = x \le \$100)\ ) &= \$60 + \$8 + \$6 + \$4 + 20\% \cdot (x - \$90) \\ &= \$78 + 20\% \cdot (x - \$90) \end{aligned}$$

Thus, $x = \$90$.

Q 6.21 With infinitely many possible outcomes, the function of expected payoffs would be a smooth increasing function. For the mathematical nitpickers: [a] We really should not allow a normal distribution, because the value of the building cannot be negative; [b] The function would increase monotonically, but it would asymptote to an upper bound.

Q 6.22 With 20% probability, the loan will pay off \$20,000; with 80% probability, the loan will pay off the full promised \$40,000. Therefore, the loan's expected payoff is $20\% \cdot \$20{,}000 + 80\% \cdot \$40{,}000 = \$36{,}000$. The loan's price is $\$36{,}000/1.06 \approx \$33{,}962$. Therefore, the promised rate of return is $\$40{,}000/\$33{,}962 - 1 \approx 17.8\%$. The expected rate of return was given: 6%.

End of Chapter Problems

Q 6.23. Is this morning's CNN forecast of tomorrow's temperature a random variable? Is tomorrow's temperature a random variable?

Q 6.24. Does a higher reward (expected rate of return) always come with more risk?

Q 6.25. Would a single individual be effectively more, equally, or less risk-averse than a pool of such investors?

Q 6.26. A bond will pay off \$100 with a probability of 99% and will pay off nothing with a probability of 1%. The equivalent risk-free rate of return is 5%. What is an appropriate promised yield on this bond?

Q 6.27. An L.A. Lakers bond promises an investment rate of return of 9%. Time-equivalent Treasuries offer 6%. Is this necessarily a good investment? Explain.

Q 6.28. A Disney bond promises an investment rate of return of 7%. Time-equivalent Treasuries offer 7%. Is the Disney bond necessarily a bad investment? Explain.

Q 6.29. Using information from a current newspaper or the WWW, what is the annualized yield on corporate bonds (high-quality, medium-quality, high-yield) today?

Q 6.30. What are the main bond rating agencies and the meanings of their ranking categories? Roughly, what are the 10-year default rate differences between investment-grade and non-investment grade bonds this month?

Q 6.31. How is a credit swap like an insurance contract? Who is the insurer in a credit swap? Why would anyone want to buy such insurance?

Q 6.32. Debt is usually safer than equity. Does the risk of the rate of return on equity go up if the firm takes on more debt, *provided* the debt is low enough to remain risk-free? Illustrate with an example that you make up.

Q 6.33. A financial instrument will pay off as follows:

Prob	50%	25%	12.5%	6.25%	3.125%	3.125%
Payoff	$100	$110	$130	$170	$250	$500

Assume that the risk-free interest rate is 0.

1. What price today would make this a fair bet?
2. What is the maximum price that a risk-averse investor would be willing to pay?

Q 6.34. Now assume that the financial instrument from Q 6.33 costs $100.

1. What is its expected rate of return?
2. If the prevailing interest rate on time-equivalent Treasuries is 10%, and if financial default happens either completely (i.e., no repayment) or not at all (i.e., full promised payment), then what is the probability p that the security will pay off? In other words, assume that full repayment occurs with probability p and that zero repayment occurs with probability $1-p$. What is the p that makes the expected rate of return equal to 10%?

Q 6.35. Go to the Vanguard website. Look at funds by asset class, and answer this question for bond funds.

1. What is the current yield-to-maturity of a taxable Vanguard bond fund invested in Treasuries?
2. What is the current yield-to-maturity of a taxable Vanguard bond fund invested in investment-grade bonds?
3. What is the current yield-to-maturity of a taxable Vanguard bond fund invested in high-yield bonds?

Q 6.36. Return to the example on Page 113, but assume that the probability of receiving full payment of $210 in one year is only 95%, the probability of receiving $100 is 4%, and the probability of receiving absolutely no payment is 1%. If the bond quotes a rate of return of 12%, what is the time premium, the default premium, and the risk premium?

Q 6.37. A project costs $19,000 and promises the following cash flows:

	Y1	Y2	Y3
Cash Flows	$12,500	$6,000	$3,000

The appropriate discount rate is 15% per annum. Should you invest in this project?

Q 6.38. A bond promises to pay $12,000 and costs $10,000. The promised discount on equivalent bonds is 25% per annum. Is this bond a good deal?

Q 6.39. Assume that the probability that the Patriots will win the Superbowl is 55%. A souvenir shop outside the stadium will earn net profits of $1.5 million if the Patriots win and $1.0 million if they lose. You are the loan officer of the bank to whom the shop applied for a loan. You can assume that your bank is risk-neutral and that the bank can invest in safe projects that offer an expected rate of return of 10%.

1. What interest rate would you quote if the owner asked you for a loan for $900,000 today?
2. What interest rate would you quote if the owner asked you for a loan for $1,000,000 today?

(These two questions require that you compute the amount that you would demand for repayment.)

Q 6.40. A new project has the following probabilities:

	Failure	Success	Buyout
Prob	10%	85%	5%
Payoff (in millions)	$50	$200	$400

Assume risk neutrality. If a bond with $100 face value collateralized by this project promises an interest rate of 8%, then what is the prevailing cost of capital, and what do shareholders receive if the buyout materializes?

Q 6.41. Assume that the correct future cash flow is $100 and the correct discount rate is 10%. Consider the value effect of a 5% error in cash flows and the effect of a 5% error (50bp) in discount rates.

1. Graph the valuation impact (both in absolute values and in percent of the correct upfront present value) as a function of the number of years from one year to twenty years.
2. Is this an accurate real-world representation of how your uncertainty about your own calculations should look? In other words, is it reasonable to assume a 5% error for cash flows in twenty years? For the appropriate discount-rate applicable to twenty-year cash flows?

Q 6.42. Under risk neutrality, a factory can be worth $500,000 or $1,000,000 in *two* years, depending on product demand, each with equal probability. The appropriate cost of capital is 6% per year. The factory can be financed with proceeds of $500,000 from loans today. What are the promised and expected cash flows and rates of return for the factory (without a loan), the loan, and the levered factory owner?

7

A First Look at Investments

Historical Rates of Return Background and Market Institutions
The subject of investments is so interesting that I first want to give you a quick tour, instead of laying all the foundation first and showing you the evidence later. I will give you a glimpse into the world of historical returns on the three main asset classes of stocks, bonds, and "cash," so that you can visualize the main patterns that matter—patterns of risk, reward, and covariation. This chapter also describes a number of important institutions that allow investors to trade equities.

7.1 Stocks, Bonds, and Cash, 1990-2016

Financial investments are often classified into just a few broad **asset classes**. The three most prominent classes are cash, bonds, and stocks.

Cash, bonds, and stocks are the most commonly studied asset classes.

Cash: The name *cash* here is actually a misnomer because it does not designate physical dollar bills under your mattress. Instead, it means debt securities that are very liquid, very low-risk, and very short-term. Other investments that are part of this generic asset class may be certificate of deposits (CDs), savings deposits, or commercial paper. (These are briefly explained in Book Appendix A.) Another common designation for cash is **money market**. To make our lives easy, we will just join the club and also use the term "cash."

Bonds: These are debt instruments that have longer maturity than cash. You already know much about bonds and their many different varieties. I find it easiest to think of this class as representing primarily long-term Treasury bonds. You could also broaden this class to include bonds of other varieties, such as corporate bonds, municipal bonds, foreign bonds, or even more exotic debt instruments.

Stocks: Stocks are sometimes all lumped together, and sometimes further categorized into different kinds of stocks. The most common subclassifications for U.S. domestic stocks are as follows:

- The asset class containing a few hundred stocks of the largest firms that trade very frequently is often called **large-cap stocks**. (**Cap** is a common abbreviation for "market capitalization," itself a fancy way of saying "market value.") Although not exactly true, you can think of the largest 500 firms as roughly the constituents of the popular **S&P 500** stock market index. (S&P is Standard and Poor's. This company invented this index in 1923 and continues to maintain it.) Stocks continuously change in value, disappear, etc. You can very easily invest in an S&P500 basket of stocks by buying a mutual fund or an exchange-traded fund.) Our chapter focuses mostly on these large-cap S&P 500 stocks and often just calls them "stocks."

- There are a few thousand other stocks. They are also sometimes put into multiple categories, such as “mid-cap” or “small-cap.” Inevitably, these stocks tend to trade less often, and some seem outright neglected. Small caps can be really small. They may have only $10 million in market cap, and not a single share may be traded for days at a time. In any case, it is so expensive to trade most small-cap stocks that large investors do not bother with them.

There are also other stock-related subclasses, such as industry stock portfolios, or a classification of stocks into “value firms” and “growth firms,” and so on. We shall ignore everything except the large-cap stock portfolio.

These asset classes are only broadly representative of similar individual investments. We are omitting many other important asset classes.

Do not take these categories too literally. They may not be representative of all assets that would seem to fit the designation. For example, most long-term bonds in the economy behave like our bond asset class, but some long-term corporate bonds behave more like stocks. Analogously, a particular firm may own a lot of bonds, and its rates of return would look like those on bonds and not like those on stocks. It would also be perfectly reasonable to include more or fewer investments in these three asset classes. (We would hope that such modifications would alter our insights only a little bit.) More importantly, there are also many other important asset classes that we do not even have time to consider, such as real estate, hedge funds, financial derivatives, foreign investments, commodities such as precious metals or orange juice, or art. Nevertheless, cash, bonds, and stocks (or subclasses thereof) are the three most studied financial asset classes, so we will begin our examination of investments by looking at their historical performances.

Graphical Representations of Historical Returns for the S&P 500

All rates of return data are in the time-series diagram.

Start with Exhibit 7.1. It shows the year-by-year rates of return (with dividends) of the S&P 500. Actually, because of how different sources treat dividends (reinvest or not?), the numbers are never exact. (Some sources even omit dividends in their total rate of return calculation—an exclusion that is definitely wrong.) The series we are using in this book take dividends into account. (And all the numbers are also on the book’s website. Obviously, I do not want to write this textbook with 8 decimal points of precision, so please be aware of—and do not worry about—rounding errors in any of the calculations that follow.) The table and the plot illustrate the same data: You would have lost 3.1% in 1990, gained 30% in 1991, gained 7.4% in 1992, and so on. The average rate of return over the 26 years from 1990-2015 was 10.7% per annum—which I have marked with a dotted line.

The histogram (statistical distribution) shows how spread out returns are.

Exhibits 7.2 and 7.3 take the same data as in Exhibit 7.1 but present it differently. Exhibit 7.2 shows a histogram that is based on the number of returns that fall within a range. This plot makes it easier to see how spread out returns were—how common it was for the S&P 500 to perform really badly, perform just about okay, or perform really well. For example, the table in Exhibit 7.1 shows that 5 years (2004, 2006, 2010, 2012, and 2014) had rates of return between 10% and 20%. In our 26 years, the most frequent returns were between 0% and 10%. Yet there were also many years that had rates of return below 10%—and even years in which you would have lost more than 20% of your money (such as 1974, 2002, and 2008). And from 2000 to 2002, you would have lost more than a third of your investment! The red triangle indicates that the average rate of return was the aforementioned 10.7%/year.

What would a $1 investment have been worth?

Most investors are interested in how much money they make and not in statistics. (As Coach Belichek likes to joke, “statistics are for losers.”) Can you take $1 and the 10.7% average return, and use the compounding formula? Well, this would indicate a final wealth of $\$1 \cdot 1.107^{26} \approx \$14$ in 2015. Unfortunately, you would have been far off the mark.

The “compound rate of return” graph shows how long-run investments would have fared.

Instead, you need a graph of the compound rate of return, which is shown in Exhibit 7.3. It plots the compounded annual returns (on a logarithmic scale). For example, by the end of 1993, the compound return of $1 invested in 1990 would have been $1.49.

	Year									
Decade	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1970	3.5%	14.1%	18.7%	–14.5%	–26.0%	36.9%	23.6%	–7.2%	6.4%	18.2%
1980	31.5%	–4.8%	20.4%	22.3%	6.0%	31.1%	18.5%	5.7%	16.3%	31.2%
1990	–3.1%	30.0%	7.4%	9.9%	1.3%	37.1%	22.7%	33.1%	28.3%	20.9%
2000	–9.0%	–11.9%	–22.0%	28.4%	10.7%	4.8%	15.6%	5.5%	–36.6%	25.9%
2010	14.8%	2.1%	16.0%	32.5%	13.5%	1.5%	(≈ 11.8%)			

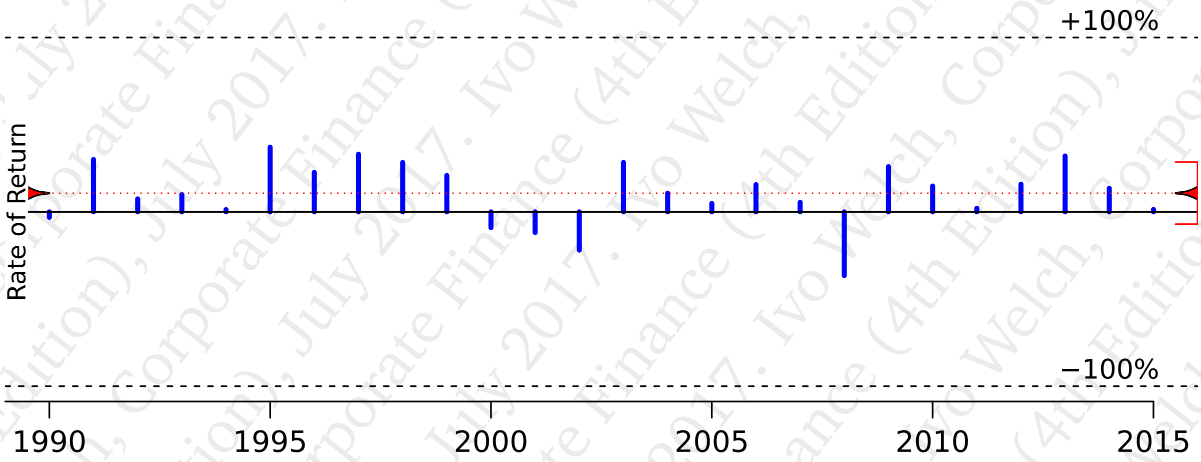


Exhibit 7.1: *The Time Series of Rates of Return on the S&P 500* with dividends. The time-series graph is a representation of rates of return of the S&P 500 index (including dividends), as shown in the table above. The average rate of return beginning in 1990 and ending in 2015 was 10.7%/year (indicated by the red triangle and the dotted line); the standard deviation was 17.8%/year. The red box on the right indicates the mean plus or minus the standard deviation.

Original source: CRSP.

$$\$1 \cdot (1 + (-3.1\%)) \cdot (1 + 30.0\%) \cdot (1 + 7.4\%) \cdot (1 + 9.9\%) \approx \$1.49$$

$$P_{1/1/1990} \cdot (1 + r_{1990}) \cdot (1 + r_{1991}) \cdot (1 + r_{1992}) \cdot (1 + r_{1993}) = P_{12/31/1993}$$

The graph also shows inflation.

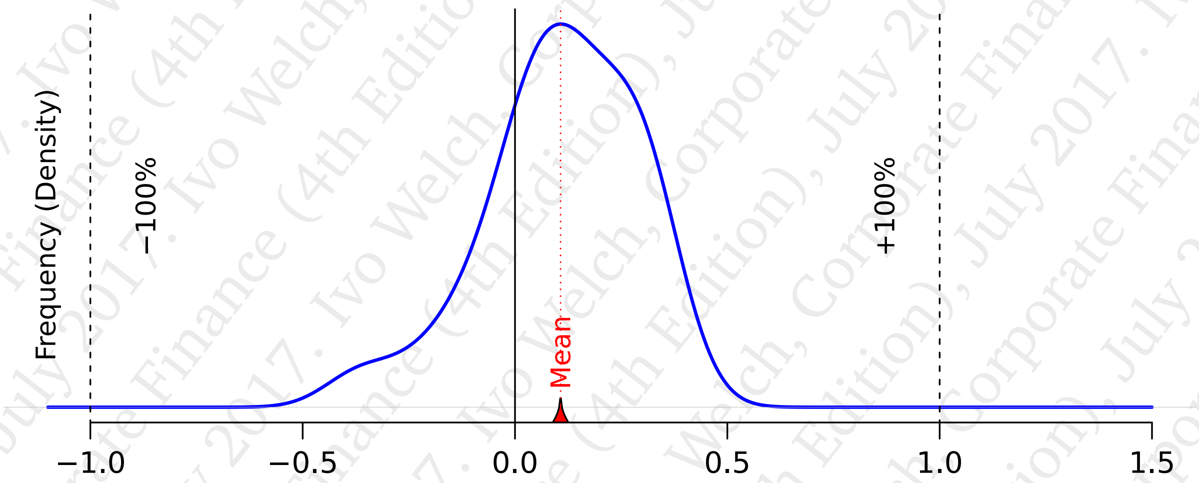
There is one further novel aspect to this graph, which is the gray-shaded area. It marks the cumulative CPI inflation. The purchasing power of $1 in 1990 was about the same as $1.87 at the end of 2015. Thus, the $9.82 nominal value in 2015 was really only worth $9.82/$1.87 ≈ $5.25 in 1990 inflation-adjusted dollars. (And, of course, none of these figures take income taxes into account.)

➤ Apples, Sect. 5.2, Pg.82.
➤ Tax Basics, Sect. 11.4, Pg.257.

How to mislead investors: quote arithmetic means for high-volatility investments.

Many long-term investors make the mistake of compounding the arithmetic average rate of return—commonly just called the mean or average. This would suggest the aforementioned $14 ($\$1 \cdot [(1 + 10.7\%)^{26} - 1]$) final wealth. However, if you compound the actual yearly returns, you find that the true compounded investment was only $9.82

Why are these numbers so different? Think of an example. If you had earned a rate of return of –50% (you lose half) followed by +100% (you double), your compounded rate of return



Return range	**<–30%**	**–30% to –20%**	**–20% to –10%**	**–10% to 0%**	**0% to 10%**	**10% to 20%**	**20% to 30%**	**>30%**
Number of years	1	1	1	2	7	5	5	4

Exhibit 7.2: *The Statistical Distribution Function of S&P 500 Rates of Return.* The graph and table are just different representations of the data in Exhibit 7.1. The x axis are the individual annual yearly rates of return. The y axis is the frequency with which these returns occur. Formally, this type of graph is called a density function. It is really just a smoothed version of a histogram.

would have been zero. However, your average rate of these two returns would have been a positive (–50% + 100%)/2 = +25%. Equivalently, if you had earned +50% followed by –50%, you would have ended up with only 1.5 · 0.5 = 75% of your investment, a negative rate of return. You will later see a real-world example in which the compound rate of return was –100% (you lost all your money) but the average rate of return was still positive. Yikes!

Arithmetic and Geometric Average Rates of Return

Geometric returns are risk-free equivalent compounding rates of return.

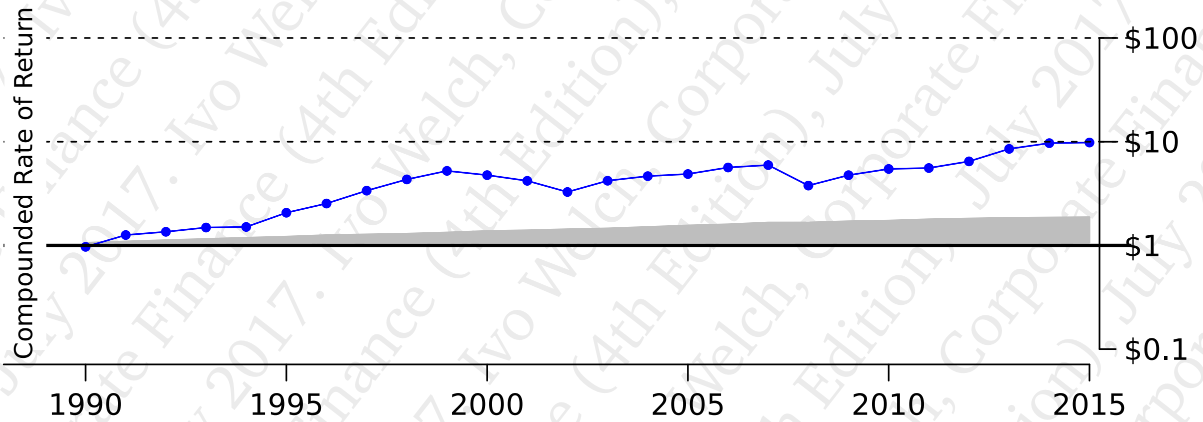
The annualized compound rate of return is often called a **geometric average**. To compute the geometric average, you uncompound (annualize). The annualized rate of return from 1990 to 2015 (26 years) for the S&P 500 investor was

$$\$1 \cdot (1 + r)^{26} \approx \$9.82 \quad \Leftrightarrow r \approx \sqrt[26]{9.82} - 1 \approx 9.2\%$$

This 9.2% is about 1.5% less than the arithmetic rate of return of 10.7%. The way to interpret this discrepancy is as follows: If there had been no volatility, then a 9.2% rate of return each and every year would have been enough to compound into $9.82. This can easily lead to misleading comparisons. The historical arithmetic average rate of return for more volatile stocks must be higher than the arithmetic equivalent for less volatile bonds just for you to end up even. If they had the same historical arithmetic rate of return, then bonds would have outperformed stocks.

How can you translate between arithmetic and geometric average returns?

Unfortunately, the *annualized* holding rate of return cannot be accurately inferred from the *average* annual rate of return, and vice-versa. The two are identical only if the rate of return is the same every period (i.e., when there is no risk). Otherwise, the geometric average rate



	Year									
Decade	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1990	\$0.97	\$1.26	\$1.35	\$1.49	\$1.51	\$2.07	\$2.53	\$3.37	\$4.33	\$5.23
2000	\$4.76	\$4.20	\$3.27	\$4.20	\$4.65	\$4.88	\$5.64	\$5.95	\$3.78	\$4.75
2010	\$5.46	\$5.57	\$6.46	\$8.53	\$9.68	\$9.82				

Exhibit 7.3: *Compound Rates of Return for the S&P 500.* This graph and table are again just different representations of the same data in Exhibit 7.1. The gray area underneath the figure is the cumulative inflation-caused loss of purchasing power.

of return is always less than the arithmetic average rate of return. The more risk, the bigger the difference. It is the geometric rate of return that makes it possible to compare returns with different volatilities in annualized terms. Fortunately, there is an approximation formula.

IMPORTANT

Rule of Thumb: If returns are approximately normally distributed, then the arithmetic mean is higher than the geometric mean by about half the variance.

In our example, the S&P had an annual standard deviation of 17.8%, which comes to a variance of $(0.178)^2 \approx 3.2\%$. Thus, the approximation formula says that the geometric rate of return should have been about 1.6% lower than the arithmetic return. In our case, the approximation is on the money (pun!). A risk-free average rate of return of about 9.2% (which is both geometric and arithmetic) would have allowed you to end up with the same return as the volatile 10.7% arithmetic average rate of return on stocks.

Counterintuitive Aspects and Tricks With Quoting

Watch out—even I am getting easily confused.

The two averages can be tricky. Let me show you an example.

A break-even example.

Say that each period, you can either win or lose 50% (W or L). Your final payoff on a \$1 investment is $(1+r_1)\cdot(1+r_2)\cdot \$1$. For example, if you lose twice, you end up with $(1-0.5)\cdot(1-0.5)\cdot\$1 = \$0.25$. Thus, your expected payoff $\mathsf{E}(V)$ is

$$\begin{array}{ccccccc} 0.25\cdot[\$0.25] & + & 0.50\cdot[\$0.75] & + & 0.25\cdot[\$2.25] & = & \$1 \\ \text{Prob}(\text{LL})\cdot[V_{\text{LL}}] & + & 2\cdot\text{Prob}(\text{LW})\cdot[V_{\text{LW}}] & + & \text{Prob}(\text{WW})\cdot[V_{\text{WW}}] & & \end{array}$$

The probability on the middle term is 50% because it does not matter whether you first win and then lose (WL) or vice-versa (LW). Your average arithmetic rate of return is –50% one quarter of the time (you earned –50% in both periods), 0% half the time (–50% and +50%, one each time), and +50% one quarter of the time. Thus, your expected arithmetic average rate of return ($\mathsf{E}[(r_1+r_2)/2]$) is

$$\begin{array}{ccccccc} 0.25\cdot[-0.5] & + & 0.50\cdot[0] & + & 0.25\cdot[0.5] & = & 0\% \\ \text{Prob}(\text{LL})\cdot[\text{Mean } r_{\text{LL}}] & + & 2\cdot\text{Prob}(\text{LW})\cdot[\text{Mean } r_{\text{LW}}] & + & \text{Prob}(\text{WW})\cdot[\text{Mean } r_{\text{WW}}] & & \end{array}$$

And, finally, even your expected two-period compounded rate of return is zero. You earn –75% one quarter of the time, –25% half the time, and +125% one quarter of the time. Thus, $\mathsf{E}(r_{0,2}) = \mathsf{E}[(1+r_{0,1})\cdot(1+r_{1,2})-1]$ is

$$\begin{array}{rl} 0.25\cdot[(1-0.5)\cdot(1-0.5)-1] + & 0.50\cdot[(1-0.5)\cdot(1+0.5)-1] \\ & +\,0.25\cdot[(1+0.5)\cdot(1+0.5)-1] \;=\; 0\% \\ \text{Prob}(\text{LL})\cdot[\text{Compounded } r_{\text{LL}}] + & 2\cdot\text{Prob}(\text{LW})\cdot[\text{Compounded } r_{\text{LW}}] \\ & +\,\text{Prob}(\text{WW})\cdot[\text{Compounded } r_{\text{WW}}] \end{array}$$

This is all just break-even. So far, so good.

But your geometric average rate of return is negative.

However, even though you are breaking even, your geometric average rate of return is less than zero. Your annualized rate of return is $\sqrt{(1-0.5)\cdot(1-0.5)}-1 = -50\%$ one quarter of the time, $\sqrt{(1-0.5)\cdot(1+0.5)}-1 = -13.4\%$ half the time, and $\sqrt{(1+0.5)\cdot(1+0.5)}-1 = +50\%$ one quarter of the time. Your expected geometric rate of return is therefore

➤ Annualization, Sect. 5.1, Pg.77.

$$\begin{array}{rl} 0.25\cdot[\sqrt{(1-0.5)\cdot(1-0.5)}-1] + & 0.50\cdot[\sqrt{(1-0.5)\cdot(1+0.5)}-1] \\ & +\;0.25\cdot[\sqrt{(1+0.5)\cdot(1+0.5)}-1] \;\approx\; -6.7\% \\ \text{Prob}(\text{LL})\cdot\text{Annualized } r_{\text{LL}} & +\;\; 2\cdot\text{Prob}(\text{LH})\cdot\text{Annualized } r_{\text{LH}} \\ & +\;\; \text{Prob}(\text{LL})\cdot\text{Annualized } r_{\text{HH}} \end{array}$$

The square-root is "at fault" here. It is why this expected geometric average rate of return is negative. You can interpret this geometric average as stating that a negative geometric rate of return of –6.7% would have been enough to keep your true expected payoff at your original investment level of \$1 (because earning a compounding 50% is really great!).

➤ Expectations of Linear Functions, Sect. A, Pg.624.

But, if you had exactly one-half of the time a rate of return of –50% and one-half of the time a rate of return of +50%, over time, you would lose money, because $(1-0.5)\cdot(1+0.5) \approx -13.4\%$. The difference between the –13.4% (well, half the time) and the 0% is the difference between simultaneously-equally-likely realizations and sequential realizations. If you get either –50% or +50% with equal probability, your expected value is 0. If you get –50% followed by +50%, your expected value is negative.

Are You Expecting Compound Arithmetic or Geometric Historical Averages?

Sampling vs. Same-Period.

Unfortunately, this is not just an academic egghead concern. If you knew the population distribution, the distinction between arithmetic and geometric returns would be just a footnote. However, you usually do not. The conceptual problem now is that the "statistical sampling" logic implicitly converts the historical sequential realizations into assumed simultaneous draws in each period. Here is what I mean. Let's say that you had observed just two periods in which investors had first earned one –50% rate of return and then one +50% rate of return. A $1 investment left them with $0.75 over this two-year period. This is all you know. If you now assume that you will receive either –50% or +50% with equal probability in each of the following two periods ($0.50 or $1.50 after one year; $0.25, $0.75, $0.75, $2.25 after two years), then you will expect to end up with $1. If you use historical realizations as equally likely samples, you are guesstimating that you will do better in the future ($1) than in the past ($0.75), purely based on the past. One way around this problem is to work with compound rates of return to begin with. Over two years, you earned –25%. This is all you know. Thus, if you have to guess what you will earn over the next two years, it is also –25%.

What do you expect to earn in the S&P500 over the next 43 years?

Let's put this insight to work on the specific question at hand. Can you estimate how much $1 invested in the S&P 500 will be worth in 43 years, given the data? In the data, the arithmetic average rate of return was 11.3% per year. If you knew the rate of return had a true population mean of 11.3% each and every year in the future (and you had no uncertainty about the population mean), then you would expect to earn $\$1 \cdot (1 + 11.3\%)^{43} - 1 \approx \$99$ on your investment. But you do not know the population mean. Do you consider this 10.7% to be an unappealing estimate? After all, investors received only $9.82 over the last 26 years. Would you not expect your next 26-year performance to be $9.82, too? (This would suggest you compound 9.2%, not 10.7%.) This is what you would guess it to be if you assumed that the last 26 years were just one grand realization, not 43 individual realizations.

➤ $9.82 figure, Pg.139.

This could be the opportunity cost of capital.

The long-standing convention in most NPV applications, where you often have to estimate an equivalent rate of return in the stock market as your opportunity cost of capital (in the PV denominator), is to compound annual or even monthly historical arithmetic rate of returns—leading NPV users to expect $14 as an opportunity cost in this case. However, it is not at all clear that this is correct. One can argue that the historical return of $10 (or $9.82 if you want to misleadingly pretend that we have this kind of accuracy) gives you a better estimate, that the historical arithmetic rate of return compounded to $14 gives you a better estimate, that something in between $10 and $14 gives you a better estimate—and, most counterintuitively (reasoning omitted), even that a value above $14 gives you a better estimate. You have been warned! In any case, don't forget your basics: the problem is your estimation uncertainty. Your goal is estimating what alternative investments would earn elsewhere compared to your own project. Statistics and math are only aids, not gospels.

Q 7.1. What can you see in a time-series graph that is not in a histogram?

Q 7.2. What can you see in a histogram that is more difficult to see in a time-series graph?

Q 7.3. What can you see in a compound return graph that is not in the time-series graph?

Q 7.4. What is the annualized holding rate of return and the average rate of return for each of the following?

1. An asset that returns 5% each year.
2. An asset that returns 0% and 10% in alternate years.
3. An asset that returns –10% and 20% in alternate years.

Is the distance between the two returns larger when there is more risk?

Q 7.5. If the risk-free rate of return is 4% per annum, how big is the difference between the arithmetic and the geometric average rate of return?

Historical Performance for a Number of Investments

Stocks, Bonds, and Cash

Explore the large comparative Exhibit 7.4.

What does history tell you about rate of return patterns on the three major investment categories—stocks, bonds, and cash? You can find out by plotting exactly the same graphs as those in Exhibits 7.1, 7.2, and 7.3. Exhibit 7.4 repeats them for cash, bonds, and stocks *all on the same scale*. You have already seen the third row, but I have changed the scale to make it easier to make direct comparisons to the other two asset classes. These mini-graphs display a lot of information about the performance of these investments.

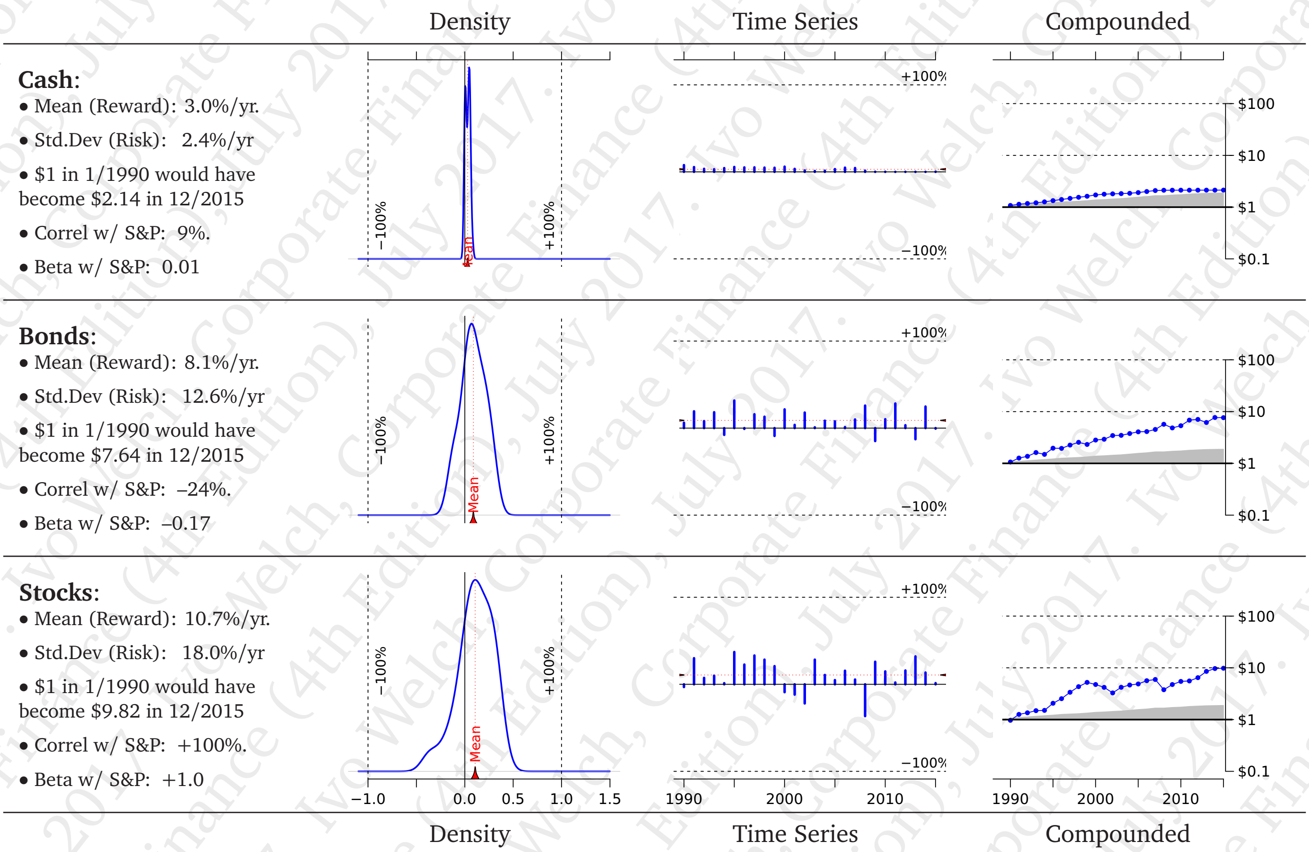


Exhibit 7.4: *Comparative Investment Class Performance.*

The first three rows show historical returns for the three asset classes.

So let's compare the first three rows:

Cash in the first row is the overnight Federal Funds interest rate. Note how tight the distribution of cash returns was around its 3% mean. You would never have lost money (in nominal terms), but you would rarely have earned much more than its mean. The value of your total investment portfolio would have steadily marched upward—although pretty slowly. Each dollar invested on January 1, 1990 would have become \$2.14 at the end of 2015. Of course, inflation would have eroded the value of each dollar. In purchasing power, your \$1 would have been the equivalent of $\$2.14/1.87 \approx \$1.14$ in 2015—and this is before the inevitable taxes you would have had to pay.

How much extra real inflation-adjusted value were these nominal returns really worth?

Bonds in the second row are long-term Treasury bonds. The middle graph shows that the bars are now sometimes slightly negative (years in which you would have earned a negative rate of return)—but there are now also years in which you would have done *much* better than cash. This is why the histogram is much wider for bonds than it is for cash: Bonds were riskier than cash. The standard deviation tells you that bond risk was 12.6% per year, much higher than the 2.4% cash risk. Fortunately, in exchange for carrying more risk, you would have also enjoyed an average rate of return of 8.1% per year, which is a lot higher than the 3.0% of cash. And \$1 invested in 1990 would have become not just \$2.14 but \$7.64 (\$4.09 in real terms)—again before taxes.

Long-term bonds offered more reward, but were more variable, too.

➤ Uncertainty and Variance, Sect. 6.1, Pg.108.

Stocks in the third row are our familiar portfolio of S&P 500 firms. Annual rates of return here are *with dividends*, and thus always more than the percent change in the widely quoted S&P500 index. The left graph shows that large stocks would have been even riskier than bonds. The stock histogram is more "spread out" than the bond histogram. The middle graph shows that there were years in which the negatives of stocks could be quite a bit worse than those for bonds, but that there were also many years that were outright terrific. And again, the higher risk of stocks also came with more reward. The S&P 500's risk of 18% per year was compensated with a mean rate of return of 10.7% per year. Your \$1 invested in 1990 would have ended up being worth \$9.82 in 2015 (\$5.25 in real terms)—again before taxes (although taxes are usually a little lower on stocks than on bonds).

Stocks offered even more reward, but were even more variable.

The difference between \$9.82 in stocks and \$2.14 in cash or \$7.64 in bonds is an understatement if you are a common taxable retail investor. *Nominal* interest payments would have been taxed each year at your full income-tax rate, somewhere between 30% and 50% per year. In contrast, the capital gains on stocks would have been taxed only at the end and at the much lower capital gains tax rate, between 15% and 30%. Roughly speaking, taking taxes into account, if you had invested in cash, you would have ended up with less real purchasing power than you started with. You would have gained real purchasing power in bonds (maybe \$2.00). And you would have roughly quadrupled your purchasing power in stocks. The sample shows good years for stocks—and perhaps even unusually good. Not every historical 43-year period would have shown this large a difference between cash and stocks. The difference between bonds and stocks were much more modest, but still considerable if you had to take income taxes into account.

Fixed-income investments performed relatively worse for taxable investors than the graphs in Exhibit 7.4 indicate at first glance.

More Asset Classes

Exhibit 7.5 shows the performance of a few other large asset classes and over a longer time period (though not ending in 2015). Small-firm stocks were riskier (and more difficult to trade), but their average rates of return were higher. Corporate bonds sat between government bonds and stocks in terms of reward, although their risk was comparable to the former. Intermediate government bonds (i.e., with about 5-year maturity) were somewhere between cash and long-term bonds. Gold was an extremely risky investment by itself, but it also did well over the sample. (Not shown in this table, it did well in years when stocks did poorly.) Moreover, unlike bonds, gold's gains were taxed at the lower capital-gains rate. Housing is the average price appreciation of residential houses. It probably understates the rate of return by about 3-6% per year, because it omits the value and other costs of living in a house. Owning real-estate (a house) from 1970 to 2010 was a good investment, especially if you take into account that tax rules now shelter some gains from *any* taxes. However, the 6-7% risk is misleading. Many economists believe that there was a housing bubble in the 2000s, which explains both the fantastic appreciation and the

Recent asset class performance.

Asset Class	1926-2010 "Reward" Geo	Ari	Risk Sdv	1970-2010 "Reward" Geo	Ari	Risk Sdv
Small-Firm Stocks (I)	12.1	16.7	32.6	12.5	15.1	23.4
Large-Firm Stocks (I)	9.9	11.9	20.4	10.0	11.6	17.9
Long-Term Corporate Bonds (I)	5.9	6.2	8.3	8.9	9.3	10.2
Long-Term Government Bonds (I)	5.5	5.9	9.5	8.7	9.3	11.7
Intermediate Government Bonds (I)	5.4	5.5	5.7	8.0	8.2	6.6
30-Day Treasuries (I)	3.6	3.7	3.1	5.6	5.6	3.1
Gold (L)	5.1	6.9	22.7	9.4	12.6	29.8
Housing *Appreciation* (S)	3.7	3.9	6.7	5.0	5.2	6.3
U.S. Inflation (I)	3.0	3.1	4.2	4.4	4.4	3.1

Other Samples		
non-U.S. OECD Equities vs. Bonds	1900-2010	≈ 3.8% geo (5.0% ari, Sdv ≈ 15.5%)
U.S. Equities vs. Bonds	1900-2010	≈ 4.4% geo (6.4% ari, Sdv ≈ 20.5%)
Buyout Funds	1984-2008	14% (ari), ≈ 1.2 × S&P500 (geo cum)
	2000-2008	10% (ari), ≈ 1.3 × S&P500 (geo cum)
VC Funds	1984-2008	17% (ari), ≈ 1.4 × S&P500 (geo cum)
	2000-2008	–1% (ari), ≈ 0.9 × S&P500 (geo cum)
Art	1976-2004	≈ 6% (Sdv ≈ 9%) vs. Stocks 12% (15%)
Wine (not consumed)	1996-2001	≈ 20% (Sdv ≈ 8%)
Commodities Futures	1959–2004	≈ 10% (Sdv ≈12%) vs. Stocks ≈ 6% (15%)
... Spot	1959–2004	≈ 4%

Exhibit 7.5: *Comparative Investment Performance for More Asset Classes and Samples.* The upper panel was calculated by Your's Truly. Original data sources, see leftmost column: L= London Gold Exchange. I=Ibbotson Stocks, Bonds, Bills and Inflation, SBBI Valuation Yearbook, Morningstar 2011. S= Robert Shiller, *Irrational Exuberance*, 2nd Ed., US National Index. Note that housing appreciation ignores the useful housing rental yield, and thus understates the rate of return. The lower panel has a potpourri of quotes from different papers with different samples and methods. The buyout funds and VC funds geometric performance are quoted over the entire 25-year sample. The commodities include metals, agriculturals, and energy, but not financials.

subsequent crash. From 1992 to 2006, there was not a single year in which prices declined. But from 2007 to 2009, residential houses lost about 30% of their values. (Real estate investment trusts [REITs] are another interesting way to invest in real-estate.) Unfortunately, I did not have the data to replicate these calculations for some further asset classes in the lower panel of Exhibit 7.5. Because they are over different intervals, I have tried to include the same-period performance of stocks. They are not only "fun" to look at, but worth contemplating as potential investments, too.

Individual Stocks

Individual stocks can offer more reward and be even more risky.

Instead of buying entire asset classes, you could also have bought just an individual stock. How would such holdings have differed from an investment in the broader asset class "stocks"? Exhibit 7.6 keeps the same scale but now shows the rates of return of a few sample stalwart firms: Coca-Cola [KO], PepsiCo [PEP], Intel [INTC], and United Airlines [UAL]. For comparison, the bottom is again the S&P 500. You can see that individual stocks' histograms are really wide: Investing in a single stock would have been a rather risky venture, even for these four household names. Indeed, it is not even possible to plot the final year for UAL in the rightmost compound return graph, because UAL stock investors lost *all* invested money in the 2003 bankruptcy, which on the logarithmic scale would have been minus infinity. And UAL illustrates another important issue: Despite losing all the money, it still had a reasonable average rate of return. If you extended the sample backwards a little, it would be positive, even though you would have still lost all your money assuming reinvestment of dividends into the stock. (You already know why: This was the difference between geometric and arithmetic averages explained on Page 141.)

Q 7.6. Rank the following asset categories in terms of risk and reward: cash (money market), long-term bonds, the stock market, and a typical individual stock.

Q 7.7. Is the average individual stock safer or riskier than the stock market?

Q 7.8. Is it possible for an investment to have a positive average rate of return, but still lose you every penny?

Comovement, Market Beta, and Correlation

Now look at the correlation of Intel (INTC) with the market, mentioned also in the leftmost column of Exhibit 7.6.

Exhibit 7.7 highlights the rates of return on the S&P 500 and one specific stock, Intel. The top rows redraw the time-series graphs for these two investments. Do you notice a correlation between these two series of rates of return? Are the years in which one is positive (or above its mean) more likely also to see the other be positive (or above its mean), and vice-versa? It does seem that way. For example, the worst rates of return for both were 2002 and 2008—and even more so for Intel investors than market investors. In contrast, 1992, 1995, and 1998 were good years for both. And again, even more so for Intel investors. The correlation is not perfect: In 2004, the S&P 500 had a good year, but Intel had a bad one; and in 2001, the market had a bad year, but Intel turned out alright. It is very common for all sorts of investments in the economy to move together with the stock market: In years of malaise, almost all assets tend to be in malaise. In years of exuberance, almost all tend to be exuberant. This tendency is called **comovement.**

Why do you care about comovement? Because you want assets that do well when everything else does poorly.

The comovement of investments is very important if you do not like risk. An investment that increases in value whenever the rest of your portfolio decreases in value is practically like "insurance" that pays off when you need it most. You might buy into such an investment even if it offers only a very low expected rate of return. In contrast, you might not like an investment that does very badly whenever the rest of your portfolio also does badly. To be included in your portfolio, such an investment would have to offer a very high expected rate of return.

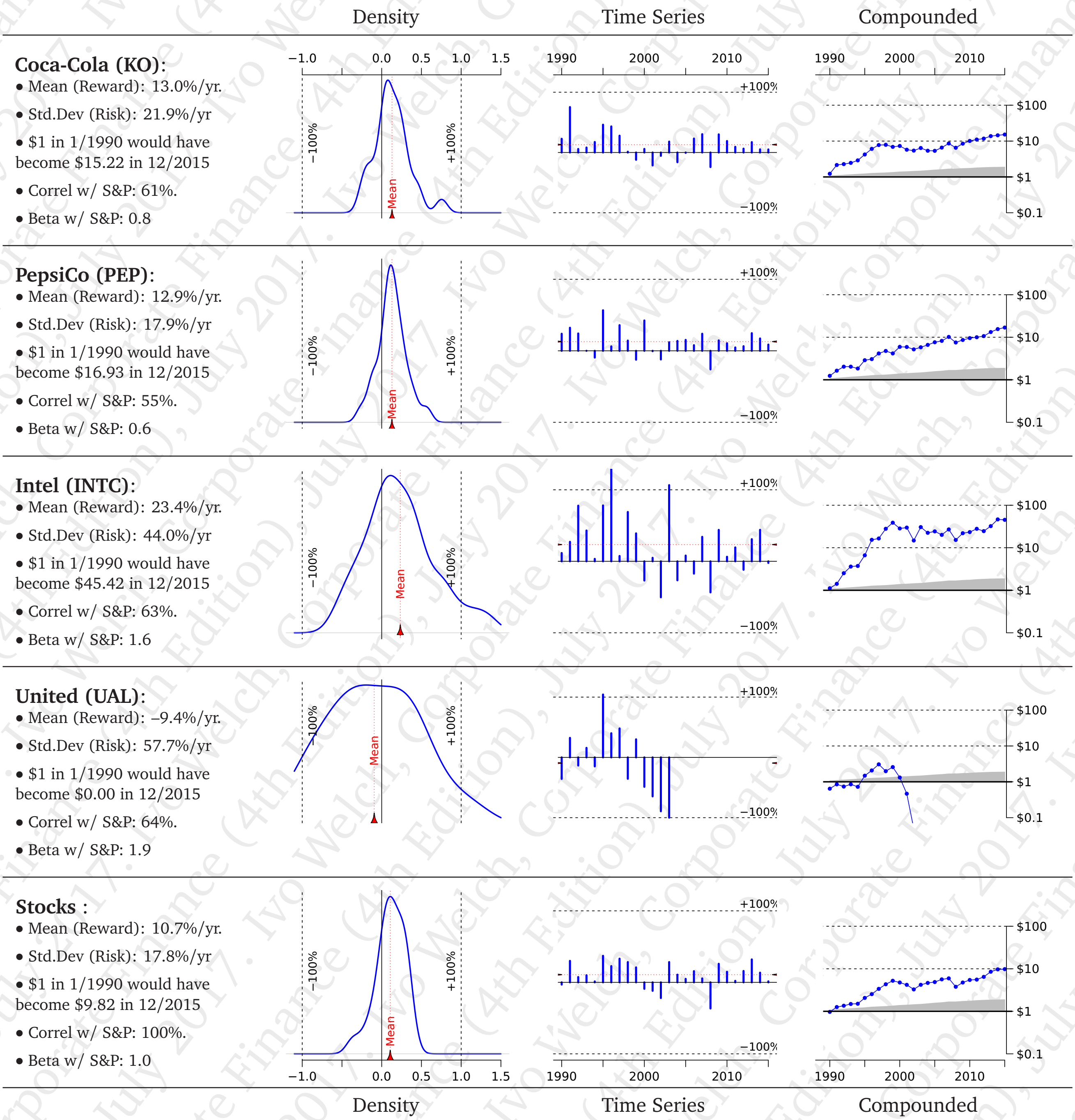


Exhibit 7.6: *Comparative Investment Performance, 1990-2015.* Data source: CRSP

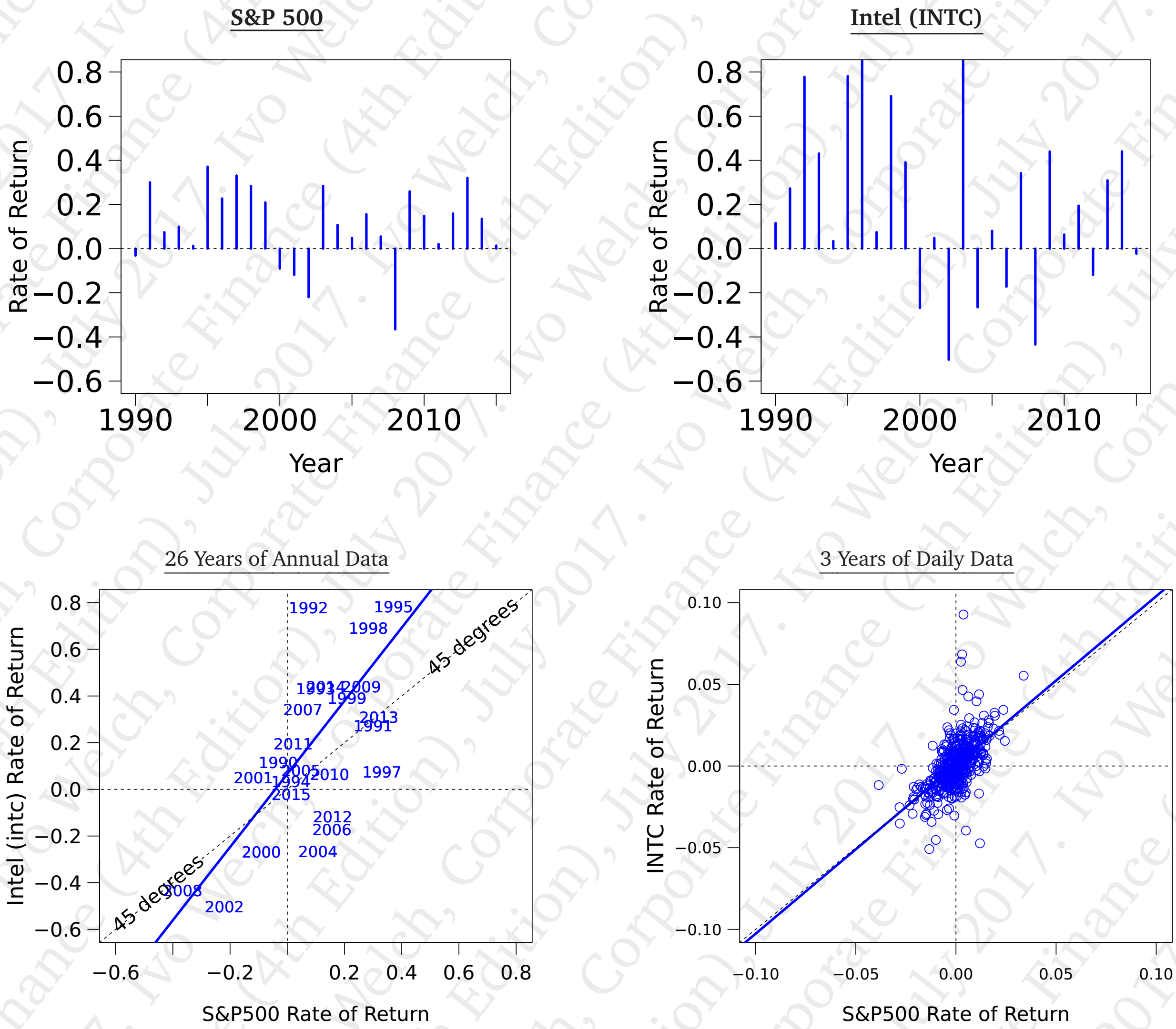


Exhibit 7.7: *Rates of Return on the S&P 500 and Intel (INTC).* The top left graph plots the annual rates of return on the S&P 500; the top right graph plots the annual rates of return on Intel. The bottom left graph combines the information. The stock market rate of return is on the x-axis, the Intel rate of return is on the y-axis. The figure shows that in years when the stock market did well, Intel tended to do well, too, and vice-versa. This can be seen in the *slope* of the best-fitting line (in fat blue), which is called the market beta of Intel. The market beta will play an important role in later investments chapters. On the bottom left, the slope is about 1.3 (steeper than 45 degrees). On the bottom right, I used 3 years of daily returns. The slope is about 1.0. Trust me when I tell you that the daily-based line on the right turns out to be a better estimator of *future* market-betas than the annual-based line on the left—especially if the historical slope is averaged with 1.0.

Quantifying comovement.

How can you measure the extent to which securities covary with others? For example, how did Intel covary with the S&P 500 (our stand-in for the market portfolio)? Did Intel also go down when the market did (making a bad situation worse), or did it go up (thereby serving as useful insurance)? How can you quantify such comovement?

Market beta is the slope of the best-fitting line (with the market's rate of return on the x-axis and the firm's rate of return on the y-axis).

You can answer this graphically. Plot the two return series against one another, as in the bottom plots in Exhibit 7.7. Then find the line that best fits between the two series. (You will learn later how to compute such "regression" lines.) The slope of this line is called the **market beta** of a stock, and it is a measure of comovement between the rate of return on the stock with the rate of return on the market. It tells an investor whether this stock moved with or against the market. It carries great importance in financial economics.

- If the best-fitting line has a slope that is steeper than the 45° diagonal (well, if the x- and y-axes are drawn with the same scale), then the market beta is greater than 1. Such a line would imply that when the stock market did better (the x-axis), on average your stock did *a lot better* (the y-axis). For example, if a stock has a very steep positive slope—say, +3—then (assuming you hold the market portfolio) if the market dropped by an additional 10%, this stock would have been expected to drop by an additional 30%. If you primarily held the market portfolio, this new stock would have made your bad situation worse.
- If the slope is less than 1 (or even 0, a plain horizontal line), it means that, on average, your stock did not move as much (or not at all) with the stock market.
- If a stock has a very negative slope such as –2, this investment would likely have "rescued" you when the market dropped by 10%. On average, it would have earned a positive 20% rate of return. Adding such a stock to your market portfolio would be like buying insurance.

Intel's annual rates of return had a slope of 1.6 against the market. That is, it was steeper than the diagonal line. In effect, this means that if you had held the stock market, Intel would have been an additional hazard for you. A 1% performance above (below) normal for the S&P would have meant you would have expected to earn 1.6% above (below) normal in your Intel holdings. However, for estimating the future market-beta in the real world, it turns out that 3 years of recent daily stock returns is better practice. (After running the statistical procedure called regression analysis to obtain the best line fit, for forecasting the future beta, you should also take the average of your regression beta estimate and 1.0.) As of 2015, it turns out that Intel's best forward-looking market-beta is much lower. In fact, it is just about 1.

Market beta is a cousin of correlation.

Instead of beta, you could measure comovement with another statistic that you may already have come across: the so-called **correlation**. Correlation and beta are related. The correlation has a feature that beta does not. A correlation of +100% indicates that two variables always perfectly move together; a correlation of 0% indicates that two variables move about independently; and a correlation of –100% indicates that two variables always perfectly move in opposite directions. (A correlation can never exceed +100% or –100%.) In Intel's case, one can work out that the correlation is +63%. The correlation's limited range from –1 to +1 is both an advantage and a disadvantage. On the positive side, the correlation is a number that is often easier to judge than beta. On the negative side, the correlation has no concept of scale. It can be 100% even if the y variable moves only very, very mildly with x (e.g., if every $y = 0.0001 \cdot x$, the correlation is still a positive 100%). In contrast, beta can be anything from minus infinity to plus infinity.

The signs of the correlation and the beta are always the same.

A positive correlation always implies a positive beta, and vice-versa. Of course, beta and correlation are only measures of *average* comovement: Even for investments with positive betas, there are individual years in which the investment and stock market do not move together (e.g., 2004 for Intel and the S&P 500). Stocks with negative betas, for which a negative market rate of return *on average* associates with a positive stock return (and vice-versa), are rare. There are only a very few investment categories that are generally thought to be negatively correlated with the market—principally gold and other precious metals. Interestingly, long-term Treasury bonds nowadays seem to have negative correlation with the market. It used to be the opposite. I am not confident to tell you what it will be in the future.

Q 7.9. How do you graph a "market beta"? What should be on the x-axis, and what should be on the y-axis? What is an individual data point?

Q 7.10. What is the market beta of the market?

Causality vs. Correlation

Causation implies correlation, but not vice-versa.

xkcd on Correlation: http://xkcd.com/552/

Allow me a brief diversion. The most important problems in finance—and economics, and statistics, and science, and theory, and practice—may well be whether correlation implies **causation**. If X causes Y, the two should be correlated. The problem is that if X does not cause Y, the two can still be correlated. People do not have very good intuition about the distinction. Thus, they often commit serious and harmful interpretation mistakes. (Some are deliberate, as in political demagoguery.) For example, just because a stock had tended to go up when there was sunspot activity (or when the overall stock market went up) does not mean that you have found that sunspots are one cause of stocks going up—this is an example of **spurious correlation**. (In fact, over long sample periods, increased sunspot activity has indeed been associated with higher stock prices.) Just because good CEOs are paid high salaries average does not mean that paying more money would make your own CEO any better—the causation may go the other way. And just because increases in government spending are associated with reductions in unemployment does not mean that the government can reduce unemployment by spending more—some other economic factor may have determined both spending and unemployment. Determining causality is important if you want to know how your strategies and policies are likely to change outcomes.

Dilbert on causality vs. correlation: 2013-04-26

Dilbert on causality vs correlation: 2012-12-12

Granger-Sims Causality.

An early answer to measuring causality in economics came from two econometricians: if unexpected changes in X predict unexpected changes in Y, then X may cause Y. If you saw an unusual sunspot (X) and it usually precedes (in time) an unusual increase in the stock return (Y), then it is a hint that X causes Y. This concept is called Granger-Sims causality. By this metric, the data reject the hypothesis that sunspots have "caused" stock returns to go up. (Both sunspots and stock prices happened to go up; it was time effects that induced the spurious correlation.) Unfortunately, Granger-Sims causality isn't perfect, either. By its metric, the weather forecast "causes" the weather. (Unusual changes in the forecast indeed predict subsequent unusual changes in weather!)

Quasi-Experimental Methods

Just when we were ready to give up, economics stumbled upon an approach that is now called "quasi-experimental." It is revolutionizing empirical economics right now (and soon economic consulting, too). Let me illustrate this with an example. Think about figuring out whether access to loans increases the success of startups. The problem is that ventures that are less likely to be successful are also less likely to attract lenders. Thus, it is not possible to conclude from the fact that funded ventures had higher success rates in the past (which they did!) that loan access played a critical role in this success. Funding may have been more like the weather forecast, itself responding to other factors (e.g., promising business plans) that ultimately determined project success. If loans to startup did not have a positive influence on survival, government programs that seek to make more loans available to more startups would probably not be a good idea.

The **Regression Discontinuity** approach.

But Fracassi, Garmaise, Kogan, and Natividad have an answer! It turns out that a particular lender employed an automated credit score algorithm with a cutoff that determined loan funding. Applicants with scores just above the cutoff (say, 4.14) received a loan. Applicants with scores below (say, 4.13) were denied. The probability of survival was 30% for the 4.14 group and 25% for the 4.13 group. Because these applicants were so close in score, their differences were probably just noise. Thus, it is likely that the entire 5% was due to the fact that the 4.14 firm got the loan and the 4.13 firm did not. (To make sure, they also compared their 5% survival difference to the survival differences for firms between 4.12 and 4.13 and firms between 4.14 and 4.15, neither of which was treated differently by the lender and neither of which showed any differences in survival.) This is convincing evidence that access to loans indeed helped improve the chances of survival for the startup firms *in their sample*.

The Big Picture Take-Aways

The main empirical regularities.

What can you learn from the performance and correlation graphs? Actually, almost everything there is to learn about investments! I will explain these facts in much more detail soon. In the meantime, here are the most important points that the graphs show:

- History tells us that stocks have offered higher average rates of return than bonds, which in turn have offered higher average rates of return than cash. However, keep in mind that this was only *on average*. In any given year, the relationship might have been reversed. For example, stocks lost 22% of their investment in 2002, while cash gained about 1.7%.
- Although stocks did well (on average), you could have lost your shirt investing in them, especially if you had bet on just one individual stock. For example, if you had invested in United Airlines in 1990, you would have lost all your money.
- Cash was the safest investment—its distribution is tightly centered around its mean, so there were no years with negative returns. Bonds were riskier. And stocks were even riskier. (Sometimes, stocks are said to be "noisy," because it is really difficult to predict how they will perform.)
- There seems to be a relationship between risk and reward: Riskier investments tended to have higher average rates of return. (However, you will learn soon that risk has to be looked at in context. Thus, please do not overread the simple relationship between the mean and the standard deviation here.)
- Large portfolios consisting of many stocks tended to have less risk than individual stocks. The S&P 500 stocks had a risk of around 15-20% per year, which was less than the risk of most individual stocks. (For example, even large-firm stalwarts like Coca-Cola, PepsiCo, and Intel had risks between 18% and 60%). This is due to "diversification," a concept we will discuss in the next chapter.

➤ Diversification, Sect. 8.2, Pg.169.

- The average rate of return is always larger than the geometric (compound) rate of return. A positive average rate of return usually, but not always, translates into a positive compound holding rate of return. For example, if investment in United Airlines had started in 1970, it would have had a positive average rate of return, despite having lost all its investors' money.
- Stocks tend to move together. For example, if you look at 2001 and 2002, not only did the S&P 500 go down, but most individual stocks tended to go down, too. In 1998, on the other hand, most stocks tended to go up (or at least not down much). The mid-1990s were good to stocks. In contrast, money-market returns had little to do with the stock market.
- On an annual frequency, the correlation between the stock market (the S&P 500) and cash was small (about 10%). The correlation between the stock market and bonds used to be positive, but now seems to be negative. The correlation between individual stocks and the stock market was around 50% to 70%. The fact that investment rates of return tend to move together is important. It is the foundation for the market beta, a measure of risk that we have touched on and that will be explained in detail in Chapter 8.

Will History Repeat Itself?

History is useful only over long horizons, not over just a few years.

As a financier, you are not interested in history for its own sake. Instead, you really want to know more about the future. History is useful only because it is your best available indicator of the future. But which history? One year? Thirty years? One hundred years? I can tell you that if you had drawn the graphs from this chapter beginning in 1926 instead of in 1970, the big conclusions would have remained the same. However, if you had started in 2001, it would have looked differently. What would you have seen? Four awful years for stock investors in the sample. You should feel intuitively that this might not be a good representative sample period. To make any sensible inferences about what is going on in the financial markets, you need many years of history, not just a decade or so—and certainly not the 6-week investment performance touted by some funds or friends (who also often display remarkable selective memory!). The flip side of this argument is that you cannot reliably say what the rate of return will be over your next year. It is easier to forecast the *average* annual rate of return over five to ten years than over one year. Your investment outcome over any single year will be very noisy.

Even over long horizons, history can sometimes be misleading. The Nikkei-225 stock index is a good example.

Instead of relying on just one year, relying on statistics computed over many years is much better. However, although twenty to thirty years of performance is the minimum number necessary to learn something about return patterns, this is still not sufficient for you to be very confident. Again, you are really interested in what will happen in the next five to ten years, not what happened in the last five to ten years. Yes, the historical performance can help you judge, but you should not trust it blindly. For example, an investor in UAL in 2000 might have guessed that the average rate of return for UAL would have been positive—and would have been sorely disappointed. Investors in the Japanese stock market in 1986 saw the Nikkei-225 stock market index rise from 10,000 to 40,000 by 1990—a 40% rate of return per year. If they had believed that history was a good guide, they would have expected $40,000 \cdot 1.40^{13} \approx 3.2$ million by the end of 2002. Instead, the Nikkei fell below 8,000 in April 2003 and has only recently recovered to 19,000 as of early 2017. History would have been a terrible guide.

➤ Peso Problem, Pg.201.

But you do not have much choice other than to rely on history.

Nevertheless, despite the intrinsic hazards in using historical information to forecast future returns, having historical data is a great advantage. It is a rich source of forecasting power, so like everyone else, you will have to use historical statistics. But please be careful not to rely too much on them. For example, if you look at an investment that had extremely high or low past historical rates of return, you may not want to believe that similar performance is likely to continue.

Historical standard deviations and variances are good estimators of their future equivalents. This is not the case for historical average rates of return.

In relative terms, what historical information can you trust more and what historical information should you trust less?

Historical risk: Standard deviations and correlations (how stock movements tend to be related or unrelated) tend to be fairly stable, especially for large asset classes and diversified portfolios. That is, say, for 2015 to 2020, you can reasonably expect PepsiCo to have a risk of about 25-30% per year, a correlation of about 50-70% with the market, and a market beta of about 0.7 to 1.1. The estimates slowly deteriorate in accuracy over multi-year horizons, though.

Historical mean reward: Historical average rates of return are not very reliable predictors of future expected rates of return. That is, you should not necessarily believe that PepsiCo will continue to earn an expected rate of return of 13% per year. And, for sure, you should not expect Intel to expect to earn 23.4% in the future.

Realizations: You should definitely not believe that past realizations are good predictors of future single-year realizations. Just because the S&P 500 had earned about 11% on average does not make it particularly likely that it will have a rate of return of 11% in any one given future year.

A lottery analogy may help you understand the last two points better. If you have played the lottery many times, your historical average rate of return is unlikely to be predictive of your future expected rate of return—especially if you have won it big at least once. Yes, you could trust it if you had millions of historical realizations, but you inevitably do not have so many. Consequently, your average historical payoff is only a mediocre predictor of your next week's payoff. And you should definitely not trust your most recent realization(s) to be indicative of the future. Just because "5, 10, 12, 33, 34, 38" won last week does not mean that it will likely win again.

To make life easier, most finance assumes that we know all the statistical distributions describing future expected rates of return. But remain mindful of this leap of faith.

Henceforth, like almost all of finance, we will just assume we know the statistical distributions from which future investment returns will be drawn. For exposition, this makes our task a lot easier. When you want to use our techniques in the real world, you will usually collect historical data and pretend that the future distribution is the same as the historical distribution. (Some investors in the real world use some more sophisticated techniques, but ultimately these techniques are also just variations on this theme.) However, always remember: historical data is an imperfect guide to the future.

7.2 Overview of Equity-Related Market Institutions

Why more info on equities?

Let's look into the institutional arrangements for equity trading. After all, from our corporate perspective, stocks are more interesting than many other financial instruments, such as foreign government bonds, even if there is more money in foreign government bonds than in corporate equity. After all, it is the equity holders who finance most of the risks of corporate projects. Moreover, although there is more money in nonequity financial markets, the subject area of investments often focuses on equities (stocks), too, because retail investors find it easier to buy stocks, and historical data for stocks is relatively easy to come by. So it makes sense to describe a few institutional details as to how investors and stocks "connect"—exchange cash for claims, and vice-versa.

Brokers

Retail brokers execute trades and keep track of portfolios. They also arrange shorts.

➤ Market maker, Sect. 7.2, Pg.155.

Most individuals place their orders to buy or sell stocks with a **retail broker**, such as *Ameritrade* (a "deep-discount" broker), *Charles Schwab* (a "discount" broker), or *Merrill Lynch* (a "full-service" broker). Discount brokers may charge only about $10 commission per trade, but they often receive "rebate" payments back from the market maker to which they route your order. This is called "payment for order flow." The market maker in turn recoups this payment to the broker by executing your trade at a price that is less favorable. Although the purpose of such an arrangement seems deceptive, the evidence suggests that discount brokers are still often cheaper in facilitating investor trades—especially small investor trades—even after taking this hidden payment into account. They just are not as (relatively) cheap as they want to make you believe. Investors can place either **market orders**, which ask for execution at the current price, or **limit orders**, which ask for execution if the price is above or below a limit that the investor can specify. (There are also many other modifications of orders, e.g., *stop-loss* orders [which instruct a broker to sell a security if it has lost a certain amount of money], *good-til-canceled* orders, and *fill-or-kill* orders.) The first function of retail brokers, then, is to handle the execution of trades. They usually do so by routing investors' orders to a centralized trading location (e.g., a particular stock exchange), the choice of which is typically at the retail broker's discretion, as is the particular individual (e.g., floor broker) engaged to execute the trade. The second function of retail brokers is to keep track of investors' holdings, to facilitate buying **on margin** (whereby investors can borrow money to buy stock, allowing them to buy more securities than they could afford on a pure cash basis), and to facilitate selling securities "short," which allows investors to speculate that a stock will go down.

Prime brokers leave execution to the client investor.

Many large institutional investors separate the two functions: The investor employs its own traders, while the broker takes care only of the bookkeeping of the investor's portfolio, margin provisions, and shorting provisions. Such limited brokers are called **prime brokers**.

How Shorting Stocks Works

Shorting is like borrowing and then issuing securities. The interest on the proceeds may be earned by the broker or by the client (or be shared).

If you want to speculate that a stock will go down, you would want to short it. This shorting would be arranged by your broker. Shorting is important enough to deserve an extended explanation:

- You find an investor in the market who is willing to lend you the shares. In a perfect market, this does not cost a penny. In the real world, the broker has to find a willing lender. Both the broker and lender usually earn a few basis points per year for doing you the favor of facilitating your short sale.
- After you have borrowed the shares, you sell them into the market to someone else who wanted to buy the shares. In a perfect market, you would keep the proceeds and earn interest on them. In the real world, your broker may force you to put these proceeds into low-yield safe bonds. If you are a small retail investor, your brokerage firm may even keep the interest proceeds altogether.
- When you want to "unwind" your short, you repurchase the shares and return them to your lender.

For example, if you borrowed the shares when they were trading for \$50 (and sold them into the market), and the shares now sell for \$30, you can repurchase them for \$20 less than what you sold them for into the market. This \$20 is your profit. In an ideal world, you can think of your role effectively as the same as that of the company—you can issue shares and use the \$50 proceeds to fund your investments (e.g., to earn interest). In the real world, you have to take transaction costs into account. (Shorting has become so common that there are now exchange-traded futures on stocks that make it even easier. Shorting is also common and easy for bonds.)

Shorting the market? Super easy!

Shorting the S&P 500 or some other market indices is even easier than shorting individual stocks. You can either short the relevant index ETF (explained below), which works the same way as shorting any other stock); or you can sell traded Futures on common stock market indexes.

Q 7.11. What are the two main functions of brokerage firms?

Q 7.12. How does a prime broker differ from a retail broker?

Q 7.13. Is your rate of return higher if you short a stock in the perfect world or in the real world? Why?

Exchanges and Non-Exchanges

The two big stock exchanges are the NYSE and NASDAQ. The NYSE is a hybrid market. The NASDAQ is solely electronic.

A retail broker would route your transaction to a centralized trading location. The most prominent are exchanges. An **exchange** is a centralized trading location where financial securities are traded. The two most important stock exchanges in the United States are the **New York Stock Exchange** (**NYSE**, also nicknamed the Big Board, established in 1792) and **NASDAQ** (originally an acronym for "National Association of Securities Dealers Automated Quotation System," established in 1971). The NYSE used to be exclusively an **auction market**, in which one designated **specialist** (assigned for each stock) managed the auction process by trading with individual brokers on the floor of the exchange. This specialist was often a monopolist. However, even the NYSE now conducts much of its trading electronically. In contrast to the NYSE's hybrid human-electronic process primarily in one physical location on Wall Street, NASDAQ has always been a purely electronic exchange without specialists. (For security reasons, its location—well, the location of its many computer systems—is secret.) For each NASDAQ stock, there is at least one **market maker**, a broker-dealer who has agreed to stand by continuously to offer to buy or sell shares—electronically of course—thereby creating a liquid and immediate market for the general public.

Moreover, market makers are paid for providing liquidity: They receive additional rebates from the exchange when they post a **bid** (short for bid price) or an **ask** (short for ask price) that is executed. (Outside investors can buy at least 100 shares at the quoted ask price or sell 100 shares at the quoted bid price. The ask price is always higher than the bid price.)

Most NASDAQ stocks have multiple market makers, drawn from a pool of about 500 trading firms (such as J.P. Morgan or ETrade), which compete to offer the best price. Market makers have one advantage over the general public: They can see the **limit order book**, which contains as-yet-unexecuted orders from investors to purchase or sell if the stock price changes—giving them a good idea at which price a lot of buying or selling activity will occur. The NYSE is the older exchange, and for historical reasons, is the most important exchange for trading most "blue chip" stocks. ("Blue chip" now means "well-established and serious." Ironically, the term itself came from poker, where the highest-denomination chips were blue.) In 2016, the NYSE listed just under 3,000 companies worth about $20 trillion (worth about half the annual U.S. GDP). On a typical day, about $170 billion change hands. NASDAQ tends to trade smaller and high-technology firms, lists about as many firms but its companies' market cap and dollar trading is only half that of the NYSE's. Some stocks are traded on both exchanges.

Auction markets, popular in other countries, have lower execution costs, but also slower execution speeds.

Continuous trading—trading at any moment an investor wants to execute—relies on the presence of the standby intermediaries (specialists or market makers), who are willing to absorb shares when no one else is available. This is risky business, and thus any intermediary must earn a good rate of return to be willing to do so. To avoid this cost, some countries have organized their exchanges into noncontinuous auction systems, which match buy and sell orders a couple of times each day. The disadvantage is that you cannot execute orders immediately but have to delay until a whole range of buy and sell orders have accumulated. The advantage is that this eliminates the risk that an (expensive) intermediary would otherwise have to bear. Thus, auctions generally offer lower trading costs but slower execution.

New alternative trading institutions: electronic communication networks (ECNs).

In the United States, innovation and change in stock trading are everywhere. For example, electronic communication networks (ECNs) have made big inroads into the trading business, often replacing exchanges, especially for large institutional trades. (They can trade the same stocks that exchanges are trading, and thus they compete with exchanges in terms of cost and speed of execution.) An ECN cuts out the specialist, allowing investors to post price-contingent orders themselves. ECNs may specialize in lower execution costs, higher broker kickbacks, or faster execution. The biggest ECNs are Archipelago and Instinet. In 2005, the NYSE merged with Archipelago and converted itself from a non-profit owned by its traders into a for-profit, and NASDAQ bought Instinet. It is hard to keep track of the most recent trading arrangements. For example, in 2006, the NYSE also merged with ArcaEx, yet another electronic trading system, and merged with Euronext (a pan-European stock exchange then based in Paris) in 2007 to become **NYSE Euronext**. In 2012, the whole exchange was acquired by the Intercontinental Exchange (ICE), a futures broker from Atlanta. Who knows who will own what in 5 years? It may not even matter—a lot of trading has become rather opaque, happening in-house instead of in-public. Even the NYSE is not really that important and irreplaceable these days any more. It may well be eclipsed or even disappear some day.

Crossing networks and more...

An even more interesting venue to buy and trade stocks are **crossing** systems, such as ITG's POSIT. ITG focuses primarily on matching large institutional trades with one another in an auction-like manner. If no match on the other side is found, the order may simply not be executed. But if a match is made, by cutting out the specialist or market maker, the execution is a lot cheaper than it would have been on an exchange. Recently, even more novel trading places have sprung up. For example, Liquidnet uses peer-to-peer networking—like the original Napster—to match buyers and sellers in real time. ECNs and electronic limit order books are now the dominant trading systems for equities worldwide, with only the U.S. exchange floors as holdouts. Similar exchanges and computer programs are also used to trade futures, derivatives, currencies, and even some bonds.

There are also informal financial markets, especially OTC (over-the-counter).

There are many other financial markets, too. There are financial exchanges handling stock options, commodities, insurance contracts, and so on. A huge segment is the **over-the-counter** (**OTC**) markets. Over-the-counter means "call around, usually to a set of traders well known to trade in the asset, until you find someone willing to buy or sell at a price you like." Though undergoing rapid institutional change, most bond transactions are still over-the-counter. Although OTC markets handle significantly more bond trading in terms of transaction dollar amounts than bond exchanges, OTC transaction costs are prohibitively high for retail investors. If you call without knowing the market in great detail, the person on the other end of the line will be happy to quote you a shamelessly high price, hoping that you do not know any better. The **NASD** (National Association of Securities Dealers) also operates a semi-OTC market for the stocks of smaller firms, which are listed on the so-called **pink sheets**. Foreign securities trade on their local national exchanges, but the costs for U.S. retail investors are again often too high to make direct participation worthwhile.

Q 7.14. How does a crossing system differ from an electronic exchange?

Q 7.15. What is a specialist? What is a market maker? When trading, what advantage do the two have over you?

Q 7.16. Describe some alternatives to trading on the main stock exchanges.

Investment Companies and Vehicles

The SEC regulates investment funds and advisors.

In 1933/1934, Congress established the *U.S. Securities and Exchange Commission* (**SEC**) through the *Securities Exchange Acts*. The SEC regulates investment advisors and funds according to the *Investment Advisers Act of 1940*. In practice, this has allowed three different types of regulated **investment companies** to operate in the public markets: open-end funds, closed-end funds, and unit investment trusts (UITs).

The "open end" feature allows investors to redeem their shares. It forces the fund's shares to trade for close to the value of its holdings.

➤ Law of One Price, Sect. 1.1, Pg.2.

In the United States, open-end fund is a synonym for mutual fund. (Elsewhere, mutual funds can include other classes.) Being **open end** means that the fund can create shares at will. Investors can also redeem their fund shares at the end of each trading day in exchange for the **net asset value** (**NAV**), which must be posted daily. This gives investors little reason to sell their fund shares to other investors—thus, mutual funds do not trade on any exchanges. The redemption right gives the law of one price a lot of bite—fund shares are almost always worth nearly exactly what their underlying holdings are worth. If an open-end fund's share price were to fall much below the value of its holdings, an arbitrageur could buy up the fund shares, redeem them, and thereby earn free money. (One discrepancy is due to some odd tax complications: the fund's capital gains and losses are passed through to the fund investors at the end of every year, but they may not be what every investor experienced.) Interestingly, in the U.S. financial markets, there are now many more stock funds than individual stocks.

Closed-end funds do not allow shares to be redeemed. This is useful for funds which are investing in illiquid assets.

In a **closed-end fund**, there is one big initial primary offering of fund shares, and investors cannot redeem their fund shares for the underlying value. The advantage of a closed-end fund is that it can itself invest in assets that are less liquid. After all, it may not be forced to sell its holdings on the whims of its own investors. Many closed-end funds are exchange-traded, so that if a closed-end fund investor needs cash, she can resell her shares. The disadvantage of the closed-end scheme is that the law of one price has much less bite. On average, closed-end funds trade persistently below the value of their underlying holdings, roughly in line with the (often high) fees that the managers of many of these closed-end funds are charging.

Mutual funds are open-ended, actively traded investment vehicles.

Both mutual funds and closed-end fund managers are allowed to trade fund holdings quite actively—and many do so. Although some funds specialize in imitating common stock market indexes, many more try to guess the markets or try to be more "boutique." Most funds are classified into a category based on their general trading motivation (such as "market timing," or "growth" or "value," or "income" or "capital appreciation").

UITs are passive "basket" investment vehicles.

A **unit investment trust** (**UIT**) is sort of closed-end in its creation (usually through one big primary offering) and sort of open end in its redemption policies (usually accepting investor redemption requests on demand). Moreover, regulatory rules forbid UITs to trade actively (although this is about to change), and UITs must have a fixed termination date (even if it is 50 years in the future). UITs can be listed on a stock exchange, which makes it easy for retail investors to buy and sell them. Some early **exchange-traded funds** (**ETFs**) were structured as UITs, although this required some additional legal contortions that allowed them to create more shares on demand. This is why ETFs are nowadays usually structured as open-end funds. ETFs can compete with mutual funds in offering a myriad of portfolio baskets. Nowadays, there are more stock ETFs and mutual funds than there are stocks themselves!

ADRs are investment vehicles, too. Many ADRs (though not all) are regulated by the SEC under different rules.

Some other investment vehicles are regulated by the SEC under different rules. The most prominent may be the **American Depositary Receipt** (**ADR**). An ADR is a passive investment vehicle that usually owns the stock of only one foreign security, held in escrow at a U.S. bank (usually the Bank of New York). ADRs make it easier for U.S. retail investors to trade in foreign securities without incurring large transaction costs. ADRs are redeemable, which gives the law of one price great bite.

Other funds are entirely unregulated.

There are also funds that are structured so that they do not need to register with the SEC. This means that they cannot openly advertise for new investors and are limited to fewer than 100 investors. This includes most **hedge funds**, **venture capital** funds, and other **private equity** funds. Many **offshore funds** are set up to allow foreign investors to hold U.S. stocks not only without SEC regulation, but also without ever having to tread into the domain of the U.S. IRS.

Q 7.17. What should happen if the holdings of an open-end fund are worth much more than what the shares of the fund are trading for? What should happen in a closed-end fund?

Q 7.18. What is the OTC market?

Q 7.19. What are the three main types of investment companies as defined by the SEC? Which is the best deal in a perfect market?

High-Frequency Trading (HFT)

The computerization of exchanges has brought us **high-frequency traders** (**HFTs**). They generate a lot of trading volume (and even more quotes), but their net price impact for buy-and-hold investors is probably modest. There is some debate about whether HFTs add liquidity to the financial markets or siphon it off. Because their activity is both anonymous and dark to independent academic inquiry, we can only guess.

Your first question should be "How can HFTs make money in a competitive market?" The answer is that if there are non-HFT traders, such as retail investors that have posted limit orders and do not revise them quickly enough when new information arrives, then the fastest HFT can pick off their limit offers. See, markets are not really competitive in nano-seconds. There is only one HFT at the very nano-second it pounces on a standing limit order, even if another trader would have loved to pay an epsilon more a nano-second later. Thus, there has been an arms race among HFTs to be faster than others—and the speed of light has literally become the constraint! Some observers have suggested bunching orders into auctions once every second, but it is hard to know what the optimum here is (1ns? 1ms? 1s? 1h?). Besides, even if HFTs were a serious problem, it is not clear whether government intervention would improve or worsen the situation. (And, as more and more HFTs have come into the market, they have begun to competing and adding more liquidity, simply by being present.)

How Securities Appear and Disappear

Inflows

Firms first sell public shares in IPOs.

Most publicly traded equities appear on public exchanges, almost always NASDAQ, through **initial public offerings** (**IPOs**). This is an event in which a privately traded company first sells shares to ordinary retail and institutional investors. IPOs are usually executed by **underwriters** (investment bankers such as Goldman Sachs or Bank of America's Merrill Lynch), which are familiar with the complex legal and regulatory process and have easy access to an investor client base to buy the newly issued shares. Shares in IPOs are typically sold at a fixed price—and for about 10% below the price at which they are likely to trade on the first day of after-market open trading. (Many IPO shares are allocated to the brokerage firm's favorite customers, and they can be an important source of profit.)

Trading Volume in the Tech Bubble

During the tech bubble of 1999 and 2000, IPOs appreciated by 65% on their opening day *on average*. Getting an IPO share allocation was like getting free money. Of course, ordinary investors rarely received any such share allocations—only the underwriter's favorite clients did. This later sparked a number of lawsuits, one of which revealed that Credit Suisse First Boston (CSFB) allocated shares of IPOs to more than 100 customers who, in return for IPO allocations, funneled between 33% and 65% of their IPO profits back to CSFB in the form of excessive trading of other stocks (like Compaq and Disney) at inflated trading commissions.

How important was this "kickback" activity? In the aggregate, in 1999 and 2000, underwriters left about $66 billion on the table for their first-day IPO buyers. If investors rebated 20% back to underwriters in the form of extra commissions, this would amount to $13 billion in excessive underwriter profits. At an average commission of 10 cents per share, this would require 130 billion shares to be traded, or an average of 250 million shares per trading day. This figure suggests that kickback portfolio churning may have accounted for as much as 10% of all shares traded!

Ritter and Welch (2002)

Money also flows into the financial markets through SEOs.

Usually, about a third of the company is sold in the IPO, and the typical IPO offers shares worth between $20 million and $100 million, although some are much larger (e.g., privatizations, like British Telecom). About two-thirds of all such IPO companies never amount to much or even die within a couple of years, but the remaining third soon thereafter offer more shares in **seasoned equity offerings** (**SEOs**). These days, however, much expansion in the number of shares in publicly traded companies—especially for large companies—comes not from seasoned equity offerings but from employee stock option plans, which eventually become unrestricted publicly traded shares.

The behavior at the IPO and subsequently is also regulated by the SEC.

The SEC is also in charge of regulating some behavior of publicly traded companies. This includes how they conduct their IPOs. It also describes how they have to behave thereafter. For example, publicly traded companies must regularly report their financials and some other information. Moreover, Congress has banned **insider trading** on unreleased *specific* information, although more general informed trading by insiders is legal (and seems to be done fairly commonly and profitably). Moreover, there are loopholes that allow smart CEOs and politicians to trade legally on inside information. (These do not apply to funds and external investors.) The SEC can only pursue civil fines. If there is fraud involved, then it is up to the states to pursue criminal sanctions, which they often do simultaneously. (Publicly traded firms also have to follow a hodgepodge of other federal and state laws.)

A reverse merger has become another common way to enter the public financial markets.

Because IPOs face unusually complex legal regulations and liability, the alternative of **reverse mergers** has recently become prominent. A larger privately-owned company simply merges with a small company (possibly just a shell) that is already publicly traded. The owners of the big company receive newly issued shares in the combined entity. And, of course, the newly issued shares in effect move private-sector assets into the public markets, where the firms' assets then appear in the form of additional market capitalization.

Outflows

Money flows out from the financial markets via dividends and share repurchases.

➤ Dividend irrelevance, Sect. 20.2, Pg.558.

Capital flows out of the financial markets in a number of ways. The most important venues are capital distributions such as dividends and share repurchases. Many companies pay some of their earnings in dividends to investors. Dividends, of course, do not fall like manna from heaven. For example, a firm worth \$100,000 may pay \$1,000, and would therefore be worth \$99,000 after the dividend distribution. If you own a share of \$100, you would own (roughly) \$99 in stock and \$1 in dividends after the payment—still \$100 in total, no better or worse. (If you have to pay some taxes on dividend receipts, you might come out for the worse.) Alternatively, firms may reduce their outstanding shares by paying out earnings in **share repurchases**. For example, the firm may dedicate the \$1,000 to share repurchases, and you could ask the firm to use \$100 thereof to repurchase your share. But even if you hold onto your share, you have not lost anything. Previously, you owned $\$100/\$100{,}000 = 0.1\%$ of a \$100,000 company, for a net of \$100. Now, you will own $\$100/\$99{,}000 \approx 1.0101\%$ of a \$99,000 company—multiply this to find that your share is still worth \$100. In either case, the value of outstanding public equity in the firm has shrunk from \$100,000 to \$99,000. We will discuss dividends and share repurchases in Chapter 20.

Shares can also shrink out of the financial markets in bankruptcies, liquidations, and delistings.

Firms can also exit the public financial markets entirely by **delisting**. Delistings usually occur either when a firm is bought by another firm or when it runs into financial difficulties so bad that they fail to meet minimum listing requirements. Often, such financial difficulties lead to Chapter 11 bankruptcy or Chapter 7 liquidation (so named for their chapters in the U.S. Bankruptcy Code). Some firms even voluntarily liquidate, determining that they can pay their shareholders more if they sell their assets and return the money to them. This is rare because managers usually like to keep their jobs—even if continuation of the company is not in the interest of shareholders. More commonly, firms make bad investments and fall in value to the point where they are delisted from the exchange and/or go into bankruptcy. Unfortunately, this usually means that equity investors usually lose all their investment. Fortunately, equity investors also enjoy **limited liability**,, which means that they can at most lose their original investment. They do not have to pay further for the sins of the firm with their other personal assets (unlike partners, such as“names” in Lloyd's of London, who are on the hook with everything they own personally).

➤ Limited liability, Sect. 6.4, Pg.125.

Q 7.20. What are the main mechanisms by which money flows from investors into firms?

Q 7.21. What are the institutional mechanisms by which funds disappear from the public financial markets back into the pockets of investors?

Q 7.22. How do shares disappear from the stock exchange?

Summary

This chapter covered the following major points:

- Exhibits 7.4 and 7.6 showed an analysis of historical rate of return patterns of investments in U.S. cash, bonds, stock indexes, and individual stocks.
  - Stocks, on average, had higher average rates of return than long-term bonds, which in turn had higher average rates of return than cash investments.
  - Individual stocks were riskiest. (Large-firm-type) stock market portfolios had lower risk than individual stock holdings. Bonds had modestly lower risk. Cash was least risky.
  - Stocks have outperformed cash by more than 5% per annum. However, they have only modestly outperformed long-term bonds.
- Stocks (and many other investments) have tended to correlate positively: When the stock market overall has had a good year, most individual stocks have also tended to have a good year (and vice-versa). No one knows why, but long-term bonds have tended to correlate negatively with the stock market over the last few decades (but not before).
- Most finance assumes that statistics are known. This is a leap of faith. In real life, historical data can help you in predicting the future, but it is not perfect. Historical risks and correlations are good predictors of their future equivalents; historical means may not be.
- Section 7.2 explained many institutional arrangements governing publicly traded equity securities. This includes the roles of retail and prime brokers, exchanges, and funds. It also described how stocks can be shorted, and how funds flow in and out of the financial markets.

Keywords

Answers

Q 7.1 A time-series graph shows how individual years matter. This can no longer be seen in a histogram.

Q 7.2 A histogram makes it easier to see how frequent different types of outcomes are—and thus, where the distribution is centered and how spread out it is.

Q 7.3 A compound return graph shows how a time series of rates of return interacts to produce long-run returns. In other words, you can see whether a long-run investment would have made or lost money. This is difficult to see in a time-series graph.

Q 7.4 Note that because the returns in (b) and (c) alternate, you just need to work out the safe two-year returns—thereafter, they will continue in their (unrealistic) patterns.

1. 5% for both.
2. Over two years, you earn $1.00 \cdot 1.10 - 1 = 10.00\%$. This means

that the annualized rate of return is $\sqrt{1.1}-1 \approx 4.88\%$. This is lower than the average rate of return, which is still 5%.

3. Over two years, you earn $0.9 \cdot 1.20 - 1 = 8.00\%$. This means that the annualized rate of return is $\sqrt{1.08}-1 \approx 3.92\%$. This is lower than the 5% average rate of return.

Yes. The difference between its annualized and its average rate of return is greater for a more volatile investment.

Q 7.5 The difference is 0, because the risk-free rate has no standard deviation.

Q 7.6 The risk is usually increasing: lowest for cash, then bonds, then the stock market portfolio, and finally individual stocks. The average reward is increasing for the first three, but this is not necessarily true for an individual stock.

Q 7.7 Usually (but not always), individual stocks are riskier.

Q 7.8 Yes. For example, look at UAL in Exhibit 7.6. It lost everything but still had a positive average arithmetic rate of return.

Q 7.9 To graph the market beta, the rate of return on the market (e.g., the S&P 500) should be on the x-axis, and the rate of return on the investment for which you want to determine the market beta should be on the y-axis. A data point is the two rates of return from the same given time period (e.g., over a year). The market beta is the slope of the best-fitting line.

Q 7.10 The market beta of the market is 1—you are plotting the rate of return on the market on both the x-axis and the y-axis, so the beta is the slope of this 45° diagonal line.

Q 7.11 Brokers execute orders and keep track of investors' portfolios. They also facilitate purchasing on margin.

Q 7.12 Prime brokers are usually used by larger investors. Prime brokers allow investors to employ their own traders to execute trades. (Like retail brokers, prime brokers provide portfolio accounting, margin, and securities borrowing.)

Q 7.13 Your rate of return is higher if you short a stock in the perfect world because you earn interest on the proceeds. In the real world, your broker may help himself to this interest.

Q 7.14 A crossing system does not execute trades unless there is a counterparty. It also tries to cross orders a few times a day.

Q 7.15 The specialist is often a monopolist who makes the market on the NYSE. The specialist buys and sells from her own inventory of a stock, thereby "making a market." Market makers are the equivalent on NASDAQ, but there are usually many and they compete with one another. Unlike ordinary investors, both specialists and market makers can see the limit orders placed by other investors.

Q 7.16 The alternatives are often electronic, and they often rely on matching trades—thus, they may not execute trades that they cannot match. Electronic communication networks are the dominant example of these. Another alternative is to execute the trade in the over-the-counter (OTC) market, which is a network of geographically dispersed dealers who are making markets in various securities.

Q 7.17 In an open-ended fund, you should buy fund shares and request redemption. (You could short the underlying holdings during the time you wait for the redemption in order not to suffer price risk.) In a closed-ended fund, you would have to oust the management to allow you to redeem your shares.

Q 7.18 The OTC is not really a market. Instead, it simply means that traders handle transactions on a one-on-one basis.

Q 7.19 UITs, open-ended funds (mutual funds), and closed-ended investment funds. In a perfect market, none is the best deal. You always get what you pay for.

Q 7.20 The main mechanisms by which money flows from investors into firms are (a) IPOs and SEOs, and (b) reverse mergers, which are then sold off to investors.

Q 7.21 Funds disappear from the public financial markets back into the pockets of investors through dividends and share repurchases.

Q 7.22 Shares can disappear in a delisting or a repurchase.

End of Chapter Problems

Q 7.23. Using the information in Exhibit 7.4, compute the discrepancy between arithmetic and geometric rates of return for cash and stocks. Which one is lower? Why?

Q 7.24. Broadly speaking, what was the average rate of return on cash, bonds, and stocks? What time period are your numbers from?

Q 7.25. Broadly speaking, what was the average risk of cash, bonds, and stocks? What time period are your numbers from?

Q 7.26. How good are historical statistics as indicators of future statistics? Which kinds of statistics are better? Which kinds are worse?

Q 7.27. Does the market beta of stocks in the market average out to zero?

Q 7.28. Give an example in which a stock had a positive average rate of return, even though it lost its investors' money.

Q 7.29. Are stock funds or bond funds that quote historical average rates of return more misleading? Would you have ended up with more money in a stock fund or a bond fund if they both quoted similar historical mean rate of return performances?

Q 7.30. Looking at the figures in this chapter, did 20-year bonds move with or against the U.S. stock market?

Q 7.31. Do individual stocks tend to move together? How could this be measured?

Q 7.32. Explain the differences between a market order and a limit order.

Q 7.33. What extra function do retail brokers handle that prime brokers do not?

Q 7.34. Describe the differences between the NYSE and NASDAQ.

Q 7.35. Roughly, how many firms are listed on the NYSE? How many are listed on NASDAQ? Then use a financial website to find an estimate of the current number.

Q 7.36. Is NASDAQ a crossing market?

Q 7.37. What are the two main mechanisms by which a privately held company can go public?

Q 7.38. When and under what circumstances was the SEC founded?

Q 7.39. Insider trading is a criminal offense. Does the SEC prosecute these charges to put violators behind bars?

Q 7.40. What is the OTC market?

Q 7.41. If a firm repurchases 1% of its shares, does this change the capitalization of the stock market on which it lists? If a firm pays 1% of its value in dividends, does this change the capitalization of the stock market on which it lists?

8

Investor Choice: Risk and Reward

We are still after the same prize: a good estimate of the corporate cost of capital $\mathsf{E}(r)$ in the NPV formula. But before you can understand the opportunity costs of capital for your firm's own projects, you have to understand your investors' other opportunities. This means that you must understand better what investors like (reward) and what they dislike (risk), how they are likely to measure their risks and rewards, how diversification works, what portfolios smart investors are likely to hold, and why it matters that "market beta" is a good measure of an investment asset's contribution to the market portfolio's risk.

8.1 Measuring Risk and Reward

We work with five assets that have four equally likely outcomes.

Put yourself into the shoes of an investor and start with the most basic questions: How should you measure the risk and reward of your portfolio? As always, we first cook up a simple example and then generalize our insights into a broader real-world context. Say you are currently investing in an asset named *M*, short for "My Portfolio," but there are also other assets you could buy, named *A* through *C*, plus a risk-free asset named *F*. These assets could themselves be portfolios, themselves consisting of many individual assets and/or yet other portfolios. (This is essentially what a mutual fund is.) So, let's just call M, A, B, C, and F themselves portfolios, too.

Historical samples can be viewed as scenarios.

We will work with four equally likely scenarios, named S1 through S4, for each of the five portfolios. The outcomes, means, and risks are laid out in the tabular portfolio of Exhibit 8.1. Each scenario gets a card deck suit to remind you that it is a random draw. (If you find it easier to think in terms of historical outcomes, you can pretend that you are analyzing historical data: scenario S1 happened at time 1, S2 at time 2, and so forth. This is not entirely correct, but it is often a helpful metaphor.) Which investment strategies do you deem better or worse, safer or riskier? If you can buy only these portfolios, what trade-offs of risk and reward are you facing?

➤ Why this is not entirely correct, Pg.183.

Graphics version of the table.

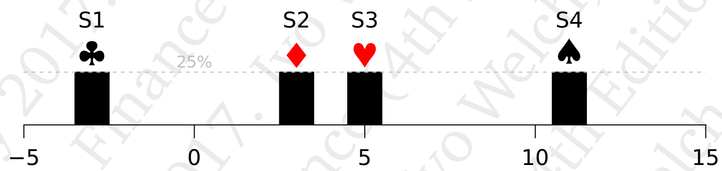
If you like visuals, Exhibit 8.1 also shows these returns in graphic form. The middle figure is the standard histogram, which you have seen many times elsewhere. However, each scenario is equally likely (the bars are equally tall), so it's more visually obvious to just put the card suit symbols where the bar is. This is what we do in the lower figure. It makes it easier to compare many different investments.

In a histogram, bars to the right mean higher returns. Bars that are more spread out indicate higher risk.

In this plot, you prefer assets that have scenario outcomes farther to the right (they have higher returns), outcomes that are *on average* farther to the right (they have higher *expected* rates of return), and outcomes that are more bunched together (they have less risk). Visual inspection confirms that investment F has outcomes perfectly bunched at the same spot, so it is

➤ Random variables *are* histograms, Pg.105.

	In S1 (♣)	In S2 (♦)	In S3 (♥)	In S4 (♠)	Reward E(r)	Variance[a] Var(r)	Risk Sdv(r)
Investment M	–3%	3%	5%	11%	4%	25%%	5%
Investment A	3%	11%	–3%	5%	4%	25%%	5%
Investment B	5%	–1%	7%	13%	6%	25%%	5%
Investment C	17%	3%	11%	–7%	6%	81%%	9%
Investment F	1%	1%	1%	1%	1%	0%%	0%



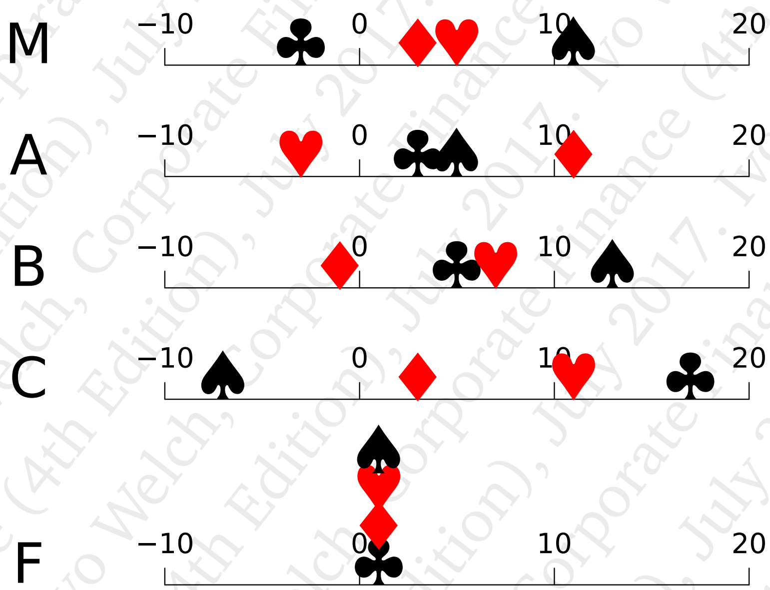


Exhibit 8.1: *Rates of Return on Five Investment Assets.* There are only four possible future scenarios, S1 through S4, each equally likely and indicated with a card suit. There are only 5 available investments (M, A, B, C, and F). (These could themselves be portfolios, of course.) The variance (Var) and standard deviation (Sdv) were explained in Section 6.1. The middle figure is a "traditional" histogram of M. The bottom figure contains the "condensed" histograms for all 5 assets.

Table note [a]: We use the '%%' notation only for variance computations. Just like '%' means 'divide by 100', '%%' means 'divide by 100 and then divide by 100 again', i.e., 'divide by 10,000'. This makes it easy to see that $\sqrt{(5\%)^2 + (5\%)^2 + (5\%)^2 + (5\%)^2}$ is $\sqrt{(25\%\% + 25\%\% + 25\%\% + 25\%\% = 100\%\%)} = 10\%$. If you find it easier to read $\sqrt{(0.0025 + 0.0025 + 0.0025 + 0.0025 = 0.01)} = 10\%$, then be my guest and use this notation instead. The answers are always the same.

not just least risky but in fact completely risk-free. It is followed by the risky M and A, then B, and finally, the riskiest investment, C.

Measuring Reward: The Expected Rate of Return

Measure reward with the expected rate of return.

Although graphical measures are helpful, we really need formulas to give us numerical measures. A good measure for the **reward** is easy: You can use the **expected rate of return**, which is the probability-weighted average of all possible returns. For example, the mean rate of return for your portfolio M is

$$\mathsf{E}(r_M) = (1/4)\cdot(-3\%) + (1/4)\cdot(+3\%) + (1/4)\cdot(+5\%) + (1/4)\cdot(+11\%) = +4\%$$
$$= \text{Sum of (each probability times its outcome)}$$

If you invest in M, you would expect to earn a rate of return of 4%. Because each outcome is equally likely, you can compute this faster as a simple average,

$$\mathsf{E}(r_M) = [(-3\%) + (+3\%) + (+5\%) + (+11\%)]/4 = 4\%$$

Measuring Risk: The Standard Deviation of the Rate of Return

Measure risk with the standard deviation of the rate of return.

➤ The standard deviation (measure of risk), Sect. 6.1, Pg.108.

A good measure of risk is less obvious than a good measure of reward, but fortunately you already learned a good measure—the standard deviation—in Section 6.1. Let's compute it in the context of our assets. We first write down how far away each point is from the center (average). You just saw that the average for M was +4%. An outcome of +3% would be closer to the mean than an outcome of –3%. The former is only 1 unit away from the mean. The latter is 7 units away from the mean.

	In S1 (♣)	In S2 (♦)	In S3 (♥)	In S4 (♠)
Asset M Rate of Return	–3%	+3%	+5%	+11%
...in deviation from its 4% mean	–7%	–1%	1%	+7%

The average deviation from the mean is always 0. It cannot measure risk.

Unfortunately, you cannot compute risk as the average deviation from the mean, which is always zero ($[-7+(-1)+1+7]/4=0$). You must first "neutralize" the sign, so that negative deviations count the same as positive deviations. The "fix" is to compute the average *squared* deviation from the mean. This is the **variance**:

$$\mathsf{Var}(r_M) = 1/4\cdot(-3\%-4\%)^2 + 1/4\cdot(3\%-4\%)^2 + 1/4\cdot(5\%-4\%)^2 + 1/4\cdot(11\%-4\%)^2$$
$$= [(-7\%)^2 + (-1\%)^2 + (+1\%)^2 + (+7\%)^2]/4 = 25\%\% \qquad (8.1)$$
$$= \text{Sum of (each probability times its squared-deviation-from-the-mean)}$$

The variance has units that are intrinsically impossible to interpret by humans (% *squared* $= 0.01\cdot0.01$, written as x%%). Therefore, the variance carries very little intuition, except that *more variance means more risk*.

The standard deviation of the portfolio's rate of return is a common measure of risk.

The measure that has more humanly meaningful (humane?) units is the **standard deviation**, which is just the square root of the variance:

$$\text{Sdv}(r_M) = \sqrt{\text{Var}(r_M)} = \sqrt{25\%\%} = 5\% \tag{8.2}$$

*The standard deviation of the portfolio's rate of return is the most common measure of overall **portfolio risk**.* Now look again at Exhibit 8.1. You can see that this standard deviation of 5% seems like a reasonable measure of how far the typical outcome of M is away from the overall mean of M. (However, 5% is more than the average absolute deviation from the mean, which in this case would be 4%; the standard deviation puts more weight on far-away outcomes than the average absolute deviation.) The last column in Exhibit 8.1 lists the standard deviations of all investments. As the visuals indicate, F is risk-free; M, A, and B are equally risky at 5%; and C is riskiest at a whopping 9%.

IMPORTANT

- You can measure investment portfolio reward by the expected rate of return on the *overall* portfolio.
- You can measure investment portfolio risk by the standard deviation of the rate of return on the *overall* portfolio.

(Warning: You will not measure the investment risk *contributions* of individual assets *inside* a portfolio via their standard deviations. This will be explained in Section 8.3.)

A preview: Smart investors eliminate unnecessary risk. After they have done so, more reward requires taking more risk.

At this point, you should begin to wonder how risk and reward are related in a reasonable world. This will be the subject of much of the next chapter. The brief answer for now is that you can speculate in dumb ways that give you high investment risk with low reward—as anyone who has gambled knows. However, if you are smart, after eliminating all investment mistakes (the low-hanging fruit), you have no choice but to take on more risk if you want to earn higher rewards.

Q 8.1. What happens if you compute the average deviation from the mean, rather than the average squared deviation from the mean?

Q 8.2. Asset M from Exhibit 8.1 offers –3%, +3%, +5%, and +11% with equal probabilities. Now add 5% to each of these returns. This new asset offers +2%, +8%, +10%, and +16%. Compute the expected rate of return, the variance, and the standard deviation of this new asset. How does it compare to the original M?

Q 8.3. Confirm the risk and reward of C in Exhibit 8.1.

Nerdnote: It would be really convenient if we could quote all gambles on the same terms. We could then easily compare them, like apples to apples. Fortunately, such a measure exists. It is called the "certainty equivalence." Unfortunately, it depends on a more complex model of the world, and it is notoriously difficult to get used to. Thus, we will cover it only in the companion Web chapter.

8.2 Diversification

In the real world, you are usually not constrained to buy assets in isolation—you can buy a little bit of many assets. This ability to buy many assets has the important consequence of allowing you to reduce your overall portfolio risk. Let's see why.

Many assets at the same time.

An Example Mixing Portfolio

Start again with your portfolio M. Now let's consider adding some of portfolio A. Why would you? It has the same risk and reward as M. However, although A has the same list of possible returns, *it offers them in different scenarios*. This rearrangement will make a lot of difference. So, let's say you have \$100 in M, but you now sell half of these holdings to buy A. You will have \$50 in M and \$50 in A. Let's call this investment portfolio MA. In this case, your \$100 investment would look like this:

Portfolios are bundles of multiple assets. Their returns can be averaged.

	In S1 (♣)	In S2 (♦)	In S3 (♥)	In S4 (♠)	Average
Return on \$50 in M:	\$48.50	\$51.50	\$52.50	\$55.50	\$52.00
Return on \$50 in A:	\$51.50	\$55.50	\$48.50	\$52.50	\$52.00
⇒ Total return in MA:	\$100.00	\$107.00	\$101.00	\$108.00	\$104.00
Rate of return in MA:	0%	7%	1%	8%	4%

You could have computed this more quickly by using the returns on M and A themselves. Your portfolio MA invests portfolio weight $w_M = 50\%$ into M and $w_A = 50\%$ in A. For example, to obtain the 7% in scenario S2, you could have computed the portfolio rate of return from M's 3% rate of return and A's 11% rate as

$$r_{MA} = r_{MA=50\% \text{ in M},50\% \text{ in A (all in S2)}} = 50\% \cdot 3\% + 50\% \cdot 11\% = 7\%$$

$$r_{MA=(w_M,w_A) \text{ in S2}} = w_M \cdot r_{M \text{ in S2}} + w_A \cdot r_{A \text{ in S2}}$$

Now let's look at these three portfolios (M, A, and MA) in a histogram. Even better, because our histogram bars are all equally tall, we can omit the bars and plot just the symbols. As Exhibit 8.2 shows, the range of M is from –3% to +11%; the standard deviation is 5%. The range of A is also from –3% to +11%; the standard deviation is also 5%. Yet the average of M and A has a much lower range (0% to 8%) and a much lower standard deviation:

Visually, the M and A combination portfolio called MA has lower variability (risk and range) than either M or A.

$$\text{Var}_{\substack{50\% \text{ in M} \\ 50\% \text{ in A}}} = \frac{(0\% - 4\%)^2 + (7\% - 4\%)^2 + (1\% - 4\%)^2 + (8 - 4\%)^2}{4} = 12.5\%\%$$

$$= \frac{[r_{S1} - \text{E}(r)]^2 + [r_{S2} - \text{E}(r)]^2 + [r_{S3} - \text{E}(r)]^2 + [r_{S4} - \text{E}(r)]^2}{N}$$

$$\Longrightarrow \quad \text{Sdv}_{50\% \text{ in M},\ 50\% \text{ in A}} = \sqrt{\text{Var}} = \sqrt{12.5\%\%} \approx 3.54\%$$

MA is simply less risky than either of its ingredients.

The reason for this reduction in risk is **diversification**—the mixing of different investments within a portfolio that reduces the impact of each one on the overall portfolio performance. More simply put, diversification means that not all of your eggs are in the same basket. If one investment component goes down, the other investment component sometimes happens to go up, or vice-versa. The imperfect correlation ("non-synchronicity") reduces the overall portfolio risk.

This is caused by diversification.

	In S1 (♣)	In S2 (♦)	In S3 (♥)	In S4 (♠)	Reward $\mathsf{E}(r)$	Variance $\mathsf{Var}(r)$	Risk $\mathsf{Sdv}(r)$
Investment M	–3%	3%	5%	11%	4%	25%%	5%
Investment A	3%	11%	–3%	5%	4%	25%%	5%
Portfolio MA	0%	7%	1%	8%	4%	12.5%%	3.54%

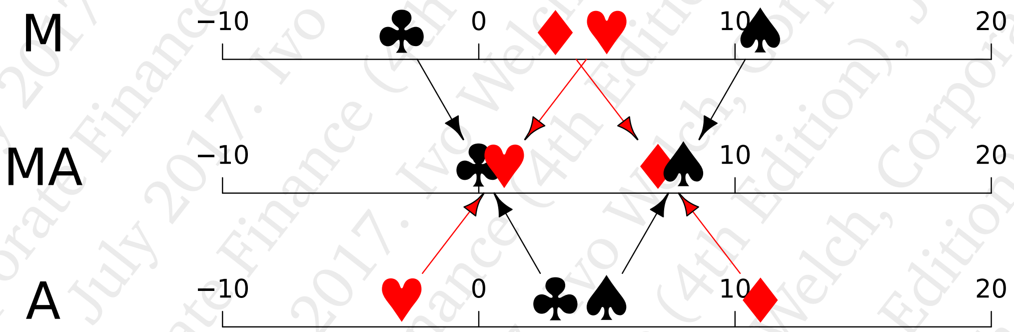


Exhibit 8.2: *Rate of return outcomes for M, A, and the (50%, 50%) combination portfolio MA.* Because each half-M/half-A point is halfway between M and A, MA has lower spread (risk) than either of its components, M and A, by themselves.

Q 8.4. The combination portfolio named MA invests 90% in M and 10% in A.

1. Compute its risk and reward.
2. In a plot similar to those in Exhibit 8.1, would this new MA portfolio look less spread out than the MA = (50%, 50%) portfolio that was worked out in the table in Exhibit 8.2?

How Risk Grows With Time

Brief important diversions.

Before we continue, I need to cover two aspects that fit more into the subfield of investments than into the subfield of corporate finance. But both are important for a general competence in finance. We will look only at them in passing.

If two variables are uncorrelated, the variance of the sum is the sum of the variances.

The first diversion is about how risk grows with time. Trust me on the following: If two random draws are independent, then the sum of these two random variables has a variance that is the sum of the two variances.

$$\mathsf{Var}(X+Y) \;=\; \mathsf{Var}(X) + \mathsf{Var}(Y) \qquad \text{if X and Y are uncorrelated}$$

(This is not true if the two variables move together!) Why should you care? Well, the rates of return of any one asset in a perfect market should be uncorrelated over time—if not, you could earn an extra rate of return by trading this asset based on its own lagged return. (If the correlation were positive, you would get rich quick by buying the asset *after* it has gone up and selling it *after* it has gone down.)

Now let's use an approximation: Ignore compounding. This means that the total return is approximately the sum of the consecutive returns. Now, if you expect a stock to earn a 10% mean expected rate of return with a standard deviation of 20% over one year ($20\% \cdot 20\% = 400\%\%$ variance), then over two years, you expect the same stock to earn 20% with a variance of $400\%\% + 400\%\% = 800\%\%$. Thus, this stock's risk (standard deviation) is $\sqrt{800\%\%} \approx 28.28\%$. In other words, its mean goes up by a factor of 2, but its risk goes up only by a factor of $\sqrt{2} \approx 1.4$.

Stocks have uncorrelated returns. Thus, with time, the risk grows more slowly than the reward.

Risk grows approximately with the squareroot of time.

IMPORTANT

Q 8.5. Please ignore compounding in this question:

1. What is the risk and reward of the "10% mean, 20% risk" investment that we just discussed in the text over 4 years? What is your reward-risk ratio? (This ratio is called the **Sharpe ratio** and often confusingly called a risk-reward ratio.)
2. What is the risk and reward of the same 10% mean, 20% risk investment over 9 years? What is the Sharpe ratio?
3. Can you guess what the risk and reward of a stock with annual mean E and risk Sdv are over T years? What is the Sharpe ratio?

The Best Mixing Portfolio — The Efficient Frontier

The second diversion is not just about how you calculate the risk and reward of a given portfolio, but how the *best* possible portfolios looks like. And how well can your best portfolios do? The details of this question are covered better in this chapter's appendix (in the companion Web chapter), but this section gives you a good though basic flavor.

Finding the best choice.

Exhibit 8.3 plots the investment performance (mean and standard deviation) of various portfolio combinations. Each portfolio has a unique spot in this coordinate system. This plot is very common and familiar to all financiers. In such a plot, you want a portfolio that is higher on the y-axis (has a higher expected rate of return) and lower on the x-axis (has a lower standard deviation). That is, you would always want to slide towards the north-west (up-left) if you can. One says a portfolio is *inside* the efficient frontier if it is south-east of another achievable portfolio, and *on* the frontier if there is no portfolio north-west

What is the typical mean-variance plot?

In the top-left plot, you can invest only in M and A. They are both at the same spot in the plot. Because both have a 4% mean rate of return, any combination of them does, too. The best (lowest risk for given mean) portfolio is the left-most one, which happens to be the equal-weighted combination. The top-right plot allows you to invest not only in M and A, but in B, too. You can see that B helps greatly, but not because you would buy it by itself. In fact, B itself is far inside the north-west boundary—the **efficient frontier**—which is the lowest-risk highest-reward set of portfolios. (Its shape is always a hyperbola.) Presumably, smart investors would buy only portfolios on this efficient frontier. Anything inside (south-east) of the frontier is worse. Anything north-west of it is not obtainable. The equal-weighted portfolio is close to, but not on the efficient frontier. This is often the case for large diversified portfolios—the S&P 500 is reasonably close, but not exactly on the efficient frontier. The bottom-left plot allows you to invest in C, too. You can see how this expands the efficient frontier even further. In fact, it is now possible to create a risk-free asset with a rate of return of about 4.5% by cleverly combining investments. (Not that clever—invest about 0.377 in M, 0.261 in A, 0.091 in B, and 0.272 in C.) But even if you do not want to play it safe, you can always do at least as well with more assets

More assets expand your opportunity set. The best investment choices are on the "efficient frontier."

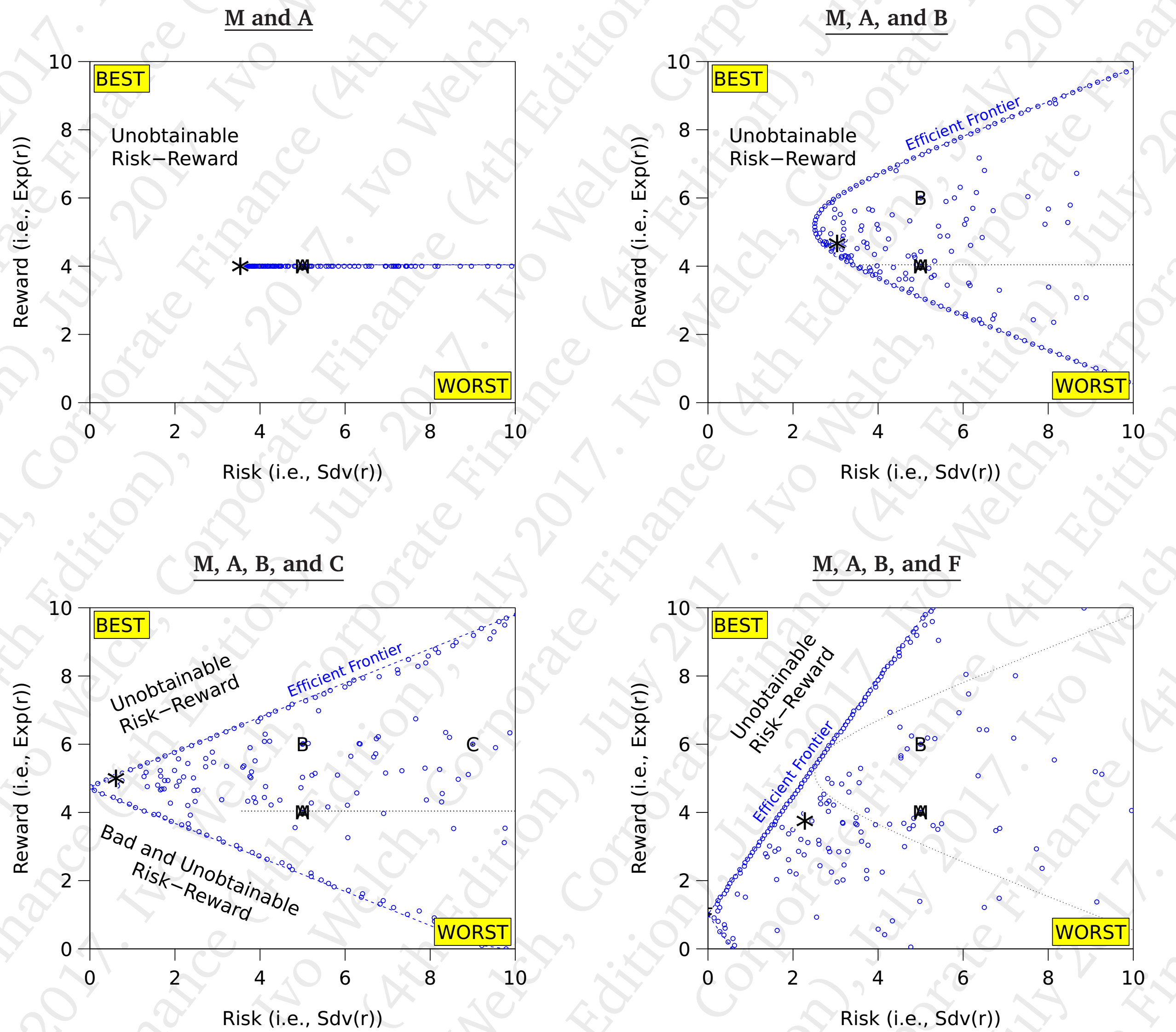


Exhibit 8.3: *The Efficient Frontier.* These plots show the mean and standard deviation of returns of portfolios composed of the stocks that are indicated in the header. (They appeared earlier, e.g., in Exhibit 8.1.) The '*' in the plots is the equal-weighted portfolio. The north-west border is the efficient frontier. (Note that the two lower plots even allow you to invest money and earn a risk-free rate. The lowest-risk portfolio is also called the **minimum-variance portfolio**.) The dotted line is the frontier from a previous plot.

than with fewer, so your efficient frontier has been pushed out further. The bottom-right plot shows your possible investments if instead of access to C, you had access to F. In both bottom figures, in which there is a risk-free asset, the efficient frontier is a line (the limit case for the hyperbola).

Q 8.6. How would the efficient frontier look if you were allowed to invest in all 5 assets, M, A, B, C, and F?

8.3 Investor Preferences and Risk Measures

You now understand that diversification can reduce risk. You still need to understand what projects the investors in your corporation—remember, this is *corporate finance*—would like you to invest in on their behalves.

The main question.

If Investors Care Only about Risk and Reward

Your intuition should now tell you that well-diversified portfolios—portfolios that invest in many different assets—tend to have lower risk. As a corporate manager, it would be reasonable for you to assume that your investors are smart. Because diversification helps investors reduce risk, you can also reasonably believe that they are indeed holding well-diversified portfolios. The most well-diversified portfolio may contain a little bit of every possible asset under the sun. Therefore, like most corporate executives, you would probably assume that your investors' portfolios are typically the overall **market portfolio**, consisting of all available investment opportunities.

Investors love diversification: the more the better. They could like the market portfolio because it is highly diversified.

Why would you even want to make any assumptions about your investors' portfolios? The answer is that if you are willing to assume that your investors are holding the market (or something very similar to it), your job as a corporate manager becomes much easier. Instead of asking what each and every one of your investors might possibly like, you can just ask, "When would my investors want to give me their money for investment into my firm's project, given that my investors are currently already holding the broad overall stock market portfolio?" The answer will be as follows:

If your investors like high reward and low risk and hold the market portfolio, you can work out how your projects affect them.

1. Your investors should like projects that offer more reward—this means higher expected rates of return.
2. Your investors should like projects that help them diversify away some of the risk in the market portfolio, so that their *overall* portfolios end up being less risky. Be careful, though. This does not mean always going for the lowest-risk projects. Instead, this may well be going for projects that behave very differently from other projects—unusual ones.

In sum, your corporate managerial task is to take those projects that your investors would like to add to their current (market) portfolios. You should therefore search for projects that have high expected rates of return and high diversification benefits with respect to the market. Let's now turn toward measuring this second characteristic: How can your projects aid your investors' diversification, and how should you measure how good this diversification is?

IMPORTANT

- Diversification is based on imperfect correlation, or "non-synchronicity," among investments. It helps smart investors reduce the overall portfolio risk.
- Therefore, as a corporate manager, in the absence of contradictory intelligence, you should believe that your investors tend to hold diversified portfolios. They could even hold portfolios as heavily diversified as the "entire market portfolio."

- As a corporate manager, your task is to think about how a little of your project can aid your investors in terms of its contribution to the risk and reward of their heavily diversified overall portfolios. (You should not think about how risky your project is in itself.)

Assume that investors hold the overall market. Now what?

If we are willing to assume that our smart investors are holding all assets in the market, then what projects offer them the best diversification?

Idiosyncratic Asset Risk (Sdv) and Risk Reduction

Comovement determines risk contribution.

Obviously, diversification does *not* help if two investment opportunities always move in the same direction. For example, if you try to diversify one $50 investment in M with another $50 investment in M (which always has the same outcomes), then your risk does not decrease. On the other hand, if two investment opportunities always move in *opposite* directions, then diversification works extremely well: One is a buffer for the other.

Pretend M is not just "My portfolio," but the market.

Let's formalize this intuition. For explanation's sake, assume that "My Portfolio" M is also the market portfolio.

Is B or C a better addition to your M portfolio?

Assume that B and C are two projects that your firm could invest in, but you cannot choose both. Both offer the same expected rate of return (6%), but B has lower risk (5%) than C (9%). As a manager, would you therefore assume that project B is better for your investors than project C?

The combination MC has almost the same risk as M.

The answer is no. Let's assume that your investors start out with the market portfolio, M. Exhibit 8.4 shows what happens if they sell half of their portfolios to invest in either B or C. You can call these two "(50,50)" portfolios MB and MC, respectively. Start with MB. If your investors reallocate half their money from M into B, their portfolios would have the following rates of return:

	in S1 (♣)	in S2 (♦)	in S3 (♥)	in S4 (♠)	Reward	Risk
MB	1%	1%	6%	12%	5%	4.5%

The upper graph in Exhibit 8.4 plots the MB rates of return, plus the rates of return for both M and B by themselves. The averages are all close to both original rates of return. There is not much change in the risk of your portfolio in moving from a pure M portfolio to the MB portfolio. The risk shrinks slightly, from 5.0% to 4.5%.

The combination MC has much lower risk than M.

Now consider the combination of MC, which is the lower graph in Exhibit 8.4. By itself, C is a very risky investment (9% risk). It also has the single-worst outcome of any investment you have seen so far. However, if your investors instead reallocate half of their wealth from M into C, their overall portfolio would have the following rates of return:

	in S1 (♣)	in S2 (♦)	in S3 (♥)	in S4 (♠)	Reward	Risk
MC	7%	3%	8%	2%	5%	2.6%

The risk is much lower! Look again at the exhibit—the MC outcomes are bunched much more closely than either M or C alone. And MB, too, has a much wider range than the MC portfolio. The MC combination portfolio is simply much safer—even though C by itself is much riskier. In sum:

	In S1 (♣)	In S2 (♦)	In S3 (♥)	In S4 (♠)	Reward E(r)	Variance Var(r)	Risk Sdv(r)
Investment M	–3%	3%	5%	11%	4%	25%%	5%
Investment B	5%	–1%	7%	13%	6%	25%%	5%
Investment C	17%	3%	11%	–7%	6%	81%%	9%
Portfolio MB	1%	1%	6%	12%	5%	20.5%%	4.5%
Portfolio MC	7%	3%	8%	2%	5%	6.5%%	2.6%

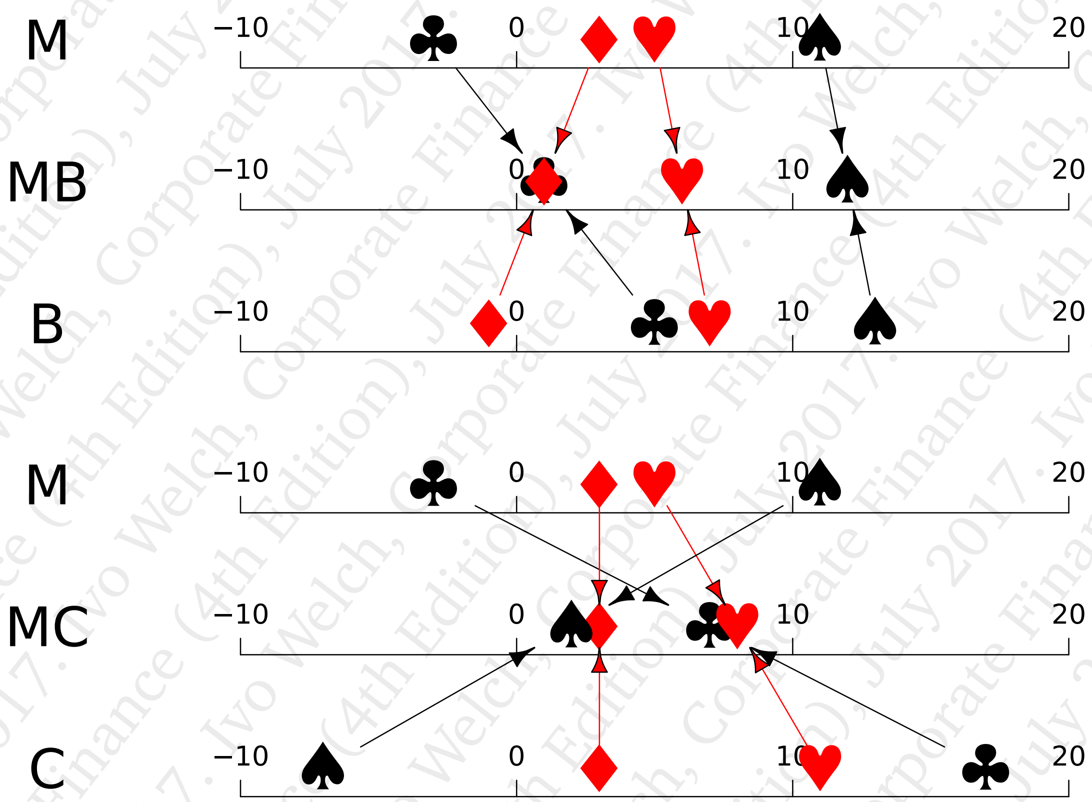


Exhibit 8.4: *Combining the Market M with Either B or C.* Although C is riskier than B by itself (look at C's one disaster outcome!), C is much better than B in reducing risk when it is added to the market portfolio M. This is because C tends to move opposite to M, especially if M turns in its worst outcome (–3%).

Portfolio	Reward	Risk	Note
M (=M) alone	4%	5.0%	Your investors' (market) portfolios
B alone	6%	5.0%	
C alone	6%	9.0%	C is riskier than B, if purchased by itself.
MB: half M, half B	5%	4.2%	Portfolio risk decreases less if B is added
MC: half M, half C	5%	2.6%	to M than when C is added to M!

The implication for your project choices as a corporate manager: Everything else equal, C could better reduce portfolio risk for your investors despite its higher idiosyncratic risk.

You now know that C's own high standard deviation compared to B's is not a good indication of whether C helps your investors reduce portfolio risk more or less than B. If your investors are primarily holding M, then a very risky project like C can allow them to build lower-risk portfolios. However, if your investors are not holding any assets other than C, they would not care about C's diversification benefits and only about its own risk. Thus, as a manager, you cannot determine whether your investors would prefer you to invest in B or C unless you know their entire portfolios. (Moreover, it could also depend on how your investors would like you to trade off more overall reward against more overall risk.)

IMPORTANT

A project's (own) standard deviation is not necessarily a good measure of how it influences the risk of your investors' portfolios. Indeed, it is possible that a project with a very high standard deviation by itself may actually help lower an investor's overall portfolio risk.

Q 8.7. Confirm the risk and reward calculations for the MB and MC portfolios in the table in Exhibit 8.4.

(Market-) Beta and (Market-) Portfolio Risk Contribution

C reduces M's risk because it tends to move in the opposite direction.

Why is portfolio C so much better than portfolio B in reducing the overall risk when held in combination with the M portfolio? The reason is that C tends to go up when M tends to go down, and vice-versa. The same cannot be said for B—it tends to move together with M. You could call this "synchronicity" or "comovement." It is why B does not help investors who are heavily invested in the overall market in their quests to reduce their portfolio risks.

Comovement can be measured by a line slope (beta). The market beta has the asset's rate of return on the y-axis and the market's rate of return on the x-axis.

Exhibit 8.5 shows the comovement graphically. The rate of return on the market is on the x-axis; the rate of return on the asset is on the y-axis. Its line slope in the plot is called the **market beta**. (It is common to write the formula for a line as $y = \alpha + \beta \cdot x$, which is where the Greek letter beta comes from.) A beta of 1 is a 45° diagonal line; a beta of 0 is a horizontal line. A positive beta slopes up; a negative beta slopes down. In statistics, you should have learned that you can find the beta by running a linear regression. If you don't remember, no worries: In Section 8.3, I will teach you again how to compute the beta. For now, take my word that the two best-fitting lines are

$$\begin{aligned} r_B &\approx 3.4\% + (+0.64) \cdot r_M \\ r_C &\approx 12.4\% + (-1.60) \cdot r_M \\ r_i &= \alpha_{i,M} + \beta_{i,M} \cdot r_M \end{aligned} \tag{8.3}$$

This formula is sometimes called the **market model**. The subscripts on the betas remind you what the variables on the x-axis and the y-axis are. The first subscript is always the variable on

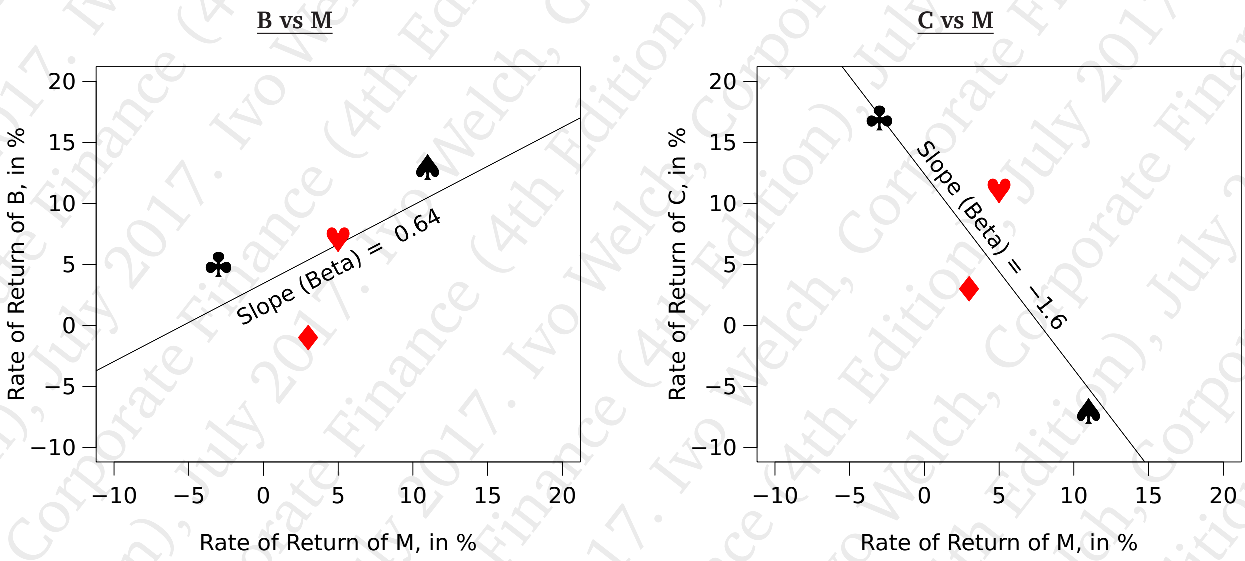


Exhibit 8.5: *Possible Outcomes: Rates of Return of C and D versus Rate of Return of M.* The four data points in each plot are taken from Exhibit 8.1 on page 166. They are the rates of return on the portfolios M, B, and C, quoted in percent. In the example, you know that these are the four true possible outcomes. In the real world, if the four points were not the true known outcomes, but just the historical outcomes (sample points), then the slope would not be the true unknown beta, but only the "estimated" beta.

the y-axis, and the second is the variable on the x-axis. Thus, $\beta_{B,M} \approx 0.64$ and $\beta_{C,M} \approx -1.60$. Market beta plays such an important role in finance that the name "beta" has itself become synonymous for "market beta," and the second subscript is usually omitted.

Market beta is a big deal in finance. It measures how your project covaries with the market. Market beta of Intel, Sect. 7.1, Pg.147.

In finance, we care about the market model line. As a corporate manager, you want to know how the rate of return on your own project comoves with that of the market. This is because you typically posit that your smart investors are on average holding the market portfolio. The best-fitting line between M and B slopes up. (It is also the same kind of line that you already saw in Section 7.1.) The positive slope means that B tends to be higher when M is higher. In contrast, the best-fitting line between M and C slopes down. The negative slope means that C tends to be lower when M is higher (and vice-versa). Again, this market slope is a common measure of expected comovement or countermovement—how much diversification benefit an investor can obtain from adding a particular new project. A higher slope means more comovement and less diversification; a lower, or even negative, slope means less comovement and more diversification.

IMPORTANT

- Diversification works better if the new investment project tends to move in the opposite direction from the rest of the portfolio than if it tends to move in the same direction.
- It is often reasonable to assume that smart investors are already holding the market portfolio and are now considering investing into just a little of one additional asset—your firm's new project.
- If this new investment asset has a negative beta with respect to the market (its "market beta"), it means that it tends to go down when the market goes up, and vice-versa. If this new investment asset has a positive beta with respect to the market, it means that it tends to move together with the market. If this new investment asset has a zero beta with respect to the market, it means that it moves independently of the market for all practical purposes.
- The market beta is a good measure of an investment asset's risk contribution for an investor who holds the market portfolio. The lower (or negative) the market beta, the more this investment helps reduce your investor's risk.
- The market beta of an asset can be interpreted as a line slope, where the rate of return on the market is on the x-axis and the rate of return on the new asset is on the y-axis. The line states how you expect the new asset to perform as a function of how the market will perform.
- You can think of market beta as a measure of "toxicity." In a reasonable equilibrium, holding everything else constant, risk-averse investors who are holding the market portfolio would agree to pay more for assets that have lower market betas. They would pay less for assets with higher market betas.

Warning: All of this beta-related risk measuring is interesting only if your investors are holding (portfolios close to) the overall market.

Before we conclude, some caveats are in order. From your perspective as the manager of a company, perhaps a publicly traded company, it is reasonable to assume that your investors are holding the market portfolio. It is also reasonable to assume that your new project is just a tiny new additional component of your investors' overall portfolios. We will staunchly maintain these assumptions, but you should be aware that they may not always be appropriate. If your investors are *not* holding something close to the market portfolio, then your project's market beta would *not* be a good measure of your projects' risk contributions. In the extreme, if your investors are holding *only* your project, market beta would not measure the project's risk contribution at all. This is often the case for entrepreneurs. They often have no choice but to put all their money into one basket. Such investors should care only about their project's standard deviation, and not about the project's market beta.

When Beta? When Standard Deviation?

Do you care about your portfolio's beta or your portfolio's standard deviation? As CFO, do you care about your firm's beta or your firm's standard deviation? Make sure you understand the answers to these questions.

IMPORTANT

- As an investor, you usually care only about your portfolio's standard deviation (risk), and not about the risk of its individual ingredients.
- Typically, you do not care about the overall market beta of your portfolio. (The individual market betas can help you design your overall portfolio.)
- If you are the CFO of a firm that wants to get into the market portfolio, so that investors willingly buy your shares, then you should care about your own firm's market beta.
- If you act purely in the interest of your diversified investors, you should not care about your firm's own standard deviation. Your investors can diversify away your firm's idiosyncratic risk. (If you care about your job or bonus, you might, however, take a different attitude towards risk. Corporate governance is the subject of companion Web chapter.)

Portfolio Alpha

Alpha has meaning, too, even though you won't use it just yet.

Although we shall not use it further in this book, the alpha intercept in Formula 8.3 also plays an important role. Together, alpha and beta help determine how attractive an investment is. For example, if the rate of return on the market will be 10%, Formula 8.3 tells you that you would expect the rate of return on C to be

$$\mathsf{E}\big(r_C \mid \text{ if } r_M = 10\%\big) \approx 12.4\% + (-1.60) \cdot 10\% \approx -3.6\%$$

The higher the alpha, the better the average performance of your investment given any particular rate of return on the market. Just as investment professionals often call the market beta just beta, they often call this specific intercept (here 12.4%) just alpha. (There is one small complication: They usually first subtract the risk-free interest rate first both r_C and r_M in their regressions—and this usually does not make much difference.)

Computing Market Betas from Historical Rates of Return

➤ Base Investment Assets, Exhibit 8.1, Pg.166.

You can compute the best-fit beta via a 4-step procedure.

So how can you actually compute beta? Let's return to the assets in Exhibit 8.1. What is the market beta of C? I have already told you that this slope is −1.6. To calculate it, I followed a tedious, but not mysterious, recipe. Here is what you have to do:

First, de-mean each rate of return. (How demeaning!)

➤ Variance calculations, Sect. 6.1, Pg.108.

1. Just as you did for your variance calculations, first translate all returns into deviations from the mean. That is, for M and C, subtract their own means from every realization.

	In S1 (♣)	**In S2 (♦)**	**In S3 (♥)**	**In S4 (♠)**
Asset M Rate of Return	−3%	+3%	+5%	+11%
...in deviation from 4% mean	−7%	−1%	+1%	+7%
Asset C Rate of Return	+17%	+3%	+11%	−7%
...in deviation from 6% mean	+11%	−3%	+5%	−13%

➤ Variance of M, Formula 8.1, Pg.167.

2. Compute the variance of the series on the X-axis. This is the variance of the rates of return on M. You have already done this in Formula 8.1: $\mathsf{Var}\big(r_M\big) = 25\%\%$.

For covariances, multiply net-of-mean returns, then average.

3. Now compute the probability-weighted average of the products of the two net-of-mean variables. In this case,

$$\text{Cov}(r_M, r_C) = \tfrac{1}{4} \cdot (-7\%) \cdot (+11\%) + \tfrac{1}{4} \cdot (-1\%) \cdot (-3\%) + \tfrac{1}{4} \cdot (+1\%) \cdot (+5\%) + \tfrac{1}{4} \cdot (+7\%) \cdot (-13\%) = -40\%\%$$
$$= \text{Sum of (each probability times the returns' products)}$$

This statistic is called the **covariance**, here between the rates of return on M and C.

The beta is the covariance divided by the variance.

4. The beta of C with respect to the market M, formally $\beta_{C,M}$ but often abbreviated as β_C, is the ratio of these two quantities,

$$\beta_C = \beta_{C,M} = \frac{-40\%\%}{25\%\%} \approx -1.6 \qquad (8.4)$$
$$= \frac{\text{Cov}(r_M, r_C)}{\text{Var}(r_M)}$$

You can confirm our calculations using a spreadsheet.

This slope of –1.6 is exactly the market beta we drew in Exhibit 8.5. Many spreadsheets and all statistical programs can compute it for you: They call the routine that does this a **linear regression**.

Think of market beta as the characteristic of an asset.

You should always think of an asset's beta with respect to a portfolio as a characteristic measure of your asset relative to an underlying base portfolio. The rate of return on portfolio P is on the x-axis; the rate of return on asset i is on the y-axis. As we stated earlier, most often—but not always—the portfolio P is the market portfolio, M, so $\beta_{i,M}$ is often just called the market beta of i, or just the beta of i (and the second subscript is omitted).

The average beta of the market (all stocks) is 1, not 0.

Now think for a moment. What is the average beta of a stock in the economy? Equivalently, what is the beta of the market portfolio? Replace C in Formula 8.4 with M:

$$\beta_M = \frac{\text{Cov}(r_M, r_M)}{\text{Var}(r_M)}$$

If you look at the definition of covariance, you can see that the covariance of a variable with itself *is* the variance. (The covariance is a generalization of the variance concept from one to two variables.) Therefore, $\text{Cov}(r_M, r_M) = \text{Var}(r_M)$, and the market beta of the market itself is 1. Graphically, if both the x-axis and the y-axis are plotting the same values, every point must lie on the diagonal. Economically, this should not be surprising, either: the market goes up one-to-one with the market.

IMPORTANT

The (value-weighted) average beta of all stocks in the market is 1 by definition.

Why torture you with computations? So you can play with scenarios.

Now that you know how to compute betas and covariances, you can consider scenarios for your project. For example, you might have a new project for which you would guess that it will have a rate of return of –5% if the market returns –10%; a rate of return of +5% if the market returns +5%; and a rate of return of 30% if the market returns 10%. Knowing how to compute a market beta therefore makes it useful to think of such scenarios. (You can also use this technique to explore the relationship between your projects and some other factors. For example, you could determine how your projects covary with the price of oil to learn about your project's oil risk exposure.)

Real-World Market Beta Estimation

In the real world, you will sometimes think in terms of such scenarios. However, you will more often have to compute a market beta from historical rates of return, using overall stock market returns and your own project (or similar project) returns. Fortunately, as we noted upfront, the beta computations themselves are exactly the same. In effect, when you use historical data, you simply assume that each time period was one representative scenario and proceed from there. Nevertheless, there are some real-world complications you should think about:

Practical advice to help you estimate market beta in the real world: Use 3-5 years of daily observations and then adjust.

1. Should you use daily, weekly, monthly, or annual rates of return? The answer is that the best market beta estimates come from daily (or weekly) data. Annual data should be avoided (except in a textbook in which space is limited). Monthly data should be used only if need be.
2. How much data should you use? Most researchers tend to use three to five years of historical rate of return data. This reflects a trade-off between having enough data and not going too far back into ancient history, which may be less relevant. If you have daily data, 2-3 years works quite well. The minimum is 1 year, and more than 5 years is not useful.
3. Is the historical beta a good estimate of the future beta? It turns out that history can sometimes be deceptive, especially if your estimated historical beta is far away from the market's beta average of 1. You should run a regression with daily historical returns and "shrink" your historical beta toward the overall market beta of 1 (or below 1 if your firm is small). This is important. For example, in the simplest such shrinker, you would simply compute an average of the overall market beta of 1 and your historical market beta estimate. If you computed a historical market beta of, say, 4 for your project, you should work with a prediction of future market beta of about $(4+1)/2 = 2.5$ for your project.

   Historical textbooks (including my own past editions) used to recommend averaging many industry projects. This seemed like a good idea but was bad advice. In practice, this approach has predicted very badly. For the most part, try to use your own historical daily returns, and not that of other firms in the industry.

   Many executives start with a statistical beta estimated from historical data (or they just look up the statistical beta on a website, such as YAHOO! FINANCE [finance.yahoo.com]) and then use their intuitive judgment to adjust it. It is unlikely that such adjustments are any good. Even trained financial economists with years of experience calculating betas cannot do this well. The only modification which tends to work is shrinking towards 1.

Q 8.8. Return to your computation of market beta of –1.6 in Formula 8.4. We called it $\beta_{C,M}$, or β_C for short. Is the order of the subscripts important? That is, is $\beta_{M,C}$ also –1.6?

Why Not Correlation or Covariance?

There is a close family relationship between covariance, beta, and correlation. The beta is the covariance divided by one of the variances. The correlation is the covariance divided by both standard deviations. The denominators are always positive. Thus, if the covariance is positive, so are the beta and the correlation; if the covariance is negative, so are the beta and the correlation; and if the covariance is zero, so are the beta and the correlation. The nice thing about the correlation, which makes it useful in many contexts outside finance, is that it has no scale and is always between –100% and +100%:

Covariance and beta (and correlation) always have the same sign.

- Two variables that always move perfectly in the same direction have a correlation of 100%.

- Two variables that always move perfectly in opposite directions have a correlation of –100%.
- Two variables that are independent have a correlation of 0%.

Such simplicity makes correlations very easy to interpret. The not-so-nice thing about correlation is that it has no scale and is always between –100% and +100%. This means that two investments, the second being a million times bigger than the first (all project rates of return multiplied by a million), have the same correlation with the stock market. Yet the second investment goes up or down with any slight tremor in the market by a million times more, which would of course mean that it would contribute much more risk. The correlation ignores this fact, which disqualifies it as a serious candidate for a project risk measure. Fortunately, beta takes care of scale—indeed, the beta for the second project would be a million times larger. This is why we prefer beta over correlation as a measure of risk contribution to a portfolio.

Spreadsheet Functions To Calculate Risk, Beta, and Reward

In real life, you can do calculations faster with a spreadsheet.

Doing all these calculations by hand is tedious. We computed these statistics within the context of just four scenarios, so that you would understand the meanings of the calculations better. However, you can do this a lot faster in the real world. Usually, you would download reams of real historical rates of return data into a computer spreadsheet, like Excel or OpenOffice. Spreadsheets have all the functions you need already built in—and you now understand what their functions actually calculate. In practice, you would use the following functions in Excel:

average computes the average (rate of return) over a range of cells.

varp (or **var.p**) computes the (population) variance. If you worked with historical data instead of known scenarios, you would instead use the **var** (or **var.s**) function. (The latter divides by N – 1 rather than by N, which I will explain in a moment.)

stdevp (or **stdev.p**) computes the (population) standard deviation. If you used historical data instead of known scenarios, you would instead use the **stdev** (or **stdev.s**) function.

covar computes the population covariance between two series. (If Excel was consistent, this function should be called covarp rather than covar.) Unlike the earlier functions, this and the next two functions require two data cell ranges, not one.

correl computes the correlation between two series.

slope computes a beta. If *range-Y* contains the rates of return of an investment and *range-X* contains the rates of return on the market, then this function computes the market beta.

Some Minor Statistical Nuances (Nuisances)

Is history a good guide?

➤ Will history repeat itself?, Sect. 7.1, Pg.153.

In this chapter, we have continued to presume (just as we did in Section 7.1) that historical data gives us a good guide to the future when it comes to means, variances, covariances, and betas (assuming you calculate them well—2 years of daily data, appropriately shrunk). Of course, this is a simplification—and remember that it can be a problematic one. I already noted that this is less of a problem for covariances, variances, and betas than it is for means. Rely on historical means as predictors of future expected rates of return only at your own risk!

When working with a sample, the (co)variance formula divides by N–1. When working with the population, the (co)variance formula divides by N.

There is a second, minor statistical issue of which you should be aware. Statisticians often use a covariance formula that divides by N – 1, not N. Strictly speaking, dividing by N – 1 is appropriate if you work with historical data. These are just sample draws and not the full population of possible outcomes. With a sample, you do not really know the true mean when you de-mean your observations. The division by a smaller number, N – 1, gives a larger but unbiased covariance estimate. It is also often called the *sample covariance*. In contrast, dividing by N is appropriate if you work with "scenarios" that you know to be true and equally likely. In this case, the statistic is often called the *population covariance*. The difference rarely matters in finance, where you usually have a lot of observations—except in our book examples where you have only four scenarios. (For example, dividing by $N = 1,000$ and by $N = 1,001$ gives almost the same number.)

This is important to keep in mind if you use a spreadsheet to check your work.

The only reason why you even needed to know this distinction is that if you use a program that has a built-in variance or standard deviation function, you should not be surprised if you get numbers different from those that you have computed in this chapter. In some programs, you can get both functions. In Excel, you can use the *varp* and *stdevp* population statistical functions to get the population statistics, not the *var* and *stdev* functions that would give you the sample statistics.

For market beta, the divisor cancels out and does not matter.

Beta is not affected by whether you divide the variance/covariance by N or N – 1, because both numerator (covariance) and denominator (variance) are divided by the same number.

My fault: Our notation should have distinguished between true population and estimated sample statistics.

Furthermore, statisticians distinguish between underlying unknown statistics and statistics estimated from the data. For example, they might call the unknown true mean μ and the sample mean m (or $\bar{x}$). They might call the unknown true beta β^{T} and the estimated sample beta a beta with a little hat ($\hat{\beta}$). And so on. Our book is casual about the difference due to lack of space, but keep in mind that whenever you work with historical data, you are really just working with sample estimates.

8.4 Interpreting Some Typical Stock Market Betas

Market beta works well when investors are holding the market and adding only a little of your project.

The market beta is the best measure of “diversification help” for an investor who holds the stock market portfolio and considers adding *just a little* of your firm’s project. From your perspective as a manager seeking to attract investors, this is not a perfect, necessarily true assumption—but it is a reasonable one. Recall that we assume that investors are smart, so presumably they are holding highly diversified portfolios. To convince your market investors to like your \$10 million project, you just need the average investor to want to buy \$10 million divided by about \$20 trillion (the stock market capitalization), which is 1/2,000,000 of their portfolios. For your investors, your corporate projects are just tiny additions to their (likely) market portfolios.

Most financial websites publish market beta estimates.

You can easily look up the market betas of publicly traded stocks on many financial websites. Exhibit 8.6 lists the betas of some randomly chosen companies in June 2016 from YAHOO! FINANCE and from Google’s finance site. Most company betas are in the range of around 0 to about 2. (American Airlines’ *historical* market beta was so high that Google even refused to admit to its own estimate.) A beta above 1 is considered risk-increasing for an investor holding the overall stock market (it is riskier than the stock market itself), while a beta below 1 is considered risk-reducing. Negative betas are rare and usually temporary. Gold is an asset that sometimes did and sometimes did not have a negative market beta (see Barrick Gold here). In this decade, long-term Treasury bonds had negative betas; but in past decades, it was positive. In almost all cases, it is better to estimate future market betas with firms’ own historical market betas (though shrunk) rather than their industry market betas.

Beta can be viewed as the marginal change of your project with respect to the market.

Market beta has yet another nice intuitive interpretation: It is the degree to which the firm’s value tends to change if the stock market changes. For example, AMD’s market beta of approximately 2 says that if the stock market will return an extra 10% next year (above and beyond its expectations), AMD’s stock will likely return an extra $2 \cdot 10\% = 20\%$ (above and beyond AMD’s expectations). For now, let’s say that the expected rate of return on the market is 6% and the expected rate of return on AMD is 9%. Then, if the market were to turn in –4% (10% less than its expected return), you would expect AMD to turn in $9\% + 2 \cdot (–10\%) = –11\%$. Conversely, if the market were to turn in 16% (10% more than its expected return), you would expect AMD to turn in $9\% + 2 \cdot (10\%) = 29\%$. AMD’s high market beta is useful because it

Company	Stock Ticker	Mkt Cap	Market Beta Yahoo	Market Beta Google	Company	Stock Ticker	Mkt Cap	Market Beta Yahoo	Market Beta Google
Intel	INTC	151	1.06	0.96	IBM	IBM	146	0.81	0.71
Coca-Cola	KO	199	0.82	0.52	PepsiCo	PEP	149	0.69	0.49
AMD	AMD	3.4	2.13	2.29	NVIDIA	NVDA	25	1.19	1.31
Ford	F	52	1.26	1.37	General Motors	GM	45	1.59	1.71
Apple Inc	AAPL	541	1.49	1.03	Google (Alphabet)	GOOG	493	1.03	N/A
Citigroup	C	129	1.54	1.97	Morgan Stanley	MS	49	1.45	1.63
Goldman Sachs	GS	66	1.32	1.69	J.P. Morgan	JPM	233	1.18	1.14
Volkswagen	VLKAY	75	1.89	1.83	Sony	SNE	35	1.49	1.83
Philip Morris	PM	157	1.02	0.95	Procter&Gamble	PG	221	0.65	0.49
American Airlines	AAL	19	3.95	N/A	Southwest	LUV	28	1.29	1.03
Boeing	BA	84	1.23	1.08	Airbus	AIR	41	N/A	N/A
Hewlett-Packard	HPQ	23	1.54	1.70	Yahoo	YHOO	35	2.16	1.48
Exxon Mobil	XOM	373	0.90	0.83	Barrick Gold	ABX	23	–0.20	–0.01

Exhibit 8.6: *Some Market Betas and Market Capitalizations in June 2016.* "MktCap" is the equity market value in billions of dollars. YAHOO! FINANCE explained its betas as follows: *The Beta used is Beta of Equity. Beta is the monthly price change of a particular company relative to the monthly price change of the S&P500. The time period for Beta is 3 years (36 months) when available.* YAHOO! FINANCE ignores dividends, but this usually makes little difference. I could not find an explanation for Google's market betas. I hope it's not a secret.

informs you that if you hold the stock market, adding AMD stock would not help you much with diversifying your market risk. Holding AMD would amplify any market swings.

Beta is not alpha.

But in any case, AMD's market beta does not tell you whether AMD is priced too high or too low on average, so that you should buy or avoid it in the first place. Market beta is not a measure of how good an investment AMD is. (This would be the aforementioned alpha [which can be interpreted as an expected rate of return]. In the next chapter, you will learn that the CAPM formula relates market beta to the expected rate of return, giving you a commonly used benchmark for alpha.)

➤ Alpha, Pg.179.

Beta is also a "hedge ratio."

Betas have another common important use. Let's say that you want to speculate that AMD will go up, but you do not want to be exposed to market risk. The AMD beta of 2 tells you that if you buy long $100 of AMD stock and go short $200 in the stock market (which you can do easily, e.g., by shorting the SPDR ETF), your overall portfolio is not likely to be subject to market-wide swings. After all, for every $1 of general decrease (increase) in the overall stock market, AMD goes up (down) on average by $2. Thus, the market-eta of 2 is also the **hedge ratio** that tells you how you can "immunize" a speculative stock position against market-wide changes.

Q 8.9. You estimate your project x to return –5% if the stock market returns –10%, and +5% if the stock market returns +10%. What would you use as the market beta estimate for your project?

Q 8.10. You estimate your project y to return +5% if the stock market returns –10%, and –5% if the stock market returns +10%. What would you use as the market beta estimate for your project?

8.5 Market Betas for Portfolios and Conglomerate Firms

Let's go back to your managerial perspective of figuring out the risk and return of your corporate projects. Many small projects are bundled together, so it is very common for managers to consider multiple projects already packaged together as one portfolio. For example, you can think of your firm as a collection of divisions that have been packaged together. If division B is worth \$1 million and division C is worth \$2 million, then a firm consisting of B and C is worth \$3 million. B constitutes 1/3 of the portfolio "Firm" and C constitutes 2/3 of the portfolio "Firm." This kind of portfolio is called a **value-weighted portfolio** because the weights correspond to the market values of the components. (A portfolio that invests \$100 in B and \$200 in C would also be value-weighted. A portfolio that invests equal amounts in the constituents—for example, \$500 in each—is called an **equal-weighted portfolio**.)

Portfolios consist of multiple assets (themselves possibly portfolios). Definitions of value-weighted and equal-weighted portfolios.

Thus, as a manager, you have to know how to work with a portfolio (firm) when you have all the information about all of its underlying component stocks (projects). If I tell you the expected rate of return and market beta of each project, can you tell me what the overall expected rate of return and overall market beta of your firm are? Let's try it. Use the B and C stocks from Exhibit 8.1 on Page 166, and call BCC the portfolio (or firm) that consists of 1/3 investment in division B and 2/3 investment in division C.

What are the expected rate of return and market beta of a portfolio?

Actually, you already know that you can compute the returns in each scenario, and then the risk and reward.

You can average actual rates of return.

	In S1 (♣)	In S2 (♦)	In S3 (♥)	In S4 (♠)	Reward $E(r)$	Variance[a] $Var(r)$	Risk $Sdv(r)$
Investment B	5%	–1%	7%	13%	6%	25%%	5%
Investment C	17%	3%	11%	–7%	6%	81%%	9%
Portfolio BCC	13%	1.67%	9.67%	–0.33%	6%	≈30%%	≈5.5%

It is also intuitive that *expected* rates of return can be averaged. In our example, B has an *expected* rate of return of 6%, and C has an *expected* rate of return of 6%. Consequently, your overall firm BCC has an expected rate of return of 6%, too. Check this.

You can average expected rates of return.

Unfortunately, you cannot compute value-weighted averages for all statistics. As the table shows, variances and standard deviations cannot be averaged ($1/3 \cdot 25\%\% + 2/3 \cdot 81\%\% \approx 62.3\%\%$, which is not the variance of 30%%; and $1/3 \cdot 5\% + 2/3 \cdot 9\% \approx 7.67\%$, which is not the standard deviation of 5.5%.)

(But you cannot average variances or standard deviations!)

But here is a remarkable and less intuitive fact: Market betas—that is, the projects' risk contributions to your investors' market portfolios—*can* be averaged! That is, I claim that the beta of BCC is the weighted average of the betas of B and C. In Formula 8.3, you already computed the market-betas for as +0.64 and –1.60. So, their value-weighted average is

News flash: You can also average market betas.

➤ Market betas of B and C, Formula 8.3, Pg.176.

$$\beta_{BCC} = \underbrace{1/3 \cdot (+0.64)}_{w_B \cdot \beta_B} + \underbrace{2/3 \cdot (-1.60)}_{w_C \cdot \beta_C} \approx -0.8533 \qquad (8.5)$$

You will be asked to confirm this conclusion in Q8.11.

IMPORTANT

- You can think of the firm as a weighted investment portfolio of components, such as individual divisions or projects. For example, if a firm named ab consists of only two divisions, a and b, then its rate of return is always

$$r_{ab} = w_a \cdot r_a + w_b \cdot r_b$$

  where the weights are the relative values of the two divisions. (You can also think of this one firm as a "subportfolio" within a larger overall portfolio, such as the market portfolio.)

- The expected rate of return ("reward") of a portfolio is the weighted average expected rate of return of its components,

$$\mathsf{E}(r_{ab}) = w_a \cdot \mathsf{E}(r_a) + w_b \cdot \mathsf{E}(r_b)$$

  Therefore, the expected rate of return of a firm is the weighted average rate of return of its divisions.

- Like expected rates of return, market betas can be weighted and averaged. The beta of a firm—i.e., the firm's "risk contribution" to the overall market portfolio—is the weighted average of the betas of its components,

$$\beta_{ab} = w_a \cdot \beta_a + w_b \cdot \beta_b$$

  The market beta of a firm is the weighted average market beta of its divisions.

- You cannot do analogous weighted averaging with variances or standard deviations.

A firm is a portfolio of debt and equity. Thus, the portfolio formulas apply to the firm (with debt and equity as its components), too!

You can think of the firm not only as consisting of divisions, but also as consisting of debt and equity. For example, say your $400 million firm is financed with debt worth $100 million and equity worth $300 million. If you own all debt and equity, you own the firm. What is the market beta of your firm's assets? Well, the beta of your overall firm must be the weighted average beta of its debt and equity. If your $100 million in debt has a market beta of, say, 0.4 and your $300 million of equity has a market beta of, say, 2.0, then your firm has a market beta of

$$\begin{array}{ccccc} 1/4 \cdot (0.4) & + & 3/4 \cdot (2.0) & = & 1.6 \\ \left(\frac{\text{Debt value}}{\text{Firm value}}\right) \cdot \beta_{\text{Debt}} & + & \left(\frac{\text{Equity value}}{\text{Firm value}}\right) \cdot \beta_{\text{Equity}} & = & \beta_{\text{Firm}} \end{array} \qquad (8.6)$$

This 1.6 is called the **asset beta** to distinguish it from the **equity beta** of 2.0 that financial websites report. Put differently, if your firm refinances itself to 100% equity (i.e., $400 million worth), then the reported market beta of your equity on YAHOO! FINANCE would fall to 1.6. The asset beta is the measure of your firm's projects' risk contribution to the portfolio of your investors. It determines the cost of capital that you should use as the hurdle rate for projects that are similar to the average project in your own firm.

Q 8.11. Let's check that the beta combination formula (Formula 8.5 on page 185) is correct. Start with the BCC line in the table on Page 185

1. Write down a table with the demeaned market rate of return and demeaned BCC rate of return in each of the four possible states.

2. Multiply the demeaned rates of return in each scenario. This gives you four cross-products, each having units of %%.
3. Compute the average of these cross-products. This is the covariance between BCC and M.
4. Divide the covariance between BCC and M by the variance of the market.
5. Which is faster—this route or Formula 8.5? Which is faster if there are a hundred possible scenarios?

Q 8.12. Confirm that you cannot take a value-weighted average of component variances (and thus of standard deviations) the same way that you can take value-weighted average expected rates of return and value-weighted average market betas.

1. What is the value-weighted average variance of BCC?
2. What is the actual variance of BCC?

Q 8.13. Consider an investment of $2/3$ in B and $1/3$ in C. Call this new portfolio BBC. Compute the variance, standard deviation, and market beta of BBC. Do this two ways: first from the four individual scenario rates of return of BBC, and then from the statistical properties of B and C itself.

Q 8.14. Assume that a firm will always have enough money to pay off its bonds, so the beta of its bonds is 0. (Being risk free, the rate of return on the bonds is obviously independent of the rate of return on the stock market.) Assume that the beta of the underlying assets is 2. What would financial websites report for the beta of the firm's equity if it changes its current capital structure from all equity to half debt and half equity? To 90% debt and 10% equity?

Q 8.15. (Advanced) Does maintaining a value-weighted or an equal-weighted portfolio require more trading? (Hint: Make up a simple example.)

Summary

This chapter covered the following major points:

- The expected rate of return is a measure of expected reward:

$$\mathsf{E}(r_P) = \frac{\text{Sum over Scenarios } [r_P \text{ in Scenario}]}{N}$$

- The variance is (roughly) the average squared deviation from the mean.

$$\mathsf{Var}(r_P) = \frac{\text{Sum over Scenarios}\left\{[r_P \text{ in Scenario}] - \mathsf{E}(r_P)\right\}^2}{N \quad (\text{or } N-1)}$$

  If you work with known scenario probabilities, divide by N. If you work with a limited number of historical observations that you use to guesstimate the future scenarios, then divide by N – 1. (With a lot of historical data, N is very large and it really makes no difference what you divide by.) The variance is an intermediate input to the more interesting statistic, the standard deviation.

- The standard deviation is the square root of the variance. The standard deviation of a portfolio's rate of return is the common measure of its risk.

$$\mathsf{Sdv}(r_P) = \sqrt{\mathsf{Var}(r_P)}$$

- Diversification reduces the risk of a portfolio.
- Corporate executives typically assume that their investors are smart enough to hold widely diversified portfolios, which resemble the overall market portfolio. The reason is that diversified portfolios offer higher expected rates of return at lower risks than undiversified ones.
- An individual project's own risk *is not* a good measure of its risk contribution to a smart diversified investor's portfolio.
- Market beta *is* a good measure of an individual asset's risk contribution for an investor who holds the market portfolio.

- Market betas for typical stocks range between 0 and 2.5.
- It requires straightforward plugging of data into formulas to compute beta, correlation, and covariance. These three measures of comovement are closely related and always share the same sign.
- Like expected rates of return, betas can be averaged (using proper value-weighting, of course). However, variances or standard deviations cannot be averaged.

Preview of the Chapter Appendix in the Companion

The appendix to this chapter explains

- how risk and reward vary for different combination portfolios.
- how one can use the "matrix" of variances and covariances to quickly recompute the overall portfolio risk of different combinations.
- what optimal combination portfolios are. This is the efficient frontier (**mean-variance efficiency** or **MVE**), which you have already briefly encountered in this chapter. It is the cornerstone of modern investment theory.
- how the availability of a risk-free asset makes the optimal portfolio always a combination of this risk-free asset and some tangency portfolio. Thus, every rational investor would buy only these two assets. The more risk-averse, the more an investor would allocate from the risk-free into the risky tangency asset.
- how market beta coincidentally affects idiosyncratic risk, and how it influences market-conditional realized rates of return.

Keywords

Answers

Q 8.1 The average deviation from the mean is always 0.

Q 8.2 The mean of portfolio M was 4%. Adding 5% to each return will give you a mean of 9%, which is 5% higher. The variance and standard deviation remain at the same level, the latter being 5%. If you think of 5% as a constant c = 5%, then you have just shown that $\mathsf{E}(r+c) = \mathsf{E}(r) + c$ and $\mathsf{Sdv}(r+c) = \mathsf{Sdv}(r)$.

Q 8.3 The reward of portfolio C is its expected rate of return, i.e., $[(17\%)+3\%+11\%+(-7\%)]/4 = 6\%$. (We can just divide by 4, rather than multiply each term by 1/4, because all outcomes are equally likely.) The variance of C is $[(11\%)^2+(3\%)^2+(5\%)^2+(-13\%)^2]/4 = 81\%\%$. The standard deviation, which is our measure of risk, is $\sqrt{81\%\%} \approx 9\%$.

Q 8.4 The combination portfolio MA of 90% in M and 10% in A has rates of return of –2.4%, 3.8%, 4.2%, and 10.4%.

1. Thus, its mean rate of return is 4%. Its variance is 20.5%%. Its standard deviation is approximately 4.528%.
2. It would look more spread out, because it has higher standard deviation.

Q 8.5

1. The reward is $4 \cdot 10\% = 40\%$. The variance is $4 \cdot 400\% = 1{,}600\%\%$. Thus, the standard deviation (risk) is $\sqrt{1{,}600\%\%} = 40\%$. The Sharpe ratio is 1.
2. The reward is 90%. The risk is $\sqrt{9 \cdot 400\%\%} = 3 \cdot 20\% = 60\%$. The Sharpe ratio is 1.5
3. The reward is $T \cdot \mathsf{E}$. The standard deviation is $\sqrt{T} \cdot \mathsf{Sdv}$. The Sharpe ratio is $(\sqrt{T} \cdot \mathsf{E})/\mathsf{Sdv}$.

Q 8.6 Exhibit 8.3 shows that by combining M, A, B, and C, you get a risk-free rate of 3.6%; and investing in F alone gets you a risk-free rate of 1%. This means that you could borrow at 1% and invest at 3.67%, both risk-free—an arbitrage. The efficient frontier would be a vertical line at 0. Obviously, this could never be the case in the real world.

Q 8.7 For the MB portfolio, the portfolio combination rates of return in the four scenarios were on the bottom of Exhibit 8.4 on Page 175. Confirm them first:

$$\begin{aligned}
\text{In S1 } (\clubsuit)&: 0.5 \cdot (-3\%) + 0.5 \cdot (5\%) = 1\% \\
\text{In S2 } (\diamondsuit)&: 0.5 \cdot (3\%) + 0.5 \cdot (-1\%) = 1\% \\
\text{In S3 } (\heartsuit)&: 0.5 \cdot (5\%) + 0.5 \cdot (7\%) = 6\% \\
\text{In S4 } (\spadesuit)&: 0.5 \cdot (11\%) + 0.5 \cdot (13\%) = 12\%
\end{aligned}$$

The expected rate of return is

$$\mathsf{E}(r_{MB}) = \frac{1\% + 1\% + 6\% + 12\%}{4} = 5\%$$

The portfolio variance is

$$\begin{aligned}
\mathsf{Var}(r_{MB}) = {} & [(1\% - 5\%)^2 + (1\% - 5\%)^2 \\
& + (6\% - 5\%)^2 + (12\% - 5\%)^2]/4
\end{aligned}$$

Therefore, $\mathsf{Sdv}(MC) = \sqrt{20.5\%\%} \approx 4.52\%$.

For the MC portfolio,

$$\begin{aligned}
\text{In S1 } (\clubsuit)&: 0.5 \cdot (-3\%) + 0.5 \cdot (17\%) = 7\% \\
\text{In S2 } (\diamondsuit)&: 0.5 \cdot (3\%) + 0.5 \cdot (3\%) = 3\% \\
\text{In S3 } (\heartsuit)&: 0.5 \cdot (5\%) + 0.5 \cdot (11\%) = 8\% \\
\text{In S4 } (\spadesuit)&: 0.5 \cdot (11\%) + 0.5 \cdot (-7\%) = 2\%
\end{aligned}$$

The expected rate of return is

$$\mathsf{E}(r_{MC}) = \frac{7\% + 3\% + 8\% + 2\%}{4} = 5\%$$

The variance is $\mathsf{Var}(MC) = [(7\%-5\%)^2+(3\%-5\%)^2+(8\%-5\%)^2+(2\%-5\%)^2]/4 = 26\%\%$ Therefore, $\mathsf{Sdv}(MC) = \sqrt{6.5\%\%} \approx 2.55\%$.

Q 8.8 The order of subscripts on market beta is important. Algebraically, $\beta_{C,M} = [\text{cov}(r_C, r_M)]/[\text{var}(r_M)]$, while $\beta_{M,C} = [\text{cov}(r_C, r_M)]/[\text{var}(r_C)]$. The denominator is different. If you work this out, $\beta_{M,C} \approx -0.49$. Fortunately, you will never ever need to compute $\beta_{M,C}$. I only asked you to do this computation so that you realize that the subscript order is important.

Q 8.9 The market beta of this project is

$$\beta_{x,M} = \frac{r_{x,2} - r_{x,1}}{r_{M,2} - r_{M,1}} = \frac{(-5\%) - (+5\%)}{(-10\%) - (+10\%)} = +0.5$$

(This is not "half as volatile" because market beta is not a measure of volatility.)

Q 8.10 Using the same formula, the market beta of y is $[(+5\%) - (-5\%)]/[(-10\%) - (+10\%)] = -0.5$.

Q 8.11

1. Start with our standard table:

	♣	♦	♥	♠	E(r)	Var(r)	Sdv(r)
BCC	13%	1.67%	9.67%	–0.33%	6%	30%%	5.5%
. . . in dev	7%	–4.33%	3.67%	–6.33%			
M	–3%	+3%	+5%	+11%	4%	25%%	5%
. . . in dev	–7%	–1%	1%	+7%			

(Variances and standard deviations are rounded.)

2. The four cross-products are –49%%, 4.33%%, 3.67%%, and –44.33%%.
3. The average (covariance) is –21.33%%.
4. The beta is $-21.33/25 \approx -0.8533$.
5. This is the more painful route—and it is more painful when there are more possible scenarios.

Q 8.12 Actually, this was already in the text. BCC has a variance of about 30%%, while the value-weighted average of the variances is about 62.3%%.

Q 8.13 The equivalent table is

	♣	♦	♥	♠	E(r)	Var(r)	Sdv(r)
B	5%	–1%	7%	13%	6%	25%%	5%
C	17%	3%	11%	–7%	6%	81%%	9%
BBC	9%	0.33%	8.33%	6.33%	6%	11.67%%	3.4%

The market beta is easiest to compute as $2/3 \cdot \beta_B + 1/3 \cdot \beta_C \approx 2/3 \cdot (0.64) + 1/3 \cdot (-1.60) \approx -0.11$.

Q 8.14 For a firm whose debt is risk free, the overall firm beta is $\beta_{\text{Firm}} = 0.5 \cdot \beta_{\text{Equity}} + 0.5 \cdot \beta_{\text{Debt}}$. Thus, $0.5 \cdot \beta_{\text{Equity}} + 0.5 \cdot 0 = 2$. Solve for $\beta_{\text{Equity}} = \beta_{\text{Firm}}/0.5 = 4$. For the (90%, 10%) case, the equity beta jumps to $\beta_{\text{Equity}} = 2/0.1 = 20$.

Q 8.15 Value-weighted portfolios usually require no trading (unless there is a payout, like a dividend). For example, using the numbers from this section, if B triples from \$1 million to \$3 million and C halves from \$2 million to \$1 million, your original value-weighted portfolio or firm would become \$3+\$1 = \$4 million. You would still be exactly value-weighted. B would now constitute 75% of the firm and C 25% of the firm. In contrast, in an originally equal-weighted portfolio, your \$1.5 million in B would become \$4.5 million, your \$1.5 million in C would become \$0.75 million, and your portfolio would be worth \$5.25 million. This means you would want to have \$2.625 million invested in each. To maintain an equal-weighted portfolio, you would have to sell some stock in your past winner to buy some stock in your loser. Only a value-weighted portfolio requires no trading. Another interesting aspect is that if you do not trade, in the very long run, any portfolio will look more and more value-weighted, because those stocks that have had large returns will automatically garner a larger weight both in your portfolio and the economy.

End of Chapter Problems

Q 8.16. Multiply each rate of return for M by 2.0. This portfolio offers –6%, +6%, +10%, and +22%. Compute the expected rate of return and standard deviation of this new portfolio. How do they compare to those of the original portfolio M?

Q 8.17. The following table contains the closing year-end prices of the Japanese stock market index, the Nikkei-225. Assume that each historical rate of return was exactly one representative scenario (independent sample draw) that you can use to estimate the future. If a Japanese investor had purchased a mutual fund that imitated the Nikkei-225, what would her annual rates of return, compounded rate of return (from the end of 1984 to the end of 2010), average rate of return, and risk have been?

Year	N-225	Year	N-225	Year	N-225
1984	11,474	1993	17,417	2002	8,579
1985	13,011	1994	19,723	2003	10,677
1986	18,821	1995	19,868	2004	11,489
1987	22,957	1996	19,361	2005	16,111
1988	29,698	1997	15,259	2006	17,225
1989	38,916	1998	13,842	2007	15,308
1990	24,120	1999	18,934	2008	8,860
1991	22,984	2000	13,786	2009	10,546
1992	16,925	2001	10,335	2010	10,229

Q 8.18. Compute the value-weighted average of $1/3$ of the standard deviation of B and $2/3$ of the standard deviation of C. Is it the same as the standard deviation of a BCC portfolio of $1/3$ B and $2/3$ C, in which your investment rate of return would be $1/3 \cdot r_B + 2/3 \cdot r_C$?

Q 8.19. Why is it so common to use historical financial data to estimate future market betas?

Q 8.20. What are the risk and reward of a combination portfolio that invests 40% in M and 60% in B?

Q 8.21. Consider the following five assets, which have rates of return in six equally likely scenarios:

	Awful	Poor	Med.	Okay	Good	Great
Asset P1	–2%	0%	2%	4%	6%	10%
Asset P2	–1%	2%	2%	2%	3%	3%
Asset P3	–6%	2%	2%	3%	3%	1%
Asset P4	–4%	2%	2%	2%	2%	20%
Asset P5	10%	6%	4%	2%	0%	–2%

1. Assume that you can only buy one of these assets. What are their risks and rewards?
2. Supplement your previous risk-reward rankings of assets P1–P5 with those of combination portfolios that consist of half P1 and half of each of the other 4 portfolios, P2–P5. What are the risks and rewards of these four portfolios?
3. Assume that P1 is the market. Plot the rates of return for P1 on the x-axis and the return for each of the other stocks on their own y-axes. Then draw lines that you think best fit the points. Do not try to compute the beta—just use the force (and your eyes), Luke. If you had to buy just a little bit of one of these P2–P5 assets, and you wanted to lower your risk, which would be best?

Q 8.22. Assume that you have invested half of your wealth in a risk-free asset and half in a risky portfolio P. Is it theoretically possible to lower your portfolio risk if you move your risk-free asset holdings into another risky portfolio Q? In other words, can you ever reduce your risk more by buying a risky security than by buying a risk-free asset?

Q 8.23. Is it wise to rely on historical statistical distributions as your guide to the future?

Q 8.24. Look up the market betas of the companies in Exhibit 8.6. Have they changed dramatically since June 2016, or have they remained reasonably stable?

Q 8.25. You estimate your project to return –20% if the stock market returns –10%, and +5% if the stock market returns +10%. What would you use as the market beta estimate for your project?

Q 8.26. Go to YAHOO! FINANCE. Obtain two years' worth of daily stock rates of return for PepsiCo, Coca Cola, and for the S&P 500 index. Use a spreadsheet to compute PepsiCo's and Coca-Cola's *historical* market betas. (Note: For future market betas, you should further shrink towards 1.)

Q 8.27. Consider the following assets:

	Bad	Okay	Good
Market M	–5%	5%	15%
Asset X	–2%	–3%	25%
Asset Y	–4%	–6%	30%

1. Compute the market betas for assets X and Y.
2. Compute the correlations of X and Y with M.
3. Assume you were holding only M. You now are selling off 10% of your M portfolio to replace it with 10% of either X or Y. Would an M&X portfolio or an M&Y portfolio be riskier?
4. Is the correlation indicative of which of these two portfolios ended up riskier? Is the market beta indicative?

Q 8.28. Compute the expected rates of return and the portfolio betas for many possible portfolio combinations (i.e., different weights) of M and F from Exhibit 8.1 on Page 166. (Your weight in M is 1 minus your weight in F.) Plot the two against one another. What does your plot look like?

Q 8.29. Are historical covariances or means more trustworthy as estimators of the future?

Q 8.30. Are geometric average rates of return usually higher or lower than arithmetic average rates of return?

Q 8.31. The following represents the probability distribution for the rates of return for next month:

Probability	Pfio P	Market M
1/6	–20%	–5%
2/6	–5%	+5%
2/6	+10%	0%
1/6	+50%	+10%

Compute by hand (and show your work) for all the following questions.

1. What are the risks and rewards of P and M?
2. What is the correlation of M and P?
3. What is the market beta of P?
4. If you were to hold 1/3 of your portfolio in the risk-free asset, and 2/3 in portfolio P, what would its market beta be?

Q 8.32. Download the historical daily stock prices for the S&P 500 index and for VPACX (the *Vanguard Pacific Stock Index* mutual fund) from YAHOO! FINANCE, beginning January 1 three years ago and ending December 31 of last year. Load them into a spreadsheet and position them next to one another. Compute the risk and reward. Compute VPACX's market beta, i.e., with respect to the S&P 500 index. How do your historical estimates compare to the Fund Risk reported by YAHOO! FINANCE and other financial websites? If you were interested not in the *historical* but *future* market beta, would this be a good estimate?

Q 8.33. Download 3 years of historical daily (dividend-adjusted) prices for Intel (INTC) and the S&P 500 from YAHOO! FINANCE.

1. Compute the daily rates of return.
2. Compute the average rates of return and risk of portfolios that combine INTC and the S&P 500 in the following proportions: (0.0, 1.0), (0.2, 0.8), (0.4, 0.6), (0.6, 0.4), (0.8, 0.2), (1.0, 0.0). Then plot them against one another. What does the plot look like?
3. Compute the historical market beta of Intel.

Q 8.34. Why do some statistical packages estimate covariances differently (and different from those we computed in this chapter)? Does the same problem also apply to expected rates of return (means) and betas?

9

Benchmarked Costs of Capital

As an investor, your problem is to form good portfolios. As a corporate manager, your problem is how to get your own firm into other investors' portfolios. So you need to know the right discount rate at which they will bite. In earlier chapters, this discount rate was just time-based and all you had to do was to offer the same expected rate of return. In this chapter, we begin adding a risk component.

We will now assume that your investors simply benchmark all investment opportunities (including your stocks, bonds, projects, etc.) to other prominent asset classes in the economy. In particular, we assume that they will evaluate your firm based on two characteristics: (1) whether your project payoffs are more like short-term or long-term investments; and (2) whether your payoffs are more like safe debt or risky equity. Safe bond-like projects can get away with offering investors lower average rates of return; risky stock-like investors must offer higher expected rates of return. This means we need to take another look at bills, bonds, and stocks in the overall economy. What is the appropriate risk-free rate of return for projects of similar durations, and what is the equity premium for the expected rate of return on stocks above bonds?

9.1 What You Already Know

You are still after an estimate for your opportunity cost of capital.

Let's take stock (pun!). You already know the right train of thought for capital budgeting purposes: As a corporate manager, your task is to determine whether you should accept or reject a project. You make this decision with the NPV formula. To determine the discount factor in the NPV formula, you need to estimate the appropriate cost of capital—or, more precisely, the *opportunity* cost of capital for your investors. This means that you need to judge what a fair expected rate of return, $E(r)$, is for your project, given your project's characteristics. When compared to "similar" projects elsewhere, if your project offers a lower expected return, then you should not put your investors' money into your own project but instead return their money to them. If your project offers a higher expected return, then you should go ahead and invest *their* money into *your* project. Put differently, your goal now is to learn what your investors, if asked, would have wanted you to invest in on their behalves. Of course, it still remains difficult to determine what "similar" is, but this is a devil in the details.

What do investors like?

Unfortunately, the perfect market assumptions of Utopia are no longer enough to proceed. You must begin to speculate more about your investors' preferences. What do investors like and dislike? You already know two relevant project attributes:

Far-Off vs. Nearby Payments: Long-term Treasury bonds have (usually) been offering higher yields *per-annum* than short-term Treasury bills. Presumably, this is because investors are more reluctant to part with their money when payment is farther down the line. In this sense, you can think of long-term as "toxic" relative to short-term. Investors (usually) seem to like getting money sooner.

Equities vs. Bonds: The stock market has offered higher average rates of return than the bond market. Presumably, this is because investors are more reluctant to part with their money when all they get is a fuzzy risky claim, like equity, with repayment depending more on success. In this sense, you can think of equity as "toxic" relative to bonds. Investors like getting money with less variance.

(A quick clarification: high expected rates of return usually mean that investors dislike an asset's attributes—this asset could *not* be sold for a high price because investors needed to be compensated extra for something.)

Et tu, Brutus?!

As an executive, you should assume that if investors dislike an attribute in the wider financial markets, they will also dislike it in your own projects. If you offer them a project that pays off more like stock-market equity, it has to offer the higher expected rate of return of stocks. If you offer them a project that pays off more like bond-market principal and interest, it can offer the lower expected rate of return of bonds. And if you compare two projects, one with payoffs farther in the future than another, the former should offer higher expected returns—just as long-term bonds offer higher expected returns than short-term bills. The focus of this chapter is therefore to assess what rates of return you can expect in these different types of investment.

Firms that are just funds are good examples where this must work.

In a perfect market, these rules must surely be correct for the most simple of all investment projects: Firms that do nothing but invest in Treasury bonds (i.e., fixed income investment funds) should offer about the same expected rates of return as their Treasury bonds. If they offer lower expected returns, investors can buy the bonds themselves. If they offer more, investors will quickly bid up the price of the fund until the expected returns become about the same. The same is true for equity. Firms that do nothing but hold S&P 500 stocks should offer about the same expected rates of return as the S&P 500. And firms that invest 50-50 should offer 50-50.

Now what?

The big question of this chapter is: how can you assess the appropriate expected rate of return on the standard benchmarks, i.e., on risk-free investments and on stocks? In the next chapter, you will learn methods to judge how similar projects are to each of these benchmarks.

9.2 The Risk-Free Rate — Time Compensation

Nominal or Real?

How do you assess the risk-free rate of return (r_F)? Most corporations want the nominal rate from U.S. Treasuries, because they want to discount nominal cash flows. In the rare case that a corporation needs to discount real cash flows, the U.S. Treasury also offers quotes on inflation-adjusted real bonds (TIPS).

Which risk-free rate?

➤ US Treasuries, Sect. 5.3, Pg.86.

There is one small issue, though—which Treasury? What if the yield curve is upward-sloping (as it usually is)? For example, in mid-2016, Treasuries yielded 0.1% per annum over one year, 1.8% over five years, and 3% over thirty years.

Advice: Pick the interest rate for a Treasury that is "most similar" to your project.

So think about the basics of your own project. You want to match your projects' cash flows to the most similar risk-free bond benchmark. You should choose the risk-free bond yield that most closely mirrors the specific expected cash flows. For example, to value a safe project that operates for three years, use the 1-year Treasury yield to discount the expected cash flow for the first year's NPV term, the 2-year Treasury for the second year's NPV term, and the 3-year Treasury for the third year's NPV term. If you had to use just one risk-free rate for multiple cash flows (because your Dilbertian boss says so), choose an average of the three rates or simply the 2-year bond. There are better duration-matching ways to do this, but unless you are a bond trader, the extra precision is rarely worth it.

Matching cash flows to similar maturity bonds is not a law of nature but a reasonable (and loose) approach. Think about the opportunity cost of capital for a small investment that does not vary systematically with anything else. If your corporation's investors are willing to commit their money for ten years, they could earn the yield on a ten-year risk-free bond instead. It is this ten-year rate that would then be more indicative of the opportunity cost of capital on your own project cash flow that will materialize in ten years than, say, a one-year or thirty-year bond. If your project's cash flow will occur in three months, your investors could alternatively only earn the lower rate of return on the three-month bill.

But don't we need formal guidance? Isn't this violating the letter of the law?

Of course, to your investors, your project's cash flows are not likely to be exactly like the analogous U.S. Treasury payments. Thus, you can consider some refinements. It may be more appropriate to use an opportunity cost more similar to corporate than to Treasury bonds. Fortunately, for short-term corporate bonds issued by investment-grade companies, after you take into account that quoted yields have to be reduced by the expected default premium, the average historical rate of return has been almost the same. For long-term non-investment-grade (i.e., high-yield) bonds (except perhaps mortgages), the cost of capital may be considerably higher.

Corporate interest rates?

➤ default premium, Sect. 6.2, Pg.113.

Q 9.1. What is today's risk-free rate for a 1-year project? For a 10-year project?

Q 9.2. If you can use only one Treasury, which risk-free rate should you use for a project that will yield \$5 million each year for 10 years?

9.3 The Equity Premium — Risk Compensation

Appropriate compensation for a risk-free investment over a given time frame is the easy part. This is the cost of risk-free capital. Now comes the hard part: appropriate compensation for taking risk. This is the cost of risky capital. Although most corporate projects are not risk free, you can think of them as some combination of a safe part (a debt-financed claim) and a risky part (an equity-financed claim). Indeed, you have already learned that you can always split a medium-risky project into claims that have safer and riskier payoffs. Therefore, you usually need to know the appropriate cost of capital on the risky part, too—the task at hand now.

Think of projects as part risk-free, part risk.

➤ Splitting payoffs, Sect. 6.4, Pg.123.

Unfortunately, the expected rate of return on risky assets is much more difficult to estimate than the risk-free rate. First, what is a good benchmark for risk? Hmmm...What is the most canonical risky asset in the economy? The stock market! We financiers usually rely on a benchmark

Work with the equity premium

$$\textbf{Equity Premium} \equiv \mathsf{E}(r_M) - r_F \,, \tag{9.1}$$

which is the extra expected rate of return that risky equity projects have to offer above and beyond what risk-free bonds are offering. (It is a difference of two rates, so you can use either two nominal or two real rates.) Later, when you want to determine the expected rate of return on a project that consists only of one asset that is the stock market, say an S&P 500 fund, you would add back the interest rate you just subtracted out here. It is easier to think about the "extra" of the risk premium above the time premium (in the risk-free rate) rate. The equity premium $[\mathsf{E}(r_M) - r_F]$ is also sometimes called the **market risk premium**. In common use, the terms can refer either to realized rates of return or expected rates of return, although the latter is more common and we will use it mostly in this sense in this chapter. (This ambiguity is not my fault.)

➤ Inflation, Sect. 5.2, Pg.82.

You want to know the equity premium

This equity premium is a number of first-order importance for everybody. It is not just the corporations want to know it for their cost-of-capital estimation. *You* also want to know it as an investor when you decide how much of your money you should invest in stocks rather than bonds. Unfortunately, in real life, the equity premium is not posted anywhere—and *no one really knows the correct number*. Worse: Not only is it difficult to estimate, but the estimate often has a large influence over all financial decision-making. *C'est la vie!*

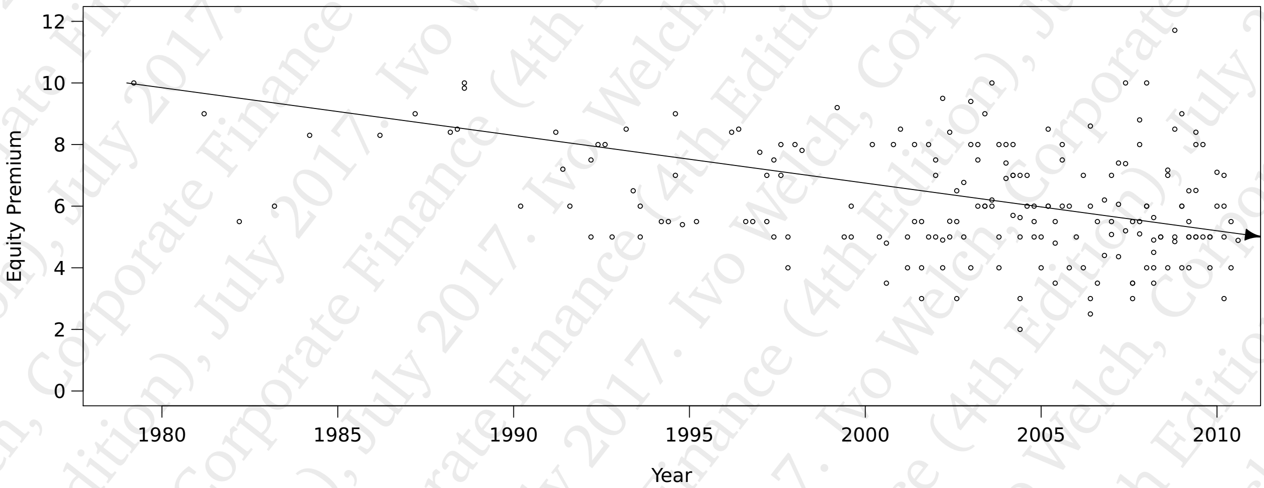


Exhibit 9.1: *Equity Premia from Different Textbooks. Source: Pablo Fernandez, SSRN, 2013.*

Should I just give it to you?

Fortunately, there are a number of methods to guestimate the equity premium. Unfortunately, for many decades now, these methods have disagreed with one another. It should thus come as no surprise to you that practitioners, instructors, finance textbook authors, and everyone else have been confused and confusing. For example, each finance textbook seems to have its own little estimate, as you can see in Exhibit 9.1. Both the disagreement and the average recommended estimate seem to have been slowly declining over the decades.

Let's show you how people are reasoning.

So "we" finance-textbook authors have two choices:

1. We can throw you one estimate, pretend it is the correct one, and hope that you won't ask questions. It would be a happy fairy tale ending. Unfortunately, it would also be a lie.
2. We can confess to the truth. We can tell you how different methods can lead to different estimates—and how we are really all in the same boat. Worse, we are not sure where the boat has holes.

In this book, I am going to take the second route. I will explain to you what each method suggests and actually means. You can then make up your own mind as to what you deem to be best. (I will tell you my own personal estimate at the end.) This also has an important advantage: you won't be surprised if your boss uses some other equity premium to come to different conclusions. At least you will understand why.

Let's discuss one-by-one—and in order of prominence—the six most prominent methods that form the bases of common equity-premium estimates.

Method 1. Historical Averages I

Historical average returns.

The first and most common guesstimation method is to assume that whatever the average equity premium was in the past will also be the case in the future. And the past century has been pretty good to us.

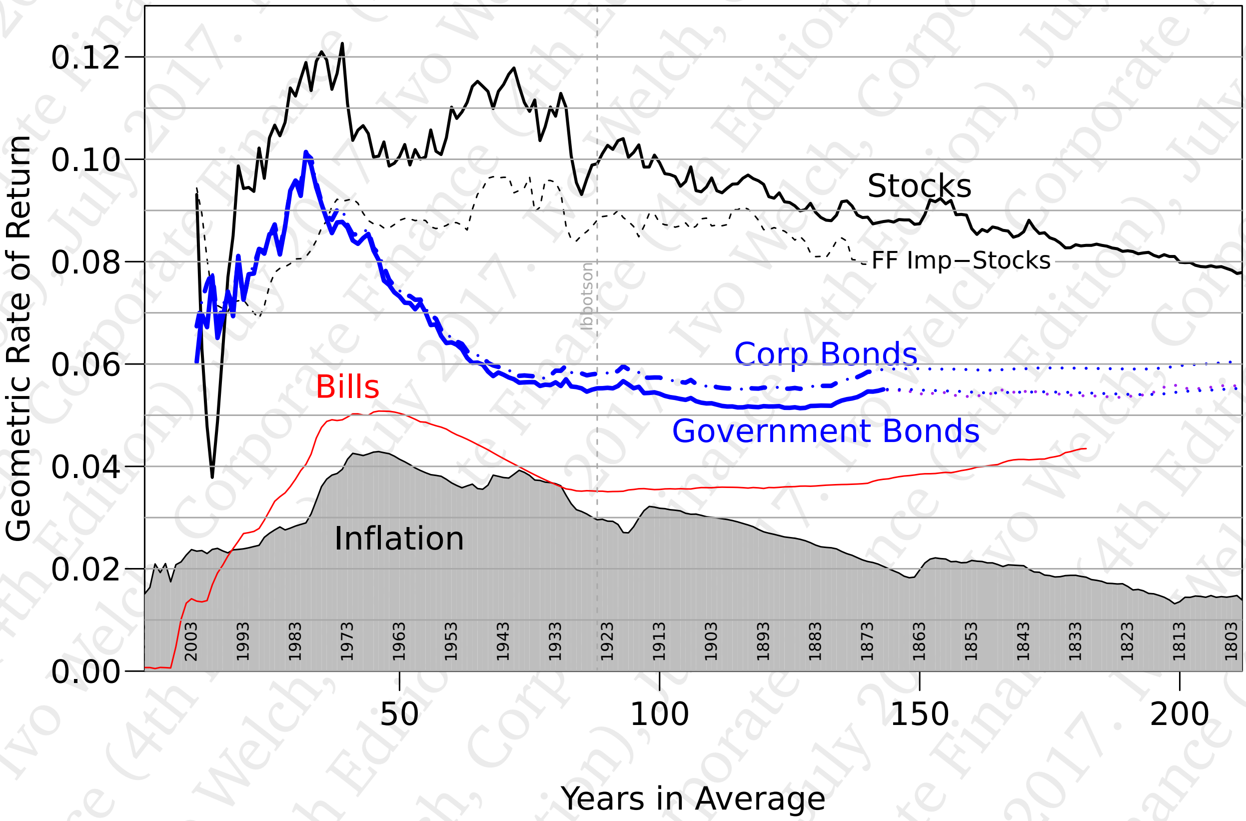


Exhibit 9.2: *Asset Class Geometric Rates of Return. Source: Levi-Welch, JFQA, 2017. This graph is* backward*-looking. If you want to know what conclusions you would draw from the data since 1963, you look at the first notch on the X-axis. If you want to know what conclusions you would draw from the data since 1863, you look at the third notch on the X-axis. The most recent 10 years were omitted, because such an experience would be too short to draw conclusions and the lines would become too jagged. The equity premium is the difference between the black “Stocks” line and either the blue “Bonds” or the red “Bills” lines. For example, the equity premium above US Treasury Bonds measured from 1975 to 2015 (i.e., about 40 years) was about 2% per annum..*

Cut to the chase.

Exhibit 9.2 plots the average *geometric* performance of the stock market (with dividends) over the last x years. You choose your point on the x axis based on how relevant you consider more recent vs. older historical data. The graph also shows the rate of return on (long-term) corporate bonds, long-term Treasury bonds, short-term Treasury bills, and inflation. The difference between the black stock market line and the red line is the short-term equity premium. The difference between the black stock market line and the blue fixed-income lines are long-term equity premia.

➤ Geometric vs. Arithmetic Returns, Sect. 7.1, Pg.140.

- Over the last 50 years, stocks have outperformed both long-term Treasury and corporate investment-grade bonds by about 2%/year (compounded). However, over the last 100 years, they have outperformed by a larger margin of about 4%/year.

- Over the last 100 years, stocks have outperformed short-term bonds by about 4-5%/year.

Let's discuss these estimates and their interpretation in more detail. In particular, we want to be clear about how to deal with these benchmarks for assessing your own short-term projects and long-term project opportunities. Most interesting corporate projects, like factories, buildings, research, or brand names, deliver cash flows over many years.

The Stock Market is a Long-Term Asset

What is the average return on the S&P 500?

A natural way to think of premium is to think of a clean decomposition into a term premium and a risk premium. The empirical evidence has shown that most of the value of stocks comes from their long-term payoffs (decades off), and not in their payoffs over the next few years. The stock market should be viewed as a (very) long-term investment.

Risk Holding Term Constant

Consequently, if you are interested in measuring the risk-premium, you should subtract the rate of return on long-term bonds from the rate of return on stocks. If you instead subtract the rate of return on short-term bills, you end up with the sum of the term premium and the risk premium.

Last 50 years: Stocks - Short-Term Bills = 3% Term + 2% Risk

According to Exhibit 9.2, the equity premium was the sum of the term premium of long-term bonds over short-term Treasury bills of about 3-4% over the last 50 years and 2% over longer histories; and the risk premium of about 2% over the last 50 years and 4% over the last 100 years. None of this is a problem. Different numbers just mean different things. The problems can arise later in the application. We have to keep in mind which is which.

Disagreeing About Estimates

The gamut of choices

➤ Geometric vs. Arithmetic Returns and Extrapolation, Sect. 7.1, Pg.140.

If you use 90 years of historical data, arithmetic rates of return, and a spread over short-term T-bills, you can settle on an equity premium estimate as high as 8%. This number is prominently quoted in many other finance textbooks. It is important to understand it, because so many people are still using it. It was etched in the minds of generations of students, practitioners, and finance professors. But it has a specific meaning and is based on a specific sample period. Worse, it is often misapplied.

Both the high 8% estimate and the lower 2% alternative estimate in Exhibit 9.2 follow from the same historical data. Let me explain the key difference between them:

Arithmetic Equity Premium 1926 to 2015 over Short-Term T-Bonds	≈ 8%
Instead use later Sample Period 1970 to 2015	–2%
Instead use Long-Term T-Bonds	–2%
Instead use Geometric Returns	–2%
Geometric Equity Premium 1970-2015 over Long-Term Bonds	≈ 2%

Equity Premium Puzzle

The 8% figure seems astonishingly high. It is often called the **equity premium puzzle**. (But can you really expect stocks to outperform bonds by a factor of $1.08^{50} \approx 50$ by the time you will retire in about 50 years? No!) This is the claimed superior performance of U.S. stocks over U.S. Treasury bills. Exhibit 9.2 is clear that since about 1970, it has been more of a term-premium puzzle. Long-term bonds outperformed short-term bills by about 3-4%. Stocks, themselves more long-term assets, have outperformed long-term bonds only by about 1-2%. Mmaybe we should argue more about the term premium puzzle and less about the equity premium puzzle. In contrast, the 2% figure seems low. For tax-exempt investments (e.g., your 401-K pension portfolio), 2% seems like a more reasonable amount of compensation for the risk.

Let's discuss the differences one by one:

1. **Sample Period?:** You have to judge what historical sample is appropriate. You probably want to end the sample recently (last year). But it is not clear whether you should start, say, in 1926 (which is when most of our common finance databases begin) or in 1970 (about half-way). Although your estimate can seem statistically more reliable if you use more years, using the long sample means that you are then leaning more heavily on a heroic assumption that the world has not changed. Are the world and its expected risk and reward choices really still the same today as they were in 1830, 1871, 1926, or 1970? (And is the United States really the right country to consider alone? Did it just happen to have had an unusually lucky streak during [the first half of] the "American Century," which is unlikely to repeat? In this case, the average country's experience may be a better forecast for today's United States, too.) No one knows the best sample choice. I prefer a shorter sample of half a century.

   Incidentally, as Exhibit 9.2 showed, the equity premium was lower in this 50-year sample not because (noisier) stocks performed worse (they did not), but because (less noisy) Treasury bonds performed better—and bonds continue to have higher yields than bills.

2. **Long-Term or Short-Term Bonds?:** You have to judge whether short-term or long-term bonds are the appropriate benchmark. From the perspective of a financial-market investor who can make daily reallocation decisions and shift effortlessly between risk-free T-bills and stocks, using the short interest rate as the benchmark makes sense. From the perspective of a manager who needs to decide about a short-term project, using short interest rates as the benchmark also makes sense. However, from the perspective of a corporate manager who needs to commit funds to a long-term project with cash flows over decades, it does not. It is not possible for corporations to quickly move in and out of decisions to build, say, power plants. Building a plant is a long-term decision. If all investors can earn higher yields in Treasuries if they commit their money for 20 years, and if your own plant requires them to commit their money for 20 years, too, then your plant should also be benchmarked to this long-term expected rate of return. Conveniently, the term spread between 1-year and 20-year risk-free rates (though not the rate of return on rolling over 1-year bills over 20 years) can be easily looked up on the web every day. There is little uncertainty.

3. **Geometric or Arithmetic?:** You have to judge whether you should use geometric or arithmetic rates of return in your benchmark cost of capital in the NPV formula. The answer is not clear, as you may recall from Section 7.1. There was a convention of assuming that past returns represent equally likely future outcomes, and many corporations compound the annual arithmetic average stock return or equity premium without much thought. However, doing so means that they expect the future multi-year stock performance relative to bonds to be better in the future than it was in the past. (For nit-pickers, the theoretically correct choice depends on the cash flow durations and suggests compounding the equity premium estimate somewhere between the arithmetic and geometric averages.)

   ➤ Geometric vs. Arithmetic Returns and Extrapolation, Sect. 7.1, Pg.140.

   But there is a simpler argument based on the rule of comparing apples to apples. How do you think about your own expected cash flows? I bet you do so in geometric terms. If you think in terms of arithmetic expected cash flows compounded over many periods—i.e., if you consider the expected cash flow on a project that first earns +200% and then –100% [for a complete overall loss] to be a success with a positive average rate of return, then you should use the arithmetic average. Hardly anyone thinks this way.

   We will return to compounding concerns in Section 9.4.

Was the 20th Century Really the "American Century?"

The compound rate of return in the United States was about 8% per year from 1920 to 1995. Adjusted for inflation, it was about 6%. In contrast, an investor who had invested in Romania in 1937 experienced not only the German invasion and Soviet domination, but also a real annual capital appreciation of about –27% per annum over its 4 years of stock market existence (1937–1941). Similar fates befell many other Eastern European countries, but even countries not experiencing political disasters often proved to be less than stellar investments. For example, Argentina had a stock market from 1947 to 1965, even though its only function seems to have been to wipe out its investors. Peru tried three times: From 1941 to 1953 and from 1957 to 1977, its stock market investors lost *all* their money. But the third time was the charm: From 1988 to 1995, its investors earned a whopping 63% real rate of return. India's stock market started in 1940 and offered its investors a real rate of return of just about –1% per annum. Pakistan started in 1960 and offered about –0.1% per annum.

Even European countries with long stock market histories and no political trouble did not perform as well as the United States. For example, Switzerland and Denmark earned nominal rates of return of about 5% per annum from 1920 to 1995, while the United States earned about 8% per annum. A book by Dimson, Marsh, and Staunton looks at 101 years of global investment returns and argues that measurement and hindsight biases can account for much of this superior return.

The U.S. stock market was an unusual above-average performer in most of the twentieth century. Will the twenty-first century be the Chinese century? And do Chinese asset prices already reflect this? Or already reflect *too much* of this?

Goetzmann and Jorion (1999)

Uncertainty About Historical Estimates and the Peso Problem

Yet another problem: your margin of error.

Forgive me, but I have not even mentioned another big problem: the large margin of error. The standard deviation of stock returns of 20%/year translates into a standard error of about $20\%/\sqrt{100} \approx 2\%$ if you use a 100-year sample. If you are willing to assume that the stock-market process has not changed over the last 100 years, and that stock returns are roughly normally distributed, then you can use some additional statistical artillery: You are then about 95% sure (a confidence range popular in statistics) that the true mean geometric stock return over long bonds was between 0% and 8% from 1926 to 2015. Frankly, this large a range on the appropriate cost of capital for equity is not the kind of accuracy you like when you have to decide where to invest your money. You already knew—or at least should have reasonably believed—that the equity premium should not have been negative.

Lucky—inference with rare outcomes.

To make matters even more complex, some economists believe that even the observed historical data are not telling the full story, either. Let me explain this by analogy. The odds in roulette are against you, with a payout of 1-in-35 when betting on a single number (out of 36 numbers). How good is a bet on the first 34 roulette numbers? Well, you will win 34/36 of the time, each time losing \$34 and getting back \$35 (i.e., 2.9%). After a run of 50 times, in which neither #35 nor #36 have showed up, you would incorrectly conclude that your expected rate of return is +2.9% per roll. Of course, this is delusional. But it's not completely impossible that you could have seen, say, 30 good rolls.

Lucky—quite possible.

Similarly, maybe we just happen to live in world in which the stock market has never rolled the worst outcomes. The true expected rate of return could be zero or even negative. Thus, these economists believe that disasters have been possible, but their probabilities have been tiny (say, 1-in-100 years)—and they just "happened not to have happened" in the last 100-200 years. The super-volcano did not blow; the asteroid did not hit. For example,

	Asteroid	Normal
Probability	0.01	0.99
Stock Return	–99%	+1%

True Average Expected Rate of Return: 0%
Average Rate of Return Given Luck of No Asteroid: 1%

Presumably, such a zero expected rate of return for a risky investment is as low as it could reasonably be. Trust me that there is about a 1-in-3 chance that over 100 years, not even one asteroid would have hit. If you happen to have lived in such a world—called "the U.S. of the last 100 years"—you would have calculated a historical average rate of return of 1%. Alas, it would be too optimstic an estimate of the true expected rate of return.

The Peso Problem

This is sometimes called the **Peso problem**, based on an otherwise obscure academic paper about the currency spread of the Mexican Peso. When you say "Peso problem," financial economists will know exactly what you mean!

Peso Problem Magnitude

There is some empirical evidence that investors behave exactly as if they fear such a Peso crash—but we do not know whether such a fear is (or was) rational and we are not sure how much of the historical equity premium it can explain. A reasonable order of magnitude is that extra compensation for crash risk could account for at most a 1-2%/year equity premium—and perhaps for nothing.

In Conclusion

A sarcastic view: History ain't what it used to be!

If your estimate of the forward-looking equity premium is based on the "historical averages I" method, then you can defend a choice of 1% (for long-term cash flows). If you are aggressive, you can defend even a choice of 8% (for short-term cash flows), and equity premium ranges from 0% to beyond 10% if need be (or, more cynically, if you are an expert witness paid to so opine). Are you in awe or disgust about out uncertainty and the wide possible range of estimates here? For me, its both.

Method 2. Historical averages II

Inverse historical averages.

The second method for estimating the equity premium is to look at historical realizations in the opposite light. Maybe stocks have become more desirable—perhaps because more investors have become less risk-averse. They would have competed to own more stocks, and thus have driven up the prices. This would imply lower expected rates of return in the future! High past rates of return would then be indicative of low future expected rates of return.

An even more extreme version of this argument suggests that high past equity returns could have been due not just to high ex-ante equity premiums, but also to historical "bubbles" in the stock market. The proponents of the bubble view usually cannot quantify the appropriate equity premium, but they do argue that it is lower after recent market run-ups—exactly the opposite of what proponents of the *Historical Averages I* guesstimation method argue. However, you should be aware that not everyone believes that there were *any* bubbles in the stock market, and few credible economists believe that the U.S. stock market over the entire century was one big bubble.

Sidenote: A **bubble** is a runaway market, in which rationality has (at least temporarily) disappeared. There is a lot of academic debate as to whether bubbles in the stock market have ever occurred. A strong case can be made that technology stocks experienced a bubble from around 1998 to 2000. It is often called the **dot-com bubble**, the **internet bubble**, or simply the **tech bubble**. I know of good fundamental-based explanations as to why the NASDAQ Index climbed from 2,280 in March 1999 to 5,000 by March 2000 and why it dropped from 5,000 back to 1,640 by April 2001—but no good non-bubble explanations for both.

Method 3. Current predictive ratios

Dividend or earnings yields.

The third method for estimating the equity premium is to try to predict the stock market rate of return actively with historical dividend yields (i.e., the dividend payments received by stockholders). Higher dividend yields should make stocks more attractive and therefore predict higher future equity premiums. This equity premium estimation is usually obtained in two steps:

1. Estimate a statistical regression that predicts next year's equity premium with this year's dividend yield
2. Substitute the currently prevailing dividend yield into your estimated regression formula in order to predict.

In mid-2016, dividend yields were so low that the predicted equity premium was negative—which makes no sense. Variations of this method have used interest rates or earnings yields, typically with similar results. In any case, the empirical evidence suggests that this method does not yield great predictions—for example, it predicted low equity premiums in the 1990s, which was a period of superb stock market performance.

Academics disagree whether such methods work for short-term equity-premium predictions (say 1-5 years). But all agree that we do not have the data to test whether this works and to predict 10-50 year equity premiums. And it is for the very-far-away expected cash flows where corporate finance managers are most in need of equity premium estimates. Therefore, most managers can neglect these regressions.

Method 4. Philosophy

Introspection and philosophy.

The fourth method is to wonder how much rate of return is required to entice reasonable investors to be indifferent between stocks and bonds. Even with an equity premium as low as 3%, over 25 years, an equity investor would end up with more than twice the money of a bond investor. Naturally, in a perfect market, nothing should come for free, and the reward for risk-taking should be just about fair. Therefore, equity premiums of 6-8% just seem too high for the amount of risk observed in the stock market. This philosophical method generally suggests reasonable equity premiums of about **1%** to **3%**.

Method 5. Surveys: Ask the Experts

Just ask!

What to choose? Welcome to the club! No one knows the true equity premium. So, the fifth method is to ask the experts—or anyone else who may *or may not* know. It's the blind leading the blind. The ranges of estimates have varied widely (and they are often also conveniently tilted in the interest of those giving them):

- The Social Security Administration sometimes uses an estimate of around 4%.
- For decades, the consulting firm McKinsey has used a standard of around 5%.

Analysts' estimates are all over the map, too. Estimates between 2% and 6% per annum seem reasonable.

- Around the turn of the millennium, the most common equity premium estimates recommended by professors of finance were **5%** for a 1-year horizon and **6%** for a 30-year horizon, both with a range from 3% to 8%. The estimates were generally similar in the United States, Spain, Germany, and the United Kingdom.
- On Monday, February 28, 2005, Jason Zweig of *The Wall Street Journal* reported some *after-inflation* forecasts from then to 2050 (per annum), as in Exhibit 9.3.

  As you already know, it matters (a) whether you quote geometric or arithmetic averages; and (b) whether you quote the equity premium with respect to a short-term or a long-term interest rate. If you want to use the short rate, then you need to add another 1-2% to the equity-premium estimates in this table. (Unrelated, for the equity premium, it does

Name	Organization	Government Stocks	Government Bonds	Corp. Bonds	Equity Premium Rel Gov	Equity Premium Rel Corp
William Dudley	Goldman Sachs	5.0%	2.0%	2.5%	3.0%	2.5%
Jeremy Siegel	Wharton	6.0%	1.8%	2.3%	4.2%	3.7%
David Rosenberg	Merrill Lynch	4.0%	3.0%	4.0%	1.0%	0.0%
Ethan Harris	Lehman Brothers	4.0%	3.5%	2.5%	0.5%	1.5%
Robert Shiller	Yale	4.6%	2.2%	2.7%	2.4%	1.9%
Robert LaVorgna	Deutsche Bank	6.5%	4.0%	5.0%	2.5%	1.5%
Parul Jain	Nomura	4.5%	3.5%	4.0%	1.0%	0.5%
John Lonski	Moody's	4.0%	2.0%	3.0%	2.0%	1.0%
David Malpass	Bear Stearns	5.5%	3.5%	4.3%	2.0%	1.2%
Jim Glassman	JP Morgan	4.0%	2.5%	3.5%	1.5%	0.5%
			Arithmetic Average (Difference):		2.0%	1.4%
	Volatility-Adjusted Geometric Average ≈ –1% :				1.0%	0.4%

Exhibit 9.3: *Jason Zweig Survey.* Some prominent equity analysts predicting.

not matter whether equity premium numbers are inflation-adjusted. Inflation cancels out, because the equity premium is itself a difference in nominal rates.)

We still have another 35 years to go before we can check the forecast, but 1-2% still looks right on the money.

- In 2005, still fairly soon after the bear markets of the early 2000s, a poll by Graham and Harvey (from Duke) and *CFO Magazine* reported an average equity premium estimate of CFOs of around 3%. By 2015, having experienced many years of bull market, Graham and Harvey reported that they then expected a 10-year relative equity premium of 4.5%.
- In mid-2008, just after the financial crisis, Merrill Lynch's survey of 300 institutional investors reported 3%.
- In 2012, Pablo Fernandez reported that analysts and companies in the United States, Spain, Germany and the United Kingdom all used average estimates of between 5% and 6%—just like finance professors, and with the same typical range from about 3% to 8%. And this estimate further increased by another 1% over the following 3 years.

➤ Pablo F., Exhibit 9.1, Pg.196.

- In 2017, the directors of the $299 billion(!) CalPERS pension fund will have to decide again whether their expected (geometric) rate of return of 7.5%—5% above the prevailing Treasury long bond—is optimistic. CalPERS also holds some non-public assets, but there is no reason to believe these assets are likely to outperform the stock market, either. If 7.5% seems unrealistic to you, it obviously is. But lowering this estimate would mean that California's politicians would have to set aside more money for their unfunded pension obligations *today*. Obviously, they would prefer to leave the optimistic estimate as is, and kick the can down the line to their successors.

  Thousands of other public employees pension funds all over the nations—and you younger taxpayers—face similar problems. Moody's estimates that politics has left public pension funds underfunded by about $7 trillion as of 2016—or about $50,000 per U.S. household. (Add social security and medicare commitments, and you can triple this.) Start saving up!

Of course, these estimates were themselves likely based on the first four methods, and they occur in echo chambers—they are what analysts, companies, consultants, students, and professors have been reading in corporate finance textbooks (like this one) for many years now. (Hmm...maybe I should try claiming 42.321% and then see how many surveyees will repeat it back in ten years.)

One aspect that does not make sense and that was already mentioned is that survey estimates seem to correlate too strongly with very recent stock market returns. For example, in late 2000, right after a huge run-up in the stock market, surveys by *Fortune* or *Gallup/Paine Webber* had investors expecting equity premiums as high as 15% per year. (They were acutely disappointed: The stock market dropped by as much as 30% over the following two years. Maybe they just got the sign wrong?!)

6. Internal Cost of Capital (ICC) and Accounting Models

Ask and Use!

A hybrid method combining survey methods and analysis is the "Internal Cost of Capital." Basically, this method uses analysts' consensus projections about S&P 500 earnings (over the next few years) with a perpetuity model to back out a cost of capital that makes the S&P 500 price equal to the analysts' discounted future earnings. Because analyst estimates vary over the business cycle, researcher usually use the average of many ICCs over many years.

In the graphs

If you glance at Exhibit 9.2 again, you will note a small line marked "FF Imp Stocks," which comes from just such an attempt to convert analysts' earnings forecasts into an expected rate of return for the stock market. Until the mid-1980s, this geometric average was generally lower than the historical average performance, consistent with the view that the 20th century was the lucky American Century. However, more recently, it has agreed more with the historical expected rate of return in suggesting much higher expected stock market rates of return for the future. (And, as with historical estimates, different variants can give estimates with a much larger range, say, from 0% all the way to 7%.)

Accounting Models?

There are some accounting-based models that are based on similar principles and are often claimed by their proponents as panaceas—or at least as better alternatives. Alas, when I looked at some of these models with a more skeptical eye, I could not share their enthusiasm for three reasons. First, these models are too "boutique": each has been tweaked just a little here and there to make it look good on their data. Second, these models tended to work well in the first halves of their samples and not so well in the second halves. Third, if they really worked half as reliably as they are claimed to work, then investment funds should flock to them like flies. Many looked at them and they did not. This is not to say that no such model works—just that those that I investigated more did not hold up.

9.4 Forward-Looking Benchmarks

The two most important numbers.

The risk-free rate and the equity premium are the two most important numbers in economics and finance. If the risk-free rate is high, you should save more and consume less. If the equity premium is high, you should allocate more of your savings into diversified risky stocks and less into bonds. The previous section has taught you about how to view the historical data.

Forward not backward!

But you are probably not interested in *historical* performance for its own sake. You are probably interested in the *future* expected performance instead. (When you want to judge whether you will have to drive uphill or downhill, looking at the rearview mirror may be better than nothing, but it is not ideal.)

Lasciate ogne speranza, voi ch'intrate

➤ Neutralizing Market Exposure, Sect. 10.4, Pg.224.

So what is the appropriate forward-looking expected equity premium *today*? Sadly, no one can tell you the authoritative estimate. Such an authority does not exist. Everyone is guessing. Unfortunately, unless your project has no (market-risk) type of exposure, you usually have to take a stance. (I will explain in Section 10.4 how you can finesse this, but doing so will have its own drawbacks.) I failed to shield you from the estimation dilemma. I can only give you the considerations that you can contemplate when you are picking *your* estimates.

No help in sight.

If you are hoping I will rescue you in future chapters, by either giving you the correct numbers or telling you that you do not really need them to make decisions, I can't. Even more involved financial models, in particular the CAPM in the next chapter, ask *you* to provide the same estimates. They just help by informing you about the expected rate of return for projects *relative* to Treasuries and the stock market. Given *your* estimate of how much risky average stock market projects should earn relative to safe projects, plus the market-beta, the CAPM tells you the benchmark cost of capital for your projects. But unless your projects have zero exposure to stock-market-type risk, the models themselves require *you* to input your equity premium estimate.

➤ Betas, Pg.181.

It is all about relative pricing, not absolute pricing.

The need for good alternatives (benchmarks) is important to capital budgeting in corporations. They measure the opportunity cost of capital. But you also need them if you are an investor on the buying side. Like everybody else, you cannot let your limited knowledge stop you from making investment decisions. You do need to be your own judge: what are your prevailing (economy-wide) opportunities? Where do you want to place your money?

Term and Risk

Not exactly chopped liver.

I admit that I could not teach you the correct premium estimates. But I am not altogether useless, either. I can teach you at least how to avoid some basic errors. You have already read about one important aspect, albeit in the context of historical averages. Short-term and long-term projects should have different benchmarks. This insight is very important and you can get this right. So let's discuss it in more detail.

Term Premia in Bonds

The correct approach is obvious for risk-free projects. If your project is short-term, the correct benchmark is the rate of return on short-term bills, not long-term bonds. If your project is long-term, the correct benchmark is the rate of return on long-term bonds, not short-term bills.

Term Premia in Stock Returns and Equity Premia?

The correct approach is less obvious for risky projects. Remember that stocks are themselves long-term cash-flow assets (even if you can sell them instantly, just as you can sell Treasury bonds at any moment).

➤ Stocks = Long-Term Asset, Pg.198.

- If you have a project with a payoff that is as risky as the stock market and with a similarly long horizon, the stock market is your correct benchmark. The stock market's expected rate of return reflects both the term and the risk premium. If you think that the last 50 years are a good representation for the future, Exhibit 9.2 tells you that you should expect a 10% average geometric rate of return, of which about 5% is the short-term benchmark (the premium for saving money), 2% is the premium for the long-term nature of payoffs, and 3% is the premium for taking risk.
- If you have a project with a payoff that is as risky as the stock market payouts or earnings, but lasts for only one period, the equity premium without the term premium is your correct benchmark. Thus, a discount rate of 7% is more appropriate.

Some finance professors believe that you should use a higher risk premium (higher than 3%) for long-term cash flows—that is, more term premium in stocks than in Treasuries. But only the Treasury term premium is easy to measure. The jury is still out, and this extra "kicker" would likely be small.

Geometric or Arithmetic Cash Flows and Benchmarks?

Geometric vs. Arithmetic matters for shorter-term return averages applied on long-term cash flows.

How does the NPV formula work under uncertainty? Over one time period, a geometric average rate of return is the same as the arithmetic average rate of return. This arithmetic average rate of return was itself calculated as the compounded (geometric) average over many smaller time intervals. Now, commonly-published benchmark rates of return are usually quoted as *annual* or even shorter-term rates of return, not as, say, 30-year rates of return. You have to translate the shorter-term rate of return statistics you are given into the expected longer rate of return statistics you need.

Arithmetic Use = Wrong

Does it make sense to compare arithmetic average returns across long-term project cash flows with different volatilities? Would you rather invest for T years in a ($100) project with an average annual rate of return of 5% and a variance of 40% (twice the stock market), or in a project with an average rate of return of 2% and zero variance? Would you take the first type of project if the financial markets offered you the opportunity of the second? If you use the NPV formula on the arithmetic averages as

$$\text{(Wrong:)} \quad -\$100 + \frac{+\$100 \cdot (1+5\%)^{T}}{1+1.02^{T}} > 0$$

you would conclude that you should take the project. But this would be wrong.

The project will not average 5% per T, but -3% per T.

➤ Geometric vs. Arithmetic Returns and Extrapolation, Sect. 7.1, Pg.140.

The reason is that the expected rate of return over T periods $\mathsf{E}\big((1+r)^{T}\big)$ is not $[1+\mathsf{E}(r)]^{T}$. Geometric rates of return are smaller than arithmetic rates of return. (Remember: a rate of return of 50% followed by one of –50% leaves you with a –25% rate of return.) As you already know, if the distribution follows a normal bell curve, then the geometric rate of return is about half the variance squared less than the arithmetic rate of return. With the mean of 5% and variance of 40%, you should really expect to earn $5\% - 40\%^2/2 \approx -3\%$ per T in your project, whereas the financial-market benchmark projects offer +2% per T. For a long-term project, you would be better off declining.

Most expected cash flows are implicitly geometric.

For long-term cash flows, NPV really makes sense only if you use the appropriately compounded, i.e., geometric, expected rates of returns. Fortunately, most investors think of the expected cash flows in the NPV numerator in geometric terms, because this is what they care about. If they use a –$100 flow today and a $150 flow in 10 years, they implicitly mean that they expect a compound rate of return of 50%, which they want to compare to geometric opportunity rates of return in the financial market elsewhere.

Term and Averaging

What do you expect as a rate of return on the stock market benchmark? If you expect the stock market to deliver 12% over the next year, with a 20% standard deviation, you should expect it to deliver about $12\% - 20\%^2/2 \approx 10\%$ over the very long run. The 2% difference is roughly the historical difference between arithmetic and geometric rates of return on the U.S. stock market over the last 50 years.

Now put together your knowledge of the term premium and risk premium when you want to benchmark your own either short-term or long-term risky cash flows. For a long-term project, you could invest either in the stock market or in Treasury bonds. As an investor, how much would you expect to earn above the stock market?

IMPORTANT

Whatever your base estimate is of the short-term market-risk premium EQPST (“equity premium, short term estimate”), the following rough adjustment is required to keep your estimate of the long-term market-risk premium consistent with your short-term market-risk premium estimate (assuming that the risk-reward tradeoffs will remain similar over the next few decades):

	Arithmetic	Geometric
Relative to Short-Term Bills	EQPST	≈ EQPST–2%
Relative to Long-Term Bonds	≈ EQPST–2%	≈ EQPST–3.5%

For example, if you believe that the stock market will outperform Treasury bills by 6% over the next one year, you should expect the stock market to outperform Treasury bonds by a (compound) ≈2-3% over the next 30 years. One can quibble whether these adjustment recommendations are off by up to 1%, but they are in the right ballpark.

Are you in the right corner?

When you evaluate short-term market-risk-level projects, you can use your EQPST base estimate in the top left corner as a reasonable benchmark. When you evaluate long-term projects, you should use the estimate in the bottom right corner. Whatever else you do, do not make the mistake of thinking they should be the same.

100% stock market is unaffected. Others are.

The decomposition of the of the stock market return into a term premium and an equity premium matters for investments that are not 100% like stocks. For investing 100% in stocks, whatever term premium you add on one end is subtracted back from the other ($\text{TP}+(\text{MRP}-\text{TP}) = \text{MRP}$). For short-term investments, you can expect a high equity premium but a low term premium. For long-term investments, you can expect a low equity premium but a high term premium. But if you have other types of investment, e.g. one that is more like 50% stock and 50% bond, it matters ($\text{TP}+0.5\cdot(\text{MRP}-\text{TP}) \neq \text{MRP}$). This will become even clearer in the next chapter.

Fund managers should also expect different excess returns over different horizons.

Investors need to think about the same kind of adjustments. When evaluating stock investments, fund managers should add the equity premium estimate and the term premium estimate, too, to arrive at what they can expect. Expecting to earn 6% over short-term Treasuries over the next year is consistent with expecting to earn 2-3% over long-term Treasuries over the long run.

Real-world hurdle rates are often set higher.

Do not take the rules too literally. It is not unusual for managers to be more conservative for long-term projects and assess higher hurdle rates on them. This is more likely related to their uncertainty about their cash flows and to imperfect market premia than the proper assessment of long-term average rates of return of stock and bond investments. For example, a tax-exempt pension fund should not expect an investment in the U.S. stock market to outperform an investment in long-term Treasuries by more than 2% per annum over the years, even if it has the perspective that the stock market will outperform Treasury bills by 6% over the next year.

Incidentally, do you remember Exhibit 9.1? Some of the disagreements over estimates stem from the fact that textbooks can mean different things by "equity premium." The most common estimate is probably the highest estimate, the EQPST.

My Personal Opinions

My recommendation.

The choice of geometric vs. arithmetic and Treasury bills vs. bonds is determined by application and not by opinion. Many earlier textbooks fail to explain the difference, resulting in miscalculated costs of capital. However, the choice of a relevant historical sample to assess the future is, in the end, opinion. For me, I tend to believe that the last 50 years are more relevant than the last 100 years. Thus, I recommend an equity premium of about 2% for long-term cash flows—which is much lower than the 5% that would be touted in other books. *Yet*, I also emphasize that I then use the 10-year term premium, which is 2-4% higher than the 1-year term premium. In Chapter 11, we will also discuss imperfect market premiums which can often further increase my long-term cost-of-capital estimates.

Remain consistent across projects.

I also emphasize that it is important to *be consistent*. Do not use 3% for investing in one project and 8% for investing in another similar project. Being consistent can sometimes reduce your relative mistakes in choosing one project over another.

Liars, liars, pants on fire

Finally, be aware that managers often care less about the scientific merits of costs of capital estimates than they care about whether they want to take or not take a project—whether they want to exaggerate or belittle its value. "Expert" witnesses often cherry-pick estimates as low as 0% or as high as 8%, depending on the paying clients' desires. I often find these estimates less believable the further away they are from my own assessment and the further they violate the spirit of the correct term adjustment. And I find anything outside this 0% to 8% range just too tough to swallow.

Q 9.3. What are appropriate equity premium estimates? What are not? What kind of reasoning are you relying on?

9.5 Asset Costs of Capital vs. Equity Costs of Capital

Equity and asset costs of capital and project hurdle rates

It is important that you always distinguish between the **asset cost of capital** and **equity cost of capital**. Debt is always safer than the underlying project and equity is always riskier. Thus, equity should have a higher cost of capital than the assets.

Comparing levered and unlevered projects.

➤ Asset and equity betas, Formula 8.6, Pg.186.

Let's work a short example. Say that you can buy a retail mall at a price that suggests an expected rate of return of 6%. However, when you look at **REITs** (real estate investment trusts, which are stock-like equity investments) of retail malls at YAHOO! FINANCE, you see that those seem to offer much higher expected rates of return, say 12%. Hands off? Not necessarily.

Levering and Unlevering

To compare the two investments, you have to take into account that REITs are typically already highly levered. It is easy to obtain a 50% mortgage on a retail mall. If an 80% mortgage has an expected rate of return of 4% per annum, then the asset cost of capital for the underlying REIT project is

$$\begin{array}{ccccccccc} \mathsf{E}\,(r_{\text{Mall}}) & = & 80\% & \cdot & (4\%) & + & 20\% & \cdot & 12\% \quad = 5.6\% \\ \mathsf{E}\,(r_{\text{Mall}}) & = & \left(\dfrac{\text{Debt value}}{\text{Firm value}}\right) & \cdot & \mathsf{E}\,(r_{\text{Mortgage}}) & + & \left(\dfrac{\text{Equity value}}{\text{Firm value}}\right) & \cdot & \mathsf{E}\,(r_{\text{REIT}}) \end{array}$$

The 6% mall looks like a great deal. This calculation is called **unlevering** the cost of capital. Alternatively, you could have calculated a **levered cost of capital** for your proposed mall, assuming you could obtain the same mortgage terms,

$$\begin{array}{ccccccccc} 6\% & = & 80\% & \cdot & (4\%) & + & 20\% & \cdot & x \\ \mathsf{E}\,(r_{\text{Mall}}) & = & \left(\dfrac{\text{Debt value}}{\text{Firm value}}\right) & \cdot & \mathsf{E}\,(r_{\text{Mortgage}}) & + & \left(\dfrac{\text{Equity value}}{\text{Firm value}}\right) & \cdot & \mathsf{E}\,(r_{\text{REIT}}) \end{array}$$

This suggests an expected rate of return of 14%.

9.6 Deconstructing Quoted Rates of Return

Reminder: Stated bond yields contain time and default premiums.

➤ Time and default premiums, Sect. 6.2, Pg.113.

Let's return to the subject of Section 6.2. You learned that in a perfect and risk-neutral world, stated rates of return consist of a time premium and a default premium. On average, the default premium would be zero, and the expected rate of return would just be the time premium. All same-timed payoffs offered the same expected rate of return.

We have the time and risk premiums.

In this chapter, when we assumed that stocks offer higher *expected* rates of return than bonds, we changed the assumptions. Expected return differences for same-timed assets only make sense if investors are risk-averse or if the world is imperfect. (Either of these two changes will do—and, incidentally, either could also contribute to higher yields for longer-term project cash flows.) Working forward, let's say that investors are risk-averse. Thus, the expected rate of return on stocks offers an extra **risk premium**.

$$\begin{aligned} \text{Promised Rate of Return} &= \text{Time Premium} + \text{Default Premium} + \text{Risk Premium} \\ \text{Actual Earned Rate} &= \text{Time Premium} + \text{Default Realization} + \text{Risk Premium} \\ \text{Expected Rate of Return} &= \text{Time Premium} + \text{Expected Risk Premium} \end{aligned}$$

Not yet default premium!

You need to be careful in distinguishing between the default premium and the risk premium. The default premium is zero, on average. Only the risk premium increases your expected rate of return in the long run. Unfortunately, the expected rate of return (or, equivalently, the risk premium) is never posted in the real world. It is always only the stated rate of return that is usually publicly posted.

A specific bond example: First compute the price necessary to make you "even" relative to the Treasury if you are risk-neutral. This price is based on the time premium and the default premium.

Here is an example. Say you want to determine the PV of a corporate project or quasi-bond that is 75% like risk-free debt and 25% like equity. Assume that the risk-free rate of return is 2% per annum and that the expected rate of return on the market is $2\% + 4\% = 6\%$. Therefore, the expected rate of return on the quasi-bond should be

$$\mathcal{E}\left(r_{\text{Quasi-Bond}}\right) = 75\% \cdot 2\% + 25\% \cdot 6\% = 3\%$$

This takes care of the time premium and the risk premium. Now assume that this quasi-bond promises to deliver \$200 next year. The price of the bond is *not* $\$200/(1+3\%) \approx \$194.17$! To understand this, continue. I have not even yet told you how likely it is that the firm goes bankrupt and what happens if it does. For example, it could be the case that with a probability of 5%, the quasi-bond pays nothing. In this case, the expected payoff on the quasi-bond is $5\% \cdot \$0 + 95\% \cdot \$200 = \$190$. Its price should be

$$\text{PV}_{\text{Quasi-Bond}} = \frac{\mathcal{E}\left(C_{\text{Quasi-Bond}}\right)}{1 + \mathcal{E}\left(r_{\text{Quasi-Bond}}\right)} = \frac{\$190}{1+3\%} \approx \$184.47$$

Given this price, you can now compute the promised (or quoted) rate of return on this bond:

$$\frac{\$200 - \$184.47}{\$184.47} \approx 8.4\%$$

$$\frac{\text{Promised Cash Flow} - \text{PV}}{\text{PV}} = \text{Promised Rate of Return}$$

The risk premium is above and beyond the time and default premiums. On average, risky investments earn more than risk-free investments now.

And you can now quantify the three components in this example. For this quasi-bond project, the time premium of money is 2% per annum—it is the rate of return that an equivalent-term Treasury offers. The specific risk premium is the extra 1% in the expected rate of return that this quasi-bond offers above the equivalent Treasury. And the rest, 5.4%, is the default premium. You do not expect to earn money from this default premium "on average." You earn it only if the bond does not default.

$$\underset{\text{Promised Interest Rate}}{8.4\%} = \underset{\text{Time Premium}}{2\%} + \underset{\text{Risk Premium}}{1\%} + \underset{\text{Default Premium}}{5.4\%}$$

In the real world, most of the premium that investment-grade corporate bonds quote above equivalent Treasuries is not due to the risk premium but more due to the default premium (and perhaps some other imperfect premiums discussed in later chapters). Corporate bonds simply won't always pay as much as they promise. However, for corporate projects and equity shares, the risk premium can be considerable.

IMPORTANT

Never forget:

- Your benchmarks should be thought of in terms of expected rates of return. If you use historical average returns, you usually assume that these averages are representative of expected rates of return.
- The expected return is not a stated (promised, quoted) return, because it does not include a default premium.
- The probability of default must be handled in the NPV numerator (through the expected cash flow), and not in the NPV denominator (through the expected rate of return).

9.7 Other Benchmarks and "The Method"

Other possible benchmarks—fixed income buckets.

Treasury bonds and stocks are not the only two benchmark assets that you can use. Depending on the project to be valued, managers often use other benchmarks, too. For example, instead of the risk-free Treasury, some corporate managers use bonds that are similar to what they can issue themselves—e.g., investment-grade or junk bonds, mortgage bonds, collateralized bonds, prime borrower bank financing, etc. In all these cases, it is important not to forget to consider that publicly quoted comparables always include default premia, and that your own firm will also have to offer default premia. This is so important that I will repeat the repeat: I beseech you never to confuse expected rates of return with promised rates of return. Just because a non-investment grade bond offers 2-5% above the risk-free rate does not mean that it expects to pay off 2-5% above the risk-free rate. Future defaults will erode the difference. Expected rates of return are much more alike.

Equity and other buckets.

Even within the small corporate segment of equity fund managers, there are many benchmarks: not just the S&P 500, but also **value-vs-growth** portfolios, **market-cap** portfolios, **momentum** portfolios, **profitability** portfolios, or industry portfolios. Some corporate managers can benchmark their expected rates of return to some underlying commodities. For example, the expected rate of return on Exxon can be closely linked to the price of oil. If the appropriate expected rate of return on oil is, say, 20%, then Exxon's oil storage operations should similarly yield an expected rate of return of 20%. Private equity, venture capital, and hedge funds often have their own set of benchmarks, too.

How do your projects measure up?

In principle, it always works the same way: as a corporate manager, first you assess the expected rate of return on some underlying benchmark portfolios. Then you assess the expected rates of return on your own internal investment opportunities. How similar are your projects and to which benchmark? Can your projects be viewed as combinations of your benchmarks? If your opportunities beat the publicly available alternatives in risk-reward, you should invest. Otherwise, you should return the funds to your investors..

Prices or expected rates of return?

Our method is essentially just comparing opportunities to the price at which your investors can buy them for elsewhere. This is also why such a model is called an **asset-pricing model**, even though the model is then phrased in terms of expected returns. Expected returns are *never* posted. Only prices are. But all the economic insights are one: "opportunities with similar characteristics—and in particular risk characteristics—should offer similar expected rates of return."

You must not offer a worse tradeoff

Again, let it sink in: as a corporate manager, you need an expected rate of return—an opportunity cost of capital—as the denominator in the NPV formula. If your project offers a lower expected return than what your investors can earn elsewhere in similarly risky projects, then you should not put your investors' money into your project but instead return their money to them. If your project offers more expected return, then you should go ahead and invest their money into your project.

Summary

This chapter covered the following major points:

- For each project cash flow, you need to estimate the expected rate of return on equivalent benchmark investments. This is the "opportunity cost of capital" that corporations can use as their costs of capital in the terms of the NPV formula.
- The most important benchmarks are the expected rate of return to low-risk assets (such as Treasury bonds) and to high-risk equity assets (such as the S&P 500).
- For r_F, you should use bonds that match the timing of your project's cash flows. Thus, cash flows farther in the future usually have higher opportunity costs

of capital.

- It is difficult to estimate the equity premium. There is no clear consensus on what it should be or how to estimate it best. Reasonable estimates for the equity premium ($\mathsf{E}(r_M) - r_F$) can range from about 1%/year for long-term payoffs to 8%/year for short-term payoffs. Estimates of about 1-3% seem common for most long-term project cash flows.
- Investors care about geometric rates of return, not arithmetic rates of return. When projects have different risk, the two averages can be very different.
- The correct benchmarks adjust properly for term and risk, but when based on historical estimates require judgment about what historical sample period is most representative of the future.
- Both bond and stock benchmarks have expected rates of return that are due to a number of factors, first and foremost risk. So do other benchmark portfolios and assets. It does not have to be bonds and stocks. By choosing better benchmarks that are more similar to their own projects, managers can often obtain better estimates for their costs of capital.

Keywords

Answers

Q 9.1 Use the 1-year Treasury rate for the 1-year project, especially if the 1-year project produces most of its cash flows at the end of the year. If it produces constant cash flows throughout the year, a 6-month Treasury rate might be more appropriate. Because the 10-year project could have a duration of cash flows much shorter than 10 years, depending on use, you might choose a risk-free Treasury rate that is between 5 and 10 years. Of course, it would be even better if you match the individual project cash flows with individual Treasuries.

Q 9.2 The duration of this cash flow is around, or a little under, 5 years. Thus, a 5-year zero-coupon U.S. Treasury would be a reasonably good guess. You should not be using a 30-day or 30-year Treasury. A 10-year zero-coupon Treasury would be a better match for a project that yields cash only once at the end of 10 years. That is, for our project, which has cash flows each year for 10 years, the 10-year Treasury as a benchmark would have too much of its payments as principal repayment at the end of its 10-year term.

Q 9.3 An estimate between 1% and 8% per year is reasonable. Anything below 0% and above 10% would seem unreasonable to me. For reasoning, please see the different methods in the chapter.

End of Chapter Problems

Q 9.4. If your projects' expected rates of return cannot meet the expected rates of return for the benchmarks, then what should you do as the manager?

Q 9.5. In a perfect world, should you take *only* the projects with the highest NPV or *all* projects with positive NPV?

Q 9.6. Explain the basic schools of thought when it comes to equity premium estimation.

Q 9.7. If you do not want to estimate the equity premium, what are your alternatives to finding a cost-of-capital estimate?

Q 9.8. Explain in 200 words or less: What are reasonable guesstimates for the market risk premium and why?

Q 9.9. Is the equity cost of capital usually higher or lower than the asset cost of capital?

Q 9.10. Assume that a comparable peer project in the financial market is financed by 50% debt and 50% equity. Its equity has an expected rate of return of 15%, its debt an expected rate of return of 5%. If your project offers an expected rate of return of 12%, should you take or leave this project?

Q 9.11. A firm has an expected rate of return of 6%. Its debt trades for the risk-free interest rate of 3%. The prevailing equity premium is 4%.

1. If the expected rate of return on the firm's equity is 7%, what is the firm's debt ratio?
2. The firm refinances itself. It repurchases one-third of its stock with debt that it issues. Assume that this debt is still risk-free. What is its new debt ratio?
3. What expected rate of return does the firm have to offer to its new *creditors*?
4. Has the firm's weighted average cost of capital changed?
5. What expected rate of return does the firm have to offer to its new *levered equity holders*?

Q 9.12. A Fortune 100 firm is financed with $15 billion in debt and $5 billion in equity. If this firm holds its underlying structure constant, would you expect the cost of capital on its equity to be higher or lower if the firm restructured its funding by repurchasing shares financed with new debt?

10

The Capital Asset Pricing Model

What expected rate of return does your project have to offer? The last chapter explained how you can determine the answer if your project is 100% like other assets—such as Treasuries, the stock market, or some other traded financial assets. But what about projects that are more like combinations? How would you judge how much of each asset you would need to mimic your project? And which other assets should you choose as your benchmarking portfolios?

This is the domain of the Capital Asset Pricing Model (CAPM). It comes with a promise that you only have to worry about Treasuries and the stock market (and nothing else) and gives you a formula that relates how much reward your investment project has to offer to compensate your investors for its risk. The risk is the market beta. The formula works for *any* kind of project. You can then use its costs of capital in your NPV calculations.

We will first briefly review what you already know. Then you will learn all about the CAPM. And you will get to apply it—and then, I will have to tell you that although the CAPM is the dominant model in practice, it is not only often poorly applied, but its empirical validity is also miserable even under the best of circumstances. I will put it all in perspective for you.

10.1 What You Already Know

We are still going at the same central question—what is a good opportunity cost of capital?

Perfect markets, diversified smart investors.

In this chapter, we will still assume but now lean more heavily on our perfect-market assumptions than we have in recent chapters. Moreover, we will also assume that investors are smart and that they diversify their portfolios to reduce their risk exposures. The types of risk that investors consider toxic can only be the parts that they cannot wash out by diversification and that remain left over even when all assets are just tiny parts of their large overall portfolios.

Investors compete for good deals.

Not being dummies, collectively, investors snatch up the best projects—those that have low risk and high expected rates of return. In fact, anyone contemplating selling a project with more reward than it deserves would attract a gazillion bidders. Anyone contemplating selling projects with too unfavorable risk contributions for its reward would not receive a single offer. There is really only one correct choice of price. Consequently, what investors purchase in the real world at the correct prices must be subject to some trade-off: Projects that drive up overall portfolio risk must offer higher expected rates of return.

You are still after an estimate for your opportunity cost of capital—and now you will get another useful measure.

Again, our perspective will be primarily that of the corporate manager, not that of a day trader. From the previous chapter, you know that investors like short-term low-risk project cash flows (like overnight Treasuries), and dislike long-term unsafe project cash flows (like the stock market). How do you determine how much of your own potential projects should be viewed "like bonds" and how much "like stocks"? This is what the CAPM will do for you—it will give you an answer to "like this much bonds, like this much stocks," and this answer is the "market beta."

We would love an integrated "beautiful" perspective.

These simplifications will leave you with a nice framework: Investors dislike risk and like reward. They care about their overall financial investments portfolio. They are diversified. If you buy into this view, as a corporate manager, you can then infer how external investors judge the risk and reward of your own corporate projects. Investors' reward is their portfolio's expected rate of return. Investors' risk is their *overall* portfolio risk, *not* your project's *own* standard-deviation risk. Your own project's contribution to investors' overall portfolio risk is then best measured by the market beta of your project. Think of beta as a measure of your project's "toxicity." A project that decreases in value when the market decreases in value (and increases when it increases) has a positive market beta. It's toxic—investors don't like it. A project that increases in value when the market decreases in value, and vice-versa, has a negative market beta. It's less toxic—investors like it more. That is, projects with lower market betas help investors (who already otherwise hold market-like portfolios) suffer less overall portfolio risk.

10.2 The Capital Asset Pricing Model (CAPM)

The CAPM gives you the cost of capital if you give it the risk-free rate, the expected rate of return on the market, and your project's market beta.

The **capital asset pricing model** (**CAPM**) gives an appropriate expected rate of return (cost of capital) for each project if you give it the project's single relevant risk characteristics (the market beta); and (just as in the previous chapter) the risk-free rate of return and equity premium. The model states that an investment's cost of capital is lower when it offers better diversification benefits for an investor who holds the overall market portfolio—less required reward for less risk contribution. Market beta is its measure of risk contribution. Projects contributing more risk (market beta) require higher expected rates of return for you to want them; projects contributing less risk require lower expected rates of return. According to the CAPM, nothing but the risk-free rate, the expected equity premium, and the market beta matters. No other financial assets need to be investigated to judge your project.

IMPORTANT

To estimate the required expected rate of return for a project or firm—that is, the cost of capital—according to the CAPM, you need three inputs:

1. The risk-free rate of return, r_F.
2. The expected rate of return on the overall market, $E(r_M)$—or, equivalently, the equity premium $E(r_M) - r_F$.
3. The project's beta with respect to the market, β_i.

The CAPM formula is

$$E(r_i) = r_F + [E(r_M) - r_F] \cdot \beta_i$$

where i is the name of your project and $E(r_i)$ is your project's expected rate of return. All model inputs are forward-looking: the risk-free rate, the equity premium, and the market beta of the asset.

You need to memorize the CAPM formula.

The CAPM formula tells you what investors care about: comovement with the market.

The CAPM specifically ignores the stand-alone risk of your project. That is, investors do not care about your projects' variance, because they are smart enough to diversify away this idiosyncratic risk. Investors care only about your project's market betas, because it is betas that measure the component of risk that your project contributes and that investors holding the wide market portfolio would not have diversified away.

On a pragmatic level, the CAPM is seductive. It limits your attention to just two benchmark assets. It gives you a coherent universal measure of where projects lie on the spectrum between stocks and bonds. More market beta means "more like stocks," and thus higher expected rates of return ("just like stocks"). Less market beta means "more like bonds," and thus lower expected rates of return ("just like bonds").

Mechanically, it looks sensible.

Without going into detail, economists also love a deep "economic equilibrium model" justification for the CAPM that I will largely spare you. In this view, financial markets are perfect, each and every investor faces the same tradeoffs and uses the model, and each and every asset is priced by it. When all the assumptions are satisfied, it implies mathematically that the CAPM must hold. Necessarily, there could then not be any benchmarks other than the risk-free rate and the stock market, and the only valid measure of risk would be the market beta. This CAPM justification, with its stringent assumptions, is too orthodox and simply not realistic.

There are deeper CAPM rationales that economists call "equilibrium models."

But more important than philosophy, the empirical data soundly rejects the CAPM, as I will explain below in more detail. For now, let me just say that you must still study the CAPM not only because it is conceptually interesting but also because every finance dinosaur in the real world is using it—and, more than likely, (s)he will growl CAPM questions in your job interview.

PS: I was brought up by dinosaurs, too—I got my degrees in the 1980s.

Q 10.1. What are the assumptions underlying the orthodox CAPM? Are the perfect market assumptions among them? Are there more?

The Security Market Line (SML)

Let's first use the CAPM formula as a recipe. If you believe that the risk-free rate is 3% and the expected rate of return on the market is 8%, then the CAPM states that

A first quick use of the CAPM formula.

$$\begin{aligned} \mathsf{E}(r_i) &= 3\% + (8\% - 3\%) \cdot \beta_i &= 3\% + 5\% \cdot \beta_i \\ \mathsf{E}(r_i) &= r_F + [\mathsf{E}(r_M) - r_F] \cdot \beta_i \end{aligned}$$

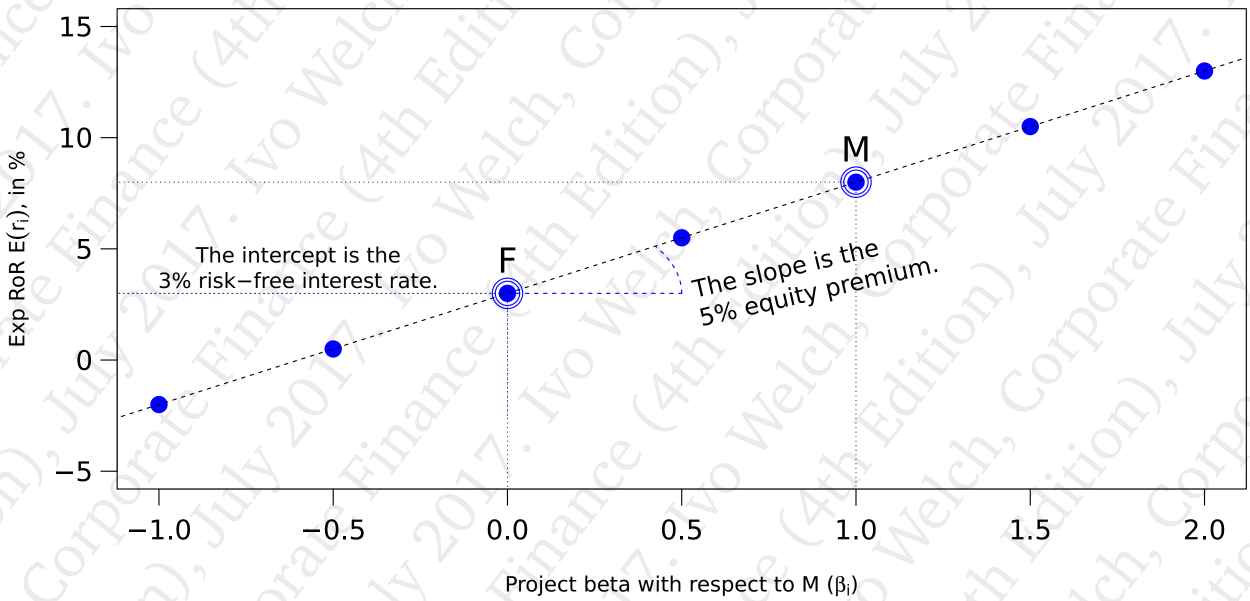
Therefore, a project with a beta of 0.5 should have a cost of capital of $3\% + 5\% \cdot 0.5 = 5.5\%$, and a project with a beta of 2.0 should have a cost of capital of $3\% + 5\% \cdot 2.0 = 13\%$. The CAPM gives the opportunity cost for your investors' capital: If the project with the beta of 2.0 cannot earn this expected rate of return of 13%, you should not take this project and instead return the money to your investors. Your project would add too much risk for its reward. Your investors have better opportunities elsewhere.

The CAPM formula is often graphed as the **security market line** (**SML**), which shows the relationship between the expected rate of return of a project and its beta. Exhibit 10.1 draws a model-perfect security market line for seven assets. Each investment asset (such as a stock or a project) is a point in this coordinate system. Because all assets in our example properly follow the CAPM formula, they must lie on a straight line. The SML is just the graphical representation of the CAPM formula. The slope of this line is the equity premium, $\mathsf{E}(r_M) - r_F$, and the intercept is the risk-free rate, r_F.

The SML is just a graphical representation of the CAPM formula.

Alas, in the real world, even if the CAPM holds, you would not have the data to draw Exhibit 10.1. The reason is that you do not know true expected returns and true expected market betas. Exhibit 10.2 plots a version where you have to rely only on what most investors have and rely on—observable historical data averages. Thus you can only fit an "estimated security market line," not the "true security market line." And you have to hope that your historical

If you know the inputs, the SML is a sharp line; if you estimate them, it is a scatterplot.



		Investment Asset						
		A	B	F	C	M	D	E
Market Beta	β_i	–1.0	-0.5	0.0	0.5	1.0	1.5	2.0
Expected Rate of Return	$\mathsf{E}(r_i)$	–2.0%	0.5%	3.0%	5.5%	8.0%	10.5%	13.0%

Exhibit 10.1: *The Security Market Line With Perfect Knowledge.* This graph plots the CAPM relation $\mathsf{E}(r_i) = r_F + [\mathsf{E}(r_M) - r_F] \cdot \beta_i = 3\% + (8\% - 3\%) \cdot \beta_i$, where β_i is the beta of an individual asset with respect to the market. In this graph, we assume that the risk-free rate is 3% and the equity premium is 5%. Each point is one asset (such as a stock, a project, or a mutual fund). The point M could be the value-weighted market portfolio or any any other security with a $\beta_i = 1$. F could be the risk-free asset or any other security with a $\beta_i = 0$.

data has provided good, unbiased estimates of the true forward-looking market beta and true forward-looking expected rates of return. (Both are big assumptions!) If the fitted line looks straight, you would not immediately throw out the CAPM. In any case, any workable version of the CAPM in real life can only state that there should roughly be a linear relationship between the data-estimated market betas and the data-estimated expected rates of return, just as drawn in Exhibit 10.2.

Q 10.2. The risk-free rate is 4%. The expected rate of return on the market is 7%. What is the appropriate cost of capital for a project that has a beta of 3?

Q 10.3. The risk-free rate is 4%. The expected rate of return on the market is 12%. What is the cost of capital for a project that has a beta of 3?

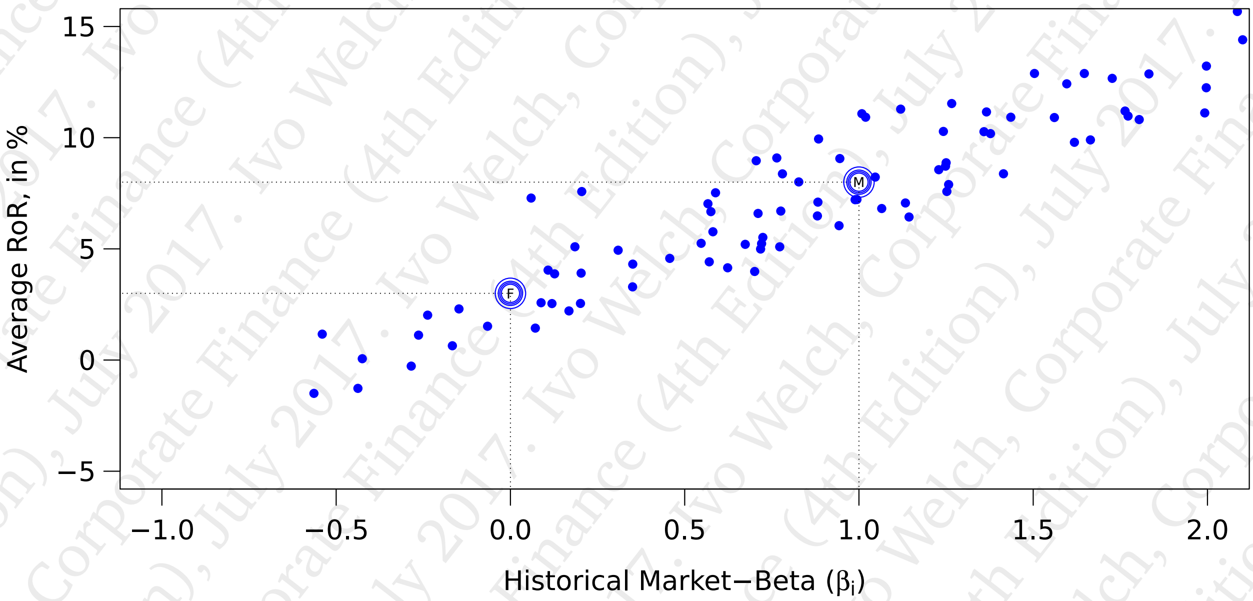


Exhibit 10.2: *The Security Market Line in an Ideal CAPM World.* This plot shows what you are, at best, confronted with: You don't know expected returns and betas. All you know are historical average returns and historical betas that are usually just data statistics. You then hope that these are unbiased representations of the underlying true historical mean returns and historical betas. In turn, you then further hope that these are also representative of future expected returns and future betas. There is always hope.

Q 10.4. The risk-free rate is 4%. The expected rate of return on the market is 12%. What is the cost of capital for a project that has a beta of –3? Does this make economic sense?

Q 10.5. Is the real-world SML with historical data a perfectly straight line?

Q 10.6. The risk-free rate is 4%. The expected rate of return on the market is 7%. A corporation intends to issue publicly traded bonds that *promise* a rate of return of 6% and offer an *expected* rate of return of 5%. What is the implicit beta of the bonds?

Q 10.7. Draw the SML if the risk-free rate is 5% and the equity premium is 9%.

Q 10.8. What is the equity premium, both mathematically and intuitively?

The CAPM in the Present Value Formula

We usually use the CAPM output, the expected rate of return, as our discount rate.

If you take the CAPM at face value, it gives you a good denominator for the NPV formula, the opportunity cost of capital, $\mathsf{E}(r)$:

$$\text{NPV} = C_0 + \frac{\mathsf{E}(C_1)}{1 + \mathsf{E}(r_1)} + \frac{\mathsf{E}(C_2)}{1 + \mathsf{E}(r_2)} + \cdots$$

➤ Asset-Pricing Model, Pg.210.

Together, the CAPM and the NPV formulas tell you that cash flows that correlate more with the overall market are of less value to your investors and therefore require higher expected rates of return ($\mathsf{E}(r)$) in order to pass muster (well, to pass the hurdle rate, which is determined by the alternative opportunities that your model presumes your investors have).

It is easier to work in required returns than in prices.

The CAPM is called an **asset-pricing model**, even though it is most often expressed in terms of a required expected rate of return rather than in terms of an appropriate asset price. Fortunately, though messy, the two are equivalent—you can always work with the CAPM return first, and then discount the expected cash flow into an appropriate price. A given expected rate of return implies a given price. (If you do not know the fair price, you will have to take two aspirins [or something more hallucinogenic] and work with a more difficult version of the CAPM formula. It is called **certainty equivalence** [CEV] and is explained in the companion chapter.)

Equity and Asset Betas

➤ Asset and equity betas, Formula 8.6, Pg.186.

As in Section 9.5, it is important that you always distinguish between asset costs of capital and equity costs of capital. Whatever worked there with the overall costs of capital also works here with market betas. Done. You can skip the rest of this section, or endure a few more examples.

Don't use the equity beta to estimate your project's hurdle rate. Use the asset beta instead.

Assume that the risk-free rate is 4% and the equity premium is 5%. You own a \$100 million project with an asset beta of 2.0 that you can finance with \$20 million of risk-free debt. Truly risk-free debt always has a beta of 0. To find your equity beta, write down the formula for your asset beta (firm beta):

$$\beta_{\text{Firm}} = \underbrace{\left(\frac{\text{Debt value}}{\text{Firm value}}\right)}^{20\%} \cdot \overbrace{\beta_{\text{Debt}}}^{(0)} + \underbrace{\left(\frac{\text{Equity value}}{\text{Firm value}}\right)}^{80\%} \cdot \overbrace{\beta_{\text{Equity}}}^{(\beta_{\text{Equity}})} \quad = 2.0$$

Solve this to find that your market beta of equity is 2.5. It is this market beta of equity that you would find reported on YAHOO! FINANCE. You would not want to base your hurdle rate for your entire firm's typical average project on your equity beta: Such a mistake would recommend you use a hurdle rate of $\mathsf{E}(r_i) = r_F + [\mathsf{E}(r_M) - r_F] \cdot \beta_i = 4\% + 5\% \cdot 2.5 = 16.5\%$. This would be too high. Instead, you should require your average projects to return $\mathsf{E}(r_i) = 4\% + 5\% \cdot 2.0 = 14\%$.

➤ Typical, average, and marginal betas, Sect. 13.3, Pg.318.

	20% Debt	80% Equity	100% Project
Beta	0.0	2.5	2.0
⇒ Cost of Capital	4%	16.5%	14.0%

In both cases, the capitalization-weighted average of debt and equity is always the overall project asset.

If you use comparables, first unlever them.

Conversely, if your project is private but the potential future owners are well-diversified, you may have to find its hurdle rate by looking at public comparables. Let's presume you find a similarly sized firm with a similar business that YAHOO! FINANCE lists with a beta of 4, or perhaps better yet, the firm's industry. Remember that financial websites always list only the equity beta. The CAPM tells you that the expected rate of return on the equity is $4\% + 5\% \cdot 4 = 24\%$. However, this is not necessarily the hurdle rate for your project. When you look further on

YAHOO! FINANCE, you may notice that your comparable is financed with 90% debt and 10% equity. (If the comparable had very little debt, a debt beta of 0 might have been a good assumption, but unfortunately, in this case it is not.) Corporate debt rarely has good historical return data that would allow you to estimate a debt beta. Consequently, practitioners often estimate the expected rate of return on debt via debt comparables based on the credit rating. Say your comparable's debt is rated BB and say that BB bonds have offered *expected* rates of return of 100 basis points above the Treasury. (This might be 200 basis points *quoted* above the Treasury). With the Treasury standing at 4%, you would estimate the comparable's cost of capital on debt to be 5%. The rest is easy. The expected rate of return on your project should be

➤ Credit ratings, Sect. 6.2, Pg.114.

$$\begin{aligned} E(r_{\text{Project}}) &= 90\% \cdot 5\% + 10\% \cdot 24\% = 6.9\% \\ &= w_{\text{Debt}} \cdot E(r_{\text{Debt}}) + w_{\text{Equity}} \cdot E(r_{\text{Equity}}) \end{aligned}$$

This would make a good hurdle rate estimate for your project.

➤ Typical, average, and marginal betas, Sect. 13.3, Pg.318.

Does Risk Reduction Create Value?

In the 1960s and 1970s, many firms became **conglomerates**, that is, companies with widely diversified and often unrelated holdings. Can firms add value through such diversification? The answer is "usually no." Diversification indeed reduces the standard deviation of the company's rate of return (diversified companies are less risky). Yet, in a perfect market, your investors can just as well diversify risk for themselves. They don't need the firm to do it for them. This is a more important insight than what follows. Again: if investors can do it without the firm, the firm cannot add value by doing it for them.

Diversification reduces risk, but does not create value.

As in the previous section, we can elaborate about this in the context of the CAPM. However, the basic idea should hold in any reasonable framework, e.g., if projects have different cash flow horizons and thus different costs of capital. Thus, you can consider it "done" and you can skip this section, too, if you already fully understand this. Otherwise, endure the example.

For example, if your \$900 million firm ABC (e.g., with a beta of 2 and a risk of 20%) is planning to take over the \$100 million firm DEF (e.g., with a beta of 1 and also a risk of 20%), the resulting firm is worth \$1 billion. ABC + DEF indeed has an idiosyncratic risk lower than 20% if the two firms are not perfectly correlated, but your investors (or a mutual fund) could just have held 90% of their portfolios in ABC and 10% in DEF and thereby achieved the very same diversification benefits. If anything, a merger takes away your investors' freedom: They no longer have the ability to buy, say, 50% of their portfolios in ABC and 50% in DEF. (In a CAPM world, this does not matter.) The CAPM makes it explicit that the cost of capital does not change unduly. Say both firms follow the CAPM pricing formula, and say that the risk-free rate is 3% and the equity premium is 5%,

A specific diversification example worked out for you, in which projects are priced fairly, and diversification neither creates nor destroys value.

$$\begin{aligned} E(r_{\text{ABC}}) &= 3\% + 5\% \cdot 2 = 13\% \\ E(r_{\text{ABC}}) &= r_F + [E(r_M) - r_F] \cdot \beta_{\text{ABC}} \end{aligned}$$

and

$$\begin{aligned} E(r_{\text{DEF}}) &= 3\% + 5\% \cdot 1 = 8\% \\ E(r_{\text{DEF}}) &= r_F + [E(r_M) - r_F] \cdot \beta_{\text{DEF}} \end{aligned}$$

The newly formed company will have an expected rate of return (cost of capital) of

➤ Value-weighted portfolios, Sect. 8.5, Pg.185.

$$E(r_{ABC+DEF}) = 90\% \cdot 13\% + 10\% \cdot 8\% = 12.5\%$$
$$E(r_{ABC+DEF}) = w_{ABC} \cdot E(r_{ABC}) + w_{DEF} \cdot E(r_{DEF})$$

and a market beta of

$$\beta_{ABC+DEF} = 90\% \cdot 2 + 10\% \cdot 1 = 1.9$$
$$\beta_{ABC+DEF} = w_{ABC} \cdot \beta_{ABC} + w_{DEF} \cdot \beta_{DEF}$$

The merged company will still follow the CAPM,

$$E(r_{ABC+DEF}) = 3\% + 5\% \cdot 1.9 = 12.5\%$$
$$E(r_{ABC+DEF}) = r_F + [E(r_M) - r_F] \cdot \beta_{ABC+DEF}$$

Its cost of capital has not unduly increased or declined. In an ideal [CAPM] world, no value has been added or destroyed—even though ABC + DEF will have a risk lower than the 20% per annum that its two constituents had.

Deconstructing Quoted Rates of Return

As in Section 9.6, the asset-pricing model provides just the expected rate of return, not the quoted rate of return. If you look at the example there again, you could view it as applying in the current context, too. The CAPM merely pins down the sources of the 75% and 25% expected debt equity components. Replace these 75/25 proportions with a beta of 0.25, and you really have the same example.

Short-Term and Long-Term Projects?

Although the CAPM formally recognizes only one SML in theory, we use different risk-free rates for different project horizons in practice. Thus, short-term projects would have lower costs of capital than long-term projects. For example, you might assess, say, a 3% equity premium; and using the prevailing yield curve in 2016, you might assess

	β_i:	–2	–1	0	1	2	3
Short-Term Projects	$E(r_i) = 1\% + 3\%$	1%	2%	3%	4%	5%	6%
Long-Term Projects	$E(r_i) = 3\% + 3\%$	–3%	0%	3%	6%	9%	12%

where the 1% is the 1-year Treasury and the 3% is the 30-year Treasury. Recall that we are not sure whether we should use the same equity premium (here 3%) for both near and far project cash flows. It is due to ignorance that we typically use the same equity-premium estimate regardless of term.

Nerdnote: If the CAPM truly held, long-term bonds would have higher expected rates of return than short-term bonds, and this could be explained exactly by their positive market beta. Alas, long-term bonds have had negative market betas for a few decades now. Nobody knows why. However, be aware that applying the CAPM to long-term bonds would so obviously contradict reality that few are tempted to use it in this context Instead, everyone uses adjusted yield-curve estimates.

Q 10.9. A corporate bond with a beta of 0.2 will pay off next year with 99% probability. The risk-free rate is 3% per annum, and the equity premium is 5% per annum.

1. What is the price of this bond?
2. What is its promised rate of return?
3. Decompose the bond's quoted rate of return into its components.

Q 10.10. Going to your school has total additional and opportunity costs of $30,000 *this year and upfront*. With 90% probability, you are likely to graduate from your school. If you do not graduate, you have lost the entire sum. Graduating from the school will increase your 40-year lifetime annual salary by roughly $5,000 per year, but more so when the market rate of return is high than when it is low. For argument's sake, assume that your extra-income beta is 1.5. Assume the risk-free rate is 3%, and the equity premium is 5%. What is the value of your education?

10.3 Estimating the Extra Input: Market Beta

Unlike the risk-free rate and the equity premium, beta is specific to each project.

We already discussed estimating the risk-free rate and equity premium in the previous chapter and beta estimation in the chapter before. Because beta is the only novel aspect relative to benchmarking, let's discuss it a little more. Beta tells you how the rate of return of your project fluctuates with that of the overall market. Unlike the previous two inputs, which are the same for every project in the economy, the beta input depends on your own specific project characteristics: Different projects have different betas.

The CAPM has three inputs. We will cover them in detail.

➤ Will history repeat itself?, Sect. 7.1, Pg.153.

Just as with the risk-free rate and the expected rate of return on the stock-market (or equivalently, the equity premium) in Chapter 9, investors are really interested in the *future* market betas of your projects and not in their historical market betas. No one really cares about the past for its own sake. But as usual, you often have no choice other than to rely on estimates, and these are usually based largely on statistical analysis of historical data. Although any estimates of future betas from historical betas tend to be better than estimates of the future equity premium from historical equity premiums, beta estimates are still not too reliable—especially over long horizons. The reason is that stock returns are very noisy, and the unobserved underlying true betas themselves also tend to move around. It's like shooting without a viewfinder at a moving target—not as good as shooting at a fixed target, but not as bad as shooting without a view. *C'est la vie.*

Market Beta Estimation Based on Historical Data

Ways to estimate beta.

The basic mechanics of finding the historical market beta for a project with historical rates of return is easy. You run a **market-model** regression. The independent variable is the rate of return on the stock market. The dependent variable is the rate of return on your project. It is also good practice to subtract the risk-free T-bill rate from both your project's and the stock market's rates of return. Any statistical software package (and common computer spreadsheet programs like Excel or Openoffice) can readily calculate the coefficients a and b in the market-model regression:

$$\underbrace{(r_{\text{Project}} - r_F)}_{\text{y variable}} = a + b \cdot \underbrace{(r_{\text{Market}} - r_F)}_{\text{x variable}}$$

The slope b is the market beta. It's a good thing that we use b as a symbol instead of β, because the b that the regression spits out is only an estimate of a true beta (β), and not the true and unknowable beta itself.

Historical market beta to forward market beta

This is only the basics. To get a better forward-looking market beta estimate, you should do the following:

1. Use daily stock returns, not monthly stock returns.
2. Use about two years' worth of data. Between one and five years of data will do.
3. "Shrink" your first-pass market beta by 30-40% towards 1, depending on the timing of the cash flow that you intend to use it on:

<1 Year	$(1-0.3)\times b + 0.3\times 1$
>5 Years	$(1-0.4)\times b + 0.4\times 1$

   This 0.3 (or 0.4) factor is used partly because it reduces historical outliers, and partly because true market betas drift over long horizons. If you want, you can shrink beta by another 10% if your project and firm are small.

   For example, if your statistical software gives you a first-pass market-beta estimate for your project of 2.0, and you want to estimate a CAPM cost of capital for a project cash flow in 1 year, then use $(1-0.3)\times 2.0 + 0.3 = 1.7$. If you want to estimate it for a project cash flow in 10 years, use 1.6. If your first-pass estimate is -1.0, and the cash flow is in 1 year, use $(1-0.3)\times -1.0 + 0.3 = -0.4$.

It does not matter much which particular stock market index you use as your independent variable. The S&P 500 with or without dividends is fine. There are also other more sophisticated methods, but the above three guidelines cover the most important basics. It is unlikely that you can improve much on them. These market betas are as good as they are going to get.

What not to do!

In practice, you may encounter two common estimation practices that dramatically worsen the quality of estimated market betas. So let me warn you:

1. If you have good daily data, do not estimate market beta with monthly return data. (And if you have no choice [as, for example, with hedge funds, which report rates of return only monthly], then shrink more—think 50-60%, not 30-40%.)
2. If you have your firm's own stock returns, do not use industry portfolio returns as stand-ins for your firm. Although industry betas move less than stock-specific betas and thus seem appealing, in reality industry betas are much worse predictors for stocks than the stocks' own market betas.

If you see either practice, tell the dinosaur using them that the mammals are taking over and they'd better evolve and adapt!

Nerdnote: If you want to estimate future market-beta even better, then shrink not towards 1, but towards a smaller constant, like 0.6-0.8, if your firm and project are small.

Market Beta Estimation Based on Theoretical Consideration

Turn the formula around to help contemplate beta

As a corporate manager, you are rarely interested in the market beta of an industry or even a stock. Usually, you are interested in the market beta of a potential project. Sometimes, your firm may not even be publicly traded, so you would not have any historical price data to begin with. In this case, corporate CAPM users sometimes rely on economic intuition rather than historical statistics. To see the logic, rearrange the CAPM formula. Now, do you think your project cash flows and its future project values (which are influenced by changes in the economy) are likely to move more or less with the overall stock market (and possibly the overall economy)?

$$\mathsf{E}(r_{\text{Project}}) = r_F + [\mathsf{E}(r_M) - r_F] \cdot \beta_{\text{project}} \iff \beta_{\text{Project}} = \frac{\mathsf{E}(r_{\text{Project}}) - r_F}{\mathsf{E}(r_M) - r_F}$$

The right side of this formula helps translate your intuition into a beta estimate. What rate of return (above the risk-free rate) will your project have if the market were to have +10% or –10% rate of return (above the risk-free rate)? Clearly, such guesswork is difficult and error-prone—but it can occasionally provide a market-beta estimate when no other is available. But be aware that such estimates are almost always poor.

Don't be so confident!

If you do not believe me that your estimate is going to be so bad that you may as well just go back to the peer benchmarks from Chapter 9, then I dare you to try. Randomly pick five stocks from YAHOO! FINANCE. Do not peek at their market betas. Explain to me what they should be, and then check your claims against their actual market betas. If you can accurately assess which market betas are far from 1, then you are a better intuitive economist than I am. In fact, I have almost no economic intuition as to why entire asset classes, such as long-term bonds, have had negative market betas over the last 20 years and positive market betas before then.

Do you even want beta for a project without historical stock returns?

Moreover, *please* stand back and think for a moment what you are really doing here. If you are dealing with a new project that has never seen the light of day and that has no historical data, would you even want to use the CAPM? And are you a fully diversified owner who cares only about market-risk and not about idiosyncratic project risk, and who has access to a perfectly competitive capital market? If you are an entrepreneur, I would like to meet you. I have never met such an entrepreneur. (And, are you even convinced so far that the CAPM is a good description of real life—something for which I have shown you zero evidence up to this point? All that you know so far is that the inputs are difficult to estimate.)

Q 10.11. According to the CAPM formula, a zero-beta asset should have the same expected rate of return as the risk-free rate. Can a zero-beta asset still have a positive standard deviation? Does it make sense that such a risky asset would not offer a higher rate of return than a risk-free asset in a world in which investors are risk-averse?

Q 10.12.A comparable firm (with similar size and business) has a YAHOO! FINANCE-listed equity beta of 2.5 and a debt/asset ratio of 2/3. Assume that the debt is risk-free.

1. Estimate the equity beta for your firm if your projects have similar betas, but your firm will carry a debt/asset ratio of 1/3.
2. If the risk-free rate is 3% and the equity premium is 2%, then what should you use as your firm's hurdle rate?
3. What do investors demand as the expected rate of return on the comparable firm's equity and on your own equity?

Q 10.13. You own a stock portfolio that has a market beta of 2.4, but you are getting married to someone who has a portfolio with a market beta of 0.4. You are three times as wealthy as your future significant other. What is the beta of your joint portfolio?

10.4 Neutralizing Equity-Premium Uncertainty?!

The three most important numbers for you.

Do you recall my claim that the risk-free rate and the equity premium were the two most important numbers in finance, regardless of whether you are using the CAPM or not? Well, you also want to know the market-beta for the same reason. It is an extremely useful number, too.

Beta has another interesting use

➤ Shorting, Sect. 7.2, Pg.155.

➤ Exchange-Traded Fund, Sect. 7.1, Pg.137.

It is very easy to short the stock market (e.g., using an S&P 500 future or ETX). This allows you to "innoculate" or "hedge" your project against overall stock-market risk. Just short the right amount of stock, which is exactly the ratio that market beta gives you. For example, if you have \$100 million invested in an asset with a market beta of 3, you can short $3 \cdot \$100 = \$300$ million in the market and thereby reduce your market risk to zero. If the stock market happens to go down by 1%, you would expect (a) your project to go down by 3% but (b) your hedge to go up by the same $3 \cdot 1\%$. The CAPM formula even suggests that your equity-premium estimate is now irrelevant.

...but it may increase idiosyncratic risk

However, a short market position can also increase the variance of your project outcomes: You may end up in a scenario in which your own project underperforms and the stock market outperforms. You may even go bankrupt because of it. Your project's idiosyncratic-risk component and your errors in estimating betas now become more important. This is not a problem if your project owners are highly diversified, and your particular project is just a tiny fraction of their wealth that they don't care a great deal about. Yet, it is a problem if they are not; or if you, as the corporate manager, care about your one specific project a great deal (or if there are bankruptcy costs, as you will learn in Chapter 19).

You can even make the CAPM come true!

Putting this together, from your perspective as the CEO of one small company in a large market, you can render a degenerate version of the CAPM formula to be nearly true by definition. If you are shorting the correct full amount of stock market, it won't matter whether you are overestimating or underestimating the equity premium. The limits to this strategy are your estimation uncertainty about beta and your idiosyncratic risk tolerance. In the real world, a full short may neither be possible nor desirable. If you do not immunize your company against market risk, then it matters to you what the equity-risk premium is—and whether the CAPM is right in the first place.

But should you be short the market?

You may object that you would not want to short the stock market—betting against the market was historically not a smart maneuver. But, as a CFO, do you really know better whether you should be long or short the stock market? If it is fairly priced, so be it. Leave this choice to your investors. If they want to bet on or against the overall stock market, they do not need you to do it for them. You are only "abusing" the insights of the CAPM to avoid or at least reduce your ignorance about your project's best cost of capital estimate.

10.5 Is the CAPM the Right Model?

The CAPM Assumptions Are Not Innocuous

Everyone can invest only in the same thing.

Although the CAPM edifice is reasonable, it does not mean that this edifice "obviously" holds. The CAPM model leans a lot more on the perfect-market assumptions (and then some) than our earlier chapters did.

Are most financial markets really so perfect? Do most investors really hold diversified stock market portfolios? Do they really care *only* about risk and reward in their financial-asset portfolios and nothing else?

Using particular stocks as insurance for you.

Stand back for a moment. How can the CAPM perspective fail? Consider the following examples:

Nerdnote: The strategy of neutralizing the market works only for a single company. If every firm did it, it would change the investment opportunity set.

- If you own a house, chances are that much of your current wealth is invested in the equity portion of your house, and you are not as diversified as you should be. You should then try to find stocks that reduce your house risk exposure, not stocks that reduce your financial market risk exposure. You should like stocks that go up when your house value goes down.
- If you are under 40 years of age, chances are that much of your lifetime wealth is in your human capital. It is not diversified. And only you can invest easily in *your* education: I cannot. You need to hedge *your* career, not mine. You should like stocks that go up when the value of your expertise goes down.
- If you are a tech engineer and work in Silicon Valley, you should short technology stocks as a hedge against their tanking. Conversely, you should not mind losing in your financial portfolio when technology stocks boom (and you end up rich from your employer's stock options, anyway). Yet many engineers in Silicon Valley are so irrationally overconfident, excited, and/or convinced of technology and their (stock-picking) abilities that they end up buying mostly technology stocks for their portfolios, instead. They double up rather than hedge. It's worked so far, but just wait...
- Do firms really live in near-perfect capital markets? Entrepreneurs often need to scratch together whatever capital they can. If they cannot easily find many capital providers, they may have to pay much higher costs of capital than suggested by the CAPM. And they may be forced to invest most of their own wealth—to the point of bankrupting themselves if their projects fail.
- Entrepreneurs are notorious for staking their entire life's savings on their startups. They are hardly ever diversified and usually highly liquidity constrained.

So, even though the theoretical CAPM assumptions are nice, their applicability is actually quite narrow—it considers a scenario in which all investors do not care about anything but the risk and return in the financial markets, and they all have (largely) the same investments and investment opportunities. Don't think the CAPM *has* to be true just because it seems reasonable at first glance.

The Scientific Evidence for the CAPM Is Negative

What equilibrium?

What if every investor were to choose portfolios for his or her own personal reasons and not with the same perspective—some looking to hedge their houses, others their job, others their industry, others their product's failure? Then it may well be that some assets offer higher or lower expected rates of return than suggested by the CAPM—the CAPM would not hold. In this case, corporate managers really should not rely on the CAPM. Instead, they should stick to a more holistic approach or the less ambitious peer benchmark approach from Chapter 9. Sadly, this always turns out to be good advice in real life: You should not use the CAPM. It does not work. Use the benchmark approach instead.

The stock market has done well...

In defense of the CAPM, it is true that the stock market has outperformed bonds. This is evidence that investors have indeed been rewarded for taking on more risk, at least across these two asset classes. However, within the asset class of stocks, the empirical evidence shows that higher market-beta stocks did not have higher average rates of return in the past than lower market-beta stocks. It's not just that benchmarks other than the equity premium *also* matter; it's that beta itself does not seem to matter. Using a market-beta of 1 on every project (the ultimate shrinkage) would not have done harm.

Beta, schmeta.

I can summarize decades worth of academic work in two sentences: At best, the empirical evidence is inconclusive about whether the CAPM should be discarded. At worst, it is conclusive and the CAPM should be completely discarded. Consequently, the common corporate use of the CAPM to obtain hurdle rates across projects is based only on wishful thinking, not on empirical evidence.

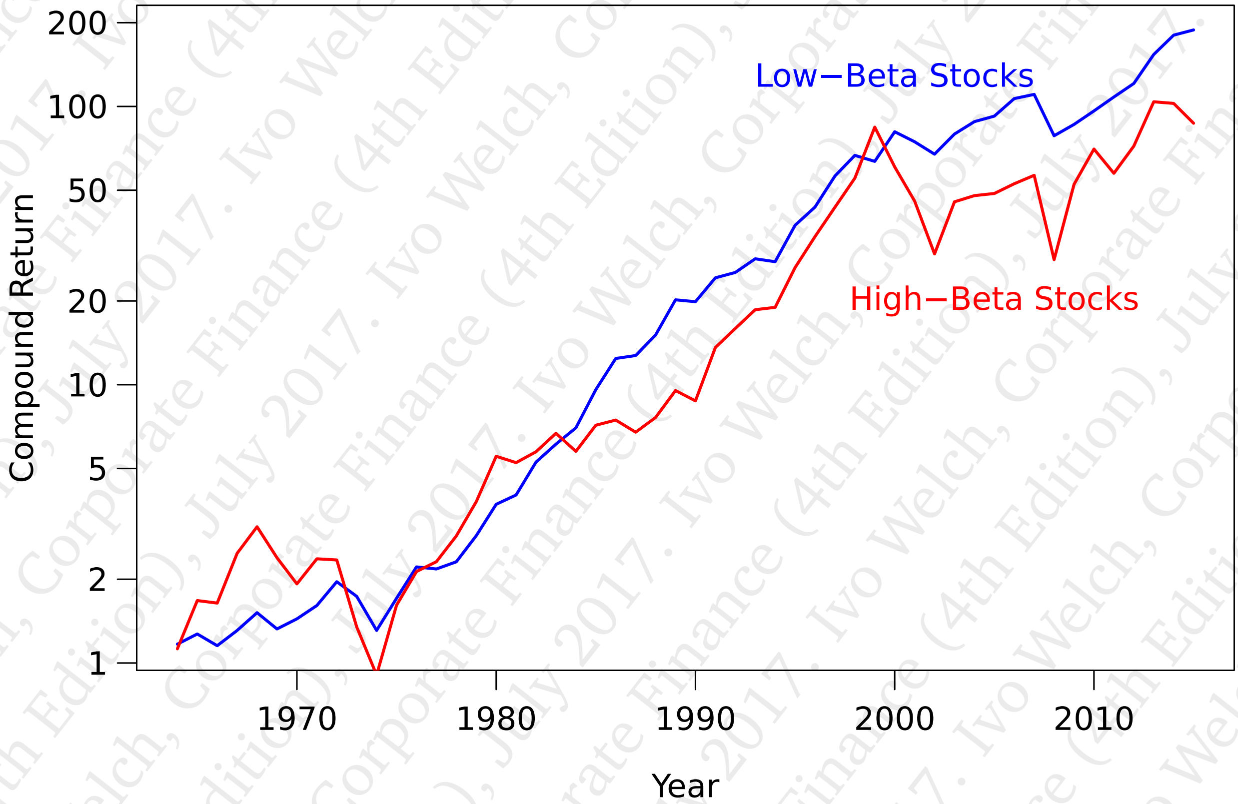


Exhibit 10.3: *Portfolio Performance of High-Beta and Low-Beta Stocks.* This graph plots the compound performance of portfolios formed from the lowest-quintile beta stocks and the highest-quintile beta stocks. If the CAPM had been right, higher-beta stocks should have offered higher expected performance. Alas, they did not.
Original Data Source: Ken French's Website.

Huh? WTH? Did you really read me right?

It does not take a PhD to see this (though it takes many to hide it). You won't find this kind of figure in other textbooks that want to obscure the truth.

Yes you did. Don't use the CAPM. The evidence is against it.

A famous finance professor, Ken French, estimates market betas for stocks each year, forms one portfolio of the quintile of stocks with the lowest betas, and another of those with the highest betas. He posts the data for everyone to see. Exhibit 10.3 plots the performance of these two portfolios. The high-beta portfolio should be toxic and thus require and deliver higher average rates of return than the low-beta portfolio. Alas, not only is the high-beta portfolio not statistically significantly superior, it isn't even superior. Low-beta stocks have outright outperformed high-beta stocks.

In Defense of the Use of the CAPM

If the evidence is against the CAPM, then why do we finance professors torture you with it? We may indeed have sadistic streaks (as our PhD students can testify), but this is not why. This "why" is much easier to answer than how stocks are priced in the real world or what the best estimate of the appropriate hurdle rates for your project should be.

***Across* asset classes:** Stocks had higher average rates of return than bonds. In this sense, high-beta assets offered higher average returns than low-beta assets. At least in this super-rough asset-class version, market beta works reasonably well. Higher-beta asset classes tended to have higher average rates of return.

Impeccable intuition: The CAPM shines through its simplicity and focus on diversification. It gets executives away from the false notion that public investors care about the idiosyncratic risk of projects that they can diversify away. Thus, corporate diversification into a conglomerate for its own sake can reduce its own risk but not market risk. It cannot add value. Investors can diversify themselves. They don't need the firm to do it for them.

Strong Belief: Many instructors and practitioners find the CAPM to be so plausible that they are willing to live with the "absence of CAPM evidence." They do not take this absence to mean "evidence of CAPM absence" (paraphrasing Rumsfeld). Thus, they adopt the CAPM based on faith and not on evidence—actually, more like despite evidence. If you do this, you must be aware that this is what you are doing.

Stand-in for Expected Cash Flow Default: The CAPM often assigns higher costs of capital to projects that are more likely to fail. If you have not fully adjusted your expected cash flow estimates downwards to adjust for failure (a common human error), the CAPM cost of capital often helps to impose a higher hurdle rate on riskier cash flows. It's a crutch.

Stand-in for Imperfect-Market Factors: The CAPM often assigns higher costs of capital to projects that do not satisfy the perfect-market assumptions and that face higher costs of capital. Again, this can accidentally result in better cost-of-capital estimates not *because* of the CAPM, but *despite* the CAPM. It's the other crutch.

Such a Great Idea: The CAPM is so intuitive and appealing that it would be "rediscovered" again and again by those who were not forced to learn it. Those who cannot remember the past are condemned to repeat it

Avoid Duplication

Everyone uses it: The CAPM is *the* standard. Exhibit 10.4 shows that 73% of the CFOs reported that they always or almost always use the CAPM. (And use of the CAPM was even more common among large firms and among CFOs with an MBA.) No alternative method was used very often. Consequently, you have no choice but to understand the CAPM model well—*if you will work for a corporation, then the CAPM is the benchmark model that your future employer will likely use and will expect you to understand well*. Chances are that you will be interrogated about it in your job interview.

Again, the CAPM is simply *the* standard. The CAPM is also used as a benchmark by many investors (e.g., to rate their investment managers), government regulatory commissions, courts (in tort cases), and so on. It is literally the dominant, if not the only, widely used model to estimate the cost of capital. There is even a whole section on the CFA exam about the CAPM!

Dinosaurs of the world—unite!

Alternatives—please stand up: The famous sociologist Lewin wrote that "there is nothing more practical than a good theory." If not the CAPM, then what else? There are alternatives, but none are universally accepted. My own recommendation is to go with the benchmarking approach from the previous chapter.

Market Hedging: Even if the market beta does not measure the average rate of return, it does guide managers about how much market risk they face—and, if they so desire, how to neutralize it and focus on their real expertise.

Method	Usage Frequency	Usage Recommendation	Explained in
CAPM	(73%)	With Caution	Chapter 9
	(87%)	Mukhlynina and Nyborg (2016)	
Historical Average Returns	(39%)	Rarely	Chapter 8
Modified CAPM	(34%)	With Caution	Chapter 9
Backed Out from Gordon Model	(16%)	Occasionally	Chapter 3
Whatever Investors Tell Us	(14%)	Occasionally	Chapter 2

Exhibit 10.4: *CFO Valuation Techniques for the Cost of Capital.* Rarely means "usually no, and often used incorrectly." Not reported, use of the CAPM is more common among managers with an MBA—and in firms who rely on consultants who in turn use the CAPM. Original Source: John Graham and Campbell Harvey, 2001.

Do you want a bedtime story that "the world is ok" in order to be able to go to sleep, or the tough truth?

Be aware that my treatment of the CAPM in an introductory corporate finance textbook borders on heresy. Most textbooks still make the CAPM their centerpiece. They do this not because the authors believe in it, but because it is dogma that new finance students (and many old finance professors) are too fragile to deserve the hard truth. I am sorry—I wish I could have told you a happy bedtime story about how the world is nice and orderly, too. But it would have been a lie.

"Cost of Capital" Expert Witnessing

When Congress tried to force the "Baby Bells" (the split-up parts of the original AT&T) to open up their local telephone lines to competition, it decreed that the Baby Bells were entitled to a fair return on their infrastructure investment—with fair return to be measured by the CAPM. (The CAPM is either the *de facto* or legislated standard for measuring the cost of capital in many other regulated industries, too.) The estimated value of the telecommunication infrastructure in the United States is about $10 to $15 billion. A difference in the estimated equity premium of 1% may sound small, but even in as small an industry as local telecommunications, it meant about $100 to $150 million a year—enough to hire hordes of lawyers and valuation consultants opining in court on the appropriate equity premium. Some of my colleagues bought nice houses with the legal fees.

I did not get the call. I lack the ability to keep a straight face while stating that "the equity premium is exactly *x* point *y* percent," which was an important qualification for being such an expert. In an unrelated case in which I did testify, the opposing expert witness even explicitly criticized my statement that my cost-of-capital estimate was an imprecise range—unlike me, he could provide an exact estimate, and it was 11% per year!

Baby-Bell History: Bradford Cornell, UCLA

If you must use it...

Never make the following errors, please.

If you still want to use the CAPM, here is my advice. As a corporate executive, you should always first think hard about why and when you want to use the CAPM. Think about whether it is useful for your own cost-of-capital estimates. Think about whether the CAPM errors seem too large to be useful for your particular needs. And understand what you are getting. Do simpler benchmarks first—do they agree with the CAPM estimate?

Accuracy: The CAPM is a poor model if you want precision. If you believe that CAPM expected rates of return should be calculated with any digits after the decimal point, then you are deluded. Please realize that, at best, the CAPM can offer only expected rates of return that are of the "right order of magnitude," plus or minus a few percentage points, perhaps.

Don't expect accuracy and don't use it for financial investing.

Actually, if accuracy is important, you are in trouble. Finance does not have *any* models that can offer precision. ortunately, you may not have to be good at estimating value; you may just need to be better than your competitors. Always remember that valuation is as much an art as it is a science. And you wouldn't be the first corporate executive who just happened to be saved by Lady Luck, even if the bet was not a particularly good one.

➤ F, Pg.??.

Investment purposes: If you are not a corporate executive looking to determine your project hurdle rate, but a financial investor looking for good investments from the universe of financial instruments, and with an ability to shift your money around every day, then please do not use the CAPM. Although the CAPM offers the correct intuition that wide diversification needs to be an important part of *any* good investment strategy, there *are* better investment strategies than just investing in the overall market index. These are discussed in advanced investments courses.

Avoid using the CAPM for short-term financial investment purposes.

Please do not confuse the CAPM with the mean-variance framework discussed in Chapter 8. Mean-variance optimization is an asset-selection technique for your individual portfolio, and it works, regardless of whether or not the CAPM holds.

➤ Mean-variance optimization in detail, Sect. 8.2, Pg.171.

Longer-Term Differences: If you are a corporate executive, be especially cautious of discount rates for expected cash flows far in the future. Look at your cost of capital more holistically. Remember that the CAPM has two terms.

➤ Corporate Time-Varying Costs of Capital, Sect. 5.5, Pg.99.

The first term is the risk-free rate, which applies to all projects, regardless of beta. Fortunately, this one is easy. You should use higher costs of capital for cash flows that will occur in the more distant future. And you have a great estimate of the premium that long-term projects need to offer over short-term projects, based on the Treasury yield curve. You don't even need historical estimates: you can use the prevailing Treasury yield curve. *Use it! It works!*

It is the second term (the beta multiplied by the risk-premium)—i.e., your beta risk-adjustment—that you must be especially suspicious about. If your cash flows will occur in many years, be modest. Do not overstate the risk assessment from the CAPM. Cut down extreme estimates. Shrink and shrink again (towards the average rate of return on risky investments). (Of course, do not forget to be similarly humble in your expected cash flow estimates.) Fortunately, you may be ok:

- As a corporate manager, compare the cost of capital on *your equity* vs. the cost of capital on *your debt* for your long-term cash flows. With an equity premium based on the performance of stocks vs. long-term Treasuries of about 1-2% from 1970 to today, it may not matter so much whether your project A has a beta of 0.8 and your project B has a beta of 1.2. The implied cost-of-capital difference between these two projects of under $(1.2-0.8)\cdot 2\% \approx 1\%$/year is already small and probably swamped by your expected cash flow estimation error.
- For long-term cash flows, your best estimate of your equity market betas should be tilted much more towards 1 than what you think your market beta is today. Thus, if you fit your historical market beta to be 0.5 for A and 1.5 for B today, you may well want to use a market beta shrunk to around 0.9 for A and 1.1 for B if those equity cash flows will occur in 10-20 years. Think about this: A and B would now have a different implied cost of equity capital of $0.2\cdot 2\% \approx 0.4\%$. This is way below your noise-and-uncertainty threshold.

  But let's continue. Say your projects are partly debt-financed, too. Now you need to calculate asset market betas rather than equity market betas. Let's say both

Asset betas are often even closer to 1—and they often give CAPM estimates some (sorely needed) time-stability.

projects have 50% debt that is almost risk-free. Then your asset beta would be $0.5 \cdot 0.0 + 0.5 \cdot 0.9 = 0.45$ for A and $0.5 \cdot 0.0 + 0.5 \cdot 1.1 = 0.55$ for B. Now you have a project cost-of-capital difference $(0.55 - 0.45) \cdot 2\% \approx 0.2\%$ between A and B.

The estimated CAPM cost of capital for long-term cash flows is fragile.

How does this expected rate-of-return difference between A and B compare to your own uncertainty about your projects' relative expected cash flows? Does the CAPM beta risk-adjustment really matter much in light of your uncertainty?

In sum, cash flows in the more distant future and cash flows that are more risky should likely be discounted more, as already explained in Chapter 9. But be humble about your capabilities in trying to distinguish between projects that are similar along time and asset-class dimensions.

Taking Advantage of CAPM Violations

The CAPM leans heavily on equilibrium market forces that are really quite weak.

Do investors have arbitrage opportunities if the CAPM fails? Absolutely not. The universe remains aligned even if the CAPM does not hold, and even in a perfect market.

Q: What happens if a stock offers too much or too little expected rate of return? A: Investor stampedes towards or away from the stock.

What would happen in the CAPM if one stock offered more than its appropriate expected rate of return? Its price would be too low. It would be too good a deal. Investors would immediately flock to it, and because there would not be enough of this stock in the economy, investors would bid up its price. This would lower its expected rate of return. The price of the stock would settle at the correct CAPM expected rate of return. Conversely, what would happen if one stock offered less than its due expected rate of return? Investors would not be willing to hold enough of this stock: The stock's price would be too high, and its price would fall. Neither situation should happen in a CAPM world.

Assets not priced according to the CAPM do not allow you to make money for nothing. However, they could imply good deals.

Is this an arbitrage—a "free money" situation? No! When stocks do not follow the CAPM formula, buying them remains risky. Yes, some stocks would offer a higher or lower expected rate of return and thus seem to be too good or too bad a deal, attracting too many or too few investors. (Or, the investors may not even flock towards better deals, perhaps because they have other needs, perhaps because they are asleep at the switch.) But these stocks would remain risky bets, and investors would want to buy just a little more or less. No investor could earn risk-free profits. There would be no arbitrage here. The market forces working on correcting any CAPM mispricing are just modest.

Maybe for some...

And also remember that there are good reasons why the CAPM would not hold in the first place. For example, as we have discussed, it relies heavily on many perfect-market assumptions. If investors are taxed or liquidity-constrained (that is, they cannot easily diversify, e.g., because the firm is a startup or family firm) or do not agree on the inputs, then it is quite plausible that some firms or even sectors (such as "value firms" or "growth firms") would offer higher or lower expected rates of return than the CAPM suggests. But not all is lost. It may mean that if *you* are an investor with CAPM preferences, you can do a little better than holding the overall market portfolio by tilting your market-like portfolio just a little towards stocks that offer higher expected rates of return than suggested by the CAPM formula and just a little away from stocks that offer lower expected rates of return.

10.6 Good CAPM Alternatives and Perspectives

You have already learned in the previous chapter about the principal alternative to the CAPM—benchmarking. The CAPM is really based on similar ideas, but it has just gone one step too far. It is too overconfident.

Madness: One Step Beyond. Can models be overconfident?

CAPM vs. Benchmarking: Widening and Narrowing Concepts

The CAPM both generalizes and narrows the idea of benchmarking. The generalization is that market beta is a more universal and objective measure of how equity-like any investment asset is than subjective judgment. It works for any asset—be it bond, stock, one specific stock or fund, equity options, gold, art, etc. The narrowing is that the CAPM is very specific about the fact that it is market beta—and market beta alone—that is the benchmark of the risk that investors care about. No other factors or exposure to other factors matter.

The CAPM goes one step too far. Stocks do seem to give higher average returns than bonds, but betas do not seem to predict higher expected returns.

- If the CAPM model is correct, then using more benchmark portfolios (à la Chapter 9) than just the stock market would still be just fine. Each benchmark portfolio would be priced according to the CAPM and lie on the SML. It is merely a convenience of the CAPM that you do not *have to* worry about these benchmark portfolios. If you do use these other benchmarks, fine. If you do not, fine, too. You will still find the same proper expected rate of return.
- If the CAPM model is incorrect, then by using it, you would have gone one step too far. You could easily get the wrong answer. For example, say, investors do not care about market risk (and market beta), but only about, say, oil risk, computer technology risk, and biotech risk. It could be the case that because the market portfolio contained some of these risks, it provided a higher expected rate of return. But it would really matter now whether your project and market beta come from oil risk (which gives you higher expected rate of return) or, say, gold risk (which does not). The CAPM would give you the right answer only if your project happened to have the same proportions as the market portfolio in its exposures.

  What you really need are the benchmark portfolios that matter as your comparison. Of course, unlike the CAPM, the benchmark portfolio method would be harder to use: What are good benchmarks? But benchmarking would still work in principle—just as long as you give this method all the right benchmark portfolios!

My Personal Opinion about Costs of Capital

Now I will give you my own educated opinion about good project cost-of-capital estimates. Different finance professors will come to different conclusions, so do not take my opinion as the gospel.

Solid Inference

The following expected-return premia are rock-solid:

What is solid empirical evidence?

- There definitely is a time value of money.
- There definitely is a term structure. Long-term cash flows usually require higher costs of capital than short-term cash flows. Your investors can earn higher expected rates of return elsewhere for longer-term commitments.
- There definitely is a credit component. Assets have to make up for higher probabilities of default with higher promised yields—that is, higher yields when they succeed.

➤ Market Imperfections, Chapter 11, Pg.241.

We have not covered the following yet. It will be explained in Chapter 11.

- Market imperfections play important roles. There are many kinds. Here are a few examples. There seems to be a liquidity premium. Assets that can be quickly liquidated (especially in general market crashes like 1987 or 2008) are more expensive, and different asset classes seem to have different degrees of liquidity. Because of their collateral, mortgage debt tends to have lower costs of capital than general bonds. Firms with less access to capital markets, such as startups, seem to pay higher costs of capital, although adjusting for default makes this difficult to measure. Investors pay more in personal income tax for interest receipts than they do for capital gains, which makes equities relatively more desirable and reduces their after-tax income. Sentiment and agency considerations also seem to play important roles in equity trading. Many of these market imperfections embody some concepts of risk, but it is not the market beta. Interestingly, courts agree with imperfect-market views. They allow as much as a 20-30% discount for the value of privately held assets relative to publicly traded peers. We may not know what the costs of capital for small, privately held firms are, but we do know that they are usually much higher.

Uncertain Inference

I wish I knew the equity risk premium—and for a lot of different reasons. The CAPM is only one of them. Benchmarking is another. Alas, I am not so confident that I have a good assessment. We are dealing with finance (with estimated probabilities), not physics (with known probabilities).

Market beta times equity premium is probably small, after other premia.

After taking into account the premia just mentioned (which includes premia that are sometimes included in and have to be captured by the risk premium, but which I already have in my number), the remaining risk premium—especially over longer horizons—is probably relatively small (1-2%). However, we do not know for sure. Our uncertainty is much larger than our certainty about its magnitude. And you need to realize that betas for cash flows far into the future are much closer to 1 than historical regressions would suggest. The "CAPM" beta-metric for measuring the project's risk impact and expected rate of return is only of modest importance.

Use reasonable risk adjustments—a little bit of beta, a little bit of idiosyncratic risk, a little bit more heuristic finesse.

So what would I do if I was not constrained by my boss? My best alternative cost-of-capital recommendation would start out just like the CAPM: As the first term in a formula, I would recommend that you use the rate of return on bonds of similar maturity as the cash flow that you want to value. Usually, this means that you assign higher costs of capital to cash flows farther in the future. It is only on the second term—the equity risk-adjustment—that I would tinker. Instead of the (shrunk) CAPM market beta multiplied by some historical equity premium (of 1-3%/year geometric above long-term Treasuries), I would recommend a more holistic approach.

- Take into consideration that projects with high volatility and/or with high leverage are more risky. The equity on these projects probably requires a higher expected rate of return to keep your investors happy. Realize that projects with higher idiosyncratic risk are also usually the same projects about which executives tend to be most overly optimistic. (Check again: are you sure your expected cash flows in the NPV numerator are not overconfident?!)
- Take into consideration whether you and your owners are well-diversified. If you are not, then you should require higher rates of return on riskier projects. In this case, it is not "beta risk" that matters, but "total risk."
- Take into consideration that your investors may "like" growth firms and are often willing to pay higher prices and thus accept lower average rates of return for some such projects. If they are willing to give you money at lower expected rates of return, take it!

There is probably little harm if you calculate a (repeatedly shrunk) CAPM market beta and apply it to a relatively low equity premium (say, 2%/year) for some heuristic orientation. Assess whether any other non-CAPM cost-of-capital assessments seem reasonably similar to such a CAPM assessment. In this sense, the CAPM can still be informative.

If Forced

What would I do if the boss liked the CAPM?

And if my boss forced me into the CAPM approach, what would I do?

- If I ran a large firm with good access to capital markets and I needed to evaluate a typical medium-term project, I would assume an equity premium of 1-3% per annum and apply this to the equity components of all my long-term cash flows. The exception would be projects for which I would have strong prior knowledge that their market betas would be very extreme—say, below –1 or greater than 3 (and I would then shrink those betas further to, say, 0 and 1.5, respectively, to account for long-term uncertainty about betas). I would consider long-term corporate debt to have a higher cost of capital than equivalent Treasuries, but a lower cost of capital than my own equity—the latter primarily because debt provides a corporate income tax shield (as you will learn in Chapter 18) and not because the equity premium over long-term corporate bonds is high.

➤ Income Taxes and Cost of Capital, Chapter 18, Pg.475.

- Deviating from the CAPM, if I ran a startup firm, I would assume a cost of capital of 2% to 6% above the *expected* rate of return on my uncollateralized debt. The expected rate of return on my equity could be very high—it could even be in the double digits. (This higher rate reflects the fact that more volatile cash flows and firms that struggle with more market imperfections must pay higher costs of capital.) Risk definitely plays a role, but not in the strict CAPM market-beta sense. Alternatively, I would abandon NPV-based models altogether and try to estimate what other similar projects are offering their investors. This is the route we take in Chapter 15.

➤ Comparables, Chapter 15, Pg.387.

And I would never use any of my schemes here (or the CAPM) for the pricing of bonds, derivatives, or other extreme kinds of projects.

NPV or Comparables? Eugene Fama thinks comparables are better.

Am I the only professor who recommends against using the CAPM? No. Many do in private, and even more do when their own money is on the line. Most are afraid to admit to our collective ignorance in front of students but prefer to proclaim knowledge (and teach the beauty that is the CAPM). Let me appeal to a higher authority for backup: Eugene Fama, the most famous finance professor alive, winner of a Nobel prize, and partly responsible for the original spread of the CAPM, nowadays strongly recommends against it. His view is that using the CAPM expected rate of return as your cost of capital in an NPV calculation effectively divides one bad uncertain number by another bad uncertain number. This practice convolutes errors and uncertainty about expected cash flows in the numerator with errors and uncertainty about expected returns in the denominator. If you get lucky, your errors cancel. If not, they do not. Yikes! Gene prefers comparables.

Conclusion

IMPORTANT

- The CAPM is *the* benchmark model in the real world. Most corporations use it.
- Every interviewer will expect you to understand the CAPM. Regardless of whether the model holds or not, you have to know it.
- The empirical evidence suggests that the CAPM is not a great model for predicting expected rates of return.
- The first CAPM term (the time adjustment) seems to hold better than the second CAPM term (the risk adjustment).
- Market betas tend to revert back towards 1. This requires you to shrink ordinary OLS beta estimates very aggressively towards 1.

- The geometric equity premium above long-term Treasuries (for evaluating long-term cash flows) has been—and is unlikely to be more in the future than—2-3% per annum.
- The CAPM never offers great accuracy.
- Mean-variance optimization (Section 8.2) works even if the CAPM does not.
- Peer portfolio benchmarking (Chapter 9) works regardless of whether the CAPM does or does not work.
- You may or may not want to immunize your project against equity-premium risk and estimation uncertainty, using its beta estimate. Immunized projects have much clearer cost-of-capital benchmarks than unimmunized projects.

Q 10.14. Does the empirical evidence suggest that the CAPM is correct?

Q 10.15. If the CAPM is wrong, why do you need to learn it?

Q 10.16. Is the CAPM likely to be more accurate for a project where the beta is very high, one where it is very low, or one where it is zero?

Q 10.17. To value an ordinarily risky project, that is, a project with a beta in the vicinity of about 1, what is the relative contribution of your personal uncertainty (lack of knowledge) about (a) the risk-free rate, (b) the equity premium, (c) the beta, and (d) the expected cash flows? Consider both long-term and short-term investments. Where are the trouble spots?

Summary

This chapter covered the following major points:

- The CAPM provides an "opportunity cost of capital" for investors, which corporations can use as the cost of capital in the NPV formula. The CAPM formula is

$$\mathsf{E}(r_i) = r_F + [\mathsf{E}(r_M) - r_F] \cdot \beta_i$$

  Thus, there are three inputs: the risk-free rate of return (r_F), the expected rate of return on the market ($\mathsf{E}(r_M)$), and the project's or firm's market beta (β_i). Only the latter is project-specific.
- The line plotting expected rates of return against market beta is called the security market line (SML).
- The CAPM provides an expected rate of return, consisting of the time premium and the risk premium. It ignores the default premium. In the NPV formula, the default risk and default premium work through the expected cash flow in the numerator, not through the expected rate of return (cost of capital) in the denominator.
- For r_F, you should use bonds that match the timing of your project's cash flows. Thus, cash flows farther in the future often require higher opportunity costs of capital. Even if you do not believe in the CAPM, term adjustment is important.
- The expected rate of return on the stock market is a critical CAPM input if the project's market beta is high—but this equity premium is difficult to guess. There are many guesstimation methods, but no one really knows which one is best. Reasonable estimates for the equity premium ($\mathsf{E}(r_M) - r_F$) can range from about 1% to 8% per annum, although 2-3% seems most common for cash flows more than a few years into the future.
- There are a number of methods to estimate market beta. Don't be too confident in betas far from 1, especially for long-term project cash flows.
- If you combine a short position in the stock market with a positive-beta project, the combined project is a lot easier to price than a project with a positive beta.

It neutralizes the effect of model and equity-premium errors.

- Never believe the CAPM blindly. Its estimates are poor. Use them more for "general direction" than as "accurate guides." Think compass, not GPS.
- Even though its estimates are poor, understand the CAPM well. Everyone will expect you to.

This negative perspective on the CAPM is so uncommon in a textbook (but not among the experts actually studying the models) that it is important that you don't misunderstand what this chapter says. So let's end this chapter with a FAQ:

- **Q**: Should riskier cash flows not require higher promised rates of return?

  A: Riskier projects have to promise higher rates of return, i.e., offer higher default premiums. This is not the same as higher risk premiums in the CAPM sense. In NPV applications, make sure to reflect the default risk in the expected cash flow numerator. Riskier projects need to pay off a lot more when they succeed, just to make up for the fact that they fail more often.

- **Q**: Should long-term and therefore riskier cash flows not require higher expected rates of return?

  A: Long-term projects command term premiums. Thus, in NPV applications, you should usually use higher required costs of capital for more distant cash flows. You should not use the CAPM for this. The U.S. Treasury Yield Curve gives you a working first estimate about how much extra premium that long-term cash flows should require above short-term cash flows.

- **Q**: Besides leverage structure and term, should riskier stocks and corporate cash flows have higher expected discount rates?

  A: Maybe, but be careful. First, make it modest. Don't be too overconfident in your ability to judge equity risks. If you can judge the risks well, make sure your estimates first flow into your expected cash flows in the NPV numerator. Second, don't be too wedded to the CAPM for the extra "risk-premium kicker." Instead, combine your cost-of-capital estimate with judgment-based and other risk measures, such as volatility (especially if your owners are not fully diversified).

Preview of the Chapter Appendix in the Companion

The appendix to this chapter explains:

- How the "certainty equivalence value" (CEV) allows you to use the CAPM for projects that you are not buying at the appropriate equilibrium price. For example, you would need the CEV to work out how to value an inheritance that will be higher if the economy does well. (Just because the inheritance is "free" to *you* does not mean that there is a zero value to it.)
- How to use the CEV formula to estimate the value of a project for which you have historical cash flows, but no market value information.
- How the CAPM is derived from the fact that the optimal portfolio is always the combination of two portfolios, one of which may be the risk-free asset.
- What a few more CAPM alternatives are and how to use them. The first alternative is the APT (arbitrage pricing theory) and its relative, the Intertemporal CAPM. The second alternative is a "Fama-French"-style model, which uses factors such as value, growth, momentum, investment, and robustness. This Fama-French model seems to predict better than any alternatives, but it is less grounded in theory (or, you may say, reason) than the former. It also often gives counterintuitive results—e.g., that small-growth stocks are safer than large-value stocks and therefore that managers should use *lower* discount rates on, say, risky tech ventures.

Keywords

Answers

Q 10.1 Yes, the perfect market is an assumption underlying the CAPM. In addition,

1. Investors are rational utility maximizers.
2. Investors care only about overall portfolio mean rate of return and risk at one given point in time.
3. All parameters are known (not discussed until later in the chapter).
4. All assets are traded. Every investor can buy every asset.

Q 10.2 With $r_F = 4\%$ and $E(r_M) = 7\%$, the cost of capital for a project with a beta of 3 is $E(r) = r_F + [E(r_M) - r_F] \cdot \beta_i = 4\% + (7\% - 4\%) \cdot 3 = 13\%$.

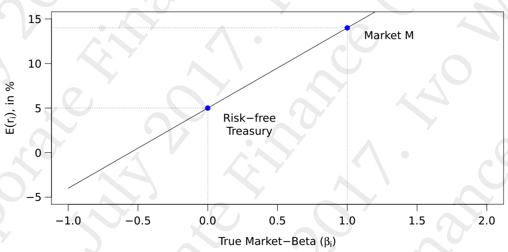
Q 10.3 With $r_F = 4\%$ and $E(r_M) = 12\%$, the cost of capital for a project with a beta of 3 is $E(r) = r_F + [E(r_M) - r_F] \cdot \beta_i = 4\% + (12\% - 4\%) \cdot 3 = 28\%$.

Q 10.4 With $r_F = 4\%$ and $E(r_M) = 12\%$, the cost of capital for a project with a beta of –3 is $E(r) = r_F + [E(r_M) - r_F] \cdot \beta_i = 4\% + (12\% - 4\%) \cdot (-3) = -20\%$. Yes, it does make sense that a project can offer a negative expected rate of return. Such projects are such great investments that you would be willing to expect losses on them, just because of the great insurance that they are offering.

Q 10.5 No—the real-world SML is based on historical data and not true expectations. It would be a scatterplot of historical risk and reward points. If the CAPM holds, a straight, upward-sloping line would fit them best.

Q 10.6 Write down the CAPM formula and solve $E(r_i) = r_F + [E(r_M) - r_F] \cdot \beta_i = 4\% + (7\% - 4\%) \cdot \beta_i = 5\%$. Therefore, $\beta_i = 1/3$. Note that we are ignoring the promised rate of return.

Q 10.7 The security market line is



Q 10.8 The equity premium, $E(r_M) - r_F$, is the premium that the market expects to offer on the risky market above and beyond what it offers on Treasuries.

Q 10.9 It does not matter what you choose as the per-unit payoff of the bond. If you choose \$100, you expect it to return \$99.

1. Thus, the price of the bond is $PV = \$99/(1 + [3\% + 5\% \cdot 0.2]) \approx \$95.19$.
2. Therefore, the promised rate of return on the bond is $\$100/\$95.19 - 1 \approx 5.05\%$.
3. The risk-free rate is 3%, so this is the time premium (which contains any inflation premium). The (expected) risk premium is 1%. The remaining 1.05% is the default premium.

Q 10.10 The cost needs to be discounted with the current interest rate. Because payment is upfront, this cost is \$30,000 now! The appropriate expected rate of return for cash flows (of your earnings) is $3\% + 5\% \cdot 1.5 = 10.5\%$. You can now use the annuity formula to determine the PV if you graduate:

$$\frac{\$5{,}000}{10.5\%} \cdot \left[1 - \left(\frac{1}{1 + 10.5\%}\right)^{40}\right] \approx \$47{,}619 \cdot 98.2\%$$
$$\approx \$46{,}741.46$$

With 90% probability, you will do so, which means that the appropriate risk-adjusted and discounted cash flow is about \$42,067.32. The NPV of your education is therefore about \$12,067.32.

Q 10.11 Yes, a zero-beta asset can still have its own idiosyncratic risk. And, yes, it is perfectly kosher for a zero-beta asset to offer the same expected rate of return as the risk-free asset. The reason is that investors hold gazillions of assets, so the idiosyncratic risk of the zero-beta asset will just diversify away.

Q 10.12 This is an asset beta versus equity beta question. Because the debt is almost risk-free, we can use $\beta_{Debt} \approx 0$.

1. First, compute an unlevered asset beta for your comparable with its debt-to-asset ratio of 2 to 3. This is $\beta_{Asset} = w_{Debt} \cdot \beta_{Debt} + w_{Equity} \cdot \beta_{Equity} = (2/3) \cdot 0 + (1/3) \cdot 2.5 \approx 0.833$. Next, assume that your project has the same asset beta, but a smaller debt-to-asset ratio of 1 to 3, and compute your own equity beta: $\beta_{Asset} = w_{Debt} \cdot \beta_{Debt} + w_{Equity} \cdot \beta_{Equity} \Rightarrow 0.833 \approx (1/3) \cdot 0 + (2/3) \cdot \beta_{Equity} \Rightarrow \beta_{Equity} = 1.25$.
2. With an asset beta of 0.83, your firm's asset hurdle rate should be $E(r_i) = 3\% + 2\% \cdot 0.83 \approx 4.7\%$.
3. Your comparable's equity expected rate of return would be $E(r_{Comps\,Equity}) = 3\% + 2\% \cdot 2.5 = 8\%$. Your own equity's expected rate of return would be $E(r_{Your\ Equity}) = 3\% + 2\% \cdot 1.25 = 5.5\%$

Q 10.13 Your combined happy-marriage beta would be $\beta_{\text{Combined}} = (3/4) \cdot 2.4 + (1/4) \cdot 0.4 = 1.9$.

Q 10.14 No, the empirical evidence suggests that the CAPM does not hold. The most important violation seems to be that value firms had market betas that were low, yet average returns that were high. The opposite was the case for growth firms.

Q 10.15 Even though the CAPM is empirically rejected, it remains the benchmark model that everyone uses in the real world. Moreover, even if you do not trust the CAPM itself, at the very least it suggests that covariance with the market could be an important factor.

Q 10.16 The CAPM should work very well if beta is about 0. The reason is that you do not even need to guess the equity premium if this is so.

Q 10.17 For short-term investments, the expected cash flows are most critical to estimate well (see Section 4.1 on Page 57). In this case, the trouble spot (d) is really all that matters. For long-term projects, the cost of capital becomes relatively more important to get right, too. The market betas and risk-free rates are usually relatively low maintenance (though not trouble-free), having only modest degrees of uncertainty. The equity premium will be the most important problem factor in the cost-of-capital estimation. Thus, the trouble spots for long-term projects are (b) and (d).

End of Chapter Problems

Q 10.18. What are the assumptions underlying the CAPM? Are the perfect market assumptions among them? Are there more?

Q 10.19. If the CAPM holds, then what should you do as a manager if you cannot find projects that meet the hurdle rate suggested by the CAPM?

Q 10.20. In a perfect world (and in the absence of externalities, which would imply that projects influence other projects), should you take only the projects with the highest NPV?

Q 10.21. Write down the CAPM formula. Which are economy-wide inputs, and which are project-specific inputs?

Q 10.22. The risk-free rate is 6%. The expected rate of return on the stock market is 8%. What is the appropriate cost of capital for a project that has a beta of 2?

Q 10.23. The risk-free rate is 6%. The expected rate of return on the stock market is 10%. What is the appropriate cost of capital for a project that has a beta of –2? Does this make economic sense?

Q 10.24. Draw the SML if the true expected rate of return on the market is 6% per annum and the risk-free rate is 2% per annum. What would the figure look like if you were not sure about the expected rate of return on the market?

Q 10.25. A junk bond with a beta of 0.4 will default with 20% probability. If it does, investors receive only 60% of what is due to them. The risk-free rate is 3% per annum and the risk premium is 5% per annum. What is the price of this bond, its promised rate of return, and its expected rate of return?

Q 10.26. What would it take for a bond to have a larger risk premium than default premium?

Q 10.27. A corporate zero-bond promises 7% in one year. Its market beta is 0.3. The equity premium is 4%; the equivalent Treasury rate is 3%. What is the appropriate bond price today?

Q 10.28. Explain the basic schools of thought when it comes to equity premium estimation.

Q 10.29. If you do not want to estimate the equity premium, what are your alternatives to finding a cost-of-capital estimate?

Q 10.30. Explain in 200 words or less: What are reasonable guesstimates for the market risk premium and why?

Q 10.31. Should you use the same risk-free rate of return both as the CAPM formula intercept and in the equity premium calculation, or should you assume an equity premium that is independent of investment horizon?

Q 10.32. Should a negative-beta asset offer a higher or a lower expected rate of return than the risk-free asset? Does this make sense?

Q 10.33. An unlevered firm has an asset market beta of 1.5. The risk-free rate is 3%. The equity premium is 4%.

1. What is the firm's cost of capital?
2. The firm refinances itself. It repurchases half of its stock with debt that it issues. Assume that this debt is risk-free. What is the equity beta of the levered firm?
3. According to the CAPM, what rate of return does the firm have to offer to its *creditors*?
4. According to the CAPM, what rate of return does the firm have to offer to its *levered equity holders*?
5. Has the firm's weighted average cost of capital changed?

Q 10.34. Download daily stock market data for Intel and the S&P 500 for calendar year 2016 from YAHOO! FINANCE.

1. What was Intel's plain stock-market-model regression beta in your sample?
2. What was Intel's shrunk stock-market beta? Use a shrinkage factor of 0.5 towards a market beta of 1.0 and your just-calculated estimate.
3. How does this compare to the Intel market beta on YAHOO! FINANCE?
4. If Intel had a debt-equity ratio of 1-to-2 and its debt was close to risk-free, what was its asset beta? (Hint: To determine the debt-to-asset ratio, make up an example in which a firm has a 30% D/E ratio.)

Q 10.35. A peer firm in a comparable business has an equity beta of 2.5 and a debt-equity ratio of 2. The debt is almost risk-free. Estimate the beta for your equity if projects have constant betas, but your firm will carry a debt-equity ratio of 1/2. (Hint: To translate a debt-equity ratio into a debt-asset ratio, make up an example.)

Q 10.36. A Fortune 100 firm is financed with $15 billion in debt and $5 billion in equity. Its historical equity beta has been 2. If the firm were to increase its leverage from $15 billion to $18 billion and use the cash to repurchase shares, what would you expect its levered equity beta to be?

Q 10.37. The prevailing risk-free rate is 5% per annum. A competitor to your own firm, though publicly traded, has been using an overall project cost of capital of 12% per annum. The competitor is financed by 1/3 debt and 2/3 equity. This firm has had an estimated equity beta of 1.5. What is it using as its equity premium estimate?

Q 10.38. Apply the CAPM. Assume the risk-free rate of return is the current yield on 5-year bonds. Assume that the market's expected rate of return is 3% per year above this. Download five years of daily rate-of-return data on four funds: NAESX, VLACX, VUVLX, and VWUSX.

- What were the historical average rates of return?
- What were the historical market betas?
- What were the historical market betas, adjusted (shrunk) toward 1 by averaging with 1?
- How do these estimates compare to the market beta estimates of the financial website from which you downloaded the data?
- Does it appear as if these funds followed a CAPM-like relationship?

Q 10.39. Draw some possible security markets lines (SML's) that would not be consistent with the CAPM. On the x axis, put the true market beta. On the y axis, put the true expected rate of return.

Q 10.40. Does the empirical evidence suggest that the CAPM is correct?

Q 10.41. Why do you need to understand the CAPM?

Q 10.42. Under what circumstances is the CAPM a good model to use? What are the main arguments in favor of using it? When is it not a good model?

Q 10.43. If you use the CAPM, explain for what kinds of projects it is important to get accurate equity-premium estimates.