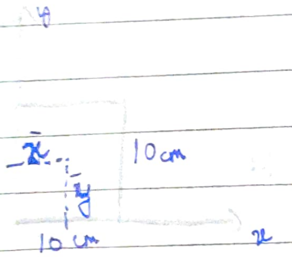
Centroid (or) Center of Gravity

Meaning:- ~~Mass of~~ Point of the structure where most of the mass is concentrated.

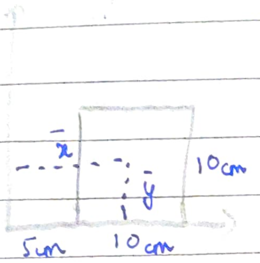
Application:-
→ Vehicle dynamics
→ Structural balance

Centroid:-
It is the point where the whole area of the plain figure is assumed to be concentrated, centroid would be the determination of the concentrated portion wrt to the reference axes.



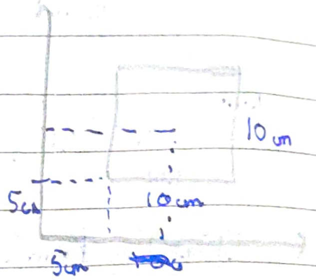
$\bar{x} = 5cm$

$\bar{y} = 5cm$



$\bar{x} = 10cm$

$\bar{y} = 5cm$



$\bar{x} = 10cm$

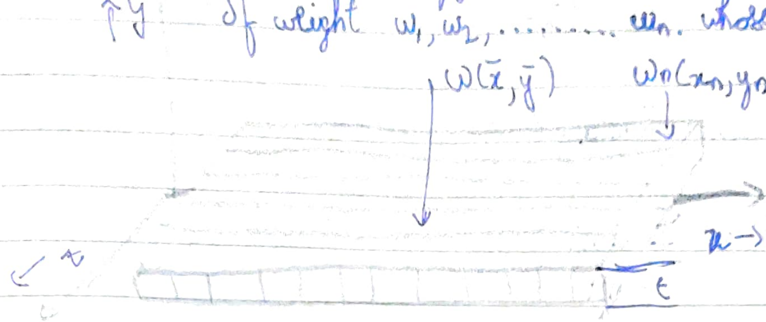
$\bar{y} = 10 cm$

Centre of Gravity (CG):-
It is the point where the whole weight is assumed to be concentrated. It is the point on which the body can be balanced, and the point where the weight of the body can act or possibly act.

Method of determining the centroid of a shape:

Generally, the centroid would be determined by the method of moments, consider a body of total weight W as shown, the centroid of the particular body would be at a distance $\bar{x}$ from the y-axis & $\bar{y}$ units away from the $\bar{x}$ axis.

Let us divide the whole figure into a number of elemental strips of weight $w_1, w_2, \ldots\ldots w_n$ whose centroid would be $x_1, x_2, x_3, \ldots\ldots x_n$ from y-axis & $y_1, y_2, y_3, \ldots\ldots y_n$ from x-axis.



Applying the theorem of moments:

$$W\bar{x} = w_1x_1 + w_2x_2 + \ldots\ldots\ldots w_nx_n$$

$$\& \quad W\bar{y} = w_1y_1 + w_2y_2 + \ldots\ldots\ldots w_ny_n.$$

$$\Rightarrow \bar{x} = \frac{w_1x_1 + w_2x_2 + \ldots\ldots\ldots w_nx_n}{W}$$

$$\& \quad \bar{y} = \frac{w_1y_1 + w_2y_2 + \ldots\ldots\ldots w_ny_n}{W}$$

The above case holds good for homogeneous & isotropic material.

For an homogenous & isotropic material, the density (γ) would be given by the formula:

$$\gamma = \frac{W}{V} = \frac{W}{A \times t}$$

Assuming, a uniformly thick material, we have:

$$W = \gamma \times A \times t \text{ (ignoring t)}$$

$\underline{III^{b}}$

$$W_1 = \gamma_1 \times A_1 \times t_1$$

we have

$$W_0 = \gamma \times A \times t$$

$$\& \quad W_1 = \gamma_1 \times A_1 \times t_1$$

$$\bar{x} = \frac{\gamma_1 a_1 t_1 + \gamma_2 a_2 t_2 + \ldots\ldots \gamma_n a_n t_n}{\gamma A t}$$

we have

$$\cancel{\bar{x} = \gamma a_1 t_1 + \gamma}$$

$$\Rightarrow \quad \bar{x} = \frac{\Sigma a_i x_i}{\Sigma a_i}$$

$$\& \quad \bar{y} = \frac{\Sigma a_i y_i}{\Sigma a_i}$$

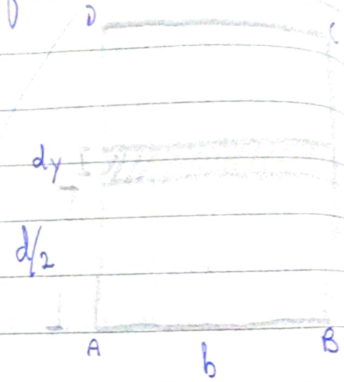
Axis of reference

Derivation for centroid of a rectangle

11/2/23

→ Let us consider a rectangular laminar of area $b \times d$ as shown.

→ Now, consider a horizontal elemental strip of area $b \times dy$ which is at a distance y from the reference axis ab as shown below.



→ Moment of area of elemental strip

$= \text{Area} \times \perp\text{r distance}$

$= b \times dy \times y$

→ Sum of moment of such elemental strips is given by:-

$$= \int_0^d b \times dy \times y$$

$$= b\left[\frac{y^2}{2}\right]_0^d = \frac{bd^2}{2} = \text{Sum of moment}$$

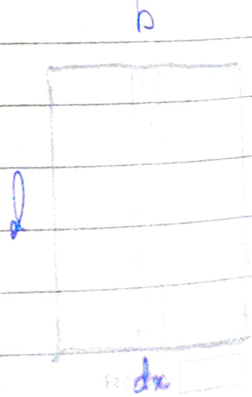
→ Moment of whole area about AB:-

$$= bd \times \bar{y}$$

(Varignon's)

→ Applying principle of moments:-

$$\cancel{bd} \times \bar{y} = \frac{\cancel{b}d^{\cancel{2}}}{2} \Rightarrow \bar{y} = \frac{d}{2}$$



|||ly

Considering an elemental strip dx:-

$$\bar{x} = b/2$$

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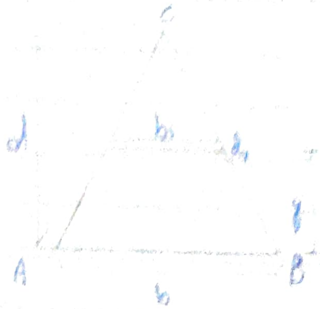
Derivation for centroid of a triangle

Consider a triangular lamina of the side ABC as shown, the area of the triangle being equal to $\frac{1}{2} \times b \times d = A$



Consider an elemental strip of area $b_1 \times dy$ which is at a distance ~~from~~ y from the reference axis.

$$A_1 = b_1 \times dy$$

* Using the property of similar triangle, we have:-

$$\frac{b_1}{b} = \frac{d-y}{d} \Rightarrow b_1 = \frac{b(d-y)}{d}$$

$$\Rightarrow A_1 = b\left(\frac{d-y}{d}\right) \times dy = \frac{(d-y)\,b \times dy}{d}$$

* Moment of elemental strip $b_1 dy$ about AB =

$$= A_1 \times y$$

$$= \frac{(d-y)\,b \times dy}{d} \times y = \frac{\cancel{d}\,b\,dy\,\cancel{d} \times y}{\cancel{d}} - \frac{y\,b\,dy \times y}{d}$$

$$= \boxed{y\,b\,dy - \frac{y^2 b\,dy}{d}} = y\,b\,dy\left(1 - \frac{y}{d}\right)$$

* ~~Moment of~~ Sum of moment of such elemental strips about reference axis AB would be equal to:

$$= \int_0^d y\,b\,dy - \int_0^d \frac{y^2 b\,dy}{d}$$

$$= \left[\frac{y^2}{2}\right]_0^d b - \left[\frac{y^3}{3}\right]_0^d \frac{b}{d} = \frac{d^2}{2}.b - \frac{d^3}{3}.\frac{b}{\cancel{d}}$$

$$= \frac{bd^2}{6}$$

Moment of total triangular area about reference axis AB

$A \times \bar{y}$

Applying principle of moments

$M_{AB} = A \times \bar{y} = \frac{1}{2} \times b \times d \times \bar{y}$

$$\boxed{\bar{y} = \frac{d}{3}}$$

Similarly $\boxed{\bar{x} = \frac{b}{3}}$

Centroid of SemiCircle

$\bar{y} = \frac{d}{2} = R$

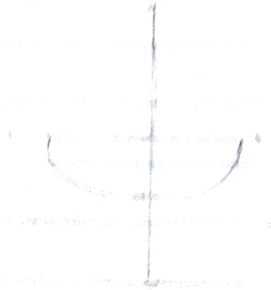
$\bar{x} = \frac{4R}{3\pi}$

$\bar{y} = \frac{4R}{3\pi}$

$\bar{x} = \frac{d}{2} = R$

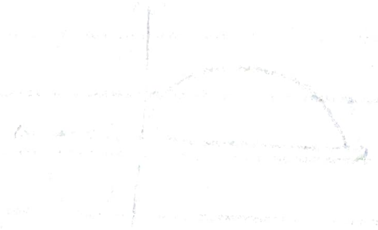
$\bar{y} = \frac{d}{2} = R$

$\bar{x} = \frac{4R}{3\pi}$



$\bar{x} = \frac{d}{2} = R$

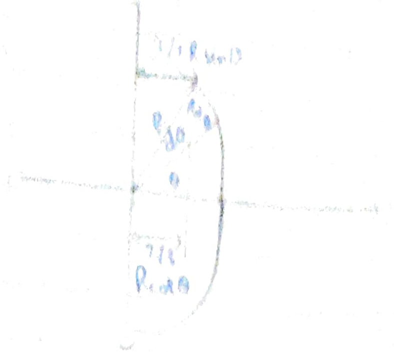
$\bar{y} = \frac{4R}{3\pi}$



$\bar{x} = \frac{d}{2} = R$

$\bar{y} = \frac{4R}{3\pi}$

Consider a semicircular lamina of area $\frac{\pi R^2}{2}$ as shown below:



Consider an elemental strip (triangle) of area

$= \frac{1}{2} \times R \times R\,d\theta = \frac{R^2 d\theta}{2}$

whose centroid is at a distance $\frac{2}{3} R$ from the origin O & its projection on the x-axis equal to $\frac{2}{3} R\cos\theta$

Moment of the area of elemental about y axis:

$$= \frac{2}{3} R\cos\theta \times \frac{R^2 d\theta}{2}$$

Sum of moments of such elemental strips about y-axis

$$= \frac{R^3}{3}\int_{-\pi/2}^{\pi/2} \cos\theta \times \cancel{\frac{1}{2} R^2 \int_{-\pi/2}^{\pi/2} d\theta}$$

$$\frac{2}{3} R(\sin\theta)_{-\pi/2}^{\pi/2} \times \cancel{\frac{1}{2} R^2 (\theta)_{-\pi/2}^{\pi/2}} = \frac{R^3}{3}(2)$$

Moment of total area about y-axis

$$= \frac{\pi R^2}{2} \times \bar{x}$$

By Varignon's theorem:-

$$\frac{\pi R^{\cancel{2}}}{2} \times \bar{x} = \frac{2R^{\cancel{3}}}{3} \Rightarrow \boxed{\frac{4R}{3\pi} = \bar{x}}$$

Moment of elemental strip about x-axis

$$\frac{R^2 d\theta}{2} \times \frac{2}{3} R\sin\theta =$$

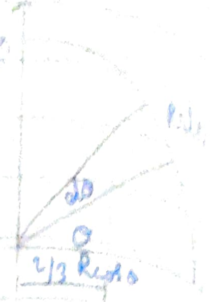
$$= \frac{R^3}{3}$$

Sum of moments of elemental strip about x-axis

$$\int_0^{\pi/2} \frac{R^2 d\theta}{2} \times \frac{2}{3}\int_0 R\sin\theta = \frac{R^3}{6} \times \frac{2}{3} = \frac{R^3}{3} \times (-\cos\theta)_0^{\pi/2}$$

Moment of:
For the above case, $\bar{y}$ is at the origin

Centroid of Quarter Cle



Consider a Quarter Cle lamina of Area $\frac{\pi R^2}{2}$, Consider an elemental triangular strip of area $\frac{R^2 d\theta}{2}$ & whose COG is at a distance from origin & its projection on x-axis $= \frac{2}{3} r\cos\theta$

& " " on y-axis $= \frac{2}{3} r\sin\theta$

Moment of elemental strip about y-axis

$$= \frac{R^2 d\theta}{2} \times \frac{2}{3} R\cos\theta = \frac{R^3 \cos\theta\, d\theta}{3}$$

Sum of moments:-

$$= \frac{R^3}{3} \int_0^{\pi/2} \cos\theta\, d\theta$$

Moment of Quarter Cle about y-axis

$$\frac{\pi R^3}{4} \times \bar{x}$$

By Varignon's

$$\Rightarrow \bar{x} \times \frac{\pi R^2}{4} = \frac{R^3}{3} \Rightarrow \frac{4R}{3\pi}$$

Similarly

$$\bar{y} = \frac{4R}{3\pi}$$

Centroid of sector of a ⊙le

Consider a sector of ⊙le as shown below

Consider a triangular elemental strip of area, at an angle from origin of area, $\frac{1}{2}R^2 d\theta$ whose COG is at a distance $\frac{2}{3}R$ from the origin and its projection on the x-axis = $\frac{2}{3}R\cos\theta$.

Moment of elemental strip about y-axis

$$\frac{R^2 d\theta}{2} \times \frac{2}{3}R\cos\theta = \frac{R^3 \cos d\theta}{3}$$

Sum of moment of elemental ~~star~~ stripe about y-axis

$$\frac{R^3}{3}\int_0^\alpha \cos\theta \, d\theta = \frac{R^3}{3}\sin\alpha$$

Area of sector of a ⊙le

$$= \frac{1}{2}\int_0^\alpha R^2 d\alpha = \frac{R^2}{2}\alpha$$

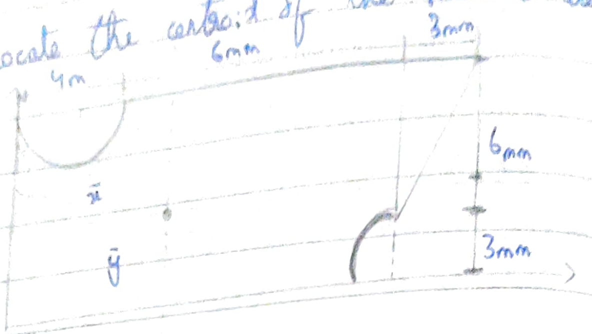
$$\frac{\alpha}{360}(\pi R^2) = \frac{\alpha}{2\pi} \times R^2 = \frac{\alpha R^2}{2}$$

Moment of sector of ⊙le about y-axis

$$\frac{\alpha R^2}{2} \cdot \bar{x} = \frac{R^3}{3}\sin\alpha$$

$$\bar{x} = \frac{2R\sin\alpha}{3\alpha}$$

~~Calculate~~ Locate the centroid of the shaded area of ...



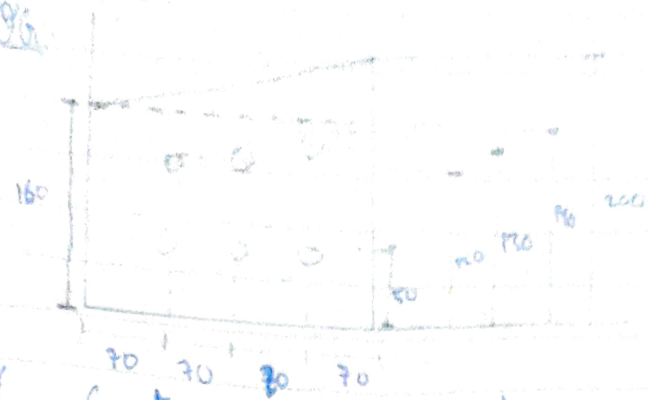
S.No	Shape	a	$\bar{x}$	$\bar{y}$	ax	ay
1)	10, 9	10×9 = 90	$\frac{13}{2}$ = 6.5	4.5	585	405
2	10, 3, 6, 9, 3	9	11	7	99	63
3	4, 7, 9	$-\frac{\pi R^2}{2}$ = -6.28	2	$9-\frac{4R}{3\pi}$ = 8.15	-12.56	+51.18
4	7, 3	-7.06	8.7	1.25	-51.42	-8.82

Σa	Σax	Σay
= 85.66	= 475	= 510.86

$\bar{x} = \frac{\Sigma ax}{\Sigma a} = 5.65$

$\bar{y} = \frac{\Sigma ay}{\Sigma a} = 5.95$

Q:- CG:



Q:- In a Gusset plate, there are six rivet holes of 21.5 mm diameter. Find CG.

S.N.	Shape	Area	$\bar{x}$	$\bar{y}$	ax	ay
1	160, 280	44800	140	80	6272000	3584000
2	40, 280	5600	186.7	173.3	1045520	970480
3	50, 70	-1451	70	50	-101570	72550
4	50, 140	-1451	140	50	-203140	-72550
5	50, 210	-1451	210	50	-304710	-72550
6	120, 70	-1451	70	120	-101570	-174120
7	130, 140	-1451	140	130	-203140	-188630
		-1451	210	140	-304710	-203140
8	140, 210	~~[illegible]~~	~~[illegible]~~	~~[illegible]~~		

$\Sigma A = 41694$ $\Sigma ax = 6098680$ $\Sigma ay = 1730940$

$\bar{x} = 14.627$ $\bar{y} = 41....$

Steps to solve the centroid problem:

① Centroid lies on the symmetrical axis, Identify the symmetrical axis & choose them as the reference axis.
② If no symmetrical axis is available, choose the bottom left as the origin so that the entire figure lies on the first quadrant (to avoid negative centroidal values)
③ Subdivide the given figure into known geometrical shapes (easy for computation) & identify their individual centroids
④ Enter the values in tabular column as shown below

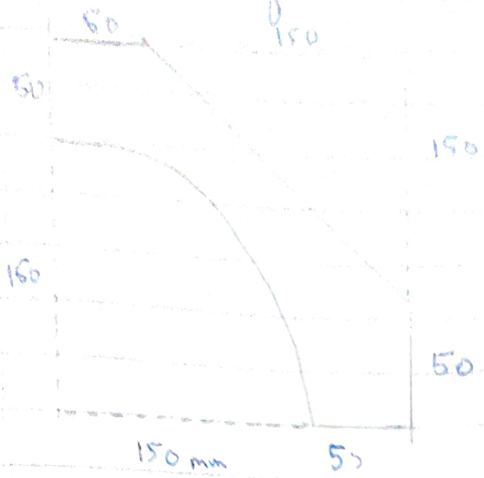
S.no.	Shape	a	$\bar{x}$	$\bar{y}$	ax	ay
		Σa			Σax	Σay

⑤ Compute the centroidal volumes as $\bar{x} = \frac{\Sigma ax}{\Sigma a}$

& $\bar{y} = \frac{\Sigma ay}{\Sigma a}$

Note:- ① The areas of hollow, void shapes can be indicated with negative sign.
②:- In case the geometrical shape lies below the x-axis & left of y-axis (III q), $\bar{x}$ & $\bar{y}$ are negative.

Q. Determine COG of the shaded area shown below.



S.No	Shape	a	$\bar{x}$	$\bar{y}$	ax	ay
1	200 × 200	40000	100	100	4000 000	4×10^6
2	150	-17662	63.7	63.7	-1125 069.4	-1125 069.4
3	50 150 150 50	-11250	150	150	-1687 500	-1687 500

Σa = ~~7676~~ 11088 ≈ 11090

Σax ~~= 1179867~~ 1187430.6

Σay ~~1174988~~ 1187 4

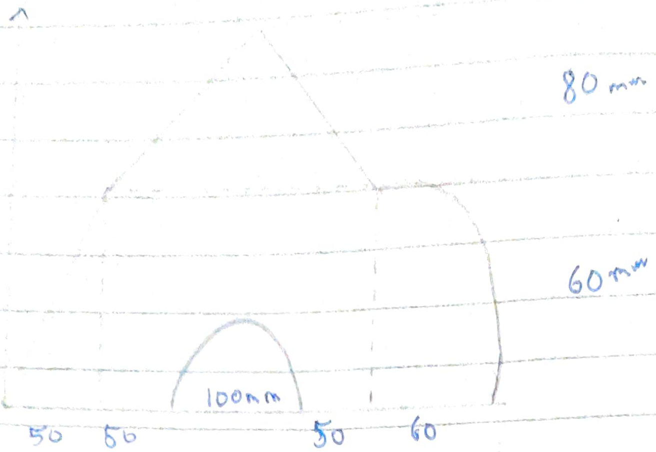
~~$\bar{x}$ = 3784.22 mm~~

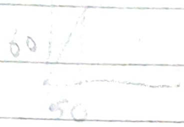
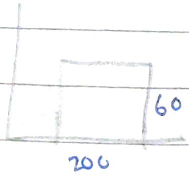
~~$\bar{y}$ = 374.22 mm~~

~~$\bar{x}$ = 157.81~~

$\bar{x}$ = 107.07 = $\bar{y}$

Q:- Determine COG of given area shown below:



SNO	Shape	a	$\bar{x}$	$\bar{y}$	ax	ay
1		1500	40	16.66	60000	24900
2		12000	30	200	360000	2400000
3		2826	251.2	251.2	709291.2	709891.2

$\Sigma a = 16326$ $\qquad \Sigma ax = 1129891.2$ $\qquad \Sigma ay = 3134791$

$\bar{x} = 69.208$ $\qquad \bar{y} = 192.01$