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RAMAIAH
Institute of Technology
(Autonomous Institute, Affiliated to VTU)
(Approved by AICTE, New Delhi & Govt. of Karnataka)
Accredited by NBA & NAAC with 'A+' Grade

SEMESTER END EXAMINATIONS – MAY 2023

Program	:	B.E :- Common to CSE / ISE / CSE(CY) / AI & DS / BT / AI & ML / CSE (AI&ML)	Semester	:	I
Course Name	:	Advanced Calculus and Modular Arithmetic	Max. Marks	:	100
Course Code	:	MAC11	Duration	:	3 Hrs

Instructions to the Candidates:

- Answer one full question from each unit.

UNIT - I

1.	a)	Write the relation between Cartesian and polar coordinates.	CO1	(02)
	b)	Find $\frac{ds}{dt}$ for the curve: $x = e^t \sin t,\ y = e^t \cos t$.	CO1	(04)
	c)	Prove with usual notation $\tan\phi = r\frac{d\theta}{dr}$.	CO1	(07)
	d)	Show that the radius of curvature of the curve $r^n = a^n \cos n\theta$ varies inversely as r^{n-1}.	CO1	(07)
2.	a)	Write the expression for radius of curvature for the curves in polar and parametric forms.	CO1	(02)
	b)	Obtain the radius of curvature for the curve $y = x^2$ at $(1,1)$.	CO1	(04)
	c)	Find the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$.	CO1	(07)
	d)	Find the angle of intersection of the pair of curves $r^n = a^n \sec(n\theta + \alpha)$ and $r^n = b^n \sec(n\theta + \beta)$.	CO1	(07)

UNIT - II

3.	a)	Show that the vector $\vec{F} = 3y^2z^2\hat{i} + 4x^3z^2\hat{j} + 3x^2y^2\hat{k}$ is solenoidal.	CO2	(02)
	b)	If $z = e^{ax+by} f(ax-by)$ then show that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$.	CO2	(04)
	c)	Find the constants *a* and *b* so that $\vec{A} = (axy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (bxz^2 - y)\hat{k}$ is irrotational and hence find a scalar function ϕ such that $\vec{A} = \nabla\phi$.	CO2	(07)
	d)	If $u = f(r,s,t)$ where $r = \frac{x}{y},\ s = \frac{y}{z},\ t = \frac{z}{x}$ then show that $xu_x + yu_y + zu_z = 0$.	CO2	(07)
4.	a)	Find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ for the function $u = \sin^{-1}\left(\frac{y}{x}\right)$.	CO2	(02)
	b)	If $\vec{F} = x^2y\hat{i} + 2x^2yz\hat{j} - 3y^2z\hat{k}$, find $div(\vec{F}), curl(\vec{F})$ at $(2,1,1)$.	CO2	(04)
	c)	If the directional derivative of $\phi = axy^2 + byz + cz^3x^3$ at $(-1,1,2)$ has a maximum magnitude of 64 units in the direction parallel to z-axis then find the values of a,b,c.	CO2	(07)

d) Find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ where $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$ CO2 (07)
and also find the functional relation between u, v and w.

UNIT - III

5. a) Evaluate $\beta\left(\frac{1}{3}, \frac{2}{3}\right)$. CO3 (02)

b) Evaluate $\int_0^1 \int_0^2 \int_1^2 xyz^2 \, dx dy dz$. CO3 (04)

c) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integration. CO3 (07)

d) Evaluate $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{y^4 - a^2x^2}} dy dx$, by changing the order of integration. CO3 (07)

6. a) Write the relation between Cartesian and cylindrical polar coordinates. CO3 (02)

b) Write the limits of integration after changing the order of integration with a neat diagram $\int_0^1 \int_0^{x^2} f(x,y) dy dx$. CO3 (04)

c) Using the transformation $x + y = u; y = uv$, show that $\int_0^1 \int_0^{1-x} e^{y/(y+x)} dy dx = \frac{1}{2}(e-1)$. CO3 (07)

d) Show that $\int_0^{\pi/2} \sqrt{\sin\theta} \, d\theta \times \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}} d\theta = \pi$. CO3 (07)

UNIT- IV

7. a) State Stoke's theorem. CO4 (02)

b) Find the total work done in moving particle in a force field $\bar{F} = 3xy \, \hat{i} - 5z \, \hat{j} + 10x \, \hat{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$ and $z = t^3$ from $t = 1$ to $t = 2$. CO4 (04)

c) State and prove Green's theorem in a plane. CO4 (07)

d) If $\bar{F} = 4xz \, \hat{i} - y^2 \, \hat{j} + yz \, \hat{k}$ then evaluate $\iint_s \vec{F} \cdot \hat{n} ds$ using divergence theorem where s is the surface of the cube bounded by $x = 0, x = 1$, $y = 0, y = 1, z = 0$ & $z = 1$. CO4 (07)

8. a) State Gauss divergence theorem. CO4 (02)

b) If $\bar{F} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{j}$ then evaluate $\int_c \vec{F} . d\vec{r}$ where c is the curve $y = x^3$ from the point (1, 1) to the point (2, 8). CO4 (04)

c) Using Stoke's theorem, evaluate $\int_c (x + y)dx + (2x - z)dy + (y + z)dz$ where c is the boundary of the triangle with vertices at $(2, 0, 0)$, $(0, 3, 0)$ and $(0, 0, 6)$. CO4 (07)

d) Verify Green's theorem for $\int_c (xy + y^2)dx + x^2 dy$ where c is bounded by $y = x$ and $y = x^2$. CO4 (07)

UNIT - V

9. a) State division algorithm. CO5 (02)
   b) Using Fermat's theorem, find the remainder When 16^{53} is divided by 7. CO5 (04)
   c) Find the general solution of the Diophantine equation $1485x+1745y=15$. CO5 (07)
   d) State and prove Wilson's theorem. CO5 (07)

10. a) Define linear congruence modulo m. CO5 (02)
    b) Prove that if $(a,b)=1=(a,c)$ then $(a,bc)=1$. CO5 (04)
    c) Solve the following system of linear congruence's by Chinese remainder theorem: CO5 (07)

$$x \equiv 1 \pmod{3}$$
$$x \equiv 2 \pmod{5}$$
$$x \equiv 3 \pmod{7}$$

    d) Using Fermat's theorem, Solve the linear congruence $37x \equiv 5 \pmod{11}$. CO5 (07)

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