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RAMAIAH
Institute of Technology

(Autonomous Institute, Affiliated to VTU)
(Approved by AICTE, New Delhi & Govt. of Karnataka)
Accredited by NBA & NAAC with 'A+' Grade

SEMESTER END EXAMINATIONS – SEPTEMBER / OCTOBER 2023

Program	:	**B.E :- Common to CSE / ISE / CSE(CY) / AI & DS / BT / AI & ML / CSE (AI&ML)**	**Semester**	: **II**
Course Name	:	**Numerical Techniques and Differential Equations**	**Max. Marks**	: **100**
Course Code	:	**MAC21**	**Duration**	: **3 Hrs**

Instructions to the Candidates:

- Answer one full question from each unit.

UNIT - I

1. a) Give the geometrical interpretation of Newton-Raphson iteration formula. CO1 (02)

   b) Expand $\sin^{-1} x$ in powers of x up to second degree term. CO1 (04)

   c) Solve the following system of non-linear equations using Newton-Raphson method (Carry out two iterations) CO1 (07)
   $x^2 + y^2 = x,\ x^2 - y^2 = y$, given that $x_0 = 0.8$ and $y_0 = 0.4$.

   d) The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. CO1 (07)

2. a) Write Taylor's series for the function of one variable. CO1 (02)

   b) Examine $x^3 + y^3 - 3axy$ for extreme values. CO1 (04)

   c) Expand the function $xy^2 + \cos(xy)$ about the point $\left(1, \frac{\pi}{2}\right)$ upto second degree terms. CO1 (07)

   d) A rectangular box open at the top is to have volume of 108 cubic ft. Find the dimension of the box if its total surface area is minimum. CO1 (07)

UNIT - II

3. a) Write the steps involved in finding the orthogonal trajectories of the curve $f(x, y, c) = 0$. CO2 (02)

   b) Suppose that an object is heated to 300^0 F and allowed to cool in a room whose air temperature is 80^0 F. After 10 minutes the temperature of the object is 250^0 F. What will be its temperature after 20 minutes? CO2 (04)

   c) Using Taylor's series method, find the particular solution of $\frac{dy}{dx} - 2y = 3e^x; y(0) = 0$ at $x = 0.2$, considering terms up to fourth degree. Compare the result with the exact solution. CO2 (07)

   d) Solve the initial value problem, $y' = 0.25y^2 + x^2, y(0) = -1$ at $x = 0.2$ by taking $h = 0.2$ using Runge - Kutta method of fourth order. CO2 (07)

4. a) Write any two differences between analytical and numerical methods. CO2 (02)

   b) A bungee jumper with a mass of 68.1 Kgs leaps from a stationary hot air balloon. Use $\frac{dv}{dt} = g - \frac{cv^2}{m}$ where $g = 9.8m/s^2, c = 0.25kg/m$ to compute velocity for the first three seconds of free fall by Euler's method in steps of 1 second. CO2 (04)

c) Solve the initial value problem $\frac{dy}{dx}=\frac{2xy+e^x}{x^2+xe^x}$, $y(1)=0$ at $x=1.2$ by taking step length of 0.2, using Modified Euler's method. Carry out 2 iterations. CO2 (07)

d) Show that the family of curves $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ is self-orthogonal, where λ is the parameter. CO2 (07)

UNIT - III

5. a) Write the steps involved in solving Cauchy's LDE. CO3 (02)

b) If $D=\frac{d}{dx}$ and $X=X(x)$, then prove that $\frac{1}{D-a}X=e^{ax}\int Xe^{-ax}dx$. CO3 (04)

c) Solve $(D^2-4D+4)y=8x^2e^{2x}\sin 2x$. CO3 (07)

d) Solve $y''+2y'+2y=e^{-x}\sec^3 x$ by the method of variation of parameters. CO3 (07)

6. a) Define linear and non-linear differential equations with example. CO3 (02)

b) If $k>0$, then show that the general solution of $y^{iv}-k^4y=0$ can be expressed as $y=C_1\cos kx+C_2\sin kx+C_3\cosh kx+C_4\sinh kx$. CO3 (04)

c) Solve $(3x+2)^2y''+3(3x+2)y'-36y=8x^2+4x+1$. CO3 (07)

d) Solve $(D^2+D)y=2+2x+x^2$, $y(0)=8$, $y'(0)=-1$. CO3 (07)

UNIT-IV

7. a) Obtain the expression for $\Delta^2 y_n$ in terms of y values. CO4 (02)

b) Construct the backward difference table representing the function $y=\cos x+x^2+2$ over the interval $(2,3)$ with step length $h=0.2$ and hence write the value of $\nabla^2 y_3$. CO4 (04)

c) Use Simpson's 1/3rd rule to evaluate $\int_0^1 \frac{1}{1+x^2}dx$ considering seven equidistant ordinates and hence find an approximate value of π. CO4 (07)

d) Use an appropriate interpolation formula to find the radius of curvature at $x=3.0$ from the following data: CO4 (07)

x	3	5	7	9	11
y	28.27	78.54	153.93	254.47	380.13

8. a) Given two points (x_0,y_0) and (x_1,y_1), write Lagrange's inverse interpolation formula. CO4 (02)

b) Evaluate $\int_0^1 e^x dx$ approximately in steps of 0.2 by using trapezoidal rule. CO4 (04)

c) Using Newton's divided difference formula find an interpolating polynomial for the following data and hence find $f(1)$. CO4 (07)

x	-1	0	2	3
$f(x)$	-8	3	1	12

d) A survey conducted in a factory reveals the following information. Estimate the probable number of persons in the income group 20 to 25. CO4 (07)

Income per hour (Rs.)	<10	10 - 20	20 - 30	30 - 40	40 - 50
No. of persons	20	45	115	210	115

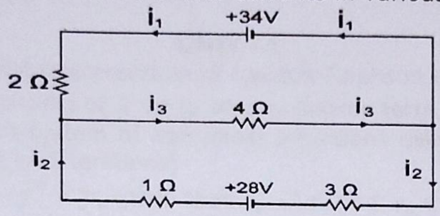
UNIT-V

9. a) Define row echelon form of a matrix. CO5 (02)

b) Use Gauss Seidel method to solve the system of equations: CO5 (04)
$2x+17y+4z=35;\ x+3y+10z=24;\ 28x+4y-z=32$
Use (0, 0, 0) as the initial approximation and carry out 2 iterations.

c) If the characteristic equation of the matrix $A=\begin{bmatrix}3 & -2 & 4\\ -2 & 6 & 2\\ 4 & 2 & 3\end{bmatrix}$ is $(\lambda-7)^2(\lambda+2)=0$, find its non-singular modal matrix. CO5 (07)

d) Write the system of linear equations from the following electrical network. Use Gauss-elimination method to find currents in various branches. CO5 (07)



10. a) Explain the geometrical interpretation of infinitely many solutions for the system of linear equations $2x+y=3$ and $4x+2y=6$. CO5 (02)

b) Use Rayleigh's power method to find the largest eigenvalue and the corresponding eigenvector of the matrix $A=\begin{bmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{bmatrix}$ by taking $\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}$ as the initial approximation to the eigenvector. Carry out two iterations. CO5 (04)

c) Find the conditions for a, b, c so that the system is solvable: CO5 (07)
$-2x+y+z=a;\ x-2y+z=b;\ x+y-2z=c$
Find all possible solutions if $a=1,\ b=1,\ c=-2$.

d) Suppose the rabbit population r and the wolf population w are governed by $\dfrac{dr}{dt}=4r-w,\quad \dfrac{dw}{dt}=2r+w$. If initially $r=240$ and $w=300$,what are the populations at time t? CO5 (07)

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