DC Circuits

Theory Questions

1. State and explain Ohm's law and discuss its limitations.
2. Sate Kirchhoff's current and voltage law and explain them with the help of an example.
3. State and explain superposition theorem with the help of an example.
4. State and explain thevenin's theorem with the help of an example.
5. State and explain maximum power transfer theorem with the help of an example.

Numericals

1. A current of 20A flows through two ammeters A and B connected in series. The potential difference across A is 0.2V and across B is 0.3V. Find how the same current will divide between A and B, when they are connected in parallel?
(Answer: I_A = 12A, I_B = 8A)

2. Two resistors of 100Ω and 200Ω are connected in series across a 4V cell of negligible internal resistance. A voltmeter of resistance 200Ω is used to measure the potential difference across each, what is the voltage reading in each case? **(Answer: V_1 = 1 Volt, V_2 = 2 Volts, *Note*: *V1+V2 is not equal to 4V*)**

3. When a certain battery with internal resistance is loaded by 60Ω resistor, its terminal voltage is 98.4V. When it is loaded by a 90Ω resistor, its terminal voltage is 98.9V. What load resistance would give a terminal voltage of 90V. **(Answer: 8.4Ω)**

4. The domestic power load in a house comprises of the following:

8 lamps of 100W each

3 fans of 80W each

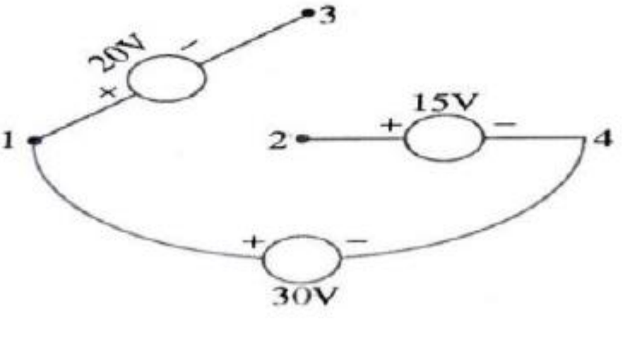
1 refrigerator of half hp

1 heater of 1000W

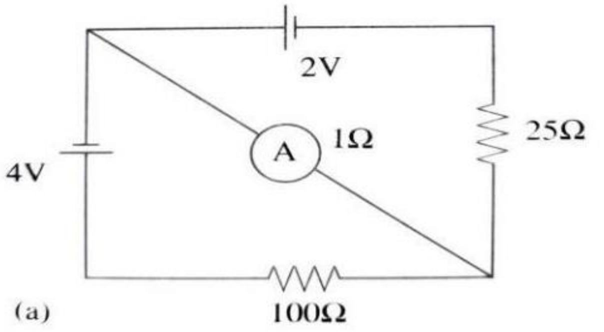
a) Calculate the total current taken from the supply of 230V.

b) Calculate the energy consumed in a day, if on an average only a quarter of the above load persists all the time. **(Answer: Total load= 2413W, I = 10.5A, E= 14.478kWh)**

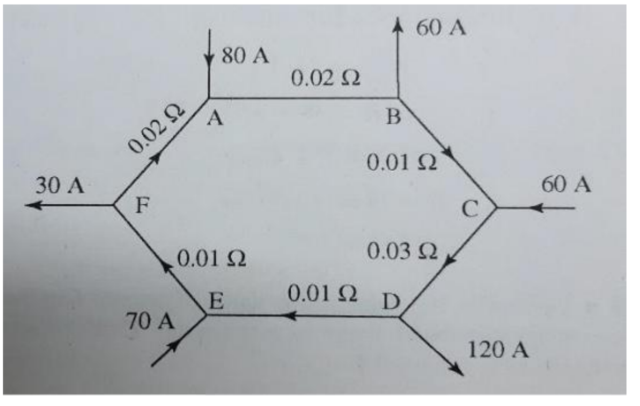
5. With reference to given figure below, find the voltages V_{12}, V_{23} and V_{34}. **(Answer: 15V, 5V, 5V)**



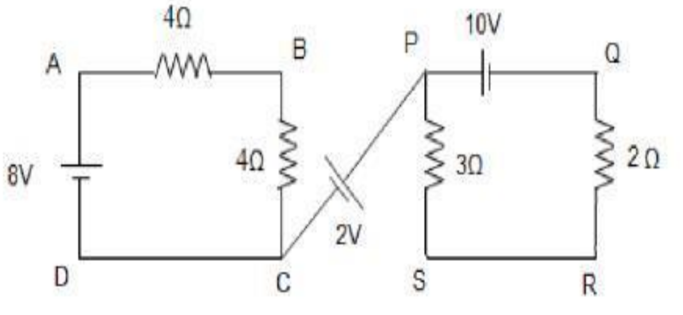
6. Find the current through ammeter for the network shown below. **(Answer: I = 38.09 mA)**



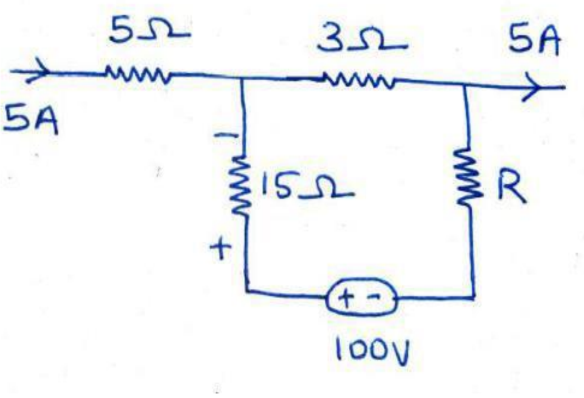
7. Find the current through all the branches in the network shown below. **(Answer: IAB = 39A, IBC= -21A, ICD= 39A, IDE= -81A, IEF= -11A, IFA= -41A**)



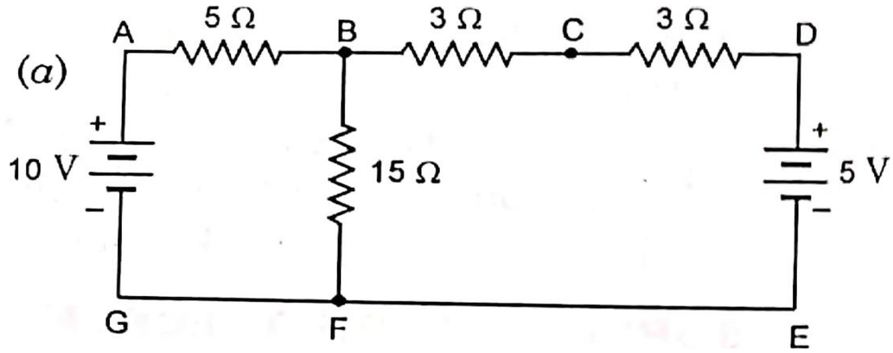
8. Find VBS, VAQ and VDR in the network shown below. **(Answer: 8V, 16V, 4V)**



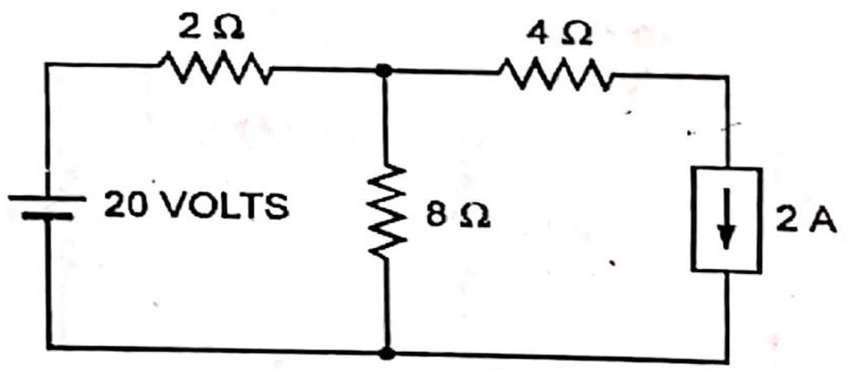
9. The voltage drop across the 15Ω resistor is 30V, having the polarity as indicated in the circuit shown below. Find R. **(Answer: R= 54.5Ω)**



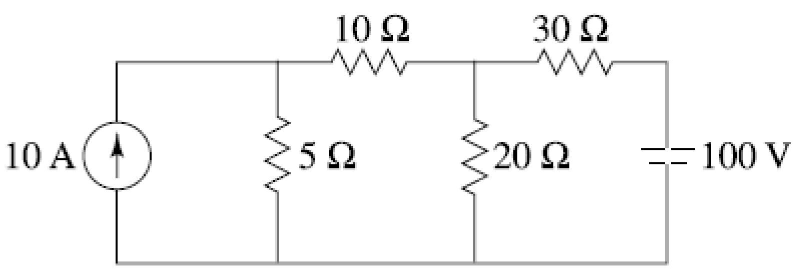
10. Find the value and direction of current in branch BF using superposition theorem. **(Answer: 0.436A)**



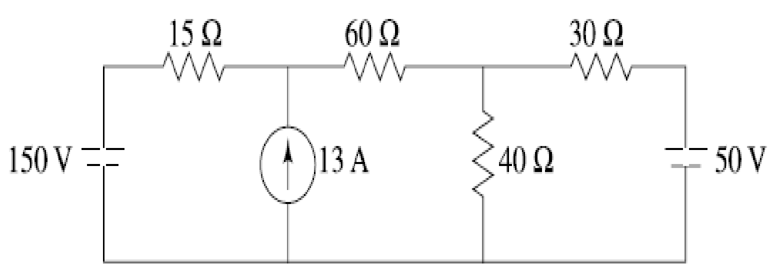
11. Determine current through 8 Ω resistor in the following network. **(Answer: 1.6A)**



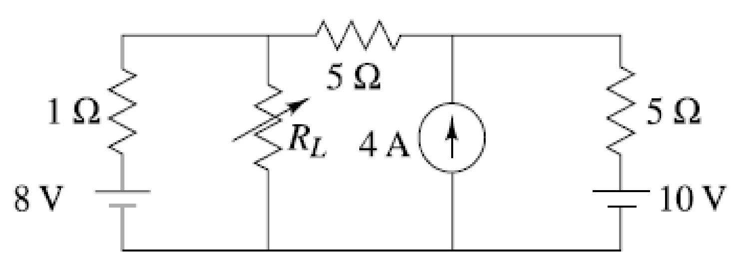
12. Find the current through the 10 Ω resistor using thevenin's theorem. **(Answer: 0.37A)**



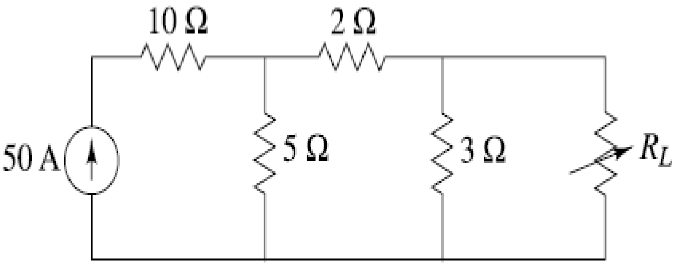
12. Find the current through the 30 Ω resistor using thevenin's theorem. **(Answer: 1.25A)**



13. For the circuit shown, find value of resistance R_L for maximum power and calculate maximum power. **(Answer: 0.91Ω, 27.47W)**

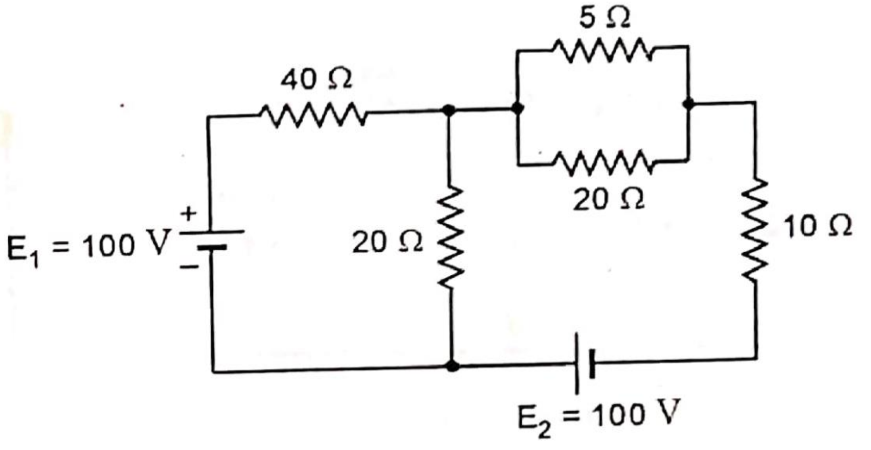


14. For the circuit shown, find value of resistance R_L for maximum power and calculate the maximum power. **(Answer: 2.1Ω, 669.64W)**

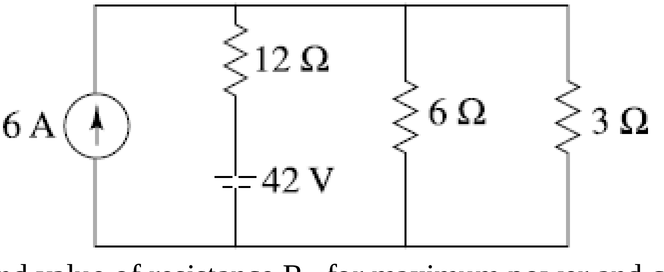


Exercise Numericals

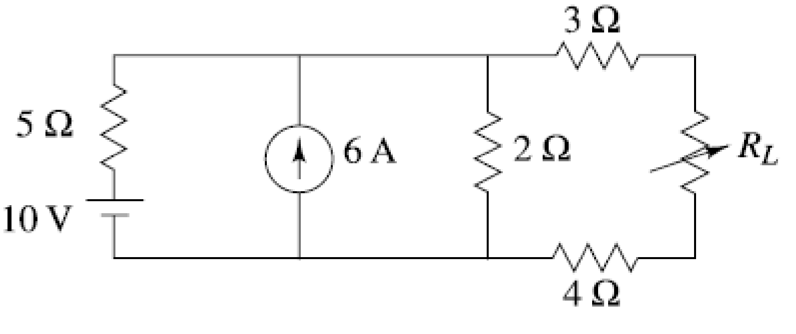
1. By mean of superposition, find the current which flows in R_2 = 20 Ω for the given circuit. **(Answer: 1.585A)**



2. Find the current through the 3 Ω resistor using thevenin's theorem. **(Answer: 5.43A)**



3. For the circuit shown, find value of resistance R_L for maximum power and calculate the maximum power. **(Answer: 8.43Ω, 3.87W)**



Charge: The charge is considered as the quantity of Electricity.

Unit of charge is Coulomb [C]

Current: Rate of flow of charge in an electric circuit due to drift of Electrons.

Current is given by, $I = \frac{dq}{dt} = \frac{Q}{t}$.

Unit of current is Ampere (A).

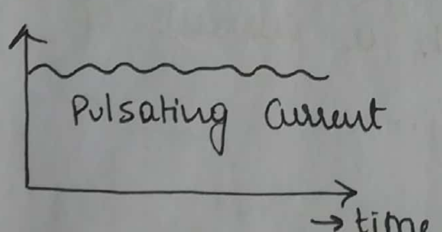
Types of Electric Current:

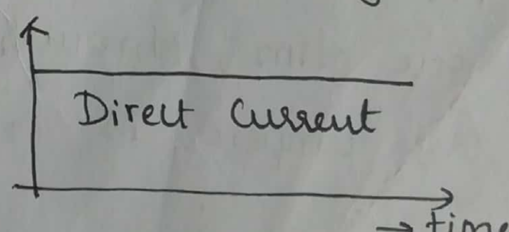
If the flow of current remains same in one direction only, it is called Unidirectional Current.

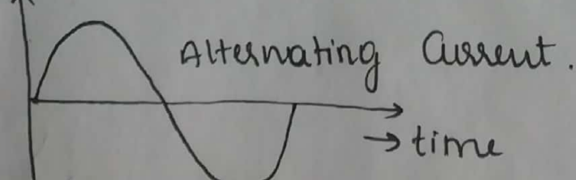
If the magnitude of the unidirectional current remains constant with time, it is called direct current or DC.

If the magnitude of the current varies continuosly with respect to time and if it is bidirectional, such type of current is called Alternating current or AC.

If the magnitude of unidirectional current varies with time is called pulsating current.







Current density [J]: The current density in a Conductor carrying current is the current per unit area of the cross section of the conductor.

Unit of current density is ampere/metre^2 [A/m^2]

The current density is given by

$$\boxed{J = \frac{I}{A}} \; A/m^2$$

Potential difference: The potential difference between any two points of charged conductor is the amount of work that has to be done to bring a unit positive charge from a point of lower potential to the point of higher potential. The unit of potential difference is Volt.

If the energy required to move a charge of 'Q' Coulomb from point 'A' to point B is 'W' joules, the voltage 'V' between A & B is given as,

$$\boxed{V = \frac{W}{Q}}$$

The unit of voltage is Volts [V]

Volt [V]: One volt is defined as the potential difference across a resistance of one ohm, through which a current of one ampere is flowing.

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Electric field in a Conductor:

When a battery is connected across two ends of a conductor, an electric field is set up at every point within the conductor. Hence the electrons keep flowing in the conductor. The flow of these electrons constitutes an electric current.

Unit of electric field is Volt/metre [V/m].

Electrical Energy:

Total amount of electrical work done in an electrical circuit.

Unit of energy is Joules [J]

Electrical Energy = Power x time | $P = V \times I$

$= V \times I\, t$ Joules or watt sec

Practically energy is measured in kWh

$1\,Wh = 3600$ Joules or W.s

$1\,kWh = 3600 \times 1000$ Joules $= 3.6 \times 10^{6}$ Joules.

Electrical Power:

The rate at which electrical work is done in an electrical circuit.

The Unit of Power is watt [W]

$P = VI$.

Note: Horse Power (hp) is also used as a unit of Power.

$\boxed{1\,hp = 746\,W}$

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Electromotive Force [EMF] :

Emf of a source is the energy imparted by the source to each coulomb of charge passing through it.

Emf is not a force, but its is the energy expended on each charge.

$$E = \frac{W}{Q} \ J/C$$

where $E \rightarrow$ Energy imparted by the source

$Q \rightarrow$ charge transferred through the source in coulomb.

Difference between emf & Potential difference

The emf of a device is a measure of the energy the battery gives to each coulomb of charge.

The potential difference between two points is a measure of the energy used by one coulomb in moving drom one point to another.

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DC CIRCUITS.

OHM'S LAW :

It states that the potential difference between the two ends of a conductor is directly proportional to the current flowing through it, provided its temperature and other physical parameters remain unchanged.

$$V \propto I \quad \text{or} \quad V = IR.$$

Constant of proportionality R is called resistance of conductor.

Resistance : The unit ohm is defined as the resistance which permits a flow of one ampere of current when a potential difference of one volt is applied to the resistance.

$$I \propto V \quad \text{or} \quad I = GV$$

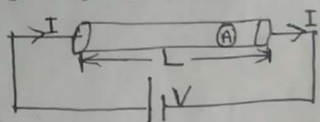
$G \rightarrow$ constant, called as conductance.

$\boxed{G = \frac{1}{R}}$ The conductance is reciprocal of the resistance

The SI unit of conductance is siemen [S].

The resistance, or opposition to the flow of current through a conductor depends on how narrow is its cross section and how long is its length.
In other words, the electrical resistance of a conductor is directly proportional to its length [L] and inversely proportional to its area of cross section [A].

$$R \propto \frac{L}{A} \quad \text{or} \quad R = \rho \cdot \frac{L}{A}$$



$\rho \rightarrow$ Constant, known as resistivity of the material.
The unit of resistivity is ohm metre [Ω m].

Limitations of ohm's Law:

i) It does not holds good for non linear devices such as semi conductor & zener diodes.

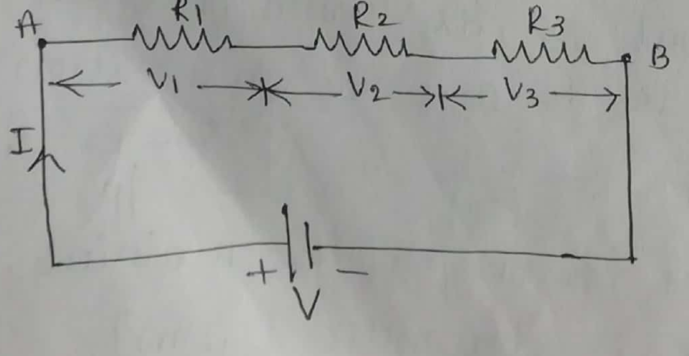
ii) It is not applicable to non metallic Conductors.

iii) Cannot be applied to arc lamps.

iv) Does not holds good when temperature raises rapidly.

Series Combination of Resistances:

Two or more resistances are said to be connected in series, if same current flows through them But there will be voltage drop across each resistor (element).



The applied voltage V must be equal to sum of three individual voltages. (voltage drops)

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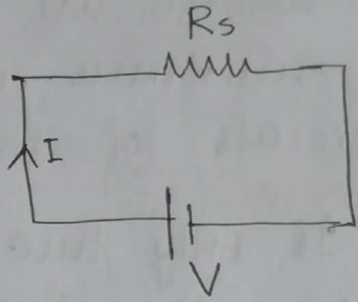
$$V = V_1 + V_2 + V_3$$
$$= IR_1 + IR_2 + IR_3$$
$$V = I(R_1 + R_2 + R_3) \rightarrow ①$$



$$\frac{V}{I} = R_1 + R_2 + R_3$$

$$\boxed{R_s = R_1 + R_2 + R_3} \rightarrow ②$$

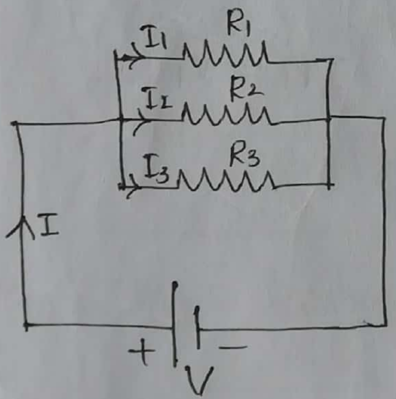
Thus, the equivalent resistance of a number of resistance connected in series is equal to the sum of individual resistances

Substitute ② in ①.

$$\boxed{V = IR_s}$$

Parallel Combination of Resistances:

All the starting point and ending point of resistors are shorted and given to a voltage source in series, same voltage exists across all the resistors. The current drawn by each resistor is different.



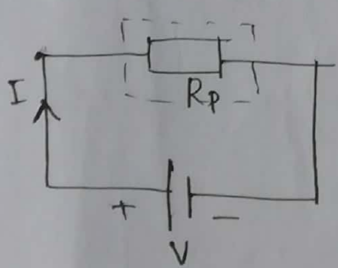
The total current I entering the circuit divides into I_1, I_2, & I_3.

$$I = I_1 + I_2 + I_3$$
$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$\because V = IR$

$\therefore I = \frac{V}{R}$

$$I = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$



$$\frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

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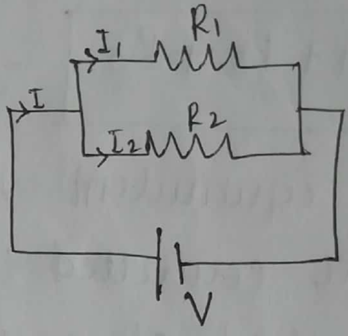
When resistors are connected in parallel, the total resistance is equal to sum of the reciprocals of the individual resistances.

If only two resistors are connected in parallel, the equivalent resistance is given by,

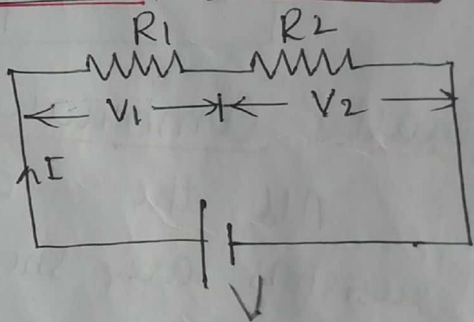
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$



$$\frac{1}{R_p} = \frac{R_1+R_2}{R_1R_2}$$

$$\boxed{R_p = \frac{R_1R_2}{R_1+R_2}}$$

Voltage division in two Resistors (In Series)



The total resistance

$$R = R_1 + R_2$$

Current in the ckt,

$$I = \frac{V}{R} = \left(\frac{V}{R_1+R_2}\right) \rightarrow ①$$

(V_1) The vtg drop across R_1, $V_1 = IR_1 \longrightarrow ②$

Replace equ ① in ②

$$IR_1 = \left(\frac{V}{R_1+R_2}\right) \cdot R_1 \qquad \Big| \qquad IR_1 = V_1$$

$$V_1 = \frac{V \cdot R_1}{R_1+R_2}$$

$$V_2 = \frac{VR_2}{R_1+R_2}$$

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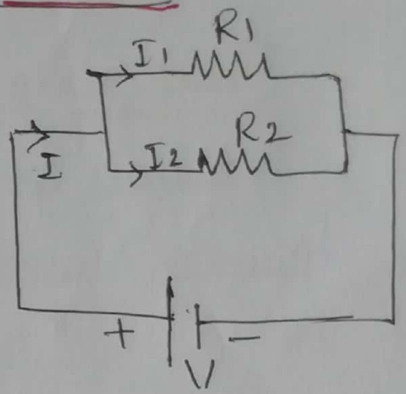
Current division in parallel Circuits



The current through R_1 is I_1.

$$\boxed{I_1 = \frac{V}{R_1}} \longrightarrow ①$$

Current through R_2 is I_2,

$$\boxed{I_2 = \frac{V}{R_2}} \longrightarrow ②$$

$\div$ ① by ②

$$\frac{I_1}{I_2} = \frac{\cancel{V}/R_1}{\cancel{V}/R_2} = \frac{R_2}{R_1}$$

$$\boxed{\frac{I_1}{I_2} = \frac{R_2}{R_1}} \longrightarrow ③$$

$I = I_1 + I_2$

$I_2 = I - I_1$

$I_1 = I - I_2$

$$\frac{I_1}{I - I_1} = \frac{R_2}{R_1}$$

$$\boxed{I_1 = \frac{R_2(I - I_1)}{R_1}} \qquad \boxed{I_2 = \frac{R_1(I - I_2)}{R_2}}$$

$I_1R_1 = R_2 I - R_2 I_1$ $\qquad$ $I_2R_2 = R_1 I - R_1 I_2$

$I_1(R_1 + R_2) = R_2 I$ $\qquad$ $I_2(R_2 + R_1) = R_1 I$

$$\boxed{I_1 = \frac{IR_2}{R_1 + R_2}} \qquad \boxed{I_2 = \frac{IR_1}{R_1 + R_2}}$$

For Ckts having three resistors in parallel,

$$I_1 = I\left[\frac{R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

$$I_2 = I\left[\frac{R_1R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

$$I_3 = I\left[\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

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Kirchoff's Laws :

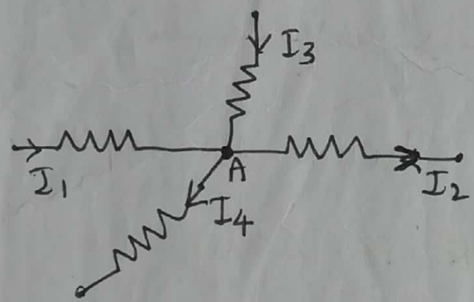
Kirchoff's first law : [Current or Node law]

"the algebraic sum of all the currents meeting at junction of an electrical circuit is zero."

or

"The sum of incoming currents towards any point is equal to the sum of outgoing currents away from that point."

Figure shows the junction 'A' of an electric ckt at which four currents I_1, I_2, I_3 & I_4 meet. All the currents entering the junction are taken as +ve and all the currents leaving the junction are taken as -ve.



According to KCL,

$$I_1 - I_2 + I_3 - I_4 = 0$$

$$I_1 + I_3 = I_2 + I_4 .$$

Kirchoff's Second Law : [Voltage law]

"In any closed electrical circuit, the algebraic sum of products of currents & Resistances [vtg drop] plus the algebraic sum of all the emf's in the circuit is zero.

Algebraic Sum of emf's + Algebraic sum of vtg drops = 0

$$\therefore \Sigma E + \Sigma IR = 0.$$

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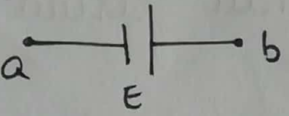
All the voltage rises are taken as +ve
all the voltage drops are taken as −ve.
Figure represents a battery of emf E volts
connected between two points a & b
which can be traced from



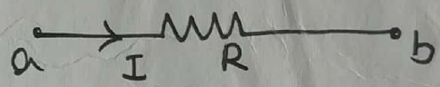
a to b or from b to a
when it is traced from a to b [−ve to +ve],
it is a voltage rise. Hence emf is +ve.

∴ E_{ab} is positive.

when the battery is traced from b to a
[+ve to −ve], it is a voltage fall. Hence
emf is −ve.

∴ E_{ba} is Negative.

Consider a resistor R connected between
two points a & b,



through which a
current I is flowing as shown in figure.
The voltage drop (V_{ab}) = IR is along the
direction of current. It is a voltage fall
hence −ve.

The voltage drop (V_{ba}) = IR is against
the direction of the current. It is a vtg rise
and hence positive.

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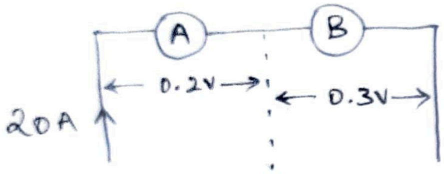
Basic Electrical Engineering

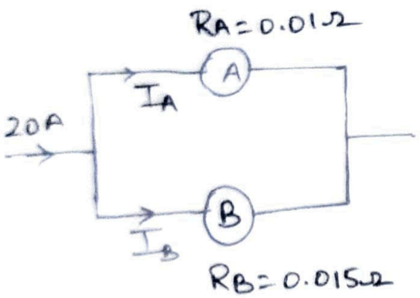
1. A Current of 20A flows through two ammeters A and B connected in Series. The potential difference across A is 0.2V and across B is 0.3V. Find how the same current will divide between A and B, when they are connected in parallel?.

Sol:



$$R_A = \frac{0.2}{20} = 0.01\,\Omega$$

$$R_B = \frac{0.3}{20} = 0.015\,\Omega$$



$$I_A = \frac{R_B}{R_A + R_B} \times 20$$

$$I_A = \frac{0.015}{0.01 + 0.015} \times 20 = 12\,A$$

$$\therefore \boxed{I_A = 12A}$$

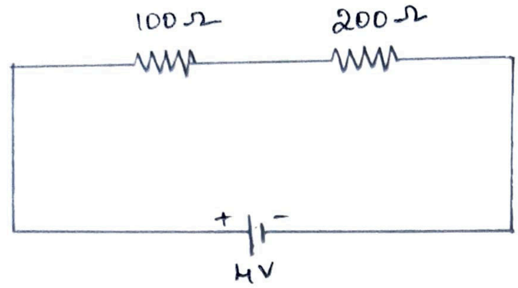
$$I_B = 20 - I_A$$

$$= 20 - 12 = 8A$$

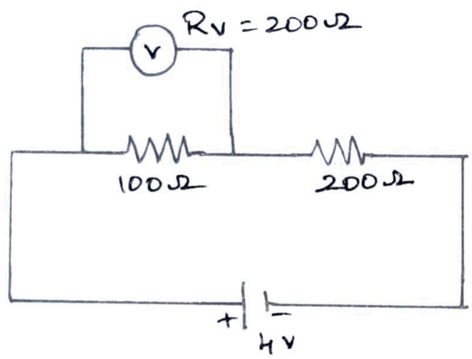
$$\therefore \boxed{I_B = 8A}$$

2) Two resistors of 100Ω and 200Ω are connected in series across a 4 V cell of negligible internal resistance. A Voltmeter of resistance 200Ω is used to measure the potential difference across each, what is the voltage in each case?

Sol

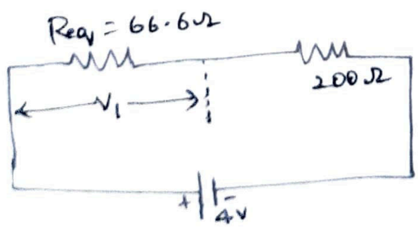


CASE i) Voltmeter connected across 100Ω resistor



Since resistance of Voltmeter (R_V) & 100Ω resistor are in parallel, their effective resistance is $R_{eq} = \dfrac{R_V \times 100}{R_V + 100}$

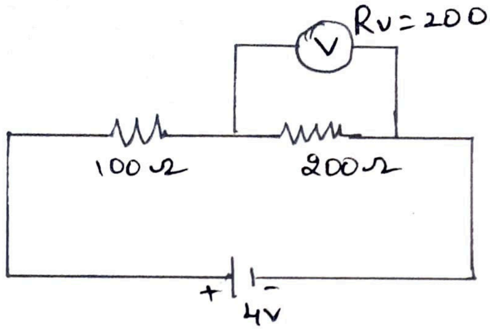
$$\therefore R_{eq} = \frac{200 \times 100}{200 + 100} = \frac{20000}{300} = 66.67\Omega$$



By Voltage Division rule $V_1 = \dfrac{66.67}{66.67 + 200} \times 4$

$$\therefore \boxed{V_1 = 1 \text{ Volts}}$$

CASE ii) Voltmeter across 200Ω resistance



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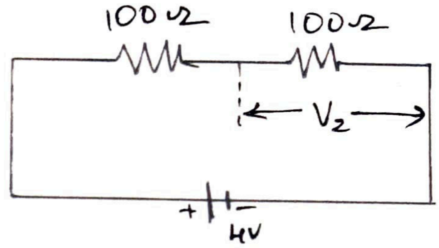


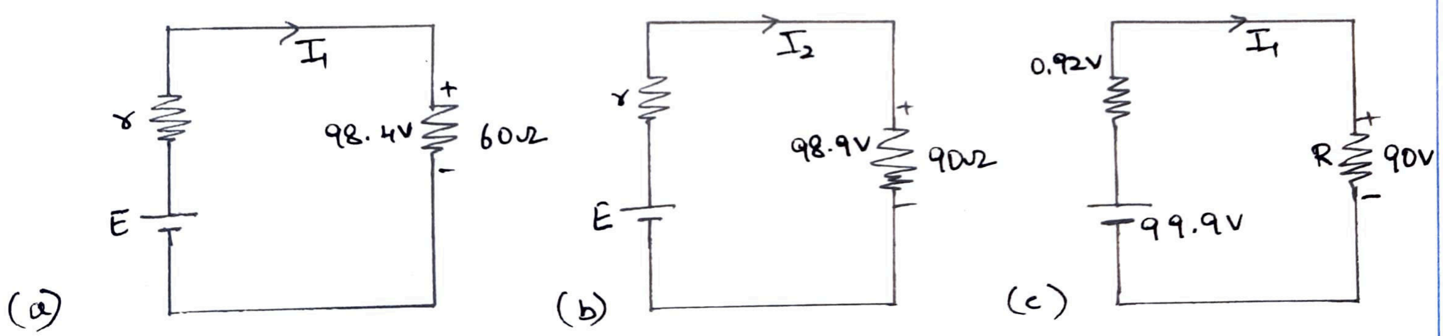
$$V_2 = \frac{100}{100+100} \times 4 = \frac{100}{200} \times 4 = 2V$$

$$\therefore \boxed{V_2 = 2 \text{ Volts}}$$

$V_1 + V_2 \neq V$ because of the error in the voltmeter.

The Error is because of the Low resistance of the Voltmeter.
In general Voltmeter should have high Resistance.

3. When a certain battery with internal resistance is loaded by 60Ω resistor, its terminal Voltage is 98.4V. When it is loaded by a 90Ω resistor, its terminal Voltage is 98.9V. What load resistance would give a terminal Voltage of 90V.



Sol The two cases are shown in Fig (a) & (b)

From (a) $\quad I_1 = \frac{98.4}{60} = 1.64A$

From (b) $\quad I_2 = \frac{98.9}{90} = 1.099A$

In (a)

$$E - I_1 \times r - 98.4 = 0$$

or

$$E - 1.64 \times r - 98.4 = 0 \quad \text{— (1)}$$

In (b)

Solving (1) & (2)

$$\boxed{\begin{array}{l} E = 99.9\,V \\ r = 0.92\,\Omega \end{array}}$$

$$E - I_r \times r - 98.9 = 0$$

or

$$E - 1.099 \times r - 98.9 = 0 \quad \text{— (2)}$$

we next calculate the load resistance which would give a terminal voltage of 90 V in fig (c)

$$99.9 - I(0.92) = 90$$

$$I = \frac{9.9}{0.92} = 10.7\,A$$

$$R = \frac{90}{10.7} = 8.411\,\Omega$$

6. The domestic power load in a house comprises of the following :
8 lamps of 100 W each , 3 fans of 80 W each , 1 refrigerator of half hp , 1 Heater of 1000 W.

a) Calculate the total current taken from the supply of 230V.

b) Calculate the energy consumed in a day, if on an average only a quarter of the above load persists all the time

Sol (a) The total load is given as .

Sl No.	Item	Load
1.	8 lamps of 100 W each	$8 \times 100 = 800\,W$
2	3 fans of 80 W each	$3 \times 80 = 240\,W$

Sl No	Item	Load	
3.	1 Refrigerator of ½ hp	$1 \times \frac{1}{2} hp = \frac{1}{2} \times 746 W$	$= 373 W$
4.	1 Heater of 1000 W	1×1000	$= 1000 W$
	Total Load		$= 2413 W$

∴ Current taken from the supply,

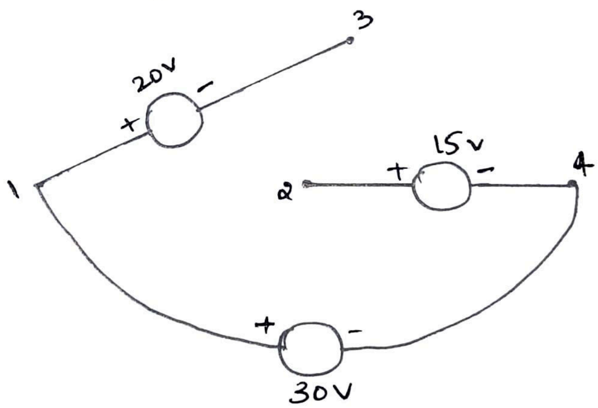
$$I = \frac{P}{V} = \frac{2413}{230} = 10.5 A$$

(b) Energy consumed per day $= 2413 W \times \frac{1}{4} \times 24$

$= 14478 Wh$

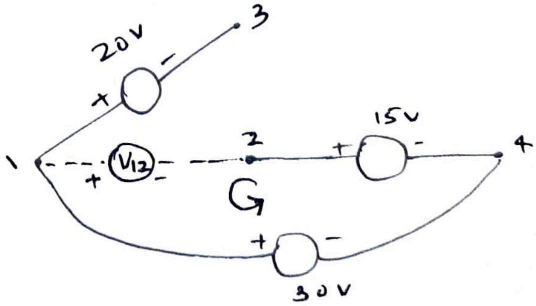
$= 14.478 kWh$

7. With reference to Fig.1, find the voltages V_{12}, V_{23} and V_{34}.



Sol: case i) To find V_{12}

connect a Voltmeter across node 1 & 2 and apply KVL to the closed path 1-4-2-1



$-30 + 15 + V_{12} = 0$

$-15 + V_{12} = 0$

$\boxed{V_{12} = 15 \text{ Volts}}$

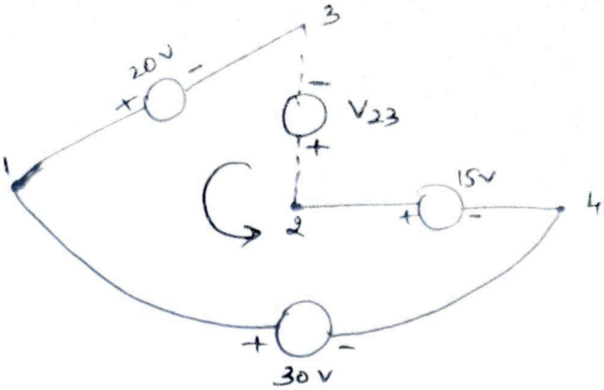
Case ii) To find V_{23}

Apply KVL to loop 1-4-2-3-1

$$-30+15-V_{23}+20=0$$

$$5-V_{23}=0$$

$$\boxed{V_{23}=5 \text{ volts}}$$



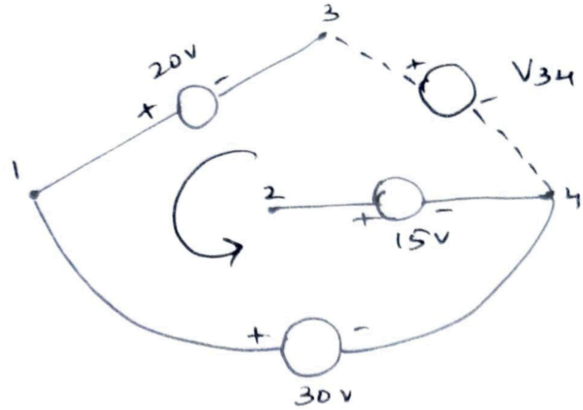
Case iii) To find V_{34}

Apply KVL to loop 1-4-3-1

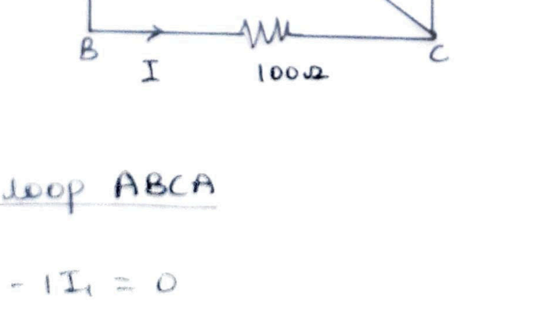
$$-30+V_{34}+20=0$$

$$V_{34}-10=0$$

$$\boxed{V_{34}=10 \text{ Volts}}$$



9. Find the current through ammeter for the network shown in figure.



Sol:

KVL to loop ABCA

$4 - 100I - 1I_1 = 0$

$\therefore 100I + I_1 = 4$ —— (1)

KVL to loop ADCA

$-2 + (I - I_1)25 - 1I_1 = 0$

$$25I - 25I_1 - I_1 = 2$$

$$25I - 26I_1 = 2 \quad \text{——— (2)}$$

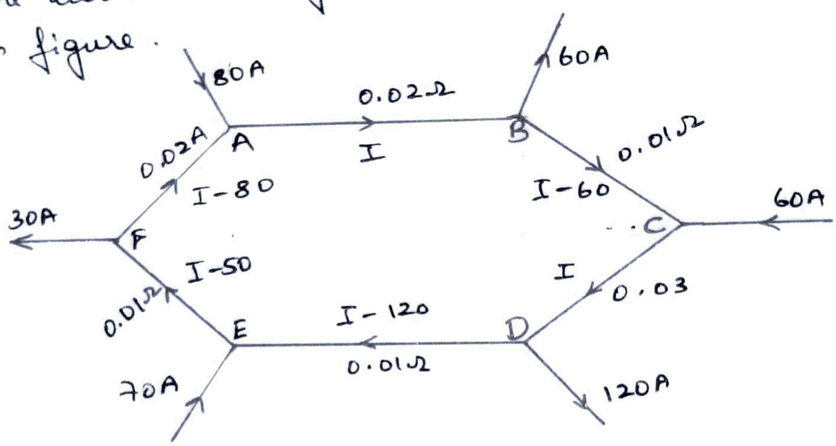
Solving equation (1) & (2), we get

$$I = 39.63\,mA$$

$$I_1 = -38.09\,mA$$

$\therefore$ Current through milliammeter is $I_1 = 38.09\,mA$ directed from A to C.

10. Find the current through all the branches in the network shown in figure.



Sol

$\therefore$ Let us assume a current I in any one of the branch.

Apply KVL to the Center loop, we get

$$0.02I + 0.01(I-60) + 0.03I + 0.01(I-120) + 0.01(I-50) + 0.02(I-80) = 0$$

$$0.1I = 3.9$$

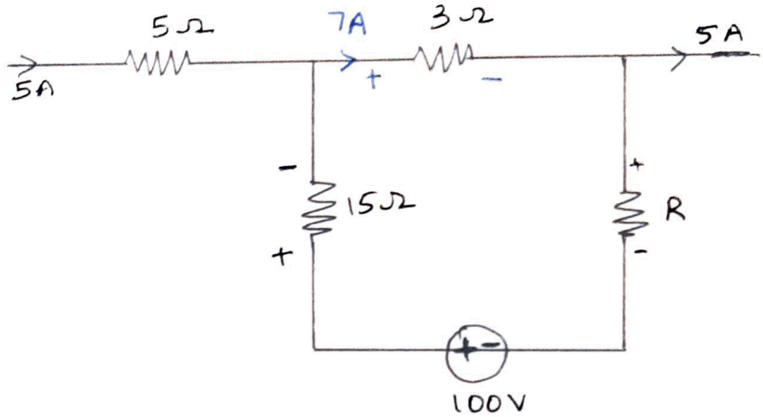
$$\therefore I = 39A$$

$\therefore$ Current in Branch $\underline{I_{AB} = 39A}$, $\underline{I_{BC} = -21A}$, $\underline{I_{CD} = 39A}$

$\underline{I_{DE} = -81A}$, $\underline{I_{EF} = -11A}$, $\underline{I_{FA} = -41A}$.

NOTE: The -ve sign implies the actual direction of current through that branch is opposite to the direction we have assumed.

11. The voltage drop across the 15 Ω resistor is 30 V, having the polarity as indicated in the circuit shown in figure. Find R.



Sol:

$$I_1 = \frac{30}{15} = 2A$$

KVL to loop − 1

$$-30 - 21 - 2R + 100 = 0$$

$$\cancel{2R = 100 + 21 - 30}$$

$$\cancel{2R = 121 - 30}$$

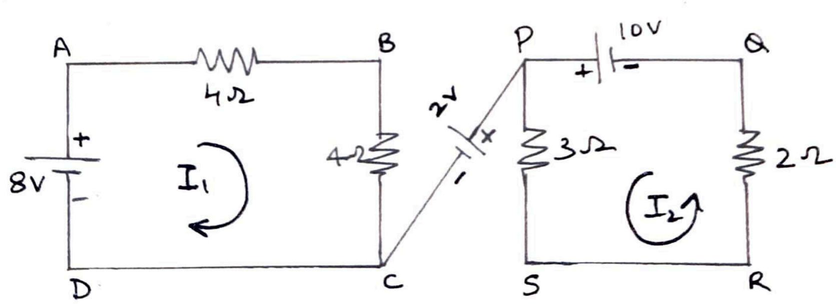
$$\cancel{R = \frac{91}{2}}$$

$$\cancel{\boxed{R = 45.5\,\Omega}}$$

$$49 = 2R$$

$$\boxed{R = 24.5\,\Omega}$$

12. Find V_{BS}, V_{AQ} and V_{DR} in the network shown in the figure.



Sol:

Loop ABCDA : $-4I_1 - 4I_1 + 8 = 0$

$$-8I_1 = -8$$

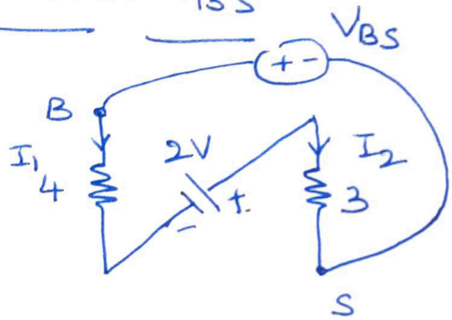
$$\boxed{I_1 = 1A}$$

KVL to loop PSRQP: $-3I_2 - 2I_2 + 10 = 0$

$$5I_2 = 10$$

$$I_2 = 2A$$

To find V_{BS}



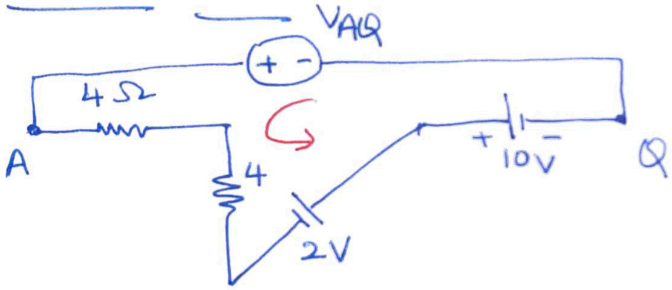
KVL to the loop

$$-4I_1 + 2 - 3I_2 + V_{BS} = 0$$

$$-4 + 2 - 6 + V_{BS} = 0$$

$$\boxed{V_{BS} = 8V}$$

To find V_{AQ}



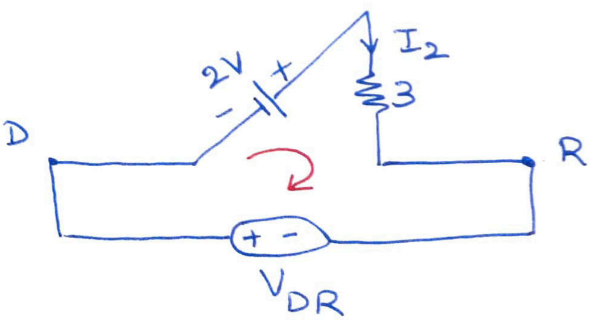
KVL to the loop

$$-4I_1 - 4I_1 + 2 - 10 + V_{AQ} = 0$$

$$-8 + 2 - 10 + V_{AQ} = 0$$

$$\boxed{V_{AQ} = 16V}$$

To find V_{DR}



KVL to the loop

$$2 - 3I_2 + V_{DR} = 0$$

$$2 - 6 + V_{DR} = 0$$

$$\boxed{V_{DR} = 4V}$$

Example 2.10. Calculate the current through the galvanometer in the following bridge (Fig. 2.35).
[U.P. Technical Univ. Electrical Engineering January 2003]

Solution : Assume current distribution in the bridge network as shown in Fig. 2.36.

Applying Kirchhoff's second law to meshes ABDA, BCDB and ABCA we have

$$I_1 + 4 I_3 - 2 I_2 = 0 \qquad ...(i)$$

$$2(I_1 - I_3) - 3(I_2 + I_3) - 4 I_3 = 0$$

$$\text{or} \quad 2 I_1 - 3 I_2 - 9 I_3 = 0 \qquad ...(ii)$$

$$\text{and} \quad I_1 + 2(I_1 - I_3) = 2$$

$$\text{or} \quad 3 I_1 - 2 I_3 = 2 \qquad ...(iii)$$

Solving Eqs. (*i*), (*ii*) and (*iii*) we get

$$I_3 = \frac{1}{44} A \; ; \; I_2 = \frac{17}{44} A \text{ and } I_1 = \frac{30}{44} A$$

So current through galvanometer

$$= I_3 = \frac{1}{44} A \textbf{ Ans.}$$

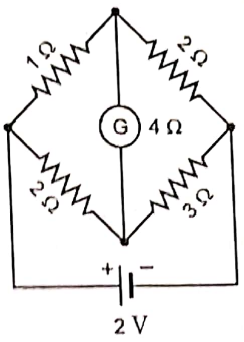


Fig 2.35

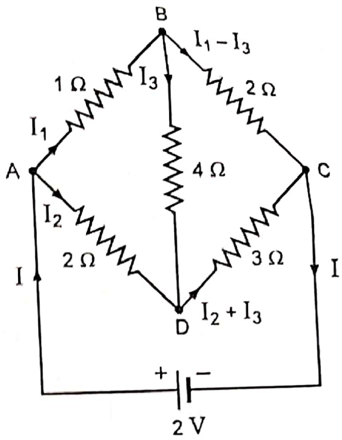


Fig 2.36

2.6. NETWORK THEOREMS

The circuit variables, we are interested in determining, generally are (*i*) current through or (*ii*) voltage across a resistance of interest or a group of resistances. Sometimes it is required to work out the necessary source voltage or source current which will cause a specific current through or voltage across a given resistor or result into a specified power dissipation by it.

The current flowing through a circuit is governed by basic law called the *Ohm's law*, already discussed in Chapter 1. Distribution of current or the voltage over a circuit is governed by Kirchhoff's laws, already discussed in the preceding Art. (Art. 2.5).

The circuit to be analysed may be simple or quite complex. In case of complex networks the solution procedure may be too tedious and time consuming. Certain techniques for solution of such networks have been developed which reduces the networks to simpler form for quick solution. This may be accomplished through the use of what are called as *network theorems*. Few of these network theorems, which are relevant at this stage, will be discussed here.

2.7. SUPERPOSITION THEOREM

This theorem is applied when we are to determine the current in one particular branch of a network containing several voltage sources or current sources or both voltage sources and current sources. This scheme is to determine how much current each of the individual source contributes to the branch in question, and then add algebraically these component currents.

If there are several sources of emfs acting simultaneously in an electric circuit, then according to this theorem emf of each source acts independently of those of other sources, *i.e.* as if the other sources of emf did not exist and current in any branch or conductor of a network is equal to the algebraic sum of the currents due to each source of emf separately, all other emfs being taken equal to zero. This theorem is applicable only in linear circuits, *i.e.* circuits consisting of resistances in which Ohm's law is valid. In circuits having nonlinear resistances such as thermionic valves and metal rectifiers, this theorem is not applicable. However, superposition theorem can be applied to a circuit containing current sources and even to circuits containing both voltage sources and current sources. To remove a current source from the circuit, circuit of the source is opened leaving in place any conductance that may be in parallel with it, just as series resistance is kept in place when voltage source is removed.

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Though the application of the above theorem requires a little more work than other methods such as the circulating current method but it avoids the solution of two or more simultaneous equations. After a little practice with this method, equations can be written directly from the original circuit diagram and labour in drawing extra diagrams is saved.

The superposition theorem can be stated as below :

In a linear resistive network containing two or more voltage sources, the current through any element (resistance or source) may be determined by adding together algebraically the currents produced by each source acting alone, when all other voltage sources are replaced by their internal resistances. If a voltage source has no internal resistance, the terminals to which it was connected are joined together. If there are current sources present they are removed and the network terminals to which they were connected are left open.

The procedure for applying superposition theorem is as follows:

1. Replace all but one of the sources of supply by their internal resistances. If the internal resistance of any source is very small as compared to other resistances existing in the network, the source is replaced by a short-circuit. In case of a current source open the circuit leaving in place any conductance that may be in parallel with it.
2. Determine the currents in various branches using Ohm's law.
3. Repeat the process using each of the emfs turn-by-turn as the sole emf each time. Now the total current in any branch of the circuit is the algebraic sum of currents due to each source.

The details of the above procedure can best be understood by examining its application to the following solved examples.

Example 2.11. Find the value and direction of current in branch BF using superposition theorem.

[R.G.P.V. Basic Elec. Engineering Jan./Feb. 2008]

Solution : Assuming 5 V source inactive or depressed, the circuit is reduced to a simple circuit shown in Fig. 2.38 (*a*).

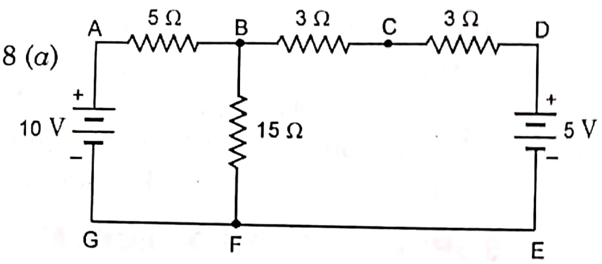

Fig. 2.37

Equivalent resistance of the circuit shown in Fig. 2.38 (*a*)

$$R' = 5 + \frac{15 \times (3+3)}{15+3+3}$$

$$= \frac{65}{7}\ \Omega$$

Current supplied by 10 V source,

$$I_1 = \frac{10}{\frac{65}{7}} = \frac{14}{13}\ \text{A}$$

Current in branch BF, $I_1' = I_1 \times \dfrac{R_{BC} + R_{CD}}{R_{BC} + R_{CD} + R_{BF}}$

$$= \frac{14}{13} \times \frac{3+3}{3+3+15} = \frac{4}{13}\ \text{A}$$

Now assuming 10 V source inactive or depressed, the circuit is reduced to that shown in Fig. 2.38 (*b*).

Equivalent resistance of the circuit shown in Fig. 2.38 (*b*).

$$R'' = 3 + 3 + \frac{5 \times 15}{5 + 15} = 9.75\ \Omega$$

Current supplied by 5 V source

$$I_2 = \frac{5}{9.75} = \frac{20}{39}\ \text{A}$$

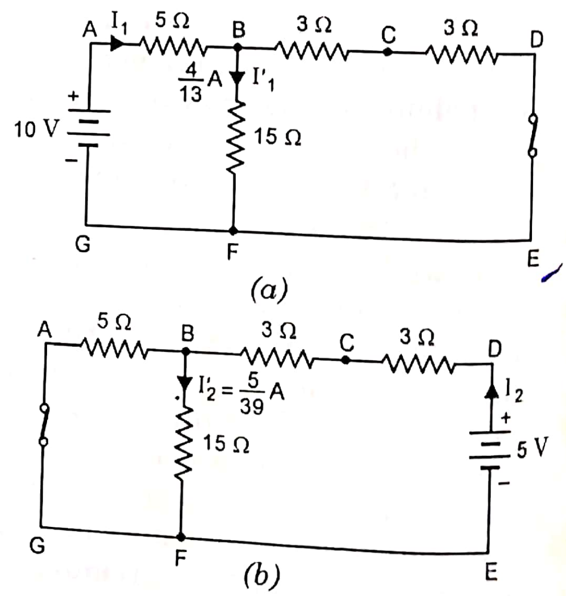

Fig. 2.38

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Current in branch BF, $I'_2 = I_2 \times \dfrac{R_{AB}}{R_{AB} + R_{BF}}$ by current division rule

$$= \frac{20}{39} \times \frac{5}{5+15} = \frac{5}{39} \text{ A}$$

Total current through branch BF, according to superposition theorem

$$I = I'_1 + I'_2 = \frac{4}{13} + \frac{5}{39} = \frac{17}{39} \text{ A from B to F} \quad \textbf{Ans.}$$

Example 2.12. By mean of superposition, find the current which flows in R_2 = 20 Ω for the circuit of Fig. 2.39. [*R.G.P.V. Basic Elec. Engineering Jan./Feb. 2007*]

Solution : Taking $E_2 = 0$, the circuit is reduced to a simple circuit shown in Fig. 2.40 (*a*).

Equivalent resistance of the circuit shown in Fig. 2.40.

$$R' = R_1 + R_2 \,||\, [R_3 \,||\, R_4 + R_5]$$

$$= 40 + 20 \,||\, [5 \,||\, 20 + 10]$$

$$= 40 + 20 \,||\left[\frac{5 \times 20}{5+20} + 10\right]$$

$$= 40 + 20 \,||\, 14$$

$$= 40 + \frac{20 \times 14}{20 + 14} = \frac{820}{17} \,\Omega$$

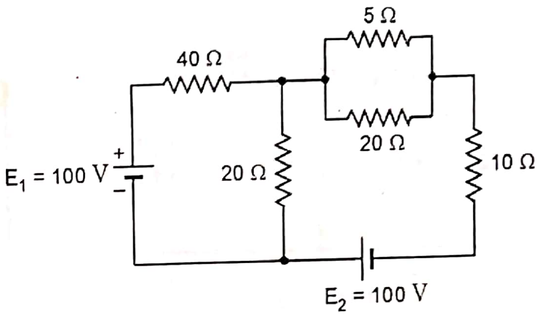

Fig. 2.39

Current supplied by source E_1, $I_1 = \dfrac{E_1}{R'} = \dfrac{100}{820/17} = \dfrac{85}{41}$ A

Current through branch BE, $I'_1 = I_1 \times \dfrac{R_3 \,||\, R_4 + R_5}{R_3 \,||\, R_4 + R_5 + R_2}$

$$= \frac{85}{41} \times \frac{\dfrac{5 \times 20}{5+20} + 10}{\dfrac{5 \times 20}{5+20} + 10 + 20}$$

$$= \frac{85}{41} \times \frac{14}{34} = \frac{35}{41} \text{ A} \quad \text{from terminal B to terminal E}$$

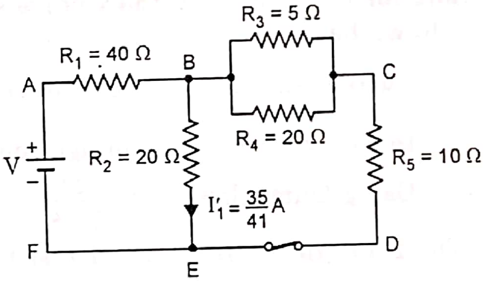

Fig. 2.40 (*a*)

Taking $E_1 = 0$, the circuit is reduced to that shown in Fig. 2.40 (*b*).

Equivalent resistance of the circuit shown in Fig. 2.40 (*b*).

$$R'' = R_5 + R_3 \,||\, R_4 + R_2 \,||\, R_1$$

$$= 10 + 5 \,||\, 20 + 20 \,||\, 40$$

$$= 10 + \frac{5 \times 20}{5 + 20} + \frac{20 \times 40}{20 + 40}$$

$$= 10 + 4 + \frac{40}{3} = \frac{82}{3} \,\Omega$$

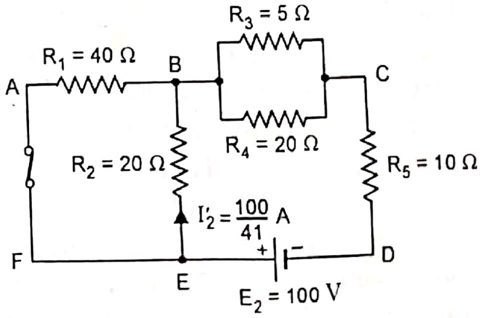

Fig. 2.40 (*b*)

Current supplied by source E_2,

$$I_2 = \frac{E_2}{R''} = \frac{100}{82/3} = \frac{300}{82} = \frac{150}{41} \text{ A}$$

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Current through branch BE,

$$I_2' = I_2 \times \frac{R_1}{R_1 + R_2} \qquad \text{By current division rule}$$

$$= \frac{150}{41} \times \frac{40}{40+20} = \frac{100}{41}\text{ A} \qquad \text{from terminal E to terminal B}$$

Total current through resistance R_2 of 20 Ω (branch BE)

$$I = I_1' + I_2'$$

$$= \left(\frac{-35}{41} + \frac{100}{41}\right)\text{A} = \frac{65}{41}\text{ A} \quad \textbf{Ans.} \qquad \text{from terminal E to terminal B}$$

Example 2.13. Determine current through 8 Ω resistor in the following network (Fig. 2.41) using superposition theorem: [*U.P. Technical Univ. Electrical Engineering, Second Semester 2002-03*]

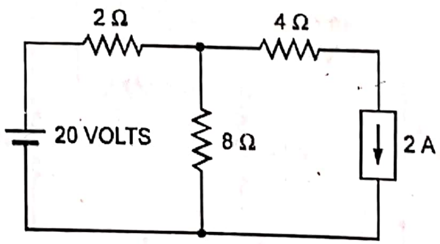

Fig. 2.41

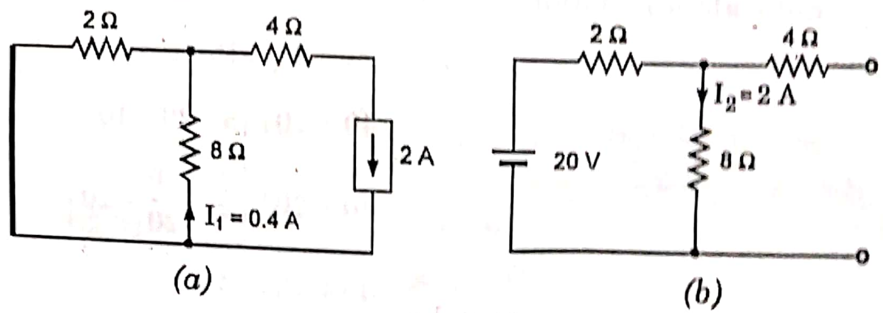

Fig. 2.42

Solution : In Fig. 2.42, the voltage source has been replaced by a short. Using the current division rule we have

Current through 8 Ω resistor, $I_1 = 2 \times \frac{2}{2+8} = 0.4$ A in the direction shown in Fig. 2.42 (*a*)

In Fig. 2.42 (*b*) only voltage source has been considered.

Using Ohm's law, $I_2 = \frac{20}{2+8} = 2$ A in the direction shown in Fig. 2.42 (*b*).

The resultant current in 8 Ω resistor, $I = I_1 + I_2 = (-0.4 + 2)$ A $= 1.6$ A **Ans.**

2.8. MAXWELL CIRCULATING CURRENT THEOREM

If a network with several sources has more than two

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3.3 THEVENIN'S THEOREM

It states that *'Any two terminals of a network can be replaced by an equivalent voltage source and an equivalent series resistance. The voltage source is the voltage across the two terminals with load, if any, removed. The series resistance is the resistance of the network measured between two terminals with load removed and constant voltage source being replaced by its internal resistance (or if it is not given with zero resistance, i.e., short circuit) and constant current source replaced by infinite resistance, i.e., open circuit.'*

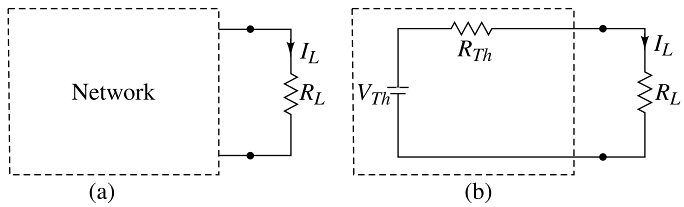

Fig. 3.61

Explanation The above method of determining the load current through a given load resistance can be explained with the help of following circuit.

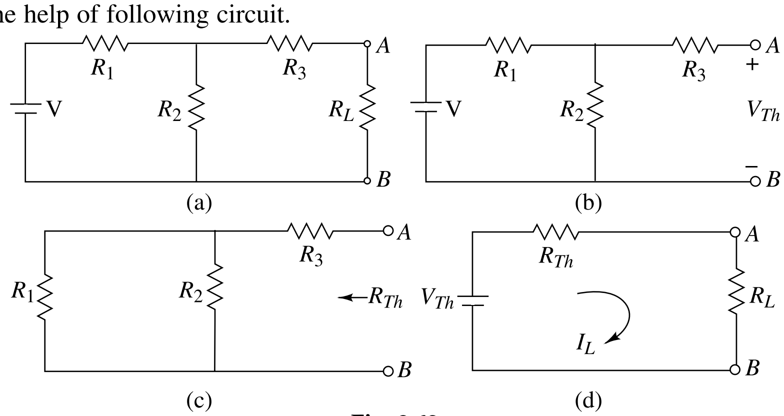

Fig. 3.62

Steps to be followed in Thevenin's theorem

1. Remove the load resistance R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B with the voltage source V replaced by a short circuit.
4. Replace the network by a voltage source V_{Th} in series with resistance R_{Th}.
5. Find the current through R_L using Ohm's law.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Example 3.15 *Find the current through the 10-Ω resistor.*

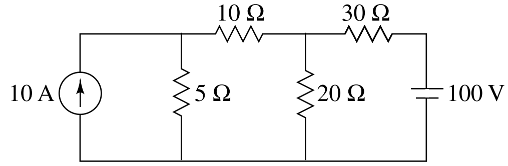

Fig. 3.63

Step I: *Calculation of V_{Th}*

Removing the 10-Ω resistor from the circuit,

For Mesh 1,

$$I_1 = 10 \text{ A}$$

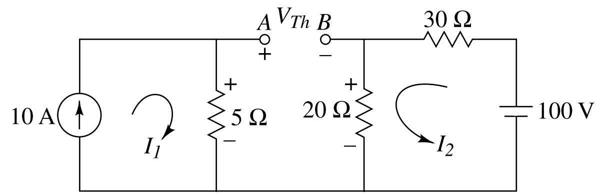

Fig. 3.64

Applying KVL to Mesh 2,

$$100 - 30I_2 - 20I_2 = 0$$

$$I_2 = 2 \text{ A}$$

Writing V_{Th} equation,

$$5I_1 - V_{Th} - 20I_2 = 0$$

$$V_{Th} = 5I_1 - 20I_2$$

$$= 5(10) - 20(2) = 10 \text{ V}$$

Step II: *Calculation of R_{Th}*

Replacing the current source of 10 A with an open circuit and the voltage source of 100 V with a short circuit,

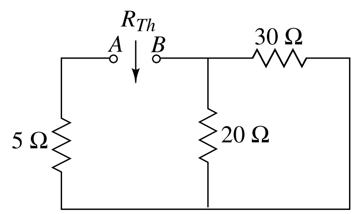

Fig. 3.65

$$R_{Th} = 5 + (20 \,||\, 30) = 17 \, \Omega$$

Step III: *Calculation of I_L*

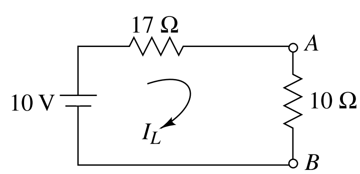

Fig. 3.66

$$I_L = \frac{10}{17+10} = 0.37 \text{ A}$$

Example 3.19 *Find the current through the 3-Ω resistor.*

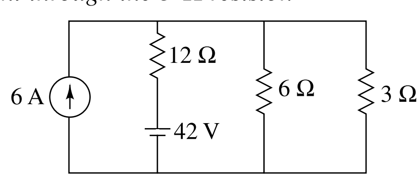

Fig. 3.87

Step I: *Calculation of* V_{Th}

Removing the 3-Ω resistor from the network,

Writing equation for Mesh 1,

$$I_1 = 6 \qquad \text{...(1)}$$

Applying KVL to Mesh 2,

$$42 - 12(I_2 - I_1) - 6\, I_2 = 0$$

$$-12\, I_1 + 18\, I_2 = 42 \qquad \text{...(2)}$$

Substituting value of I_1 in Eq. (2),

$$I_2 = 6.33 \text{ A}$$

Writing V_{Th} equation,

$$V_{Th} = 6\, I_2 = 38 \text{ V}$$

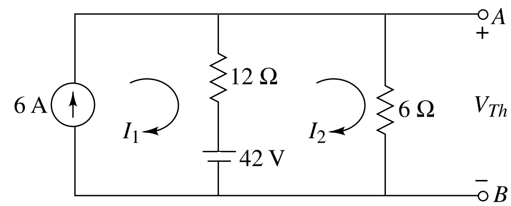

Fig. 3.88

Step II: *Calculation of R_{Th}*

Replacing voltage source by short circuit and current source by open circuit,

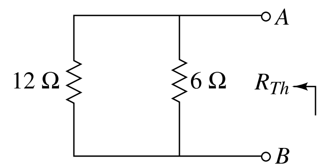

Fig. 3.89

$$R_{Th} = 6 \parallel 12 = 4\ \Omega$$

Step III: *Calculation of I_L*

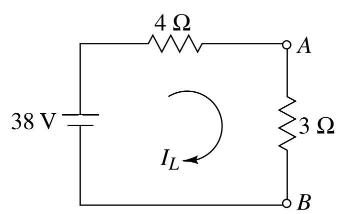

Fig. 3.90

$$I_L = \frac{38}{4+3} = 5.43 \text{ A}$$

Example 3.20 *Find the current through the 30-Ω resistor.*

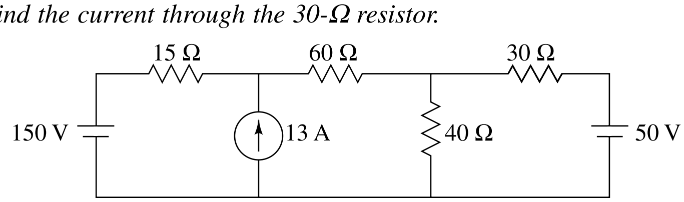

Fig. 3.91

Step I: *Calculation of V_{Th}*

Removing the 30-Ω resistor from the network,

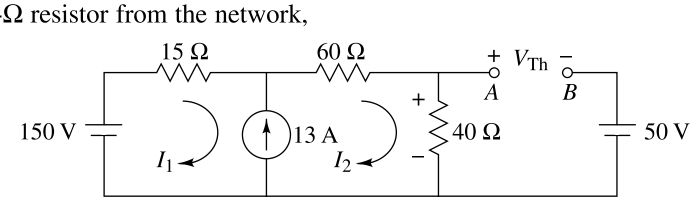

Fig. 3.92

Meshes 1 and 2 form a supermesh.

Writing current equation for supermesh,

$$I_2 - I_1 = 13 \qquad \text{...(1)}$$

Writing voltage equation for supermesh,

$$150 - 15I_1 - 60I_2 - 40I_2 = 0$$

$$15I_1 + 100I_2 = 150 \qquad \text{...(2)}$$

Solving Eqs (1) and (2),

$$I_1 = -10 \text{ A}$$

$$I_2 = 3 \text{ A}$$

Writing V_{Th} equation,

$$40I_2 - V_{Th} - 50 = 0$$

$$V_{Th} = 40I_2 - 50 = 40(3) - 50 = 70 \text{ V}$$

Step II: *Calculation of R_{Th}*

Replacing the voltage sources by short circuits and the current source by an open circuit,

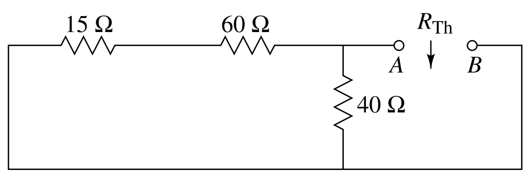

Fig. 3.93

$$R_{Th} = 75 \parallel 40 = 26.09\ \Omega$$

Step III: *Calculation of I_L*

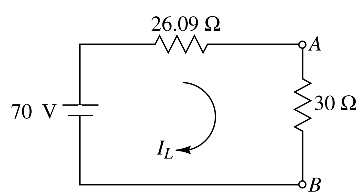

Fig. 3.94

$$I_L = \frac{70}{26.09 + 30} = 1.25 \text{ A}$$

3.6 MAXIMUM POWER TRANSFER THEOREM

It states that the maximum power is delivered from a source to a load when the load resistance is equal to the source resistance.

$$I = \frac{V}{R_S + R_L}$$

Power delivered to the load $R_L = P = I^2 R_L = \dfrac{V^2 R_L}{(R_S + R_L)^2}$

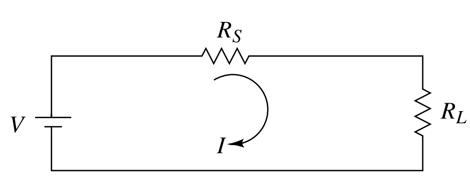

Fig. 3.176

To determine the value of R_L for maximum power to be transferred to the load,

$$\frac{dP}{dR_L} = 0$$

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\frac{V^2}{(R_S+R_L)^2}R_L$$

$$= \frac{V^2[(R_S+R_L)^2-(2R_L)(R_S+R_L)]}{(R_S+R_L)^4}$$

$$(R_S+R_L)^2 - 2R_L(R_S+R_L) = 0$$

$$R_S^2 + R_L^2 + 2R_S R_L - 2R_L R_S - 2R_L^2 = 0$$

$$R_S = R_L$$

Hence, the maximum power will be transferred to the load when load resistance is equal to the source resistance.

Steps to be followed in maximum power transfer theorem

1. Remove the variable load resistor R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B with voltage source and current source replaced by internal resistance.
4. Find the resistance R_L for maximum power transfer.
$$R_L = R_{Th}$$
5. Find the maximum power.

$$I_L = \frac{V_{Th}}{R_{Th}+R_L} = \frac{V_{Th}}{2R_{Th}}$$

$$P_{\max} = I_L^2 R_L$$

$$= \frac{V_{Th}^2}{4R_{Th}^2}\times R_{Th} = \frac{V_{Th}^2}{4R_{Th}}$$

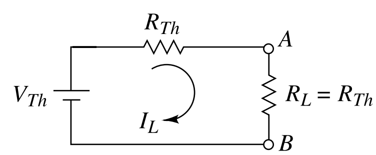

Fig. 3.177

Example 3.41 *For the circuit shown, find value of resistance R_L for maximum power and calculate maximum power.*

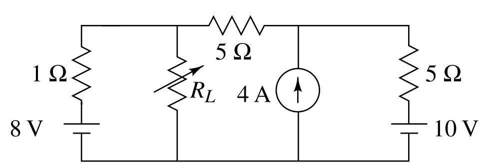

Fig. 3.178

Step I: *Calculation of V_{Th}*

Removing the variable resistor R_L from the network,

$$I_2 - I_1 = 4 \qquad ...(1)$$

Applying KVL to the outer path,

$$8 - I_1 - 5I_1 - 5I_2 - 10 = 0$$

$$-6I_1 - 5I_2 = 2 \qquad ...(2)$$

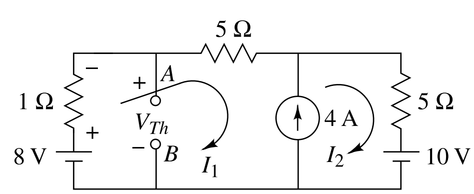

Fig. 3.179

Solving Eqs (1) and (2),

$$I_1 = -2 \text{ A}$$
$$I_2 = 2 \text{ A}$$

Writing V_{Th} equation,

$$8 - 1(I_1) - V_{Th} = 0$$
$$V_{Th} = 8 - I_1 = 8 - (-2) = 10 \text{ V}$$

Step II: *Calculation of R_{Th}*

Replacing the voltage sources by short circuits and current source by an open circuit,

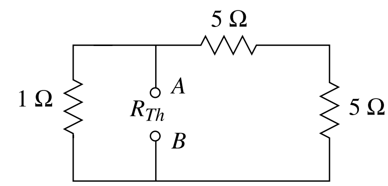

Fig. 3.180

$$R_{Th} = 10\ \Omega \parallel 1\ \Omega = 0.91\ \Omega$$

Step III: *Value of R_L*

For maximum power transfer,

$$R_L = R_{Th} = 0.91\ \Omega$$

Step IV: *Calculation of P_{max}*

$$P_{\max} = \frac{V_{Th}^{\;2}}{4R_{Th}}$$

$$= \frac{(10)^2}{4 \times 0.91} = 27.47 \text{ W}$$

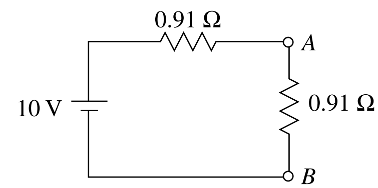

Fig. 3.181

Example 3.42 *For the circuit shown, find the value of the resistance R_L for maximum power and calculate the maximum power.*

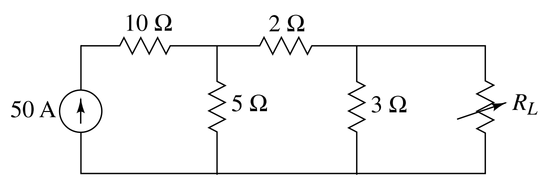

Fig. 3.182

Step I: *Calculation of V_{Th}*

Removing the variable resistor R_L from the circuit,

For Mesh 1,

$$I_1 = 50 \text{ A}$$

Applying KVL to Mesh 2,

$$-5\,(I_2 - I_1) - 2I_2 - 3I_2 = 0$$
$$5I_1 - 10I_2 = 0$$
$$I_1 = 2I_2$$
$$I_2 = 25 \text{ A}$$
$$V_{Th} = 3I_2 = 3(25) = 75 \text{ V}$$

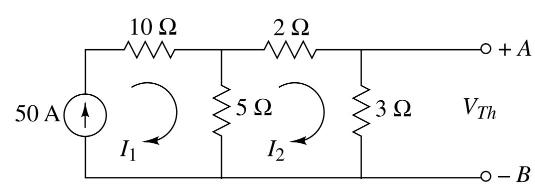

Fig. 3.183

Step II: *Calculation of R_{Th}*

Replacing the current source of 50 A with an open circuit,

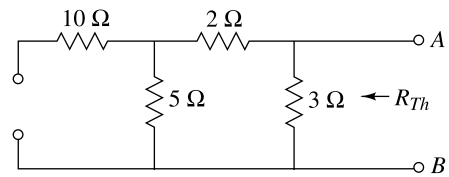

Fig. 3.184

$$R_{Th} = 7 \parallel 3 = 2.1\ \Omega$$

Step III: *Value of R_L*

For maximum power transfer,

$$R_L = R_{Th} = 2.1\ \Omega$$

Step IV: *Calculation of P_{max}*

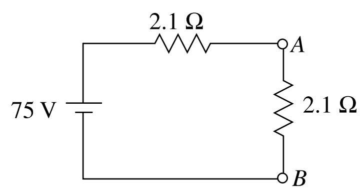

Fig. 3.185

$$P_{\max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(75)^2}{4 \times 2.1} = 669.64\ \text{W}$$

Example 3.43 *For the circuit shown, find value of resistance R_L for maximum power and calculate maximum power.*

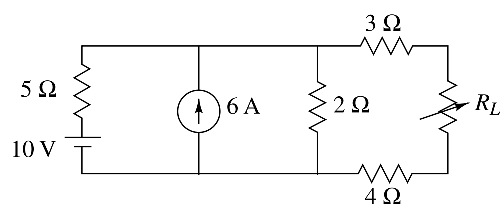

Fig. 3.186

Step I: *Calculation of V_{Th}*

Removing the variable resistor R_L from the circuit,

Writing the current equation for the supermesh,

$$I_2 - I_1 = 6 \quad \text{...(1)}$$

Applying KVL to the supermesh,

$$10 - 5I_1 - 2I_2 = 0$$

$$5I_1 + 2I_2 = 10 \quad \text{...(2)}$$

Solving equations (1) and (2),

$$I_1 = -0.29\ \text{A}$$

$$I_2 = 5.71\ \text{A}$$

Writing V_{Th} equation,

$$V_{Th} = 2I_2 = 11.42\ \text{V}$$

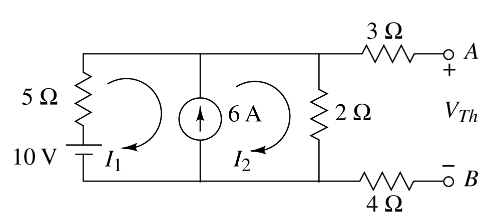

Fig. 3.187

Step II: *Calculation of* R_{Th}

Replacing the voltage source by a short circuit and the current source by an open circuit,

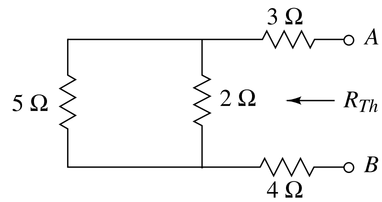

Fig. 3.188

$$R_{Th} = (5 \parallel 2) + 3 + 4 = 8.43\ \Omega$$

Step III: *Calculation of* R_L

For maximum power transfer

$$R_L = R_{Th} = 8.43\ \Omega$$

Step IV: *Calculation of* P_{max}

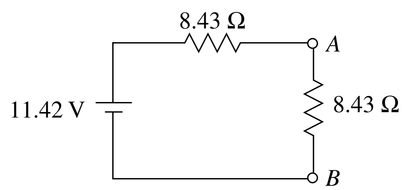

Fig. 3.189

$$P_{\max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(11.42)^2}{4 \times 8.43} = 3.87\ \text{W}$$