MAM11

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RAMAIAH
Institute of Technology

(Autonomous Institute, Affiliated to VTU)
(Approved by AICTE, New Delhi & Govt. of Karnataka)
Accredited by NBA & NAAC with 'A+' Grade

SEMESTER END EXAMINATIONS – MAY 2023

Program	: **B.E :- Common to ME / IM / CH**	**Semester**	: **I**
Course Name	: **Advanced Calculus**	**Max. Marks**	: **100**
Course Code	: **MAM11**	**Duration**	: **3 Hrs**

Instructions to the Candidates:

- Answer one full question from each unit.

UNIT - I

1.	a)	Find the angle between radius vector and the tangent for $r = a(1+\cos\theta)$	CO1	(02)
	b)	Find $\frac{ds}{dt}$ for the curve $x = e^t \sin t,\ y = e^t \cos t$.	CO1	(04)
	c)	Find the angle of intersection of the pairs of curves $r^2 \sin 2\theta = 4$ and $r^2 = 16\sin 2\theta$.	CO1	(07)
	d)	Show that the radius of curvature of the curve $r^n = a^n \cos n\theta$ varies inversely as r^{n-1}.	CO1	(07)
2.	a)	Write the formula to find derivative of Arc length in cartesian and parametric forms.	CO1	(02)
	b)	Prove that the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1, 1) is $\frac{\sqrt{2}}{3}$.	CO1	(04)
	c)	Find the pedal equation to the curve $r^2 = a^2 \sin 2\theta$.	CO1	(07)
	d)	Find the length of perpendicular from pole to the tangent to the curves $r^2 \cos 2\theta = a^2$ at $\theta = \frac{\pi}{6}$.	CO1	(07)

UNIT - II

3.	a)	State Euler's Theorem for function of 2 variables and 3 variables.	CO2	(02)
	b)	Find $div\vec{F}$ at the point $(1,2,3)$ where $\vec{F} = (y^2+z^2-x^2)\hat{i} + (z^2+x^2-y^2)\hat{j} + (x^2+y^2-z^2)\hat{k}$.	CO2	(04)
	c)	If $u = x+y+z, v = y+z, w = z+x$ then find $\frac{\partial(u,\upsilon,\omega)}{\partial(x,y,z)}$.	CO2	(07)
	d)	If the directional derivative of $\phi = axy^2 + byz + cz^3x^3$ at $(-1,1,2)$ has a maximum magnitude of 32 units in the direction parallel to y-axis then find a,b,c.	CO2	(07)
4.	a)	Define Solenoidal and irrotational vectors.	CO2	(02)
	b)	If $u = x\log(xy)$ where $x^3+y^3+3xy=1$ then find $\frac{du}{dx}$.	CO2	(04)
	c)	If $z = f(x,y)$ and $x = e^u + e^{-v}, y = e^{-u} - e^v$ then show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.	CO2	(07)

d) Show that $\vec{F}=(x^2-y^2+x)\hat{i}-(2xy+y)\hat{j}$ is irrotational. Also find a scalar function ϕ such that $\vec{F}=\nabla\phi$. CO2 (07)

UNIT - III

5. a) Evaluate $\int_0^{\frac{\pi}{2}} Sin^5(x)dx$. CO3 (02)

b) Write the limits of integration with respect to r, θ while evaluating the integrals: $\int_0^{a}\int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} f(x,y)\,dy\,dx$. CO3 (04)

c) Evaluate $\iint_R xydxdy$ x where R is the region bounded by $\frac{x}{a}+\frac{y}{b}=1$, $x=0$ and $y=0$. CO3 (07)

d) Evaluate $\iiint(x^2+y^2+z^2)dx\,dy\,dz$ taken over the volume enclosed by the sphere $x^2+y^2+z^2=1$, by transforming into spherical polar coordinates. CO3 (07)

6. a) Evaluate $\int_0^1\int_0^1\int_0^1 xyz\ dxdydz$. CO3 (02)

b) Evaluate $\int_0^{\pi}\frac{Sin^4x}{(1+Cosx)^2}dx$. CO3 (04)

c) Show that $\int_0^6\int_{y/2}^{3}\frac{1}{x}e^{y/x}\,dy\,dx=3(e^2-1)$ by changing the order of integration. CO3 (07)

d) Find the area bounded by the curves $y^2=4x$ and $x^2=4y$. CO3 (07)

UNIT- IV

7. a) State Gauss divergence theorem. CO4 (02)

b) If $\vec{F}=3xy\hat{i}-5y^2\hat{j}$ evaluate $\int_c \vec{F}\cdot d\vec{r}$ where c is the curve $y=2x^2$ in xy-plane from (0,0) to (1,2). CO4 (04)

c) State and prove Green's theorem in a plane. CO4 (07)

d) Using Stoke's theorem evaluate $\int_c (x+y)dx+(2x-z)dy+(y+z)dz$ where c is the boundary of the triangle with vertices at $(2,0,0)$, $(0,3,0)$ and $(0,0,6)$. CO4 (07)

8. a) State Stoke's theorem. CO4 (02)

b) If $\bar{F} = xy\hat{i} + yz\hat{j} + zx\hat{k}$ then evaluate $\int_c \bar{F}.d\bar{r}$ where c is the curve represented by $x = t, y = t^2, z = t^2, -1 \le t \le 1$. CO4 (04)

c) Evaluate $\int_s \vec{F} \cdot \hat{n} ds$ where $\vec{F} = yz\hat{i} + 2y^2\hat{j} + xz^2\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 9$ contained in the first octant between $z = 0$ and $z = 2$ using Gauss divergence theorem. CO4 (07)

d) By using Green's theorem evaluate $\int_c xy\,dx + xy^2\,dy$ where 'c' is the square in the xy-plane with vertices $(1,0), (0,1), (-1,0)$ and $(0,-1)$. CO4 (07)

UNIT - V

9. a) Write any two properties of eigen values of a square matrix A. CO5 (02)

b) Test for consistency of the system of linear equations and solve $3x_1 + 2x_2 + 4x_3 = 7$; $2x_1 + x_2 + x_3 = 4$; $x_1 + 3x_2 + 5x_3 = 2$. CO5 (04)

c) Solve the system of equations $5x + 2y + z = 12$; $x + 4y + 2z = 15$; $x + 2y + 5z = 20$ using Gauss-Seidel method taking $(1, 0, 0)$ as initial approximation. (Perform two iterations). CO5 (07)

d) Diagonalize the matrix $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ and hence find A^5. CO5 (07)

10. a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$. CO5 (02)

b) Apply Gauss Elimination method to solve the systems of equations $x + y + z = 9$; $x - 2y + 3z = 8$; $2x + y - z = 3$. CO5 (04)

c) Find the values of λ and μ such that the equations $2x + 3y + 5z = 9$ $7x + 3y - 2z = 8$; $2x + 3y + \lambda z = \mu$ have (i) no solution (ii) unique solution (iii) an infinite number of solutions. CO5 (07)

d) Find the Eigenvector corresponding to the largest Eigenvalue of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by taking $[1 \quad 1 \quad 1]^T$ as the initial approximation by using Rayleigh's Power method. (Perform four iterations). CO5 (07)

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