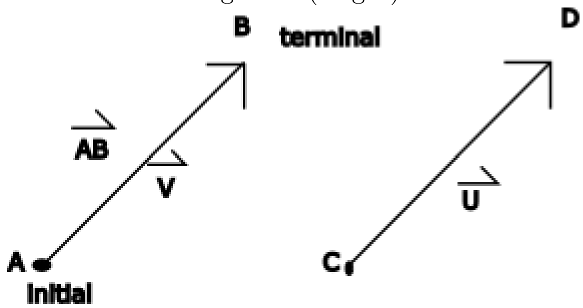
1 Vectors

Vectors have a magnitude(length) and direction.

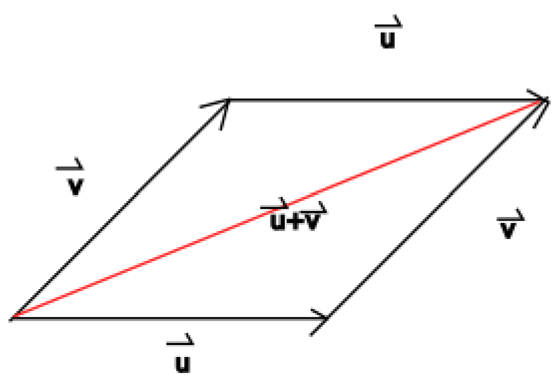


$\vec{u}$ & $\vec{v}$ have the same direction and magnitude, $\therefore$ they are equivalent.

Zero Vector $\vec{O}$ has length zero.

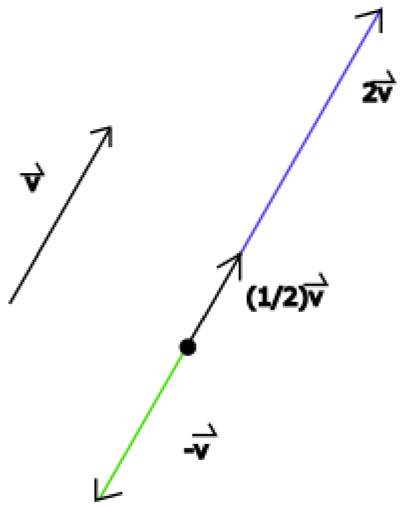
Vectors appear in forces, position, velocity, acceleration, torque, displacement, images.

Sums $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

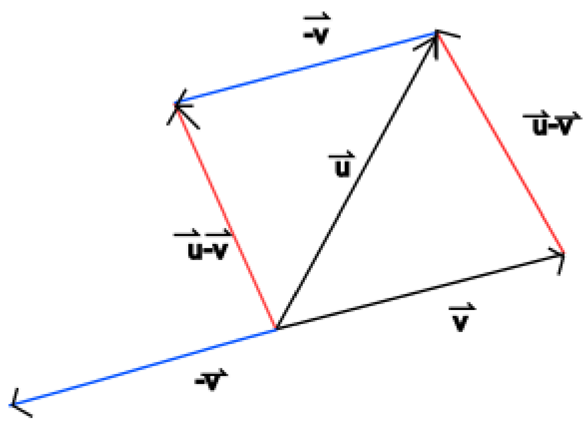


Scalar Multiplication

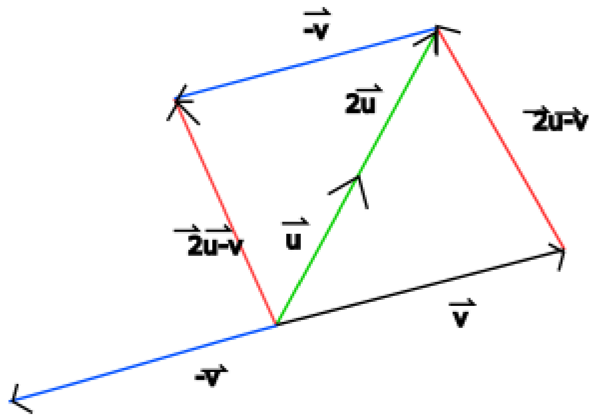
- If $c \in \mathbb{R}$, then vector $c\vec{v}$ has length —c— times the length of $\vec{v}$ and
  - the same direction as $\vec{v}$ if $c > 0$
  - opposite direction as $\vec{v}$ if $c < 0$
- If $c = 0$ or $\vec{v} = \vec{o}$, then $c\vec{v} = 0$



<u>Differences</u> $\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$

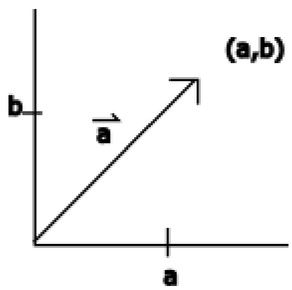


<u>Ex:</u> Consider $\vec{u}$ *and* $\vec{v}$. Sketch $2\vec{u} - \vec{v}$

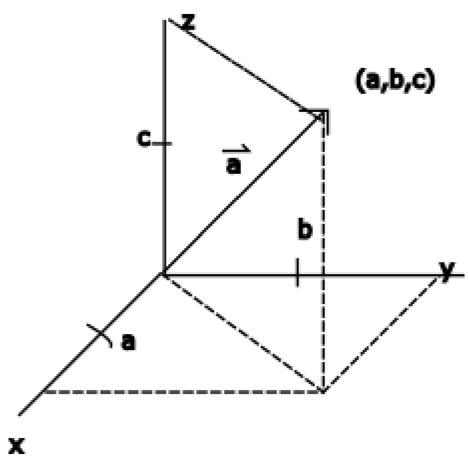


Components

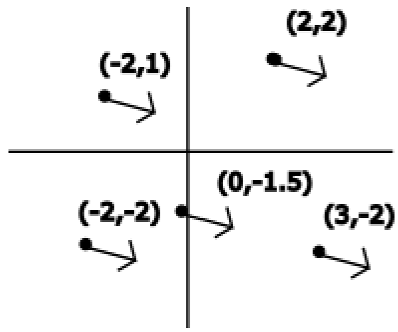
2D: $\vec{a} = <, a, b>$



3D: $\vec{a} = <a, b, c>$



Sketch vectors equivalent to $\vec{a} = <2, -1>$
Choose any initial position in the graph. So long as it obeys the magnitude and direction of the vector it is valid.



Unmarked in the graph is the point $o\vec{P}$ which is the position vector for point P, otherwise known as the origin.

Find components of the vector $\vec{a}$ that has the following:
Initial Point: $(3, 1)$
Terminal Point: $(-2, 5)$

Vector $\vec{a}$ has point $(-2-3, 5-1) = (-5, 4)$

Find components of the vector $\vec{b}$ that has the following:
Initial Point: $(1, 2, 3)$
Terminal Point: $(-2, 5, -7)$

Vector $\vec{a}$ has point $(-2-1, 5-2, -7-3) = (-3, 3, -10)$

To sum up, In general $A\vec{B}$ has components $B(x_2, y_2),\ A(x_1, y_1)$ and is the result of $A\vec{B} =< x_2 - x_1, y_2 - y_1 >$

Vector $A\vec{B}C$ has components $B(x_2, y_2, z_2),\ A(x_1, y_1, z_1)$.
It is the result of $A\vec{B}C =< x_2 - x_1, y_2 - y_1, z_2 - z_1 >$

Practice:

EX: Let $\vec{a} =< 0, 4, 3 >$ *and* $\vec{b} =< 1, -2, 5 >$. Find:

1. $|\vec{a}| \ = \sqrt{0^2 + 4^2 + 3^2} = \sqrt{25} = 5$
2. $\vec{a} + \vec{b} \ =< 0, 4, 3 > + < 1, -2, 5 >$
   $=< 0 + 1, 4 + (-2), 3 + 5 >$
   $=< 1, 2, 8 >$
3. $\vec{a} - \vec{b} \ =< 0 - 1, 4 - (-2), 3 - 5 >=< -1, 6, -2 >$
4. $4\vec{b} \ =< 4(1), 4(-2), 4(5) >$
   $=< 4, -8, 20 >$

5. $3\vec{a}+2\vec{b} \; = 3 < 0,4,3 > +2 < 1,-2,5 >$
   $=< 0+2, 12+(-4), 9+10 >$
   $=< 2,8,19 >$

<u>Notation</u>

- V_2 =\{all 2D vectors\}$= \mathbb{R}^2 = \{< a,b >: a,b \in \mathbb{R}\}$
- V_3 =\{all 3D vectors\}$= \mathbb{R}^3$
- V_n =\{all n-dimensional vectors\}$= \mathbb{R}^n$
- $\mathbb{R}^\infty = \{(a_1,a_2,a_3,\ldots) : a_i \in \mathbb{R}\}$

<u>Ideas:</u> If $\vec{a} \in V_n$ then $\vec{a} =< a_1,a_2,\ldots,a_n >$

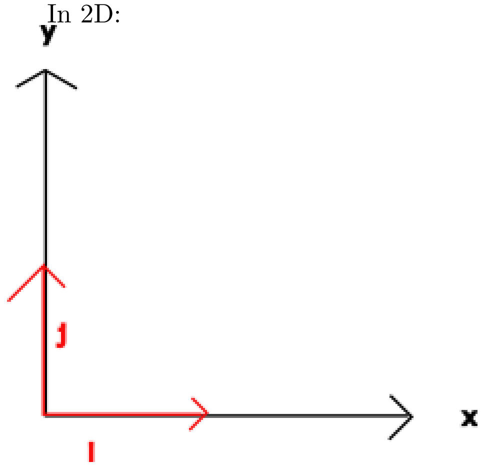
<u>Properties</u>

If $\vec{a},\vec{b},\vec{c} \in V_n$ *and* $c\&d$ are scalars, then:

- $\vec{a}+\vec{b}=\vec{b}+\vec{a}$: commutative property
- $\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c}$: associative property
- $\vec{a}+\vec{0}=\vec{a}$: additive identity
- $\vec{a}+(\vec{-a})=\vec{0}$: additive inverse
- $c(\vec{a}+\vec{b})=c\vec{a}+c\vec{b}$: distributive property
- $(c+d)\vec{a}=c\vec{a}+d\vec{a}$: distributive property
- $(cd > \vec{a}=c(d\vec{a})$: associative property
- $1\vec{a}=\vec{a}$

<u>Standard Basis Vectors</u>
Set of vectors whose components are all zero except one which equals 1.

In 2D:

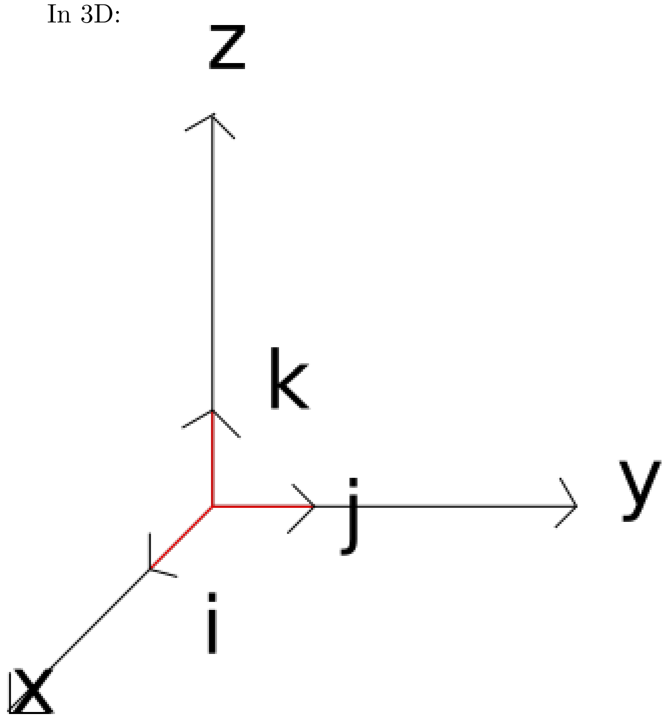


$\hat{i} = < 1, 0 >, \; \hat{j} = < 0, 1 >$

The following equation describes the components of a vector $\vec{a}$ using standard basis vectors.

$\vec{a} = < a_1, a_2 >$
$= a_1 < 1, 0 > + a_2 < 0, 1 >$
$= a_1\hat{i} + a_2\hat{j}$

In 3D:



The following equation describes the components of a vector $\vec{a}$

$$\vec{a} = < a_1, a_2, a_3 > \tag{1}$$

$$= a_1 < 1, 0, 0 > + a_2 < 0, 1, 0 > + a_3 < 0, 0, 1 > \tag{2}$$

$$= a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k} \tag{3}$$