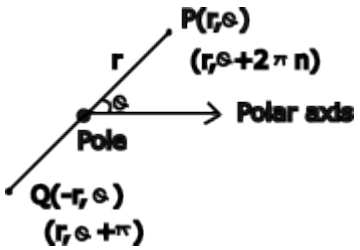
1 Curves in Polar Coordinates



Polar coordinates are constructed by the distance from the origin a (radius) and the angle θ.
Previously, we used Rectangular Coordinates
Conversions
$x = r\cos\theta$
$y = r\sin\theta$
$\tan\theta = \frac{y}{x}$

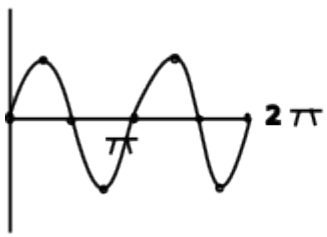
1.1 Example Polar 1

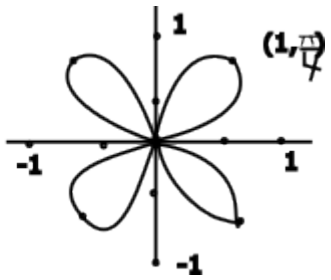
Ex: Convert $(r, \theta) = (2, \frac{\pi}{3})$ to Cartesian coordinates.

$$x = r\cos\theta, y = r\sin\theta \tag{1}$$
$$x = 2\cos\frac{\pi}{3}, y = 2\sin\frac{\pi}{3} \tag{2}$$
$$x = 1, y = \sqrt{3} \tag{3}$$

1.2 Example Polar 5

Graph: $r = \sin 2\theta$





2 Parametric Equations in Polar Coordinates

Recall: Parametric Equations
if $r = f(\theta)$, then:
$x = r\cos\theta = f(\theta)\cos\theta$
$y = r\sin\theta = f(\theta)\sin\theta$
$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-f'(\theta)\sin\theta}{f'(\theta)\cos\theta}$

2.1 Parametrc Example 1

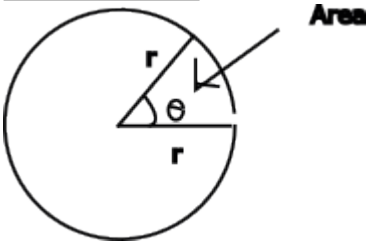
2.2 Parametric Example 2

Find values of θ where the tangent line is horizontal or vertical.
Hor: $\frac{dy}{dx} = 0 = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

2.3 Parametric Example Indeterminate Form

3 Areas and Lengths in Polar Coordinates

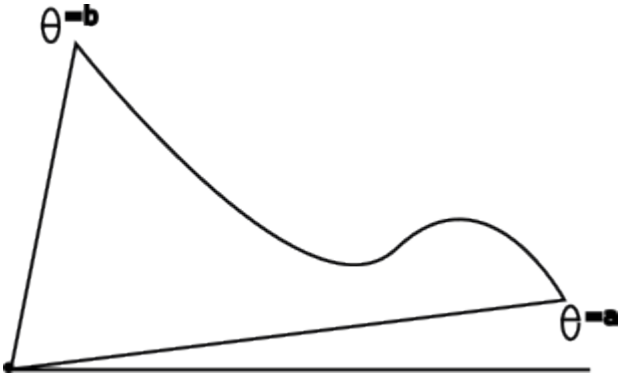
Area of Sector



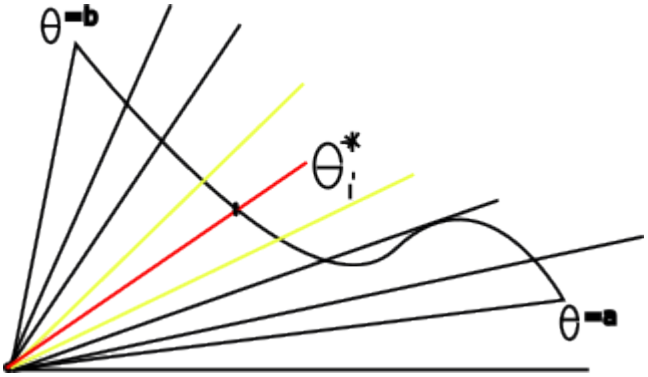
Area $= A$

Area of a Polar Region

$r = f(\theta)$



Divide polar region into sections.



3.1 Sector area Example 1

Consider $r = \sin 2\theta$ Find the area of one leaf/petal.

$$1 + 2 = 3 \tag{1}$$

3.2 Sector area Example 2

Find the area inside the circle $r = 3\cos\theta$ and outside the cardiod $r = 1 + \cos\theta$
Find intersection points

$3\cos\theta = 1 + \cos\theta$
$\cos\theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{3}, \frac{-\pi}{3}$

3.3 Sector area Example 3

Find all points of intersection between:
a) $r = \frac{1}{2}$ and $r = \sin 2\theta$