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Time series analysis of Cryptocurrency returns and volatilities

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Abstract

There is a significant interest in the growth and development of cryptocurrencies, the most notable ones being Bitcoin and Ripple. Global trading in these cryptocurrencies has led to highly speculative and "bubble-like" price movements. Since these cryptocurrencies trade like stocks, provide a feasible alternative to gold and appreciate during uncertain times, it can be hypothesized that their prices are partly determined by the global stock indices, gold prices, and fear gauges such as the VIX and the US Economic Policy Uncertainty Index. In this paper, we test this hypothesis by conducting a time series analysis of returns and volatilities of Bitcoin and of Ripple. We use the Autoregressive-moving-average model with exogenous inputs model (ARMAX), Generalized Autoregressive Conditionally Heteroscedastic (GARCH) model, Vector Autoregression (VAR) model, and Granger causality tests to determine linkages between returns and volatilities of Bitcoin and of Ripple. We find that the Bitcoin crash of 2018 could have been explained using these time series methods. We also find that returns of global stock markets and of gold do not have a causal effect on Bitcoin returns, but we do find returns on Ripple have a causal effect on Bitcoin prices.

Keywords Asset management · Alternative investments · Digital currency · Cryptocurrency · Bitcoin, ripple, BTC, XRP, economic uncertainty index

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1 Introduction

Bitcoin (largest 'cryptocurrency', abbreviated as BTC) is the first decentralized peer-to-peer payment network that is powered by its users with no central authority or middlemen.[1] It has had an astounding rise in popularity in recent times. Many believe that it is the currency of the future, and the rise in its price has been attributed to its limited supply. Nakamoto (2008) is said to be the pioneer behind this cryptocurrency. New transactions are announced on a computer network by BTC users connected via the internet, and these transactions are verified by network nodes. The transactions are then recorded in a public distributed ledger called the Blockchain. BTC is one of the many digital currencies that uses Blockchain as its underlying platform. BTCs are awarded to miners or users who offer their computing power to verify and record transactions into the Blockchain.

As of January 2019, more than 100 million transactions from 32 million digital wallets based in 140 countries are processed by the Blockchain.[2] Since BTC's introduction in 2009, the BTC market capitalization grew rapidly from less than a billion USD in 2013 to more than $323 billion in December 2017, after contracting to $67 billion in January 2019.[3] Daily Blockchain transactions had increased from a negligible amount in the year 2010 to $5.7 Billion in December 2017, then fell to 0.5 Billion USD in January 2019.[4] BTC has the largest market capitalization among all cryptocurrencies, but its market capitalization is still a small fraction of all other currencies in circulation. As an example, more than $1.7 trillion of non-cryptocurrencies are in circulation in January 2019.[5]

Other cryptocurrencies have emerged following the success of BTC.[6] There are 2086 such cryptocurrencies in the global market as of January 2019. Ethereum is the second most popular currency with a market capitalization of $16.4 Billion, and Ripple (popularly known as XRP) is third on the list with a market capitalization of $14.5 billion as of January 2019. The remaining cryptocurrencies are an order of much smaller magnitude of market capitalization; consequently, we do not include them in our study. Since data for Ethereum (the second-biggest cryptocurrency in terms of market capitalization) is not available beyond five years, it restricts the generalizability of results in our study. Consequently, we do not include Ethereum in our analysis. In this paper, we focus on two major cryptocurrencies: BTC (abbreviated as BTC) and XRP, primarily due to data availability constraints. Together, BTC and XRP are bigger than the next 800 cryptocurrencies combined in terms of market capitalization. The combined market capitalization of BTC and XRP is 77% of the entire 2170 cryptocurrency market, and this is displayed in Fig. (1).

This figure plots the historical market capitalization of BTC and XRP, the two largest cryptocurrencies in 2019. Visible in the figure is a twenty-fold increase in the

[1] What is Bitcoin? https://Bitcoin.org/en/faq#what-is-Bitcoin
[2] Blockchain by the numbers, https://www.blockchain.com/about/index.html
[3] Bitcoin Market Capitalization, https://coinmarketcap.com/
[4] Estimated USD transaction value, https://blockchain.info/charts/estimated-transaction-volume-usd
[5] USD in circulation, https://fred.stlouisfed.org/series/WCURCIR
[6] List of major cryptocurrencies by market capitalization, https://coinmarketcap.com/all/views/all/

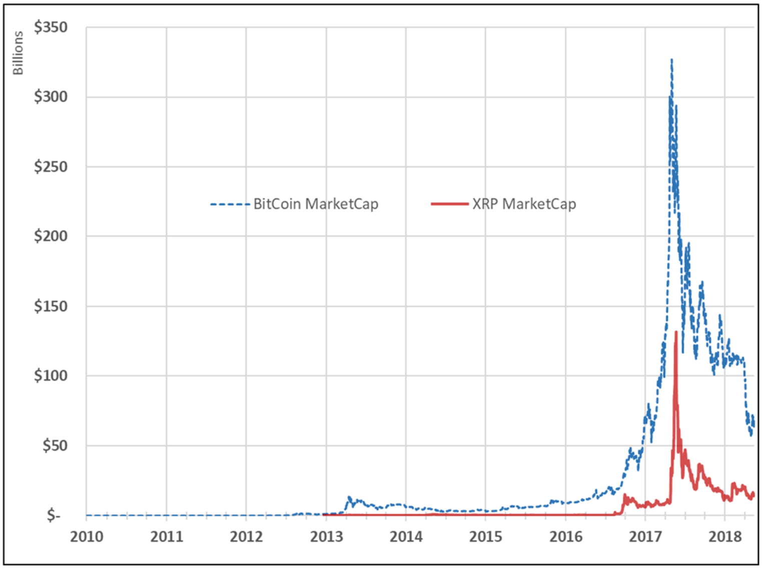

Fig. 1 Market capitalization of BTC and XRP in billion $USD (as of Jan 2019)

BTC market capitalization in the year 2017, followed by a 50% crash in the first two months of 2018. During the same period, XRP went up by 550-fold and down by 75%.

In this study, we analyze the returns and volatilities of cryptocurrencies, and specifically, we model and examine the 2018 crash. We also analyze the linkages between the BTC, XRP, S&P500 index, global stock market indices (i.e., MSCI Emerging Markets, MSCI World), VIX, gold, US economic policy uncertainty index (abbreviated as EUI), and their respective returns and volatilities. In the following section, we provide an overview of BTC literature. Section 3 explains the description of the data and methodology used in this study. This is followed by results in Section 4 and conclusions and directions for future research in the final section.

2 The literature on CRYPTOCURRENCIES

Recently, many studies have been conducted to understand the origin, behavior of, and the mechanism behind cryptocurrencies. Narayanan et al. (2016) provides a comprehensive overview of BTC, Blockchain, and the ecosystem around cryptocurrencies, whereas Noga (2018) addresses the role of money and artificial currencies. A technical survey of BTC, Blockchain, Security, Network, and Privacy is provided in Conti et al. (2018) and Tschorsch and Scheuermann (2016).

With the advent of cryptocurrencies, researchers have extended the investment asset universe to include cryptocurrencies. Financial asset return predictability is of great interest in the financial literature, as summarized by Golez and Koudijs (2018). Empirical evidence suggests that stock returns are indeed partially predictable (Campbell and Shiller 1988; Fama and French 1988; Cochrane 2007; Binsbergen et al. 2010). For a detailed overview of risk-return characteristics of cryptocurrencies, see Liu and Tsyvinski (2018). Malladi et al. (2019) have studied BTC predictability with a smaller set of predictors and forecasted the direction of returns accurately but missed the magnitude of returns. In this study, we include several other predictors such as XRP prices, global stock markets, and economic uncertainty index in our analysis of BTC and XRP prices. As a result, we use these variables to explain the magnitude and direction of BTC and XRP returns and volatilities.

2.1 Factors influencing returns of cryptocurrencies

There have been few studies on cryptocurrencies prices and their price movements. Van Wijk (2013) found that most of the BTC price influencing variables are related to the US economy. Using daily and weekly data within a Dynamic Conditional Correlation (DCC) model (Engle 2002), Bouri et al. (2017) showed that BTC could serve as an effective diversifier for most of the cases. Ciaian et al. (2016) found that market forces of BTC supply and demand, the arrival of additional information (trust), and speculators are three key drivers of BTC prices. Besides, they did not support previous findings that global macro-financial development might be driving BTC's price.

Other studies, namely Cheah and Fry (2015) and Katsiampa (2017) report that the recent volatility in cryptocurrency prices is an outcome of market sentiment, where the latter can be associated with significant "memory." According to those studies, the "memory" of shocks of cryptocurrency prices are semi-important determinants of cryptocurrency prices. Dyhrberg (2016a) found that BTC can be an ideal tool for risk-averse investors as a buffer against negative shocks to the market, whereas Dyhrberg (2016b) found that BTC can serve as a hedge against market-specific risk. The most recent study on BTC prices was conducted by Cheah et al. (2018). They model cross-market BTC prices as long-memory processes and dynamic inter-dependence in a fractionally cointegrated VAR framework. Their findings suggest that there is long-memory in both individual market and five-market systems, indicating non-homogeneous informational inefficiency and a cointegration relationship with slow adjustment of shocks.

In this paper, we extend the existing literature by adding the influence of fear and uncertainty in the markets as measured by the VIX and EUI, and physical gold spot prices on BTC and XRP prices. This analysis is conducted using three popular time series forecasting techniques: (a) ARMAX, (b) GARCH, and (c) VAR.

2.2 Factors influencing volatilities of cryptocurrencies

Financial market volatility is an important input for investment, option pricing, and financial market regulation. For a detailed review of volatility forecasting, refer to (Engle 1982; Poon and Granger 2003; Satchell and Knight 2011). Using GARCH analysis, Dyhrberg (2016a) found that cryptocurrencies can combine some of the advantages of both commodities and currencies. Guo and Antulov-Fantulin (2018) investigate the volatility of cryptocurrency and try to predict short-term prices using volatility and trade order book data. Catania et al. (2018) tried to predict the conditional volatility of four major cryptocurrencies (BTC, XRP, Ethereum, and Litecoin). Considering the factors mentioned in these studies, we extend the analysis further by explicitly incorporating variables that can be a proxy for fear and uncertainty. Specifically, we use VIX, realized volatilities of BTC prices (BVOL) as computed by Bitmex,[7] of XRP prices (XVOL), and

[7] BVOL computations, https://www.bitmex.com/app/index/.BVOL

of gold prices (GVOL), To be consistent, XVOL and GVOL are computed in the same manner as BVOL, as explained in Section 3.2.

3 Data and methodology

3.1 Data

We use the following four data sources in our study. We collect daily data on BTC and XRP prices from Coindesk[8,9] VIX closing prices from CBOE,[10] Gold prices from the World Gold Council,[11] and the US Economic Policy Uncertainty Index (USEPUINDXD) from the FRED, Federal Reserve Bank of St. Louis.[12] The window of analysis is from 08/01/2013 (earliest data point for XRP) to 01/01/2019 (most recent). During this window, BTC prices in USD went up by 220% in one month between November 2017 and December 2017; then, they crashed by 85% between December 2017 and December 2018. We could have used other cryptocurrencies (such as Ethereum, and Litecoin), but decided against it deliberately for two reasons: (a) to have a wider data window since some of these digital currencies were released after 2015; and (b) other cryptocurrencies have significantly lower market capitalizations and can be safely ignored.

3.2 Methodology

BTC and XRP prices are available on all seven days of the week. However, gold prices and VIX data are available only on US trading days. Consequently, VIX dates (from CBOE trading days) are used as a baseline, and weekend data for cryptocurrencies are removed from the dataset. As a result, we have 1363 daily price points between 08/01/2013 and 01/01/2019. Daily returns are computed using ln (P_1/P_0) formula, where P_1 is today's closing price, and P_0 is the previous trading day's closing price. BTC volatility is computed using the annualized realized volatility approach of the Bitmex.[13] We computed the 30-day historical volatility index (hereafter referred to as the BVOL Index). The BVOL index is a rolling 30-day annualized (365-day) volatility of the daily (11:30 UTC to 12:00 UTC) Time Weighted Average Price (TWAP) of BTC in USD. To be consistent in the implied volatility computations, we use the same BVOL approach in computing GVOL and XVOL. Due to 30-day rolling average calculations, the sample size of volatilities is reduced to 1332.

Summary statistics of BTC, XRP, Gold, Stock markets (S&P500, MSCI World, and MSCI Emerging Market), volatilities (BVOL, XVOL, GVOL, and VIX), and EUI are provided in Tables (1) and (2). We find that the returns of XRP and BTC to be very volatile compared to other asset classes in our study. As an example, Table (1) shows

[8] Bitcoin (BTC) Prices, https://www.coindesk.com/price/bitcoin
[9] Ripple (XRP) prices, https://www.coindesk.com/price/xrp
[10] VIX historical data, http://www.cboe.com/products/vix-index-volatility/vix-options-and-futures/vix-index/vix-historical-data
[11] Gold historical data, https://www.gold.org/data/gold-price
[12] https://fred.stlouisfed.org/series/USEPUINDXD/
[13] BVOL computations, https://www.bitmex.com/app/index/.BVOL

Table 1 Summary Statistics of BTC, Gold, and S&P500 returns and volatilities. This table provides summary statistics of all 17 variables in our study between 08/01/2013 and 01/01/2019

	Mean	Median	Max.	Min.	Std. Dev.	Skew	Kurtosis	N
BITCOIN_R	0.25%	0.21%	28.00%	−26.69%	5.14%	(0.26)	8.38	1332
XRP_R	0.30%	−0.41%	75.08%	−54.74%	9.00%	1.42	16.69	1332
GOLD_R	0.00%	0.00%	4.84%	−3.19%	0.86%	0.25	5.75	1332
SP500_R	0.03%	0.04%	4.84%	−4.18%	0.83%	(0.48)	6.77	1332
MSCI_EM_R	0.00%	0.05%	3.22%	−5.13%	0.89%	(0.32)	4.86	1332
MSCI_W_R	0.01%	0.05%	2.61%	−5.03%	0.70%	(0.74)	7.22	1332
EUI_30DMA_R	0.02%	−0.25%	34.90%	−21.53%	3.32%	1.21	16.26	1332
BITCOIN	2441.94	628	19,396	100.81	3429.12	1.87	6.28	1332
XRP	0.17	0.01	3.20	0.00	0.33	3.82	24.72	1332
GVOL	0.16	0.15	0.32	0.07	0.05	0.84	3.63	1332
GOLD	1242.37	1248.13	1385.00	1049.40	68.32	(0.57)	2.89	1332
SP500	2227.76	2108.20	2930.75	1655.45	324.79	0.53	2.11	1332
MSCI_EM	978.46	986.25	1273.07	688.52	113.43	(0.09)	2.65	1332
EUI_30D_MA	83.98	79.97	182.77	48.67	23.72	1.31	5.04	1332
VIX	14.84	13.72	40.74	9.14	4.16	1.67	6.90	1332
BVOL	0.86	0.78	2.66	0.16	0.44	1.08	4.61	1332
XVOL	1.42	1.12	5.02	0.29	0.94	1.61	5.36	1332

that the range of daily returns is 129.83% for XRP, 54.69% for BTC, 8.03% for gold, and 9.02% for the S&P500. Table (2) shows that the daily returns of both BTC and XRP are uncorrelated (close to zero) with those of stocks, gold, and EUI. However, we

Table 2 Correlations of BTC, XRP, Gold, S&P500, MSCI, and EUI. The top panel shows correlations of daily returns, and the bottom panel shows correlations of volatility. Negative correlations are provided in () in red color, correlations above 0.5 are in green color

	Bitcoin_R	XRP_R	Gold_R	SP500_R	MSCI_EM_R	MSCI_W_R	EUI_30DMA_R
Bitcoin_R	1.00						
XRP_R	0.01	1.00					
Gold_R	(0.00)	0.01	1.00				
SP500_R	0.07	0.03	(0.12)	1.00			
MSCI_EM_R	0.02	0.02	0.05	0.42	1.00		
MSCI_W_R	0.07	0.03	(0.07)	0.92	0.62	1.00	
EUI_30DMA_R	(0.05)	(0.01)	0.03	(0.02)	(0.04)	(0.05)	1.00

	BVOL	XVOL	GVOL	VIX
BVOL	1.00			
XVOL	0.53	1.00		
GVOL	(0.06)	(0.03)	1.00	
VIX	(0.06)	(0.12)	0.16	1.00

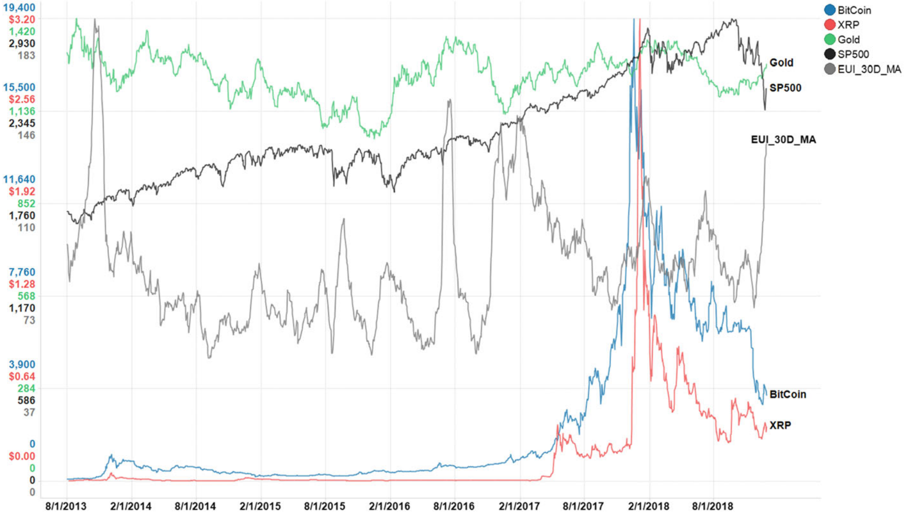

Fig. 2 Historical price charts of BTC, XRP, Gold, S&P500, and EUI. Four line charts (one per each return) are labeled on the right-hand side of the chart window. A 30-day moving average return is used for a smooth curve

see that the volatility of daily returns of BTC and XRP are correlated. We also notice that the volatility of daily returns of gold and stocks are uncorrelated with those of cryptocurrencies. Historical returns are shown in Fig. (2), and historical volatilities are shown in Fig. (3). Fig. (2) shows the BTC bubble (rapid rise of BTC price by 220% in one month between 11/2017 and 12/2017) followed by BTC bust (crash by 85% between 12/2017 and 12/2018). Fig. (3) shows that jumps of BVOL and XVOL do

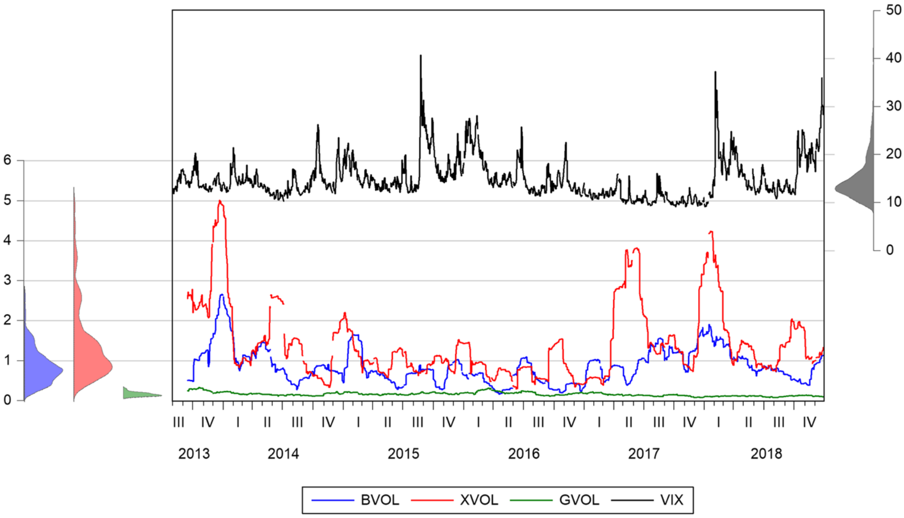

Fig. 3 Historical volatility of daily returns of BTC, XRP, Gold, and VIX. Four line charts (one per each volatility) are labeled at the bottom of the chart window. BTC, XRP, and Gold volatilities are shown on the left axis. VIX is shown on the right axis. The four frequency distribution charts are shown on the y-axis

not mimic those of VIX. When we compare the volatility of gold with the volatility of cryptocurrencies, we notice that the volatility of gold is several magnitudes smaller than those of cryptocurrencies.

We use time-series techniques such as the ARMAX, GARCH, and VAR to explain the returns and volatilities of BTC. The rationale and the five-step process used is explained below.

First, we employ the backward elimination method of stepwise regression. In this process, we start with all explanatory variables, as shown in Eq. (1), and see if they can explain BTC's daily returns. We deploy multiple asset classes as proxies to BTC (such as stocks, commodities, and other cryptocurrencies) to explain BTC returns. In the beginning, all six return variables (XRP_R, SP500_R, MSCI_EM_R, MSCI_W_R, GOLD_R, and EUI_30DMA_R) are entered into the regression equation, and the ones that do not contribute to the regression equation are removed one at a time.

Granger and Newbold (1974) posit that spurious regression problems occur if there is non-stationarity in data, and this leads to unreliable correlations within regression analysis. A stationary series can be defined as one with a constant mean, constant variance, and constant autocovariance for each given lag. We test for stationarity of time series returns using the early and pioneering work done by Dickey and Fuller (1979) on testing for unit roots in time series data.

$$
\begin{aligned}
BTC\_R_t = {} & XRP\_R_t + SP500\_R_t + MSCI\_EM\_R_t + MSCI\_W\_R_t + GOLD\_R_t \\
& + \; EUI\_30DMA\_R_t
\end{aligned}
\tag{1}
$$

Second, after establishing stationarity of time series data, we employ the Autoregressive Moving Average (ARMA) model, one of the most popular methodologies popularized by Box and Jenkins (1976) for analyzing time series. ARMA offers great flexibility in analyzing and forecasting various time series. ARMA's other advantage is that for analyzing a time series, the technique uses that series' historical data. Since ARMA lacks economic intuition, sometimes explanatory variables (X) are added, thus making it an ARMAX model. Brooks (2014) provides a detailed background of the ARMA model, and Peter and Silvia (2012) provide a detailed use case of the ARMAX model. We use ARMAX (p, q, b) model from Baillie (1980) to explain BTC returns using Eq. (2), where p is an autoregressive term, q is moving average term, and b is exogenous inputs term. We split the available data from $t = 0, 1, \ldots, T$ into two periods. Data from $t = 0, 1, \ldots, N - 1$ are used for ARMAX model estimation, and from $t = N, N + 1, \ldots, T$ is used for generating ARMAX forecasts, which are subsequently evaluated against actual values.

$$
Y_t(p,q,b) = \sum_{i=1}^{p} \phi_i Y_{t-i} + \sum_{i=1}^{q} \theta_i \upsilon_{t-q} + \upsilon_t + \sum_{i=1}^{b} \beta_i X_{it} \tag{2}
$$

where Y_t, or $Bitcoin\_R_t$, is a stationary time series,

υ_t is a white noise process with $E(\upsilon_t) = 0; E\left(\upsilon_t^2\right) = \sigma^2; E(\upsilon_t \upsilon_s) = 0, t \neq s$

and X_t is an exogenous variable.

Andersen et al. (2006) states that return volatility is, of course, central to financial economics. The trade-off between risk and expected return, where risk is associated with some notion of price volatility, constitutes one of the key concepts in modern finance—as such, measuring and forecasting volatility is arguably among the most important pursuit in empirical asset pricing finance and risk management. As described in detail by Brownlees et al. (2011), realized volatility models often demonstrate excellent forecasting performance. Hence, we turn our attention to forecasting volatilities.

Volatility is inherently unobserved, or latent, and evolves stochastically through time. Not only is there nontrivial uncertainty to deal with in financial markets, but the level of uncertainty is latent. The current interest in volatility modeling and forecasting was spurred by Engle's (1982) ARCH paper, which set out the basic idea of modeling and forecasting volatility as a time-varying function of current information. A useful generalization of the ARCH model is the GARCH model introduced by Bollerslev (1986) and discussed independently by Taylor (1987). GARCH models have become the workhorses of volatility modeling. A detailed summary of these models is provided in Engle (2001). Andersen et al. (2006) provide a comprehensive theoretical overview of volatility forecasting.

Third, we split the available data from $t = 0, 1, \ldots, T$ into two periods. Data from $t = 0, 1, \ldots, N - 1$ are used for GARCH model estimation, and from $t = N, N + 1, \ldots, T$ are used for generating GARCH forecasts, which are subsequently evaluated against actual values. GARCH (p, q) model is provided as Eq. (3).

$$\sigma^2_{BVOL(t)} = \alpha_0 + \textstyle\sum_{i=1}^{p} \beta_i \sigma^2_{BVOL(t-i)} + \textstyle\sum_{j=1}^{q} \alpha_j \mu^2_{t-j} \tag{3}$$

where $\sigma^2_{BVOL(t)}$is the conditional variance of μ_t since it is one period ahead estimate based on past information, and $\mu_t \sim N\left(0, \sigma^2_{BVOL(t)}\right)$. α_j and β_i are positive to ensure that conditional variance is positive. When q = 0, the GARCH model reduces to the ARCH model. $Y_t = C + \mu_t$, where C is the mean of Y_t and $\mu_t$is i.i.d. with mean zero. To allow for conditional heteroscedasticity $Var_{t-1}[\mu_t] = \sigma^2_t$.

It is quite common in economics to have models in which some variables are not only explanatory variables for a given dependent variable but are also explained by the variables that they are used to determine. In these cases, we have models of simultaneous equations, in which it is necessary to identify clearly which are endogenous and which are the exogenous variables. The decision regarding such differentiation among variables was heavily criticized by Sims (1980). According to him, if several variables are simultaneous, then all these variables should be treated in the same way. In other words, there should be no distinction between endogenous and exogenous variables. Therefore, once this distinction is abandoned, all variables are treated as endogenous. This means that in its general reduced form, each Equation has the same set of regressors, which leads to the development of Vector Autoregression (VAR) models.

We use the VAR technique to understand better the interrelationships between returns of cryptocurrencies and returns of stocks, gold, and the EUI. The VAR model approach has some desirable characteristics, as outlined in Asteriou and Hall (2011). VAR models generalize the univariate AR model by allowing for more than one evolving variable. All variables in a VAR enter the model in the same way. In essence, each variable has an equation explaining its evolution based on its own lagged values, the lagged values of the other variables, and an error term. VAR model does not require as much knowledge about the forces influencing a variable as do structural models with simultaneous equations. The only prior knowledge required is a list of variables that can be hypothesized to affect each other intertemporally. We use a Pth order VAR, denoted by VAR(p) or Y_t, as shown in Eq. (4).

$$Y_t = C + \prod_1 Y_{t-1} + \prod_2 Y_{t-2} + \ldots + \prod_p Y_{t-p} + \mu_t, t = 1, \ldots, T \tag{4}$$

where $Y_t = \left(Y_{1,t}, Y_{2,t}, \ldots, Y_{n-t}\right)'$is a $k \times 1$ vector of time series variables,

$\prod_i$ are $k \times k$ matrices of coefficients, C is a $k \times 1$ vector of constants, and $\mu_t$is a k

$\times$ 1unobservable white noise vector process.$E(\mu_t)$

$= 0, E\left(\mu_t \mu_{t-k}'\right) = 0$ for any non−zero k.

VAR models have three advantages: (a) they are simple - the econometrician does not have to worry about which variables are endogenous or exogenous; (b) estimation is also very simple, in the sense that each Equation can be estimated separately with the usual OLS method; (c) forecasts obtained from VAR models are in most cases better than those obtained from the far more complex simultaneous equation models. Therefore, a VAR is a systems regression model (i.e., there is more than one dependent variable).

Finally, in order to understand the causal forces between our variables, we search for variables that can "explain" or "cause" other variables. In many realistic economic situations, one suspects that feedback and causality are occurring simultaneously. The causality test by Granger (1969) method has been used extensively to test the direction of causality between variables. This method, which is explained in Eqs. (5) and (6), provides a useful way of describing the relationship between two (or more) variables when one is causing the other(s). After establishing that the data is stationary, we conduct unit root tests using Augmented Dickey-Fuller Tests. Based on the results of unit root tests, we then test for cointegration using Johansen's (1991) methodology. Estrada (2017) found that there exists a bidirectional Granger-causality relationship between BTC realized volatility and the VIX at the 5% significance level.

Let X_t and Y_t be two stationary time series with zero mean.A simple causal model is

$$X_t = \sum_{j=0}^{m} a_j X_{t-j} + \sum_{j=0}^{m} b_j Y_{t-j} + \varepsilon_t, \tag{5}$$

$$Y_t = \sum_{j=0}^{m} c_j X_{t-j} + \sum_{j=0}^{m} d_j Y_{t-j} + \eta_t \tag{6}$$

where ε_t, η_t are two uncorrelated white−noise series, i.e.,

$$E[\varepsilon_t \varepsilon_s] = 0 = E[\eta_t \eta_s], s \neq t, \text{and } E[\varepsilon_t \varepsilon_s] = 0 \text{ for all } s \text{ and } t.$$

The definition of causality in Eqs. (5) and (6) imply that Y_t is causing X_t, provided some b_j is not zero. Similarly, X_t is causing Y_t, provided some c_j is not zero. If both events occur, there is said to be a feedback relationship between X_t and Y_t.

4 Results

In this section, we summarize the results based on the methodology described in Section 3.2 in three distinct subsections. First, we develop an out-of-sample model to explain the BTC returns based on training data and well-established time series methods (i.e., GARCH and ARMAX). Then, using the model returns, we forecast BTC returns and prices and validate them against the actuals. Next, we repeat the process to model and validate BTC volatility. Finally, we provide evidence of causality between cryptocurrency returns and other financial variables in our study.

4.1 Estimate and forecast of BTC daily returns and prices

In Table (3), we provide results of the stepwise regression using Eq. (1). We find that MSCI World daily return is the only statistically significant variable at a 95% confidence interval that can explain daily BTC returns, albeit at a very low adjusted R-square of 0.2%. Since we show that the stepwise regression method does not offer much help in explaining daily BTC returns, we then move on to employ time series forecasting methods to yield better forecasts.

Since a reliable forecast is based on the time series variables being stationary (Hamilton 1994), we check for stationarity of variables before proceeding with the time series analysis. As expected, all-time series variables in this study (i.e., BTC, XRP, S&P500, Gold, MSCI-W, and MSCI-EM) are non-stationary at level and stationary at the first difference (i.e., $I(1)$ series) at 5% significance level. However, returns of all variables are stationary at the 5% significance level. Also, BTC returns contain both the autoregressive (AR) and moving average (MA) terms up to three lags. Consequently, we use an ARMAX model, as shown in Eq. (2), containing AR, MA, and explanatory variables (X) guided by economic theory (ex: MSCI World) to forecast daily BTC returns.

Table 3 Backward stepwise regression results of BTC daily returns. The only significant independent variable is shown in this Table. Dependent variable: BITCOIN_R. The full dataset from 08/01/2013 to 01/01/2019 with 1360 observations are used in this regression

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MSCI_W_R	0.523	0.198	2.639	**0.008**
R-squared	0.002		SE of regression	0.051
Adjusted R-squared	0.002		Durbin-Watson stat	1.95

We split the available data into two groups: data from 08/01/2013 to 10/31/2018 are used for training (ARMAX model estimation), and data from 11/01/2018 to 01/01/2019 are used for the test (ARMAX model forecast). We chose 10/31/2018 as a cutoff date so that we can produce BTC return forecasts for two months.

Compared to a multiple regression model, ARMAX (3,3,1) model is a better predictor of BTC returns, as shown in Table (4). We chose the (3,3,1) model since it produces the highest log-likelihood value and significant *p*-values after experimenting with different combinations of p and q in the ARMAX (p, q, b) model. All the variables shown are statistically significant, indicating that we have arrived at a good forecasting model for the given training data.

BTC prices are estimated using all three methods (i.e., simple regression, GARCH(1,1), and ARMAX(3,3,1)). The absolute differences between the forecast and the actual price are shown in Fig. (4) as a stacked chart. ARMAX method, as shown in Eq. (2), has performed well at forecasting BTC prices, closely followed by the GARCH method, leaving the regression method as the least accurate. The sum of squared error (SSE) for the ARMAX is 94.7% of the SSE of the regression model. Similarly, SSE for the GARCH is 96.1% of the regression model's SSE. So, ARMAX and GARCH models provide 5.3% and 4.9% better explanation of cryptocurrency returns compared to the standard regression models. As expected, the forecasted returns are smoother compared to the actual since it is difficult to forecast inflection points in any time series estimation. However, as shown in Fig. (5), the forecast of the next-day BTC price, based on the estimated ARMAX model return, is very close to the actual BTC price. We try to demonstrate that the BTC price crash between 11/01/2018 and 01/01/2019, during which the BTC price lost more than 50% of the value, can be successfully explained using these time series techniques.

4.2 Estimate and forecast of BTC volatility of daily returns

The conditional variance of financial time series is important for pricing derivatives, calculating measures of risk, and hedging. Conditional variance has sparked enormous

Table 4 BTC Return model estimates based on the ARMAX (3,3,1) model. The economic variable is shown first, followed by AR and MA components. Dependent Variable: BITCOIN_R, Method: ARMA Maximum Likelihood. Training dataset from 08/01/2013 to 10/31/2018, observations: 1323, convergence is achieved after 111 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MSCI_W_R	0.488	0.223	2.184	0.0291
AR (1)	0.783	0.156	5.025	0.0000
AR (2)	(0.421)	0.193	(2.184)	0.0029
AR (3)	0.470	0.152	3.105	0.0019
MA (1)	(0.770)	0.164	(4.692)	0.0000
MA (2)	0.387	0.201	1.920	0.0551
MA (3)	(0.374)	0.155	(2.414)	0.0159
SIGMASQ	0.003	0.0001	47.240	0.0000
Log-likelihood	2076.19		Prob(F-statistic)	0.0001

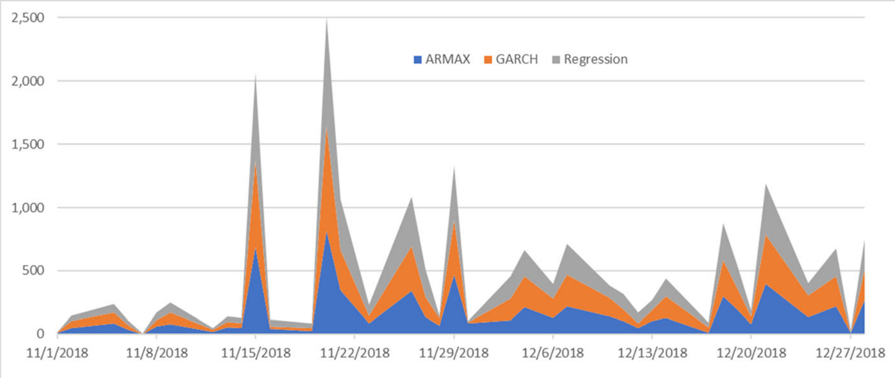

Fig. 4 Difference in the forecast and actual BTC daily price from 11/01/18 to 01/01/19. Forecasts from three estimation methods (simple regression, GARCH (1,1), and ARMAX (3,3,1) are subtracted from the actual daily price. The absolute difference (error in the forecast) is shown below. ARMAX method has the lowest error (i.e., best estimation) of all three methods

interest, and a large number of volatility models have been developed since the seminal paper of Engle (1982). See Poon and Granger (2003) for an extensive review of 93 ARCH models used in volatility forecasting. After comparing 330 ARCH-type models in terms of their ability to describe the conditional variance, Hansen and Lunde (2005) found no evidence that a GARCH(1,1) is outperformed by more sophisticated models. Also, Katsiampa (2017) has found that the GARCH family of models is optimal in terms of goodness-of-fit to the BTC data. So, we use the GARCH(1,1) model for BTC daily return volatility estimation and forecast.

We augment the GARCH(1,1) volatility forecast of Eq. (3) by the ARMAX(1,1,0) model of Eq. (2) since the ARMAX model was found to be effective in forecasting

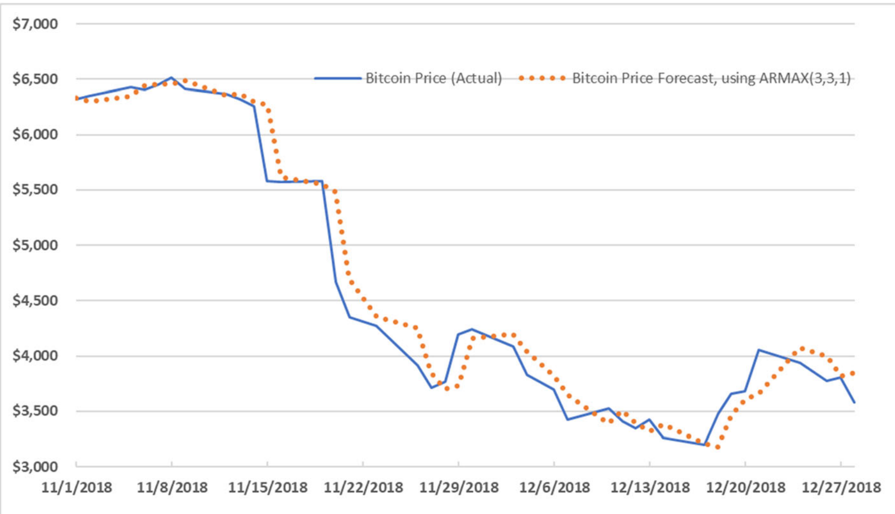

Fig. 5 Actual and forecasted BTC Price from 11/01/18 to 01/01/19. In this figure, BTC daily price forecast, in the dotted line, from the best estimation method (i.e., ARMAX (3,3,1)) is compared with the actual BTC daily price in solid line. The estimates closely mimic actuals

Table 5 BTC volatility forecast based on the GARCH (1,1) model. In this table, BVOL is the dependent variable. ML ARCH method is used for estimation. Data from 08/01/2013 to 10/31/2018 is used for training, with 1323 observations. Convergence is achieved after 78 iterations: GARCH = C (1) + C (2) *RESID (−1) ^2 + C (3) *GARCH (−1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
MSCI_W_R	0.430	0.146	2.955	0.003
AR(1)	1.029	0.148	6.945	0.000
AR(2)	(0.789)	0.127	(6.191)	0.000
AR(3)	0.113	0.033	3.464	0.001
MA(1)	(0.967)	0.143	(6.741)	0.000
MA(2)	0.713	0.120	5.944	0.000
Variance Equation				
C	0.000	0.000	8.326	0.000
RESID(−1)^2	0.116	0.012	9.383	0.000
GARCH(−1)	0.850	0.014	62.957	0.000

BTC returns in the previous section. The details of volatility forecasting of cryptocurrencies (BVOL) are previously described in Section 3.2. The results of the GARCH (1,1) model are shown in Table (5) and of the ARMAX (1,1,0) model in Table (6). We use the data from 08/01/2013 to 10/31/2018 for training the BVOL estimation model and data from 11/01/2018 to 01/01/2019 for the test (BVOL model forecast).

While analyzing the volatility of BTC's daily returns, we notice that the volatility of BTC's daily returns is clustered, as indicated by the GARCH coefficient of 0.850 in Table (5). A high GARCH coefficient (larger than 0.7) signals that large values of $\sigma^2_{BVOL(t-1)}$ are followed by large values of $\sigma^2_{BVOL(t)}$, and small values of $\sigma^2_{BVOL(t-1)}$ are followed by small values of $\sigma^2_{BVOL(t)}$. The high GARCH coefficient indicates evidence of volatility clustering in the Cryptocurrency asset class. Clustered volatility and ARCH effects are ubiquitous in financial data (Lux and Marchesi 2000). Other researchers, such as Chu et al. (2017), also found evidence of volatility clustering in Cryptocurrencies. The ARMAX (1,1,0) model results shown in Table (6) show a high

Table 6 BTC volatility forecast based on the ARMAX (1,1,0) model. In this table, BVOL is the dependent variable. ARMA Maximum Likelihood method is used for estimation. Data from 08/01/2013 to 10/31/2018 is used for training, with 1323 observations. Convergence is achieved after 14 iterations.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR (1)	0.997	0.0016	612.27	0.0000
MA (1)	0.209	0.0214	9.75	0.0000
SIGMASQ	0.003	0.0004	67.15	0.0000
Log-likelihood	1909.23		Prob(F-statistic)	0.0001
Adjusted R^2	98.4%		Durbin-Watson Stat.	1.982

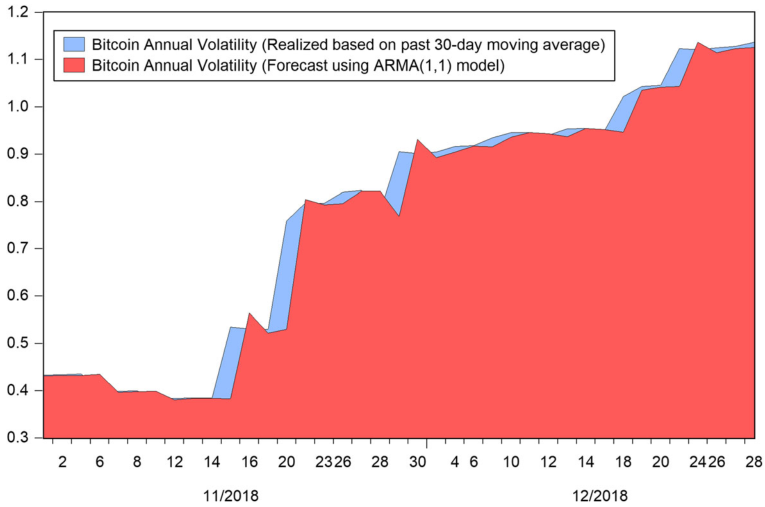

Fig. 6 Actual and forecasted BTC annualized volatility from 11/01/18 to 01/01/19. In this figure, BTC annualized volatility forecast based on the ARMAX (1,1,0) in red color is compared with the realized BTC annual volatility in blue color. The estimates closely mimic actuals

adjusted R^2 of 98.4%. So, it is used to forecast BVOL. The actual and forecasted annualized BVOL are shown as a daily chart in Fig. (6). Our model forecasts both the direction as well as the magnitude of the BTC volatility.

4.3 Results of Granger causality tests between BTC daily returns and other variables

The results from the VAR model, which can uncover simultaneous interdependencies between variables, as specified in Eqs. (4), (5), and (6), are shown in Table (7). We only include results for the one-lag model since it produced the best fit[14] with the training data from 08/01/2013 to 10/31/2018. We observe in these results that the cryptocurrency returns may be interdependent—the previous day's XRP returns have a statistically significant and positive impact (i.e., t-stat of 11.5) on the current day's BTC returns. BTC return is not influenced by the S&P500 market returns. Our findings are similar to Dyhrberg (2016b), who found that the return on BTC is not affected by the changes in the stock market. Thus, BTC creates a possibility for investors to hedge some of the market risks. We show that BTC does not exhibit a safe-haven property— investors are not rushing to trade BTCs when the stock market turns volatile. Also, our results show that BTC return has no impact on the S&P500 market return, possibly because BTC is too small to make a directional impact on the S&P500. Finally, we show that the previous day's BTC return has a negative and significant impact on the current day's gold return. Our finding is similar to that of Klein et al. (2018) who found that while gold prices increase in the flight-to-quality, BTC prices are decreasing with the markets.

Our last step in analysis deals with determining the direction of causality, if any, between returns and volatilities of all variables in our study. Granger causality test results, as shown in Table (8) reveal the following:

[14] VAR model order selection: https://sccn.ucsd.edu/wiki/Chapter_3.5._Model_order_selection

Table 7 VAR (1) model estimation of linkages between lagged returns. In this table, linkages between the current day's daily returns and the previous day (one lag) returns are shown. All seven return variables, standard errors and t-statistics are included for BTC, XRP, Gold, S&P500, MSCI Emerging Markets, MSCI World, and EUI. The total number of observations is 1359

	BITCOIN_R	XRP_R	GOLD_R	SP500_R	MSCI_EM_R	MSCI_W_R	EUI_30DMA_R	
BITCOIN_R(−1)	0.021	0.054	(0.010)	(0.002)	0.001	(0.001)	(0.012)	<-Coefficient
	(0.026)	(0.048)	(0.005)	(0.004)	(0.004)	(0.004)	(0.017)	<-StdErr
	0.791	1.116	(2.063)	(0.506)	0.241	(0.282)	(0.675)	<−t-statistics
XRP_R(−1)	0.167	0.119	(0.001)	0.003	(0.002)	0.001	(0.009)	<-Coefficient
	(0.015)	(0.027)	(0.003)	(0.002)	(0.002)	(0.002)	(0.010)	<-StdErr
	11.523	4.411	(0.434)	1.254	(0.002)	0.607	(0.886)	<−t-statistics
GOLD_R(−1)	0.090	0.453	(0.020)	(0.015)	0.073	(0.005)	0.088	<-Coefficient
	(0.155)	(0.287)	(0.027)	(0.026)	(0.026)	(0.022)	(0.102)	<-StdErr
	0.578	1.579	(0.738)	(0.563)	2.786	(0.213)	0.858	<−t-statistics
SP500_R(−1)	0.480	(1.176)	0.064	(0.098)	0.283	0.125	(0.082)	<-Coefficient
	(0.463)	(0.858)	(0.082)	(0.078)	(0.079)	(0.065)	(0.306)	<-StdErr
	1.038	(1.370)	0.784	(1.252)	3.596	1.919	(0.268)	<−t-statistics
MSCI_EM_R(−1)	(0.059)	(0.459)	0.079	0.021	(0.006)	0.007	(0.043)	<-Coefficient
	(0.217)	(0.403)	(0.039)	(0.037)	(0.037)	(0.031)	(0.144)	<-StdErr
	(0.273)	(1.138)	2.041	0.585	(0.156)	0.240	(0.299)	<−t-statistics
MSCI_W_R(−1)	(0.383)	1.625	(0.124)	0.104	0.177	0.023	(0.350)	<-Coefficient
	(0.633)	(1.175)	(0.112)	(0.107)	(0.108)	(0.089)	(0.418)	<-StdErr
	(0.605)	1.383	(1.106)	0.972	1.642	0.258	(0.836)	<−t-statistics
EUI_30DMA_R(−1)	0.042	0.019	0.003	0.006	0.013	0.007	0.222	<-Coefficient
	(0.040)	(0.074)	(0.007)	(0.007)	(0.007)	(0.006)	(0.026)	<-StdErr
	1.049	0.253	0.414	0.925	1.937	1.258	8.411	<−t-statistics
C	0.002	0.003	0.000	0.000	(0.000)	0.000	0.000	<-Coefficient
	(0.001)	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	<-StdErr
	1.513	1.146	0.118	1.285	(0.409)	0.576	0.483	<−t-statistics

Table 8 Pairwise Granger Causality test results. In the table below, only statistically significant results (i.e., *p* value <5%) are reported. The total number of observations is 1351. Three variables (GVOL, VIX, and XRP daily returns) have a Granger causal effect on BTC daily returns. Whereas, BTC daily returns appear to have Granger causal effect on gold daily returns and volatility)

Granger Cause	Effect	F-Statistic	Prob.
GVOL	BITCOIN_R	5.2	**0.02**
VIX	BITCOIN_R	5.0	**0.03**
XRP_R	BITCOIN_R	133.8	**0.00**
XRP_R	BVOL	17.8	**0.00**
XVOL	BVOL	8.6	**0.00**
BITCOIN_R	GOLD_R	4.5	**0.03**
BITCOIN_R	GVOL	5.9	**0.02**
VIX	SP500_R	4.2	**0.04**
VIX	XVOL	6.3	**0.01**
XRP_R	XVOL	28.9	**0.00**

- BTC returns are impacted by GVOL, VIX, and XRP_R
- Returns on XRP Granger cause both returns on BTCs and volatility of BTCs
- Returns on BTCs Granger cause return on gold prices
- Bidirectional causality exists between GVOL and returns on BTCs
- VIX Granger causes return on BTC and volatility of XRP.

An important finding of this causality test suggests that the main drivers behind BTC prices are some measures of fear (as proxied by GVOL and VIX) and other cryptocurrency returns (as proxied by XRP). We also find that gold returns and volatilities are influenced by the BTC returns. This finding agrees with Klein et al. (2018), who found that BTCs do not reflect any distinctive properties of gold other than the asymmetric response in variance. It appears that BTC is showing signs of a global asset (as opposed to a country-specific asset) since its returns are influenced by the MSCI World stock returns and other global events that normally influence the VIX and GVOL. Recent studies have tried to unearth the BTC and other cryptocurrency price changes without much economic intuition using machine learning (McNally et al. 2018; Alessandretti et al. 2018) and neural network autoregression (NNAR) models (Munim et al. 2019).

Even though XRP is a smaller cryptocurrency than the BTC, its returns and volatilities influence returns and volatilities of BTCs. This result is surprising, as one would expect the direction of causality to be the opposite, given the larger market capitalization of BTCs. The stock market volatility, as measured by VIX, Granger causes the volatility of XRP (but not BTCs). This result is expected since smaller cryptocurrencies are more sensitive to stock market volatility. Finally, the stock market returns of the MSCI Emerging Market Index, the MSCI World Index, and the SP500 do not seem to influence either the return or volatilities of cryptocurrencies.

5 Conclusion

In this paper, we conduct time series analysis of the returns and volatilities of BTC, XRP (XRP), Stock markets (S&P 500 index, MSCI World Index, MSCI Emerging Markets Index), gold prices, and fear gauges such as the VIX and the US Economic Policy Uncertainty Index. We use the Autoregressive-moving-average model with exogenous inputs model (ARMAX), Generalized Autoregressive conditionally heteroscedastic (GARCH) model, Vector autoregression (VAR) model, and Granger causality tests to determine linkages between returns and volatilities.

Our findings suggest (a) the BTC crash of 2018 could have been modeled and explained accurately using these methods; (b) forecasts of both the direction and the magnitude of BTC volatility are more precise than those of forecasted BTC returns; (c) returns of global stock markets and gold do not have a causal effect on BTC returns. However, smaller cryptocurrencies (ex: XRP) are more sensitive to gold prices and general stock market volatility, and can have a causal effect on BTC prices; (d) primary factors influencing BTC prices are some measures of fear (as proxied by GVOL and VIX), and other cryptocurrency returns (as proxied by XRP).

Future research in the area of cryptocurrency returns and volatility can cover several other variables in different markets and different regimes. Besides, it is still not clear how central monetary authorities of various countries will regulate (if at all) or monitor the emerging cryptocurrency markets and their consequent impact on their domestic economic environment. Investors of all asset classes need to be aware of the linkages between the emerging group of cryptocurrencies and the traditional asset classes such as stocks and bonds.

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