Pontryagin minimum fuel orbit transfer and docking with model predictive control

Objective

- Minimum fuel orbit transfer to geostationary orbit with safety constraints for proximity operations.

Approach

- Apply Pontryagin's Minimum Principle to Orbit Transfer (Level 1)
- Achieve docking from radially outward direction under applicable safety constraints (Level 2)

$$\mathbf{f}(\mathbf{x},\mathbf{u}) = \dot{\mathbf{x}} = \begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{v}} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ -\frac{\mu}{r^3}\mathbf{r} + \frac{1}{m}\mathbf{u} \\ -\alpha\|\mathbf{u}\|_2 \end{bmatrix}$$

$$\Gamma = \begin{cases} u_{\max}, & \text{if } S < 0 \\ 0, & \text{if } S > 0 \end{cases} \qquad S = 1 - \frac{1}{m}\|\lambda_v\|_2 - \lambda_m \alpha$$

$$\Psi(\mathbf{z}) = \begin{bmatrix} \mathbf{r}_s - \mathbf{r}_t \\ \mathbf{v}_s - \mathbf{v}_t \\ \lambda_m(t_f) \end{bmatrix} \in \mathbb{R}^7, \quad \mathbf{z} = \lambda_0$$

$$J = \sum_{k=1}^{N} [\ \mathbf{x}_k Q \mathbf{x}_k + \mathbf{u}_k R \mathbf{u}_k + w s_k\ ] + \mathbf{x}_N T \mathbf{x}_N$$

$$\mathbf{x}_{k+1} = A_k \mathbf{x}_k + B_k \mathbf{u}$$

$$\sqrt{y^2 + z^2} \le x \tan\theta + s$$

$$n^\top \mathbf{r}_s \ge n^\top \mathbf{r}_0$$

$$\|\mathbf{u}\|_\infty \le u_{\max}$$

Discussion

- Best case orbit transfer achieved with low efficiency engine, minimized fuel consumption with low transfer time
- Constrained docking scenario with negligible cost, handles realistic constraints with quadratic programming formulation

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Results

Result	Orbit Transfer Phase	Proximity Phase	Total	Unit
delta-v	2.1834	0.0349	2.2183	km/s
elapsed time	571.392	3.733	575.125	minutes
fuel mass spent	1,002.36	11.03	1,013.39	kg