AC FUNDAMENTALS

Theory Questions

1 Define waveform, time period, frequency, amplitude, average value, RMS value, form factor and peak factor.

2 With a neat diagram, derive an expression for the relationship between voltage and current in a pure inductive (pure capacitive) circuit and show that the average power dissipated is zero. Draw the waveforms of Voltage, Current and Power.

3 Define power factor of a circuit and mention its importance. What are the power factors of pure resistance, pure inductance and pure capacitance.

4 With a neat diagram, derive an expression for the relationship between voltage and current in a series RL (series RC circuit) and show that the average power dissipated is VIcosΦ. Draw the waveforms of Voltage, Current and Power.

5 Show that current in a series RL (series RC) circuit lags (leads) the supply voltage with an appropriate vector diagram. Hence, derive an expression for power consumed in the circuit and also draw the waveforms of Voltage, Current and Power.

6 With a neat diagram, derive an expression for the relationship between voltage and current in a series RLC circuit for XL = XC case and show that the average power dissipated is VI. Draw the waveforms of Voltage, Current and Power.

7 With a neat diagram, derive an expression for the relationship between voltage and current in a series RLC circuit for XL < XC (XL > XC) case and show that the average power dissipated is VIcosΦ. Draw the waveforms of Voltage, Current and Power.

8 Draw the impedance triangle, voltage triangle and power triangle of a series RL, RC and RLC circuit.

Numericals

1. An alternating current ‘i’ is given by i=141.4sin(314t) A. find
   i) The maximum valure
   ii) Time Period
   iii) Instantaneous value when t= 3msec.

(Answer: 141.4A, 20msec, 114.39A)

2. The equation of an alternating current is i = 42.42sin(628t)A. Calculate its
   i) Maximum value,
   ii) Frequency,
   iii) rms value,
   iv) Average value and
   v) Form factor.

(Answer: 42.42A, 100Hz, 30A, 27A, 1.11)

3. An ac circuit consists of a pure resistance of 10Ω and is connected to an ac supply of 230V, 50Hz. Calculate the (i) current (ii) power consumed and iii) equations for voltage and current. (Answer: 23A, 5260W, V=325.25sin314t and I=32.52sin314t)

4. A pure inductive coil allows a current of 10A to flow from a 230V, 50Hz supply. Find (i) inductance of the coil (ii) power absorbed and (iii) equations for voltage and current. (Answer: 0.073H, 0W, V=325.25sin314t and I=14.14sin(314t- π/2))

5. A coil having a resistance of 7Ω and an inductance of 31.8mH is connected to 230V, 50Hz supply. Calculate (i) the circuit current (ii) phase angle (iii) power factor (iv) power consumed. (Answer: 18.85A, 55^0 lag, 0.573 lag, 2484.24W)

6. A capacitor of capacitance 79.5μF is connected in series with a non-inductive resistance of 30Ω across a 100V, 50Hz supply. Find (i) impedance (ii) current (iii) phase angle (iv) equation for the instantaneous value of current. (Answer: 50Ω, 2A, 53^0 lead, I=2.828sin(314t + 53^0))

7. A current of average value 18.019A is flowing in a circuit to which a voltage of peak value 141.42V is applied. Determine the impedance in polar form and power. Assume voltage lags current by 30^0.

   (Answer: Z= $5\angle -30^0$, P=1732.05W)

8. An AC voltage (80+j60)V is applied to a circuit. The current in the circuit is (10+j4)A. Find the impedance and power consumed. (Answer: Z=(8.96+j2.41)Ω, P=1040W)

9. In a series R-L circuit, voltage and current are expressed by e = 15sin(314t+5 π/6)V, i=5sin(314t+2 π/3)A. Find a) impedance, b) resistance, c) inductance, d) average power, e) power factor and f) voltage across R and X_L.

   (Answer: 3Ω, 2.6Ω,4.77mH, 32.5W, 0.866, 9.2V, 5.3V)

10. Resistor R in series with a capacitor C is connected to a 50Hz, 240V supply. Find the value of C so that R absorbs 300W at 100V. (Answer: 43.76μF)

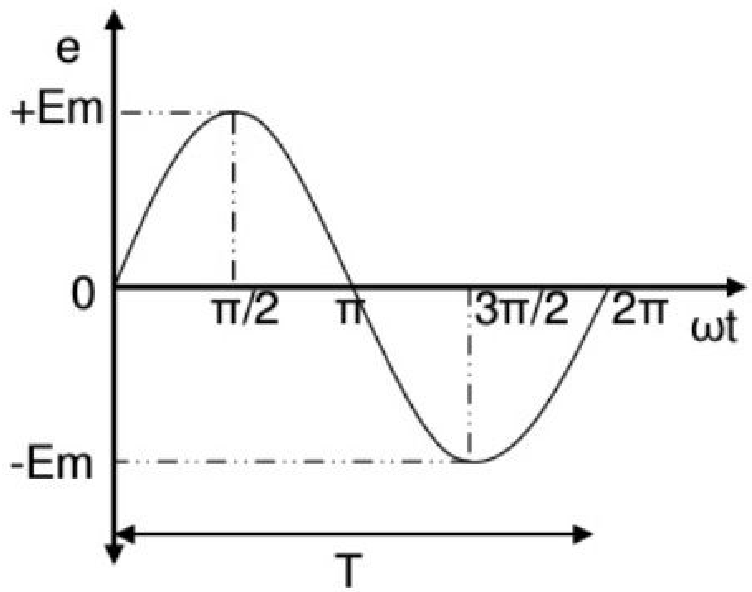
11. A circuit is made up of 10Ω resistance, 12mH inductance and 281.5μF capacitance in series. Supply voltage is 10V. calculate the value of current when supply frequency is (i) 50Hz, (ii) 150Hz.

12. Two circuits of impedance Z1 = (10+j15)Ω and Z2 = (6+j8)Ω are connected in parallel. If the total current is 15A, what will be the power taken by each branch. (Answer: 286W, 559W)

****END****

SINGLE PHASE AC CIRCUITS

Definition of Alternating Quantity



An alternating quantity changes continuously in magnitude and alternates in direction at regular intervals of time. Important terms associated with an alternating quantity are defined below.

1. Amplitude

It is the maximum value attained by an alternating quantity. Also called as maximum or peak value

2. Time Period (T)

It is the Time Taken in seconds to complete one cycle of an alternating quantity

3. Instantaneous Value

It is the value of the quantity at any instant

4. Frequency (f)

It is the number of cycles that occur in one second. The unit for frequency is Hz or cycles/sec. The relationship between frequency and time period can be derived as follows.

Time taken to complete f cycles = 1 second

Time taken to complete 1 cycle = 1/f second

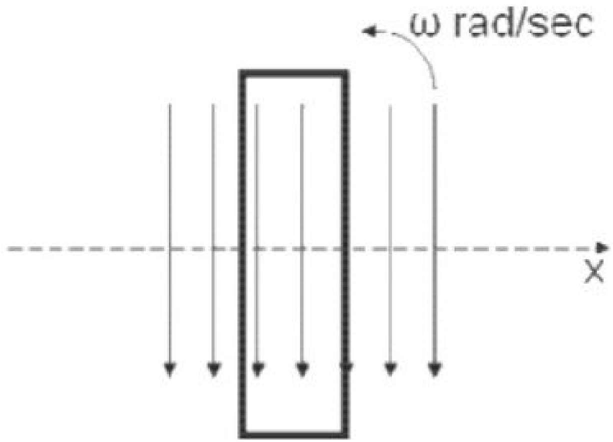
$$T = 1/f$$

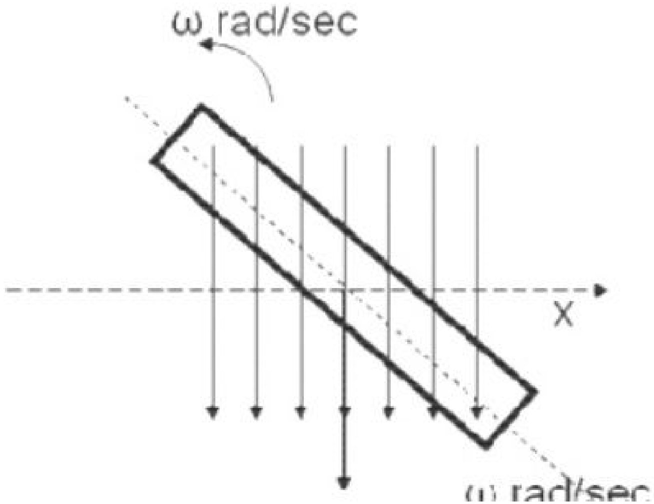
Advantages of AC system over DC system

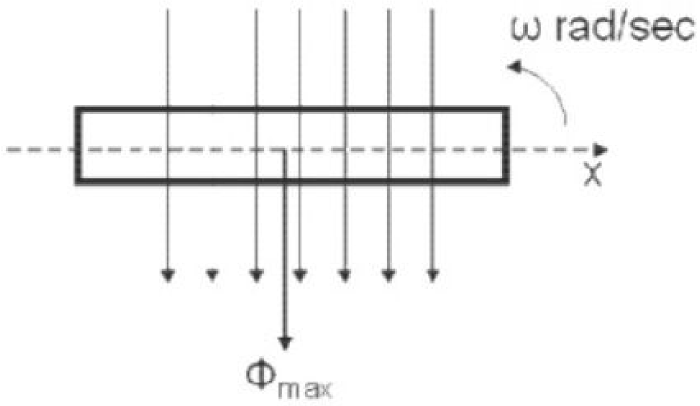
1. AC voltages can be efficiently stepped up/down using transformer
2. AC motors are cheaper and simpler in construction than DC motors
3. Switchgear for AC system is simpler than DC system

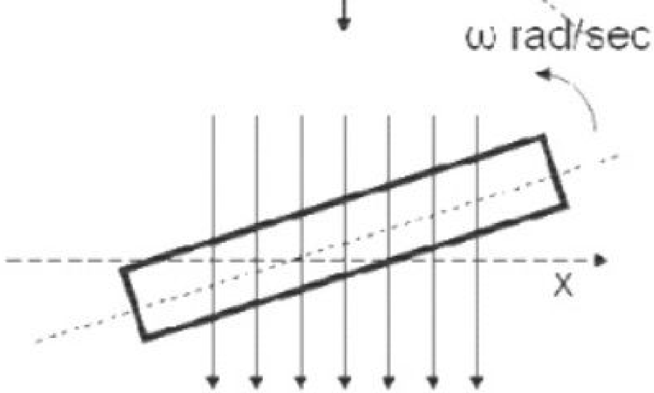
Generation of sinusoidal AC voltage

Consider a rectangular coil of N turns placed in a uniform magnetic field as shown in the figure. The coil is rotating in the anticlockwise direction at an uniform angular velocity of ω rad/sec.



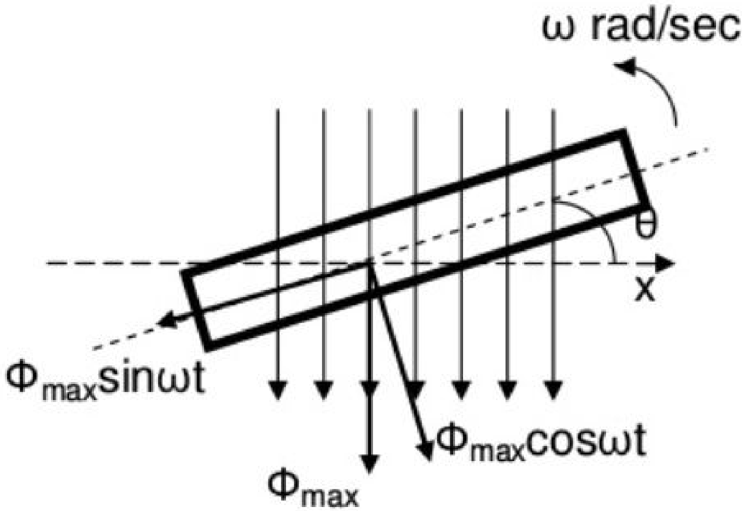






When the coil is in the vertical position, the flux linking the coil is zero because the plane of the coil is parallel to the direction of the magnetic field. Hence at this position, the emf induced in the coil is zero. When the coil moves by some angle in the anticlockwise direction, there is a rate of change of flux linking the coil and hence an emf is induced in the coil. When the coil reaches the horizontal position, the flux linking the coil is maximum, and hence the emf induced is also maximum. When the coil further moves in the anticlockwise direction, the emf induced in the coil reduces. Next when the coil comes to the vertical position, the emf induced becomes zero. After that the same cycle repeats and the emf is induced in the opposite direction. When the coil completes one complete revolution, one cycle of AC voltage is generated.

The generation of sinusoidal AC voltage can also be explained using mathematical equations. Consider a rectangular coil of N turns placed in a uniform magnetic field in the position shown in the figure. The maximum flux linking the coil is in the downward direction as shown in the figure. This flux can be divided into two components, one component acting along the plane of the coil $\Phi_{max}\sin\omega t$ and another component acting perpendicular to the plane of the coil $\Phi_{max}\cos\omega t$.



The component of flux acting along the plane of the coil does not induce any flux in the coil. Only the component acting perpendicular to the plane of the coil ie $\Phi_{max}\cos\omega t$ induces an emf in the coil.

$$\Phi = \Phi_{\max} \cos \omega t$$

$$e = -N \frac{d\Phi}{dt}$$

$$e = -N \frac{d}{dt} \Phi_{\max} \cos \omega t$$

$$e = N\Phi_{\max} \omega \sin \omega t$$

$$e = E_m \sin \omega t$$

Hence the emf induced in the coil is a sinusoidal emf. This will induce a sinusoidal current in the circuit given by

$$i = I_m \sin \omega t$$

Angular Frequency (ω)

Angular frequency is defined as the number of radians covered in one second(ie the angle covered by the rotating coil). The unit of angular frequency is rad/sec.

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Problem 1

An alternating current i is given by

i = 141.4 sin 314t

Find i) The maximum value

ii) Frequency

iii) Time Period

iv) The instantaneous value when t=3ms

i = 141.4 sin 314t

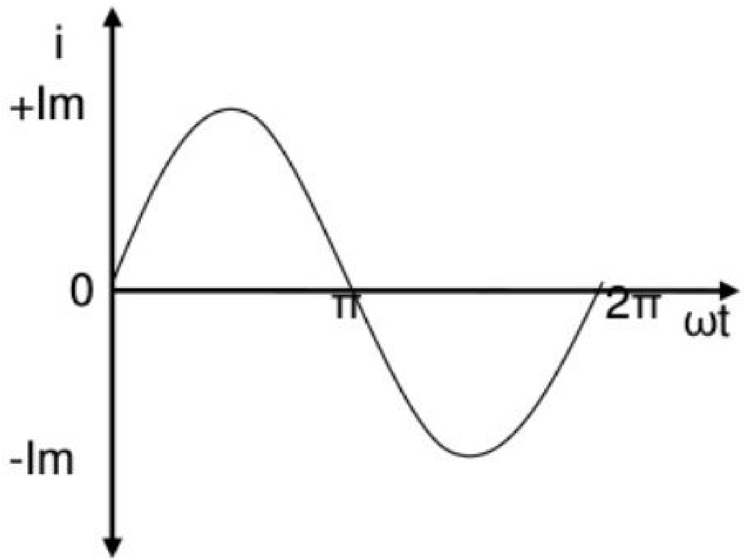
$$i = I_m \sin \omega t$$

i) Maximum value Im=141.4 V

Average value = $\dfrac{\text{Area under one half cycle}}{\text{Base}}$

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} v\,d(\omega t)$$

Average value of a sinusoidal current



$$i = I_m \sin \omega t$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i\,d(\omega t)$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} I_m \sin \omega t\,d(\omega t)$$

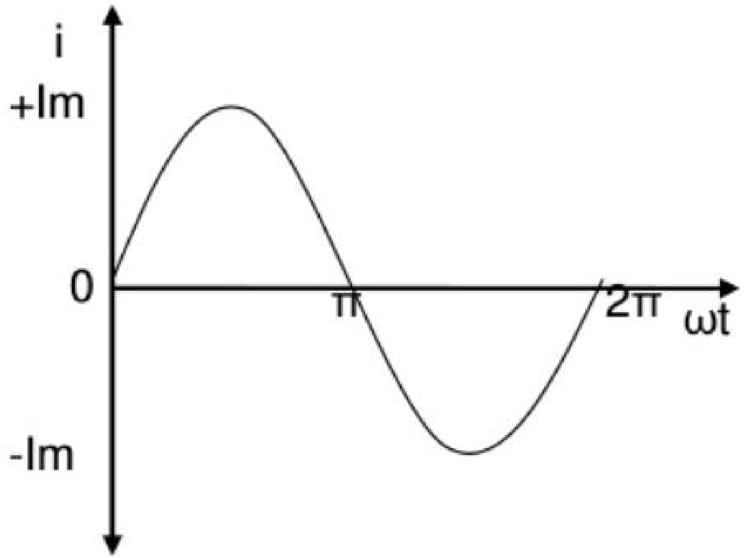
$$I_{av} = \frac{2I_m}{\pi} = 0.637\,I_m$$

The effective or RMS value of an alternating quantity is that steady current (dc) which when flowing through a given resistance for a given time produces the same amount of heat produced by the alternating current flowing through the same resistance for the same time.

$$RMS = \sqrt{\frac{\text{Area under squared curve}}{base}}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_{0}^{2\pi} v^2 d(\omega t)}$$

RMS value of a sinusoidal current



$$i = I_m \sin \omega t$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)}$$

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t d(\omega t)}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

Form Factor

The ratio of RMS value to the average value of an alternating quantity is known as Form Factor

$$FF = \frac{RMSValue}{AverageValue}$$

Peak Factor or Crest Factor

The ratio of maximum value to the RMS value of an alternating quantity is known as the peak factor

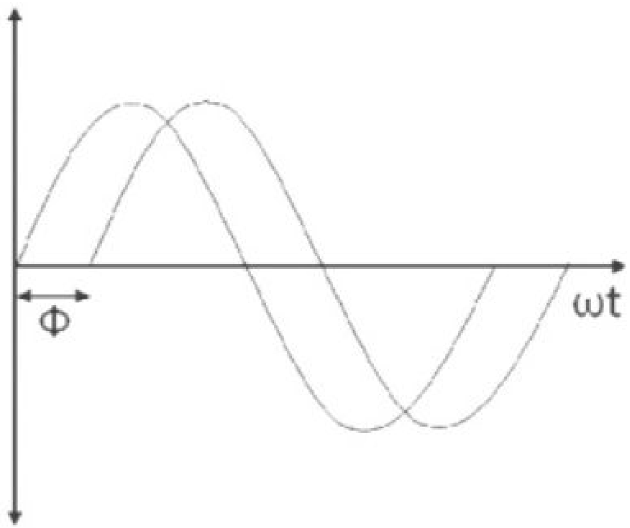
$$PF = \frac{MaximumValue}{RMSValue}$$

For a sinusoidal waveform

$$I_{av} = \frac{2I_m}{\pi} = 0.637 I_m$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

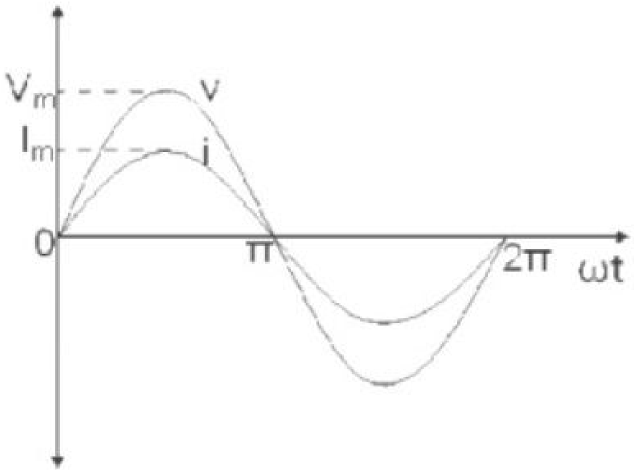
Phase Difference



When two alternating quantities of the same frequency have different zero points, they are said to have a phase difference. The angle between the zero points is the angle of phase difference.

In Phase

Two waveforms are said to be in phase, when the phase difference between them is zero. That is the zero points of both the waveforms are same. The waveform, phasor and equation representation of two sinusoidal quantities which are in phase is as shown. The figure shows that the voltage and current are in phase.

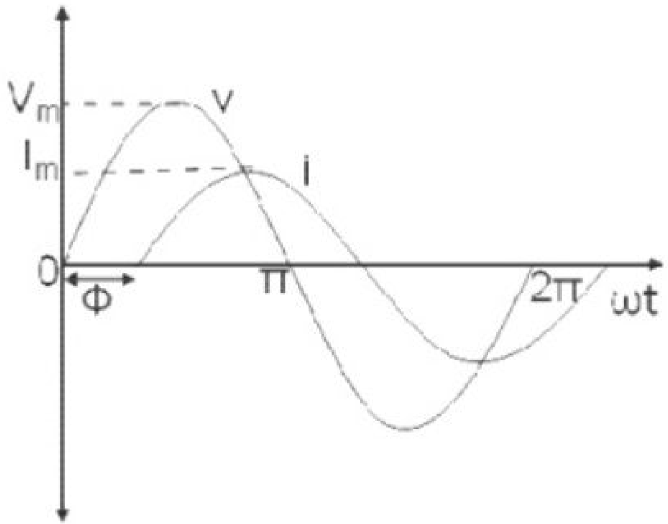


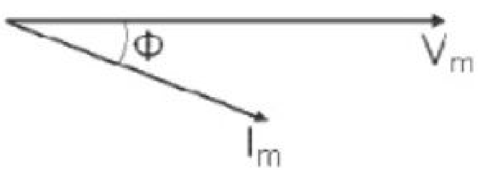
$$v = V_m \sin \omega t$$

$$i = I_m \sin \omega t$$

Lagging

In the figure shown, the zero point of the current waveform is after the zero point of the voltage waveform. Hence the current is lagging behind the voltage. The waveform, phasor and equation representation is as shown.





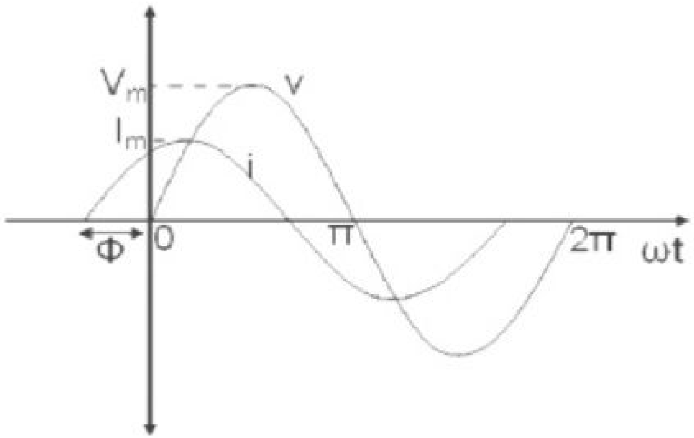
$$v = V_m \sin \omega t$$

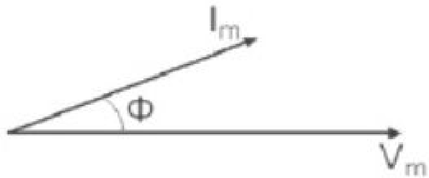
$$i = I_m \sin(\omega t - \Phi)$$

$$i = I_m \sin(\omega t - \Phi)$$

Leading

In the figure shown, the zero point of the current waveform is before the zero point of the voltage waveform. Hence the current is leading the voltage. The waveform, phasor and equation representation is as shown.

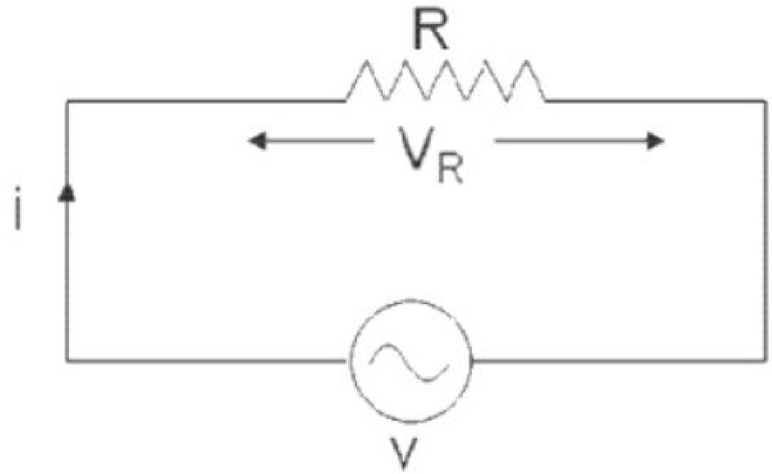




$$v = V_m \sin \omega t$$

$$i = I_m \sin(\omega t + \Phi)$$

AC circuit with a pure resistance



Consider an AC circuit with a pure resistance R as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t \qquad \text{---------- (1)}$$

The current flowing in the circuit is i. The voltage across the resistor is given as V_R which is the same as v.

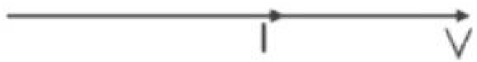
Using ohms law, we can write the following relations

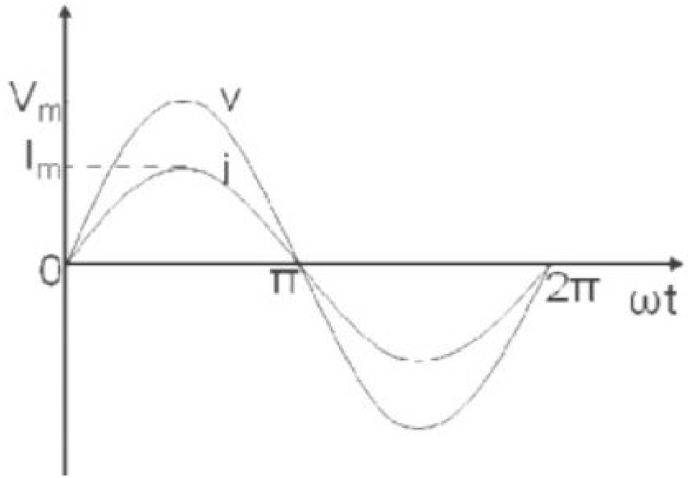
$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R}$$

$$i = I_m \sin \omega t \qquad \text{---------------(2)}$$

Where $\quad I_m = \dfrac{V_m}{R}$

From equation (1) and (2) we conclude that in a pure resistive circuit, the voltage and current are in phase. Hence the voltage and current waveforms and phasors can be drawn as below.





Instantaneous power

The instantaneous power in the above circuit can be derived as follows

$$p = vi$$

$$p = (V_m \sin \omega t)(I_m \sin \omega t)$$

$$p = V_m I_m \sin^2 \omega t$$

$$p = \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$p = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$$

The instantaneous power consists of two terms. The first term is called as the constant power term and the second term is called as the fluctuating power term.

Average power

From the instantaneous power we can find the average power over one cycle as follows

$$P = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t\right]d\omega t$$

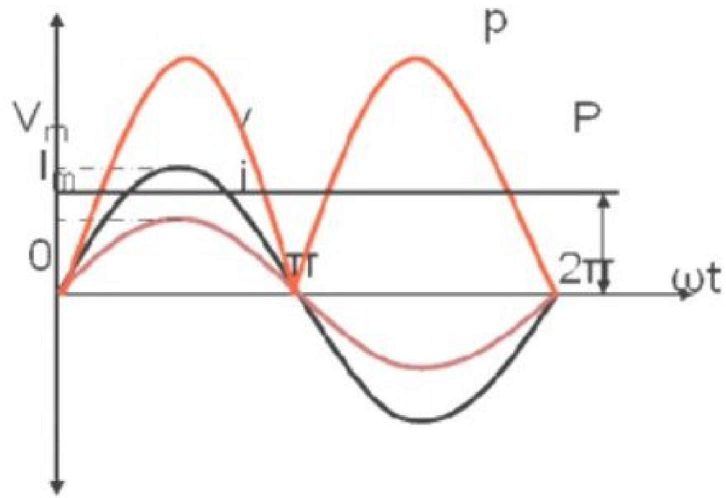
$$P = \frac{V_m I_m}{2} - \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2}\cos 2\omega t\right]d\omega t$$

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}$$

$$P = V.I$$

As seen above the average power is the product of the rms voltage and the rms current.

The voltage, current and power waveforms of a purely resistive circuit is as shown in the figure.



As seen from the waveform, the instantaneous power is always positive meaning that the power always flows from the source to the load.

Phasor Algebra for a pure resistive circuit

$$\overline{V} = V\angle 0^\circ = V + j0$$

$$\overline{I} = \frac{\overline{V}}{R} = \frac{V + j0}{R} = I + j0 = I\angle 0^\circ$$

Problem 2

An ac circuit consists of a pure resistance of 10Ω and is connected to an ac supply of 230 V, 50 Hz. Calculate the (i) current (ii) power consumed and (iii) equations for voltage and current.

$$(i) I = \frac{V}{R} = \frac{230}{10} = 23A$$

$$(ii) P = VI = 230 \times 23 = 5260W$$

$$(iii) V_m = \sqrt{2}V = 325.27V$$

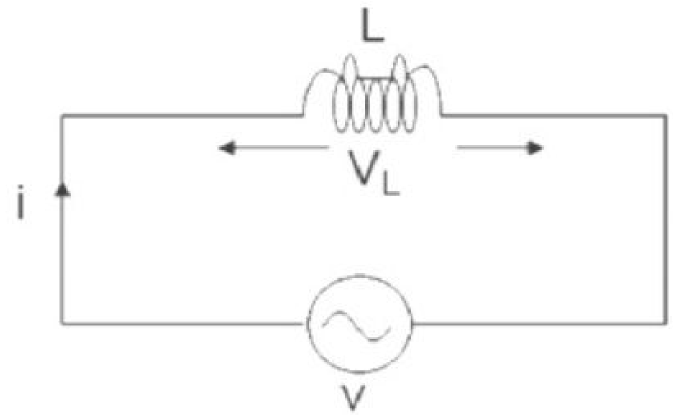
$$I_m = \sqrt{2}I = 32.52A$$

$$\omega = 2\pi f = 314 rad / \sec$$

$$v = 325.25 \sin 314t$$

$$i = 32.52 \sin 314t$$

AC circuit with a pure inductance



Consider an AC circuit with a pure inductance L as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t \qquad \text{----------} \ (1)$$

The current flowing in the circuit is i. The voltage across the inductor is given as V_L which is the same as v.

We can find the current through the inductor as follows

$$v = L\frac{di}{dt}$$

$$v = L\frac{di}{dt}$$

$$V_m \sin \omega t = L\frac{di}{dt}$$

$$di = \frac{V_m}{L}\sin \omega t dt$$

$$i = \frac{V_m}{L}\int \sin \omega t dt$$

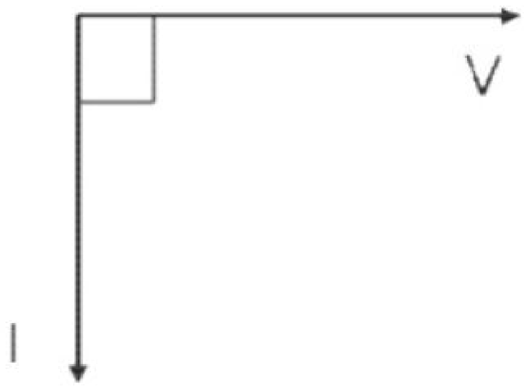
$$i = \frac{V_m}{\omega L}(-\cos \omega t)$$

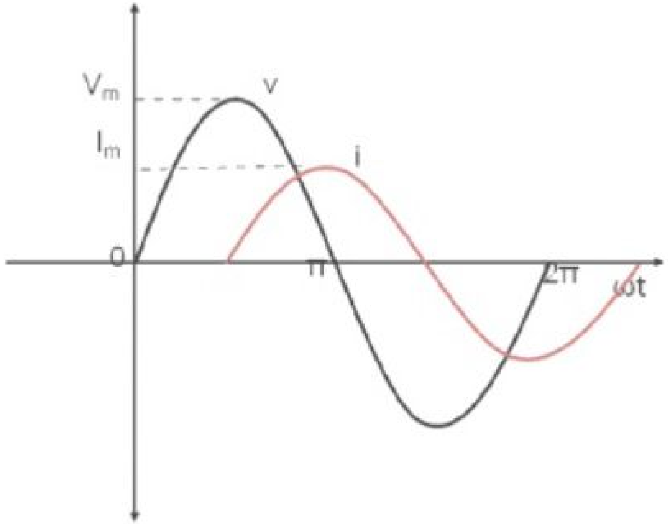
$$i = \frac{V_m}{\omega L}\sin(\omega t - \pi/2)$$

$$i = I_m \sin(\omega t - \pi/2) \qquad \text{-------------------(2)}$$

Where $I_m = \frac{V_m}{\omega L}$

From equation (1) and (2) we observe that in a pure inductive circuit, the current lags behind the voltage by 90°. Hence the voltage and current waveforms and phasors can be drawn as below.





Inductive reactance

The inductive reactance X_L is given as

$$X_L = \omega L = 2\pi f L$$

$$I_m = \frac{V_m}{X_L}$$

It is equivalent to resistance in a resistive circuit. The unit is ohms (Ω)

Instantaneous power

The instantaneous power in the above circuit can be derived as follows

$$p = vi$$

$$p = (V_m \sin \omega t)(I_m \sin(\omega t - \pi / 2))$$

$$p = -V_m I_m \sin \omega t \cos \omega t$$

$$p = -\frac{V_m I_m}{2} \sin 2\omega t$$

As seen from the above equation, the instantaneous power is fluctuating in nature.

Average power

From the instantaneous power we can find the average power over one cycle as follows

$$P = \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2}\sin 2\omega t\, d\omega t$$

$$P = 0$$

The average power in a pure inductive circuit is zero. Or in other words, the power consumed by a pure inductance is zero.

The voltage, current and power waveforms of a purely inductive circuit is as shown in the figure.

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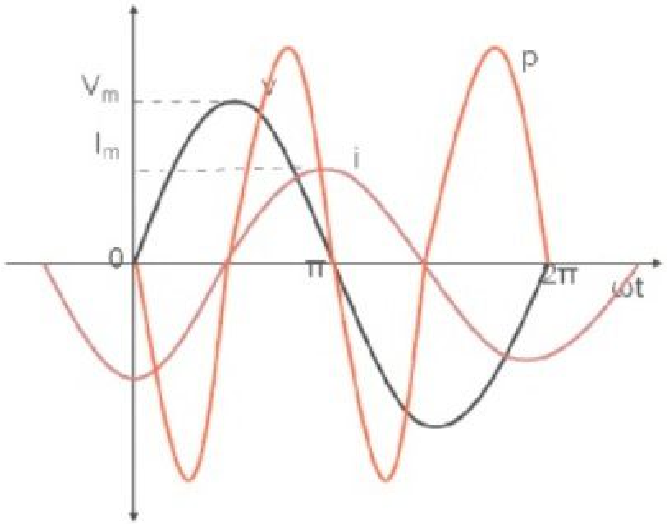
The voltage, current and power waveforms of a purely inductive circuit is as shown in the figure.



As seen from the power waveform, the instantaneous power is alternately positive and negative. When the power is positive, the power flows from the source to the inductor and when the power in negative, the power flows from the inductor to the source. The positive power is equal to the negative power and hence the average power in the circuit is equal to zero. The power just flows between the source and the inductor, but the inductor does not consume any power.

As seen from the power waveform, the instantaneous power is alternately positive and negative. When the power is positive, the power flows from the source to the inductor and when the power in negative, the power flows from the inductor to the source. The positive power is equal to the negative power and hence the average power in the circuit is equal to zero. The power just flows between the source and the inductor, but the inductor does not consume any power.

Phasor algebra for a pure inductive circuit

$$\overline{V} = V\angle 0^\circ = V + j0$$

$$\overline{I} = I\angle -90^\circ = 0 - jI$$

$$\frac{\overline{V}}{\overline{I}} = \frac{V\angle 0^\circ}{I\angle -90} = X_L\angle 90^\circ$$

$$\overline{V} = \overline{I}(jX_L)$$

Problem 3

A pure inductive coil allows a current of 10A to flow from a 230V, 50 Hz supply. Find (i) inductance of the coil (ii) power absorbed and (iii) equations for voltage and current.

$$(i) X_L = \frac{V}{I} = \frac{230}{10} = 23\Omega$$

$$X_L = 2\pi f L$$

$$L = \frac{X_L}{2\pi f} = 0.073H$$

$$(ii) P = 0$$

$$(iii) V_m = \sqrt{2}V = 325.27V$$

$$I_m = \sqrt{2}I = 14.14A$$

$$(i) X_L = \frac{V}{I} = \frac{230}{10} = 23\Omega$$

$$X_L = 2\pi f L$$

$$L = \frac{X_L}{2\pi f} = 0.073H$$

$$(ii) P = 0$$

$$(iii) V_m = \sqrt{2}V = 325.27V$$

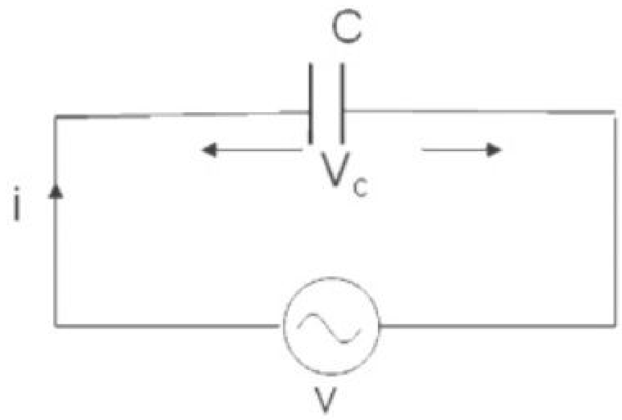
$$I_m = \sqrt{2}I = 14.14A$$

$$\omega = 2\pi f = 314 rad / \sec$$

$$v = 325.25 \sin 314t$$

$$i = 14.14 \sin(314t - \pi / 2)$$

AC circuit with a pure capacitance



Consider an AC circuit with a pure capacitance C as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t \qquad \text{---------- (1)}$$

The current flowing in the circuit is i. The voltage across the capacitor is given as V_C which is the same as v.

We can find the current through the capacitor as follows

$$q = Cv$$

$$q = CV_m \sin \omega t$$

$$i = \frac{dq}{dt}$$

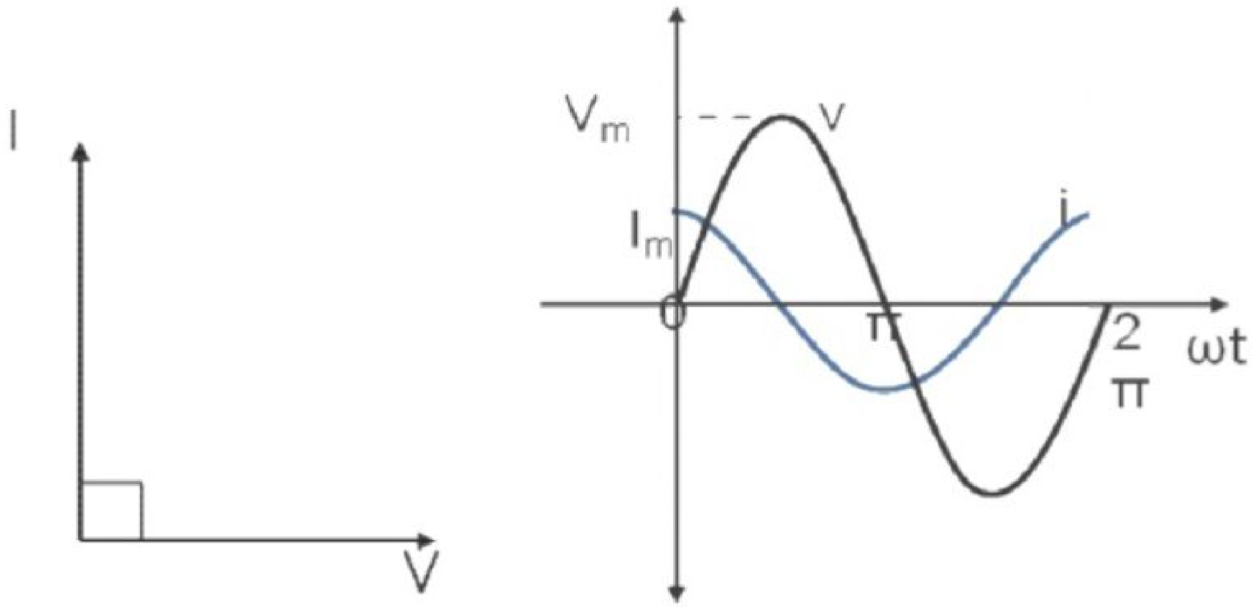
$$i = CV_m \omega \cos \omega t$$

$$i = \omega CV_m \sin(\omega t + \pi / 2)$$

$$i = I_m \sin(\omega t + \pi / 2) \qquad \text{-----------------(2)}$$

Where $I_m = \omega CV_m$

From equation (1) and (2) we observe that in a pure capacitive circuit, the current leads the voltage by 90°. Hence the voltage and current waveforms and phasors can be drawn as below.



Capacitive reactance

The capacitive reactance X_C is given as

$$X_L = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$I_m = \frac{V_m}{X_C}$$

It is equivalent to resistance in a resistive circuit. The unit is ohms (Ω)

Instantaneous power

The instantaneous power in the above circuit can be derived as follows

$$p = vi$$

$$p = (V_m \sin \omega t)(I_m \sin(\omega t + \pi / 2))$$

$$p = V_m I_m \sin \omega t \cos \omega t$$

$$p = \frac{V_m I_m}{2} \sin 2\omega t$$

Average power

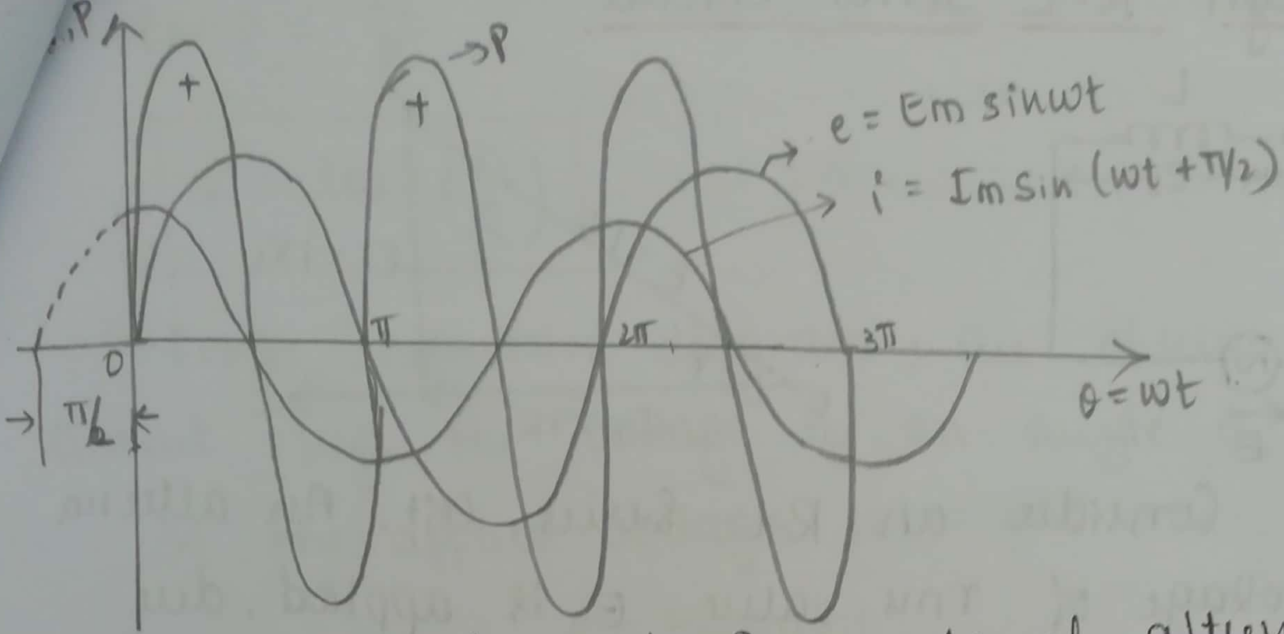
From the instantaneous power we can find the average power over one cycle as follows

$$P = \frac{1}{2\pi}\int_0^{2\pi}\frac{V_m I_m}{2}\sin 2\omega t\, d\omega t$$

$$P = 0$$

The average power in a pure capacitive circuit is zero. Or in other words, the power consumed by a pure capacitance is zero.

The voltage, current and power waveforms of a purely capacitive circuit is as shown in the figure.



The waveform of p consists of alternate lobes of +ve & -ve power whose areas are equal whose average value is also zero. Hence a pure capacitor does not consume any power.

Average power :

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} p.d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{P_m}{2}\sin 2\theta \, d\theta.$$

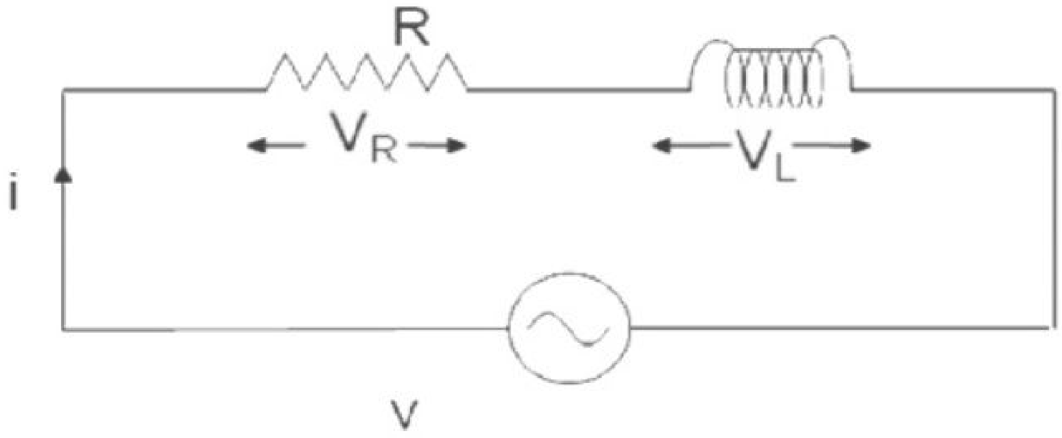
$$= \frac{P_m}{4\pi}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{-P_m}{8\pi}\left[\cos 4\pi - \cos 0\right]$$

$$= -\frac{P_m}{8\pi}[0]$$

$$\boxed{P_{av} = 0.}$$

R-L Series circuit



Consider an AC circuit with a resistance R and an inductance L connected in series as shown in the figure. The alternating voltage v is given by

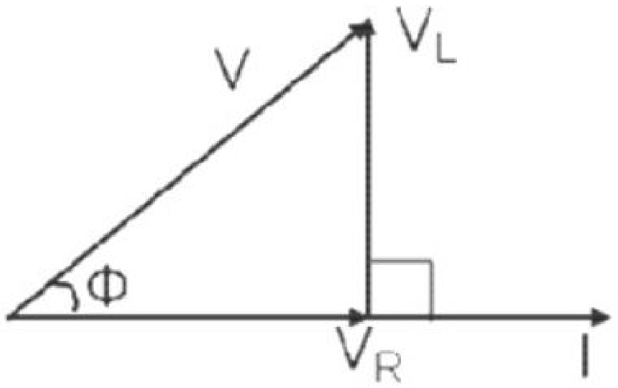
$$v = V_m \sin \omega t$$

The current flowing in the circuit is i. The voltage across the resistor is V_R and that across the inductor is V_L.

V_R=IR is in phase with I

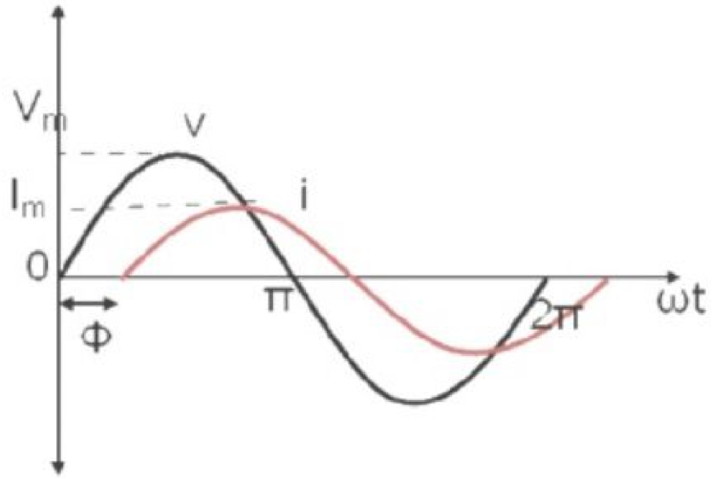
V_L=IX_L leads current by 90 degrees

With the above information, the phasor diagram can be drawn as shown.



The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_L leads the current by 90^0. The resultant voltage V can be drawn as shown in the figure. From the phasor diagram we observe that the voltage leads the current by an angle Φ or in other words the current lags behind the voltage by an angle Φ.

The waveform and equations for an RL series circuit can be drawn as below.



$$V = V_m \sin \omega t$$

$$I = I_m \sin(\omega t - \Phi)$$

From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + V_L^2}$$

From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + V_L^2}$$

$$V_R = IR$$

$$V_L = IX_L$$

$$V = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2}$$

$$V = IZ$$

Where impedance $Z = \sqrt{R^2 + X_L^2}$

The impedance in an AC circuit is similar to a resistance in a DC circuit. The unit for impedance is ohms (Ω).

$$\Phi = \tan^{-1}\left(\frac{V_L}{V_R}\right)$$

$$\Phi = \tan^{-1}\left(\frac{IX_L}{IR}\right)$$

$$\Phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\Phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Instantaneous power

The instantaneous power in an RL series circuit can be derived as follows

$$p = vi$$

$$p = (V_m \sin \omega t)(I_m \sin(\omega t - \Phi)$$

$$p = \frac{V_m I_m}{2} \cos \Phi - \frac{V_m I_m}{2} \cos(2\omega t - \Phi)$$

The instantaneous power consists of two terms. The first term is called as the constant power term and the second term is called as the fluctuating power term.

Average power

From the instantaneous power we can find the average power over one cycle as follows

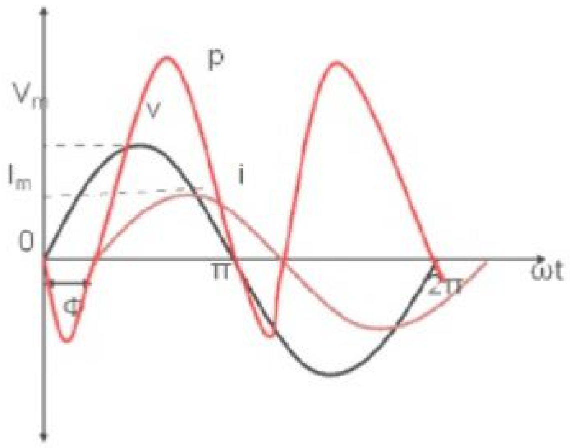
$$P = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2}\cos\Phi - \frac{V_m I_m}{2}\cos(2\omega t - \Phi)\right]d\omega t$$

$$P = \frac{V_m I_m}{2}\cos\Phi$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\Phi$$

$$P = VI\cos\Phi$$

The voltage, current and power waveforms of a RL series circuit is as shown in the figure.



As seen from the power waveform, the instantaneous power is alternately positive and negative. When the power is positive, the power flows from the source to the load and when the power in negative, the power flows from the load to the source. The positive power is not equal to the negative power and hence the average power in the circuit is not equal to zero.

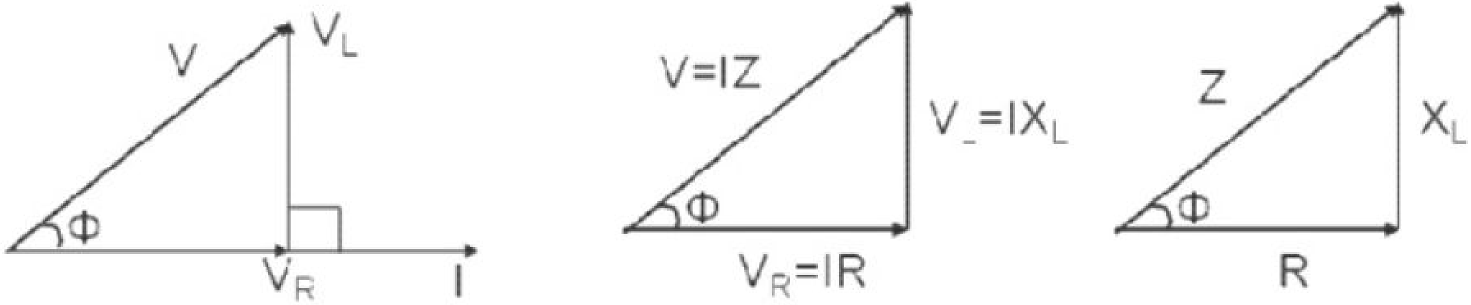
From the phasor diagram,

$$\cos\Phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

$$P = VI\cos\Phi$$

Impedance Triangle

We can derive a triangle called the impedance triangle from the phasor diagram of an RL series circuit as shown



The impedance triangle is right angled triangle with R and X_L as two sides and impedance as the hypotenuse. The angle between the base and hypotenuse is Φ. The impedance triangle enables us to calculate the following things.

calculate the following things.

1. Impedance $Z = \sqrt{R^2 + X_L^2}$

2. Power Factor $\cos\Phi = \dfrac{R}{Z}$

3. Phase angle $\Phi = \tan^{-1}\left(\dfrac{X_L}{R}\right)$

4. Whether current leads or lags behind the voltage

Power

In an AC circuit, the various powers can be classified as

1. Real or Active power
2. Reactive power
3. Apparent power

Real or active power in an AC circuit is the power that does useful work in the cicuit. Reactive power flows in an AC circuit but does not do any useful work. Apparent power is the total power in an AC circuit.

Real Power

The power due to the active component of current is called as the active power or real power. It is denoted by P.

$P = V \times I\mathrm{Cos}\Phi = I^2R$

Real power is the power that does useful power. It is the power that is consumed by the resistance. The unit for real power in Watt(W).

Reactive Power

The power due to the reactive component of current is called as the reactive power. It is denoted by Q.

$Q = V \times I\mathrm{Sin}\Phi = I^2X_L$

Reactive power does not do any useful work. It is the circulating power in th L and C components. The unit for reactive power is Volt Amperes Reactive (VAR).

Apparent Power

The apparent power is the total power in the circuit. It is denoted by S.

$S = V \times I = I^2Z$

$$S = \sqrt{P^2 + Q^2}$$

The apparent power is the total power in the circuit. It is denoted by S.

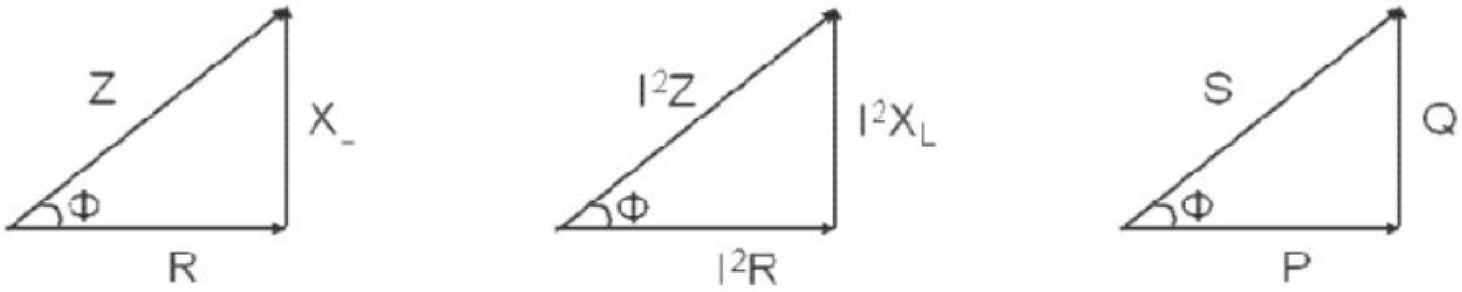
$$S = V \times I = I^2Z$$

$$S = \sqrt{P^2 + Q^2}$$

The unit for apparent power is Volt Amperes (VA).

Power Triangle

From the impedance triangle, another triangle called the power triangle can be derived as shown.



The apparent power is the total power in the circuit. It is denoted by S.

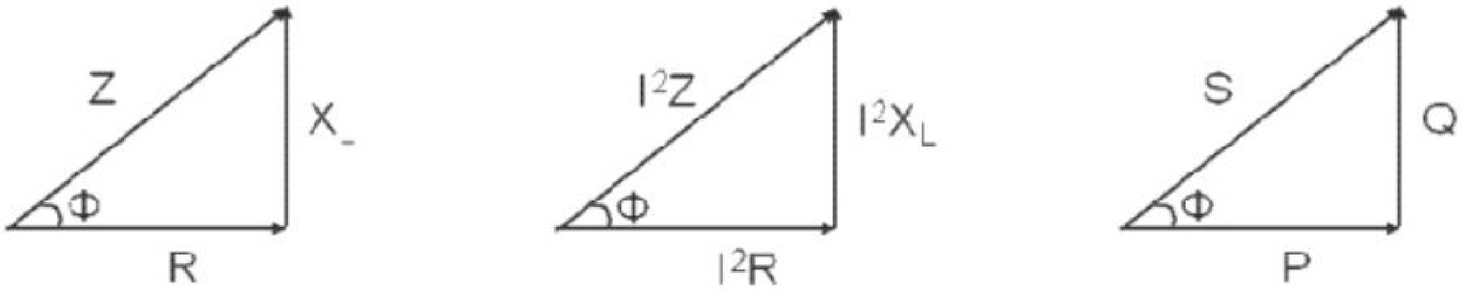
$$S = V \times I = I^2Z$$

$$S = \sqrt{P^2 + Q^2}$$

The unit for apparent power is Volt Amperes (VA).

Power Triangle

From the impedance triangle, another triangle called the power triangle can be derived as shown.



Phasor algebra in a RL series circuit

$$V = V + j0 = V\angle 0^\circ$$

$$\bar{Z} = R + jX_L = Z\angle\Phi$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V}{Z}\angle -\Phi$$

$$\bar{S} = VI^* = P + jQ$$

Problem 5

A coil having a resistance of 7Ω and an inductance of 31.8mH is connected to 230V, 50Hz supply. Calculate (i) the circuit current (ii) phase angle (iii) power factor (iv) power consumed

$$X_L = 2\pi fL = 2\times3.14\times50\times31.8\times10^{-3} = 10\Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{7^2 + 10^2} = 12.2\Omega$$

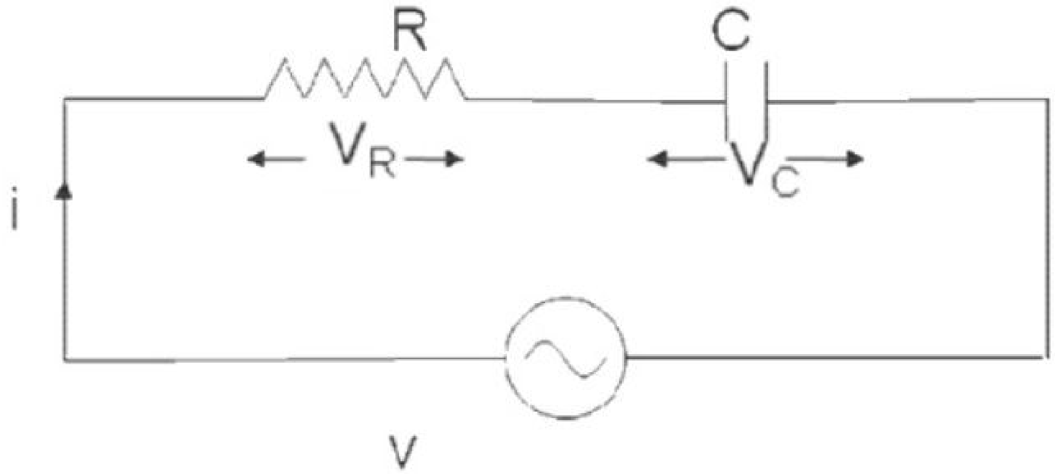
$$(i) I = \frac{V}{Z} = \frac{230}{12.2} = 18.85A$$

$$(ii)\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{10}{7}\right) = 55^\circ\, lag$$

$$(iii) PF = \cos\Phi = \cos(55^\circ) = 0.573 lag$$

$$(iv) P = VI\cos\Phi = 230\times18.85\times0.573 = 2484.24W$$

R-C Series circuit



Consider an AC circuit with a resistance R and a capacitance C connected in series as shown in the

Consider an AC circuit with a resistance R and a capacitance C connected in series as shown in the figure. The alternating voltage v is given by

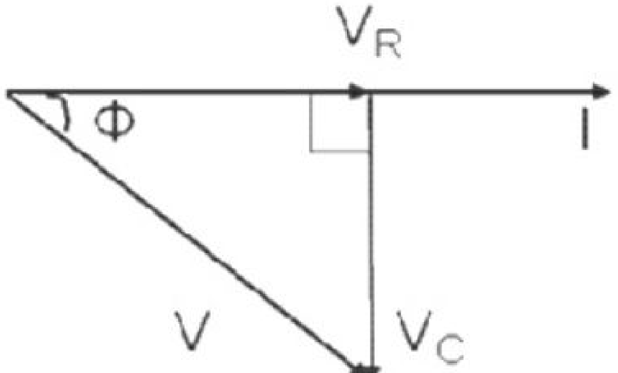
$$v = V_m \sin \omega t$$

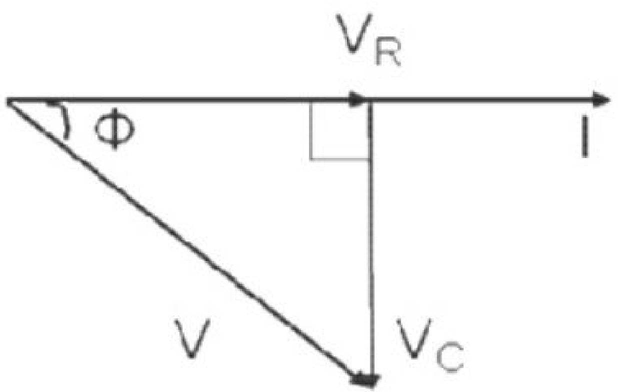
The current flowing in the circuit is i. The voltage across the resistor is V_R and that across the capacitor is V_C.

V_R=IR is in phase with I

V_C=IX_C lags behind the current by 90 degrees

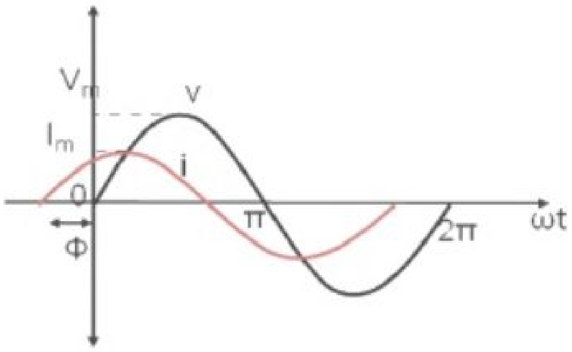
With the above information, the phasor diagram can be drawn as shown.





The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_C lags behind the current by 90^o. The resultant voltage V can be drawn as shown in the figure. From the phasor diagram we observe that the voltage lags behind the current by an angle Φ or in other words the current leads the voltage by an angle Φ.

The waveform and equations for an RC series circuit can be drawn as below.



$$V = V_m \sin \omega t$$

$$I = I_m \sin(\omega t + \Phi)$$

$$V = \sqrt{V_R^2 + V_C^2}$$

$$V_R = IR$$

$$V_C = IX_C$$

$$V = \sqrt{(IR)^2 + (IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C^2}$$

$$V = IZ$$

Where impedance $Z = \sqrt{R^2 + X_C^2}$

Phase angle

$$\Phi = \tan^{-1}\left(\frac{V_C}{V_R}\right)$$

$$\Phi = \tan^{-1}\left(\frac{IX_C}{IR}\right)$$

$$\Phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

Average power

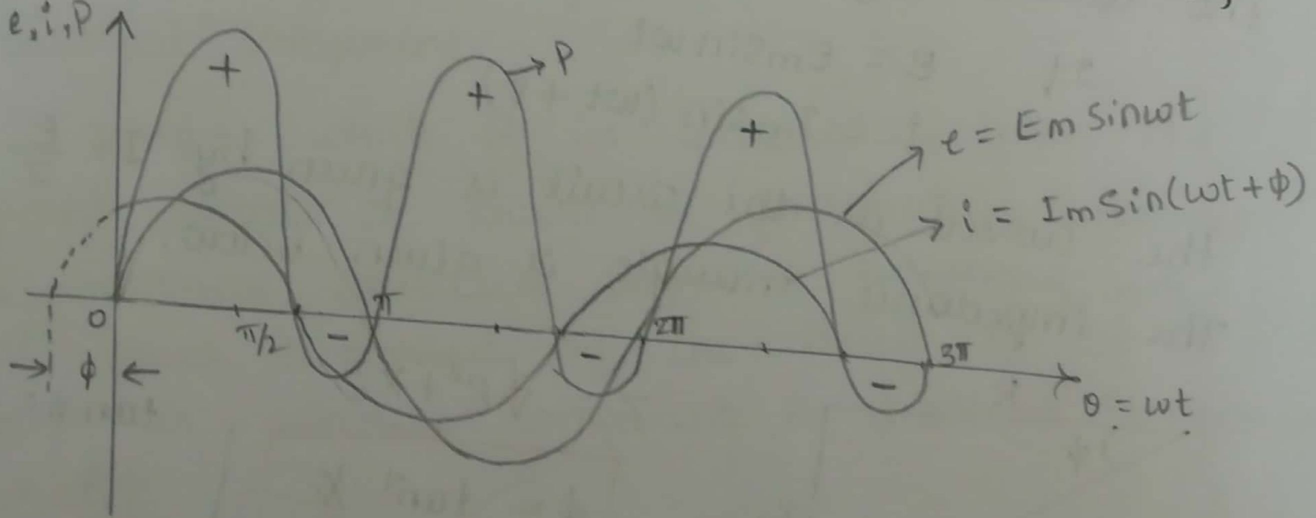
$$P = VI\cos\phi$$

$$P = (IZ)\times I\times\frac{R}{Z}$$

$$P = I^2R$$

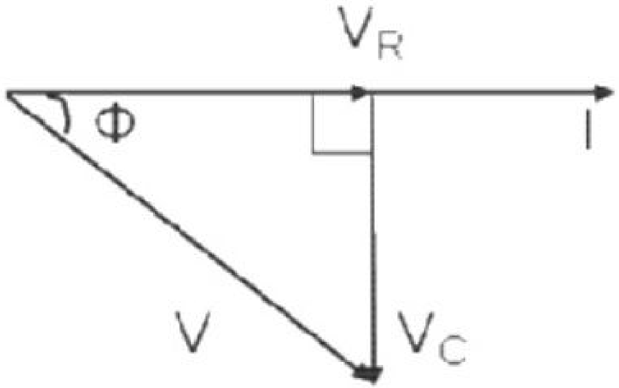
Hence the power in an RC series circuit is consumed only in the resistance. The capacitance does not consume any power.

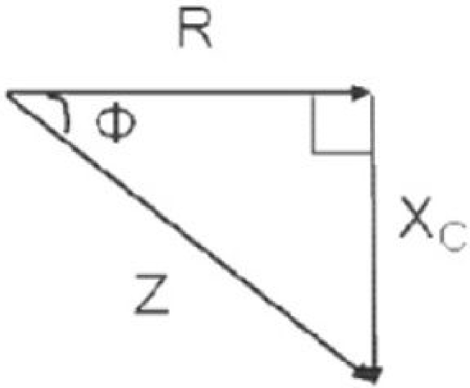
The waveform of e, i & P is shown below,



Impedance Triangle

We can derive a triangle called the impedance triangle from the phasor diagram of an RC series circuit as shown





Phasor algebra for RC series circuit

$$V = V + j0 = V\angle 0^{\circ}$$

$$\bar{Z} = R - jX_C = Z\angle -\Phi$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V}{Z}\angle +\Phi$$

Problem 7

A Capacitor of capacitance 79.5μF is connected in series with a non inductive resistance of 30Ω across a 100V, 50Hz supply. Find (i) impedance (ii) current (iii) phase angle (iv) Equation for the instantaneous value of current

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times 3.14\times 50\times 79.5\times 10^{-6}} = 40\Omega$$

$$(i) Z = \sqrt{R^2 + X_C^2} = \sqrt{30^2 + 40^2} = 50\Omega$$

$$(ii) I = \frac{V}{Z} = \frac{100}{50} = 2A$$

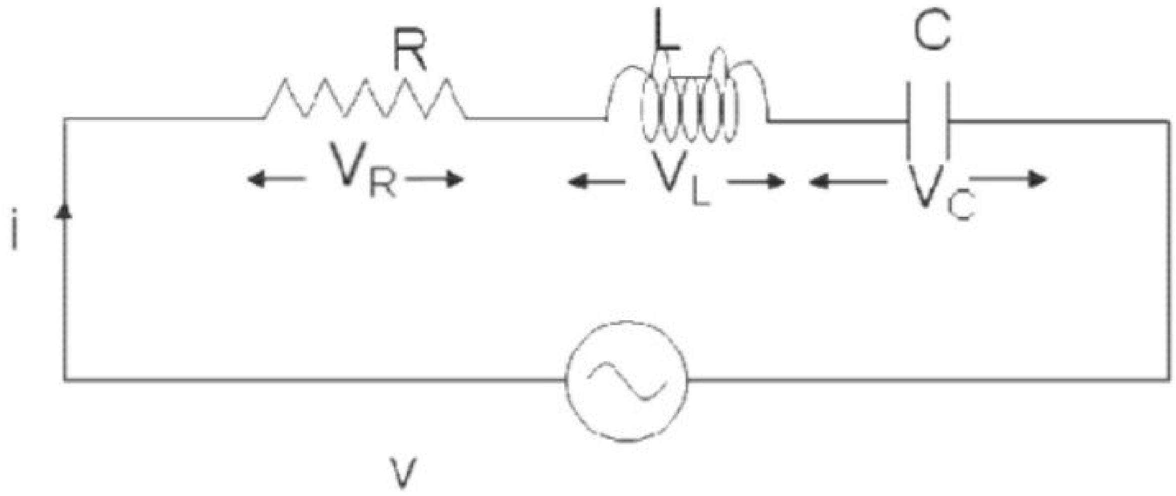
$$(iii) \Phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{40}{30}\right) = 53^\circ \, lead$$

$$(iv) I_m = \sqrt{2} I = \sqrt{2}\times 2 = 2.828A$$

$$\omega = 2\pi f = 2\times 3.14\times 50 = 314 rad/\sec$$

$$i = 2.828 \sin(314t + 53^\circ)$$

R-L-C Series circuit



Consider an AC circuit with a resistance R, an inductance L and a capacitance C connected in series as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t$$

$$v = V_m \sin \omega t$$

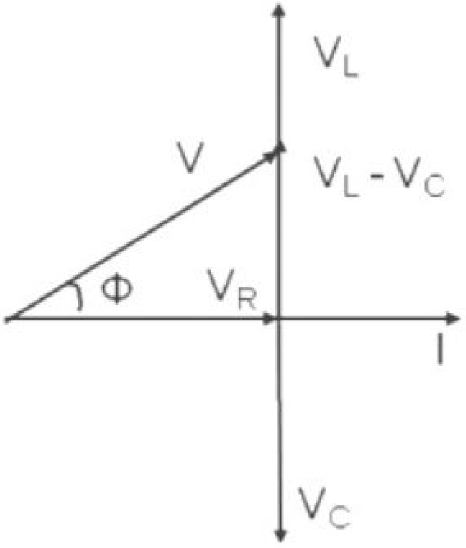
The current flowing in the circuit is i. The voltage across the resistor is V_R, the voltage across the inductor is V_L and that across the capacitor is V_C.

V_R=IR is in phase with I

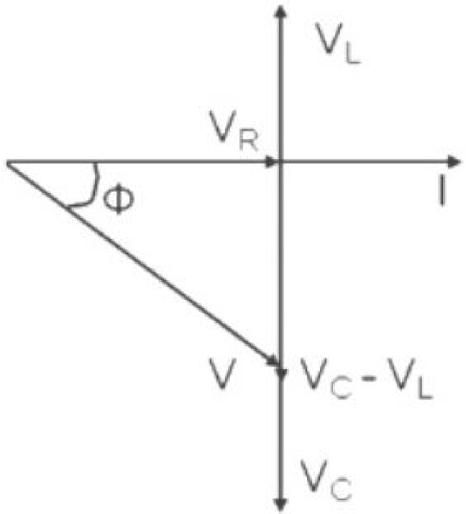
V_L=IX_L leads the current by 90 degrees

V_C=IX_C lags behind the current by 90 degrees

With the above information, the phasor diagram can be drawn as shown. The current I is taken as the reference phasor. The voltage V_R is in phase with I, the voltage V_L leads the current by 90^o and the voltage V_C lags behind the current by 90^o. There are two cases that can occur $V_L>V_C$ and $V_L<V_C$ depending on the values of X_L and X_C. And hence there are two possible phasor diagrams. The phasor V_L-V_C or V_C-V_L is drawn and then the resultant voltage V is drawn.



$V_L > V_C$



$V_L < V_C$

From the phasor diagram we observe that when $V_L > V_C$, the voltage leads the current by an angle Φ or in other words the current lags behind the voltage by an angle Φ. When $V_L < V_C$, the voltage lags behind the current by an angle Φ or in other words the current leads the voltage by an angle Φ.

From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived

From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$V = IZ$$

Where impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Phase angle

$$\Phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$$

$$\Phi = \tan^{-1}\left(\frac{IX_L - IX_C}{IR}\right)$$

$$\Phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

From the expression for phase angle, we can derive the following three cases

Case (i): When $X_L > X_C$

The phase angle Φ is positive and the circuit is inductive. The circuit behaves like a series RL circuit.

Case (ii): When XL<XC

The phase angle Φ is negative and the circuit is capacitive. The circuit behaves like a series RC circuit.

Case (iii): When XL=XC

The phase angle Φ = 0 and the circuit is purely resistive. The circuit behaves like a pure resistive circuit.

The voltage and the current can be represented by the following equations. The angle Φ is positive or negative depending on the circuit elements.

$$V = V_m \sin \omega t$$

$$I = I_m \sin(\omega t \pm \Phi)$$

Average power

Average power

$$P = VI\cos\phi$$

$$P = (IZ)\times I\times\frac{R}{Z}$$

$$P = I^2R$$

Hence the power in an RLC series circuit is consumed only in the resistance. The inductance and the capacitance do not consume any power.

Phasor algebra for RLC series circuit

$$V = V + j0 = V\angle 0^\circ$$

$$\bar{Z} = R + j\left(X_L - X_C\right) = Z\angle\Phi$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V}{Z}\angle -\Phi$$

Problem 8

A 230 V, 50 Hz ac supply is applied to a coil of 0.06 H inductance and 2.5 Ω resistance connected in series with a 6.8 μF capacitor. Calculate (i) Impedance (ii) Current (iii) Phase angle between current and voltage (iv) power factor (v) power consumed

$$X_L = 2\pi fL = 2\times 3.14\times 50\times 0.06 = 18.84\,\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times 3.14\times 50\times 6.8\times 10^{-6}} = 468\,\Omega$$

$$(i)Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{2.5^2 + (18.84 - 468)^2} = 449.2\Omega$$

$$(ii)I = \frac{V}{Z} = \frac{230}{449.2} = 0.512\,A$$

$$(iii)\Phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{18.84 - 468}{30}\right) = -89.7^\circ$$

$$(iv)\,pf = \cos\Phi = \cos 89.7 = 0.0056\,lead$$

$$(v)P = VI\cos\Phi = 230\times 0.512\times 0.0056 = 0.66W$$

A resistance R, an inductance L=0.01 H and a capacitance C are connected in series. When an alternating voltage v=400sin(3000t-20°)is applied to the series combination, the current flowing is $10\sqrt{2}$ sin(3000t-65°). Find the values of R and C.

$$\Phi = 65^\circ - 20^\circ = 45^\circ\ lag$$

$$X_L = \omega L = 3000 \times 0.01 = 30\Omega$$

$$\tan\Phi = \tan 45^\circ = 1$$

$$\tan\Phi = \frac{X_L - X_C}{R} = 1$$

$$R = X_L - X_C$$

$$Z = \frac{V_m}{I_m} = \frac{400}{10\sqrt{2}} = 28.3\Omega Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + R^2}$$

$$\sqrt{2}R = 28.3$$

$$R = 20\Omega$$

$$X_L - X_C = 20\Omega$$

$$X_C = 30 - 20 = 10\Omega$$

$$C = \frac{1}{\omega X_C} = \frac{1}{3000 \times 10} = 33.3\mu F$$

SINGLE PHASE AC CIRCUITS

1. An Alternating current 'i' is given by $i = 141.4 \sin(314t)$. Find
   i) The Maximum Value ii) Time Period
   iii) Instantaneous Value when $t = 3$ msec.

Sol:

i) $i = I_m \sin \omega t$

$\therefore \boxed{I_m = 141.4 A}$

ii) $T = \frac{1}{f} = \frac{1}{50} = 20$ m sec

$\boxed{T = 20 \text{ m sec}}$

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi} = \frac{314}{2 \times 3.14}$

$\boxed{f = 50 \text{ Hz}}$

iii) $i = 141.4 \sin(314 \times 3 \times 10^{-3})$

$= 141.4 \sin(0.942) \times \frac{180}{\pi}$

$= 141.4 \sin(54)$

$\boxed{i = 114.39 A}$

---

2. The Equation of an alternating current is $i = 42.42 \sin(628t)$ A.
   Calculate its i) Maximum value, ii) Freq iii) RMS value
   iv) Average Value and v) Form Factor.

Sol: $i = 42.42 \sin 628t$

i) $I_m = 42.42 A$

ii) $\omega = 2\pi f$

$2\pi f = 628$

iii) $I = \frac{I_m}{\sqrt{2}} = 30 A$

iv) $I_{av} = \frac{2 I_m}{\pi}$

$= \frac{2(42.42)}{\pi}$

$= 27.00 A$

v) Form Factor $= \frac{I}{I_{av}} = \frac{30}{27.00}$

$= 1.11 A$

form factor: $f.f = \frac{e_{rms}}{e_{avg}} = \frac{\cancel{e_m}}{\sqrt{2}} \times \frac{\pi}{2\cancel{e_m}}$

$\boxed{f.f = 1.11}$

Peak factor:

$K_p = \frac{e_m}{e_{rms}} = \frac{\cancel{e_m}}{\cancel{e_m}/\sqrt{2}}$

$\boxed{K_p = 1.414}$

---

4. A Current of Average Value 18.019 A is flowing in a circuit to which a voltage of peak value 141.42 V is applied. Determine the impedance in polar form and power. Assume voltage lags current by 30°.

Sol

$I_{avg} = 18.019\,A$

$V_m = 141.42\,V$

$Z\angle\theta = ?$ $P = ?$ $\phi = 30^\circ$

- $V_{rms} = \frac{141.42}{\sqrt{2}}$

$\boxed{V_{rms} = 100\,V}$

- $I_{rms} = I_{avg} \times f.f$

$= 18.019 \times 1.11$

$\boxed{I_{rms} = 20A}$

- $Z = \frac{100}{20} = 5\Omega$

$Z = \frac{V}{I} = \frac{100\angle -30^\circ}{20\angle 0^\circ}$

$\boxed{Z = 5\angle -30^\circ}$

- $P = V_{rms}\, I_{rms} \cos\phi$

$= 100 \times 20 \times \cos(30)$

$\boxed{P = 1732.05\,W}$

5. An AC Voltage $(80+j60)V$ is applied to a circuit. The current in the circuit is $(10+j4)A$. Find the impedance and power consumed.

Sol:

i) $Z = \dfrac{V}{I} = \dfrac{80+j60}{10+j4} = 8.96+j2.41$

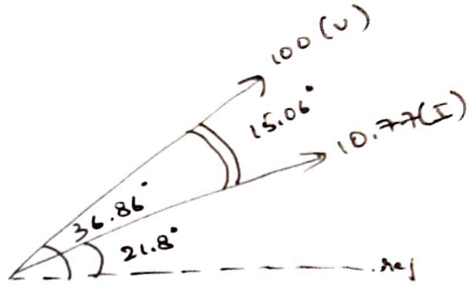
$= 9.27 \angle 15.06^\circ$

$Z = (8.96+j2.41)\Omega$

$V_{rms} = (80+j60)V$

$= 100 \angle 36.86^\circ$

$I_{rms} = 10.7 \angle 21.8^\circ$



ii) $P = V_{rms} I_{rms} \cos\phi$

$= 100 \times 10.77 \times \cos(15.06^\circ)$

$\boxed{P = 1040\,W}$

6. In a Series R-L Circuit, Voltage and Current are expressed by $e = 15\sin(314t + 5/6)V$, $i = 5\sin(314t + 2/3)A$. Find a) Impedence b) resistance c) Inductance d) Average power, e) power factor and f) Voltage across R and X_L.

Sol:

$E = 15/\sqrt{2}$ $\quad \phi = 30^\circ$ $\quad \cancel{E = 10.606 \angle 150^\circ}$

$I = 5/\sqrt{2}$ $\quad \cancel{I = 3.535 \angle 120^\circ}$

a) $Z = \dfrac{E}{I} = 3.0\,\Omega$

b) $R = Z\cos\phi$

$= 3.0\cos(30^\circ)$

$= 2.59\,\Omega$

c) $X_L = \sqrt{Z^2 - R^2}$

$= 1.513\,\Omega$

$L = 4.81 \times 10^{-3} H.$

d) $P = V.I\cos\phi$

$= 32.475\,W$

e) $pf = 0.866$

f) $V_R = I.R.$

$= 9.157\,V$

$V_L = I.X_L$

$= 5.349.\,V$

$E = 10.606 \angle 150^\circ$

$I = 3.535 \angle 120^\circ$

a) $\vec{Z} = \dfrac{\vec{E}}{\vec{I}} = 2.598 + j\,1.50\,\Omega$

b) $R = 2.598\,\Omega$ $X_L = 1.50\,\Omega$

c) $L = 4.77 \times 10^{-3}\,H$

d) $P = V \cdot I \cos\phi$
$= 32.469\,W$

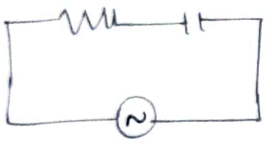
e) $pf = 0.866$

f) $V_R = I \cdot R$
$= 9.157\,V$

$V_L = I \cdot X_L$
$= 5.349\,V$

10. Resistor R in Series with a Capacitor C is Connected to a 50Hz, 240V supply. Find the Value of C so that R absorbs 300W at 100V.

Sol
:



240V

$P = VI.$

$P = I^2 R$

$\therefore R = \frac{300}{9} = 33.33\,\Omega$

$I = \frac{P}{V} = \frac{300}{100} = 3A$

$Z = \frac{E}{I} = \frac{240}{3} = 80A.$

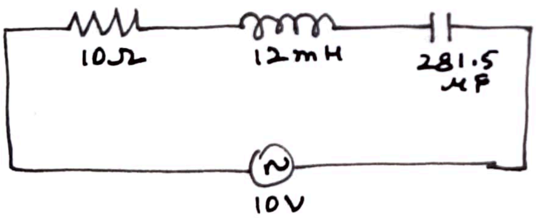
$X_c = \sqrt{Z^2 - R^2} = 72.726\,\Omega$

$= \frac{1}{2\pi f c}$

$\therefore \boxed{C = 0.437 \times 10^{-6} F}$

12. A circuit is made up of $10\,\Omega$ resistance, 12 mH inductance and 281.5 μF capacitance in series. Supply Voltage is 10V. Calculate the value of current when supply frequency is i) 50Hz ii) 150Hz.

Sol:



i) $f = 50\,Hz$

$X_L = 2\pi f L$

$= 2 \times 3.14 \times 50 \times 12 \times 10^{-3}$

$\boxed{X_L = 3.768\,\Omega}$

$X_C = \frac{1}{2\pi f C}$

$= \frac{1}{2 \times 3.14 \times 50 \times 281.5 \times 10^{-6}}$

$\boxed{X_C = 11.31\,\Omega}$

$I = \frac{10\angle 0}{(10 - j7.542)}$

$= \frac{10\angle 0}{12.52\angle -37.02}$

$\boxed{I = 0.8\angle 37^\circ}$

$Z = R + j(X_L - X_C)$

$= 10 + j(-7.542)$

$= (10 - j7.542)$

$\boxed{Z = 12.52\angle -37.02^\circ\,\Omega}$

ii) $X_L = 2\pi f C$

$= 2 \times 3.14 \times 150 \times 12 \times 10^{-3}$

$\boxed{X_L = 11.3\,\Omega}$

$X_c = \frac{1}{2\pi f c}$

$= \frac{1}{2 \times 3.14 \times 150 \times 281.5 \times 10^{-6}}$

$\boxed{X_c = 3.77\,\Omega}$

$Z = R + j(X_L - X_c)$

$= 10 + j(7.53)$

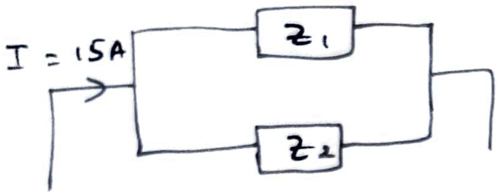
$\boxed{Z = 12.51 \angle 36.97}$

$I = \frac{10 \angle 0}{12.51 \angle 36.97}$

$\boxed{I = 0.8 \angle -37^\circ}$

14. Two circuit of impedance $Z_1 = (10+j15)\,\Omega$ and $Z_2 = (6+j8)\,\Omega$ are connected in parallel. If the total current is 15 A, what will be the power taken by each branch.

Sol :



Let $\vec{I} = 15\angle 0^\circ$ A

$$\therefore \vec{I_1} = \frac{\vec{I} \cdot \vec{Z_2}}{\vec{Z_1} + \vec{Z_2}}$$

$$= \frac{15\angle 0^\circ \cdot (6+j8)}{(10+j15)(6+j8)}$$

$$= (5.513 - j\,0.081)\ \text{A}$$

$$= (5.514\angle -0.842)\ \text{A}$$

$$\therefore \vec{I_2} = 15\angle 0^\circ - (5.154\angle -0.84)$$

$$= (9.846 + j\,0.075)\ \text{A}$$

$$= (9.846\angle 0.439)\ \text{A}$$

$$P_1 = I_1^2 \cdot R_1 = (5.514)^2 \cdot (10)$$

$$= 304.04\ \text{W}$$

$$P_2 = I_2^2 R_2 = (9.846)^2 \times 6$$

$$= 581.66\ \text{W}$$

Power Factor [PF] of a Circuit:

The power factor of a ckt can be defined in the following three ways.

(i) $p.f = \cos\phi$

Power factor of a ckt is the cosine of the angle between voltage & current.

(ii) $p.f = \frac{R}{Z}$

The power factor of a ckt is the ratio of the resistance to the impedance of the ckt.

(iii) $p.f = \frac{P}{EI}$

The power factor of a ckt is the ratio of the real power to the apparent power.

The maximum value of a p.f is UNITY

Practical Importance of Power factor:

The active power consumed by the load in ac ckt is given by $P = EI\cos\phi$.

* If the power factor of the load is small, the active power generated by an alternator & the active power transmitted & received by the consumer decreases.

* To generate the same power from generator at poor p.f as at high p.f, the capacity of the generator has to be increased which involves additional investment on generation.

* If the p.f is small, for transmitting a particular power, the current in the transmission line increases and hence the copper losses will increase and efficiency of transmission decreases.

* Due to low power factor, the current carrying capacity of the conductor has to be increased. Hence large sized conductors have to be used for transmission of electrical power which involves larger investment.

* Most of the loads used by consumers are inductive in nature and normally their power factor are low. Hence for the effective use of supplied energy, the supplying agencies always insist on the consumer to improve the p.f of their loads to 0.85 or 0.9 by using static condensers of suitable capacities across their loads.

* The supplying agencies will give some incentives in the tariff to the consumers for improving the p.f of their loads.