$400 \, 225$
$\tan(270 - 45)$
$\cos 135$

MAE11

RAMAIAH
Institute of Technology

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(Autonomous Institute, Affiliated to VTU)
(Approved by AICTE, New Delhi & Govt. of Karnataka)
Accredited by NBA & NAAC with 'A+' Grade

SEMESTER END EXAMINATIONS – MAY 2023

Program	: **B.E :- Common to ECE / EEE / EIE / ETE / MLE**	**Semester**	: **I**
Course Name	: **Advanced Calculus and Linear Algebra**	**Max. Marks**	: **100**
Course Code	: **MAE11**	**Duration**	: **3 Hrs**

Instructions to the Candidates:

- Answer one full question from each unit.

UNIT - I

1. a) Write the formula for the radius of curvature of the curve in parametric form and explain the terms involved in it. CO1 (02)
   b) Find the pedal equation of the curve $r = a + b\cos\theta$. CO1 (04)
   c) Show that the curves $r^2 = a^2\cos 2\theta$ & $r = a(1+\cos\theta)$ intersect at angle $3\sin^{-1}\left(\frac{3}{4}\right)^{1/4}$. CO1 (07)
   d) Show that the radius of curvature of the curve $x^2 y = a(x^2 + y^2)$ at $(-2a, 2a)$ is 2a. CO1 (07)

2. a) Write the formula for the derivative of arc length in parametric form. CO1 (02)
   b) Find the slope of the tangent to the curve $r\sec^2\left(\frac{\theta}{2}\right) = 4$ at $\theta = \frac{\pi}{2}$. CO1 (04)
   c) Show that the radius of curvature of the curve $r^n = a^n \cos n\theta$ varies inversely as r^{n-1}. CO1 (07)
   d) Show that the pedal equation of the curve $r^n = a^n \sin n\theta + b^n \cos n\theta$ is $p^2\left(a^{2n} + b^{2n}\right) = r^{2n+2}$. CO1 (07)

UNIT - II

3. a) Define row echelon form of a matrix A. CO2 (02)
   b) Use Rayleigh's power method to find the largest eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ by taking $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ as the initial approximation to the eigenvector. Carry out two iterations. CO2 (04)
   c) Apply Gauss ellimination method to solve the following system of equations $x + y + z = 9, \; x - 2y + 3z = 8, \; 2x + y - z = 3$. CO2 (07)
   d) Diagonalize the matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ and hence find A^5. CO2 (07)

4. a) Explain the geometrical interpretation of no solution for the system of linear equations $x+3y=4$ and $2x+6y=6$. CO2 (02)

b) Find the value of λ for which the system of equations $x+2y+3z=14,\ x+4y+7z=30,\ x+y+z=\lambda$. are consistent. CO2 (04)

c) Use Gauss - Seidel method to solve $5x-y=9$; $x-5y+z=4$, $y-5z=6$, by taking $(0,0,0)$ as an initial approximation. Carry out three iterations. CO2 (07)

d) Solve the following system of linear differential equations using matrix method $x_1'=3x_1$, $x_2'=x_1+x_2$ given $x_1(0)=1,\ x_2(1)=-1$. CO2 (07)

UNIT - III

5. a) Define: (i) solenoid vector (ii) irrotational vector. CO3 (02)

b) The altitude of a right circular cone is 15cm and it is increasing at 0.2cm/sec. The radius of the base is 10cm and it is decreasing at 0.3cm/sec. How fast is the volume changing? CO3 (04)

c) If $u=\operatorname{cosec}^{-1}\left[\dfrac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right]$ then find i) xu_x+yu_y, ii) $x^2u_{xx}+2xyu_{xy}+y^2u_{yy}$ CO3 (07)

d) Find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$, where $u=x^2+y^2+z^2,\ v=xy+yz+zx,\ w=x+y+z$ and also find the functional relation between u,v and w. CO3 (07)

6. a) For $u=\sin(x-ct)$ then find u_{tt}. CO3 (02)

b) Find the directional derivative of $u(x,y,z)=xy^2+yz^3$ at the point (2, -1, 1) in the direction of the vector $\hat{i}+2\hat{j}+3\hat{k}$. CO3 (04)

c) Show that $\vec{F}=(\sin z+y)\hat{i}+x\hat{j}+x\cos z\hat{k}$ is irrotational. Also find a scalar function ϕ such that $\vec{F}=\nabla\phi$. CO3 (07)

d) If F is a function of x,y,z and $x=u+v+w,\ y=uv+wu+vw,\ z=uvw$ then show that $uF_u+vF_v+wf_w=xF_x+2yF_y+3zF_z$. CO3 (07)

UNIT- IV

7. a) With a help of neat diagram mark the region of integration in the integral $\int_0^1\int_{1-x}^{1-x^2} dxdy$. CO4 (02)

$\frac{1}{2}\cdot\frac{1}{3}=\frac{1}{6}$

b) Evaluate $\int_0^1\int_0^2\int_1^2 x^2yz\,dxdydz$. CO4 (04)

c) Evaluate the following integral by changing to polar coordinates CO4 (07)

$$\int_0^a\int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}}\frac{xy}{x^2+y^2}e^{-(x^2+y^2)}dy\,dx.$$

d) Find the volume of the sphere $x^2+y^2+z^2=a^2$ by changing to spherical polar coordinates. CO4 (07)

8. a) With the help of a neat diagram mark the region of integration in $\int_0^{\pi/2}\int_0^{\infty} f(r,\theta)\,dr\,d\theta$. CO4 (02)

b) Evaluate $\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y}\,dy\,dx$ by changing the order of integration. CO4 (04)

c) Evaluate $\iiint_R (x+y+z)\,dx\,dy\,dz$ where R is the region bounded by $z=0,\ y=0,\ x=0$ and $x+y+z=1$. CO4 (07)

d) Find the area lying inside the cardioid $r=a(1+\cos\theta)$ and outside the circle $r=a$. CO4 (07)

UNIT - V

9. a) State Stokes theorem. CO5 (02)

b) If $\vec{F}=(3x^2+6y)\,\hat{i}-14yz\,\hat{j}+20xz^2\,\hat{k}$ then evaluate $\int_c \vec{F}.d\vec{r}$ along the straight line c from (0, 0, 0) to (2, 1, 3). CO5 (04)

c) State and prove Greens theorem in a plane. CO5 (07)

d) Evaluate $\int_s \vec{F}.\hat{n}\,ds$, where $\vec{F}=4x\hat{i}-2y^2\,\hat{j}+z^3\,\hat{k}$ and S is the surface bounded by $x^2+y^2=25$ and the planes $z=0$ and $z=6$ using Gauss divergence theorem. CO5 (07)

10. a) Define line integral of a vector function and give its physical interpretation. CO5 (02)

b) If $\vec{F}$ is irrotational then show that $\int_c \vec{F}.\,d\vec{r}=0$ for any closed curve c. CO5 (04)

c) Verify Green's theorem for $\int_c (x^2-y^2)dx+2xydy$, where c is the rectangle bounded by $y=0, x=0, y=b$ and $x=a$. CO5 (07)

d) Evaluate $\int_c \vec{F}.d\vec{r}$, where $\vec{F}=(2x-y)\hat{i}-yz^2\,\hat{j}-y^2z\,\hat{k}$ and c is the boundary of the upper half of the sphere $x^2+y^2+z^2=1$ using Stoke's theorem. CO5 (07)

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