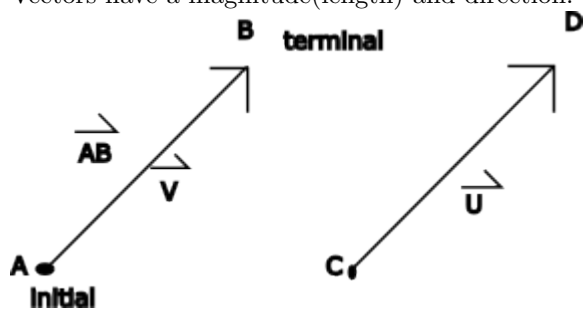


# 1 Vectors

Vectors have a magnitude(length) and direction.

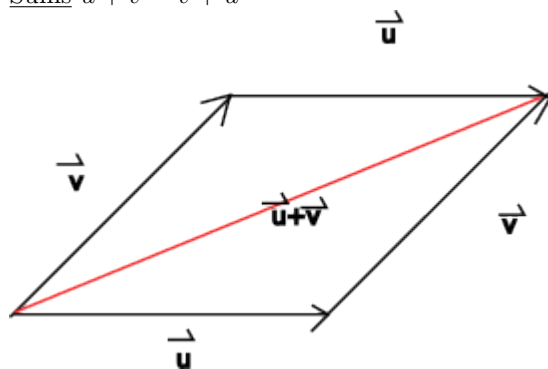


$\vec{u}$  &  $\vec{v}$  have the same direction and magnitude,  $\therefore$  they are equivalent.

Zero Vector  $\vec{O}$  has length zero.

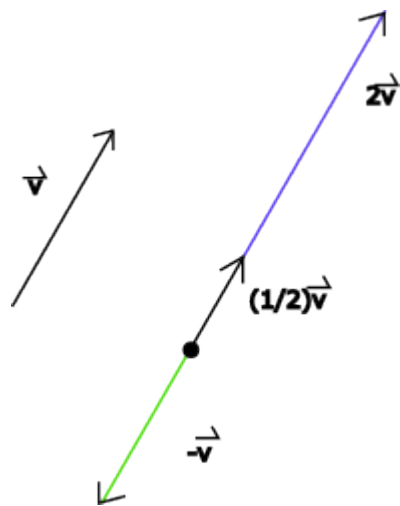
Vectors appear in forces, position, velocity, acceleration, torque, displacement, images.

Sums  $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

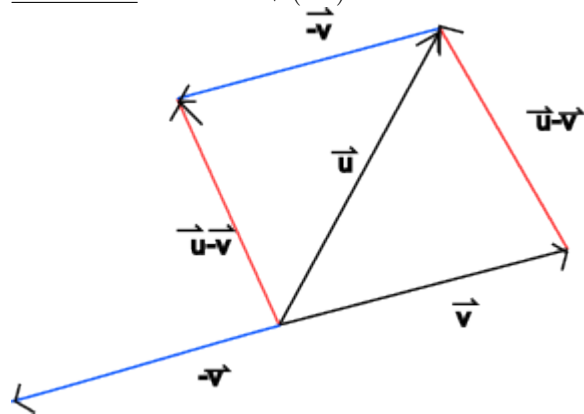


Scalar Multiplication

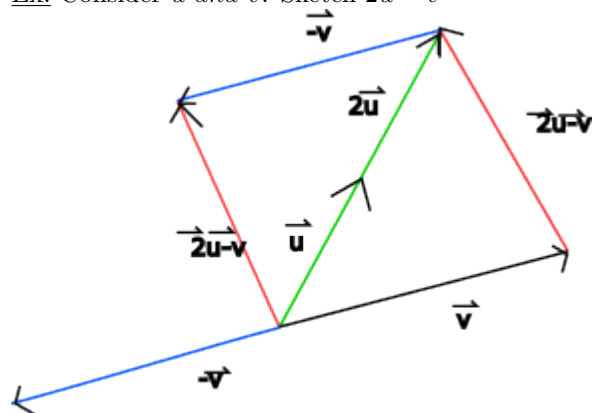
- If  $c \in \mathbb{R}$ , then vector  $c\vec{v}$  has length  $|c|$  times the length of  $\vec{v}$  and
  - the same direction as  $\vec{v}$  if  $c > 0$
  - opposite direction as  $\vec{v}$  if  $c < 0$
- If  $c = 0$  or  $\vec{v} = \vec{O}$ , then  $c\vec{v} = \vec{O}$



Differences  $\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$

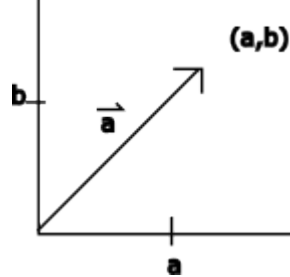


Ex: Consider  $\vec{u}$  and  $\vec{v}$ . Sketch  $2\vec{u} - \vec{v}$

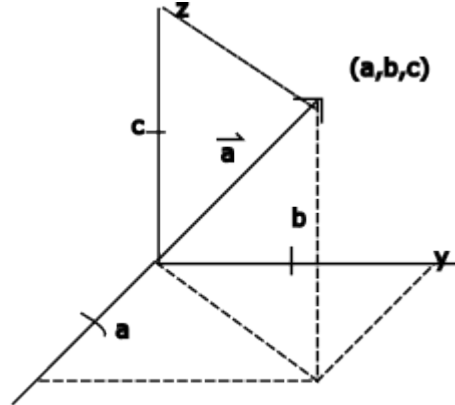


Components

2D:  $\vec{a} = \langle a, b \rangle$



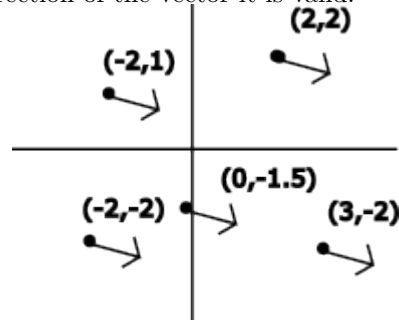
3D:  $\vec{a} = \langle a, b, c \rangle$



**x**

Sketch vectors equivalent to  $\vec{a} = \langle 2, -1 \rangle$

Choose any initial position in the graph. So long as it obeys the magnitude and direction of the vector it is valid.



Unmarked in the graph is the point  $o\vec{P}$  which is the position vector for point  $P$ , otherwise known as the origin.

Find components of the vector  $\vec{a}$  that has the following:

Initial Point:  $(3, 1)$

Terminal Point:  $(-2, 5)$

Vector  $\vec{a}$  has point  $(-2 - 3, 5 - 1) = (-5, 4)$

Find components of the vector  $\vec{b}$  that has the following:

Initial Point:  $(1, 2, 3)$

Terminal Point:  $(-2, 5, -7)$

Vector  $\vec{a}$  has point  $(-2 - 1, 5 - 2, -7 - 3) = (-3, 3, -10)$

To sum up, In general  $\vec{AB}$  has components  $B(x_2, y_2)$ ,  $A(x_1, y_1)$  and is the result of  $\vec{AB} = \langle x_2 - x_1, y_2 - y_1 \rangle$

Vector  $\vec{ABC}$  has components  $B(x_2, y_2, z_2)$ ,  $A(x_1, y_1, z_1)$ .  
It is the result of  $\vec{ABC} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$