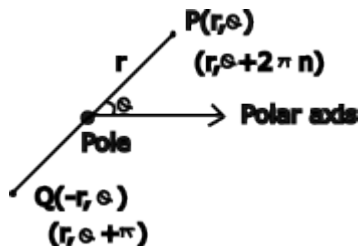


# 1 Curves in Polar Coordinates



Polar coordinates are constructed by the distance from the origin  $a$  (radius) and the angle  $\theta$ .

Previously, we used Rectangular Coordinates

Conversions

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\tan \theta = \frac{y}{x}$$

## 1.1 Example Polar 1

Ex: Convert  $(r, \theta) = (2, \frac{\pi}{3})$  to Cartesian coordinates.

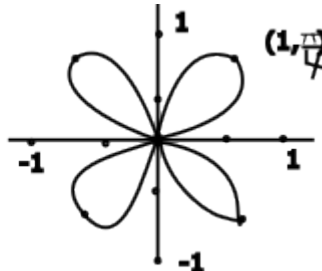
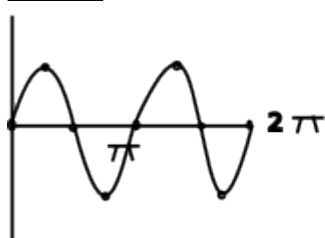
$$x = r \cos \theta, y = r \sin \theta \quad (1)$$

$$x = 2 \cos \frac{\pi}{3}, y = 2 \sin \frac{\pi}{3} \quad (2)$$

$$x = 1, y = \sqrt{3} \quad (3)$$

## 1.2 Example Polar 5

Graph:  $r = \sin 2\theta$



## 2 Parametric Equations in Polar Coordinates

Recall: Parametric Equations

if  $r = f(\theta)$ , then:

$$x = r \cos \theta = f(\theta) \cos \theta$$

$$y = r \sin \theta = f(\theta) \sin \theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-f'(\theta) \sin \theta}{f'(\theta) \cos \theta}$$

### 2.1 Parametric Example 1

### 2.2 Parametric Example 2

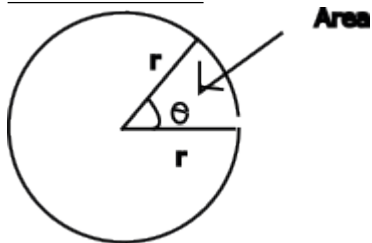
Find values of  $\theta$  where the tangent line is horizontal or vertical.

Hor:  $\frac{dy}{dx} = 0 = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

### 2.3 Parametric Example Indeterminate Form

## 3 Areas and Lengths in Polar Coordinates

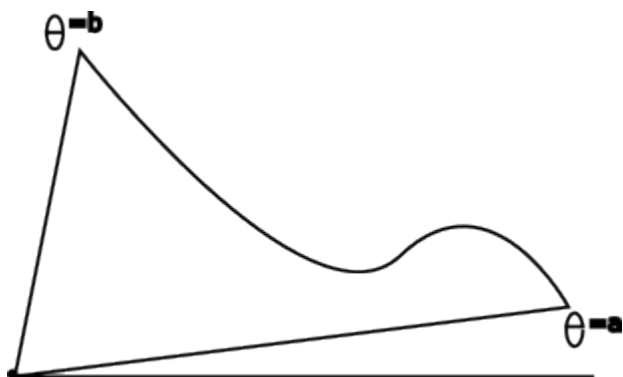
Area of Sector



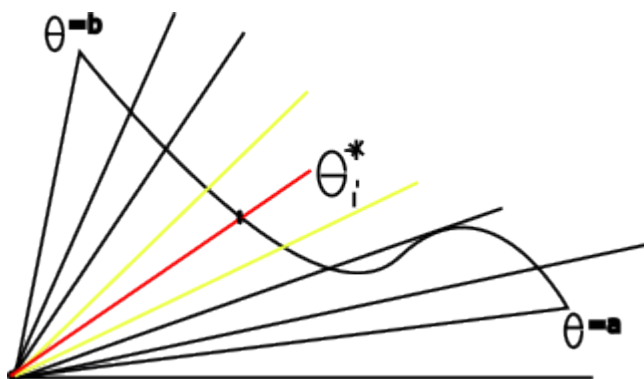
Area =  $A$

Area of a Polar Region

$$r = f(\theta)$$

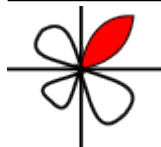


Divide polar region into sections.



### 3.1 Sector area Example 1

Consider  $r = \sin 2\theta$  Find the area of one leaf/petal.



$$1 + 2 = 3 \quad (1)$$

### 3.2 Sector area Example 2

Find the area inside the circle  $r = 3 \cos \theta$  and outside the cardioid

$r = 1 + \cos \theta$

Find intersection points

$$3 \cos \theta = 1 + \cos \theta$$

$$\cos \theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{3}, \frac{-\pi}{3}$$

### 3.3 Sector area Example 3

Find all points of intersection between:

a)  $r = \frac{1}{2}$  and  $r = \sin 2\theta$