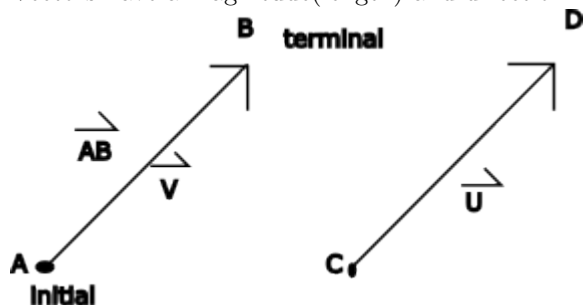# 1 Vectors

Vectors have a magnitude(length) and direction.
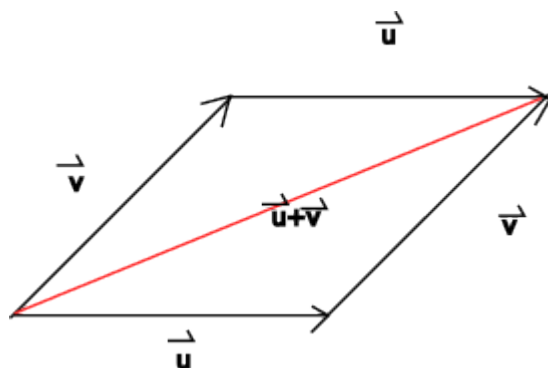

$\vec{u}$ & $\vec{v}$ have the same direction and magnitude, $\therefore$ they are equivalent.

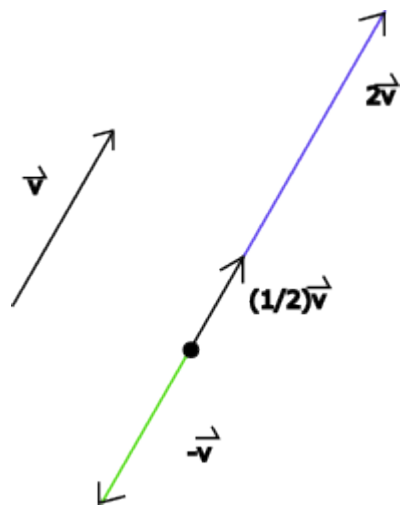<u>Zero Vector</u> $\vec{O}$ has length zero.

Vectors appear in forces, position, velocity, acceleration, torque, displacement, images.

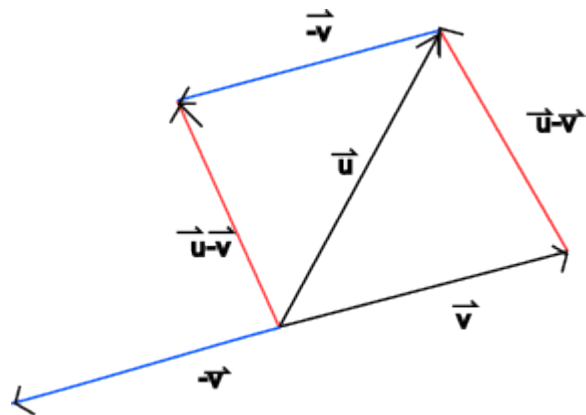<u>Sums</u> $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

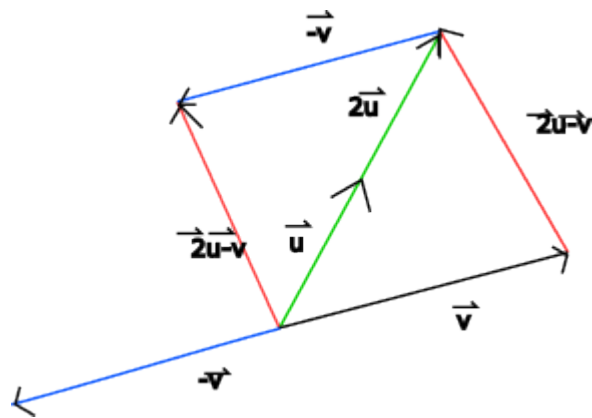<u>Scalar Multiplication</u>

- If $c \in \mathbb{R}$, then vector $c\vec{v}$ has length —c— times the length of $\vec{v}$ and
    - the same direction as $\vec{v}$ if $c > 0$
    - opposite direction as $\vec{v}$ if $c < 0$
- If $c = 0$ or $\vec{v} = \vec{o}$, then $c\vec{v} = 0$
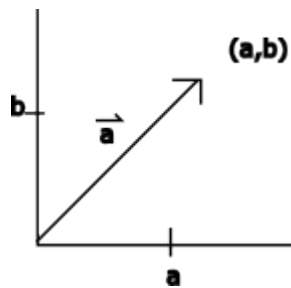
Differences $\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$

Ex: Consider $\vec{u}$ and $\vec{v}$. Sketch $2\vec{u} - \vec{v}$

<u>Components</u>
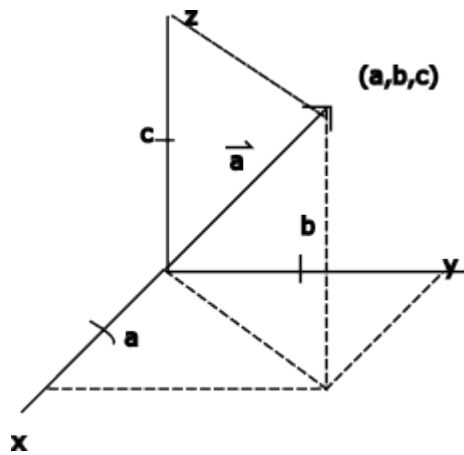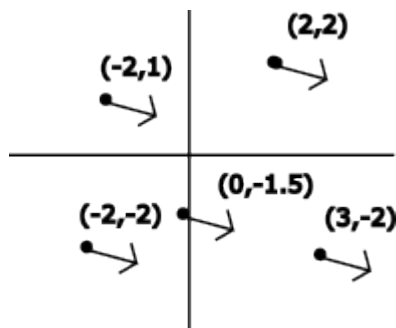
<u>2D:</u> $\vec{a} =<, a, b>$

<u>3D:</u> $\vec{a} =< a, b, c >$

Sketch vectors equivalent to $\vec{a} =< 2, -1 >$

Choose any initial position in the graph. So long as it obeys the magnitude and direction of the vector it is valid.

Unmarked in the graph is the point $o\vec{P}$ which is the position vector for point P, otherwise known as the <u>origin</u>.

Find components of the vector $\vec{a}$ that has the following:
<u>Initial Point</u>: $(3, 1)$
<u>Terminal Point</u>: $(-2, 5)$

Vector $\vec{a}$ has point $(-2 - 3, 5 - 1) = (-5, 4)$

Find components of the vector $\vec{b}$ that has the following:
<u>Initial Point</u>: $(1, 2, 3)$
<u>Terminal Point</u>: $(-2, 5, -7)$

Vector $\vec{a}$ has point $(-2 - 1, 5 - 2, -7 - 3) = (-3, 3, -10)$

To sum up, In general $\vec{AB}$ has components $B(x_2, y_2), \ A(x_1, y_1)$ and is the result of $\vec{AB} = < x_2 - x_1, y_2 - y_1 >$

Vector $\vec{ABC}$ has components $B(x_2, y_2, z_2), \ A(x_1, y_1, z_1)$.
It is the result of $\vec{ABC} = < x_2 - x_1, y_2 - y_1, z_2 - z_1 >$

Practice:

<u>EX:</u> Let $\vec{a} = < 0, 4, 3 > \ and \ \vec{b} = < 1, -2, 5 >$. Find:

1. $|\vec{a}| = \sqrt{0^2 + 4^2 + 3^2} = \sqrt{25} = 5$

2. $\vec{a} + \vec{b} \ = < 0, 4, 3 > + < 1, -2, 5 >$
   $= < 0 + 1, 4 + (-2), 3 + 5 >$
   $= < 1, 2, 8 >$

3. $\vec{a} - \vec{b} \ = < 0 - 1, 4 - (-2), 3 - 5 > = < -1, 6, -2 >$

4. $4\vec{b} \ = < 4(1), 4(-2), 4(5) >$
   $= < 4, -8, 20 >$

4

5. $3\vec{a} + 2\vec{b} = 3 < 0, 4, 3 > +2 < 1, -2, 5 >$
   $=< 0 + 2, 12 + (-4), 9 + 10 >$
   $=< 2, 8, 19 >$

Notation

- $V_2 = \{$all 2D vectors$\} = \mathbb{R}^2 = \{< a, b >: a, b \in \mathbb{R}\}$

- $V_3 = \{$all 3D vectors$\} = \mathbb{R}^3$

- $V_n = \{$all n-dimensional vectors$\} = \mathbb{R}^n$

- $\mathbb{R}^\infty = \{(a_1, a_2, a_3, \ldots) : a_i \in \mathbb{R}\}$

Ideas: If $\vec{a} \in V_n$ then $\vec{a} =< a_1, a_2, \ldots, a_n >$

Properties

If $\vec{a}, \vec{b}, \vec{c} \in V_n$ and $c \& d$ are scalars, then:

- $\vec{a} + \vec{b} = \vec{b} + \vec{a}$: commutative property

- $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$: associative property

- $\vec{a} + \vec{0} = \vec{a}$: additive identity

- $\vec{a} + (-\vec{a}) = \vec{0}$: additive inverse

- $c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$: distributive property

- $(c + d)\vec{a} = c\vec{a} + d\vec{a}$: distributive property

- $(cd > \vec{a} = c(d\vec{a})$: associative property
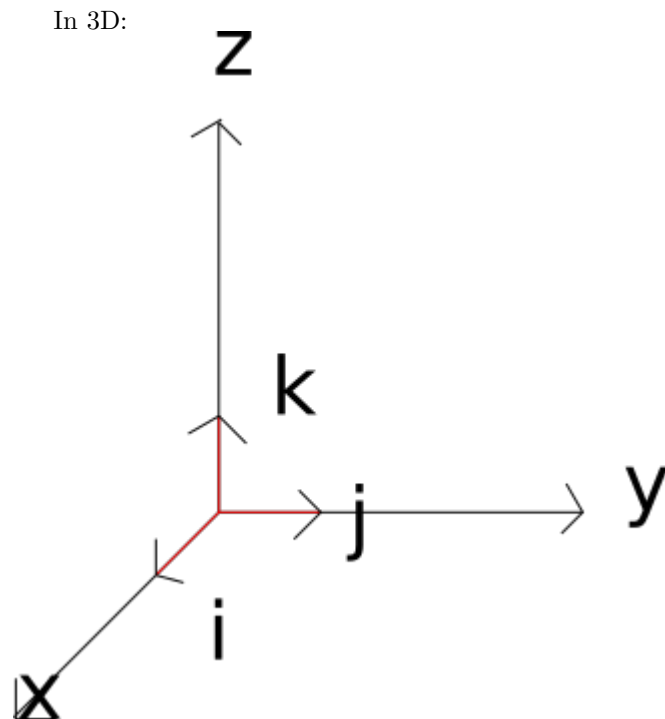
- $1\vec{a} = \vec{a}$

Standard Basis Vectors
Set of vectors whose components are all zero except one which equals 1.
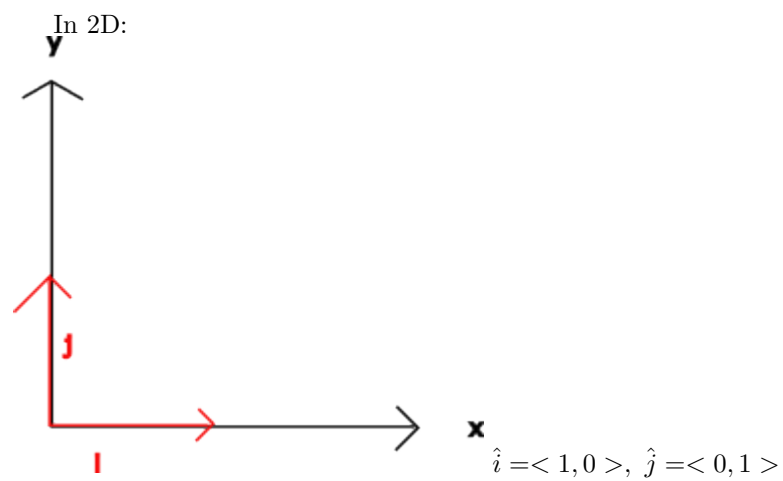
In 2D:

$$\hat{i} =< 1,0 >, \ \hat{j} =< 0,1 >$$

The following equation describes the components of a vector $\vec{a}$ using standard basis vectors.

$$\vec{a} =< a_1, a_2 >$$
$$= a_1 < 1,0 > +a_2 < 0,1 >$$
$$= a_1\hat{i} + a_2\hat{j}$$

In 3D:

The following equation describes the components of a vector $\vec{a}$ using standard basis vectors.

$$\vec{a} = <a_1, a_2, a_3> \qquad (1)$$
$$= a_1 <1, 0, 0> + a_2 <0, 1, 0> + a_3 <0, 0, 1> \qquad (2)$$
$$= a_1\hat{i} + a_2\hat{j} + a_3\hat{k} \qquad (3)$$