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Can cryptocurrencies be a safe haven: a tail risk perspective analysis

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ABSTRACT
Cryptocurrencies are one of the most promising financial innovations of the last decade. Different from major stock indices and the commodities of gold and crude oil, the cryptocurrencies exhibit some characteristics of immature market assets, such as auto-correlated and non-stationary return series, higher volatility, and higher tail risks measured by conditional Value at Risk (VaR) and conditional expected shortfall (ES). Using an extreme-value-theory-based method, we evaluate the extreme characteristics of seven representative cryptocurrencies during 08 August 2015–01 August 2017. We find that during the sub-period of 01 August 2016–01 August 2017, there are finite loss boundaries for most of the selected cryptocurrencies, which are similar to the commodities, and different from the stock indices. Meanwhile, we find that left tail correlations are much stronger than right tail correlations among the cryptocurrencies, and tail correlations increased after August 2016, suggesting high and growing systematic extreme risks. We also find that cryptocurrencies to be both left tail independent, and cross tail independent with four selected stock indices, which implies part of the safe-haven function of the cryptocurrencies, indicating their ability to be a great diversifier for the stock market as gold, but not enough to be a tail hedging tool like gold.

KEYWORDS
Cryptocurrency; tail risk; tail correlation; diversification; hedge

JEL CLASSIFICATION
C13; C22; G10; G11; G15

I. Introduction

Cryptocurrencies, as a main subset of digital currencies, are a remarkable financial and technological innovation of the last decade, and have been attracting lots of attentions from investors, criminals, as well as regulatory authorities worldwide. At the time of 18 February 2018, the total market capitalization of cryptocurrencies is more than 500 billion. The first and largest capped cryptocurrency is Bitcoin. Since its invention by Satoshi Nakamoto in 2008, Bitcoin has become more and more widely accepted as an alternative payment method by many retailers and internet companies, such as Subway, Microsoft, and Expedia. The most important distinctions of cryptocurrencies from the traditional flat currencies or traditional financial assets are their decentralization property and transaction anonymity based on the blockchain technology, as well as their roller-coaster-ride like price movement and high risks.

Previous financial academic researches on cryptocurrencies mostly concentrate on the fair value of Bitcoin and bubble existence in this market (Cheung, Roca, and Su 2015; Cheah and Fry 2015), the main drivers of Bitcoin return and volatility (Kristoufek 2015; Bouoiyour et al. 2016; Polasik et al. 2015; Ciaian, Rajcaniova, and Kancs 2016; Bouri, Azzi, and Dyhrberg 2017; Balcilar et al. 2017), and the efficiency of the Bitcoin market (Urquhart 2016; Bartos 2015). However, different researchers hold different opinions on these topics and haven't reached a consensus.

If cryptocurrencies are considered as a new kind of investment assets or alternative assets, two topics that investors care about are: first, the investment risks, and second, their relationship with other assets, most importantly, the diversifying and hedging ability of them. According to the modern portfolio theory (Markowitz 1952), exposure to individual asset risks can be reduced by holding a diversified portfolio of assets; the less correlated (or more independent) the assets are, the lower systematic risk the portfolio has, and the better diversified the portfolio is. In practice, a common way to diversify the portfolio is international diversification (Heston and Rouwenhorst 1994), simply by holding global stocks. However, as the global stock market still share some common risk factors and stock crashes are contagious, this

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diversification could not well remove the systematic extreme risks. Meanwhile, hedging is also important in risk management. A hedge is a position expected to offset potential losses of a companion investment. For example, some hedge funds keep buying cheap deep out-of money put option to hedge against black swan events.

In this article, we concentrate on three problems about cryptocurrencies, from the perspective of extreme risks or equivalently, tail risks. The first problem is to assess the tail risks of cryptocurrencies; the second problem is to evaluate the tail correlations and hence the extreme systematic risks inside the cryptocurrency market; the third problem is to estimate the tail diversifying and tail hedging ability of the cryptocurrencies.

In asset or portfolio risk management, we often concentrate on the tail, namely the extreme quantiles of the return distribution, and analyse how much could we lose under extreme circumstances (for example, the 10% worst cases). As sudden and extreme price drops are very common in the cryptocurrency market (Figure 1), the potential risks behind cryptocurrencies are a topic that the investors and the regulatory authorities care about.

We evaluate the extreme risks of seven representative cryptocurrencies, that is, Bitcoin, Ethereum, Ripple, Litecoin, Dash, NEM and Monero. We adopt the method proposed by McNeil and Frey (2000) and fit the generalized Pareto distribution (GPD) to the innovation in the ARMA-GARCH model. Compared to GPD fitting for raw data, this method is robust to deal with data with autocorrelation and heteroscedasticity (McNeil and Frey 2000), which is the case for the cryptocurrencies (Gronwald 2014; Urquhart 2016). In this way, we can study the real tail risks of the innovations rather than the price movement caused by dynamic variance. We estimate the tail shape of each of the selected cryptocurrencies. Meanwhile, to quantitatively evaluate the extreme losses, we also calculate the dynamic conditional VaR, ES, and extremal boundary series implied by the extreme value model, for the selected cryptocurrencies.

We test the systematic extreme risks in the cryptocurrency market, as well as the cryptocurrencies' ability to diversify and hedge traditional assets' risks under extremal circumstances, using the tail correlation estimation. The tail correlation or tail dependency measures the extremal co-movement between multiple assets or asset classes. Briefly speaking,

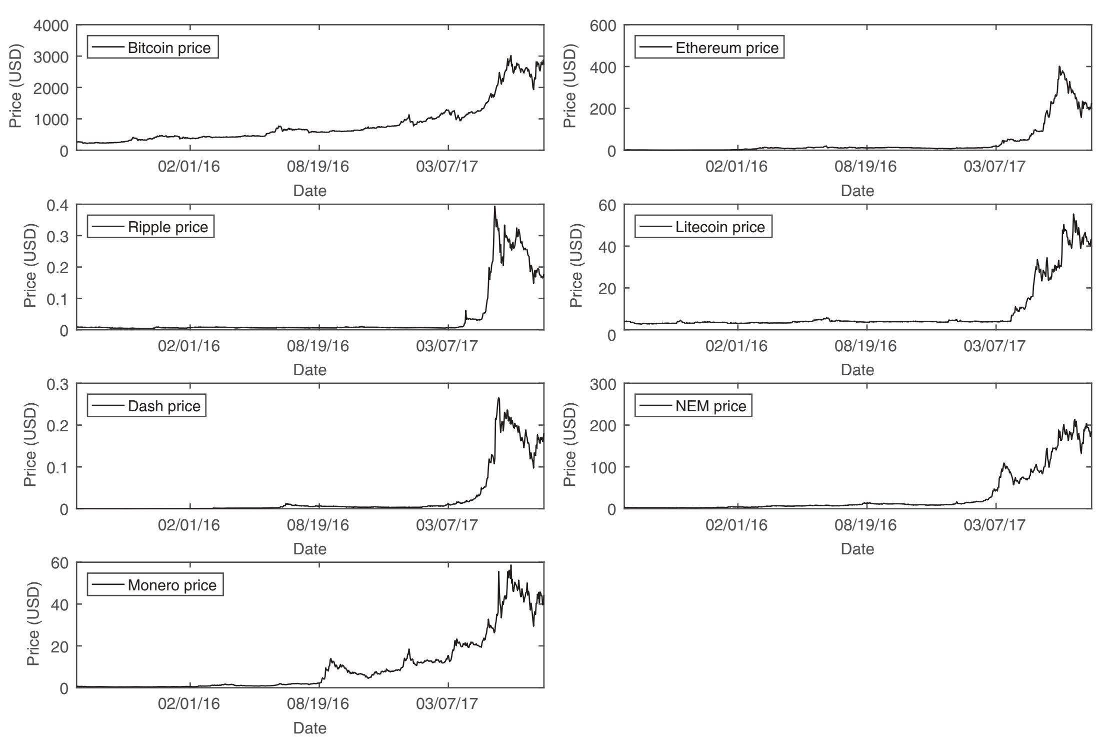

Figure 1. Prices of seven selected cryptocurrencies (Bitcoin, Ethereum, Ripple, Litecoin, Dash, NEM, Monero), 08/08/2015–08/01/2017.

when one asset experience large depreciation, is another asset more likely to experience large depreciation, large appreciation, or more likely to show no extremal price changes? The tail correlation analysis is useful not only for studying the extreme systematic risks in the market, but also for managing a portfolio and diversifying it to avoid devastating losses in extreme circumstances, like financial crises. It is worthwhile to mention that tail correlation only measures the extreme co-movement and is a different concept from linear correlation. For example, for a bivariate normal distributed vector, the two components are always tail independent to each other, as long as their Pearson correlation $\rho \in [0, 1)$.

Different cryptocurrencies are exposed to some common extreme risk factors; meanwhile, cryptocurrencies and traditional assets are exposed to different extreme risk factors; moreover, they are substitutive investment assets for each other, at least to some extent. The stock markets are more exposed to macro economics factors, for example, macroeconomic (Cutler, Poterba, and Summers 1989), government fiscal or monetary policy (Chatziantoniou, Duffy, and Filis 2013), oil price shock (Sadorsky 1999), and investor sentiments (Fisher and Statman 2000); the investment commodities are also exposed to the macroeconomic factors and industry factors. However, the cryptocurrencies are pretty sensitive to government cryptocurrency policies, regulatory events, technological events, hacking events, and market events (Feng, Wang, and Zhang, forthcoming), as well as speculations (Cheah and Fry 2015). During the extremal events of the cryptocurrency market (or traditional financial markets), the investors may pull their money out and invest them in other substitutive markets, especially the markets which are free from the common extreme risk factors or contagious crashes, possibly leading to large appreciations in the substitutive markets.

Based on the above reasoning, the cryptocurrencies may be highly tail correlated to each other due to the similar extreme risk factors; but they may be tail independent to the traditional assets or even can hedge them under extremal circumstances, due to the independent extreme factors and substitutive characteristics. If the hypotheses of tail independence or tail hedging hold, cryptocurrencies could exhibit safe haven property for the traditional assets, to some extent.

To test these intuitive hypotheses, we use the logistic model to quantitatively analyse tail correlations among the cryptocurrencies, to measure the systematic risks in this market. Meanwhile, we choose six representative stock indices and commodity indices and measure the correlations between cryptocurrencies and these traditional assets. In this way, we estimate cryptocurrencies' tail diversifying and tail hedging ability for traditional assets.

These tail correlation analyses are very important issues before investing or adding cryptocurrencies to the portfolios which consist of traditional assets. If the cryptocurrencies are tail independent with the traditional assets, they can be a good diversifying asset for traditional portfolios consisting of global stock indices and commodities, and adding the cryptocurrencies to the traditional portfolio can moderate the extreme risks of the portfolio, as when the stock market crash comes, the cryptocurrencies may keep their values and are free from the contagious price drop; if significantly positive left tail correlations exist, then the cryptocurrencies and traditional assets are either exposed to the same extreme factors or they are rapidly contagious to each other during extremal events; if the cryptocurrencies' right tails are correlated with the traditional assets' left tails, which means that when traditional assets experience extreme price drops, cryptocurrencies were likely to appreciate at the same time, then cryptocurrencies could be a good hedging tool for the traditional assets, in the tail risk sense.

Our article relates to the literature on the extreme risks of cryptocurrencies. Gronwald (2014) used an autoregressive jump-intensity GARCH model and found that Bitcoin prices are marked by extreme price movements. Osterrieder and Lorenz (2017) used both GPD and generalized extreme value (GEV) distribution to fit Bitcoin return data and analyse its risks. Osterrieder, Strika, and Lorenz (2017) computed the tail dependency among several cryptocurrencies using two copula-based methods; they also used historical simulation and normal distributions to compute the static VaR and ES. Compared to Osterrieder and Lorenz (2017) and Osterrieder, Strika, and Lorenz (2017), our methods are robust and better deal with emerging and immature assets, such as cryptocurrencies, whose return data are not independently and identically distributed (i.i.d.); as both direct GEV or direct GPD fitting are good for i.i.d. cases. The dynamic conditional

VaR and ES we use, which are based on the extreme value models, can better describe the time-varying risks of the leptokurtic cryptocurrencies' returns, compared to the static risk indexes based on historical simulation or normal distribution.

Our article is also a part of a growing literature that researches cryptocurrencies' ability to serve as either a hedge or a diversifier for traditional financial assets, and our work complements the literature from the perspective of tail correlations. Bouri, Molnár et al. (2017) used the dynamic conditional correlation (DCC) model and found that Bitcoin is a good diversifier but a poor hedge for stock and bond indices. Bouri, Jalkh et al. (2017) also used the DCC model and found that Bitcoin can hedge against the general commodity indice, and for energy commodities, before the December 2013 crash, whereas in the post-crash period Bitcoin is no more than a diversifier for these two kinds of indices. Dyhrberg (2016) used an asymmetric GARCH method and found that Bitcoin can hedge against stocks in the Financial Times Stock Exchange Index, and that Bitcoin can also hedge the American Dollars in the short term. Bouri, Gupta et al. (2017) found that Bitcoin can hedge global uncertainty.

The main findings of this article can be summarized as follows.

First, we notice that, compared to the traditional assets, cryptocurrencies exhibit characteristics of emerging and immature assets. Using conditional VaR and ES to measure the tail risks, we find that the cryptocurrencies show much higher tail risks caused by high volatility; some cryptocurrencies are exposed to autocorrelated return series; different time trends, different tail shapes stand for different periods; tail correlations in the market are also time-varying. Dividing the whole sample approximately into two equal-length sub-periods (1st: 08 August 2015–31 July 2016; 2nd: 01 August 2016–01 August 2017), we find that statistically significant tail shape changes between the two sub-periods, for almost all the selected cryptocurrencies. Some of the cryptocurrencies' tails turn from Fréchet type (long and heavy tail) to Weibull type (truncated tail), or the other way around. However, we find there are finite boundaries of losses for most of the selected cryptocurrencies in the latter period, which is similar to the gold and crude oil commodity, and different from the four selected stock indices. Moreover, significant asymmetry commonly exists between the tail shape of right tails and left tails, for the cryptocurrencies.

Second, we find that statistical significant left tail dependency exists for most of the cryptocurrency pairs, while only few pairs are right tail correlated. The results signify that, when one of the cryptocurrencies suffers from bad extremal events, other cryptocurrencies are exposed to simultaneous sharp drops in price. This kind of tail dependence leads to systematic risks in the cryptocurrency market. However, when one of the cryptocurrencies experience large appreciation, few other cryptocurrencies will experience large appreciation at the same time. Moreover, we find that the tail correlations in the cryotocurrency market increase in the latter period (01 August 2016–01 August 2017), indicating the rising market systematic extreme risks. Overall, Bitcoin has the highest average tail correlation with other cryptocurrencies. One possible explanation is the leading and fundamental role of Bitcoin in the cryptocurrency market.

Third, we find that in the tail sense, cryptocurrencies can serve as great diversification assets for stocks, like gold do; but cryptocurrencies cannot serve as good hedging tools for the stocks as gold. We test the bivariate left tail dependence between Bitcoin, Ethereum and six selected stock and commodity indices. We find that both Bitcoin and Ethereum's left tails are statistically tail independent with all these six indices' left tails, which implies part of the cryptocurrencies' safe-haven property. The estimation results show that, a large shortfall in one stock market implies the higher probability of shortfall for foreign stock market, but not for the cryptocurrencies. However, although cryptocurrencies can serve as a good diversifier for stock indices, they cannot hedge the tail risk of the stock market, as when we test the left-right cross tail dependency between the traditional assets and cryptocurrencies, they are uncorrelated; that is to say, large shortfalls in the stock market does not necessarily mean simultaneous cryptocurrencies' large appreciation, and vice versa. These conclusions agree with Bouri, Molnár et al. (2017), who used the DCC model and drove to similar conclusions, although their work is for the whole sample, and ours is in the tail or extreme value sense.

Our findings are important to investors who consider adding cryptocurrencies to their portfolio and hedge funds which needs to diversify their portfolio and hedge the extreme risks. Our work is also meaningful to financial practitioners and regulators who are concerned about the extreme risks of cryptocurrencies.

The rest of the article is organized as follows. Section II draws the theoretical framework. Section III gives an overview of the data we use. Section IV demonstrates the empirical result. Section V concludes.

II. Theoretical framework

We use the method proposed by McNeil and Frey (2000) to analyse the univariate tail characteristics of cryptocurrencies, and then we use the logistic function to model and estimate the tail dependency among the cryptocurrencies or the assets we are interested in.

Measure the tail risk of a single cryptocurrency

Cryptocurrency return series are exposed to autocorrelation and stochastic volatility (Table 2). Accordingly, we use the method proposed by McNeil and Frey (2000) to assess the tail risk of cryptocurrencies' returns. This approach combines extreme value theory with ARMA-GARCH model, using pseudo-maximum-likelihood (PML) approach to fit the ARMA-GARCH model, and get the pseudo return series, which is the ARMA residual divided by estimated GARCH volatility; the next step is using GPD to fit the pseudo returns. In this way, we can remove the effects of autocorrelation and time-varying variance, and analyse the tail risk in the innovations. Further, we can get the dynamic conditional VaR and conditional ES (conditioning on $\mathcal{F}_{t-1}$ and measure the risk at time t), which could better describe dynamic risks.

Specifically, let X_t denote the negative log returns (if we are concerned about large losses) or log returns (if we are concerned about large appreciations) of a cryptocurrency. Let $F_X(x)$ denote the distribution function of X_t. Assuming the following GARCH model can describe the dynamic variance of X_t,[1]

$$X_t = \mu_t + \varepsilon_t, \tag{1}$$

$$\varepsilon_t = v_t Z_t, \tag{2}$$

$$v_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i v_{t-i}^2, \tag{3}$$

where μ_t is the mean return and may consists of ARMA terms; lags of ARMA terms should be selected to effectively remove autocorrelation[2]; if no autocorrelation exists, we can simply use a constant. $Z_t \sim F_z(z)$ is a white noise process, which has zero mean and unit variance. We first use PML to estimate the model (1)-(3). That is, we estimate the model using maximum likelihood estimation (MLE) but assuming Z_t to be i.i.d. normal series. Gourieroux, Monfort, and Trognon (1984) proved that if the real distribution of Z_t belongs to the family of linear exponential family, the estimators of PML are consistent and asymptoticly normal. After fitting the ARMA-GARCH model, we can get the estimated volatility $\{\hat{v}_t\}$ and pseudo return series $\left\{\hat{z}_t = (x_t - \hat{\mu}_t)/\hat{v}_t\right\}$.

Next, we use GPD to model the tail of the pseudo return series Z_t. Define the conditional excess distribution function of Z_t to be

$$F_{Z,u}(y) = P\left\{Z - u \le y | Z > u\right\} = \frac{F(y+u) - F(u)}{1 - F(u)}, \tag{4}$$

where $0 \le y \le z_F - u$ and z_F is the supremum of the support of $F_Z(z)$. For the tail part, namely for $z > u$, we have

$$1 - F_Z(z) = (1 - F_{Z,u}(z-u))(1 - F_Z(u)). \tag{5}$$

For the first part of the right hand of (5), we can use GPD to approximate $F_{Z,u}(z-u)$, as the famous Pickands-Balkema-de Haan theorem pointed out that when $u \to Z_F$, $F_{Z,u}(y)$ can be approximated by GPD:

$$\lim_{u \to z_F} \sup_{0 \le y \le z_F - u} \left|F_{Z,u}(y) - G_{\xi,\sigma}(y)\right| = 0, \tag{6}$$

where $G_{\xi,\sigma}(y)$ is the GPD function, described by the shape parameter ξ and scale parameter σ:

$$G_{\xi,\sigma}(y) = \begin{cases} 1 - (1 + \xi y/\sigma)^{-1/\xi} \text{ if } \xi \neq 0, \\ 1 - \exp(-y/\sigma) \text{ if } \xi = 0, \end{cases} \tag{7}$$

[1]Usually GARCH(1,1) is good enough and is a robust model; more ARCH or GARCH terms can be added if necessary.

[2]If the price movement exhibit apparent different patterns over different periods, we have to divide the sample into two or more sub periods and analyse them separately, otherwise serial correlation may exist even if including ARMA terms into the model.

where ξ is the shape parameter of GPD. A positive ξ means a long and heavy tail distribution with domain $y \geq 0$; the bigger ξ is, the heavier the tail and the higher probability of tail events. For exponential decaying function like normal and log normal, $\xi = 0$. A negative ξ means a truncated tail, and the domain is $0 \leq y \leq -\sigma/\xi$. We can use MLE to get the estimator of $\hat{\xi}$ and $\hat{\sigma}$, the consistency and asymptotic normality of the estimators are supported by Smith (1987); and then we use $G_{\hat{\xi},\hat{\sigma}}(y)$ to approximate $F_{Z,u}(z-u)$.

For the second part of the right hand of (5), we can use the portion of tail samples to approximate it, namely $\#(Z \leq u)/N$. In practice, the choice of u is a trade-off between bias and estimation variance. There are plenty of discussions on the threshold choice (e.g. Yang 1978; Smith 1987; Davison and Smith 1990). In this article, we follow a common choice in the literature (e.g. DuMouchel 1983; Chavez-Demoulin 1999) and choose u to be the 10% upper quantile of $F_X(x)$.

So far, we can estimate the tail of Z using

$$\hat{F}_Z(z) = 1 - 0.1 \times (1 + \hat{\xi}\frac{z-u}{\hat{\sigma}})^{-1/\hat{\xi}} \; (z > u), \tag{8}$$

where u is the upper 10% quantile of Z's samples.

Measure the tail correlation

What we are concerned about is the tail correlations among the cryptocurrencies, as well as the tail correlations between the cryptocurrencies and traditional financial assets, under extremal events. Specifically, the tail correlation (e.g. McNeil, Frey, and Embrechts 2015) of two stochastic variables is defined to be

$$\lambda_{i,j} = \lim_{q \to 1} P(Z_j \geq F_{Z_j}^{\leftarrow}(q) | Z_i \geq F_{Z_i}^{\leftarrow}(q)), \tag{9}$$

where $\lambda_{i,j} \in [0,1]$. When $\lambda_{i,j} = 0$, Z_i and Z_j are asymptotic independent or tail independent, otherwise they are asymptotic dependent or tail dependent.

To study the bivariate dependence structure, we need to convert the variables into the same scale and eliminate the effect of different marginal distributions. Following the common procedure of literature (e.g. Poon, Rockinger, and Tawn 2003), we make the Fréchet transformation of the (negative/positive) pseudo return series Z_t. Mathematically, for asset i, we make the following transformation:

$$S_i = -\frac{1}{logF_{Z_i}(Z_i)}, \tag{10}$$

and for notational convenience, here we drop the time subscript. In this way, the distribution function for S_i $(\forall i)$ is $F(s) = e^{-1/s}$, $s > 0$. And this transformation will not change the relative correlation of variables, as

$$\begin{aligned} P(q) = &Prob(F(S_j) \\ &\geq q \Big| F(S_i) \geq q) = Prob(Z_j \geq F_{Z_j}^{\leftarrow}(q) \Big| Z_i \\ &\geq F_{Z_i}^{\leftarrow}(q)). \end{aligned} \tag{11}$$

We could easily get that,

$$\lambda_{i,j} = \lim_{q \to 1} P(q) = \lim_{s \to \infty} Prob(S_j \geq s | S_i \geq s). \tag{12}$$

To estimate the bivariate correlation, we need to use a model to describe the joint distribution of the two variables. For simplicity, we will use the logistic model following Davison and Smith (1990), Ledford and Tawn (1997), Longin and Solnik. (2001), and Poon, Rockinger, and Tawn (2003). This model uses only one parameter to describe the tail dependence structure. Alternatively, there are many other tail correlation estimation methods in the literature, for example, the tail quotient correlation coefficient (TQCC) method proposed by Zhang (2008), and Zhang, Zhang, and Cui (2017).

Suppose that for any Z_i, its marginal distribution satisfy,

$$\begin{cases} Prob(Z_i < u_i) = F_{Z_i}(u_i) & \text{ifz } < \text{u}, \\ F_{Z_i}(z) = F_{Z_i}(u_i) + (1 - F_{Z_i}(u_i))G_{\hat{\xi}_i,\hat{\sigma}_i}(z-u_i) & \text{ifz } \geq \text{u}, \end{cases} \tag{13}$$

where $\hat{\xi}$ and $\hat{\sigma}$ are the estimated parameters of the univariate GPD model described in "Measure the tail risk of a single cryptocurrency". Because what we care about are the tail characteristics and tail correlations, we do not make assumptions about the non-tail part. Instead, we only assume that Z_i falls into the non-tail region with some certain probability and do not model the non-tail distribution. Following Ledford and Tawn (1997), Longin and Solnik (2001), and Poon, Rockinger, and Tawn (2003), we use the logistic function to connect the distribution of S_i and S_j, namely the Fréchet transformed Z_i and Z_j. Mathematically, assuming the joint distribution of (S_i, S_j) to be

$$Prob(S_i < s_i, S_j < s_j) = exp(-(s_i^{-1/\gamma_{i,j}} + s_j^{-1/\gamma_{i,j}})^{\gamma_{i,j}}), \quad (14)$$

using the definition of $\lambda_{i,j}$, we could easily prove that $\lambda_{i,j} = 2 - 2^{\gamma_{i,j}}$. Using (13) and (14), we could use MLE to get the estimated $\hat{\lambda}_{i,j}$.

If two or multiple assets are estimated to be left tail independent (the null hypothesis of $\lambda_{i,j} = 0$ cannot be rejected), when one asset suffer from extreme price drops, the other assets are immune to the same extreme risk factors; holding these tail independent assets together can well diversify the portfolio in regards to the tail risks. If multiple assets are left tail correlated ($\lambda_{i,j} > 0$ significantly), they are exposed to the same extreme factors or contagious to each other in the case of extremal events; holding these tail correlated assets can not diversify the portfolio in the tail sense; meanwhile, tail correlations in a market also lead to systematic tail risks, as many assets are exposed to extreme co-movement. If asset A's right tail is correlated with the asset B's left tail, then asset A can hedge asset B under asset B's extreme circumstances.

III. Data

We choose seven cryptocurrencies out of the top ten largest-capped cryptocurrencies (at the time of 02 August 2017), and use their daily price data for this research. Our data of cryptocurrencies are from www.coinmarketcap.com. The related data period is from 08/08/2015 to 08/01/2017.[3] The selected cryptocurrencies are: Bitcoin, Ethereum, Ripple, Litecoin, NEM, Dash and Monero. We omitted the other three top-capped cryptocurrencies, that is Etherem Classsic, IOTA and EOC, because they became trading cryptocurrencies on exchanges in July 2016, June 2017 and July 2017, respectively, and the price data are not enough for daily frequency statistical researches. Meanwhile, the selected seven cryptocurrencies came into the market no later than August 2015 and about 2-year price data are available. At the time of 02 August 2017, the seven selected cryptocurrencies' total market capitalization are over 78 billion, and they dominated over 78% of the whole cryptocurrency market's capitalization (Table 1). The large market capitalizations make price manipulation more difficult, making statistical researches more solid. Based on the above reasons, we believe the seven selected cryptocurrencies can well proxy the cryptocurrency market. Our selection is similar to Osterrieder, Strika, and Lorenz (2017).

Table 1. Basic information of the selected cryptocurrencies.

Rank	Name	Market Cap	Dominance	Circulating Supply	Volume (24 h)
1	Bitcoin	$44.78 Bn	44.55%	16.48 MM BTC	$1.13 Bn
2	Ethereum	$20.84 Bn	20.73%	93.74 MM ETH	$722.52 MM
3	Ripple	$6.72 Bn	6.68%	38.33 Bn XRP	$74.56 MM
5	Litecoin	$2.22 Bn	2.21%	52.29 MM LTC	$115.55 MM
6	NEM	$1.93 Bn	1.92%	9 Bn XEM	$14.08 MM
8	Dash	$1.37 Bn	1.36%	7.46 MM DASH	$21.6 MM
10	Monero	$0.65 Bn	0.65%	14.88 MM XMR	$10.39MM

This table shows some basic information of seven selected cryptocurrencies. The data in this table are from www.coinmarketcap.com (08/02/2017, 15:00, UTC-6). MM and Bn are short for million and billion. Dominance are measured by a cryptocurrency's market capitalization divided by the total cryptocurrency market capitalization.

Although the selected cryptocurrencies are exposed to some common risk factors, they have different advantages and characteristics, leading to different demand and sources of risks. Bitcoin is the first, most famous and largest-capped cryptocurrency; it is a preferred tool for cross-border transactions and blackmailing payment, for example, the notorious WannaCry ransomware attack in 2017 used Bitcoin as the only way to pay ransom. Ethereum is remarked as an archetype of 'Blockchain 2.0' for its programability, or in other words, smart contract; it enables multiple parties to freely define their trading logic and trade on the blockchain with fairness; Ethereum provides an efficient platform for conveniently raising funds (i.e. initial coin offering, ICO) and it also acts as the funding currency of ICO. Ripple is a centrally-controlled cryptocurrency and is welcomed for its good liquidity; Ripple features fast payment and low transaction fees; and it also have got some support from traditional banks and financial institutions.[4] Litecoin is famous for its emulation to Bitcoin; compared to Bitcoin, Litecoin achieves better decentralization, lower cost and faster confirmation time. Dash and Monero are famous for good anonymity. NEM

[3]The data period is about a two-year period of 725 observations per cryptocurrency. The start point is selected so that the seven selected cryptocurrencies have all come to be traded and quoted in the exchanges. Before 08/08/2015, the total market capitalization of the cryptocurrency market had been less than 16 billion dollars, the daily transaction volume was also low and the liquidity was not good.

[4]https://ripple.com.

Table 2. Descriptive statistics of the cryptocurrencies' and traditional financial assets' return.

	Mean	Variance	Skewness	Kurtosis	p(Q test)	p(Engle) (lag = 1)	p(Engle) (lag = 6)
Bitcoin	0.0032	0.0011	−0.4267	9.5427	0.6585	0.0003	0.0000
Ethereum	0.0079	0.0057	0.6672	7.3931	0.1028	0.0000	0.0000
Ripple	0.0042	0.0055	4.0390	64.2827	0.0001	0.0000	0.0000
Litecoin	0.0033	0.0025	2.2985	23.6778	0.0005	0.1822	0.0000
Dash	0.0099	0.0083	1.3365	9.3879	0.0904	0.0000	0.0000
NEM	0.0057	0.0030	1.0445	8.2822	0.5360	0.0003	0.0000
Monero	0.0058	0.0051	1.5612	13.0138	0.0027	0.0001	0.0000
S&P 500	0.0004	0.0001	−0.4302	6.5630	0.6550	0.0000	0.0000
Euro STOXX 50	−0.0001	0.0002	−0.7633	8.7275	0.1045	0.0000	0.0001
Nikkei 225	−0.0001	0.0002	−0.0818	8.4639	0.5812	0.0000	0.0000
CSI 300	−0.0001	0.0002	−1.4630	11.5015	0.3692	0.0000	0.0000
Gold Fixing (PM)	0.0003	0.0001	0.3394	5.0153	0.0998	0.1900	0.0063
WTI Crude Oil	0.0003	0.0007	0.4665	4.6764	0.8434	0.0000	0.0000

This table presents a statistical description of seven cryptocurrencies' and six traditional financial assets' return series. Column 2–5 are the one to four moments. Column 6 presents the *p*-values of Ljung–Box (Q) autocorrelation tests, with the null hypothesis of no serial correlation. We use $ln(T)$ (where T is the length of the time series) as the lag of Ljung-Box tests (following Tsay 2005). Columns 7 and 8 present the *p*-values of Engle's LM tests for heteroscedasticity, with the null hypothesis of no ARCH effects, at lag = 1 and lag = 6, respectively.

provides a trading platform to record and trade digital assets; it features high performance and a novel consensus algorithm.

Table 1 listed the basic information of the cryptocurrencies we selected. To research on the tail dependence between cryptocurrencies and traditional financial assets, we also use four major stock indices across the world, that is, S&P 500 (USA), Euro Stoxx 50 (Europe), Nikkei 225 (Japan) and CSI 300 (China), and two traditional commodities, of gold and crude oil. The stock index data are available on Yahoo! Finance or Google Finance; the gold prices come from quandl.com and are based on the London Gold Fixing (PM), which are the spot prices quoted in USD/ounce; the crude oil prices are represented by the WTI spot prices, which come from the US Energy Information Agency and are quoted in USD/barrel.

Figures 1 and 2 plot the prices of the seven selected cryptocurrencies and the six selected stocks and commodity indices during the sample period. It is obvious that all the seven

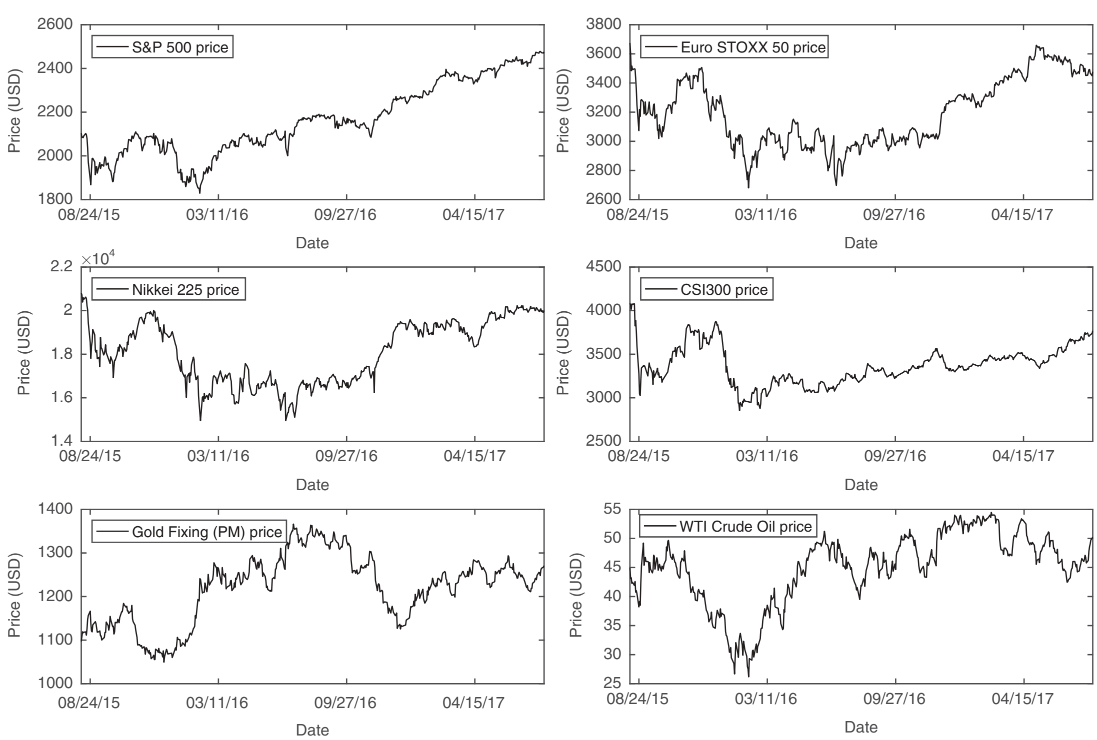

Figure 2. Prices of six selected traditional assets (S&P 500, Euro Stoxx 50, Nikkei 225, CSI 300, Gold PM Fixing, WTI Crude Oil indices), 08/08/2015–08/01/2017.

cryptocurrencies' prices and volatility both rise largely in 2017. There are two possible reasons. First, the WannaCry ransomware attack in 2017 uses Bitcoin as the only payment method, and it made a natural advertisement for the cryptocurrencies. Market speculation activities follow quickly. Second, numerous ICOs have launched in 2017, which raise demand and attract lots of market attentions.

In Table 2 we present the one to four moments of the cryptocurrencies' log returns. To test whether the log returns of cryptocurrencies can be approximated by i.i.d. assumption, we carried out the Ljung–Box Portmanteau test (Q test) to see if the autocorrelation is statistically significant (Table 2); we also implement the Engle's LM test for heteroscedasticity (Table 2).[5] To make a comparison, we also list these statistical properties of the four stock indices we choose, as well as for gold and crude oil. From Table 2 we can see that, overall, the whole sample mean and variance of the cryptocurrencies are higher than their stock/commodity counterparts. This phenomenon is largely due to the samples of year 2017. Among the seven selected cryptocurrencies, Dash has the highest whole sample return as well as volatility, while Bitcoin has the lowest average return and volatility. As for skewness, all the cryptocurrencies except Bitcoin are positively skewed, which means long and thick tail are more evident for the positive side than for the negative side. This phenomenon is very different from the stocks and is similar to the commodities. It is well known that stock return distributions tend to be asymmetric and negative extreme returns are more frequent and more extremal than the positive returns. But this is not the case for most of the cryptocurrencies and the commodities, as extremal returns are more often in their right tails. Concerning the kurtosis, we can see that all the assets we discuss are leptokurtic, while Ripple and Litecoin's kurtosis are apparently higher than other cryptocurrencies or traditional assets, while the two commodities have the lowest kurtosis.

Using the Ljung–Box Q test, we can see that under the significance level of 0.05, Ripple, Litecoin and Monero cannot reject the null hypothesis of no autocorrelation (Table 2). This is a phenomenon commonly existing in emerging and inefficient markets, which means statistical arbitrage can be carried out in these markets. The last test of Table 2 is about heteroscedasticity. We can see that heteroscedasticity exists for all the cryptocurrencies if we include 6 lags. The autocorrelation and heteroscedasticity imply that when we use extreme value theory to analyse the tail characteristics of cryptocurrencies, it is necessary to use a robust method. We achieve this target by using the ARMA-GARCH pseudo returns instead of the raw log returns to remove autocorrelation and heteroscedasticity without affecting the tail property, as described in "Measure the tail risk of a single cryptocurrency". Meanwhile, we also partition the whole sample into two or three subsets and get the pseudo returns piecewise; this is also to remove part of the non-stationarity, as different price trends exist for different periods.[6] After the above handling, the pseudo returns, which are free of autocorrelation and heteroscedasticity, are more close to the i.i.d. assumption, and classic extreme analysis can be applied.

IV. Empirical estimation

We first estimate the ARMA-GARCH-PML-GPD model, and analyse the tail risk of the selected cryptocurrencies. Then we estimate the bivariate tail correlations among the cryptocurrencies and assess the tail diversification and hedging ability of cryptocurrency.

Tail characteristics

In Table 3, we present the MLE estimation results of the GPD model of the pseudo returns. We estimate the tail shape parameters ξ and the dispersion parameters σ; both left tails (extremal negative returns) and right tails are estimated. We approximately divide the samples into two equal-length halves, that is 08 August 2015–31 July 2016 for the first half and 01 August 2016–01 August 2017 for the second half. There are two reasons for this division. First, the cyptocurrencies are emerging and rapidly developing assets (for example, Bouri, Azzi, and Dyhrberg 2017; Bouri, Jalkh et al. 2017), their price movement behaviours apparently change in 2017, and we would like to see if tail characteristics are

[5]We use ln(T) (where T is the length of the time series) as the lag of Ljung-Box tests, which is suggested by Tsay (2005) and followed by other papers (For example, Fang and Miller 2007). We use lag = 1 and lag = 6 for Engle tests of ARCH effect. In the literature, researchers usually choose 2 or 3 different (low order) lags to see if ARCH effect exists (for example, Engle 1982).

[6]A further test suggests that this autocorrelation cannot be removed by purely using ARMA, and we have to divide the sample into two or more sub-periods and then use ARMA to remove this autocorrelation for extreme value analysis to be applied. For all the cryptocurrencies, we use 03/01/2017 as a breakpoint; for Ripple, we use 05/01/2017 as another breakpoint, to fit the ARMA-GARCH model and get the pseudo return.

Table 3. Tail estimations of the cryptocurrencies.

	Period 1: 08 August 2015–31 July 2016					Period 2: 01 August 2016–01 August 2017					LR test	
	Left tail		Right tail		LR test, H_0:	Left tail		Right tail		LR test, H_0:	H_0:	H_0:
	$\xi_{1,L}$	$\sigma_{1,L}$	$\xi_{1,U}$	$\sigma_{1,U}$	$\xi_{1,U} = \hat{\xi}_{1,L}$	$\xi_{2,L}$	$\sigma_{2,L}$	$\xi_{2,U}$	$\sigma_{2,U}$	$\xi_{2,U} = \hat{\xi}_{2,L}$	$\xi_{2,L} = \hat{\xi}_{1,L}$	$\xi_{2,U} = \hat{\xi}_{1,U}$
Bitcoin	0.3814	0.6340	−0.0473	0.8158	5.8059	−0.4036	1.2569	−0.2723	0.9628	Inf	16.9517	101.6139
	[1.7504]	[3.7470]	[−0.2742]	[4.1713]	(0.016)	[−2.7686]	[4.8070]	[−1.6244]	[4.3656]	(0.0000)	(0.0000)	(0.0000)
Ethereum	0.0776	0.6470	0.0778	0.7910	32.5836	−0.1895	0.5818	0.0779	0.6701	38.9625	4.0557	50.1588
	[0.3786]	[3.7914]	[0.4660]	[4.2461]	(0.0000)	[−1.4842]	[4.8983]	[0.4400]	[4.1697]	(0.0000)	(0.0440)	(0.0000)
Ripple	0.4625	0.3211	0.1799	0.8274	2.2599	−0.3236	0.7802	0.2530	0.6513	Inf	16.7376	4.0521
	[1.4685]	[2.8746]	[0.7628]	[3.4812]	(0.1328)	[−2.0455]	[4.5369]	[1.4905]	[4.3257]	(0.0000)	(0.0000)	(0.0441)
Litecoin	0.0321	0.9777	0.1315	0.8408	163.2684	0.1423	0.5635	0.4231	0.5680	6.0452	41.7427	8.7988
	[0.1749]	[4.0373]	[0.7307]	[4.0932]	(0.0000)	[0.6530]	[3.6939]	[1.7455]	[3.5274]	(0.0139)	(0.0000)	(0.0030)
Dash	0.0686	0.5587	−0.2456	1.2923	58.9728	0.0711	0.4418	0.0465	0.7678	42.9903	1.0692	Inf
	[0.4340]	[4.3599]	[−1.2092]	[3.8534]	(0.0000)	[0.4717]	[4.5223]	[0.2772]	[4.2886]	(0.0000)	(0.3011)	(0.0000)
NEM	0.1238	0.4575	0.0752	0.7129	2.3385	−0.1523	0.4557	−0.2137	1.0906	6.7402	3.9391	117.2281
	[0.7208]	[4.1949]	[0.4474]	[4.2294]	(0.1262)	[−0.8235]	[4.0863]	[−1.7767]	[5.0264]	(0.0094)	(0.0472)	(0.0000)
Monero	0.2346	0.4742	0.0739	0.7704	1.6639	−0.4956	0.6391	0.4279	0.5009	Inf	18.6059	103.0555
	[0.9962]	[3.4980]	[0.3786]	[3.8996]	(0.1971)	[−3.1363]	[4.7235]	[0.0094]	[3.8679]	(0.0000)	(0.0000)	(0.0000)

This table gives the two-periods ARMA-GARCH-PML-GPD estimation results of the parameters described in 'Measure the tail risk of a single cryptocurrency'. Both left tails (extremal negative returns) and right tails (large appreciation) are estimated. The threshold set for GPD estimation is the 10% upper quantile. Column 2–6 correspond to the results of period 1, that is 08 August 2015–31 July 2016, and column 7–11 corresponds to the results of period 2, namely 01 August 2016–01 August 2017. Column 2–5 and column 7–10 report the estimation results of the tail shape parameters ξ and the tail dispersion parameters σ for each of the seven cryptocurrencies. t-statistics of estimations are given below in brackets. Column 6 and 11 give the likelihood ratio (LR) test results of whether asymmetry exists between right tail and left tail's shape; the null hypothesis is H_0: $\xi_{i,U} = \hat{\xi}_{i,L}$,($i =$ period 1 or 2). The last two columns reports the LR test results of whether there are statistically significant difference between the tail shape of the two periods, and the null hypothesis is H_0: $\xi_{2,j} = \hat{\xi}_{1,j}$, ($j =$ right tail or left tail).The p-values of the tests are given below in parentheses.

changing; second, we need to keep enough samples in each sub-periods for statistical estimation. We report the parameters for each sub-periods. Note that all the results are based on pseudo returns, and are free of the time trend effects and GARCH-predictable time-varying volatility. In other words, the estimations are for the innovations, and are for unpredictable tail risks.

The results are very interesting. Looking at the first sub-period, all the seven cryptocurrencies' left tails are estimated to have positive tail shape parameters (Table 3, column 2), which is similar to the behaviour of stock indices (e.g. Longin and Solnik 2001). All the seven selected cryptocurrencies' left tail distributions fall into Fréchet domain of attraction and the potential left tail risks are unlimited as the lower bounds of the tails are negative infinity. Ripple's left tail has the biggest tail shape parameter and thus the thickest left tail, while Litecoin's left tail has the smallest shape parameter. Still looking at the first period, for the right tail, Bitcoin and Dash's right tails are estimated to have negative shape parameters, while the rest of the selected cryptocurrencies' right tail shape parameters are estimated to be positive (Table 3, column 4). It means that Bitcoin and Dash have finite upper bounds for positive returns for the first period while the rest of the coins do not have a limit for appreciation. We find some of the parameters are not estimated with great precision, which is commonly observed in the literature of extreme value fitting with financial data (e.g. Longin and Solnik 2001). Looking at the second sub-period, things become very different. Five out of seven cryptocurrencies, that is, Bitcoin, Ethereum, Ripple, NEM, and Monero are estimated to have negative tail shape parameters for the left tails, while Bitcoin, Ripple, and Monero's results are significantly negative (Table 3, column 7). Although cryptocurrencies' price movements seem much more volatile in the second sub-period, the large movements are mostly due to the rising volatility. From the GPD analysis we can see that the negative return risks for the innovations get finite lower bounds for most of the cryptocurrencies in the second sub-period. Still looking at the second sub-period, for the right tails, Bitcoin and NEM show negative tail shape parameters (with finite upper bound of positive pseudo returns) while the rest are estimated to have positive tail shape parameters (with infinite upper bound of positive pseudo returns) (Table 3, column 9).

We conduct four likelihood ratio tests to see whether statistically significant differences stand between the shapes of the left tails and the right tails (the null hypothesis is H$_0$: $\xi_{i,U} = \hat{\xi}_{i,L}$, $i =$ period 1 or 2), and whether significant differences stand between the shape parameters of two periods (the null hypothesis is H$_0$: $\xi_{2,j} = \hat{\xi}_{1,j}$, $j =$ right tail or left tail). For the first period, for Bitcoin, Ethereum, Litecoin and Dash, asymmetry exists and their right tails have statistically significant different tail shapes with their left tails; but no significant asymmetry stands between the two tails of Ripple, NEM or Monero (Table 3, column 6). However, for the latter period, all the seven cryptocurrencies' two tails are significantly different (Table 3, column 11). In the second sub-period, for Ethereum, Ripple and Monero, these three cryptocurrencies are estimated to have Weibull-type left tails (with finite upper bounds) but Fréchet type right tails (with no finite upper bound). Regarding the tail shape difference between the two periods, the likelihood ratio test results suggest that changes mostly exist (Table 3, the last two columns). The only exception is the left tail of Dash, as there is no significant difference between Dash's left tail shape parameters of the two periods. The overall significant changes in the tail shapes might be largely related to the rapid innovations and changes in the cryptocurrencies market in 2017, the immaturity of the market, and the growing market speculations.

Conditional VaR, conditional ES and conditional extremal boundary

VaR and ES are two major risk metrics in the literature and for financial industry risk managements and for regulators. Conditioning on the information set of $\mathcal{F}_{t-1}$, using GARCH implied volatility and the tail distribution implied in the GPD model, we can compute the dynamic conditional VaR and conditional ES series, which are proposed by McNeil and Frey (2000) and can well measure the dynamic tail risk at time t. Specifically, for the interested quantile $q > 90\%$, we can solve z_q, the q-quantile of $F_Z(z)$ using (8):

$$\hat{z}_q = u + \frac{\hat{\sigma}}{\hat{\xi}}\left\{[10 \times (1-q)]^{-\hat{\xi}} - 1\right\}. \tag{15}$$

Defining the one-step ahead expected conditional VaR and conditional ES as,

$$VaR_q^t = inf\{x \in R : F_{X_t|\mathcal{F}_{t-1}}(x) > q\}, \tag{16}$$

$$ES_q^t = E[X_t | X_t > VaR_q^t, \mathcal{F}_{t-1}], \tag{17}$$

where X_t is the log return. For the concerned $q > 90\%$, under the model described in 'Measure the tail risk of a single cryptocurrency', we can derive the estimator of conditional VaR and conditional ES[7] as

$$\widehat{VaR}_q^t = \hat{\mu}_t + \hat{v}_t \hat{z}_q, \tag{18}$$

$$\begin{aligned} \widehat{ES}_q^t &= \hat{\mu}_t + \hat{v}_t E[Z|Z > \hat{z}_q] \\ &= \hat{\mu}_t + \hat{v}_t \frac{\hat{z}_q + \hat{\sigma} - \hat{\xi} u}{1 - \hat{\xi}}, \end{aligned} \tag{19}$$

where $\hat{z}_q$ is the upper qth quantile of the distribution of Z_t implied by (8); $\hat{\mu}_t$ and $\hat{v}_t$ are the one-step-ahead conditional mean and conditional volatility of ARMA-GARCH model, as described in 'Measure the tail risk of a single cryptocurrency', and are $\mathcal{F}_{t-1}$ measurable; while $\widehat{VaR}_q^t$ and $\widehat{ES}_q^t$ are the one step ahead expectations for the extremal negative return risk of t, and they are also $\mathcal{F}_{t-1}$ measurable.

Under the assumption that the model specification is correct, there should be a finite extremal boundary for each of the Weibull-type tails, which is $(u - \frac{\sigma}{\xi})$ for pseudo returns and $\mu_t + v_t(u - \frac{\sigma}{\xi})$ for log returns, conditioning on $\mathcal{F}_{t-1}$. We can use the sample counterpart $\hat{\mu}_t + \hat{v}_t(u - \frac{\hat{\sigma}}{\hat{\xi}})$ to approximate the conditional extremal boundaries of the log returns in the Weibull-type tail cases. We denote the conditional extremal boundary series as $\{B_t\}$. Note that the estimated series of $\left\{\widehat{VaR}_t, \widehat{ES}_t, \hat{B}_t\right\}$ measure the potential risks at t, and are all $\mathcal{F}_{t-1}$ measurable if we formally consider the estimated $\hat{\xi}$ and $\hat{\sigma}$ equal the true values and can be computed using historical realized returns.

We present the *negative* log return, conditional VaR, conditional ES and conditional extremal boundary series for the two largest cryptocurrencies, Bitcoin and Ethereum in Figure 3. Note that we only draw the conditional extremal boundary for the second sub-period (Aug.1, 2016–Aug.1,2017) for these two cryptocurrencies, because their tails of period one are

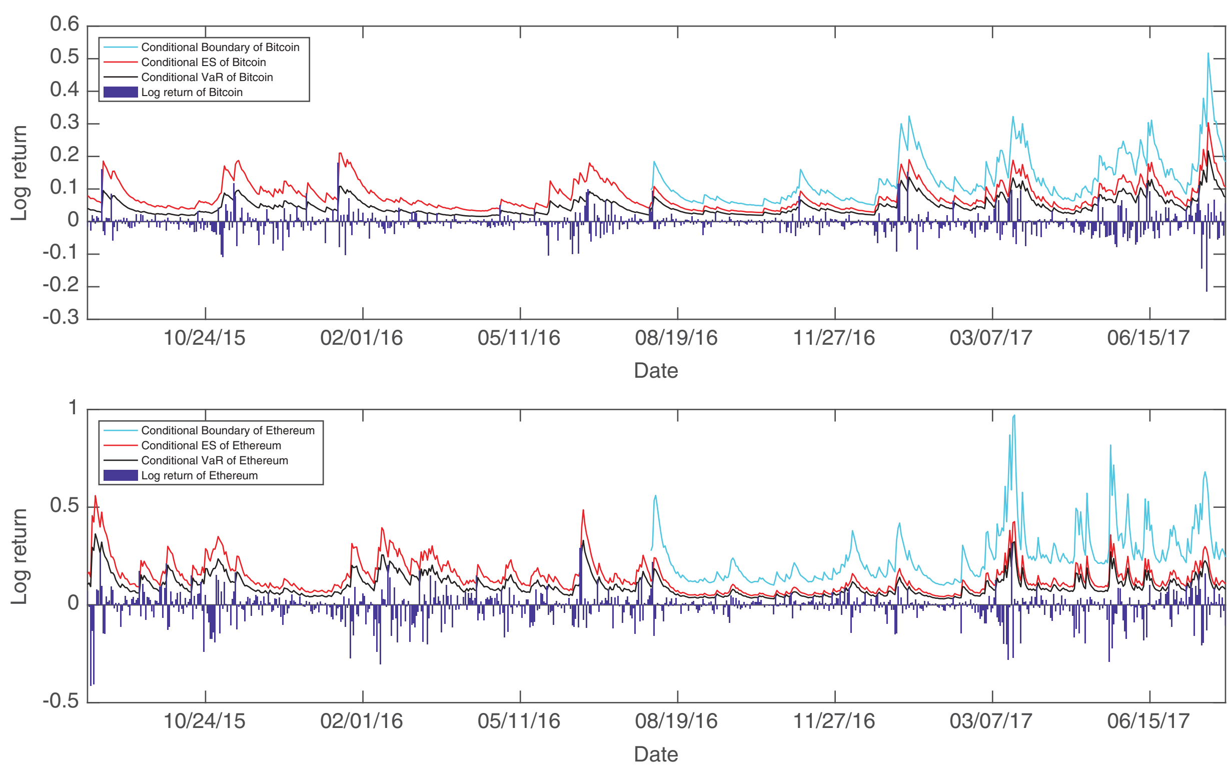

Figure 3. Negative log return, conditional VaR, conditional expected shortfall (08/08/2015–08/01/2017), and conditional extremal boundaries (08/01/2016–08/01/2017) of Bitcoin and Ethereum.

[7]We can prove that when $Z > z_q$, and under the assumption of GPD, we have $E[Z|Z > z_q] = \frac{z_q + \sigma - \xi u}{1 - \xi}$.

estimated to be Fréchet type with no finite boundaries. The probability for computing conditional VaR and conditional ES are both 5%. For comparison, we also plot the conditional VaR, conditional ES, and conditional extremal boundaries (if applicable) for the six selected traditional assets.[8] (Figure 4)

Looking at Figures 3 and 4, we can see that the PML-GPD model fits the tails of cryptocurrencies very well, and the computed conditional VaR, ES and extremal boundary well describe the risks. Compared to the traditional assets, first, we can see that for both Bitcoin and Ethereum, positive returns are more intense than negative returns, and this is in accordance with the overall rising trend of cryptocurrencies' price as shown in Figure 1. Second, overall, the volatilities of cryptocurrencies are much higher than the volatility of stock indices and commodities, leading to much higher VaR and ES for cryptocurrencies. The average daily volatility for Bitcoin and Ethereum during the sample period are 3.17 and 7.13%, respectively; for S&P 500 it is only 0.77%. The average daily VaR for Bitcoin and Ethereum are 4.78 and 9.82%, respectively; and it is 1.22% for S&P 500; The average ES, which measures the expected loss when the return exceed VaR, are 7.92 and 14.12% for Bitcoin and Ethereum; and it is only 1.86% for S&P 500. Third, for Bitcoin and Ethereum, although the volatility are higher, there are boundaries' for possible daily loss in the second period (Aug.1, 2016 to Aug.1, 2017), which are 6.69 and 12.58% on average respectively. The two cryptocurrencies' property of left tail truncation are similar to gold and crude oil, but different from the four stock indices, whose possible losses could reach 100% as their left tails are Fréchet type.

Define a tail event as,

$$TE_{i,t} = \mathbf{1}(-r_{i,t} \geq VaR_{i,t}), \tag{20}$$

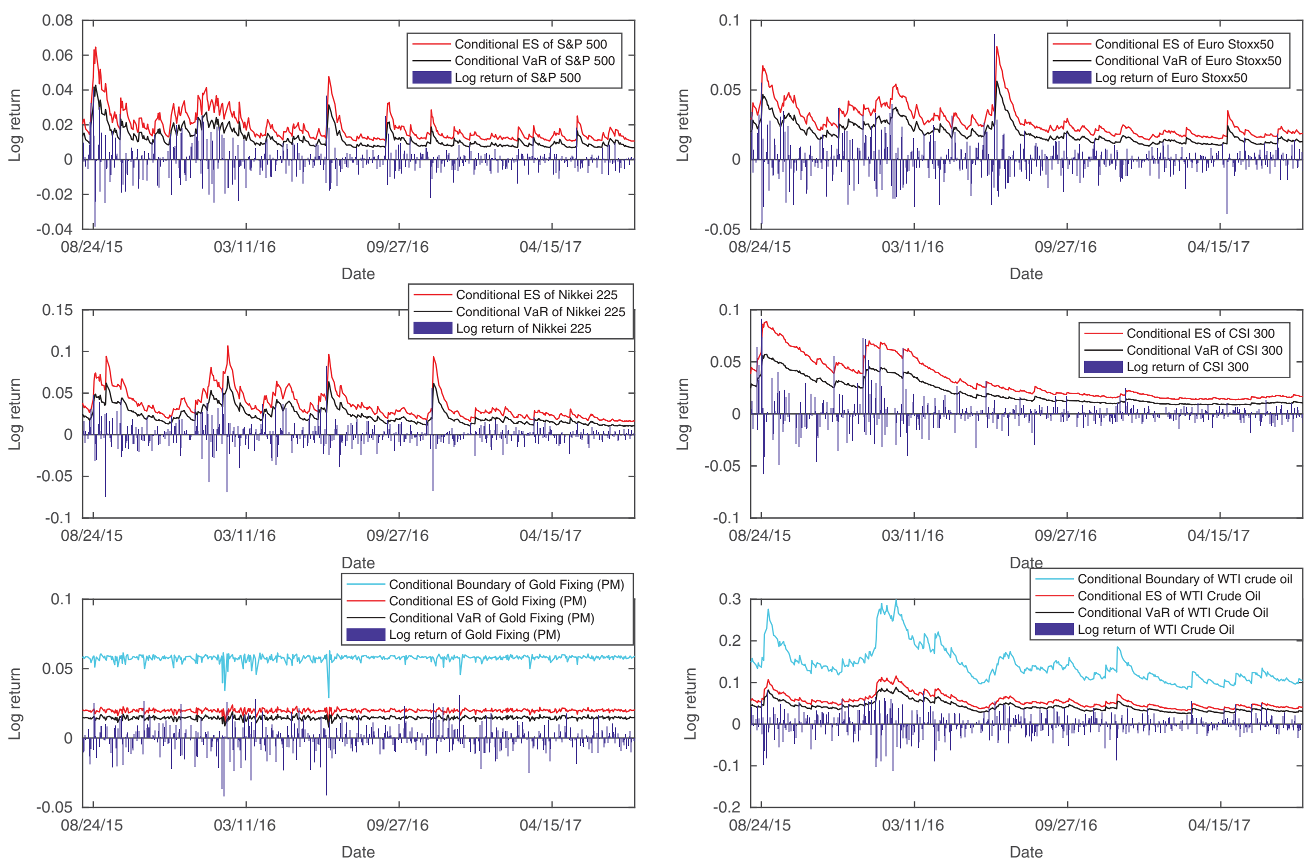

Figure 4. Negative log return, conditional VaR, conditional expected shortfall for S&P 500, Euro Stoxx 50, Nikkei 225, CSI 300, Gold Fixing (PM), and WTI Crude Oil indices, and conditional extremal boundaries for Gold Fixing (PM) and WTI Crude Oil (08/08/2015–08/01/2017).

[8]The left tail distributions of the four stock indices (S&P 500, Euro Stoxx 50, Nikkei 225, CSI 300) are estimated to be Fréchet type (long tails) during the sample period, and the two commodities (Gold PM Fixing and WTI Crude oil Indices) are estimated to be Weibull type tails (truncated tail distribution), so conditional extremal boundaries only apply to the two commodities.

where $\mathbf{1}(\cdot)$ is the indicator function and $r_{i,t}$ is the log return. Define $TE_t = \sum_{i=1}^{7} TE_{i,t}$, which measures how many cryptocurrencies (among the seven selected ones) experience tail events on day t. We find that among the sample period of 724 days, at the probability level of 5% for computing VaR, there are 119 days when $TE_t = 1$, 28 days when $TE_t = 2$, 8 days when $TE_t = 3$, 8 days when $TE_t = 4$, 2 days when $TE_t = 5$, and 4 days when $TE_t = 6$. This is a strong indication of tail dependency of the cryptocurrency market as simultaneous negative extremal events happen much more frequent than in the independent case.

Tail correlations, diversification, and hedging abilities of the cryptocurrencies

Tail correlation measures the correlation between two variables under extremal circumstances, and is a different measure from the traditional Pearson correlation. In financial context, if the left tails of the return series of two assets are uncorrelated or tail correlation coefficient is low, they are good diversification assets for each other, as when one asset drops significantly, it would not indicate the other asset's large depreciation. On the other side, if asset A's right tail shows significant correlation with B's left tail, then A can hedge B's tail to some extent, as if one holds a portfolio consisting of A and B, when she suffers big loses from B's price drop, she may gain from A's large appreciation.

We use the method described in 'Measure the tail correlation' to estimate the following issues.

(1) **We estimate both the left tail correlation, and the right tail correlations respectively, among the seven selected cryptocurrencies, of the two sub-periods** (08 August 2015–31 July 2016, 01 August 2016–01 August 2017; Table 4). This is to see whether there are correlations among the bad extremal events or positive extremal events of different cryptocurrencies. There are some interesting findings. First, the left tails of the cryptocurrencies are more correlated than the right tails, and the correlation tends to be higher and more significant in the second sub-period than in the first sub-period. At the significance level of 0.05, 13 out of 21 pairs have statistically significant left tail correlations for the first sub-period, and all the 21 pairs have statistically

Table 4. Bivariate tail correlation estimations of the cryptocurrencies.

	Period 1: Aug.08,2015-July 31,2016						Period 2: Aug.01,2016-Aug 01,2017					
	Ethereum	Ripple	Litecoin	Dash	NEM	Monero	Ethereum	Ripple	Litecoin	Dash	NEM	Monero
Panel A: left tail correlation estimations												
Bitcoin	0.0229	0.1877	0.5997	0.1756	0.2005	0.3066	0.3688	0.1777	0.4905	0.2084	0.2005	0.4260
	[0.2756]	[2.8396]	[13.4055]	[2.5871]	[2.9787]	[4.6469]	[5.8439]	[2.7895]	[8.9083]	[3.0563]	[3.0770]	[7.0307]
Ethereum	-	0.0650	0.0000	0.0633	0.0406	0.0469	-	0.2043	0.2598	0.2563	0.2074	0.3763
		[1.0125]	[0.0000]	[0.8825]	[0.5560]	[0.6124]		[3.2160]	[4.2395]	[3.8588]	[3.2461]	[6.1788]
Ripple	-	-	0.2199	0.1338	0.0708	0.1304	-	-	0.2109	0.1242	0.1963	0.3119
			[3.3013]	[2.0308]	[1.3072]	[2.0620]			[3.3502]	[2.0196]	[3.1212]	[5.0701]
Litecoin	-	-	-	0.2517	0.2090	0.3254	-	-	-	0.2411	0.1872	0.2596
				[3.7039]	[3.0961]	[4.8684]				[3.7870]	[2.9872]	[4.1414]
Dash	-	-	-	-	0.0858	0.1500	-	-	-	-	0.2213	0.2384
					[1.4168]	[2.2270]					[3.4337]	[3.6455]
NEM	-	-	-	-	-	0.1739	-	-	-	-	-	0.3000
						[2.5973]						[4.8641]
Panel B: right tail correlation estimations												
Bitcoin	0.0850	0.1084	0.4072	0.0280	0.0964	0.1087	0.0377	0.0000	0.1879	0.1265	0.1617	0.1631
	[1.3921]	[1.5951]	[7.1404]	[0.6199]	[1.5136]	[1.7461]	[0.6603]	[0.000]	[2.947]	[2.0438]	[2.5064]	[2.5935]
Ethereum	-	0.0000	0.0760	0.0000	0.1029	0.0334	-	0.0310	0.0000	0.0234	0.1487	0.1044
		[0.000]	[1.2871]	[0.000]	[1.5933]	[0.6198]		[0.5855]	[0.000]	[0.4499]	[2.4103]	[1.6372]
Ripple	-	-	0.1112	0.0000	0.0402	0.0000	-	-	0.0484	0.0423	0.0000	0.0000
			[1.6729]	[0.000]	[0.6733]	[0.000]			[0.9128]	[0.7491]	[0.000]	[0.000]
Litecoin	-	-	-	0.0886	0.0727	0.0000	-	-	-	0.0000	0.0330	0.1222
				[1.4719]	[1.3148]	[0.000]				[0.0000]	[0.6089]	[1.9168]
Dash	-	-	-	-	0.0625	0.0364	-	-	-	-	0.1068	0.1458
					[1.0047]	[0.6164]					[1.7692]	[2.3221]
NEM	-	-	-	-	-	0.1032	-	-	-	-	-	0.1575
						[1.7227]						[2.5061]

This table gives the pairwise tail correlation coefficient estimations of the seven selected cryptocurrencies. Panel A corresponds to left tail correlations and Panel B corresponds to right tail correlations. The 2–7 columns correspond to tail correlation of 08 August 2015–31 July 2016 and column 8–13 correspond to the results of 01 August 2016–01 August 2017. The *t*-statistics of each estimation are given below in brackets.

significant left tail correlation for the second sub-period. Meanwhile, only 1 (for the first sub-period) and 7 (for the second sub-period) pairs show significant right tail correlations. The average estimated tail correlation coefficients are 0.1647 (for the first sub-period) and 0.2603 (for the second sub-period) for the left tails, and are only 0.0743 (for the first sub-period) and 0.0781 (for the second sub-period) for the right tails. These phenomena can probably explained by that bad news from market or regulatory authorities tend to influence the whole cryptocurrency market together, leading to contagious and systematic risks in cryptocurrency market; while the value appreciations of different cryptocurrencies are more probably led by its own unanticipated high demand and good news. Second, if we just take arithmetic average over the two periods, Bitcoin has the highest average correlations with other cryptocurrencies, for both left and right tail. This is due to its leading role in the market, as well as its high liquidity. Third, among all the pairs, Bitcoin and Litecoin has the highest tail correlations, for both right tails and left tails, and for both two sub-periods.

(2) **We estimate the left tail correlations among Bitcoin/Ethereum and six traditional assets** (Table 5). The six selected traditional indices are S&P 500, Euro Stoxx 50, Nikkei 225, CSI 300, the London Gold Fixing (PM) spot prices, and the WTI Crude Oil spot prices. This is to see whether cryptocurrencies and traditional financial assets can be a good diversification asset for each other. Here we only include two major cryptocurrencies instead of all because of their large market cap percentage, high liquidity, and representativeness. The results are indicative of part of the virtual gold property or safe haven property of the cryptocurrencies. At the significance level of 0.05, we can see that both Bitcoin and Ethereum do not have significant left tail correlations with all the four major stock indices or the two commodity indices, for both of the two periods (Table 5, row 2–5). To make a comparison, we can see that the four stock markets are highly and significantly tail correlated themselves (Table 5, row 6–9, column 2–5). Cryptocurrencies as well as gold are good diversification assets for stock assets as their left tails are independent with the stock market, while crude oil does not satisfy this property as it is tail correlated with three of the selected stock market (Table 5, the last column). This means that when one of the stock market experience large shortfall, foreign stock market as well as crude oil price are also likely to experience large shortfall, while

Table 5. Left tail correlations between the cryptocurrencies and the traditional financial assets (diversification effect).

	S&P 500 (USA)	Euro Stoxx 50 (Europe)	Nikkei 225 (Japan)	CSI 300 (China)	Gold Fixing (PM)	WTI (Crude Oil)
Bitcoin (t = 1)	0.0000	0.0099	0.0000	0.0674	0.0351	0.0637
	[0.0000]	[0.2175]	[0.0000]	[1.2620]	[0.5785]	[0.9729]
Bitcoin (t = 2)	0.0000	0.0576	0.0000	0.0314	0.0000	0.0770
	[0.0000]	[0.8333]	[0.0000]	[0.4165]	[0.0000]	[1.1307]
Ethereum (t = 1)	0.0730	0.0000	0.0000	0.0000	0.0319	0.0000
	[1.2439]	[0.0000]	[0.0000]	[0.0000]	[0.5352]	[0.0000]
Ethereum (t = 2)	0.0000	0.0188	0.0000	0.1168	0.0000	0.1082
	[0.0000]	[0.3340]	[0.0000]	[1.3780]	[0.0000]	[1.5174]
S&P 500	–	0.3853	0.1648	0.1050	0.0000	0.1374
		[7.4891]	[2.9387]	[2.0520]	[0.0000]	[2.5166]
Euro Stoxx 50	–	–	0.2466	0.1397	0.0000	0.1304
			[4.2088]	[2.5177]	[0.0000]	[2.4311]
Nikkei 225	–	–	–	0.1554	0.0000	0.1179
				[2.8327]	[0.0000]	[2.2168]
CSI 300	–	–	–	–	0.0103	0.0650
					[0.2658]	[1.2796]
Gold Fixing (PM)	–	–	–	–	–	0.0319
						[0.8431]

This table gives the pairwise left tail correlation estimation results between cryptocurrencies and six traditional financial assets, as well as between the six selected financial assets. Because the tails of the two selected cryptocurrencies do not keep a same shape during the whole sample period, we estimate their tail correlations with others in two sub-sample periods separately, that is, 08 August 2015–31 July 2016 (t = 1), and 01 August 2016–01 August 2017 (t = 2). We assume the other 6 mature traditional assets' tail shapes do not change over the sample period. The t-statistics of the estimations are given below in brackets.

cryptocurrencies or gold are not expected to experience a higher probability of shortfall than in normal conditions. These findings suggest that adding cryptocurrencies to investors' portfolio can help to achieve better diversification than using global diversification, and these results are meaningful for asset allocation theory.

(3) **We estimate the left-right cross tail correlations among the selected six traditional assets and Bitcoin/Ethereum** (Table 6). Mathematically, that is,

$$\eta_{i,j} = \lim_{q\to 1} P(Z_j \geq F^{\leftarrow}_{Z_j}(q) | -Z_i \geq F^{\leftarrow}_{-Z_i}(q)). \tag{21}$$

This is to see whether cryptocurrencies and traditional financial assets can hedge each other in the extremal sense. If $\eta_{i,j}$ is significantly larger than zero, asset j will be a good hedging asset for asset i in the tail sense. The estimation results suggest that, among all the selected cryptocurrencies, stock indices and commodities, gold is the only asset that possesses some tail hedging ability, as its right tail is significantly correlated with the left tail of S&P 500, Euro Stoxx 50, and Nikkei 225 indices (at the significance level of 0.05). This means when USA, Europe or Japan stock market experience a large shortfall, the gold prices are more likely to experience a large appreciation. However, Bitcoin or Ethereum does not show this kind of tail hedging ability. On the other hand, neither the stock indices nor the two selected commodities can hedge the tail risks of the selected digital currencies.

V. Conclusion

Cryptocurrencies are a new and promising financial innovation in financial market. This study is one of the pioneers who research on the tail risks of the cryptocurrencies. Our main findings are summarized as follows.

(1) About tail characteristics: for the seven selected cryptocurrencies, the likelihood ratio tests suggest that asymmetry commonly exist between the tail shapes of the right tails and the left tails. Moreover, dividing the sample period into two sub-periods (1st: 08 August 2015–31 July 2016; 2nd: 01 August 2016–01 August 2017), almost all the selected cryptocurrencies have statistically significant different tail shapes between the two sub-periods. For the left tails, all the seven selected cryptocurrencies' pseudo returns have long and heavy left tails (Fréchet type) in sub-period one, but five of them turn to a truncated

Table 6. Cross tail correlations between the cryptocurrencies and the traditional financial assets (hedging effect).

	Bitcoin (t = 1)	Bitcoin (t = 2)	Ethereum (t = 1)	Ethereum (t = 2)	S&P 500 (USA)	Euro Stoxx 50 (Europe)	Nikkei 225 (Japan)	CSI 300 (China)	Gold Fixing (PM)	WTI (Crude Oil)
Bitcoin (t = 1)	-	-	-	-	0.0718	0.0000	0.0940	0.0382	0.0000	0.0381
					[1.1977]	[0.0000]	[1.3582]	[0.7353]	[0.0000]	[0.5572]
Bitcoin (t = 2)	-	-	-	-	0.1024	0.0000	0.0012	0.0143	0.0000	0.0000
					[1.3552]	[0.0000]	[0.0140]	[0.1962]	[0.0000]	[0.0000]
Ethereum (t = 1)	-	-	-	-	0.0000	0.0514	0.0880	0.0000	0.0112	0.1011
					[0.0000]	[0.8389]	[1.2958]	[0.0000]	[0.1432]	[1.3287]
Ethereum (t = 2)	-	-	-	-	0.0000	0.0000	0.0472	0.0000	0.0000	0.0270
					[0.0000]	[0.0000]	[0.5800]	[0.0000]	[0.0000]	[0.6563]
S&P 500 (USA)	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0496	0.0000
	[0.0000]	[0.0000]	[0.0000]	[0.0000]		[0.0000]	[0.0000]	[0.0000]	[0.9503]	[0.0000]
Euro Stoxx 50 (Europe)	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0009	0.0937	0.0000
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]		[0.0000]	[0.0230]	[1.7633]	[0.0000]
Nikkei 225 (Japan)	0.0722	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0819	0.0000
	[0.9524]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]		[0.0000]	[1.4859]	[0.0000]
CSI 300 (China)	0.0000	0.0517	0.0000	0.0005	0.0000	0.0000	0.0000	-	0.0000	0.0376
	[0.0000]	[0.7403]	[0.0000]	[0.0076]	[0.0000]	[0.0000]	[0.0000]		[0.0000]	[0.9935]
Gold Fixing (PM)	0.0000	0.0097	0.0000	0.0000	0.1630	0.2190	0.1404	0.0175	-	0.0760
	[0.0000]	[0.1503]	[0.0000]	[0.0000]	[2.9073]	[3.8535]	[2.4732]	[0.3820]		[1.5238]
WTI (Crude Oil)	0.0131	0.0000	0.0405	0.0273	0.0000	0.0000	0.0000	0.0000	0.0000	-
	[0.2898]	[0.0000]	[0.7408]	[0.3157]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	

This table estimates the left-right cross tail correlation coefficients, which are defined by $\eta_{i,j} = \lim_{q\to 1} P(Z_j \geq F^{\leftarrow}_{Z_j}(q) | -Z_i \geq F^{\leftarrow}_{-Z_i}(q))$ among six traditional assets (S&P 500, Euro Stoxx 50, Nikkei 225, CSI 300, the London Gold Fixing (PM), the WTI Crude Oil), and two cryptocurrencies (Bitcoin and Ethereum). Asset j is the row asset while asset i is the column asset. The t-statistics of the estimations are given below in brackets.

left tail (Weibull type) in the latter sub-period.

(2) About the bivariate left tail correlations among the cryptocurrencies: we find that left tails of the cryptocurrencies are more correlated than the right tails; and the tail correlations increase when time passes, which is indicative of the rising systematic risks. Among the seven selected cryptocurrencies, Bitcoin has the highest average correlations with other cryptocurrencies, and this is a reflection of Bitcoin's leading role in the market.

(3) About the diversification and hedging abilities: Using the two largest cryptocurrencies, Bitcoin and Ethereum as examples, we find that cryptocurrencies are very good diversification assets for stocks, as their left tails are uncorrelated with the left tails of S&P 500, Euro Stoxx 50, Nikkei 225 and CSI 300 Index. Moreover, their left tails are also uncorrelated with the commodities of gold and crude oil. The tail independence suggests part of the safe-haven property of the cryptocurrencies, as cryptocurrencies are not exposed to the financial contagion of the stock market. Therefore, adding cryptocurrencies to the portfolio can help investors to achieve better diversification. In the tail risk sense, although the cryptocurrencies and gold can both well diversify the stock portfolios, cryptocurrencies cannot hedge stock portfolios' tail risks like the gold commodity does. When the stock market experience extremal shortfall, cryptocurrencies are not exposed to a higher probability of large appreciation, but gold is.

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