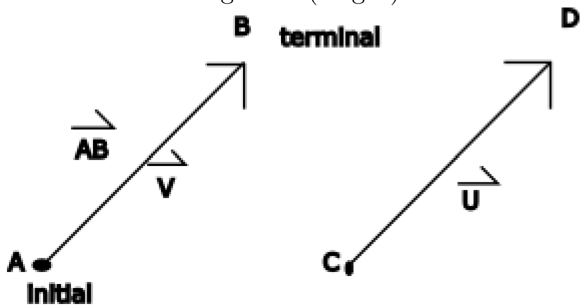
1 Vectors

Vectors have a magnitude(length) and direction.

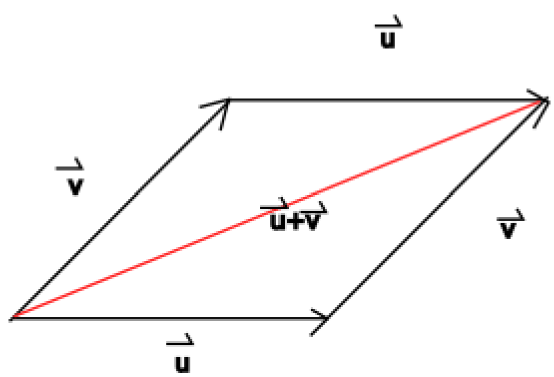


$\vec{u}$ & $\vec{v}$ have the same direction and magnitude, $\therefore$ they are equivalent.

Zero Vector $\vec{O}$ has length zero.

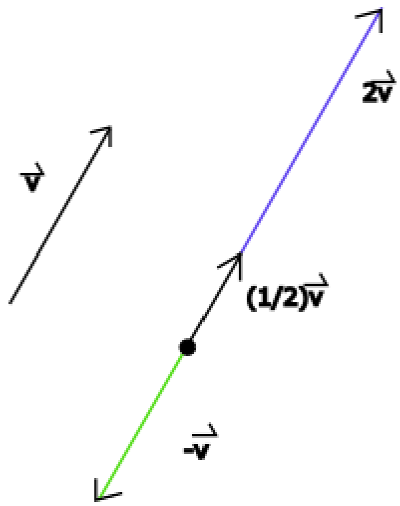
Vectors appear in forces, position, velocity, acceleration, torque, displacement, images.

Sums $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

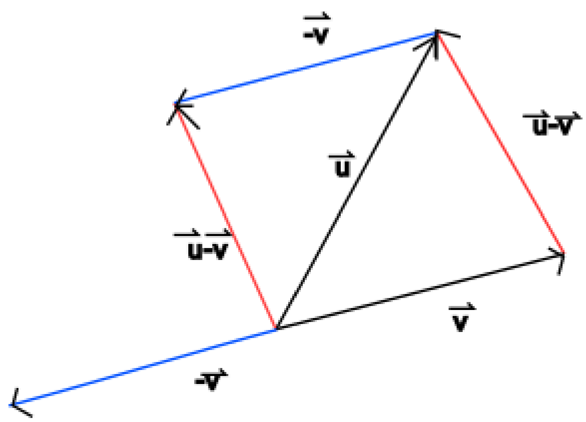


Scalar Multiplication

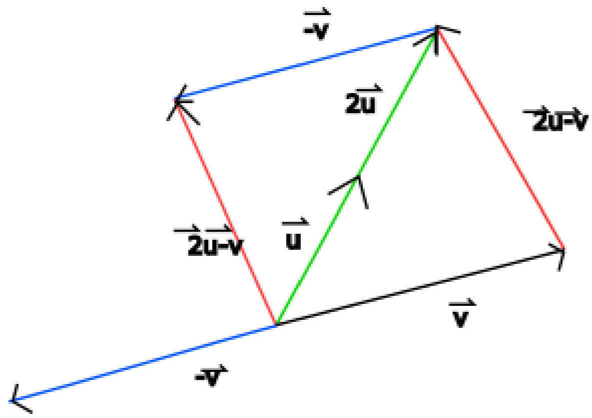
- If $c \in \mathbb{R}$, then vector $c\vec{v}$ has length —c— times the length of $\vec{v}$ and
  - the same direction as $\vec{v}$ if $c > 0$
  - opposite direction as $\vec{v}$ if $c < 0$
- If $c = 0$ or $\vec{v} = \vec{o}$, then $c\vec{v} = 0$



Differences $\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$

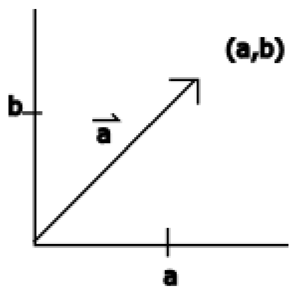


Ex: Consider $\vec{u}$ *and* $\vec{v}$. Sketch $2\vec{u} - \vec{v}$

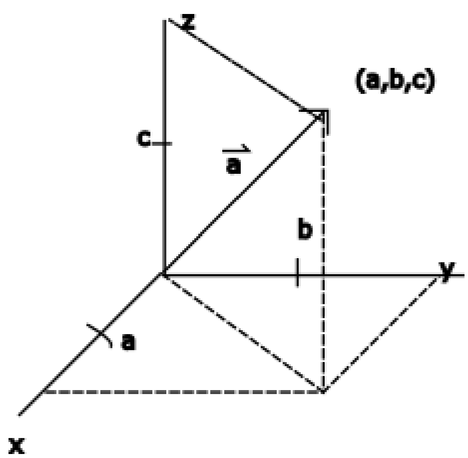


Components

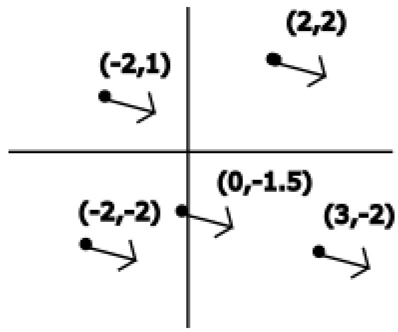
2D: $\vec{a} =<, a, b >$



3D: $\vec{a} =< a, b, c >$



Sketch vectors equivalent to $\vec{a} =< 2, -1 >$
Choose any initial position in the graph. So long as it obeys the magnitude and direction of the vector it is valid.



Unmarked in the graph is the point $o\vec{P}$ which is the position vector for point P, otherwise known as the origin.

Find components of the vector $\vec{a}$ that has the following:
Initial Point: $(3, 1)$
Terminal Point: $(-2, 5)$

Vector $\vec{a}$ has point $(-2-3, 5-1) = (-5, 4)$

Find components of the vector $\vec{b}$ that has the following:
Initial Point: $(1, 2, 3)$
Terminal Point: $(-2, 5, -7)$

Vector $\vec{a}$ has point $(-2-1, 5-2, -7-3) = (-3, 3, -10)$

To sum up, In general $\vec{AB}$ has components $B(x_2, y_2)$, $A(x_1, y_1)$ and is the result of $\vec{AB} =< x_2 - x_1, y_2 - y_1 >$

Vector $A\vec{B}C$ has components $B(x_2, y_2, z_2)$, $A(x_1, y_1, z_1)$.
It is the result of $A\vec{B}C =< x_2 - x_1, y_2 - y_1, z_2 - z_1 >$

Practice:

EX: Let $\vec{a} =< 0, 4, 3 >$ *and* $\vec{b} =< 1, -2, 5 >$. Find:

1. $|\vec{a}| = \sqrt{0^2 + 4^2 + 3^2} = \sqrt{25} = 5$
2. $\vec{a} + \vec{b} =< 0, 4, 3 > + < 1, -2, 5 >$
   $=< 0 + 1, 4 + (-2), 3 + 5 >$
   $=< 1, 2, 8 >$
3. $\vec{a} - \vec{b} =< 0 - 1, 4 - (-2), 3 - 5 >=< -1, 6, -2 >$
4. $4\vec{b} =< 4(1), 4(-2), 4(5) >$
   $=< 4, -8, 20 >$

5. $3\vec{a} + 2\vec{b}\ = 3 < 0, 4, 3 > +2 < 1, -2, 5 >$
   $=< 0 + 2, 12 + (-4), 9 + 10 >$
   $=< 2, 8, 19 >$

Notation

- V_2 ={all 2D vectors}= $\mathbb{R}^2 = \{< a, b >: a, b \in \mathbb{R}\}$
- V_3 ={all 3D vectors}= $\mathbb{R}^3$
- V_n ={all n-dimensional vectors}= $\mathbb{R}^n$
- $\mathbb{R}^\infty = \{(a_1, a_2, a_3, \ldots) : a_i \in \mathbb{R}\}$

Ideas: If $\vec{a} \in V_n$ then $\vec{a} =< a_1, a_2, \ldots, a_n >$

Properties

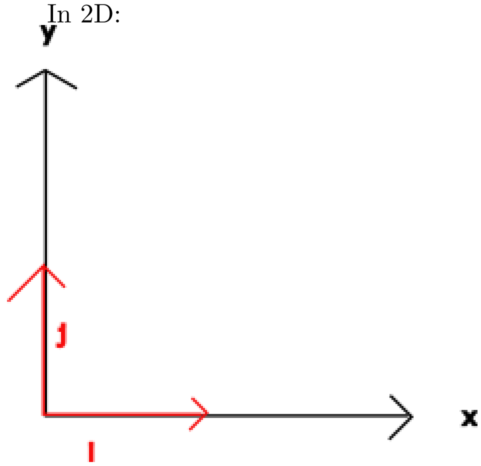
If $\vec{a}, \vec{b}, \vec{c} \in V_n$ *and* $c\&d$ are scalars, then:

- $\vec{a} + \vec{b} = \vec{b} + \vec{a}$: commutative property
- $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$: associative property
- $\vec{a} + \vec{0} = \vec{a}$: additive identity
- $\vec{a} + (\vec{-a}) = \vec{0}$: additive inverse
- $c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$: distributive property
- $(c + d)\vec{a} = c\vec{a} + d\vec{a}$: distributive property
- $(cd > \vec{a} = c(d\vec{a})$: associative property
- $1\vec{a} = \vec{a}$

Standard Basis Vectors

Set of vectors whose components are all zero except one which equals 1.

In 2D:



$\hat{i} =< 1, 0 >$, $\hat{j} =< 0, 1 >$

The following equation describes the components of a vector $\vec{a}$ using standard basis vectors.

$\vec{a} =< a_1, a_2 >$
$= a_1 < 1, 0 > + a_2 < 0, 1 >$
$= a_1\hat{i} + a_2\hat{j}$