Starting in calculus 3, we use 3d coordinate system rather than 2d.

Z

P(3,3,3)

**Y**

X
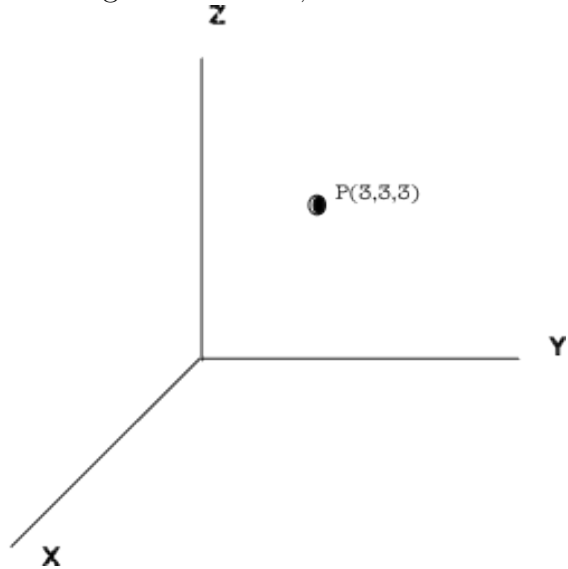
To form the graph, refer to the chart below.

Point index to pinky finger in direction of 1; this shows the x-axis. Then curl fingers towards 2 for the y-axis. The thumb indicates the z axis.

3

1

2
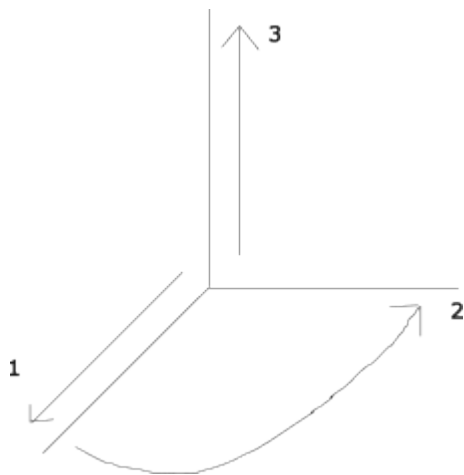
We use the **right hand rule** to get this (we use the right hand)

We can create a projection of a point onto xy, xz, yz plane by setting z, y, or x to 0.

Z

Projection onto xz-axis

Projection onto zy-axis

P(5,4,2)

Y

Projection onto xy-axis

X
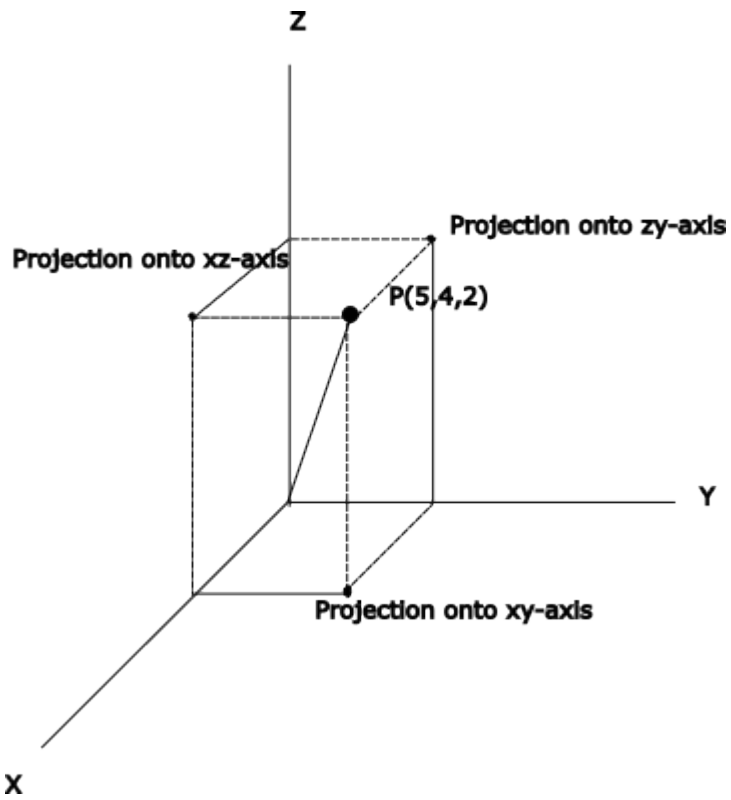
Here the point P(5,4,2) is plotted and the projections onto the 3 planes
are displayed. From left to right: (0,4,2), (5,0,2), (5,4,0)
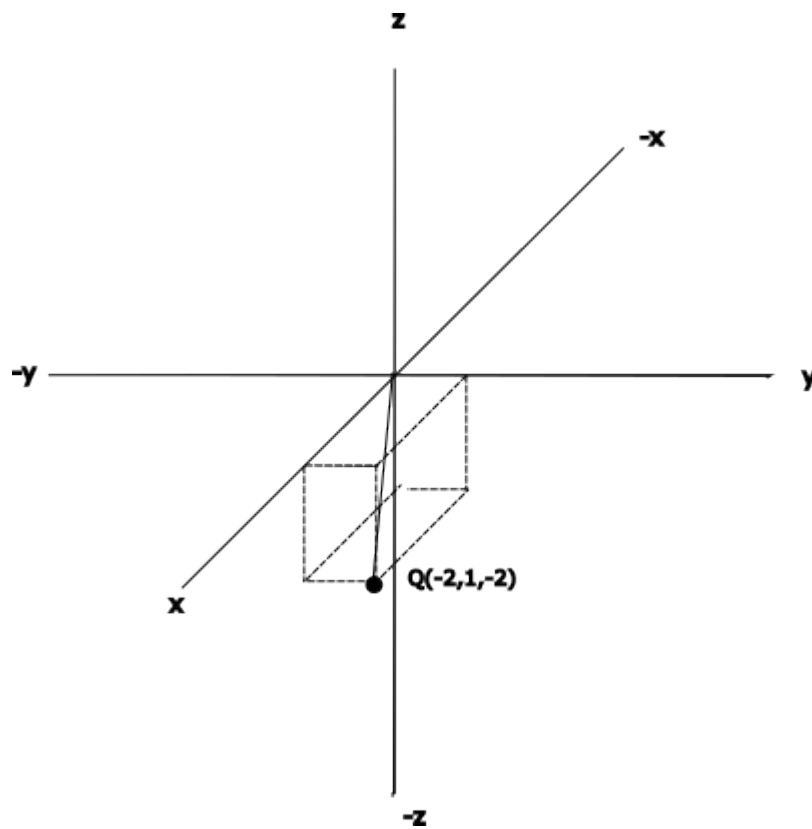
The graph above demonstrates a point Q(2,1,-2) where the x coordinates
are now negative. An error was made where the x coordinate in the graph is
negative when it is in fact positive.
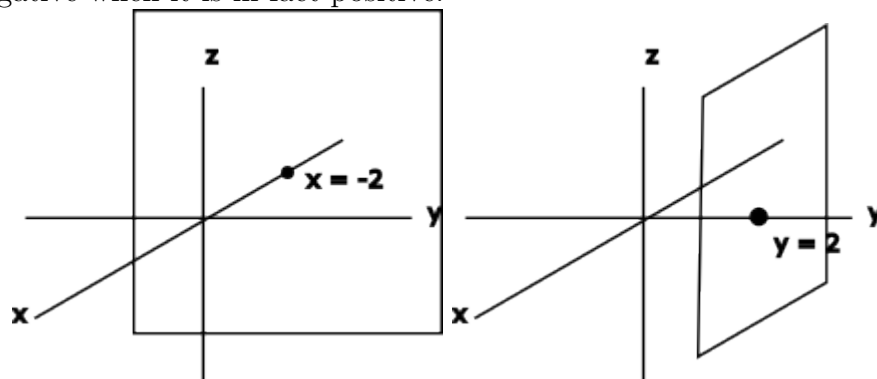
The preceding graphs demonstrate planes parallel to the zy and xz planes
respectively. Notice for each of the graphs, there is a single fixed point and
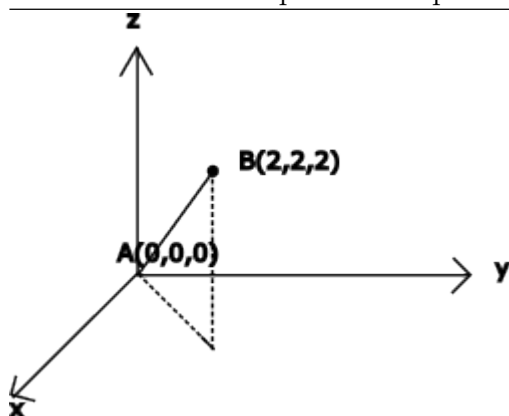domains for unfixed coordinates are $\in R$.

Finally, below is a demonstration of a plane parallel to the xy plane.

H(0,0,2)

z

y

x
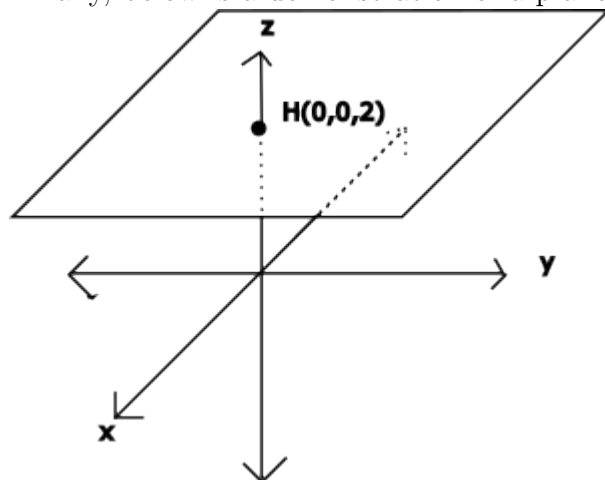
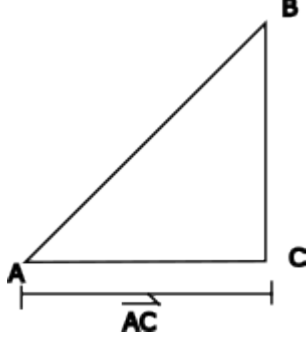Below is a comparison between $x^2 + y^2 = 1$, the equation for a circle, on a 2d graph vs on a 3d graph.

y

x

z

y

x
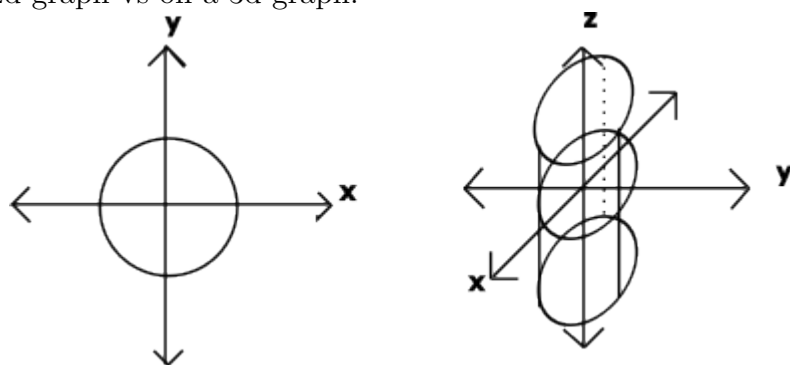
Find distance from point A to point B

z

B(2,2,2)

A(0,0,0)

y

x

Visualize: a triangle where you are finding hypotenuse, distance between

4

point $A(0,0,0)$ and point $B(0,0,0)$, otherwise known as the magnitude of $\vec{AB}$, which can be represented as $|\vec{AB}|$.

We can find $|\vec{AC}|$ by taking the result of applying pythagorean function on difference between the X and Y coordinates on the points B and A.

$$Pythagoras = a^2 + b^2 = c^2 \tag{1}$$
$$a = x_b - x_a, \ b = y_b - y_a \tag{2}$$
$$2^2 + 2^2 = |\vec{AC}|^2 \tag{3}$$
$$\therefore |\vec{AC}| = \sqrt{8} \tag{4}$$

.

Thus, we can solve for $|\vec{AB}|$ by then applying the pythagorean formula once more this time using $|\vec{AC}|$.

$$|\vec{AB}| = \sqrt{|\vec{AC}|^2 + (z_b - z_a)^2} \tag{1}$$
$$|\vec{AC}|^2 = (x_b - x_a)^2 + (y_b - y_a)^2 = 2^2 + 2^2 \tag{2}$$
$$|\vec{AB}| = \sqrt{2^2 + 2^2 + 2^2} \tag{3}$$
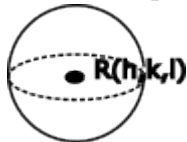$$\therefore |\vec{AB}| = \sqrt{12} \tag{4}$$

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As can be seen in (3), the equation for the distance between any given point $P(x, y, z)$ and the origin$(0, 0, 0)$ can be represented as:
$|\vec{P}| = \sqrt{x^2 + y^2 + z^2}$
We can also say that the distance between any 2 points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ can be represented by the equation:
$|\vec{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here is a sphere centered on point $R(h, k, l)$

The formula for a circle is $x^2 + y^2 = 1$

Similarily, the formula for a sphere centered at the origin $(0, 0, 0)$ is:
$x^2 + y^2 + z^2 = 1$

Standard form of a sphere:
$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$

...

Exercise: find the centre and radius of a sphere with equation:
$x^2 + y^2 + z^2 + 6x - 4y - 2z + 6 = 0$

Group like terms.

$$x^2 + 6x + y^2 - 4y + z^2 - 2z + 6 = 0 \tag{1}$$

Complete the square.

$$x^2 + 6x + 9 - 9 + y^2 - 4y + 4 - 4 + z^2 - 2z + 1 - 1 + 6 = 0 \tag{2}$$

Factor

$$(x + 3)^2 + (y - 2)^2 + (z - 1)^2 = 8 \tag{3}$$

Thus, the center of the sphere lies at coordinates $(-3, 2, 1)$ and the radius of the sphere is $\sqrt{8}$.