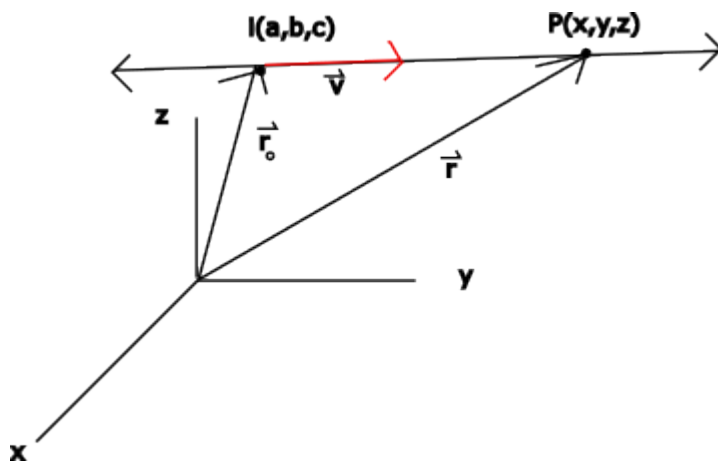


1 Equations of Lines and Planes



$$\vec{r} = \vec{r}_0 + t\vec{v}$$

t is the parameter of the equation.

Reiterate: $\vec{r} = \vec{r}_0 + t\vec{P}$

Let $\vec{v} = \langle a, b, c \rangle$

then $\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t \langle a, b, c \rangle$

$\langle x, y, z \rangle = \langle x_0 + ta, y_0 + tb, z_0 + tc \rangle$

So the parametric equations for a line are as follows:

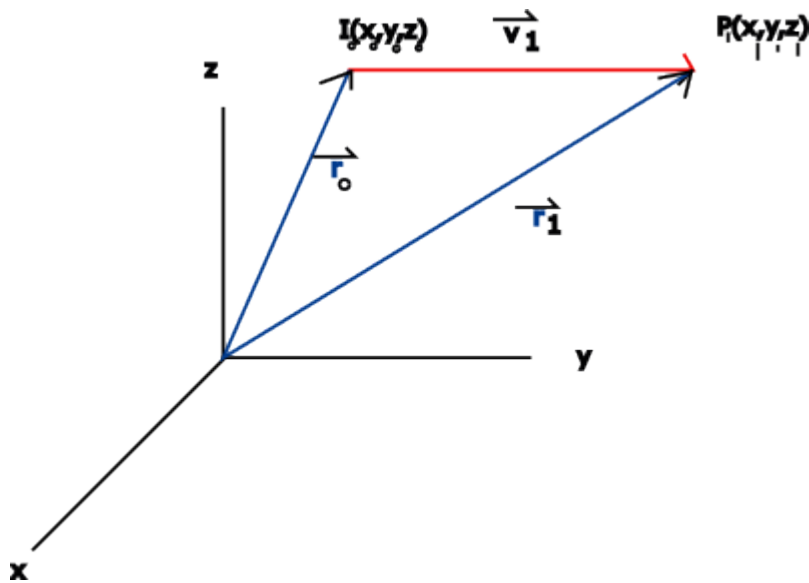
- $x = x_0 + ta$
- $y = y_0 + tb$
- $z = z_0 + tc$

When you solve for t , you get the symmetric equations.

$$t = \frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

If $b = 0$, then you get:

$$\frac{x-x_0}{a} = \frac{z-z_0}{c}, y = y_0$$



Lines in a 3D space can be parallel, intersecting, or skew.

The figure above demonstrates \vec{v}_1 which travels from $I(x, y, z)$ to $P(x, y, z)$

Previously, we know $\vec{r} = \vec{r}_0 + t\vec{v}$

Substitute $\vec{v} = \vec{r}_1 - \vec{r}_0$

We get $\vec{r} = \vec{r}_0 + t(\vec{r}_1 - \vec{r}_0)$

$\vec{r} = (1 - t)\vec{r}_0 + t\vec{r}_1$

$0 \leq t \leq 1$, since the parameter t limits the points to between I and P

1.1 Do the lines L_1 and L_2 intersect, are they parallel, or are they skew?

$$L_1 : x = 2 + 5t, y = -1 + 3t, z = 5 - t$$

$$L_2 : x = 1 + 2u, y = 4 + u, z = -2 + 4u$$

$$\vec{v}_1 = \langle 1, 3, -1 \rangle$$

$$\vec{v}_2 = \langle 2, 1, 4 \rangle$$

incomplete do later

Ex. Find where line L intersects plane $5x - 2y + 4z = 18$

$$L : x = -4t, y = 5 + t, z = 2 + 3t$$

$$5(-4t) - 2(5 + t) + 4(2 + 3t) = 18 \quad (1)$$

$$-20t - 10 - 2t + 8 + 12t = 18 \quad (2)$$

$$-10t = 20 \quad (3)$$

$$t = -2 \quad (4)$$

1. Two planes are parallel if their normal vectors are parallel.
2. Two planes that are not parallel intersect along a line
3. The angle between intersecting planes is the angle between their normal vectors

Ex.: Consider planes $x + y + z = 1$ and $3x + y - 2z = 1$

a) Find the angle between the planes

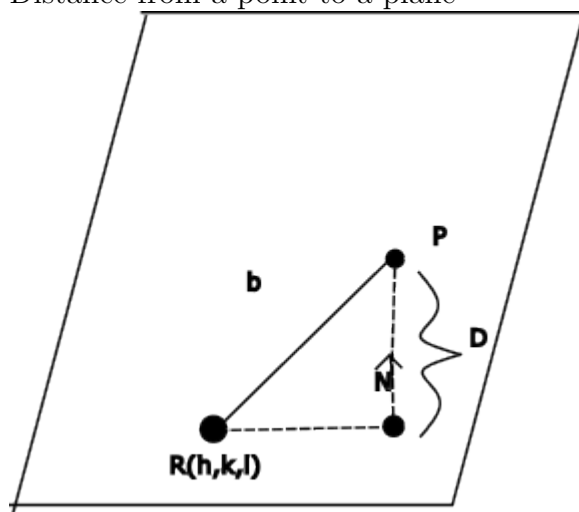
1.2 Continued

$$\vec{n}_1 = \langle 1, 1, 1 \rangle, \vec{n}_2 = \langle 3, 1, -2 \rangle \quad (1)$$

$$\vec{n}_1 \cdot \vec{n}_2 = |\vec{n}_1| |\vec{n}_2| \cos \theta \quad (2)$$

Use the equations of two planes to describe a line

Distance from a point to a plane



$$P_1(x_1, y_1, z_1)$$

$$ax + by + cz + d = 0$$

EX: Find the distance between the parallel planes

1.3 E

x: Find the distance between the lines L_1 and L_2

The distance between L_1 and L_2 is the same as the distance between the two parallel planes that contain these lines.

The normal vector \vec{n} for these two planes must be orthogonal to \vec{v}_1 and \vec{v}_2