



**DEPARTMENT OF MATHEMATICS**

Sub Code:	MAM11	Sub:	ADVANCED CALCULUS	Test:	01
Time:	9.30 to 10.30 am	Term:	30-11-2022 TO 10-03-2023	Marks:	30
Date:	19-1-2023	Semester:	1	Sections:	M, N, O, P

**Note: Answer any TWO full questions. Each main question carries 15 marks**

Q.No.	Questions	Bliss Level	CO's	Marks
1.	a) Write the formula for the derivative of arc length in Cartesian form.	L1	CO1	2
	b) If $u = \sin^{-1} \left[ \frac{ax+by}{\sqrt{x^2+y^2}} \right]$ , prove that $xu_x + yu_y = \left(1 - \frac{n}{2}\right) \tan u$ .	L2	CO2	3
	c) Derive the expression for radius of curvature in Cartesian form.	L3	CO1	5
	d) If $x = r \cos \phi \sin \theta$ , $y = r \sin \phi \sin \theta$ , $z = r \cos \theta$ then show that $\frac{\partial(x,y,z)}{\partial(r,\phi,\theta)} = r^2 \sin \theta$ .	L4	CO2	5
2.	a) State Euler's theorem on homogenous function for two and three independent variables.	L1	CO2	2
	b) Show that $\frac{ds}{d\theta} = r\sqrt{8r-3}$ for the curve $2r \cos^2 \theta = 1$ .	L2	CO1	3
	c) Find the values of the constants $a, b, c$ such that $\vec{F} = (x+2y+az)\hat{i} + (bx-3y-z)\hat{j} + (4x+cy+2z)\hat{k}$ is conservative. Also find its scalar potential.	L3	CO2	5
	d) Find the angle of intersection of the following pair of curves $r = \frac{a\theta}{1+\theta}$ & $r = \frac{a}{1+\theta^2}$	L5	CO1	5
3.	a) Write the relation between Cartesian and Polar coordinates.	L1	CO1	2
	b) Find the directional derivative of $\phi = x^2 - 2xy + z^3$ at the point $(1, -2, -1)$ along the vector $2\hat{i} - 4\hat{j} + 4\hat{k}$ .	L2	CO2	3
	a) Find the pedal equation to the curve $r^n = a^n \cos n\theta$ .	L3	CO1	5
	d) If $\vec{A} = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$ find (a) $\nabla \cdot \vec{A}$ ; (b) $\nabla \times \vec{A}$ ; (c) $\nabla(\nabla \times \vec{A})$ at $(1, -1, 1)$ .	L4	CO2	5