Task 1

a.) is $2^{n+1} = O(2^n)$?

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} \frac{2^{n+1}}{2^n} = \lim_{n \to \infty} \frac{2 \cdot 2^n}{2^n} = 2$$

Since the dominant terms of the top and bottom have the same degree then the limit is the ratio of the coefficients

Since $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$ or $c$, for $O(n)$ then we can say that $2^{n+1} = O(2^n)$.

b.) is $2^{2n} = O(2^n)$?

$$\lim_{n \to \infty} \frac{2^{2n}}{2^n} = \lim_{n \to \infty} \frac{(2^n)^n}{2^n} = \ln \lim_{n \to \infty} \frac{(2^n)^n}{2^n} = \lim_{n \to \infty} \frac{\ln(2^n)^n}{\ln(2^n)} = \lim_{n \to \infty} \frac{n\ln(2^n)}{\ln(2^n)}$$

$$\lim_{n \to \infty} \frac{n\ln(2^n)}{\ln(2^n)} = \lim_{n \to \infty} \frac{n^2 \cancel{\ln(2)}}{n \cancel{\ln}(2)} = \lim_{n \to \infty} \frac{n^2}{n} = \lim_{n \to \infty} \frac{n}{1} = \infty$$

Thus since the limit resulted in something other than $0$ or $c$ the $2^{2n} \neq O(2^n)$.

Task 2 Let $f(n) = (4/9)^0 + (4/9)^1 + (4/9)^2 + \ldots + (4/9)^n$. Find $\Theta$ for $f(n)$.

$$\sum_{k=0}^{n} (4/9)^k = \frac{(4/9)^{n+1} - 1}{x-1} = \Theta\left(\left(\tfrac{4}{9}\right)^n\right)$$