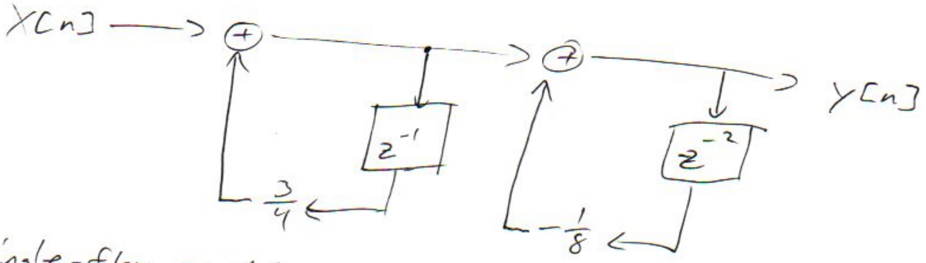
Matthew Irvine

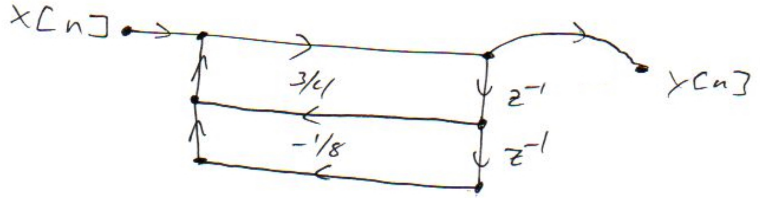
1. $y[n] - \frac{3}{4} y[n-1] + \frac{1}{8} y[n-2] = x[n]$

$$y[n] = \frac{3}{4} y[n-1] - \frac{1}{8} y[n-2] + x[n]$$

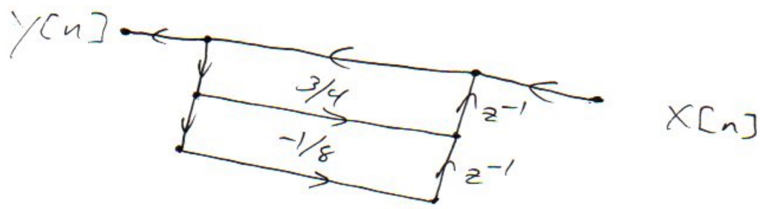
a) block-diagram:



b) single-flow graph:



c)

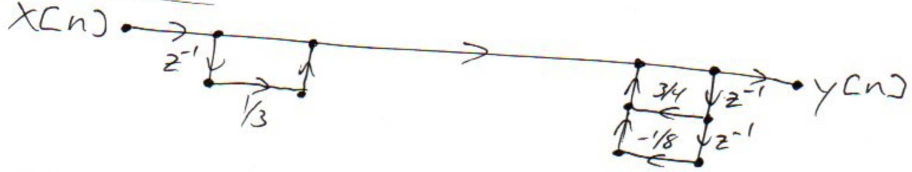


2.

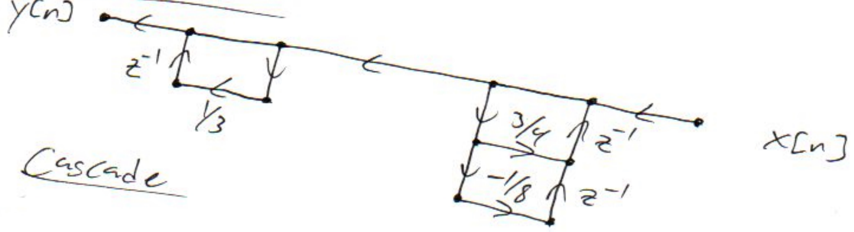
$$y[n] - \frac{3}{4} y[n-1] + \frac{1}{8} y[n-2] = x[n] + \frac{1}{3} x[n-1]$$

$$y[n] = \frac{3}{4} y[n-1] - \frac{1}{8} y[n-2] + x[n] + \frac{1}{3} x[n-1]$$

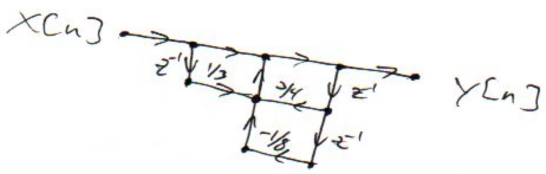
a) Direct Form I



b) Direct form II

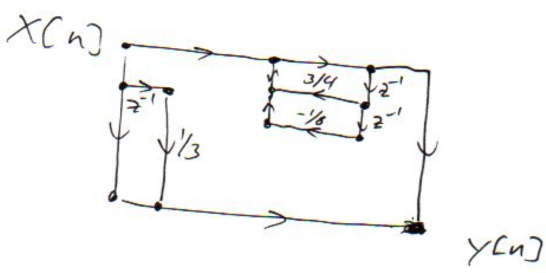


c) Cascade

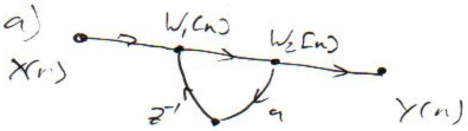


d) Parallel

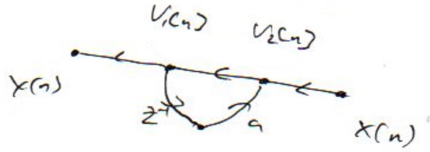
$$H(z) = \frac{\frac{3}{4} z^{-1} Y[z] - \frac{1}{8} z^{-2} Y(z)}{X(z) + \frac{1}{3} z^{-1} X(z)}$$



3. a)



Trans. $\Rightarrow$

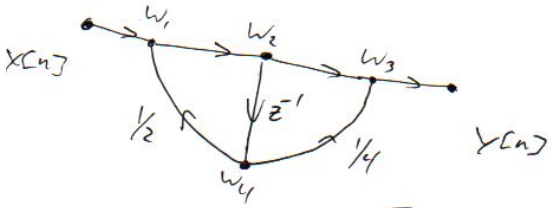


These are the same transfer function

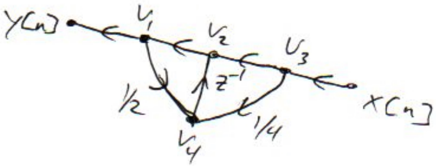
$Y[n] = aY[n-1] + X[n]$

$Y[n] = aY[n-1] + X[n]$

b)

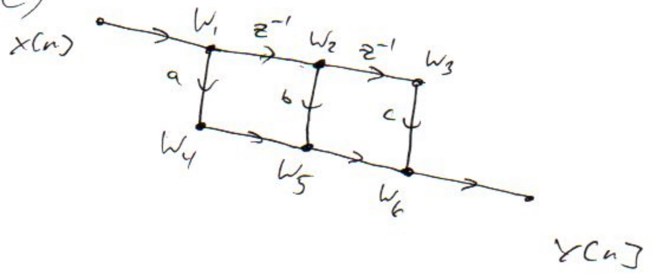


Trans. $\Rightarrow$

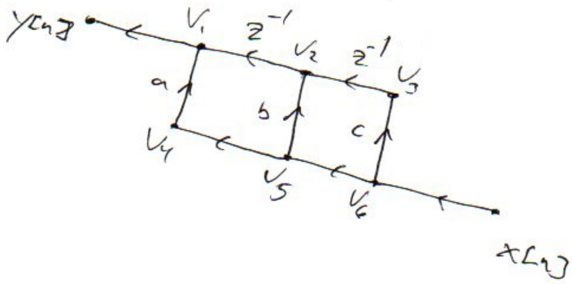


These are the same

c)



Trans. $\Rightarrow$



Calculations are made on the next page(s).

3. a2) original:

$W_1[n] = X[n] + a z^{-1} W_2[n]$

$W_2[n] = W_1[n]$

$Y[n] = W_2[n] \Rightarrow Y[n] = W_1[n] \Rightarrow Y[n] = a z^{-1} W_2[n] + X[n]$

Transposed:

$V_1[n] = V_2[n]$

$V_2[n] = X[n] + a z^{-1} V_1[n]$

$Y[n] = V_1[n] \Rightarrow Y[n] = V_2[n] \Rightarrow Y[n] = a z^{-1} V_1[n] + X[n]$

these are equivalent

basically $V_1[n] = W_2[n]$ and thus $V_2[n] = W_1[n]$ meaning they are equivalent.

b2) Or.:

$W_1 = \frac{1}{2} W_4 + x[n]$ $\quad W_2 = W_1$

$W_4 = z^{-1} W_2 \quad W_3 = \frac{1}{4} W_4 + W_2$

$Y[n] = W_3 \Rightarrow Y[n] = \frac{1}{4} W_4 + W_2 \Rightarrow Y[n] = \frac{1}{4} z^{-1} W_2 + W_1$

$\Rightarrow Y[n] = \frac{1}{4} z^{-1} W_1 + \frac{1}{2} W_4 + X[n]$

Tr.:

$V_1 = V_2 \quad V_2 = V_3 + z^{-1} V_4$

$V_4 = \frac{1}{2} V_1 + \frac{1}{4} V_3 \quad V_3 = X[n]$

$Y[n] = V_1 \Rightarrow Y[n] = V_2 \Rightarrow Y[n] = V_3 + z^{-1} V_4 \Rightarrow Y[n] = X[n] + z^{-1}[\frac{1}{2} V_1 + \frac{1}{4} V_3]$

$Y[n] = \frac{1}{4} z^{-1} V_3 + \frac{1}{2} z^{-1} V_1 + X[n]$

These are the same.

both $Y[n]$ follow a similar format for the same nodes.

3. c2) Or.:

$W_1 = x[n] \quad W_2 = z^{-1}W_1 \quad W_3 = z^{-1}W_2$

$W_4 = aW_1 \quad W_5 = W_4 + bW_2 \quad W_6 = W_5 + cW_3$

$y[n] = W_6$

$= W_5 + cW_3 \Rightarrow W_4 + bW_2 + cz^{-1}W_2$

$= aW_1 + bz^{-1}W_1 + cz^{-2}W_1$

$= W_1(a + bz^{-1} + cz^{-2})$

$= x[n](a + bz^{-1} + cz^{-2})$

Trans.:

$V_1 = aV_4 + z^{-1}V_2 \quad V_2 = z^{-1}V_3 + bV_5 \quad V_3 = cV_6$

$V_4 = V_5 \quad V_5 = V_6 \quad V_6 = x[n]$

$y[n] = V_1$

$= aV_4 + z^{-1}V_2$

$= aV_5 + z^{-1}(z^{-1}V_3 + bV_5) = aV_5 + z^{-2}V_3 + bz^{-1}V_5$

$= aV_6 + cz^{-2}V_6 + bz^{-1}V_6$

$= V_6(a + bz^{-1} + cz^{-2})$

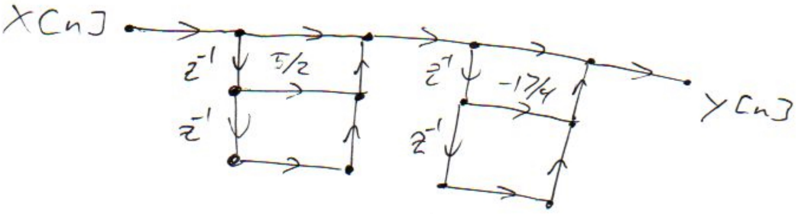
$= x[n](a + bz^{-1} + cz^{-2})$

These are the same

4. $H(z) = \left(1 + \frac{1}{2} z^{-1}\right)\left(1 + 2z^{-1}\right)\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - 4z^{-1}\right)$

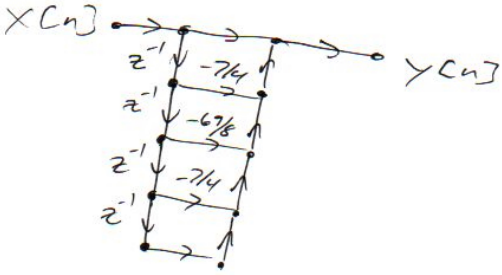
a) Cascade

$$H(z) = \left(1 + 2z^{-1} + \tfrac{1}{2} z^{-1} + z^{-2}\right)\left(1 - 4z^{-1} - \tfrac{1}{4} z^{-1} + z^{-2}\right)$$
$$= \left(1 + 5/2\, z^{-1} + z^{-2}\right)\left(1 - 17/4\, z^{-1} + z^{-2}\right)$$



b) Direct form

$$H(z) = \left(1 + 5/2\, z^{-1} + z^{-2}\right)\left(1 - 17/4\, z^{-1} + z^{-2}\right)$$
$$= \left(1 - 17/4\, z^{-1} + z^{-2} + 5/2\, z^{-1} - \tfrac{85}{8} z^{-2} + 5/2\, z^{-3} + z^{-2} - 17/4\, z^{-3} + z^{-4}\right)$$
$$= \left(1 - 7/4\, z^{-1} - \tfrac{69}{8} z^{-2} - 7/4\, z^{-3} + z^{-4}\right)$$

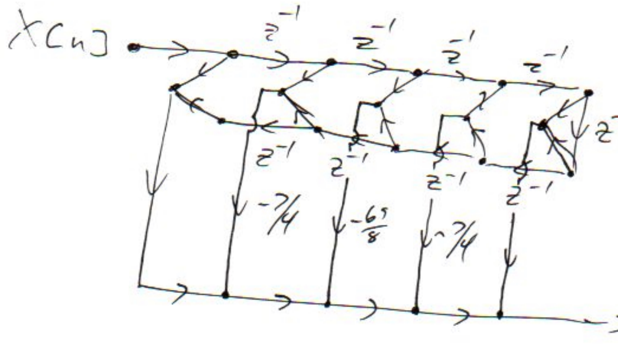


4. c) $H(z) = 1 - 7/4\, z^{-1} - \frac{69}{8} z^{-2} - 7/4\, z^{-3} + z^{-4}$

$4 = \frac{N}{2} - 1$

$5 = \frac{N}{2}$

$10 = N$



$H(z) = \sum_{n=0}^{4} h[n]\left[z^{-n} + z^{-(9-n)}\right]$

d) Did not complete