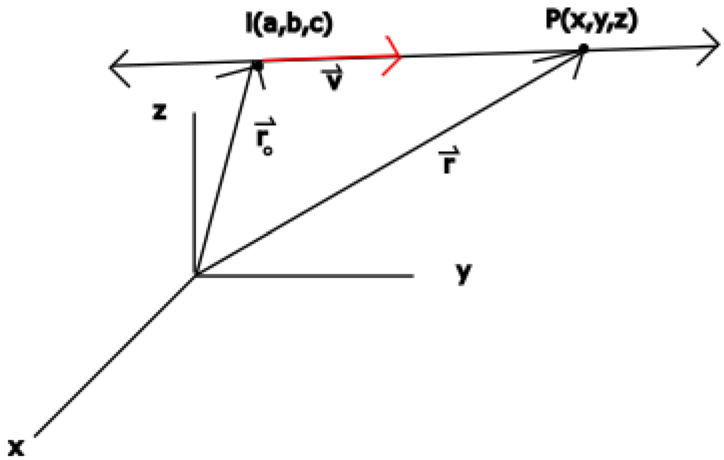
1 Equations of Lines and Planes



$\boxed{\vec{r} = \vec{r_0} + t\vec{v}}$

t is the paramater of the equation.

Reiterate: $\vec{r} = \vec{r_0} + ti\vec{P}$

Let $\vec{v} =< a, b, c >$

then $< x, y, z >=< x_0, y_0, z_0 > +t < a, b, c >$

$< x, y, z >=< x_0 + ta, y_0 + tb, z_0 + tc >$

So the parametric equations for a line are as follows:

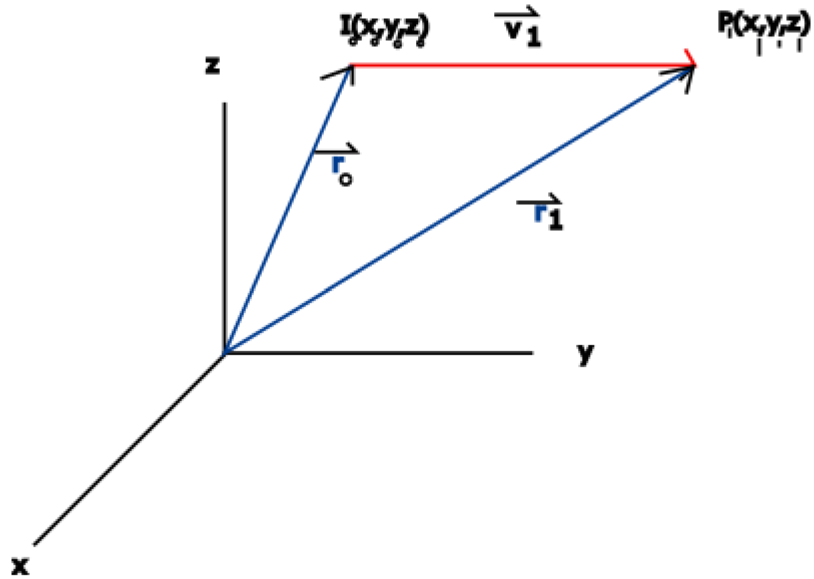
- $x = x_0 + ta$
- $y = y_0 + tb$
- $z = z_0 + tc$

When you solve for t, you get the symmetric equations.

$t = \frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$

If $b = 0$, then you get:

$\frac{x-x_0}{a} = \frac{z-z_0}{c}, y = y_0$



Lines in a 3D space can be parallel, intersecting, or skew.
The figure above demonstrates $\vec{v_1}$ which travels from $I(x, y, z)$ to $P(x, y, z)$
Previously, we know $\vec{r} = \vec{r_0} + t\vec{v}$
Substitute $\vec{v} = \vec{r_1} - \vec{r_0}$
We get $\vec{r} = \vec{r_0} + t(\vec{r_1} - \vec{r_0})$
$\vec{r} = (1 - t)\vec{r_0} + t\vec{r_1}$
$0 \leq t \leq 1$, since the parameter t limits the points to between I and P

1.1 Do the lines L_1 and L_2 intersect, are they parallel, or are they skew?

$L_1 : x = 2 + 5,\ y = -1 + 3t,\ z = 5 - t$
$L_2 : x = 1 + 2u,\ y = 4 + u,\ z = -2 + 4u$
$\vec{v_1} =< 1, 3, -1 >$
$\vec{v_2} =< 2, 1, 4 >$

incomplete do later
Ex. Find where line L intersects plane $5x - 2y + 4z = 18$

$L: x=-4t,\ y=5+t,\ z=2+3t$

$$5(-4t)-2(5+t)+4(2+3t)=18 \tag{1}$$
$$-20t-10-2t+8+12t=18 \tag{2}$$
$$-10t=20 \tag{3}$$
$$t=-2 \tag{4}$$

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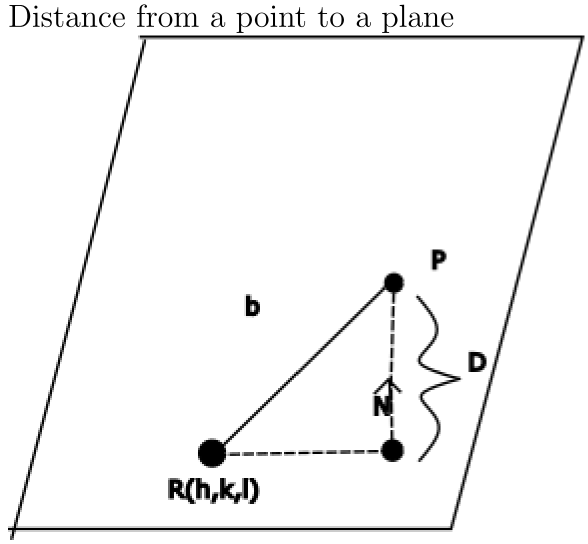
1. Two planes are parallel if their normal vectors are parallel.
2. Two planes that are not parallel intersect along a line
3. The angle between intersecting planes is the angle between their normal vectors

Ex.: Consider planes $x+y+z=1$ and $3x+y-2z=1$
a) Find the angle between the planes

1.2 Continued

$$\vec{n_1} = <1,1,1>, \vec{n_2} = <3,1,-2> \tag{1}$$
$$\vec{n_1}\cdot\vec{n_2} = |\vec{n_1}||\vec{n_2}|\cos\theta \tag{2}$$

Use the equations of two planes to describe a line
Distance from a point to a plane



$P_1(x_1, y_1, z_1)$
$ax + by + cz + d = 0$

EX: Find the distance between the parallel planes

1.3 E

x: Find the distance between the lines L_1 and L_2

The distance between L_1 and L_2 is teh same as teh distance between the two parallel planes that contain these lines.
The normal vector $\vec{n}$ for these two planes must be orthogonal to $\vec{v_1}$ and $\vec{v_2}$