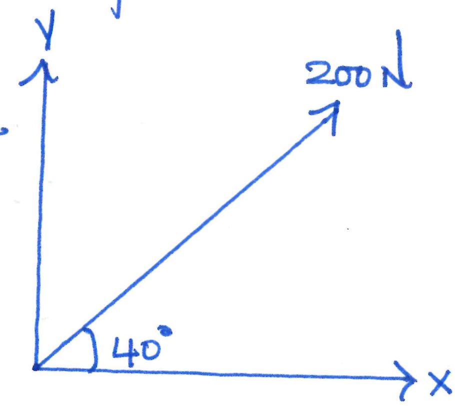
Unit 3 - Numerical Problems

Resultant of Coplanar - Concurrent forces

1) A force of 200N is acting at a point making an angle of 40° with the horizontal. Determine the components of this force along the x and y directions.



Soln:- Component along the x-direction,

$$F_x = F\cos\theta$$
$$= 200\cos 20^\circ$$
$$\boxed{F_x = 153.20\,N}$$

Component along the y-direction,

$$F_y = F\sin\theta$$
$$= 200\sin 40^\circ$$
$$\boxed{F_y = 128.55\,N}$$

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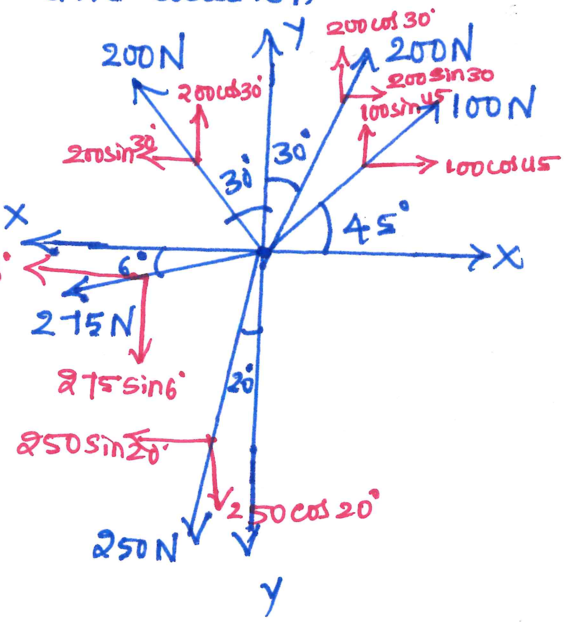
2) Five coplanar forces are acting at a point as shown in figure. Determine the resultant in magnitude and direction.



Soln:

$$\Sigma F_x = -200\sin 30^\circ + 200\sin 30^\circ + 100\cos 45^\circ - 275\cos 6^\circ - 250\sin 20^\circ$$

$$\boxed{\Sigma F_x = -288.28\,N}$$

$$\Sigma F_y = 200\cos 30^\circ + 200\cos 30^\circ + 100\sin 45 - 275\sin 6^\circ - 250\cos 20^\circ$$

$$\boxed{\Sigma F_y = 153.45\,N}$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

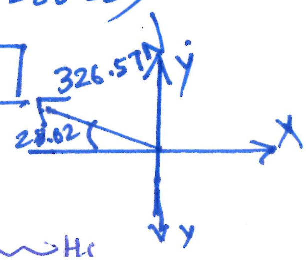
$$R = \sqrt{(-288.28)^2 + (153.45)^2}$$

$$\boxed{R = 326.57\,N}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{153.45}{-288.28}\right)$$

$$\boxed{\theta = -28.02}$$



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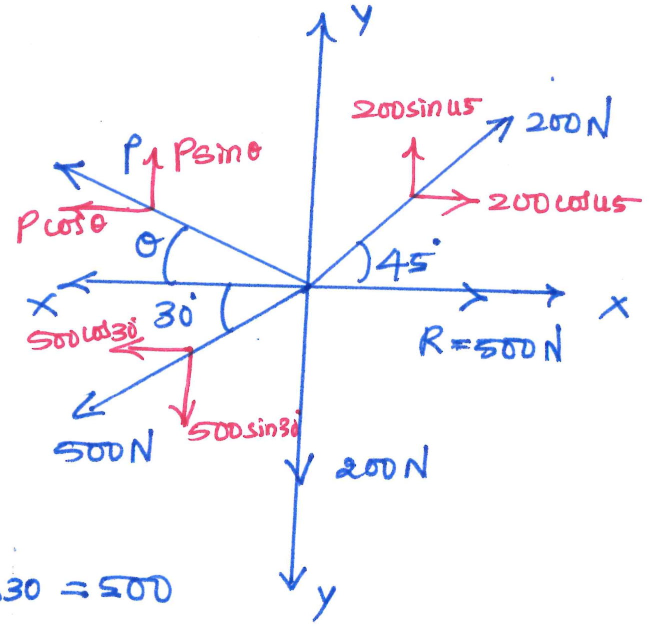
3. Four coplanar forces acting at a point are shown in figure. One of the force is unknown and its magnitude is shown by P. The resultant has a magnitude of 500N and is acting along the X axis. Determine the unknown force P and its inclination with the X axis.



<u>Sol</u>

Solⁿ :- W.K.T

$$\Sigma F_x = R_x$$
$$\Sigma F_y = R_y$$

Resolving the forces along x direction

$$\Sigma F_x = R_x = R\cos\theta = R$$
$$\underline{\Sigma F_x = 500N.}$$

$\therefore$

$$\Sigma F_x = -P\cos\theta + 200\cos 45° - 500\cos 30 = 500$$
$$\Rightarrow -P\cos\theta = 791.59$$
$$\boxed{P\cos\theta = -791.59N} \text{ --- (1)}$$

Also, $\Sigma F_y = R_y = 0$

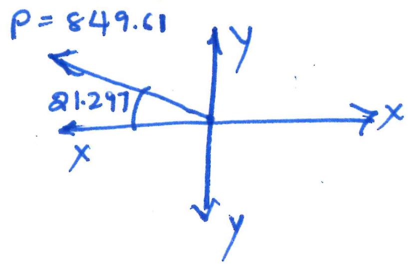
$$\Rightarrow P\sin\theta + 200\sin 45° - 500\sin 30 - 200 = 0$$
$$\boxed{P\sin\theta = 308.579N} \text{ --- (2)}$$

Squaring both (1) & (2) and then adding, we have

$$P^2\cos^2\theta + P^2\sin^2\theta = (-791.59)^2 + (308.579)^2$$
$$P^2 = 721835.72$$
$$\boxed{P = 849.61}$$

Dividing Eq (2) by (1), $\frac{P\sin\theta}{P\cos\theta} = \frac{308.579}{-791.59}$

$$\tan\theta = -0.389$$
$$\theta = \tan^{-1} 0.389$$
$$\boxed{\theta = 21.297}$$



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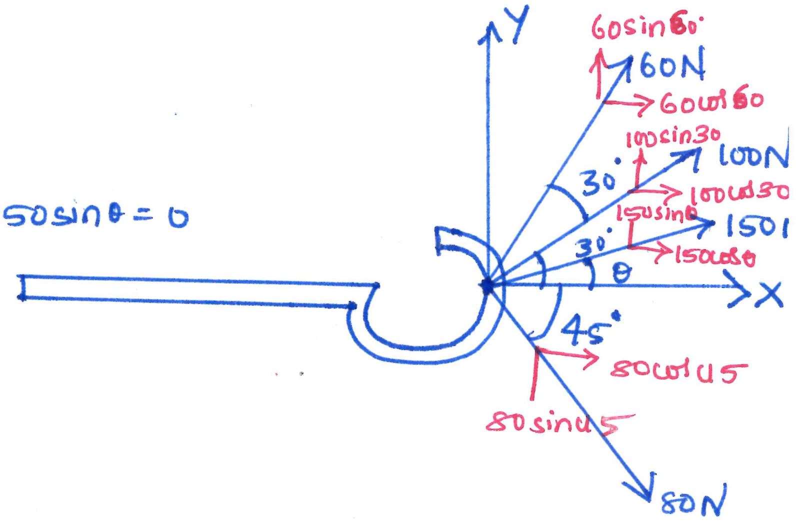
4. Four forces acting on a hook are shown in the figure. Determine the direction of the force 150 N such that the hook is pulled in X direction. Determine the resultant force in the x direction

<u>Sol^n:</u>

$$\Sigma F_x = R$$
$$\Sigma F_y = 0$$



for $\Sigma F_y = 0$, we have

$$-80\sin45° + 60\sin60 + 100\sin30 + 150\sin\theta = 0$$

$$150\sin\theta = 45.39$$

$$\theta = \sin^{-1}\left(45.39/150\right)$$

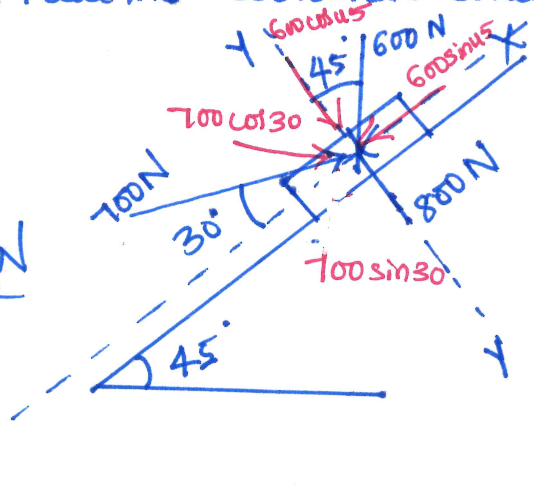
$$\boxed{\theta = 17.61°}$$

for $\Sigma F_x = R$, we have

$$80\cos45 + 60\cos60 + 100\cos30 + 150\cos17.61 = R$$

$$\Rightarrow \boxed{R = 316.142}$$

5.) Determine the resultant of the system of forces acting on a body as shown in the figure. Take the coordinate directions as shown in the figure



<u>Sol^n:</u> $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

$$\Sigma F_x = 700\cos30 - 600\cos45° = \underline{181.95\text{ N}}$$

$$\Sigma F_y = -700\sin30 - 600\sin45° + 800$$

$$\underline{\Sigma F_y = 25.73\text{ N}}$$

$$R = \sqrt{181.95^2 + 25.73^2}$$

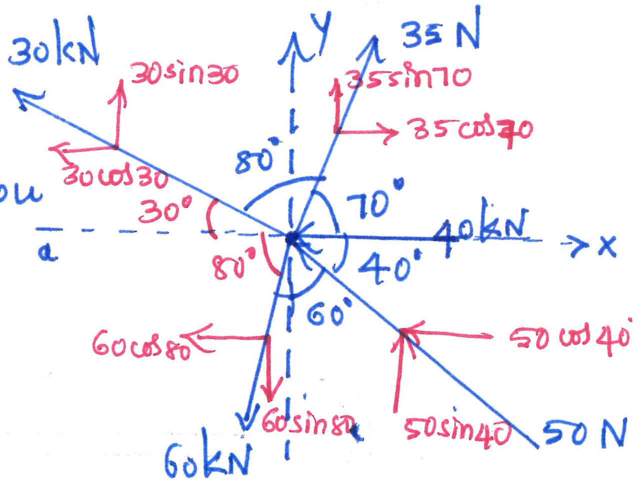
$$\boxed{R = 183.76\text{ N}}$$

$$\theta = \tan^{-1}\left(\frac{25.73}{181.95}\right) = \underline{8.048}$$

$$\boxed{\theta = 8.048}$$

6. Determine the equilibrant of the force system shown in fig

Sol^n: To find the equilibrant, we have to first calculate the Resultant. Equilibrant is force which is having same magnitude and opposite direction



$$R_x = \Sigma F_x = -40 - 30\cos 30 + 35\cos 70 - 60\cos 80 - 50\cos 40$$

$$\underline{R_x = -102.73N}$$

$$R_y = \Sigma F_y = 35\sin 70 + 30\sin 30 - 60\sin 80 + 50\sin 40$$

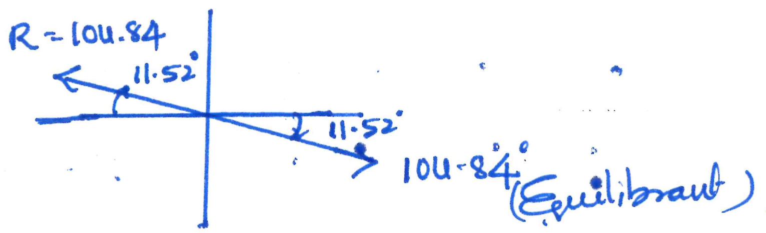
$$\underline{R_y = 20.94kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-102.73)^2 + 20.94^2}$$

$$\therefore \boxed{R = 104.84N}$$

$$\theta = \tan^{-1}\left(\frac{|R_y|}{|R_x|}\right) = \tan^{-1}\frac{20.94}{-102.73} = \underline{11.52^\circ}$$

Equilibrant is force of magnitude 104.84kN acting @ an angle of 11.52° in 4th quadrant which is exactly opposite to Resultant



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7. Forces are transmitted by two members as shown in figure. If the Resultant of these forces is 1400N directed vertically upward, find angles α and β.

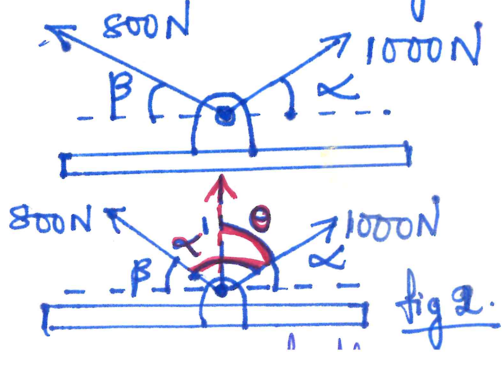

fig 2.

Sol^n:- As two angles are unknown, the parallelogram law will be convenient to use. Let $P = 1000N$, $Q = 800N$.

Their Resultant R = 1400N as shown in the figure. The angle between P & Q is α' and angle made by R with P is θ as shown in the figure 2.

$$R^2 = P^2 + Q^2 + 2PQ\cos\alpha'$$

$$1400^2 = 1000^2 + 800^2 + 2(1000)(800)\cos\alpha'$$

$$\underline{\underline{\alpha' = 78.46^\circ}}$$

$$\tan\theta = \frac{Q\sin\alpha'}{P + Q\cos\alpha'}$$

$$\tan\theta = \frac{800\sin 78.46^\circ}{1000 + 800\cos 78.46^\circ}$$

$$\underline{\underline{\theta = 34.04^\circ}}$$

$$\alpha = 90 - \theta = 90 - 34.05 = \underline{\underline{55.95}}$$

$$\beta = 180 - \alpha - \alpha' = 180 - 55.95 - 78.46$$

$$\underline{\underline{\beta = 45.59}}$$

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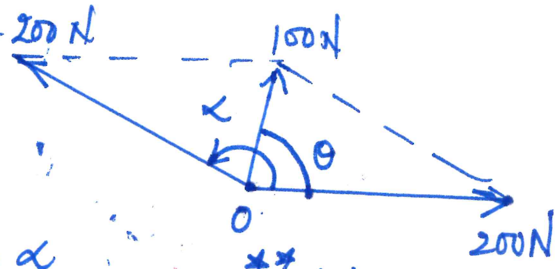
8) Resolve a force of 100 N into two Components of 200 N each Show the orientation of these components on a sketch

Sol^n: As all the magnitudes are known, to find the angles parallelogram law will be the most Convenient

The 100 N force will be the Resultant of two 200 N forces



$P = 200$ N, $Q = 200$ N, $R = 100$ N

$$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$$

$$100^2 = 200^2 + 200^2 + 2(200)(200)\cos\alpha$$

$$\boxed{\alpha = 151.04^\circ}$$

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha} = \frac{200\sin 151.04^\circ}{200 + 200\cos 151.04^\circ}$$

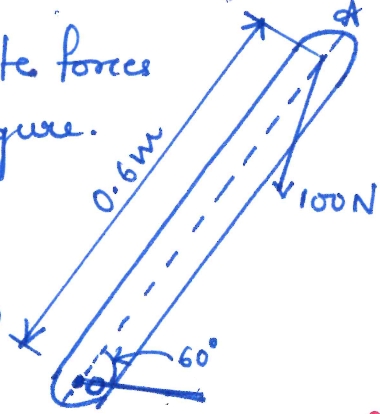
$$\boxed{\theta = 75.52^\circ}$$

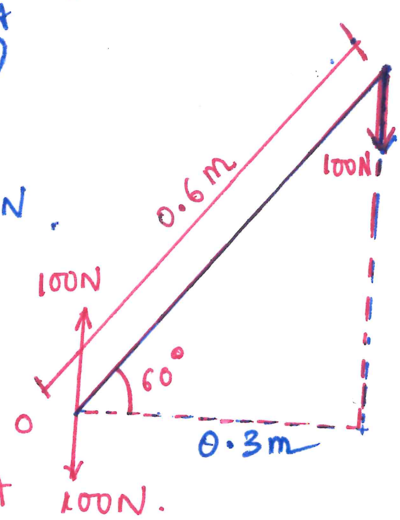
** Note:- Sketch will be assuming one of the components to be horizontal.

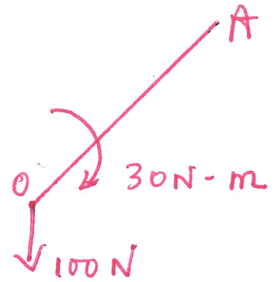
9. A 100N Vertical force is applied to a shaft at A as shown in figure. Determine the effect of 100N force at O

Soln:- Apply Equal and opposite forces of 100N at O as shown in figure.
The effect produced as shown in figure.

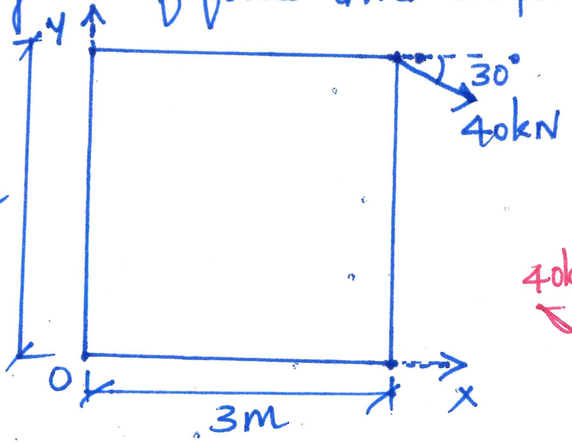
a) A downward force 100N @ O
b) clockwise couple of 100×0.3
$= \underline{\underline{30 \text{ N-m}}}$

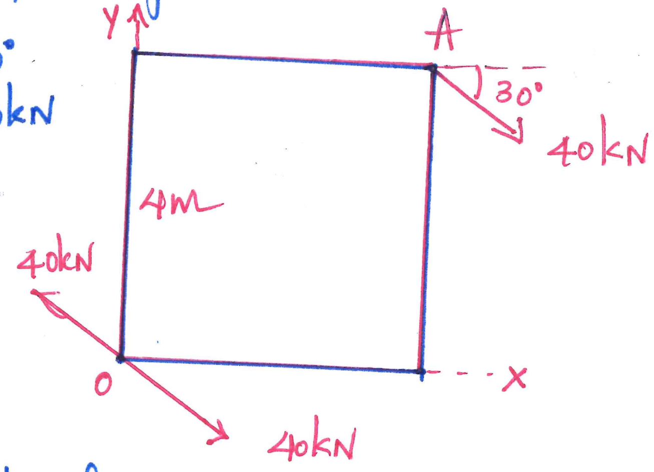






10. Reduce the force acting at A as shown in the figure into a system of force and couple at point O.

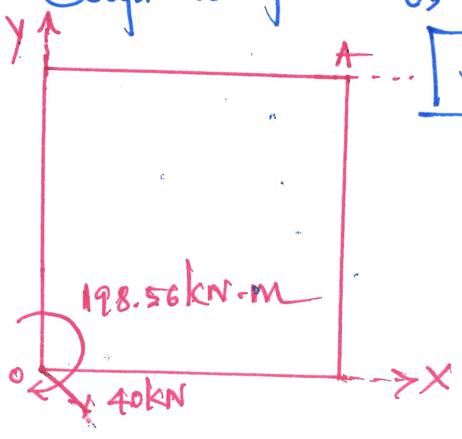




Soln: Apply equal and opposite forces of 40kN at O parallel to the given force.

Couple at point O, $M_o = 40\cos 30° \times 4 + 40\sin 30 \times 3$

$\boxed{M_o = 198.56 \text{ kN-m}}$ clockwise drn



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Numerical Problems on Coplanar Non-concurrent force System

If two or more forces are acting in a single plane, but not passing through the single point, such a force system is known as coplanar Non-concurrent force system.

Magnitude of the Resultant, $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

Direction of the Resultant, $\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$

Position of the Resultant :- The position of the Resultant means the calculation of d, or x and y intercepts as shown in the figure

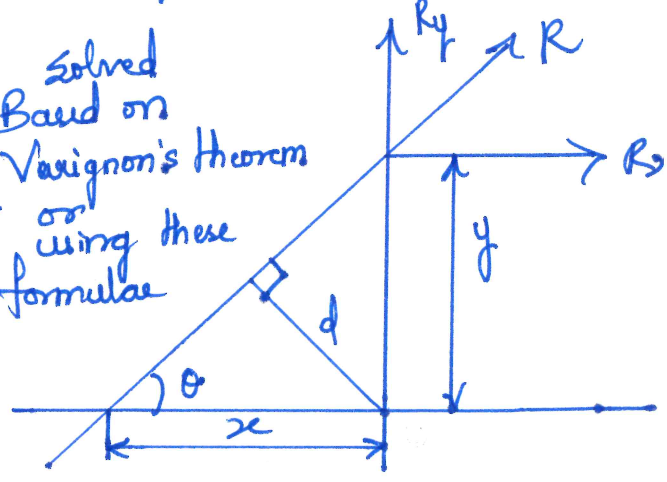
$R \times d$ = algebraic sum of moments of number of forces about that point.

$d = \frac{\Sigma M}{R}$

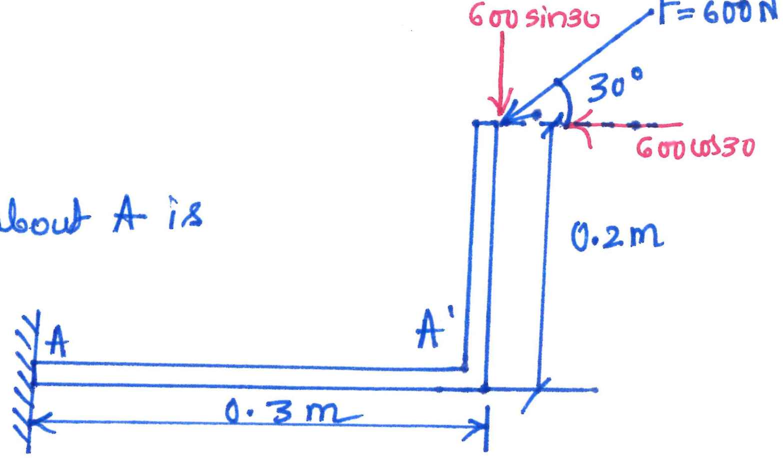
and x intercept : $x = \left|\frac{\Sigma M}{\Sigma F_y}\right|$

and y intercept : $y = \left|\frac{\Sigma M}{\Sigma F_x}\right|$

Solved Based on Varignon's theorem or using these formulae



1. Find the moment of the force F = 600N about A as shown in the figure



Solution :

Moment of force F = 600N about A is

$M_A = 600 \sin 30 (0.3) - 600 \cos 30 (0.2)$

$\boxed{M_A = -13.923 \text{ N-m}}$

or

$\boxed{M_A = 13.923 \text{ N-m} \circlearrowright}$

* Red pen marks will not be given in the question. It is the part of a Soln to be done by us.

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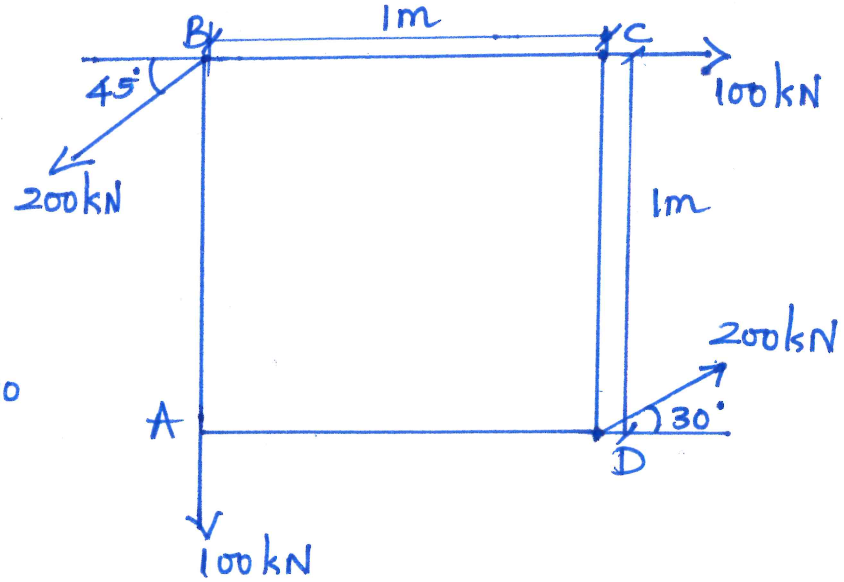
2) A rigid plate ABCD is subjected to forces as shown in the figure. Compute the magnitude, direction and line of action of the resultant of the system with reference to the point A.



Soln:-

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

$\Sigma F_x = 100 - 200\cos 45° + 200\cos 30$

$\Sigma F_x = 131.78\,kN$

$\Sigma F_y = -100 - 200\sin 45 + 200\sin 30$

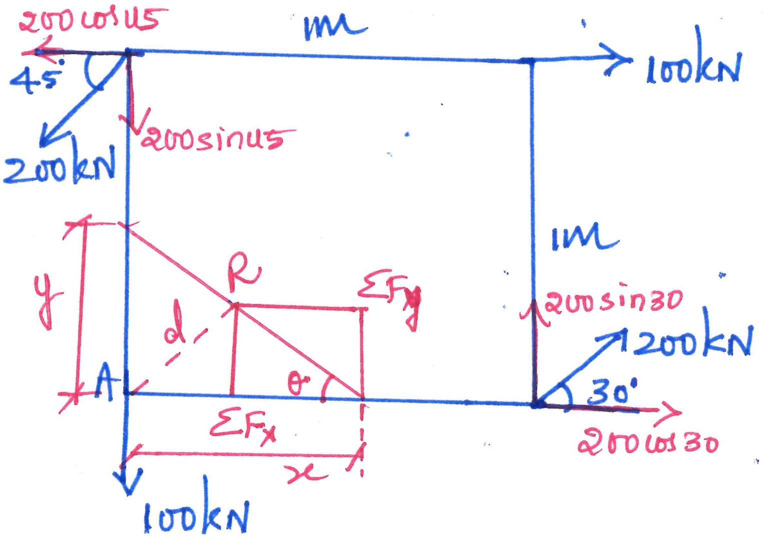
$\Sigma F_y = -141.42\,kN$

Also,

$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{-141.42}{131.78}$

$\theta = \tan^{-1}(-1.073)$

$\theta = -42.02°$



∴ The line of action 100kN is directly passing through A, ∴ the moment produced by 100kN force about A is zero.

$\Sigma M_A = \cancel{100 \times 0} + \cancel{200\sin 45(0)} - 200\cos 45(1) + 100(1) + \cancel{200\cos 30(0)}$
$- 200\sin 30(1)$

$= -200\cos 45 + 100 - \overset{100}{\cancel{200}}(1/2)$

$= -200\cos 45 + \cancel{100} - \cancel{100}$

$= -200\cos 45$

$\boxed{\Sigma M_A = -141.42\,kN}$

$d = \frac{\Sigma M_A}{R} = \left|\frac{-141.42}{193.3}\right| = 0.732\,m$

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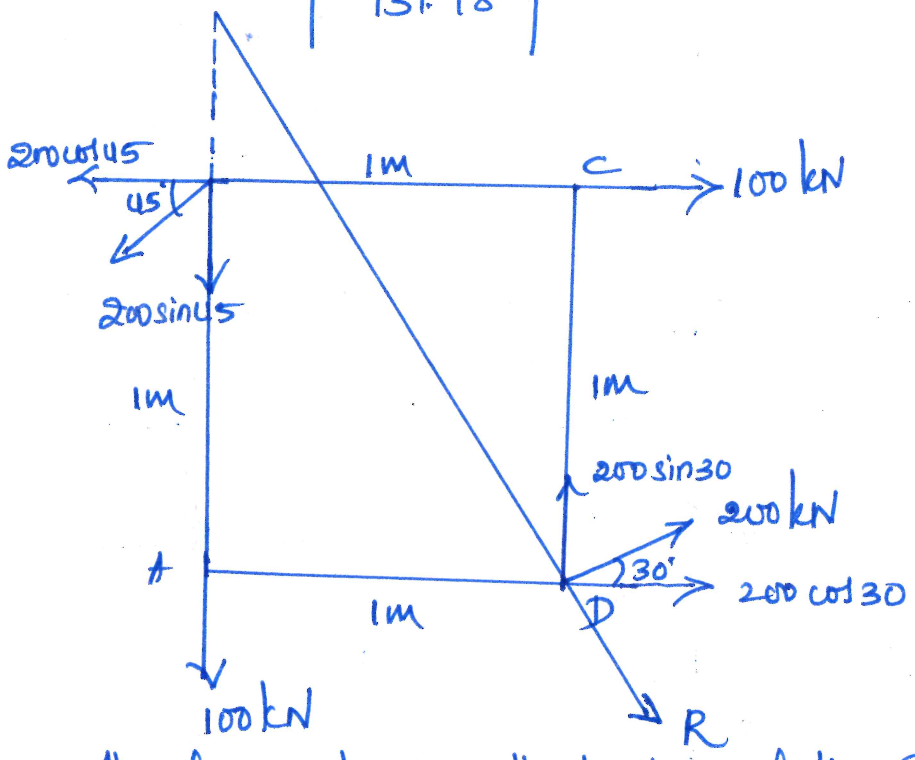
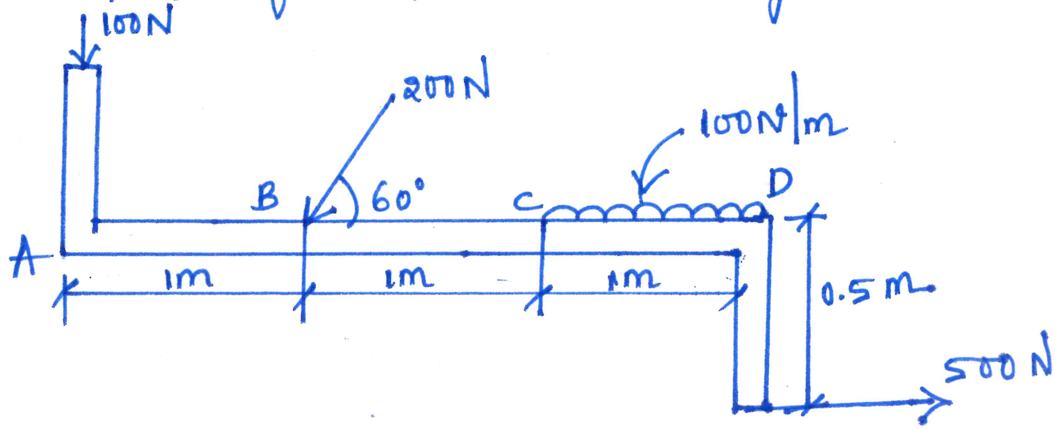
$$x \text{ intercept} = \left|\frac{\Sigma M_A}{\Sigma F_y}\right| = \left|\frac{-141.42}{-141.42}\right| = \underline{\underline{1m}}$$

$$y \text{ intercept} = \left|\frac{\Sigma M_A}{\Sigma F_x}\right| = \left|\frac{-141.42}{131.78}\right| = \underline{\underline{1.073m}}$$



The figure showing the position of the resultant.

3) Find the magnitude, direction and position of the resultant force with reference to A of the forces shown in figure



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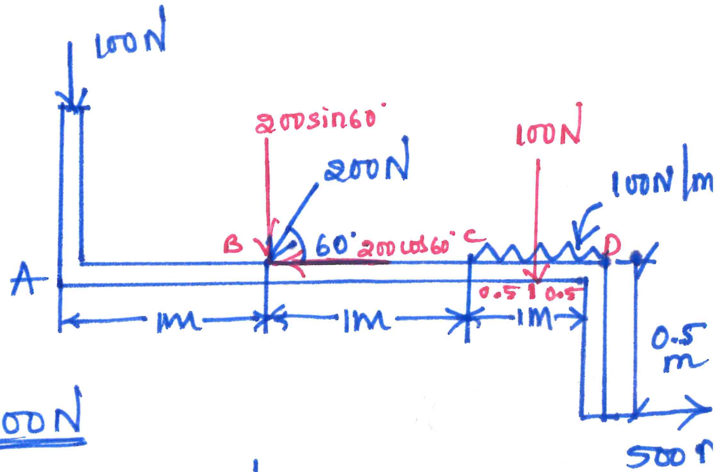
Soln:

convert the uniformly distributed load (UDL) into point load,

i.e $100\,N/m \times 1m = 100\,N$

which is acting at the centre of the span CD.



$$\Sigma F_x = -200\cos 60 + 500 = \underline{400\,N}$$

$$\Sigma F_y = -100 - 200\sin 60 - 100 = \underline{-373.21\,N}$$

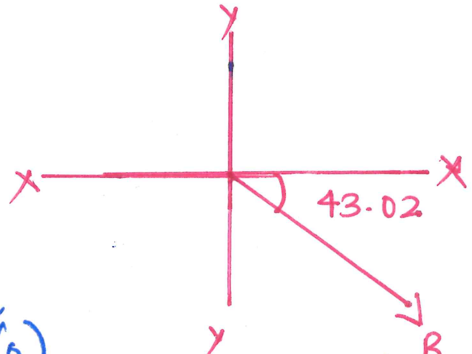
$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$R = \sqrt{400^2 + (-373.21)^2}$$

$$\boxed{R = 547.07\,N}$$

and $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{-373.21}{400}$

$$\boxed{\theta = -43.02}$$



showing the direction of the resultant.

Taking moments about A:

$$\Sigma M_A = 100(0) + 200\sin 60(1) + 200\cos 60(0) + 100(2.5) - 500(0.5)$$

$$\Sigma M_A = \underline{173.21\,N\text{-}m}$$

$$d = \frac{\Sigma M_A}{R} = \frac{173.21}{547.07}$$

$\cancel{d = 173.21}$ $\boxed{d = 0.316\,m}$

$$x \text{ intercept} = \left|\frac{\Sigma M_A}{\Sigma F_y}\right| = \left|\frac{173.21}{-373.21}\right| = \underline{0.464\,m}$$

$$y \text{ intercept} = \left|\frac{\Sigma M_A}{\Sigma F_x}\right| = \left|\frac{173.21}{400}\right| = \underline{0.433\,m}$$

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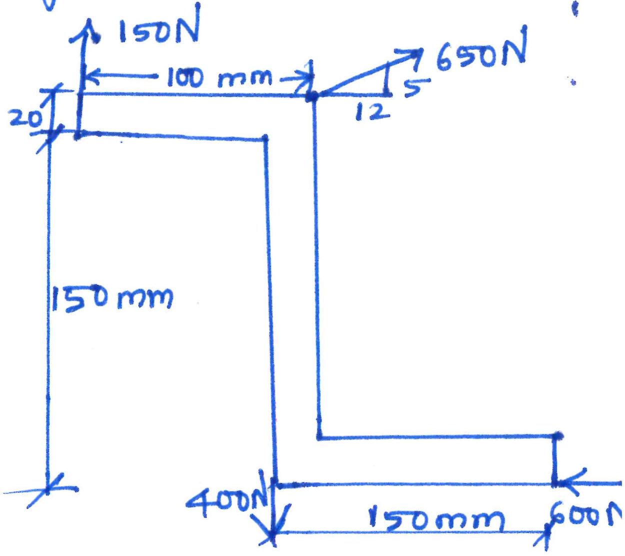
4) A Z shaped lamina of uniform width of 20 mm is subjected to four forces as shown in figure. Find the Equilibrant in magnitude and direction.



Soln: To find the Equilibrant, we need to find the Resultant,

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$\Sigma F_x = 650 \cos 22.6 - 600$$

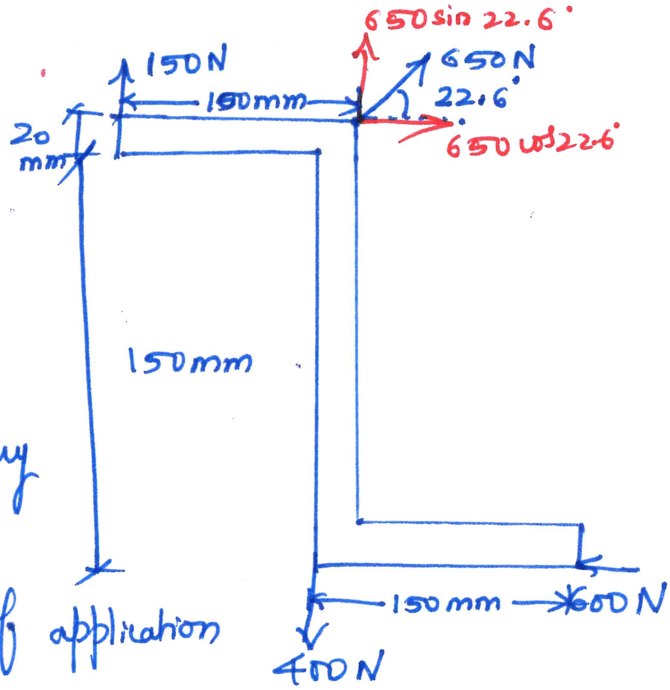
$$\boxed{\Sigma F_x = 0.08 \simeq 0}$$

$$\Sigma F_y = +150 + 650 \sin 22.6 - 400$$

$$\boxed{\Sigma F_y = 0}$$

$$R = \sqrt{0^2 + 0^2} = 0$$

$$\boxed{R = 0}$$



when R=0, the Resultant can be a moment which be same about any point in plane.

Taking moment about the point of application of 650 N force.

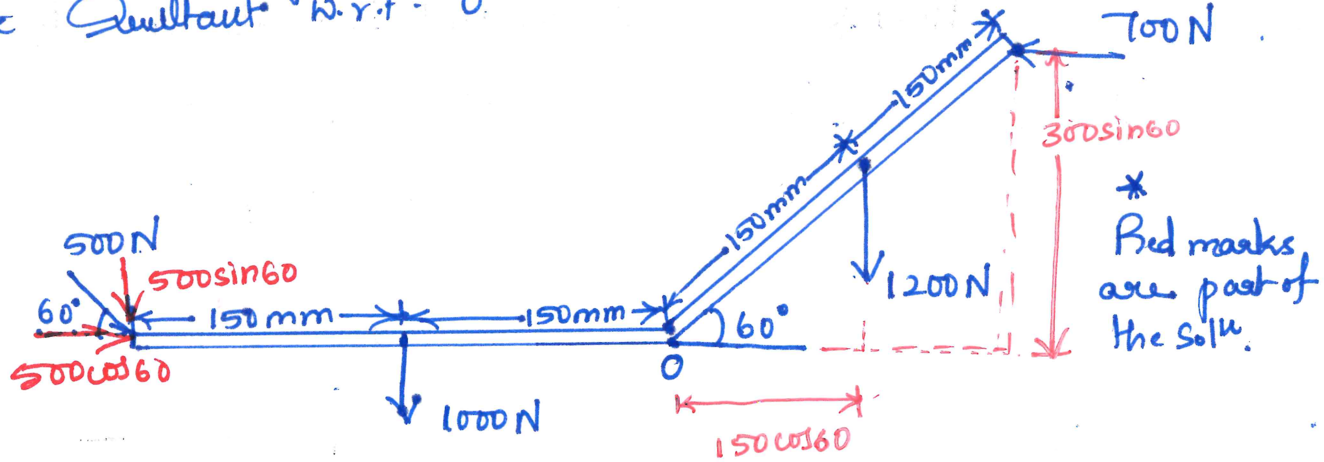
$$M = 650 \cos 22.6 (0) + 650 \sin 22.6 (0) + 150(100) - 400(20) + 600(170)$$

$$M = 109\,000 \text{ N mm}$$

$$\boxed{M = 109 \text{ Nm}}$$

∴ Equilibrant is a moment of 109 N-m.

3) A System of forces acting on a bell crank as shown in figure. Determine the magnitude, direction and the point of application of the Resultant w.r.t. 'O'



Soln:

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$\Sigma F_x = +500\cos 60 - 700$$

$$\Sigma F_x = -450\ N$$

$$\Sigma F_y = -500\sin 60 - 1000 - 1200$$

$$\Sigma F_y = -2633.01\ N$$

$$R = \sqrt{(-450)^2 + (-2633.01)^2}$$

$$\boxed{R = 2671.18\ N}$$

$$\theta = \tan^{-1}\left(\left|\frac{\Sigma F_y}{\Sigma F_x}\right|\right) = \tan^{-1}\left(\frac{-2633.01}{-450}\right)$$

$$\boxed{\theta = 80.30^\circ}$$

To find moment @ O.

$$M_o = 500\cos 60(0) - 500\sin 60(300) - 1000(150) + 1200(150\cos 60) - 700(300\sin 60)$$

$$M_o = -371,769\ N\text{-}mm$$

$$\boxed{M_o = 371.769\ N\text{-}m}$$

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$$d = \left|\frac{\Sigma M_o}{R}\right| = \left|\frac{-371.769}{2671.18}\right|$$

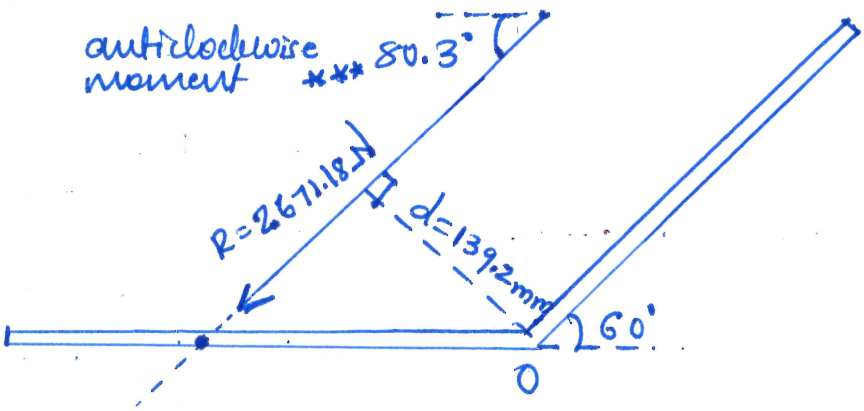
$\boxed{d = 0.139\, m}$ or $\boxed{d = 139\, mm}$

$$x \text{ intercept} = \left|\frac{\Sigma M_o}{\Sigma F_y}\right| = \left|\frac{-371.769}{-2633.01}\right| = \underline{0.141\, m}$$

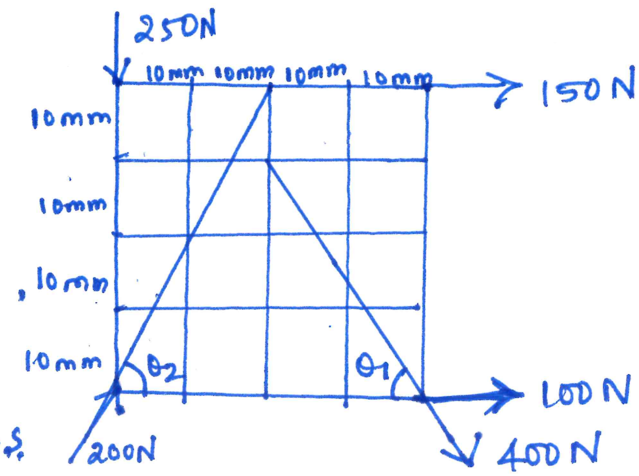
x intercept $= 0.141$ m or 141 mm

$$y \text{ intercept} = \left|\frac{\Sigma M_o}{\Sigma F_x}\right| = \left|\frac{-371.769}{-450}\right| = \underline{0.8261\, m}$$

y intercept $= 0.8261$ m or 826.1 mm



6. Determine the resultant of system of forces as shown in fig acting on a 40mm × 40mm size lamina. Each grid is of size 10mm × 10mm. Determine the x and y intercepts also.



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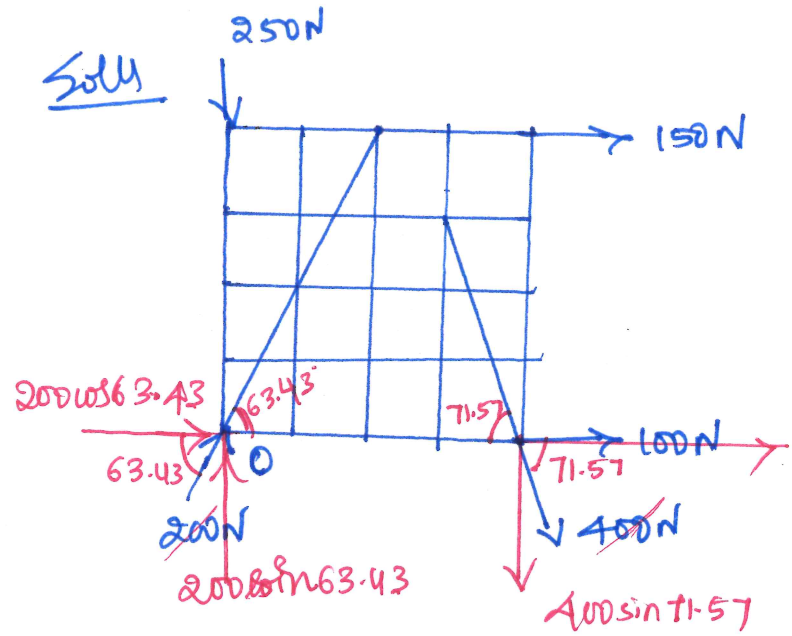
Soln



$\tan\theta_2 = \frac{40}{20} = \theta_2 = \tan^{-1} 2$

$\boxed{\theta_2 = 63.43}$

$\tan\theta_1 = 30/10 = , \theta_2 = \tan^{-1} 3$

$\boxed{\theta_2 = 71.57^\circ}$

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

$\Sigma F_x = 150 + 400\cos 71.57 + 200\cos 63.43 + 100$

$\Sigma F_x = 465.916\,N$

$\Sigma F_y = -250 - 400\sin 71.57 + 200\sin 63.43$

$\Sigma F_y = -450.588\,N$

$R = \sqrt{(465.916)^2 + (-450.588)^2}$

$R = 648.169\,N$

$\theta = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x} = \frac{-450.58}{465.916} = -44.04^\circ$

x intercept $= \left|\frac{\Sigma M_o}{\Sigma F_y}\right|$ y intercept $= \left|\frac{\Sigma M_o}{\Sigma F_x}\right|$

$\Sigma M_o = 200\cos 63.43(0) + 200\sin 63.43(0) - 100(0)$
$+ 400\cos 71.57(0) + 400\sin 71.57(40) + 250(0)$
$+ 150(40)$

$\Sigma M_o = 21178.933\ N\text{-}mm = 211.78\ N\text{-}m$

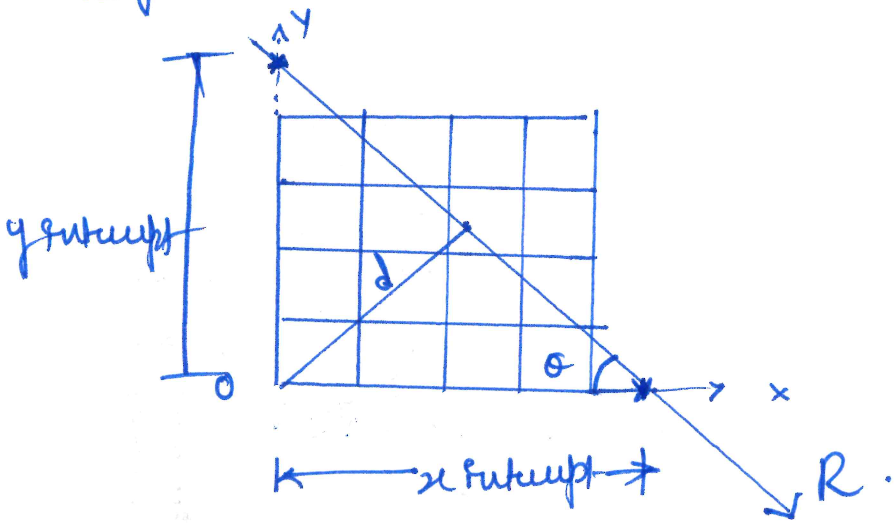
Lakshmi H.S

$$x\text{-intercept} = \left| \frac{21178.933}{-450.61} \right|$$

$$\boxed{x\text{-intercept} = 47\,mm}$$

$$y\text{ intercept} = \left| \frac{21178.933}{460.93} \right|$$

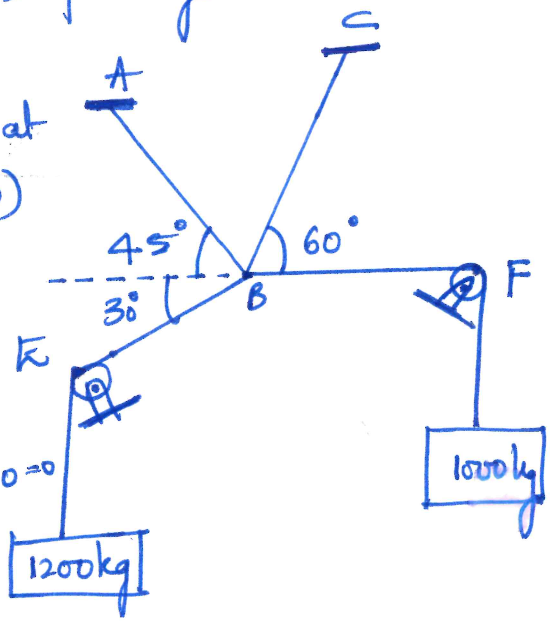
$$\boxed{y\text{-intercept} = 45.455\,mm}$$



Unit-4

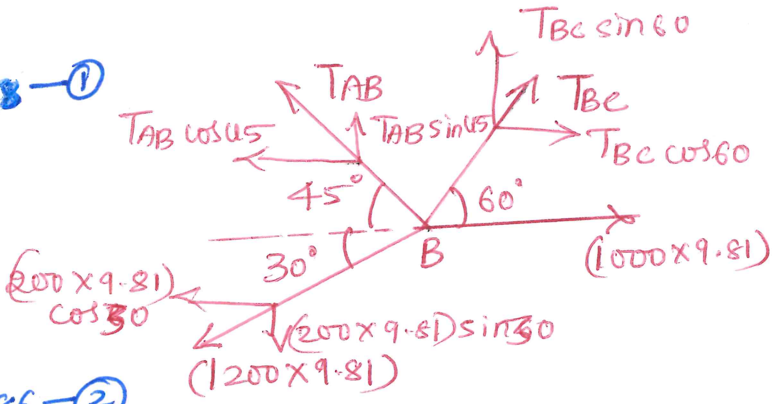
Numerical Problems on Equilibrium of Coplanar forces

1. Find the forces in cables AB and CB shown in figure. The remaining two cables pass over frictionless pulleys E & F and support masses 1200kg and 1000kg respectively



Soln.

All the forces are concurrent at B. The Free body diagram (FBD) of B as shown in the figure



$\Sigma F_x = 0$

$\Rightarrow -T_{AB}\cos 45 + T_{BC}\cos 60 + 1000(9.81) - (1200\times 9.81)\cos 30 = 0$

$\Rightarrow -T_{AB}\cos 45 + T_{BC}\cos 60 + 9810 - 10194.8 = 0$

$\Rightarrow -T_{AB}\cos 45 + T_{BC}\cos 60 = 384.8$ — ①

$\Sigma F_y = 0$

$\Rightarrow T_{AB}\sin 45 + T_{BC}\sin 60 - (1200\times 9.81)\sin 30 = 0$

$\Rightarrow T_{AB}\sin 45 + T_{BC}\sin 60 = 5886$ — ②

Solving ① & ②, we have

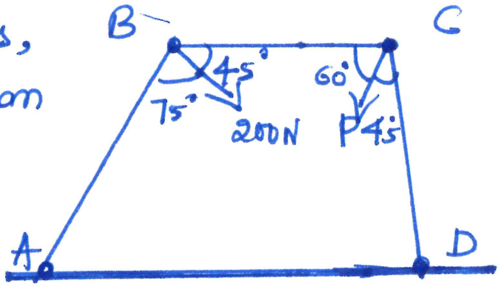
$\boxed{T_{AB} = 2701.81\ N}$

$\boxed{T_{BC} = 4590.54\ N}$

2. In a four bar mechanism shown in figure, bars are hinged at A, B, C and D. Find the force P that will prevent movement of bars. Also find forces in bars AB, BC and CD.

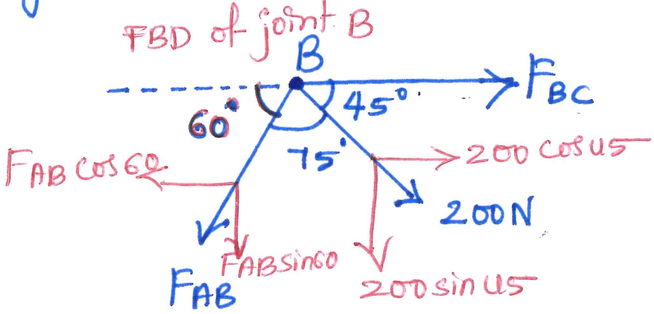
Lakshmi H.S.

2. . . . continued



Soln :- As the rods AB, BC and CD are subjected to forces only at their ends, the rods will have axial forces which can be either tensile or compressive.

Assuming tensile forces, draw F.B.D of point B as shown in figure.



$\Sigma F_y = 0$

$-F_{AB}\sin 60 - 200\sin 45 = 0$

$F_{AB} = -\dfrac{200\sin 45}{\sin 60}$

$\boxed{F_{AB} = 163.3N}$

$\boxed{F_{AB} = -163.3N}$

Negative sign indicates that F_{AB} is compressive.

$\boxed{F_{AB} = 163.3N\ (C)}$

$\Sigma F_x = 0$

$F_{BC} + 200\cos 45 - F_{AB}\cos 60 = 0$

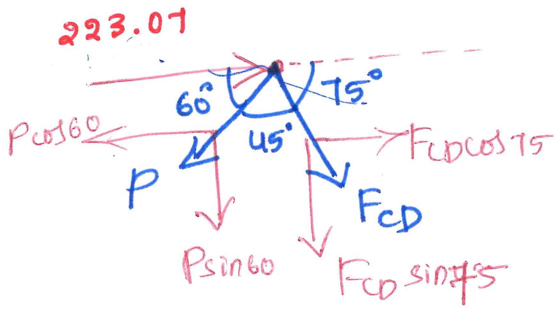
$F_{BC} + 200\cos 45 - (-163.3)\cos 60 = 0$

$\boxed{F_{BC} = -223.07N}$

-ve sign indicates that F_{BC} is compressive

$\boxed{F_{BC} = 223.07N\ (C)}$

As F_{BC} is compressive we show this force towards C in FBC of joint C as shown in the figure



$\Sigma F_x = 0$

$223.07 - P\cos 60 + F_{CD}\cos 75 = 0$ —— (3)

$\Sigma F_y = 0$

$-P\sin 60 - F_{CD}\sin 75 = 0$ —— (4)

Lakshmi H-S.

Solving ③ and ④

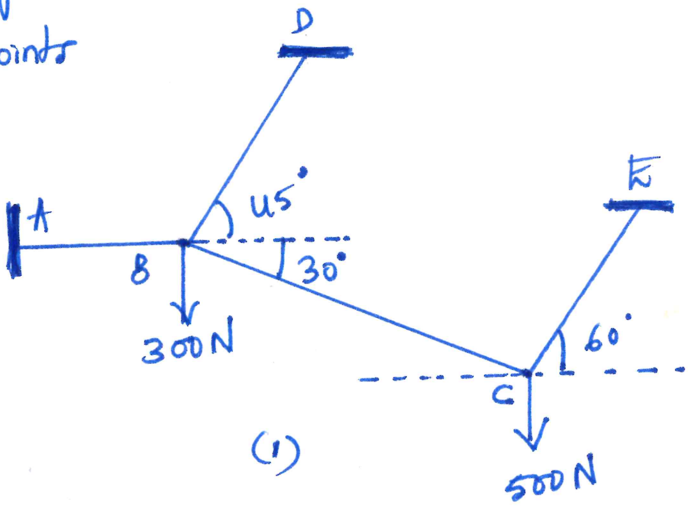
$$\boxed{P = 304.71\,N}$$

$F_{CD} = -273.20$

$$\boxed{F_{CD} = 273.20\,N\ (C)}$$

3. Below figure shows a System of cables in equilibrium condition under two vertical loads of 300N and 500N. Determine the forces developed in the different segments.

Free body diagrams of points B and C are shown in the figure (1) (a) and 1 (b) respectively



(1)

For joint B,

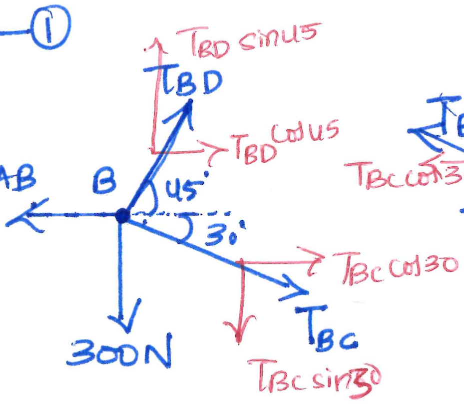
$\Sigma H = 0$

$\Rightarrow -T_{AB} + T_{BD}\cos 45 + T_{BC}\cos 30° = 0$ —①

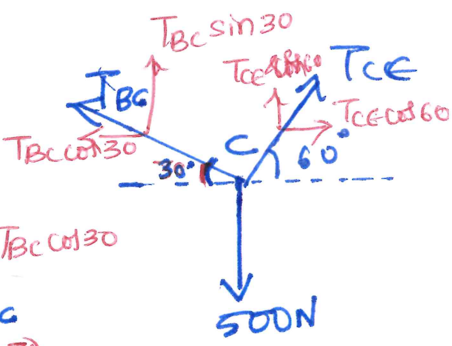
$\Sigma V = 0$

$T_{BD}\sin 45 - T_{BC}\sin 30 - 300 = 0$

$T_{BD}\sin 45 - T_{BC}\sin 30 = 300$ —②



1(a)



1(b)

For joint C

$\Sigma H = 0$

$-T_{BC}\cos 30 + T_{CE}\cos 60 = 0$

$-T_{BC}\cos 30 = -T_{CE}\cos 60$

$T_{BC} = \cos 60/\cos 30\ T_{CE}$

$$\boxed{T_{BC} = 0.577\,T_{CE}}$$ —③

Lakshmi H.S

$\Sigma V = 0$

$+ T_{BC} \sin 30 + T_{CE} \sin 60 - 500 = 0$ — ④

Apply T_{BC} value from Eq ③ in ④; we have

$(0.577 T_{CE} \sin 30) + T_{CE} \sin 60 = 500$

$1.154 T_{CE} = 500$

$\boxed{T_{CE} = 433.07 \text{ N}}$

Apply T_{CE} in Eq ③ to get T_{BC} i-e

$T_{BC} = 0.577(433.07)$

$\boxed{T_{BC} = 249.88 \text{ N}}$

Apply T_{BC} value in Equation ② to get T_{BD} i-e

$T_{BD} \sin 45 - (249.88) \sin 30 = 300$

$T_{BD} \sin 45 = \cancel{546.40}\ 424.94$

$\boxed{T_{BD} = 600.95 \text{ N}}$

Apply T_{BD} Value in Eqn ① to get T_{AB}, i-e

$-T_{AB} + (600.95) \cos 45 + T_{BC} \cos 30 = 0$

$T_{AB} = (600.95) \cos 45 + (249.88) \cos 30$

$\boxed{T_{AB} = 641.33 \text{ N}}$

4) Two identical rollers, each of weight 500 N, are supported by an inclined plane making an angle of 30° to the horizontal and a vertical wall as shown in the figure.

i) Sketch the free body diagram of the two rollers

ii) Assuming smooth surfaces, find the reactions at the support points.

Lakshmi H.S.

Soln:

The F.B.D of each roller is shown in figure 1(a) and 1(b), and 1(c)

The reaction between the two rollers makes an angle 30° with the horizontal as the line joining the centres is parallel to the inclined plane

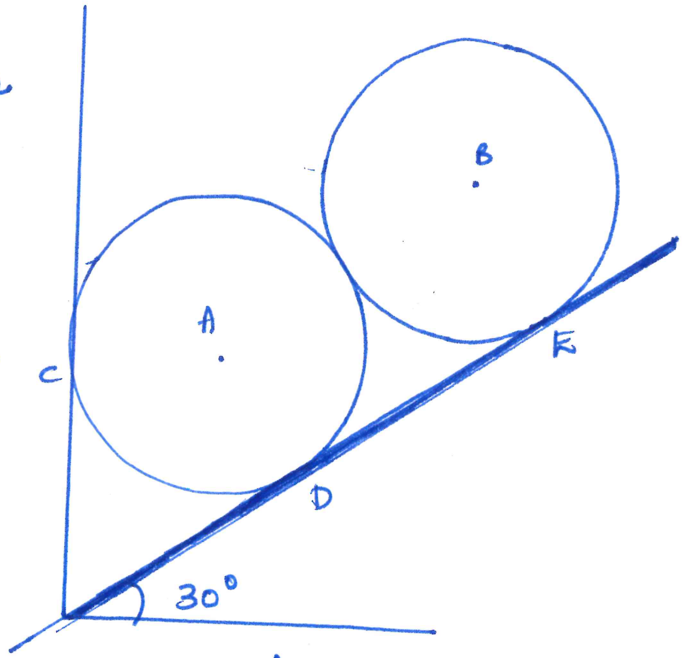
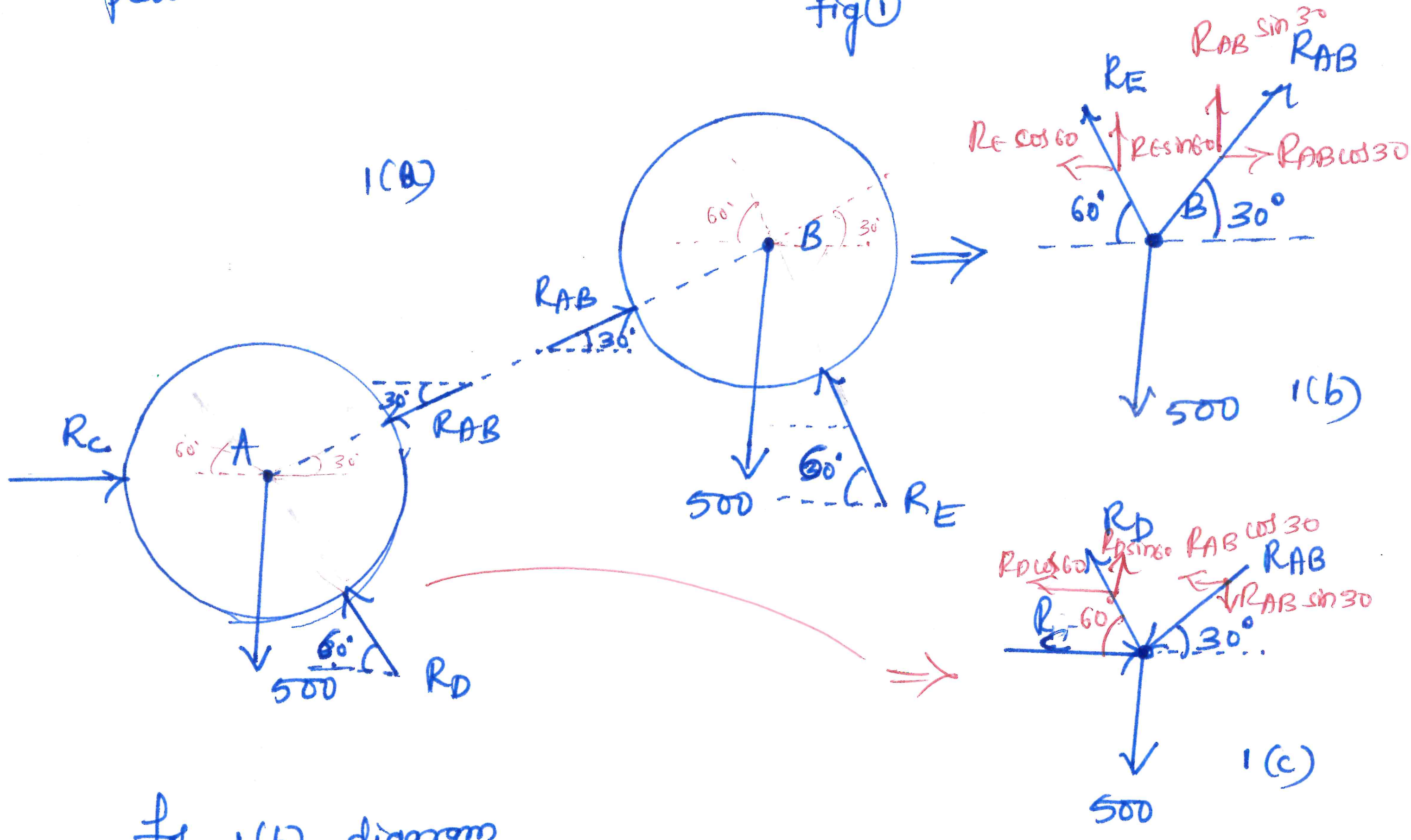


fig 1



For 1(b) diagram

$\Sigma V = 0$

$-500 + R_{AB}\sin 30 + R_E \sin 60 = 0$ —— (1)

$\Sigma H = 0$

$R_{AB}\cos 30 - R_E\cos 60 = 0$

$R_{AB} = R_E\cos 60/\cos 30$

$\boxed{R_{AB} = 0.577 R_E}$ —— (2)

Lakshmi H-S

Applying ② in ①, we get

$-500 + (0.577 R_E) \sin 30 + R_E \sin 60 = 0$

$\boxed{R_E = 433.01 N}$

Applying R_E value in ① we get,

$-500 + R_{AB} \sin 30 + (433.01) \sin 60 = 0$

$\boxed{R_{AB} = 250 N}$

for (i) c.

$\Sigma H = 0$

$\Rightarrow -R_{AB} \cos 30 - R_D \cos 60 + R_C = 0$

$R_C - R_D \cos 60 = 250 \cos 30$

$R_C - R_D \cos 60 = 216.50$ — ③

$\Sigma V = 0$

$R_D \sin 60 - R_{AB} \sin 30 - 500 = 0$

$R_D \sin 60 - 250 \sin 30 - 500 = 0$

$\boxed{R_D = 721.68 N}$

Apply R_D in Eqn ③ to get R_C

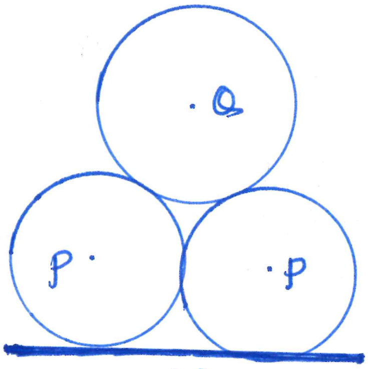
$R_C - (721.68) \cos 60 = 216.50$

$\boxed{R_C = 577.35 N}$

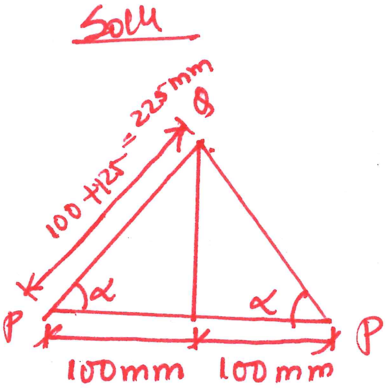
Walder HS

5) Two round solid identical cylinders each of radius 100 mm and weight 50 N, tied together by a thread of length 200 mm, rest on a horizontal surface. They hold a third homogeneous cylinder of radius 125 mm and weight 75 N, placed as shown in the figure. Calculate the tension in the thread and the reactions at all contact surfaces.

Soln

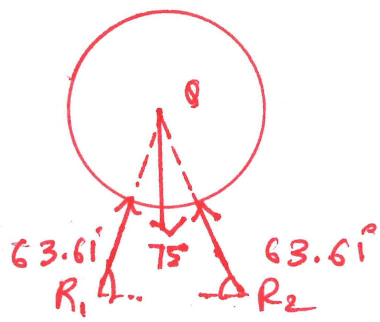


(1)

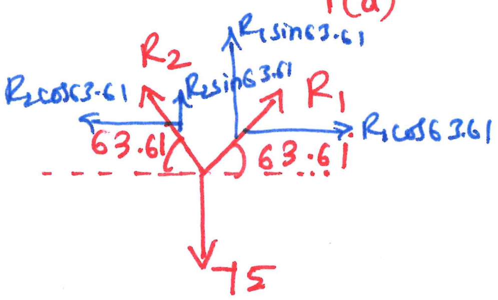


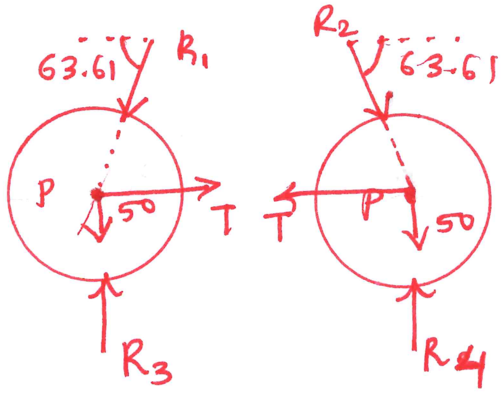
1(a)

1(b)



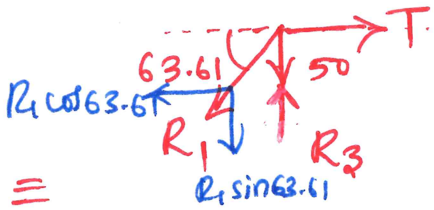
(1) b ≡



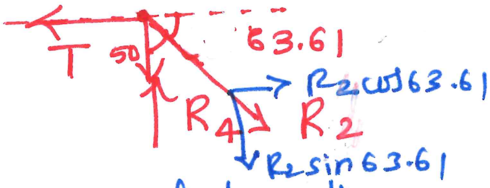


1(c) 1(d)

(1) c ≡



(1) d ≡



The triangle joined by lines joining the centres of three cylinders as shown in the figure 1(a)

$\cos\alpha = 100/225$

$\boxed{\alpha = 63.61^\circ}$

This is the angle made by the reactions between the cylinders with horizontal. The FBD of the cylinders are shown in the figure 1(b), 1(c) and 1(d)

Note that there is no reaction between the two lower cylinders as they tend to move away from each other due to upper cylinder.

Lakshmi H.S

from FBD (b)

$\Sigma H = 0$

$-R_2 \cos 63.61 + R_1 \cos 63.61 = 0$

$R_2 \cos 63.61 = R_1 \cos 63.61$

$\boxed{R_2 = R_1}$ —— ①

$\Sigma V = 0$

$R_1 \sin 63.61 + R_2 \sin 63.61 - 75 = 0$

$R_1 \sin 63.61 + R_2 \sin 63.61 = 75$ —— ②

Apply ① in ② we get

$2R_1 \sin 63.61 = 75$

$\boxed{R_1 = R_2 = 41.86\ N}$

from FBD 1(c)

$\Sigma F_x = 0$

$\Rightarrow T - R_1 \cos 63.61 = 0$

$T = R_1 \cos 63.61$

$T = 41.86 \cos 63.61$

$\boxed{T = 18.6\ N}$

$\Sigma F_y = 0$

$\Rightarrow -50 + R_3 - R_1 \sin 63.61 = 0$

$R_3 - R_1 \sin 63.61 = 50$

$R_3 - 41.86 \sin 63.61 = 50$

$\boxed{R_3 = 87.49\ N}$

from FBD (1) (d)

$\Sigma H = 0$

$-T + R_2 \cos 63.61 = 0$

$R_2 \cos 63.61 = T$ —— (3)

$\Sigma V = 0$

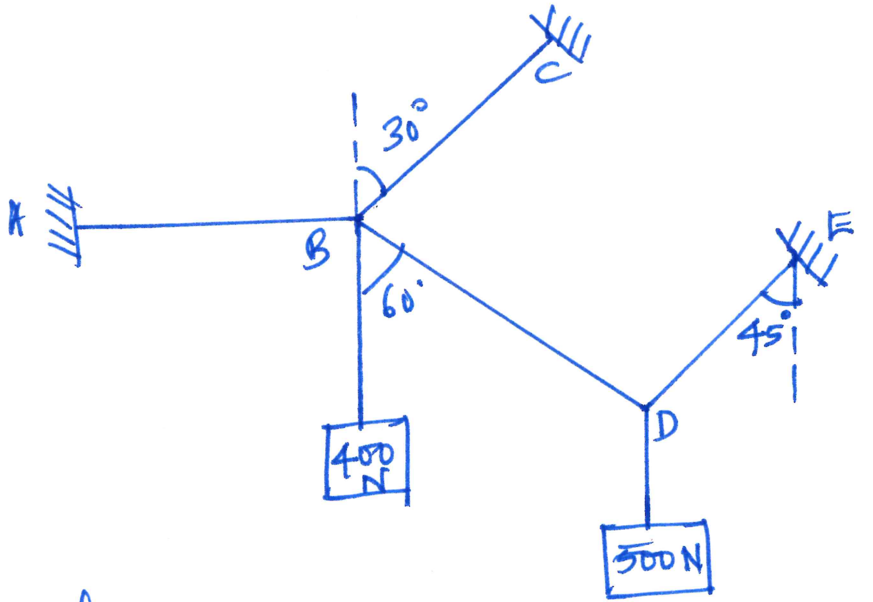
$R_4 - 50 - R_2 \sin 63.61 = 0$

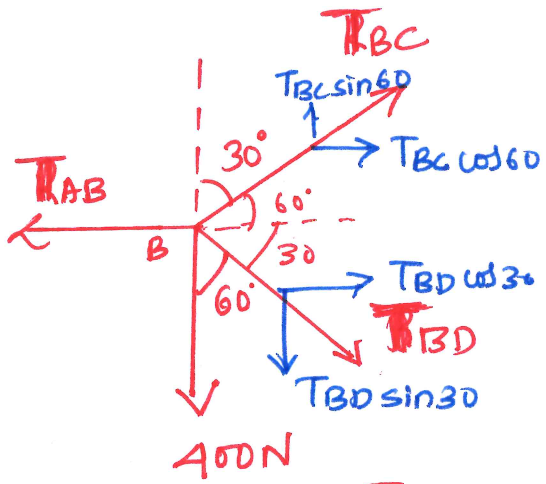
$R_4 - 50 - 41.86 \sin 63.61 = 0$

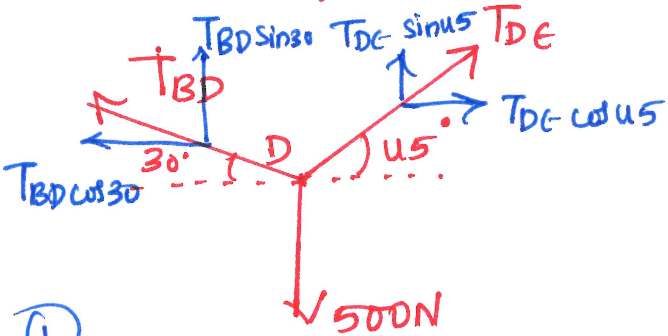
$\boxed{R_4 = 87.49\,N}$

6) The System of Connected flexible cables shown in the figure supporting 2 loads of 400 N and 500 N at points B and D respectively. Determine the tensions in the various segments of the cable

Sol^n:







for FBD @ joint B

$\Sigma V = 0$

$\Rightarrow T_{BC} \sin 60 - T_{BD} \sin 30 - 400 = 0$

$T_{BC} \sin 60 - T_{BD} \sin 30 = 400$ —— (1)

$\Sigma H = 0$

$T_{BC}\cos 60 + T_{BD}\cos 30 - T_{AB} = 0$ —— (2)

from FBD @ D

$\Sigma V = 0$

$T_{BD}\sin 30 + T_{DG}\sin 45 - 500 = 0$

$T_{BD}\sin 30 + T_{DG}\sin 45 = 500$ —— (3)

$\Sigma H = 0$

$T_{DE}\cos 45 - T_{BD}\cos 30 = 0$

$T_{DE} = T_{BD}\cos 30/\cos 45$

$\boxed{T_{DE} = 1.22\, T_{BD}}$ —— (4)

apply Eq (4) value in Eq^n (3), we will have,

$T_{BD}\sin 30 + 1.22\, T_{BD}\sin 45 = 500$

$\boxed{T_{BD} = 366.92\, N}$

$T_{DE} = 1.22(366.92)$

$\boxed{T_{DE} = 447.64\, N}$

Apply T_{DE} & T_{BD} in (1) & (2), we have

$T_{BC}\sin 60 - 366.92\sin 30 = 400$

$\boxed{T_{BC} = 673.72\, N}$

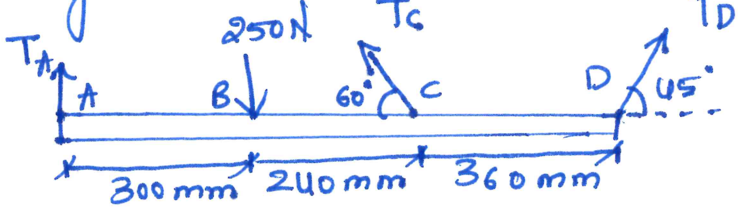
$$673.72 \cos 60 + 366.92 \cos 30 = T_{AB}$$

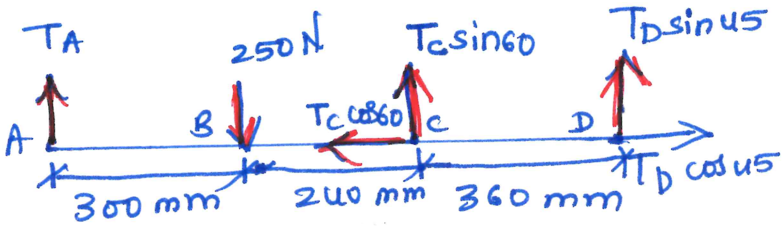
$$\boxed{T_{AB} = 654.2\,N}$$

Unit-4 — Numerical Problems on Equilibrium of Coplanar Non Concurrent forces

1. A light beam ABCD carries a load 250 N at B as shown in fig. It is supported by three strings at A, C and D. Determine the tensions in the strings.

Soln:-





$$\Sigma F_x = 0$$

$$-T_C \cos 60 + T_D \cos 45 = 0$$

$$T_C \cos 60 = T_D \cos 45$$

$$T_C = \frac{\cos 45}{\cos 60} T_D$$

$$\boxed{T_C = 1.414 T_D} \quad —(1)$$

$$\Sigma F_y = 0$$

$$+T_A - 250 + T_C \sin 60 + T_D \sin 45 = 0$$

$$T_A + T_C \sin 60 + T_D \sin 45 = 250 \quad —(2)$$

$$\Sigma M_A = 0$$

$$T_A \cancel{(0)} + 250(300) - T_C \cancel{\cos 60}(0) - T_C \sin 60 (540) + T_D \cancel{\cos 45}(0)$$

$$- T_D \sin 45 (900) = 0$$

$$75000 - 467.65 T_C - 636.39 T_D = 0$$

$$75000 - 467.65(1.414 T_D) - 636.39 T_D = 0$$

$$1297.64 T_D = 75000$$

$$\boxed{T_D = 57.79 N}$$

Apply T_D in Eq (1) to get T_C, $T_C = 1.414(57.79)$

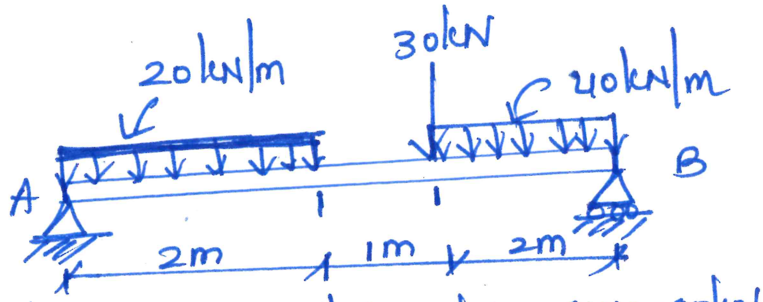
$$\boxed{T_C = 81.71 N}$$

Applying T_C and T_D in Eq (2) to get T_A

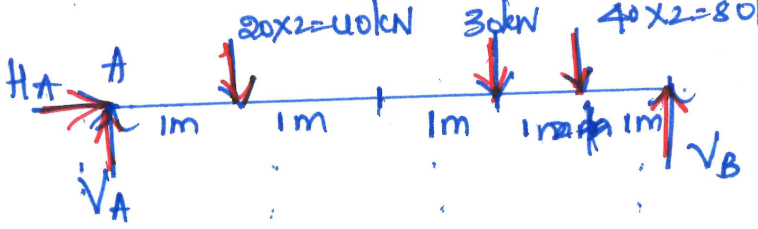
$$T_A = 250 - 81.71 \sin 60 - 57.79 \sin 45$$

$$\boxed{T_A = 138.37 N}$$

2) Determine the Support Reactions for beam AB loaded and supported as show in figure



Soln:



$\Sigma F_x = 0$

$$\boxed{H_A = 0}$$

$\Sigma F_y = 0$

$+V_A - 40 - 30 - 80 + V_B = 0$

$V_A + V_B = 150$ —— ①

$\Sigma M_A = 0$

$H_A(0) + V_A(0) + 40(1) + 30(3) + 80(4) = V_B(5) = 0$

$+V_B(5) = +40 + 90 + 320$

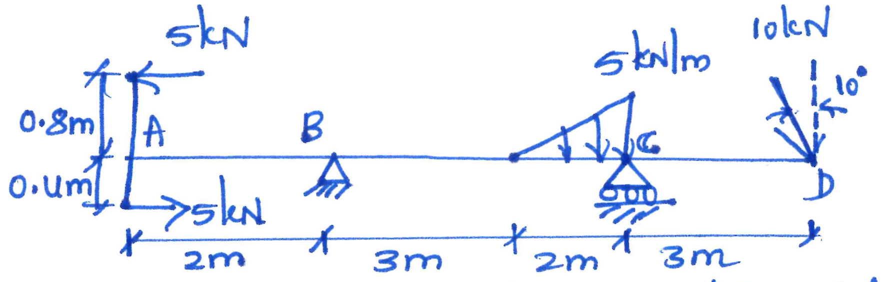
$$\boxed{V_B = 90\ kN \uparrow}$$

Applying V_B in Eq ① to get V_A

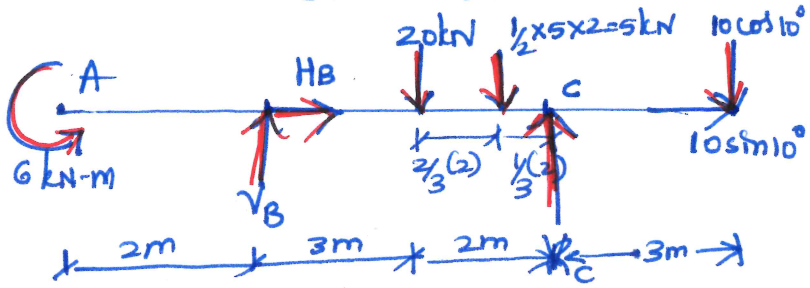
$V_A = 150 - 90$

$$\boxed{V_A = 60\ kN \uparrow}$$

3) A beam ABCD having self weight 2kN/m is subjected to additional load as shown in the figure. Find the support reaction at B and C.



Soln: To draw FBD.
The two forces of 5kN form a couple moment
$5 \times 1.2 = 6$ kN-m
The weight of beam 2kN/m acting throughout so
$= 2 \times 10m = 20kN$
acting at centre.



$\Sigma F_x = 0$

$H_B + 10\sin 10° = 0$

$H_B = -10\sin 10°$

$\underline{H_B = 1.736 \text{ kN}}$

$\Sigma F_y = 0$

$+V_B - 20 - 5 - 10\cos 10° + V_c = 0$

$V_B + V_c = \underline{34.84 \text{ kN}}$

$\Sigma M_B = 0$

$-6 + V_B(\cancel{0}) + H_B(\cancel{0}) + 20(3) + 5(4.333) - V_c(5) + 10\sin 10°(0) + 10\cos 10°(8) = $

$-6 + 60 + 21.65 - 5V_c + 78.78 = 0$

$5V_c = 154.43$

$\boxed{V_c = 30.886 \text{ kN}}$

$\boxed{V_B = 3.96 \text{ kN}}$

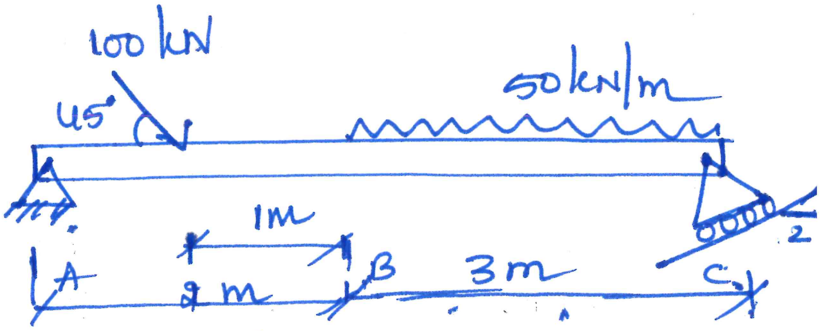
4) For the beam with the loading shown in fig. Determine the reactions at the supports.

Soln

1) FBD

angle made by reaction with horizontal is $\tan^{-1}(2/1)$

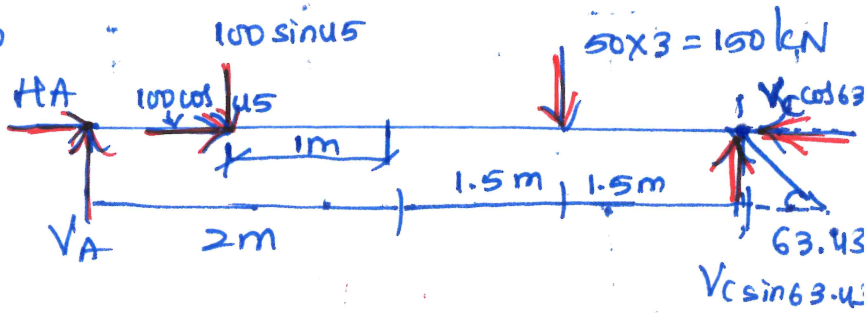
$= \underline{\underline{63.43^\circ}}$



$\Sigma f_x = 0$

$+H_A + 100\cos 45 - V_c\cos 63.43 = 0$

$H_A - V_c\cos 63.43 = -70.71$ —①



$\Sigma f_y = 0$

$+V_A - 100\sin 45 - 150 + V_c\sin 63.43 = 0$

$V_A + V_c\sin 63.43 = 220.71$ —②

$\Sigma M_c = 0.$

$\cancel{H_A(0)} + V_A(5) - 100\sin 45(4) - 150(1.5) + \cancel{V_c\cos 63.43(0)} + \cancel{V_c\sin 63.43} = 0$

$5V_A = 507.84$

$\boxed{V_A = 101.56\ kN}$

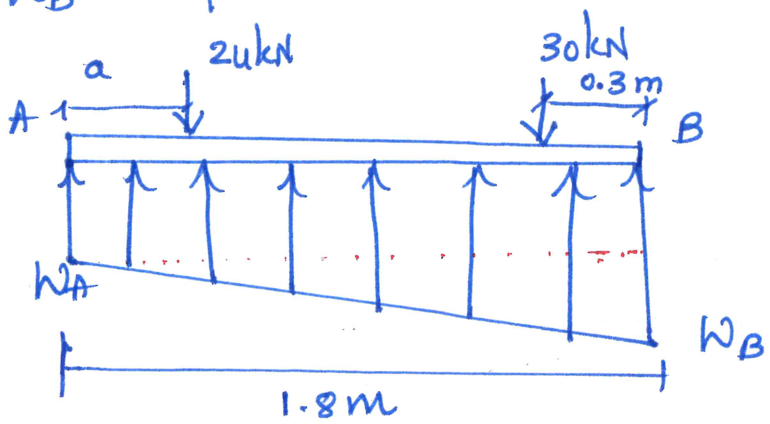
$V_c = \dfrac{220.71 - 101.56}{\sin 63.43}$

$\boxed{V_c = 133.21\ kN}$

$H_A = -70.71 + (133.21)\cos 63.43$

$\boxed{H_A = -11.12\ kN}$

5) The beam supports two concentrated loads and rests on the soil which exerts a linearly distributed reaction as shown in the figure. If $W_A = 18$ kN/m determine i) Distance 'a'
ii) Corresponding value of W_B in kN/m.



Sol^n:

$\Sigma F_y = 0$

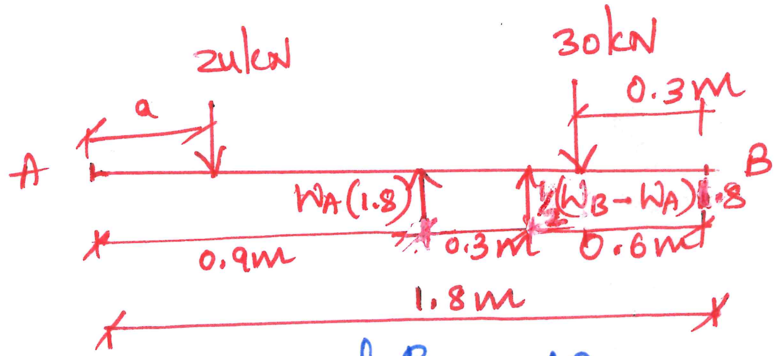
$\Rightarrow -24 - 30 + 1.8(W_A) + (W_B - W_A)\frac{1}{2}\times 1.8 = 0$

Given $W_A = 18$ kN/m

$\Rightarrow -24 - 30 + 1.8(18) + (W_B - 18)\frac{1}{2}\times 1.8 = 0$

$\Rightarrow -24 - 30 + 32.4 + 0.9W_B - 16.2 = 0$

$\Rightarrow \boxed{W_B = 42\text{ kN/m}}$



FBD of Beam AB

$\Sigma M_A = 0$

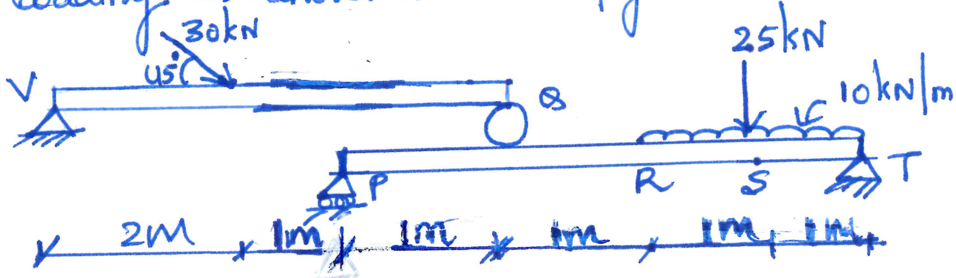
$\Rightarrow 24(a) - W_A(1.8)(0.9) - (W_B - W_A)\frac{1}{2}\times 0.8\times 1.2 + 30(1.5) = 0$

$\Rightarrow 24a - 18(1.8)(0.9) - (42-18)\frac{1}{2}\times 1.8\times 1.2 + 30(1.5) = 0$

$24a = 29.16 + 25.92 - 45$

$\boxed{a = 0.42\text{ m}}$

6) Determine the reactions at supports P, Q, T and V for the beam subjected to loading as shown in the figure.



Soln

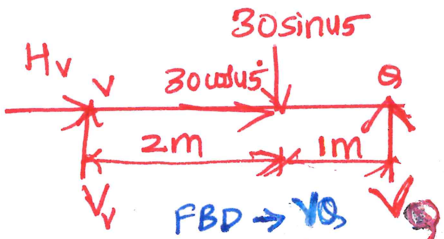
For FBD VQ

$\Sigma f_x = 0$

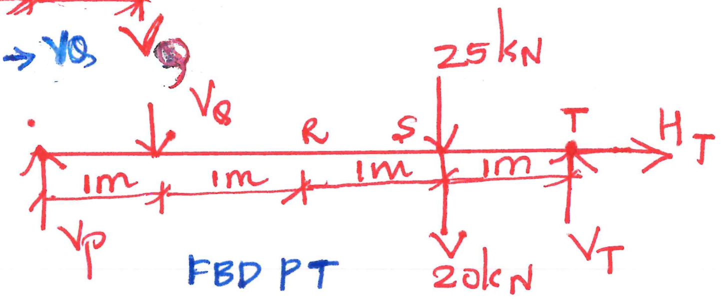
$+ H_V + 30\cos 45° = 0$

$\boxed{H_V = -21.21\ kN}$

$\boxed{H_V = 21.21\ kN \leftarrow}$



FBD → VQ



FBD PT

$\Sigma F_y = 0$

$V_V + V_Q - 30\sin 45 = 0$

$V_V + V_Q = 30\sin 45° \quad \text{—— (1)}$

$\Sigma M_V = 0$

$\cancel{H_V(0)} + \cancel{V_V(0)} + \cancel{30\cos 45(0)} + 30\sin 45°(2) - V_Q(3) = 0$

$V_Q = \dfrac{30\sin 45(2)}{3}$

$\boxed{V_Q = 14.14\ kN} \uparrow$

Applying V_Q in Eq (1) to get V_V

$V_V = 30\sin 45 - 14.14$

$\boxed{V_V = 7.073\ kN \uparrow}$

For FBD PT,

$\Sigma F_x = 0$

$\Rightarrow H_T = 0$

$\Sigma F_y = 0$

$V_P - V_Q - 25 - 20 + V_T = 0$

$V_P - 14.14 - 25 - 20 + V_T = 0$

$V_P + V_T = 59.14 \quad \text{—— (2)}$

$\Sigma M_T = 0$

$V_P(4) - V_Q(3) - 25(1) - 20(1) + H_T\cancel{(0)} + V_P\cancel{(0)} = 0$

$V_P(4) - 14.14(3) - 25 - 20 = 0$

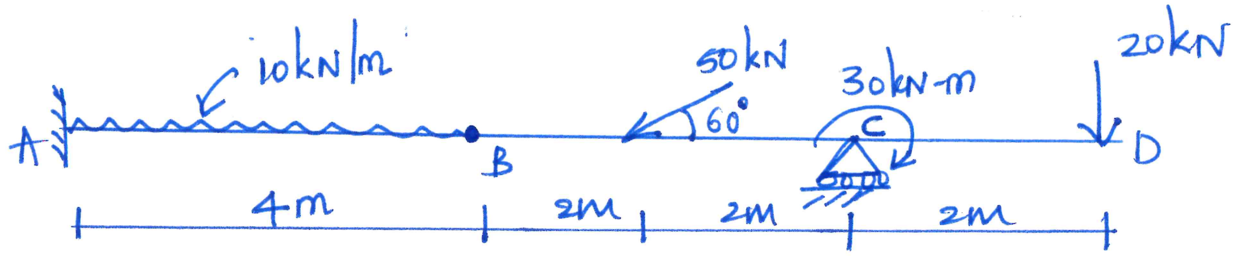
$\boxed{V_P = 21.85\,kN \uparrow}$

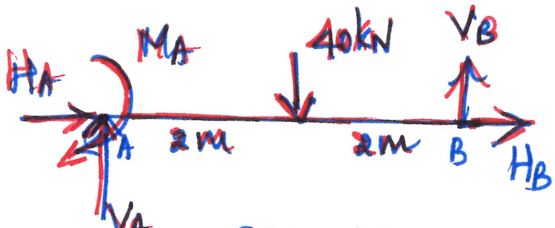
Applying V_P in Eqn ② to get V_T

$V_T = 59.14 - 21.85$

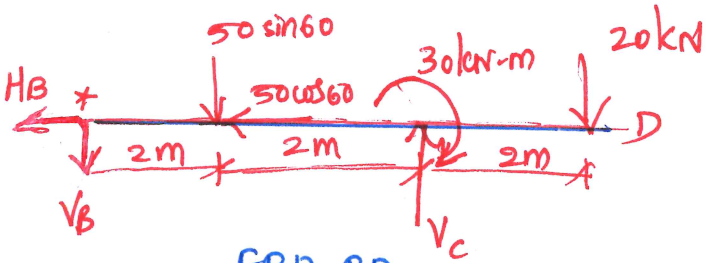
$\boxed{V_T = 37.29\,kN \uparrow}$

7) ~~Analyse~~ the compound beam ABCD shown in the figure to ~~find~~ the reaction @ the internal hinge B and @ supports ~~A~~ and C.





FBD AB



FBD BD

Soln

for FBD AB

$\Sigma F_x = 0$

$H_A + H_B = 0$ —— ①

$\Sigma F_y = 0$

$V_A - 40 + V_B = 0$

$V_A + V_B = 40$ —— ②

* → When you replace internal hinge & put reactions in FBD for 2nd part you have to take reactions in opp to first part

$\Sigma M_A = 0$

$\Rightarrow \cancel{H_A(0)} + \cancel{V_A(0)} + M_A + 40(2) - V_B(4) + \cancel{H_B(0)} = 0$

$\Rightarrow M_A - 4V_B = -80$ —— ③

$\sum F_x = 0$ for FBD BD $\rightarrow$

$-H_B - 50\cos 60 = 0$

$H_B = -50\cos 60$

$\boxed{H_B = -25\,kN}$ or $\boxed{H_B = +25\,kN}$ $(\leftarrow)$

$\boxed{H_A = 25\,kN}\ \rightarrow$

$\sum F_y = 0$

$-V_B - 50\sin 60 + V_c - 20 = 0$

$-V_B + V_c = 63.60$ —— (4)

$\sum M_B = 0$

$\cancel{H_B(0)} + \cancel{V_B(0)} + \cancel{50\cos 60(0)} + 50\sin 60(2) + 30 - V_c(4) + 20(6) = 0$

$\boxed{V_c = 59.15\,kN \uparrow}$

Applying V_c in Eq (4) to get V_B

$-V_B + 59.15 = 63.30$

$-V_B = 4.15$

$\boxed{V_B = -4.15\,kN}$ or $\boxed{V_B = 4.15\,kN \downarrow}$

Applying V_B in Eq^n (2) to get V_A

$\cancel{-(-4.15) + V_c = 63.60}$

$V_A = -V_B + 40$

$V_A = -(-4.15) + 40$

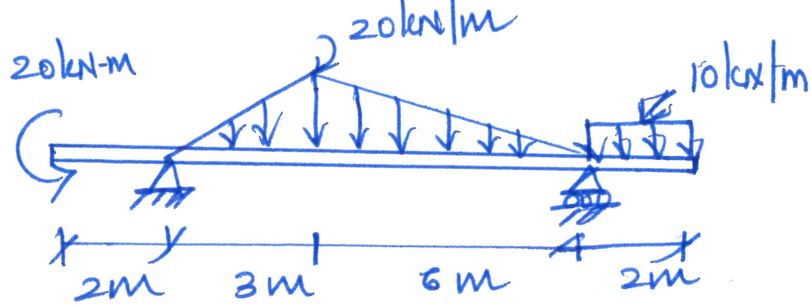
$\boxed{V_A = 44.15\,kN \uparrow}$

Applying V_B in Eq^n (3) to get M_A

$M_A = -80 + 4(-4.15)$

$\boxed{M_A = 96.6\,kN\text{-}m}$ or $\boxed{M_A = -96.6\,kN\text{-}m}$

8) Find the Reactions at the supports of the beam loaded as shown in the figure



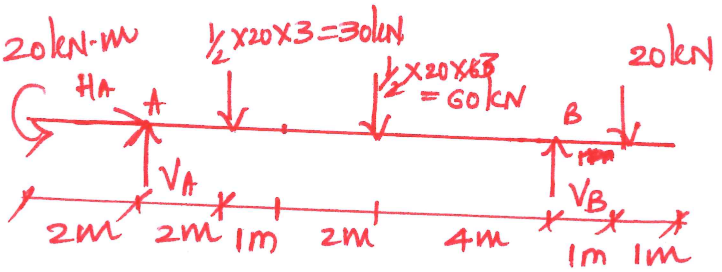
Soln

$\Sigma F_x = 0$ (No H force/load) $\therefore$ $\boxed{H_A = 0}$

$\Sigma F_y = 0$

$\Rightarrow V_A - 30 - 60 - 20 + V_B = 0$

$V_A + V_B = 110$ —— ①



$\Sigma M_A = 0$

$\cancel{H_A(0)} + \cancel{V_A(0)} - 20 + 30(2) + 60(5) - V_B(9) + 20(10) = 0$

$-20 + 60 + 300 + 200 = V_B(9)$

$\boxed{V_B = 60\,kN \uparrow}$

$\boxed{V_A = 50\,kN \uparrow}$

9) Find analytically the support reaction at B and load P for the beam as shown in fig., if reaction @ support A is zero

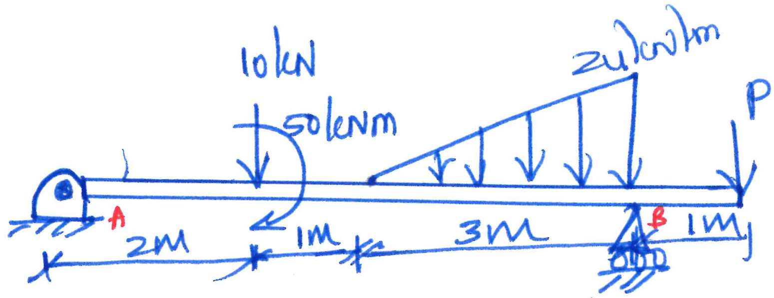
Soln

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$-10 - 36 + V_B + P = 0$

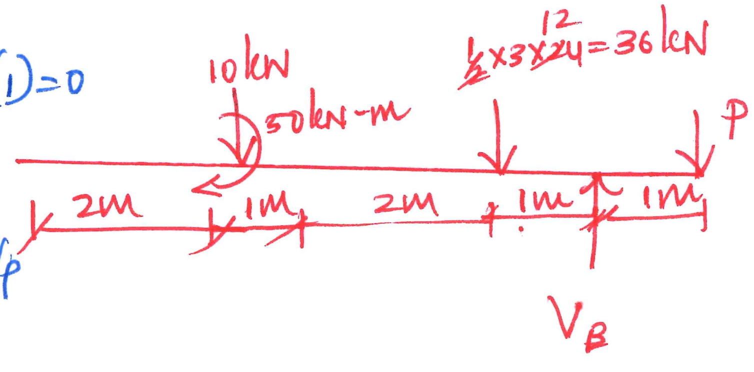
$V_B + P = 46$ —— ①



$\Sigma M_B = 0$

$+50 - 10(4) - 36(1) + \cancel{V_B(0)} + P(1) = 0$

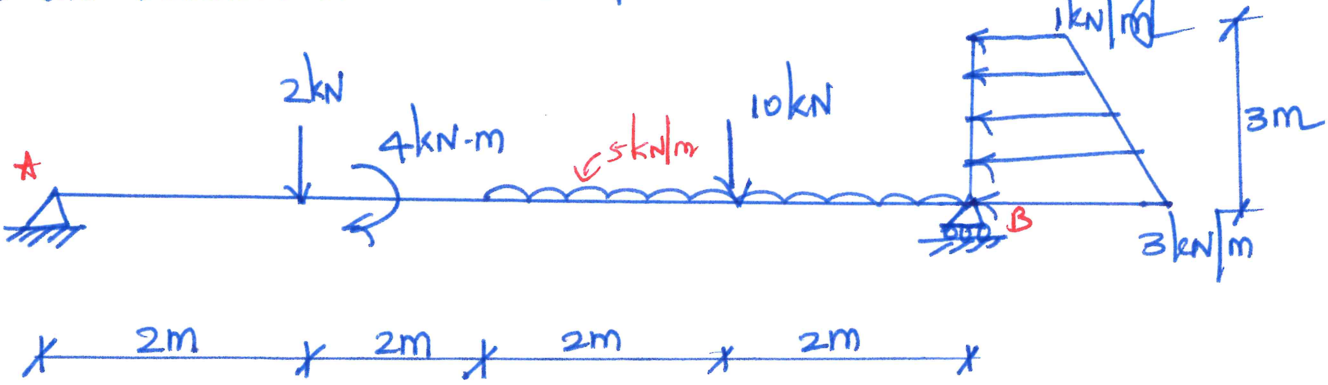
$\boxed{P = 26\,kN}$

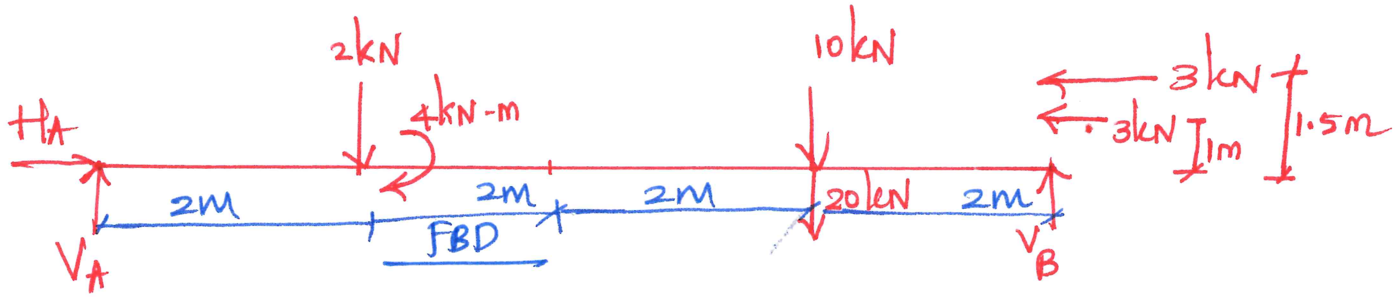


Applying P in Eq ① to get V_B

$\boxed{V_B = 12\,kN}$

10) Find the reactions at A and B for beam shown in fig.





<u>Soln</u>

$$\Sigma F_x = 0$$

$$H_A - 3 - 3 = 0$$

$$\boxed{H_A = 6\ kN \rightarrow}$$

$$\Sigma F_y = 0$$

$$V_A - 2 - 10 - 20 + V_B = 0$$

$$V_A + V_B = 32 \quad \text{——} (1)$$

$$\Sigma M_A = 0$$

$$\cancel{H_A(0)} + \cancel{V_A(0)} + 2(2) + 4 + 10(6) + 20(6) - V_B(8) - 3(1) - 3(1.5) = 0$$

$$\Rightarrow \quad 4 + 4 + 60 + 120 - 8V_B - 3 - 4.5 = 0$$

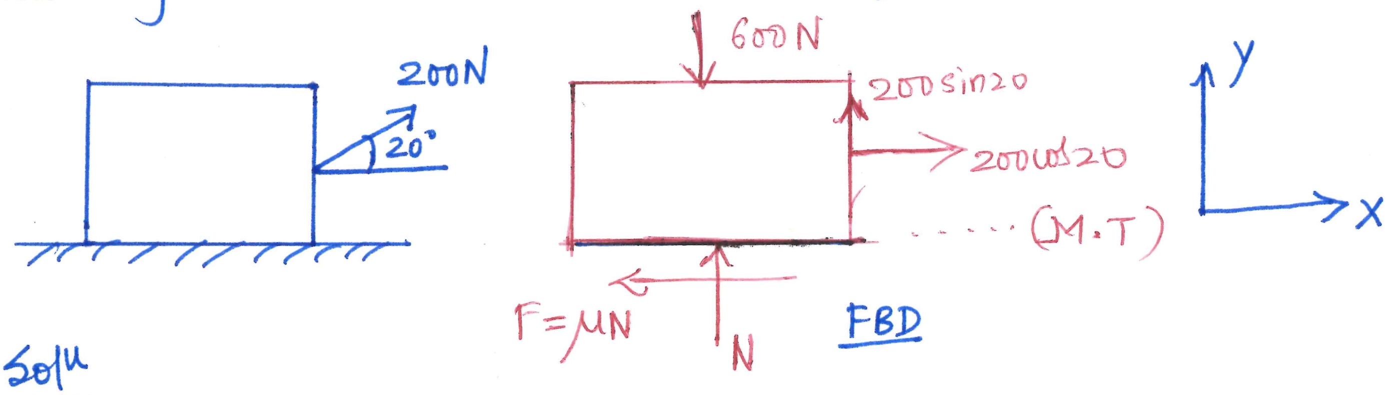
$$\boxed{V_B = 22.56\ kN \uparrow}$$

Applying V_B in Eqn (1) to get V_A

$$\boxed{V_A = 9.44\ kN \uparrow}$$

Numerical Problems on Friction

1) A block shown in figure is just moved by a force of 200 N. The weight of the block is 600 N. Determine the coefficient of static friction between the block and the floor.



Soln

Considering Equilibrium Conditions

$$\Sigma F_x = 0$$

$$200 \cos 20 - \mu N = 0$$

$$\mu N = 187.93 \quad \text{—— (1)}$$

$$\Sigma F_y = 0$$

$$+N - 600 + 200 \sin 20 = 0$$

$$\boxed{N = 531.59\,N}$$

Substituting the value of N in Eqn (1)

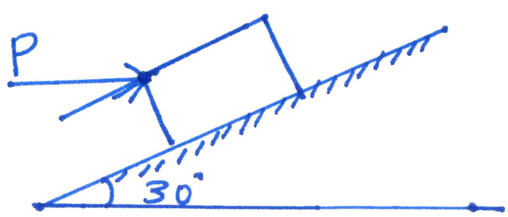
$$\mu = \frac{187.93}{531.59}$$

$$\boxed{\mu = 0.35}$$

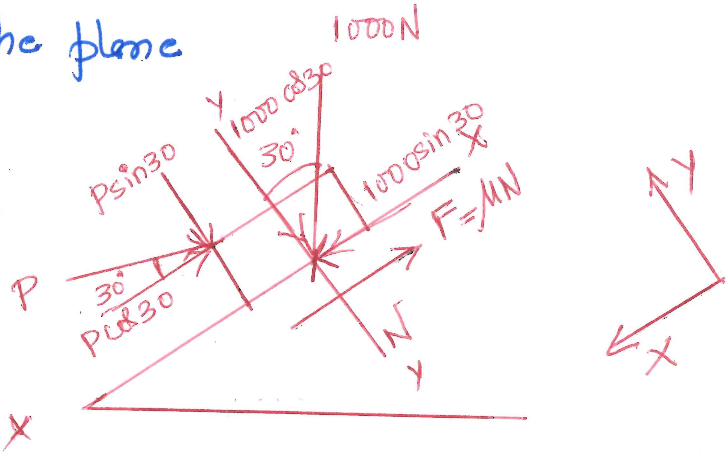
2) A Small block of weight 1000 N as shown in figure is placed on 30° inclined plane with μ = 0.25. Determine the horizontal force to be applied for:

i) Impending motion down the plane

ii) Impending motion up the plane



1) Impending motion down the plane



$$\Sigma F_x = 0$$

$$+P\cos 30 - 1000\sin 30 + \mu N = 0 \quad \text{—(1)}$$

$$\Sigma F_y = 0$$

$$+N - 1000\cos 30 - P\sin 30 = 0$$

$$N = 866.02 + 0.5P \quad \text{—(2)}$$

Applying N Value in Eq (1).

$$P\cos 30 - 1000\sin 30 + \mu(866.02 + 0.5P) = 0$$

$$P\cos 30 - 1000\sin 30 + 866.02\mu + 0.5\mu P = 0$$

$$0.867P + 866.02\mu + 0.5\mu P = 500$$

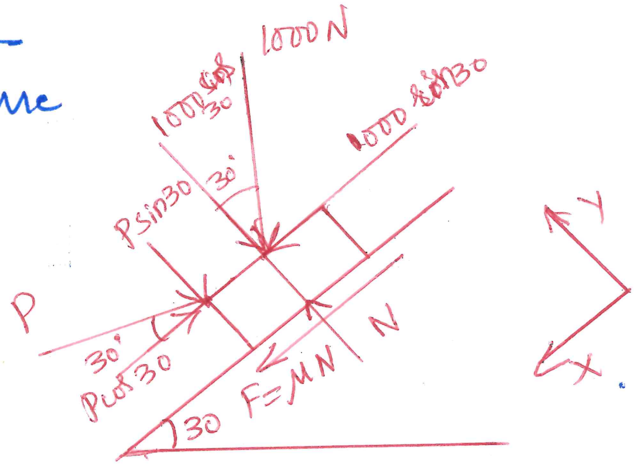
$\mu = 0.25$ (given data)

$$0.867P + 866.02(0.25) + 0.5(0.25)P = 500.$$

$$0.992P = 283.495$$

$$\boxed{P = 286.06N}$$

2) Impending motion up the plane



$$\Sigma F_x = 0$$

$$-\mu N - 1000\sin 30 + P\cos 30 = 0 \quad \text{—(3)}$$

$$\Sigma F_y = 0$$

$$-P\sin 30 - 1000\cos 30 + N = 0$$

$$N = 0.5P + 866.02 \quad \text{—(4)}$$

Applying N Value in Equ (3)

$$-\mu(0.5P + 866.02) - 500 + 0.867P = 0$$

$$-0.5P\mu - 866.02\mu + 0.867P = 500$$

$$-0.5P(0.25) - 866.02(0.25) + 0.867P = 500$$

$$0.742P = 716.505$$

$$\boxed{P = 965.64N}$$

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3) A body resting in plane (horizontal) required a pull of 18 kN inclined at 30° to plane just to move it. It was also found that a push of 22 kN inclined at 30° to the plane just moved the body. Determine the weight of the body and coefficient of friction.

Soln When a pull of 18 kN is applied. FBD fig(1)

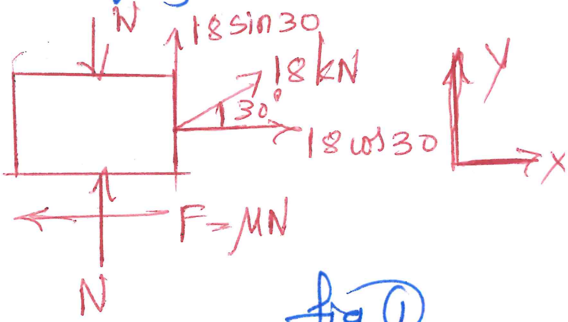


fig ①

$\Sigma F_x = 0$

$-\mu N + 18\cos 30 = 0$ —— ①

$\Sigma F_y = 0$

$-W + N + 18\sin 30 = 0$

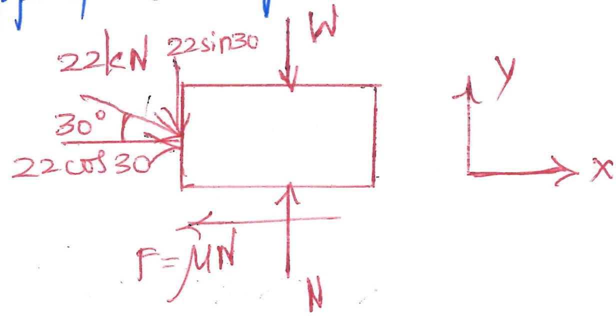
$N = W - 9$ —— ②

Applying N value in Eq ①

$-\mu(W-9) + 15.58 = 0$

$\boxed{\mu = \dfrac{15.58}{W-9}}$ —— ③

When a pull of 22 kN is applied (FBD) fig ②



$\Sigma F_x = 0$

$+22\cos 30 - \mu N = 0$ —— ④

$\Sigma F_y = 0$

$-W + N - 22\sin 30 = 0$

$N = W + 11$ —— ⑤

Applying N value in Eqn ④

$19 - \mu(W+11) = 0$

$19 = \mu(W+11)$

$\boxed{\mu = \dfrac{19}{W+11}}$ —— ⑥

Equating ③ & ⑥

$\dfrac{15.58}{W-9} = \dfrac{19}{W+11}$

$15.58W + 171.38 = 19W - 171$

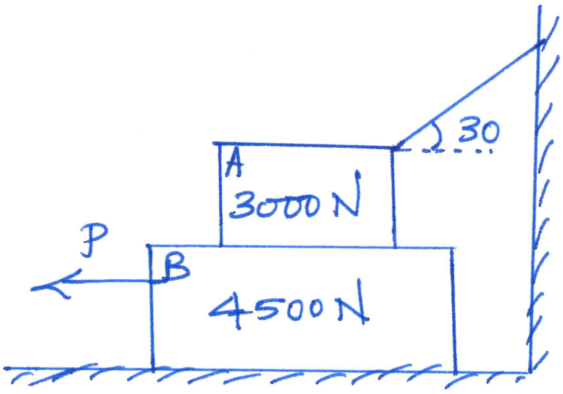
$3.42W = 342.38$

$\boxed{W = 100.1\,kN}$

Applying W in Eq ⑤ or ⑥ to get μ

$\boxed{\mu = 0.171}$

4) A block weighing 4500 N resting on horizontal surface supports another block of 3000 N as shown in the figure. Find the horizontal force P required to just move the block to the left. Take the coefficient of friction for all contact surface as 0.3.



For FBD A block

$\Sigma F_x = 0$

$T\cos 30 - 0.3N_2 = 0$ — ①

$\Sigma F_y = 0$

$T\sin 30 - 3000 + N_2 = 0$

$N_2 = 3000 - 0.5T$ — ②

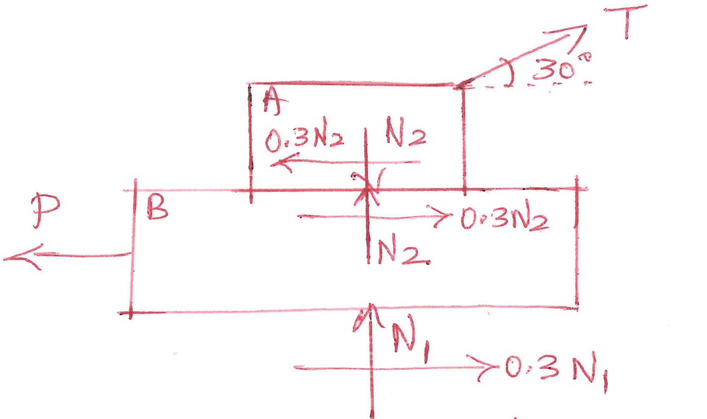
Applying N in Eq ①

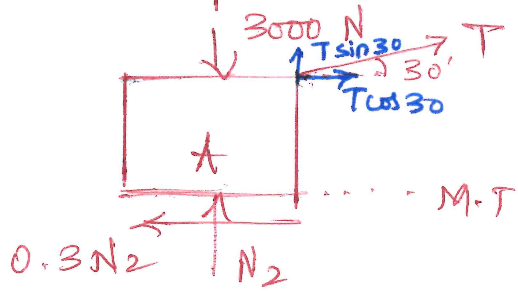
$T\cos 30 - 0.3(3000 - 0.5T) = 0$

$0.867T - 900 + 0.15T = 0$

$1.017T = 900$

$\boxed{T = 885\ N}$





$N_2 = 3000 - 0.5(885)$

$\boxed{N_2 = 2557.52\ N}$

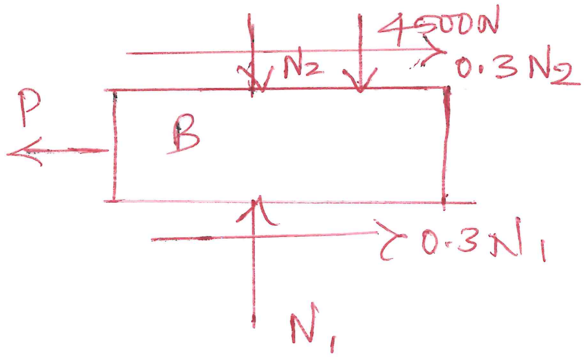
For FBD B block

$\Sigma F_x = 0$

$\Rightarrow 0.3N_2 + 0.3N_1 - P = 0$

$0.3(2557.52) + 0.3N_1 - P = 0$

$P = 0.3N_1 + 767.25$ — ③



$\Sigma F_y = 0$

$-N_2 - 4500 + N_1 = 0$

$N_1 = 2557.52 + 4500$

$\boxed{N_1 = 7057.52\ N}$

Applying N_1 in Eq ③

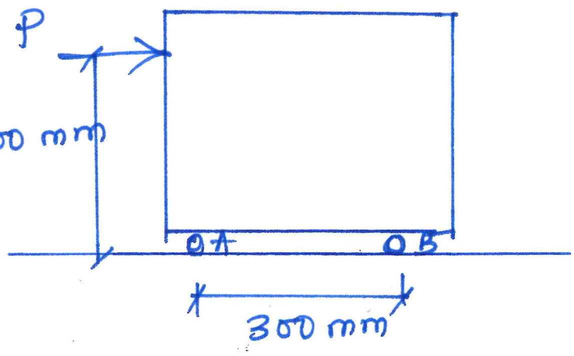
$P = 0.3(7057.52) + 767.25$

$\boxed{P = 2884.50\ N}$

5) A 100 kg cupboard is mounted on small caster wheels which can be locked to prevent their rotation. If $\mu_s = 0.3$ between wheels and floor, determine P to move cupboard to the right when

a) All casters are locked b) Only casters @ B are locked

c) Only casters @ A are locked.



Sol

Soln — for case A, FBD ①

If all casters are locked, there will be frictional force as rolling friction is negligible

$$\Sigma F_x = 0$$

$$+P - 0.3N_A - 0.3N_B = 0$$

$$P = 0.3N_A + 0.3N_B \quad — ①$$

$$\Sigma F_y = 0$$

$$-981 + N_A + N_B = 0$$

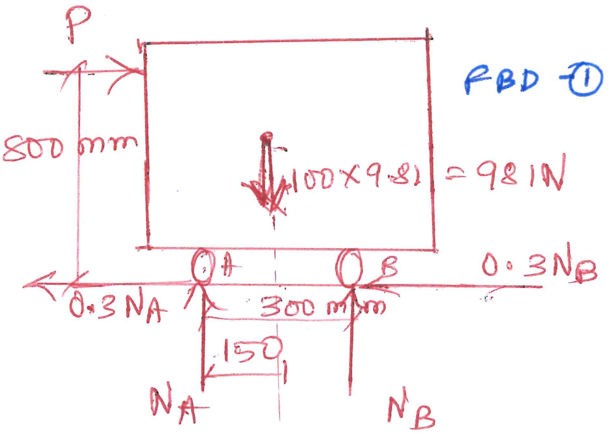
$$N_A + N_B = 981 \quad — ②$$

Applying ② in ①

$$P = 0.3(N_A + N_B)$$

$$P = 0.3(981)$$

$$\boxed{P = 294.3N}$$



for case B, FBD ②

As caster B is locked, there will be frictional force @ B & 0 @ A as these casters are free to roll.

$$\Sigma F_x = 0$$

$$+P - 0.3N_B = 0$$

$$N_B = P/0.3 \quad — ③$$

$$\Sigma F_y = 0$$

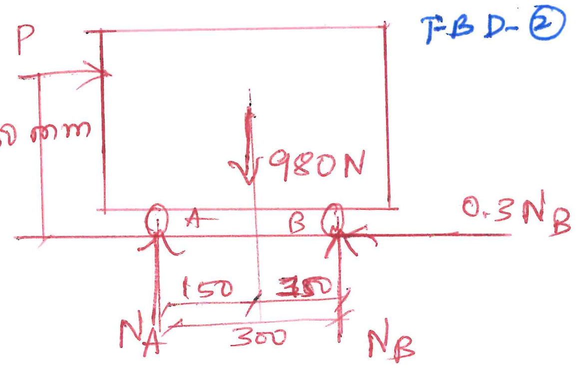
$$-980 + N_A + N_B = 0 \quad — ④$$

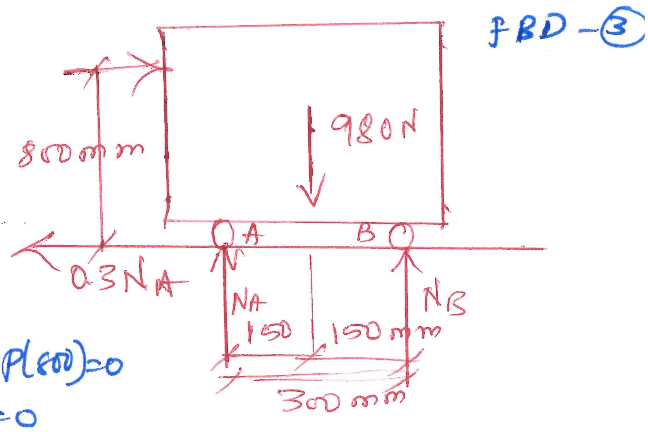
$$\Sigma M_A = 0$$

$$N_A(0) - N_B(300) + 980(150) - 0.3N_B(0) + P(800) = 0$$

$$-P/0.3(300) + 147000 + 800P = 0$$

$$\boxed{P = 735N}$$





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For case c FBD (3)

$\sum F_x = 0$

$+P - 0.3N_A = 0$

$N_A = P/0.3$ —— (5)

$\sum F_y = 0$

$-980 + N_A + N_B = 0$ —— (6)

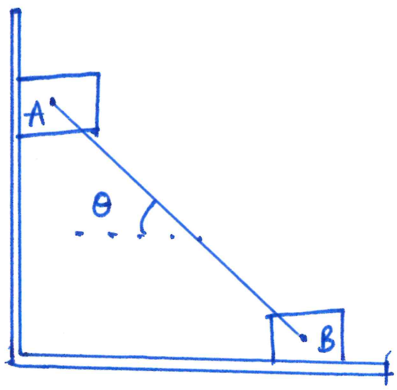
$\sum M_B = 0$

$-0.3N_A\cancel{(0)} + N_A(300) + \cancel{N_B(0)} - 980(150) + P(800) = 0$

$\frac{P}{0.3}(300) + 800P = 147000$

$\boxed{P = 81.67\ N}$

6) Two identical blocks A and B are connected by rod and rest against vertical and horizontal planes respectively as shown in the figure. If sliding impends when $\theta = 45°$, determine the coefficient of friction μ assuming it to be same @ the floor and wall both!



Soln. The weight of each block is assumed to be W $\therefore$ $W_A = W_B = W$. **

for block A $\sum F_x = 0$

$+N_A - F_{AB}\cos 45 = 0$

$N_A = F_{AB}\cos 45$ —— (1)

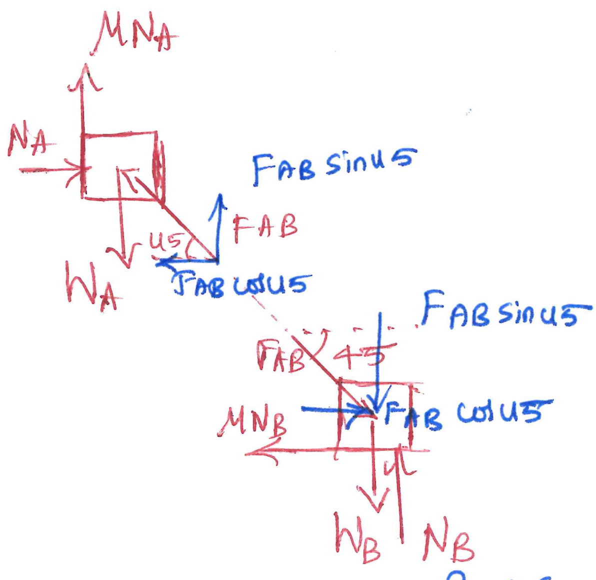
$\sum F_y = 0$

$-W + \mu N_A + F_{AB}\sin 45 = 0$

$-W + \mu(F_{AB}\cos 45) + F_{AB}\sin 45 = 0$

$-W + F_{AB}(\mu\cos 45 + \sin 45) = 0$

$F_{AB} = \frac{W}{\mu\cos 45 + \sin 45}$ —— (2)



For B, $\sum F_x = 0$

$-\mu N_B + F_{AB}\cos 45 = 0$ —— (3)

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$\Sigma F_y = 0$

$-W + N_B - F_{AB} \sin 45 = 0$

$N_B = W + F_{AB} \sin 45$ —— (4)

~~FAB cos~~ Applying N_B in Eq (3)

$F_{AB} \cos 45 - \mu (W + F_{AB} \sin 45) = 0$

$F_{AB} \cos 45 - \mu W + \cancel{\mu} F_{AB} \sin 45 = 0$

$F_{AB} = \dfrac{\mu W}{\cos 45 - \mu \sin 45}$ —— (5)

Equating (2) & (5)

$$\frac{W}{\mu \cos 45 + \sin 45} = \frac{\mu W}{\cos 45 - \mu \sin 45}$$

$\cos 45 = \sin 45 = 1/\sqrt{2}$

$$\frac{1}{\mu(1/\sqrt{2}) + 1/\sqrt{2}} = \frac{\mu}{1/\sqrt{2} - 1/\sqrt{2}\,\mu}$$

$$\frac{1}{\cancel{1/\sqrt{2}}(\mu+1)} = \frac{\mu}{\cancel{1/\sqrt{2}}(1-\mu)}$$

$\mu(\mu+1) = 1(1-\mu)$

$\Rightarrow \mu^2 + \mu = 1 - \mu$

$\Rightarrow \mu^2 + 2\mu - 1 = 0$

$\mu = 0.4142$ or -2.4142

As μ cannot be -ve

$\boxed{\mu = 0.4142}$

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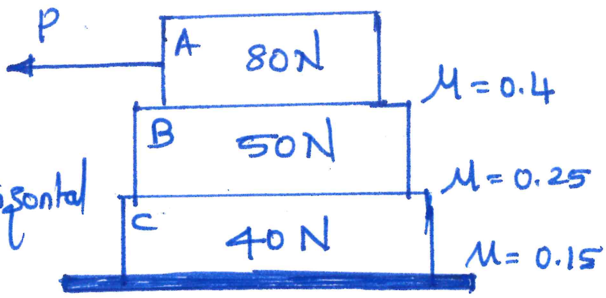
7) Three blocks are placed on the Surface one above the other as shown in the figure. The Static coefficient friction between the Surfaces are shown. Determine the minimum value of P that can be applied before any slipping takes place.



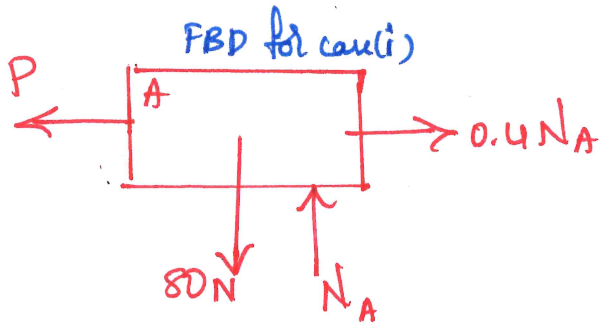
Soln: When force P is applied, slipping can take place in the following three ways.

i) A slips on B

ii) A & B together slips on C

iii) A, B, & C together slip on the horizontal

The FBD for case (i)



$\Sigma F_y = 0$

$+N_A - 80 = 0$

$\boxed{N_A = 80N}$

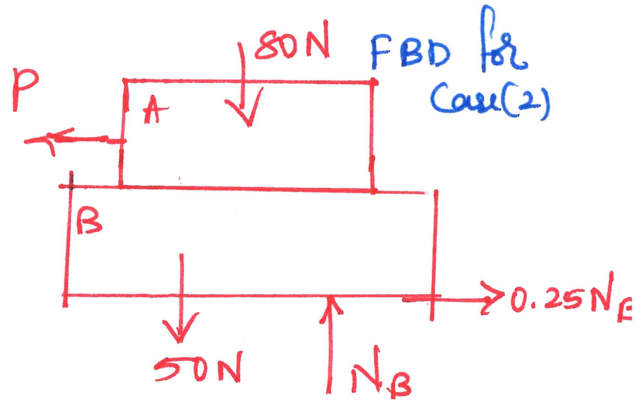
$\Sigma F_x = 0$

$-P + 0.4N_A = 0$

$P = 0.4(80)$

$\boxed{P = 32N}$

The FBD for case (ii)



$\Sigma F_y = 0$

$-80 - 50 + N_B = 0$

$\boxed{N_B = 130N}$

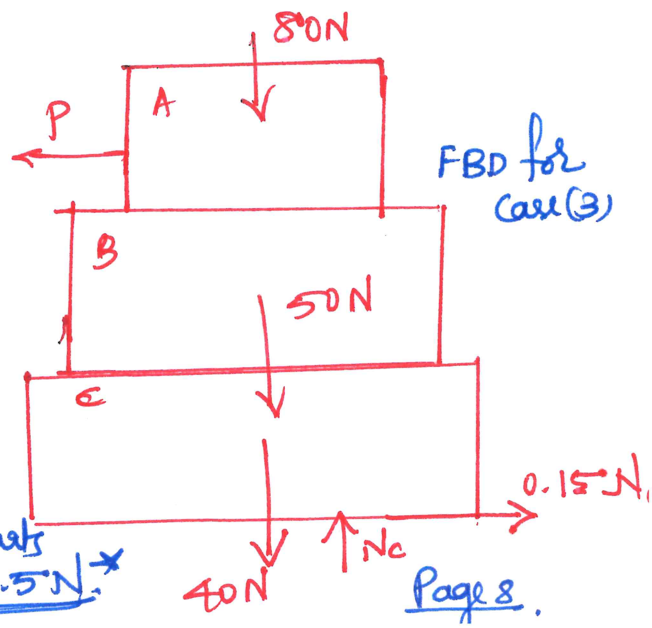
$\Sigma F_x = 0$

$+0.25N_B - P = 0$

$P = 0.25(130)$

$\boxed{P = 32.5N}$

The FBD for case ③



$\Sigma F_y = 0$

$-80 - 50 - 40 + N_c = 0$

$\boxed{N_c = 170N}$

$\Sigma F_x = 0$

$-P + 0.15N_c = 0$

$P = 0.15(170) = 25.5N$ **

$\boxed{P = 25.5N}$ ∴ slipping starts from 25.5N.*

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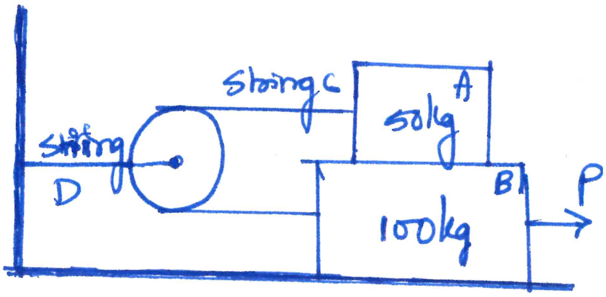
8) Two blocks A and B of mass 50 kg and 100 kg respectively are connected by a string C which passes through a frictionless pulley connected with the fixed wall by another string D as shown in figure.

Find the force P required to pull the block B. Also find the tension in the string D. Take coefficient of friction at all contact surfaces as 0.3.



Soln For A, FBD

* $T_C = T_D = T$.

$\Sigma F_y = 0$

$+N_1 - 490.5 = 0$

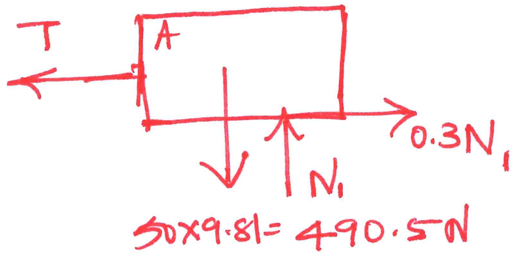
$\boxed{N_1 = 490.5\ N}$

$\Sigma F_x = 0$

$0.3 N_1 - T = 0$

$T = 0.3(490.5)$

$\boxed{T = 147.15\ N}$



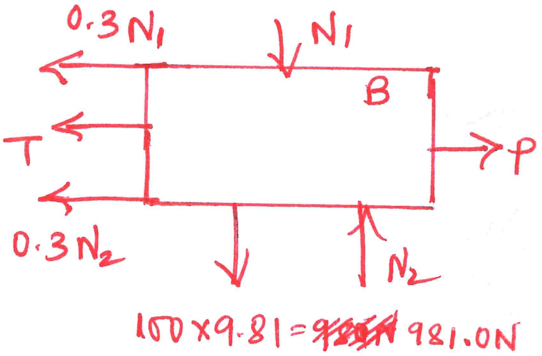
For B, FBD

$\Sigma F_y = 0$

$N_2 - N_1 - (100 \times 9.81) = 0$

$N_2 = 490.5 + 981.0$

$\boxed{N_2 = 1471.5\ N}$



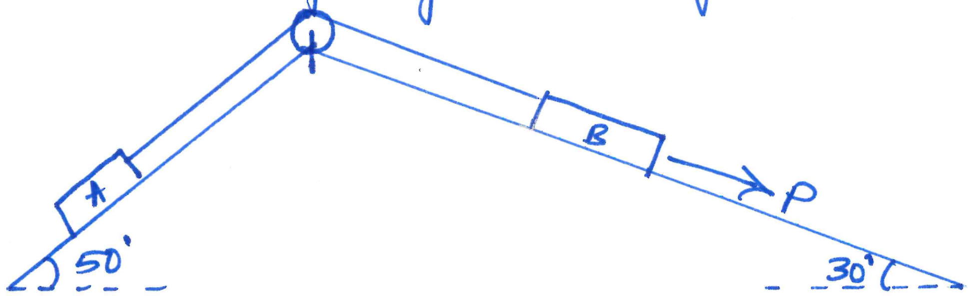
$\Sigma F_x = 0$

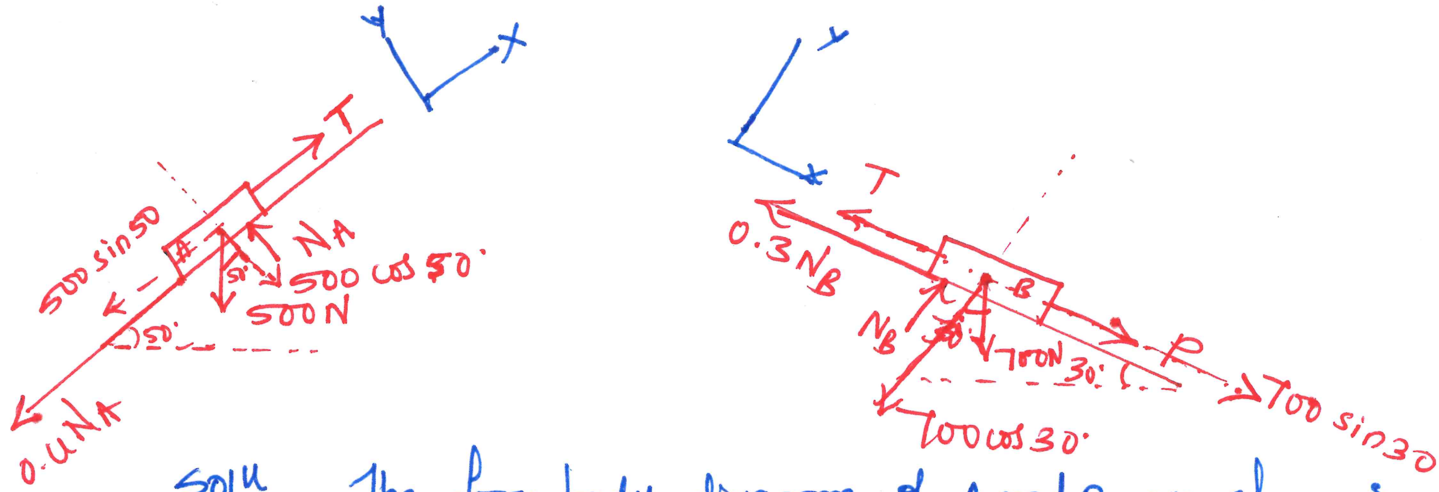
$-0.3N_1 - T - 0.3N_2 + P = 0$

$P = 0.3(490.5) + 0.3(1471.5) + 147.15$

$\boxed{P = 735.75\ N}$

9) Two blocks A and B of weight 500 N and 700 N respectively are connected by a cord that passes over a frictionless pulley as shown in figure. The coefficient of friction between the block A and the inclined plane is 0.4 and that between the block B and the incline is 0.3. Determine the force P to be applied to block B to produce the impending motion of block B down the plane.





Soln. The free body diagram of A and B are shown in the figure.

For A,

$\sum F_y = 0$

$-500\cos 50 + N_A = 0$

$\boxed{N_A = 321.39\,N}$

$\sum F_x = 0$

$T - 500\sin 50 - 0.4\,N_A = 0$

$T = 500\sin 50 + 0.4(321.39)$

$\boxed{T = 511.58\,N}$

For B,

$\sum F_y = 0$

$-700\cos 30 + N_B = 0$

$\boxed{N_B = 606.21\,N}$

$\sum F_x = 0$

$P - T + 700\sin 30 - 0.3\,N_B = 0$

$P = 511.58 - 700\sin 30 + 0.3(606.2)$

$\boxed{P = 343.44\,N}$

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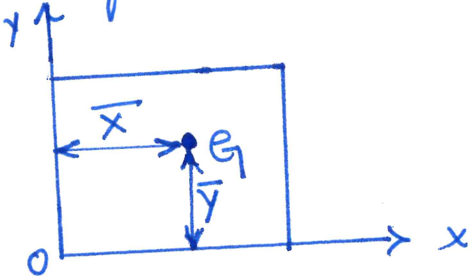
Unit 5 → Centroid and Moment of Inertia

Centre of gravity → It is the point where the whole weight of the body is assumed to be concentrated. It is the point on which the body can be balanced. It is the point through which the weight of the body is assumed to act. This point is usually denoted by 'C.G' or 'G'

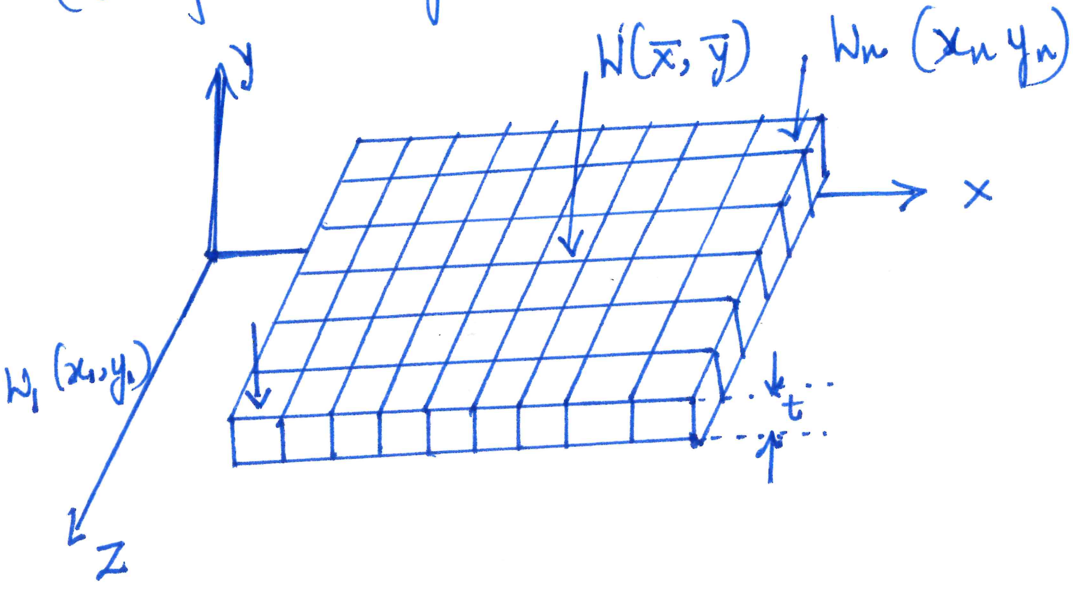
Centroid (C.G. or G)

Centroid is the point where the whole area of the plane figure is assumed to be concentrated. The determination of $\overline{x}$ and $\overline{y}$ is itself the calculation of centroid.



Determination of Centroid by the method of moments

Let us consider a body of total weight W as shown in the figure. The centre of gravity of the whole figure is located at a distance $\overline{x}$ from the y axis and at a distance $\overline{y}$ from the x axis (the point through which the total weight W acts)



Let us divide the whole figure into a number of elemental strips of weights $w_1, w_2, w_3, w_4, \ldots w_n$ whose centroids are located at distance $x_1, x_2, x_3, \ldots x_n$ from the y axis and $y_1, y_2, y_3 \ldots y_n$ from the x axis.

Applying the theorem of moments about the y-axis.

$$W\bar{x} = w_1 x_1 + w_2 x_2 + w_3 x_3 + \cdots + w_n x_n$$

or
$$\bar{x} = \frac{w_1 x_1 + w_2 x_2 + w_3 x_3 + \cdots w_n x_n}{W}$$

But if the body is homogeneous and isotropic, then the specific weight of the material is given by

$$\gamma = \frac{W}{V} = \frac{W}{A \times t}$$

where W is the weight of the body and V is the volume of the body, A is the cross-sectional area and t is the thickness which is constant.

Hence we have

$$\bar{x} = \frac{\gamma a_1 t x_1 + \gamma a_2 t x_2 + \gamma a_3 t x_3 + \cdots \gamma a_n t x_n}{\gamma a_1 t + \gamma a_2 t + \gamma a_3 t + \cdots \gamma a_n t}$$

$$\bar{x} = \frac{\cancel{\gamma t}\,(a_1 x_1 + a_2 x_2 + a_3 x_3 + \cdots a_n x_n)}{\cancel{\gamma t}\,(a_1 + a_2 + a_3 + \cdots a_n)}$$

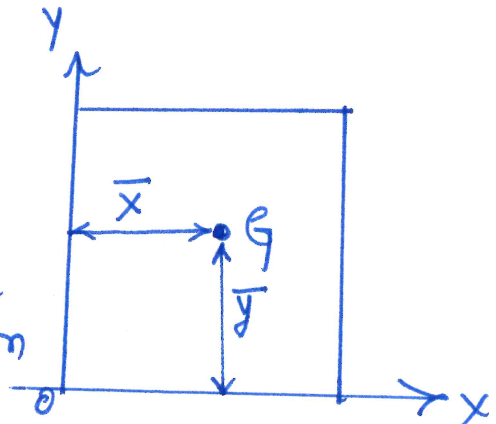
$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + \cdots a_n x_n}{a_1 + a_2 + a_3 + \cdots a_n}$$

$$\boxed{\bar{x} = \frac{\Sigma a x}{\Sigma a}}$$

Similarly $\boxed{\bar{y} = \frac{\Sigma a y}{\Sigma a}}$

Axis of Reference

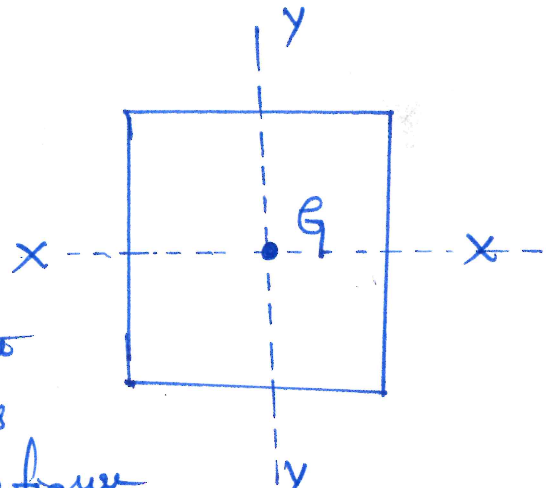
The axes with respect to which the centroid of a given figure is determined is called axes of reference.



Generally the left-hand bottom corner of the plane figure is considered as the origin so that the left extreme edge and the bottom line are considered reference axes, with respect to which the centroid of the given figure is determined.

Centroidal axis

The axis which passes through the centroid of the given figure is known as centroidal axis, such as X-X and Y-Y as shown in the figure.



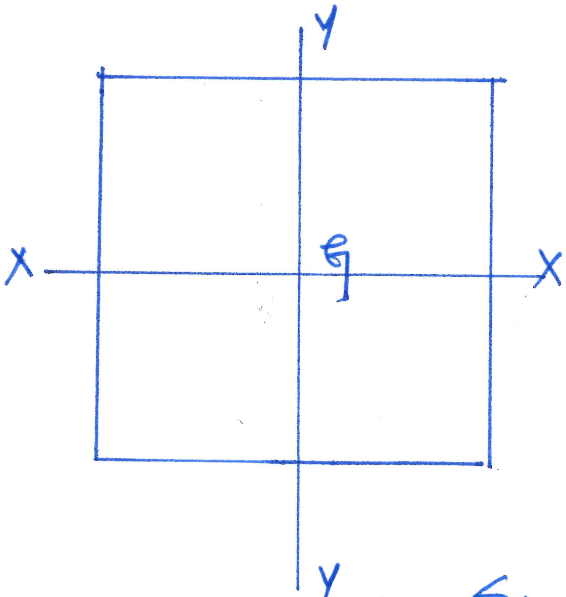
Symmetrical axis

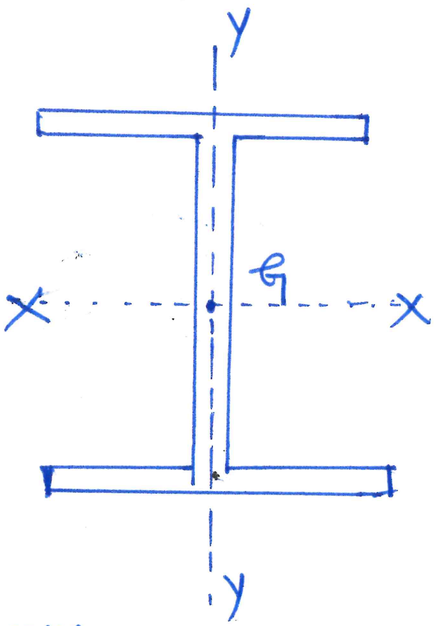
It is the axis which divides the whole figure into two equal parts, such as the ~~X-X~~ X and X and Y-Y shown in figure

a) For a figure which is symmetrical about the both the axes $\bar{x}=0$ and $\bar{y}=0$

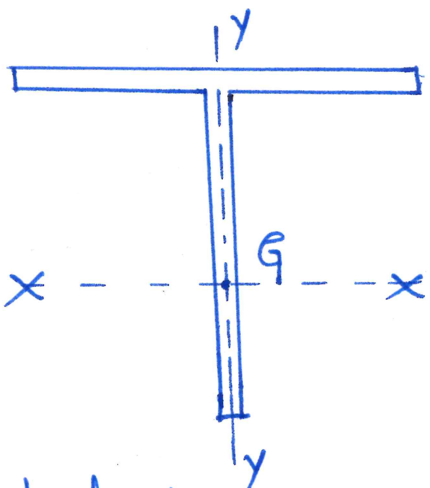
b) For a figure which is symmetrical about the Y-Y axis, $\bar{x}=0$. Such a figure which is symmetrical about the Y-Y axis is shown in figure. The area on the left side of the Y-Y axis is equal to the area on the right side of the Y-Y axis.

c) For a figure which is symmetrical about the X-X axis, $\bar{y}=0$, Such a figure which is symmetrical about the X-X axis is shown in figure

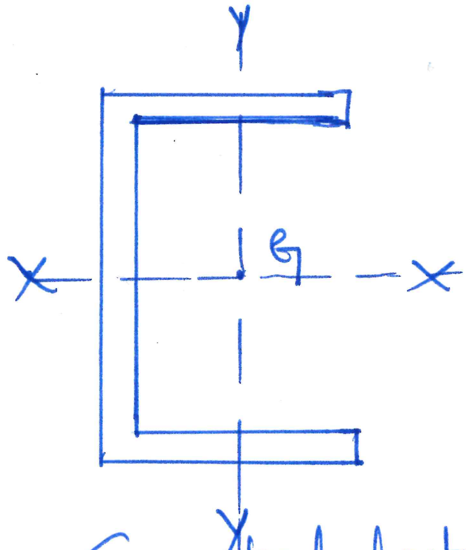




Symmetrical axes

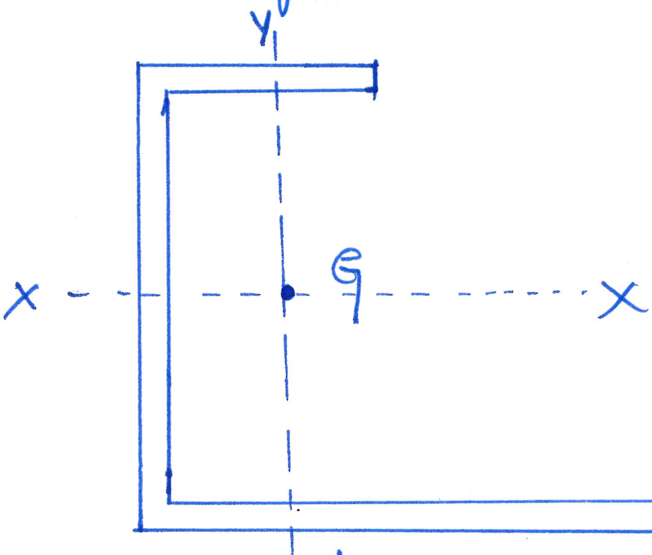


Symmetrical about Y-Y axis



Symmetrical about X-X axis

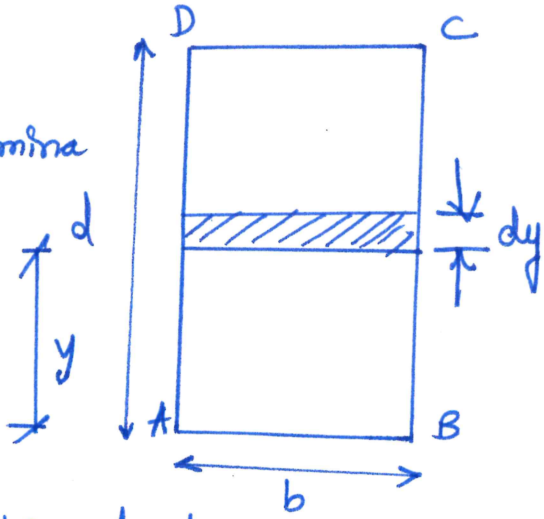
For a figure which doesn't have any axis of symmetry, we calculate both $\bar{x}$ and $\bar{y}$.



Neither the x-x axis nor the Y-Y axis of symmetry

Derivation of Centroid of some important geometrical figures

1) Rectangle



Let us consider a rectangular lamina of area $b \times d$ as shown in the figure.

Now consider a horizontal strip (Elementary strip) of area $b \times dy$, which is at a distance y from the reference axis AB.

Moment of area of elementary strip about AB

$= b \times dy \times y$ {area of an elementary strip $\times$ ⊥r dist to AB}

Sum of moments of such elementary strips about AB is given by

$$\int_0^d b \times dy \times y$$

$$= b \int_0^d y \cdot dy$$

$$= b \times \left[\frac{y^2}{2}\right]_0^d$$ {Integrating and applying the limits}.

$$= b\left[\frac{d^2}{2} - \cancel{\frac{0^2}{2}}\right]$$

$$= \frac{bd^2}{2}$$

Moment of total area about AB $= bd \times \bar{y}$

Apply the principle of moment about AB,

$$\frac{bd^2}{2} = bd \times \bar{y}$$

$$\Rightarrow \bar{y} = \frac{\cancel{b}d^2}{2b\cancel{d}} = d/2 \quad \therefore \boxed{\bar{y} = d/2}$$

By considering vertical strip, similarly we can proove that $\boxed{\bar{x} = b/2}$.

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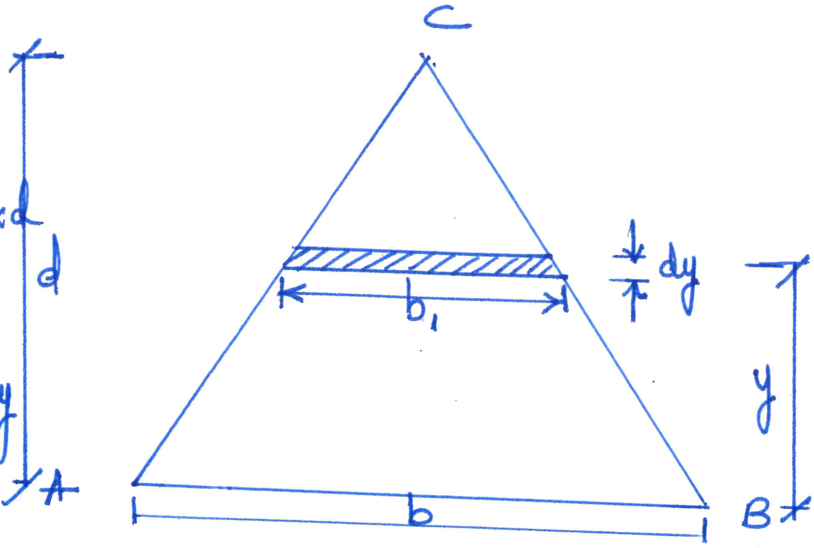
2) Triangle

Sol^n :-

Consider a triangular lamina of area $(\frac{1}{2}) \times b \times d$ as shown in the figure



Now consider an elementary strip of area $b_1 \times dy$ which is at a distance y from the reference axis AB.

Using the property of similar Δles, we have

$$\frac{b_1}{b} = \frac{d-y}{d}$$

or

$$b_1 = \frac{(d-y)b}{d}$$

Area of the elementary strip is $= \cancel{\tfrac{1}{2} \times b}\ b_1 \times dy$

$$= \frac{(d-y)b}{d} \cdot dy$$

Area of the Elementary strip $= \dfrac{(d-y)\, b \cdot dy}{d}$

Moment of area of Elementary strip about AB

$$= \text{area} \times y$$

$$= \frac{(d-y) b \cdot dy \cdot y}{d}$$

$$= \frac{b \cdot dy \cdot d \cdot y}{d} - \frac{by^2 \cdot dy}{d}$$

$$= by \cdot dy - \frac{by^2 \cdot dy}{d}$$

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Sum of moments of such Elementary strips is given by

$$= \int_0^d by \cdot dy - \int_0^d \frac{by^2}{d} \cdot dy$$

$$= \left[\frac{by^2}{2}\right]_0^d - \left[\frac{by^3}{3d}\right]_0^d$$

$$= b\left[\frac{y^2}{2}\right]_0^d - \frac{b}{d}\left[\frac{y^3}{3}\right]_0^d$$

$$= \frac{bd^2}{2} - \frac{bd^{\cancel{3}\,2}}{3\cancel{d}}$$

$$= \frac{bd^2}{2} - \frac{bd^2}{3}$$

$$= \frac{bd^2}{6}$$

Moment of total area about AB $= \frac{1}{2} \times b \times d \times \bar{y}$

Applying theorem of moments,

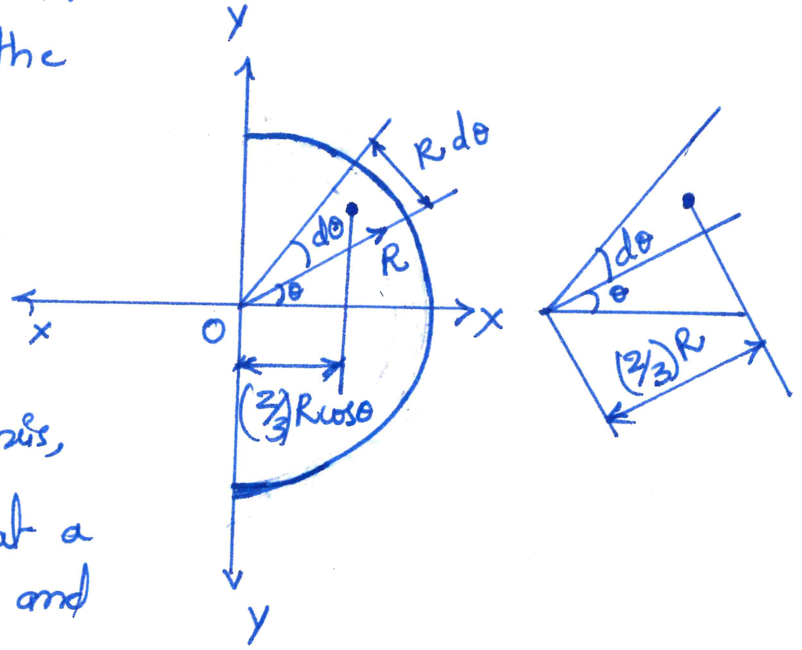
$$\frac{bd^2}{6} = \frac{1}{2} \times b \times d \times \bar{y}$$

$$\bar{y} = \frac{\cancel{2}\,\cancel{b}\,d^{\cancel{2}}}{3\,\cancel{6}\,\cancel{b}\,\cancel{d}}$$

$$\boxed{\therefore\ \bar{y} = d/3}$$ from the base AB.

3) Semi Circle

Consider a Semicircular lamina of area $\frac{\pi r^2}{2}$ as shown in the figure.



Now Consider a triangular elementary strip of area $\frac{1}{2} \times R \times R \times d\theta$ at an angle of θ from the x axis, whose center of gravity is at a distance of $\frac{2}{3}R$ from O and it's projection on the x axis $= \left(\frac{2}{3}\right)R\cos\theta$.

Moment of area of Elementary strip about the y axis

$$= \frac{1}{2} \times R^2 \times d\theta \times \left(\frac{2}{3}\right)R\cos\theta$$

$$= \frac{R^3 \cdot \cos\theta \cdot d\theta}{3}$$

Sum of moments of such elementary strips about the y axis

$$= \int_{-\pi/2}^{\pi/2} \frac{R^3 \cdot \cos\theta \cdot d\theta}{3}$$

$$= \frac{R^3}{3} \left(\sin\theta\right)_{-\pi/2}^{\pi/2}$$

$$= \frac{R^3}{3}\left[\sin \pi/2 - \sin(-\pi/2)\right] = \frac{R^3}{3}\left[\cancel{\sin \pi/2}^{\,1} + \cancel{\sin \pi/2}^{\,1}\right]$$

$$= \frac{2R}{3}$$

Moment of total area about the y axis $= \frac{\pi R^2}{2} \times \bar{x}$

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Using the principle of moments,

$$\frac{2R^3}{3} = \frac{\pi R^2}{2} \times \bar{x}$$

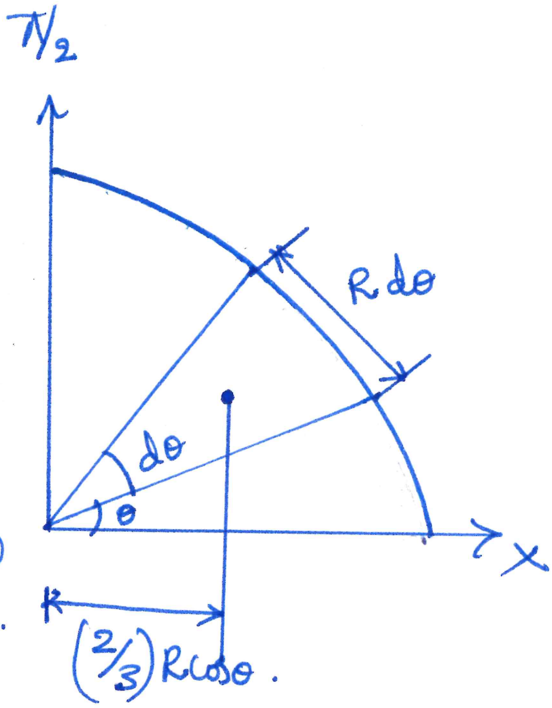
$$\bar{x} = \frac{2 \times 2R^{\cancel{3}}}{3\pi \cancel{R^2}}$$

$$\boxed{\bar{x} = \frac{4R}{3\pi}}$$

4) <u>Quarter</u> <u>Circle</u>

Consider a quarter circular lamina of area $\frac{\pi R^2}{4}$ as shown in the figure.

Consider a ~~triangular~~ elementary strip of area $\frac{1}{2} \times R \times R \times d\theta$ at an angle of θ from the x axis, whose centre of gravity is at a distance of $\frac{2}{3}R$ from O and its projection on x axis $= \left(\frac{2}{3}\right) R\cos\theta$.



Moment of area of elementary strip about the y axis

$$= \frac{\cancel{2}}{3} R\cos\theta \times \frac{1}{\cancel{2}} \times R^2 \times d\theta$$

$$= \frac{R^3 . \cos\theta . d\theta}{3}$$

Sum of moments of such elementary strips about the y-axis

$$= \int_0^{\pi/2} \frac{R^3}{3} . \cos\theta . d\theta$$

$$= \frac{R^3}{3} \Big[\sin\theta \Big]_0^{\pi/2}$$

$$= \frac{R^3}{3} \left[\cancel{\sin \pi/2}^{\,1} - \cancel{\sin 0}^{\,0} \right]$$

$$= \frac{R^3}{3} //.$$

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Moment of total area about the y axis

$$= \frac{\pi R^2}{4} \times \bar{x}$$

Using the principle of moments,

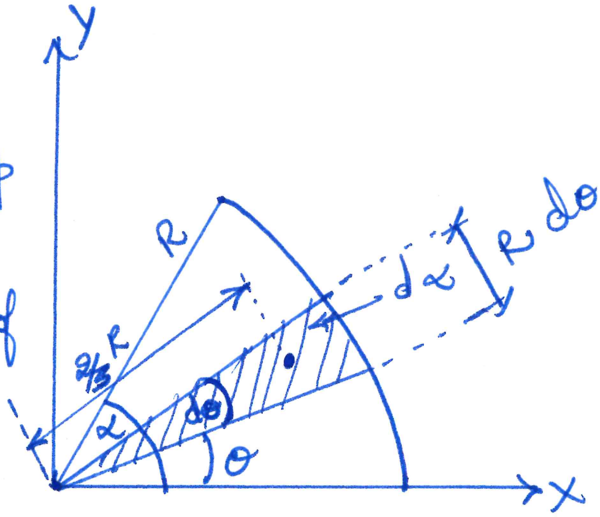
$$\frac{R^3}{3} = \frac{\pi R^2}{4} \times \bar{x}$$

$$\bar{x} = \frac{4R^{\cancel{3}}}{3\pi \cancel{R^2}}$$

$$\boxed{\bar{x} = \frac{4R}{3\pi}}$$

or Similarly, we can prove that $\boxed{\bar{y} = \frac{4R}{3\pi}}$

5) Sector of a Circle



Consider a sector of circular lamina as shown in the figure.

Consider a triangular elementary strip of area $\frac{1}{2} \times R \times R\,d\theta$ at an angle of θ from the x axis, whose centre of gravity is at a distance of $\frac{2}{3}$ from o and its projection on x axis $= \frac{2}{3} R\cos\theta$.

$$\text{Area of strip} = \frac{1}{2} \times R \times R\,d\theta = \frac{1}{2} R^2 d\theta$$

Area of sector is given by

$$\int_0^{\alpha} \frac{1}{2} \cdot R^2 \cdot d\theta = \frac{1}{2} \cdot R^2 [\theta]_0^{\alpha} = \underline{\underline{\frac{1}{2} R^2 \alpha}}$$

Moment of area of Elementary strip about y axis

$$= \frac{2}{3} R \cos\theta \times \frac{1}{2} \times R^2 d\theta$$

$$= \frac{R^3 . \cos\theta . d\theta}{3}$$

Sum of moments of such Elementary strips about y axis

$$= \int_0^{\alpha} \frac{R^3 . \cos\theta . d\theta}{3} = \frac{R^3}{3} \int_0^{\alpha} \cos\theta . d\theta$$

$$= R^3/3 \left[\sin\theta \right]_0^{\alpha} = R^3/3 \left[\sin\alpha - \cancel{\sin 0} \right]$$

$$= \frac{R^3}{3} \sin\alpha$$

Moment of total area about y- axis

$$= \frac{R^2 \alpha}{2} \times \bar{x}$$

Using the principle of moments

$$\frac{R^3}{3} \sin\alpha = \frac{R^2 \alpha}{2} \times \bar{x}$$

$$\bar{x} = \frac{2 R^{\cancel{3}} \sin\alpha}{3 \cancel{R^2} \alpha}$$

$$\bar{x} = \frac{2R}{3} \frac{\sin\alpha}{\alpha}$$

Hy

Moment of area of elementary strip about x-axis

$$= \frac{2}{3} R \sin\theta \times \frac{1}{2} \times R^2 . d\theta = \frac{R^3 . \sin\theta . d\theta}{3}$$

Sum of moments of such Elementary strips about y axis

$$= \int_0^{\alpha} \frac{R^3}{3} \sin\theta . d\theta = \frac{R^3}{3} \left[-\cos\theta \right]_0^{\alpha} \frac{R^3}{3} \left[-\cos\alpha - (-\cos 0) \right]$$

$$= \frac{R^3}{3} \left[1 - \cos\alpha \right]$$

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Moment of total area about y axis $= \frac{R^2\alpha}{2} \times \bar{y}$

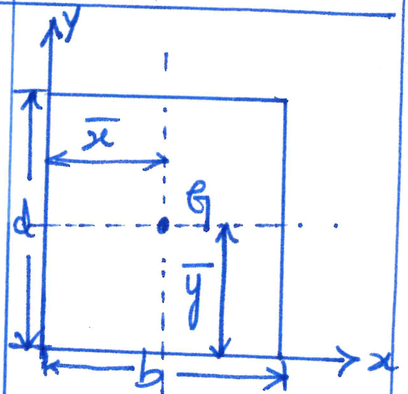
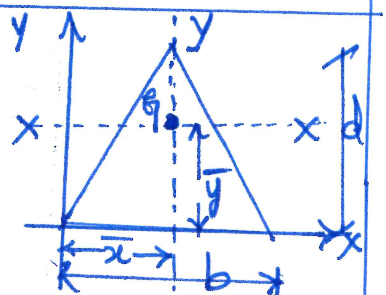
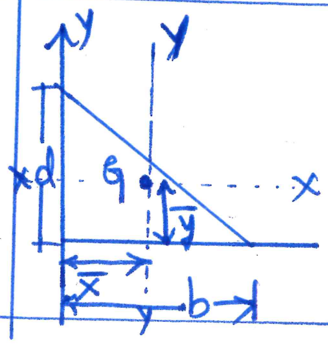
Using the principle of moments,

$$\frac{R^3}{3}(1-\cos\alpha) = \frac{R^2\alpha}{2} \times \bar{y}$$

$$\bar{y} = \frac{2R^{\cancel{3}}(1-\cos\alpha)}{3\cancel{R^2}\alpha}$$

$$\bar{y} = \frac{2R}{3}\left(\frac{1-\cos\alpha}{\alpha}\right)$$

Centroids of some important Geometrical figures

Shape	Area	$\bar{x}$	$\bar{y}$	figure
Rectangle (Same for Square)	$b.d$	$b/2$	$d/2$	
Triangle	$\left(\frac{1}{2}\right)b.d$	$b/2$	$\left(\frac{1}{3}\right)d$ from base.	
Right angled Triangle	$\left(\frac{1}{2}\right)b\,d$	$\left(\frac{1}{3}\right)b$	$\left(\frac{1}{3}\right)d$	

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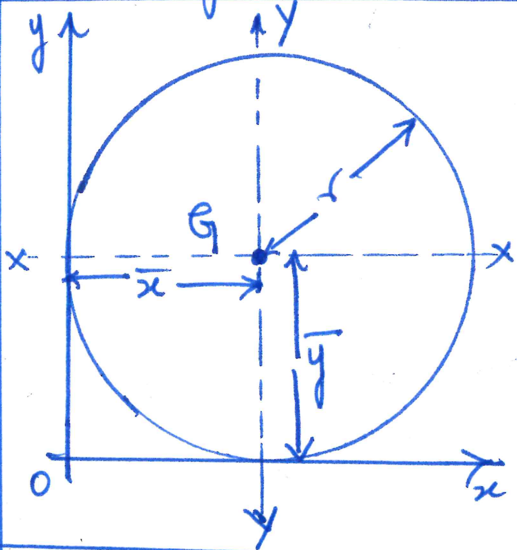
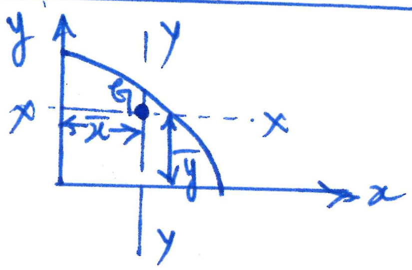
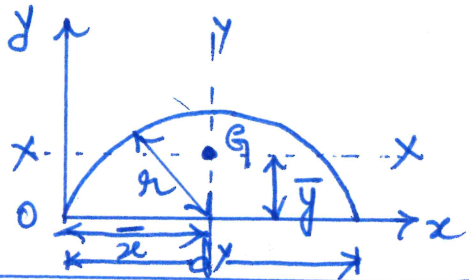
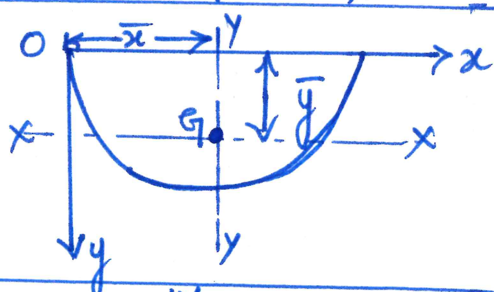
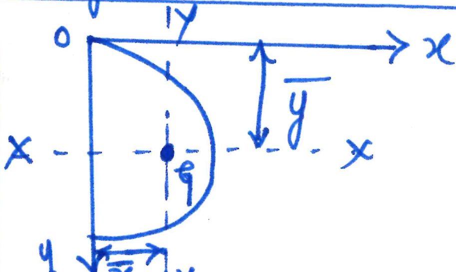
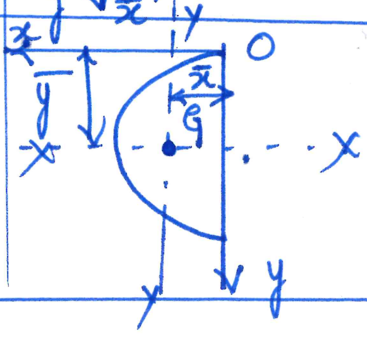
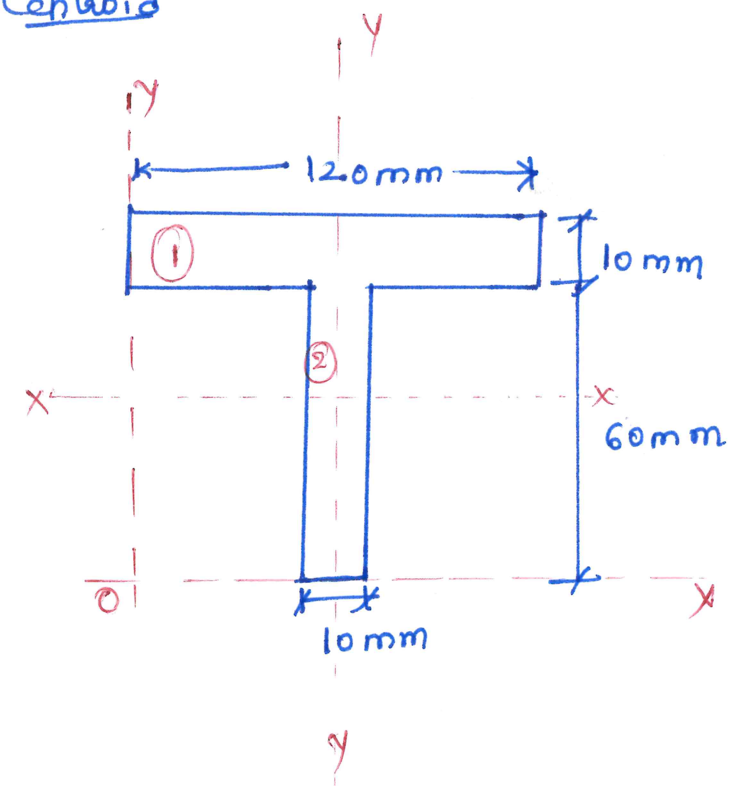
Shape	Area	$\bar{x}$	$\bar{y}$	figure
Circle	πr^2	$\bar{x}=r$	$\bar{y}=r$	
Quarter Circle	$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	
Semi Circle	$\frac{\pi r^2}{2}$	$d/2$	$\frac{4r}{3\pi}$	
		$d/2$	$-\frac{4r}{3\pi}$	
		$\frac{4r}{3\pi}$	$-d/2$	
		$-\frac{4r}{3\pi}$	$d/2$	

Numerical Problems on Centroid

1) Find the centroid of figure



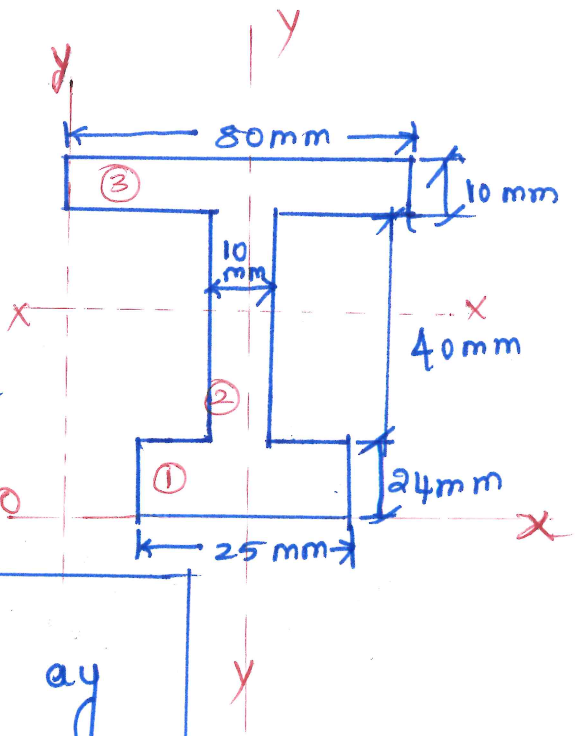
Component	Area (mm^2)	Centroidal distance from y axis (x) mm	Centroidal distance from x axis (y) mm	ax mm^3	ay mm^3.
b×d Rectangle 1 (120 × 10)	120×10 =1200	$\frac{120}{2} = 60$	$60 + \frac{10}{2} = 65$	72,000	78,000
b×d Rectangle 2 (10 × 60)	10×60 = 600	$\frac{120}{2} = 60$	$\frac{60}{2} = 30$	36,000	18,000
sum	$\Sigma a = 1800$			1,08,000	96,000

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{1,08,000}{1800} = 60\ mm$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{96000}{1800} = 53.33\ mm$$

Note:- This figure is symmetrical about y axis. so $\bar{x}$ can be directly written as 60 mm instead of doing calculation of ax and x in the tabular column.

2) Find the centroid of the figure



Sol: The given figure is symmetrical about y-axis and hence we consider

$$\boxed{\bar{x} = 80/2 = 40 \text{ mm}}$$

To calculate $\bar{y}$, we need to determine y and ay.

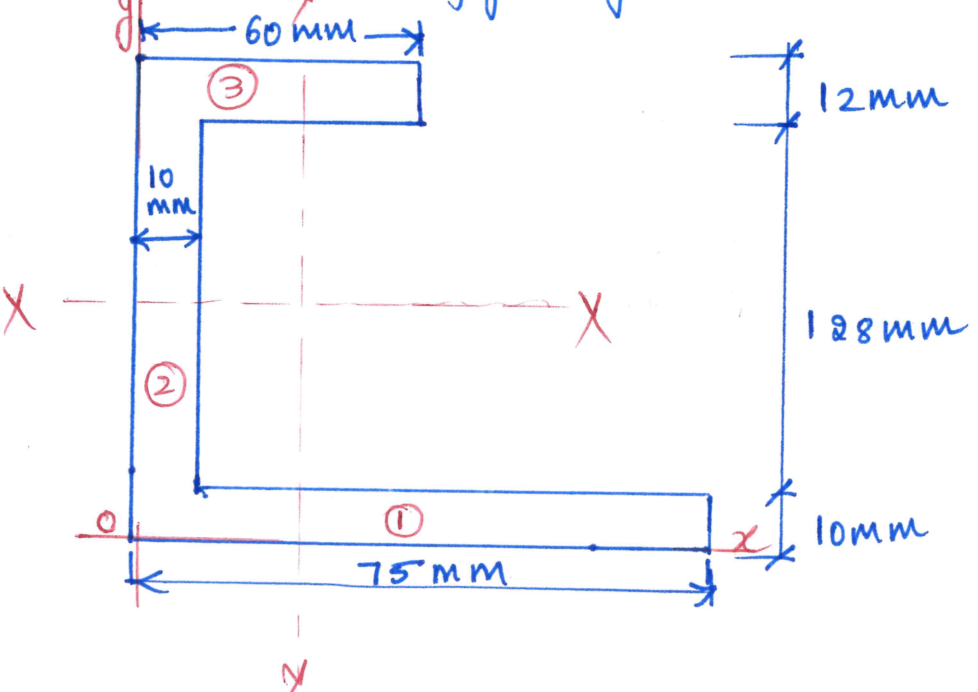
component b×d	Area (a) mm^2	Distance of centroid from x axis, y mm	ay mm^3
Rectangle 1 25×24	25×24 = 600	$\frac{24}{2} = 12$	7200
Rectangle 2 10×40	10×40 = 400	$24 + \frac{40}{2} = 44$	17,600
Rectangle 3 80×10	80×10 = 800	$24 + 40 + \frac{10}{2} = 69$	55,200
Sum	$\Sigma a = 1800$		$\Sigma ay = 80,000$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a}$$

$$\bar{y} = \frac{80,000}{1800}$$

$$\boxed{\bar{y} = 44.44 \text{ mm}}$$

3) Determine the centroid of figure given.



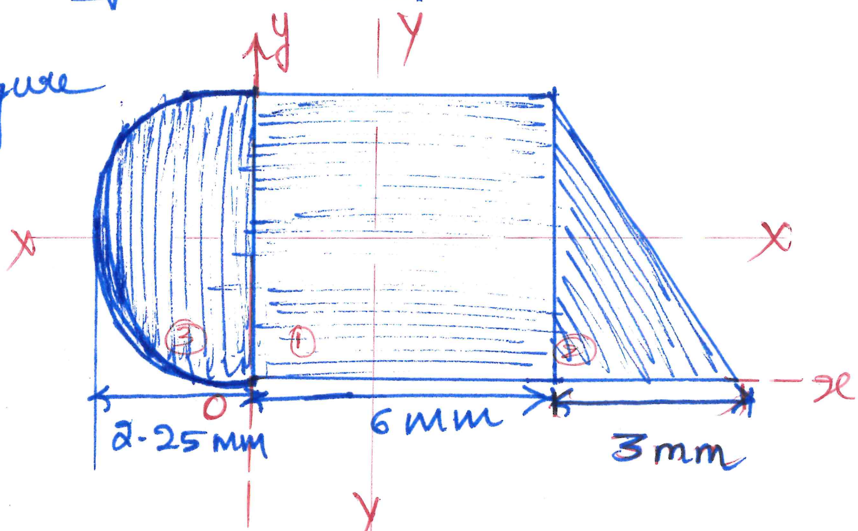
Lakshmi H.S

component	Area, a mm^2	Depth of centroid from y axis, x, mm	Depth of centroid from x axis y, mm	a x mm^3	a y mm^3
Rectangle 1 75×10	75×10 = 750	$75/2 = 37.5$	$10/2 = 5$	28,125	3750
Rectangle 2 10×128	10×128 = 1280	$10/2 = 05$	$10+\frac{128}{2} = 74$	6400	94,720
Rectangle 3 60×12	60×12 = 720	$60/2 = 30$	$10+128+\frac{12}{2} = 144$	21,600	1,03,680
Sum	$\Sigma a = 2750$			$\Sigma ax = 56,125$	$\Sigma ay = 2,02,150$

$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{56,125}{2750}$; $\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{2,02,150}{2750}$

$\boxed{\bar{x} = 20.409}$ mm $\boxed{\bar{y} = 73.509 \text{ mm}}$

4) Calculate the centroid of figure

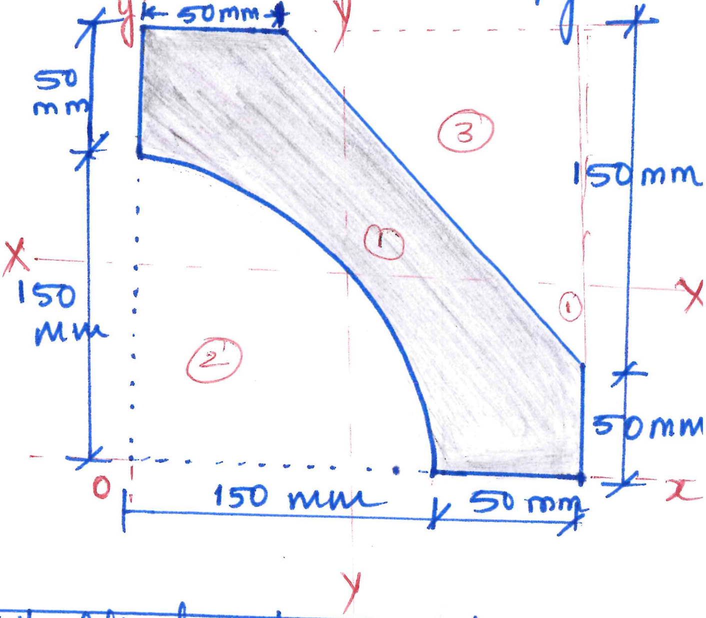


component	Area, a, mm^2	Dist of centroid from y axis x, mm	Distance of centroid from x axis, y mm	a x mm^3	a y mm^3
Rectangle 6×4.5	6×4.5 = 27	$6/2 = 3$	$\frac{4.5}{2} = 2.25$	81	60.75
Triangle b=3 d=4.5	$\frac{1}{2}$×3×4.5 = 6.75	$6+\frac{1}{3}(3) = 7$	$\frac{1}{3}(4.5) = 1.5$	47.25	10.125
Semicircle r = 2.25 (Behind Y axis) **	$\frac{\pi(2.25)^2}{2} = 7.95$	$-\frac{4r}{3\pi} = -\frac{4(2.25)}{3\pi} = -0.955$	$\frac{4.5}{2} = 2.25$	-7.592	17.88
Sum.	$\Sigma a = 41.7$			$\Sigma ax = 120.65$	$\Sigma ay = 88.75$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{120.65}{41.7} \qquad \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{88.75}{41.7}$$

$$\boxed{\bar{x} = 2.893\,mm} \qquad \boxed{\bar{y} = 2.129\,mm}$$

5) Determine the centroid of the shaded area shown in figure

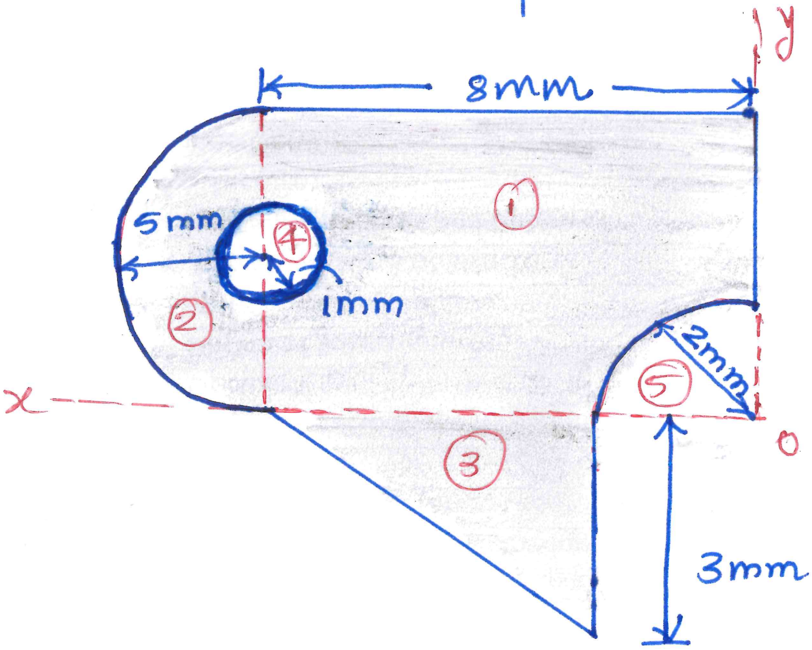


Component	Area a, mm^2	Depth of centroid from y axis x, mm	Depth of centroid from x axis, y, mm	ax	ay
Rectangle/ Square 200X200	$200 \times 200 = 40,000$	$\frac{200}{2} = 100$	$\frac{200}{2} = 100$	40,00,000	40,00,000
Deductions Quarter circle $r = 150$	$-\frac{\pi(150)^2}{4}$ $= 17,671.45$ $= -17,671.45$	$\frac{4 \times 150}{3\pi}$ $= 63.662$	$\frac{4 \times 150}{3\pi}$ $= 63.662$	-1124999.85	-1124999.85
Triangle $b = 150$ $d = 150$	$\frac{1}{2} \times 150 \times 150$ $= 11250$ $= -11250$	$50 + \frac{2}{3}(150)$ $= 150$	$50 + \frac{2}{3}(150)$ $= 150$	-1687500	-1687500
	$\Sigma a = 11,078.55$			1187500.15	1187500.15

$$\bar{X} = \frac{\Sigma ax}{\Sigma a} = \frac{11,87500.15}{11078.55} \qquad \bar{Y} = \frac{1187500.15}{11,078.55}$$

$$\boxed{\bar{X} = 107.189\,mm} \qquad \boxed{\bar{Y} = 107.189\,mm}$$

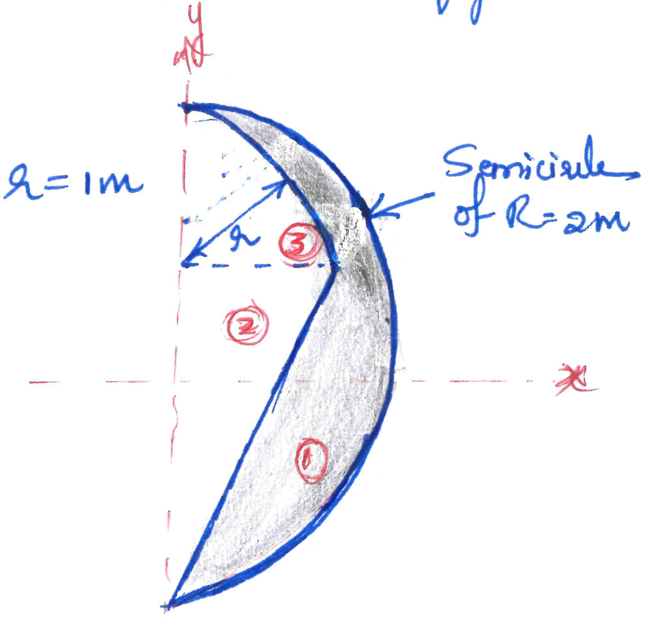
6) Locate the centroid of the plane shown in the figure.



Components	Area (a) mm^2	depth of centroid from y axis x, mm	depth of centroid from x axis y, mm	ax mm^3	ay mm^3
Rectangle 8×10	8×10 =80	$-8/2 = -4$	$\frac{10}{2} = 5$	−320	400
Semicircle r=5mm	$\frac{\pi \times 5^2}{2}$ =39.26	$-\left(8+\frac{4(5)}{3\pi}\right)$ = −10.12	$\frac{10}{2} = 5$	−397.31	+196.34
Triangle b=6 d=3	½×6×3 = 9	$-\left(2+\frac{1}{3}(6)\right)$ = −4	$-\left(\frac{1}{3}\times 3\right)$ = −1	−36	−9
Deductions Circle r=1mm	$-\pi(1)^2$ = −3.141	−8	$\frac{10}{2} = 5$	+25.12	−15.705
Quarter circle r=2mm	$-\frac{\pi(2)^2}{4}$ = −3.141	$-\left(\frac{4\times 2}{3\pi}\right)$ = −0.849	$\frac{4\times 2}{3\pi}$ = 0.849	+2.676	−2.676
	Σa=121.978			Σax= −725.514	Σay = 568.968

$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{-725.514}{121.97} = -5.948\ mm$ $\quad \bar{y} = \frac{568.968}{121.97} = 4.66\ mm$

7) Determine the centroid of the shaded area shown in the figure.



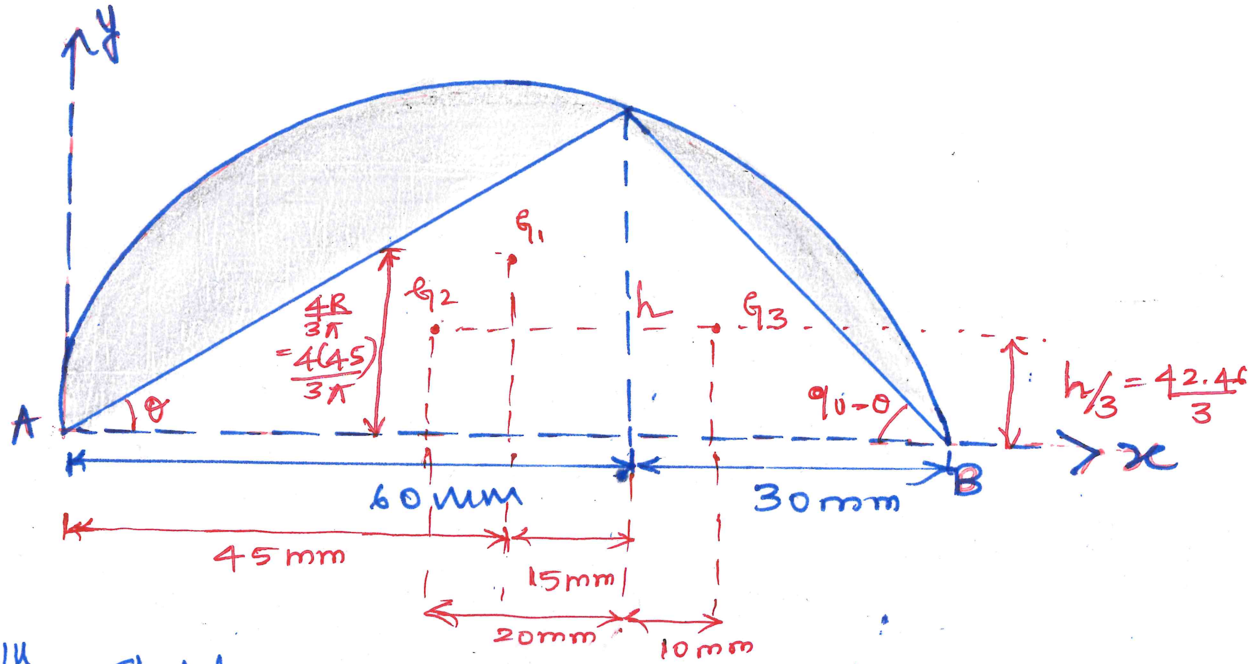
Component	Area a, mm^2	Depth of centroid from y axis x, mm	Depth of centroid from x axis y, mm	ax mm^3	ay mm^3
Semicircle $r = 2m$	$\frac{\pi(2)^2}{2} = 6.288$	$\frac{4(2)}{3\pi} = 0.848$	0	5.327	0
Deductions Triangle b = 1m d = 3m	$-\frac{1}{2}\times1\times3 = -1.5$	$\frac{1}{3}(1) = 0.333$	0	-0.4995	0
Quarter circle $r = 1m$	$-\frac{\pi(1)^2}{4} = -0.785$	$\frac{4(1)}{3\pi} = 0.424$	$1+\frac{4(1)}{3\pi} = 1.424$	-0.3328	-1.1178
	$\Sigma a = 3.990$			$\Sigma ax = 4.494$	$\Sigma ay = -1.1178$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} \qquad \bar{y} = \frac{\Sigma ay}{\Sigma a}$$

$$\bar{x} = \frac{4.494}{3.990} \qquad \bar{y} = \frac{-1.1178}{3.990}$$

$$\boxed{\bar{x} = 1.126\ m} \qquad \boxed{\bar{y} = -0.280\ m}$$

8) Locate the centroid of the shaded portion of lamina if AB = 90mm is diameter of semicircle.



Soln. Shaded area can be obtained by substracting the two right angled triangle from the semicircle.

from the figure,

$$\tan\theta = h/60 \quad \text{--- (1)}$$

and, $\tan(90-\theta) = h/30$

$$\therefore \cot\theta = h/30$$

$$\therefore \tan\theta = 30/h \quad \text{--- (2)}$$

from (1) and (2), we have

$$h/60 = \frac{30}{h}$$

$$\therefore \boxed{h = 42.46 \text{ mm}}$$

The centroid of three components are shown in figure

component	Area mm^2	x mm	y (mm)	Ax mm^3	Ay mm^3
1 Semicircle	$\frac{\pi R^2}{2}$ $= \frac{\pi(45)^2}{2}$ $= 3180.86$	45	$\frac{4(45)}{3\pi}$ $= 19.09$	+143138.7	+60722.
2) Triangle deduction	$= -\frac{1}{2} \times 60 \times 42.46$ $= 1273.0$	40	$= \frac{42.426}{3}$ $= 14.142$	-50920	-18002.
3) Triangle deduction	$= -\frac{1}{2}(30)(42.46)$ $= 636.9$	70	$= \frac{42.426}{3}$ $= 14.142$	-44583	-9007.
	$\Sigma A = 1270.1\ mm^2$			$\Sigma Ax = 47635.7$	33712.

$$\bar{X} = \frac{\Sigma Ax}{\Sigma A} = \frac{47635.7}{1270.1} = 37.50 mm$$

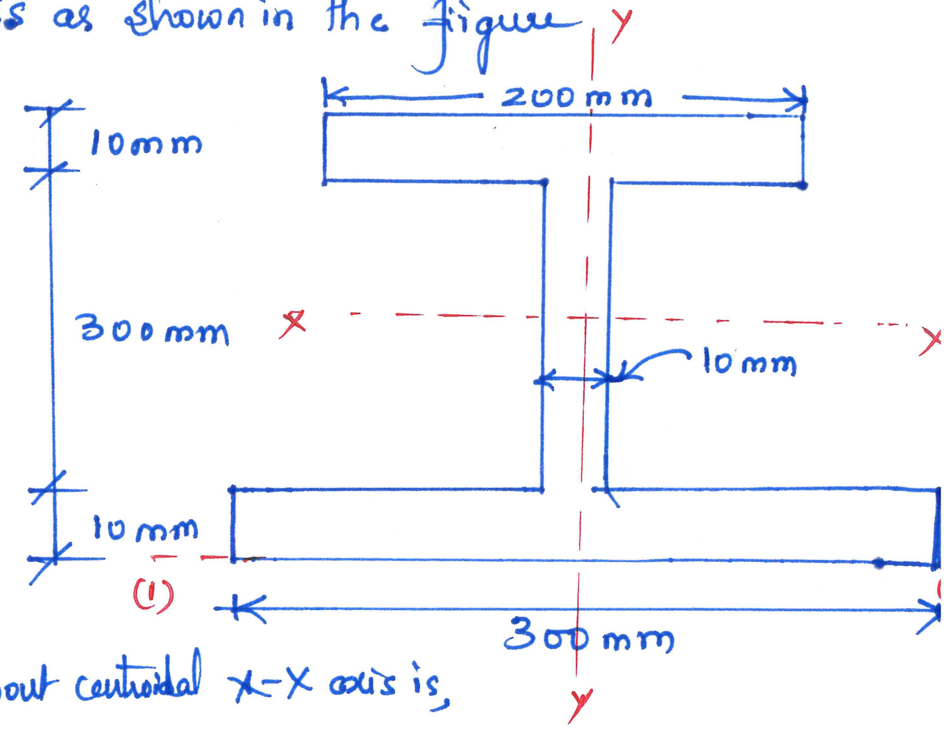
$$\bar{Y} = \frac{33712.87}{1270.1} = 26.54 mm$$

$$\therefore \boxed{\bar{X} = 37.50 mm}$$

$$\boxed{\bar{Y} = 26.54 mm}$$

Unit 5 – Numerical Problems on Moment of Inertia

1. Determine the moment of inertia of the unequal I Section about its centroidal axis as shown in the figure.



Soln.:

The given figure is symmetrical about y axis. Therefore the centroidal y axis coincides with the reference axis y axis.

Hence $\bar{x} = 0$.

∴ Moment of inertia about centroidal x-x axis is,

$I_{(1)-(1)} = \sum I_x + \sum A y^2$

$\bar{I}_x = I_{(1)-(1)} - A\bar{y}^2$ ||ly. $I_{2-2} = \sum I_y + \sum A x^2$

$\bar{I}_y = I_{2-2} - A\bar{x}^2$

x-x, y-y } centroidal axis.
(1)-(1) → Reference axis.

Component	Area mm^2	Depth of centroid from (1)-(1) axis y (mm)	Ay mm^3	Ay^2 mm^4	I_x mm^4 $bD^3/12$	I_y mm^4 $db^3/12$
1) Rectangle 300 × 10	300×10 = 3000	$10/2 = 5$	0.15×10^5	75×10^3	$\frac{300\times10^3}{12}$ = 25000	$\frac{10\times300^3}{12}$ = 22500000
2) Rectangle 10 × 300	10×300 = 3000	$10+\frac{300}{2}$ = 160	4.8×10^5	0.76×10^8	$\frac{10\times300^3}{12}$ = 22500000	$\frac{300\times10^3}{12}$ = 25000
3) Rectangle 200 × 10	200×10 = 2000	$10+300+10/2$ = 315	6.3×10^5	1.98×10^8	$\frac{200\times10^3}{12}$ = 16666.7	$\frac{10\times200^3}{12}$ = 6,666,666.7
	$\Sigma A = 8000$	8/1	$\Sigma Ay = 11.25\times10^5$	$\Sigma Ay^2 = 2.74\times10^8$	$\Sigma I_x = 0.25\times10^8$	$\Sigma I_y = 0.291\times10^8$

$$\bar{x} = 0$$

$$\bar{y} = \frac{\Sigma A y}{\Sigma A} = \frac{11.25 \times 10^5}{8000} = \underline{140.625\ \text{mm}}$$

$$I_{(1)-(1)} = \Sigma \bar{I}_x + \Sigma A y^2$$

$$= 0.258 \times 10^8 + 2.74 \times 10^8$$

$$I_{(1)-(1)} = \underline{2.99 \times 10^8\ \text{mm}^4}$$

$$\bar{I}_x = I_{(1)-(1)} - A\bar{y}^2$$

$$= 2.99 \times 10^8 - 8000(140.625)^2$$

$$\bar{I}_{x} = 1.416 \times 10^8\ \text{mm}^4$$

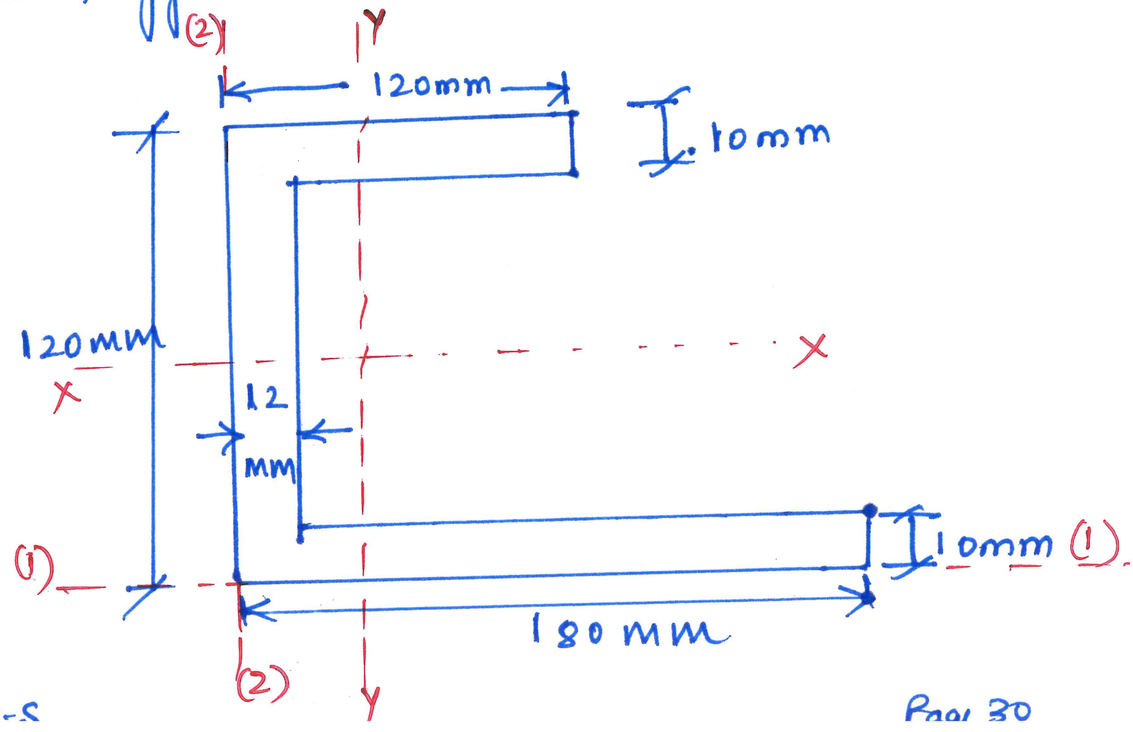
$$\bar{I}_x = \underline{1.416 \times 10^8\ \text{mm}^4}$$

$$\bar{I}_y = \Sigma \bar{I}_y = \underline{0.291 \times 10^8\ \text{mm}^4}$$

2. Determine the moment of inertia of the section as shown in figure about its centroidal axes. Calculate the least radius of gyration as well.

(1) − (1)
(2) − (2)
Reference axes

(X) − (X)
(Y) − (Y)
Centroidal axes.



Soln

Component	Area (A) mm^2	Depth of centroid from (2)-(2) axis x, mm	Depth of centroid from (1)-(1) axis, y, mm	Ax	Ay	Ax^2	Ay^2
1) Rectangle 180 × 10	180×10 = 1800	$\frac{180}{2} = 90$	$\frac{10}{2} = 5$	162×10^3	9×10^3	14.58×10^6	45×10^3
2) Rectangle 12×120	= 12×120 = 1440	$\frac{12}{2} = 6$	$10 + \frac{120}{2} = 60$	8.64×10^3	86.4×10^3	51.84×10^3	5.04×10^6
3) Rectangle 120×10	120 × 10 = 1200	$\frac{120}{2} = 60$	$120 - \frac{10}{2} = 115$	72×10^3	138×10^3	4.32×10^6	15.87×10^6
	$\Sigma A = A = 4440$			$\Sigma Ax = 2.42 \times 10^5$	$\Sigma Ay = 2.334 \times 10^5$	$\Sigma Ax^2 = 18.93 \times 10^6$	$\Sigma Ay^2 = 20.95 \times 10^6$

	$\bar{I}_x$ (mm⁴)	$\bar{I}_y$ (mm⁴)
1) Rectangle	$\frac{180\times10^3}{12} = 15000$	$\frac{10\times180^3}{12} = 4860000$
2) Rectangle	$\frac{12\times120^3}{12} = 1728000$	$\frac{120\times12^3}{12} = 17280$
3) Rectangle	$\frac{120\times10^3}{12} = 10000$	$\frac{10\times120^3}{12} = 1440000$
	$\Sigma I_x = 1753000$	$\Sigma I_y = 6317280$

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{2.42\times10^5}{4440} \qquad \bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{2.334\times10^5}{4440}$$

$$\bar{x} = \underline{54.54\,mm} \qquad \bar{y} = \underline{52.56\,mm}$$

$$I_{(1)-(1)} = \Sigma I_x + \Sigma Ay^2 = 1753000 + 20.95\times10^6 = \underline{22.70\times10^6\,mm^4}$$

$$\bar{I}_x = I_{(1)-(1)} - A\bar{y}^2 = 22.70\times10^6 - 4440(52.56)^2 = \underline{10.43\times10^6\,mm^4}$$

$$I_{(2)-(2)} = \Sigma I_y + \Sigma Ax^2 = 6317280 + 18.93\times10^6 = \underline{25.24\times10^6\,mm^4}$$

$$I_y = I_{(2)-(2)} - A\bar{x}^2 = 25.24\times10^6 - 4440(54.54)^2$$

$$\bar{I}_y = \underline{12.03\times10^6\,mm^4}$$

∴ Radius of gyration

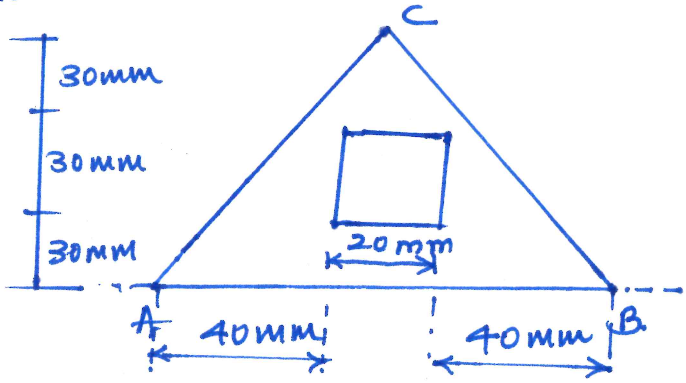
$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{10.43\times10^6}{4440}} \qquad k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{12.03\times10^6}{4440}}$$

$$k_x = \underline{48.46\,mm} \qquad k_y = \underline{52.05\,mm}$$

Least radius of gyration $= \underline{48.46}$ mm

Parallel axis

3) Determine the moment of inertia and radius of gyration of the area as shown in the figure about base AB and centroidal axes parallel to AB.



Soln

Component	A mm^2	depth of centroid from AB, y mm	Ay	Ay^2	$\bar{I}_x$
1) Triangle	$\frac{1}{2} \times 100 \times 90 = 4500$	$\frac{1}{3}(90) = 30$	135×10^3	4.05×10^6	$\frac{bh^3}{36} = 2.02 \times 10^6$
2) Rectangle Deduction	$30 \times 20 = 600$	$30 + 30/2 = 45$	-27×10^3	-1.21×10^6	$-\frac{bd^3}{12} = 45 \times 10^3$
	$\Sigma a = A = 3900$		$\Sigma Ay = 108 \times 10^3$	$\Sigma Ay^2 = 2.84 \times 10^6$	$\Sigma \bar{I}_x = 1.97 \times 10^6$

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{108 \times 10^3}{3900} = 27.69\,mm$$

$$I_{AB} = \bar{I}_x + A\bar{y}^2$$

$$I_{AB} = \Sigma \bar{I}_x + \Sigma Ay^2 = 1.97 \times 10^6 + 2.84 \times 10^6 = 4.81 \times 10^6\,mm^4$$

$$I_{AB} = 4.81 \times 10^6\,mm^4 = I_x$$

Moment of inertia about centroidal x-x axis

$$= \bar{I}_x = I_{1-1} - A\bar{y}^2$$

$$\bar{I}_x = 4.81 \times 10^6 - 3900\,(27.69)^2 = 1.81 \times 10^6\,mm^4$$

$$K_{AB} = \sqrt{\frac{I_{AB}}{A}} = 35.14\,mm \qquad K_x = \sqrt{\frac{\bar{I}_x}{A}} = 21.54\,mm$$