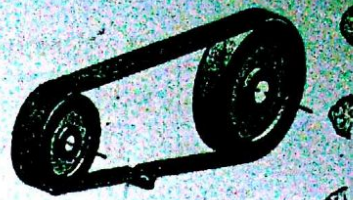
16 POWER TRANSMISSION



Power is transmitted from the prime mover to a machine by means of intermediate mechanism called *drives*. The most common are the rotary motion drives. These intermediate mechanisms are necessary due to the following reasons:

1. The optimal speed of the prime mover or that of the standard motor may be different from the speed required to operate the machine.
2. To regulate or change the speed of the machine when the speed of the prime mover is kept constant.
3. To operate several machines from one prime mover.
4. Ease of maintenance, convenience, safety, etc.

Flexible machine elements such as belts, ropes and chains are used for the transmission of power over comparatively long distances. They are adapted for transmitting a pull but are incapable of carrying a thrust. Belts and ropes do not give positive drive, because their ability to transmit power depends on friction between the belt and pulleys. Chain drives are used where positive action is required.

BELT DRIVES

Belt drives are used to transmit power form one shaft to another when the shafts are some distance apart and it is not required that their velocity ratio be absolutely constant. In its simplest form, the belt drive consists of an endless belt which is wrapped tightly over two pulleys known as *driving pulley* and the *driven pulley* which in turn are fixed on the driving (motor) and driven (machine) shafts respectively. The motion from the driving pulley is transmitted to the driven pulley by frictional resistance between the belt and the surface of the pulleys. Since there will be slip between the belt and the surface of the pulleys, the velocity ratio in the belt drives is never absolutely constant.

Belt materials

Belts used for power transmission must be strong, flexible, and durable and must have a high coefficient of friction. The most common belt materials are leather, fabric, rubber, balata, camel's hair and woven cotton, Belts are available with flat, round or V-cross sections, but flat belts and V-belts are the most commonly used belts.

Advantages and disadvantages of flat belt drives

Advantages:

1. Flexibility, shock absorption, efficiency at high speeds, protection against over load, resistance to abrasive and other harmful environments.
2. Simplicity, smoothness of operations, low cost, low maintenance and long life.

3. It can be used to connect widely spaced shafts and it does not require precise alignment of the shafts and pulleys.

Disadvantages:

1. It is not a positive drive.
2. Less efficiency due to slip and creep.
3. Not suitable for short center distances.
4. Comparatively large size.
5. Stretching of the belt calls for a tensioning device.
6. High bearing loads and belt stresses.
7. Belt joints reduce the life of the belt.

Types of belt drives: There are two common types of belt drives: 1. Open belt drive, and 2. Cross belt drive.

Open belt drive: Fig. 16.1(a) shows an open belt drive. It is used to connect shafts, which are parallel and rotating-in-the-same direction. The part of the belt leaving the follower and approaching the driver is the *tight side*, and the part of the belt leaving the driver and approaching the follower is the *slack side*. When the center distance between the shafts is quite large, the lower side of the belt should be tight one and the upper - the slack one.

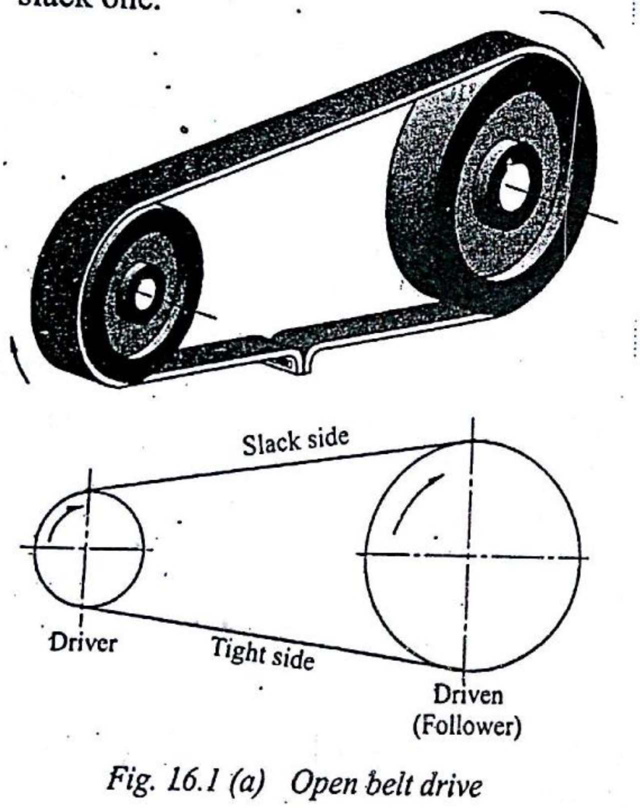


Fig. 16.1 (a) Open belt drive

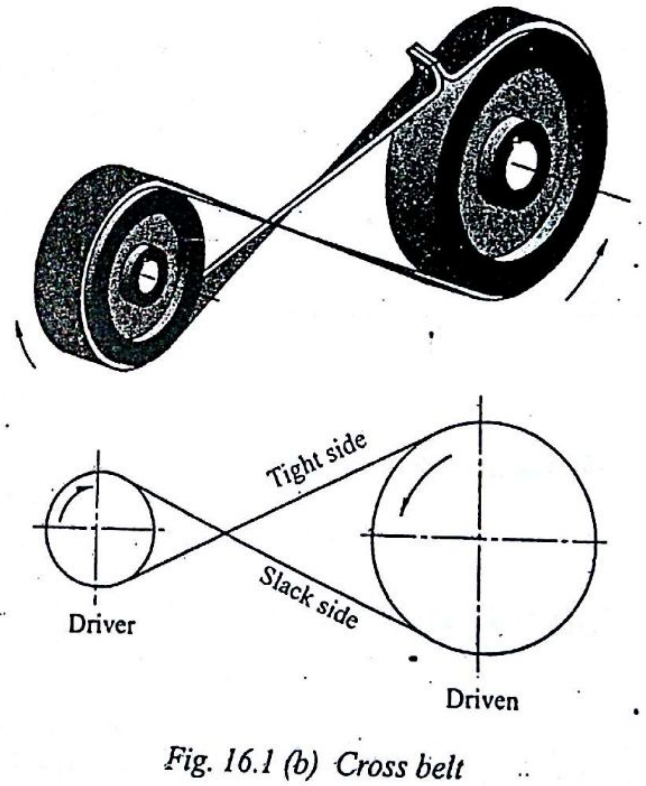


Fig. 16.1 (b) Cross belt

Cross belt drive: Cross belt drive is used to connect shafts which are parallel and rotating in opposite directions as shown in fig. 16.1(b). At the point where the belt crosses, it rubs against itself and wears. To avoid excessive wear, the center distance between the shafts must be greater than $20b$, where b is the width of the belt and operate at low velocity.

Velocity ratio of belt drives

For positive drives, the ratio of angular velocity of the driver pulley to the angular velocity of the driven pulley is called *velocity ratio* or *speed ratio* or *transmission ratio.*

Let d_1 = Diameter of the driver pulley in mm

d_2 = Diameter of the driven pulley in mm

r_1 = Radius of the driver pulley in mm

r_2 = Radius of the driven pulley in mm

n_1 = Speed of the driver pulley in rpm

n_2 = Speed of the driven pulley in rpm

Assuming that there is no slip between the belt and the pulley rim, the linear speed of the belt over the two pulleys must be the same.

i.e., $\pi d_1 n_1 = \pi d_2 n_2$

or $d_1 n_1 = d_2 n_2$

$$\therefore \text{ The velocity ratio } = \frac{n_1}{n_2} = \frac{d_2}{d_1} = \frac{r_2}{r_1}$$

$$\text{i.e., Velocity ratio } = \frac{\text{Speed of the driver}}{\text{Speed of the driven}} = \frac{\text{Diameter of the driven pulley}}{\text{Diameter of the driver pulley}}$$

Thus in the belt drive, the speeds are inversely proportional to the diameter of the pulleys.

Effect of belt thickness on velocity ratio

By considering the velocity of the belt at the neutral section, the velocity ratio of the belt is

$$\frac{n_1}{n_2} = \frac{d_2 + t}{d_1 + t} = \frac{r_2 + t/2}{r_1 + t/2}$$

where t is the thickness of the belt.

Slip

When a belt is transmitting power, there is always a small amount of slip between the belt and the pulleys so that actual velocity of the belt is slightly less than the surface speed of the driving pulley and slightly greater than that of the driven pulley. This is due to insufficient frictional grip between the belt and the rim of the pulleys. Therefore slip may be defined as the relative motion between the pulley and the belt on it. The difference in linear speeds of the pulley rim and the belt on it is the measure of slip and is generally expressed as a percentage.

Effect of slip on velocity ratio

Let s_1 = Percentage of slip between driver pulley rim and the belt.

s_2 = Percentage of slip between the belt and driven pulley rim.

Then velocity ratio $\frac{n_1}{n_2} = \frac{d_2}{d_1(1-s_1/100)\,(1-s_2/100)}$

By considering the thickness of belt,

Velocity ratio $\frac{n_1}{n_2} = \frac{d_2+t}{(d_1+t)\,(1-s_1/100)\,(1-s_2/100)}$

If s is total the percentage of slip in the belt drive,

Velocity ratio $\frac{n_1}{n_2} = \frac{d_2}{d_1(1-s/100)}$

By considering the belt thickness and slip,

Velocity ratio $\frac{n_1}{n_2} = \frac{(d_2+t)}{(d_1+t)\,(1-s/100)}$

Belt length for open belt drive

Let r_1 = Radius of the larger pulley

r_2 = Radius of the smaller pulley

c = Distance between the centers of the two pulleys

L_o = Length of open belt

The length of the belt is calculated from the geometry of the fig. 16.2.

The length of the belt comprises three portions namely,

1. Length of belt l_1 in contact with larger pulley,
2. Length of belt l_2 in contact with smaller pulley, and
3. Length of belt l_3 not in contact with either of the pulleys.

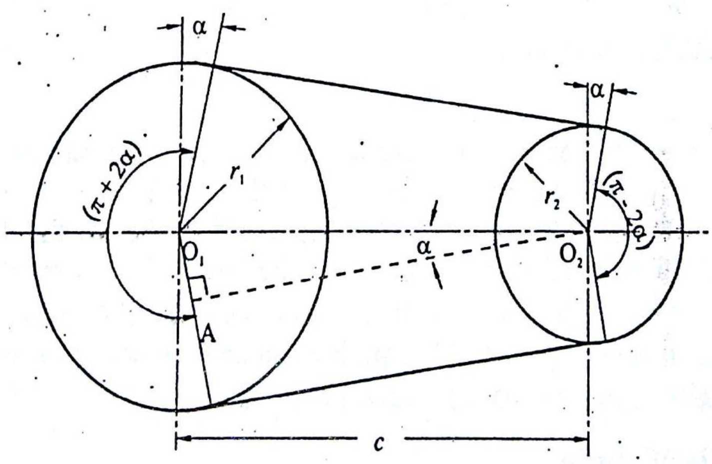


Fig. 16.2 Open belt drive

From the figure,

$$l_1 = (\pi+2\alpha)r_1$$

$$l_2 = (\pi-2\alpha)r_2$$

From O_2 draw a line O_2A parallel to the belt, which is not in contact with either of the pulleys.

From triangle O_1O_2A

$$O_2A = [O_1O_2^2 - O_1A^2]^{1/2} = [c^2-(r_1-r_2)^2]^{1/2} = c\left[1-\left(\frac{r_1-r_2}{c}\right)^2\right]^{1/2}$$

Expanding the term within the brackets by binomial theorem and neglecting the higher power terms being small, we get

$$\left[1-\left(\frac{r_1-r_2}{c}\right)^2\right]^{1/2} = 1-\frac{1}{2}\left(\frac{r_1-r_2}{c}\right)^2$$

$$\therefore \quad O_2A = c\left[1-\frac{1}{2}\left(\frac{r_1-r_2}{c}\right)^2\right]$$

Length $\quad l_3 = 2\,O_2A = 2c\left[1-\frac{1}{2}\left(\frac{r_1-r_2}{c}\right)^2\right] = 2c - \frac{(r_1-r_2)^2}{c}$

Length of the belt $\quad L_o = l_1+l_2+l_3$

i.e., $\quad L_o = (\pi+2\alpha)r_1+(\pi-2\alpha)r_2+2c-\frac{(r_1-r_2)^2}{c}$

$$= \pi(r_1+r_2)+2\alpha(r_1-r_2)+2c-\frac{(r_1-r_2)^2}{c} \qquad \text{.....(1)}$$

Also from triangle O_1O_2A

$$\sin\alpha = \frac{O_1A}{O_1O_2} = \frac{r_1-r_2}{c}$$

For small values of α, $\sin\alpha = \alpha$

$$\therefore \ \alpha = \frac{r_1-r_2}{c}$$

Substituting the value of α in the equation (1) we get,

$$L_o = \pi(r_1+r_2)+ +2c-\frac{2(r_1-r_2)^2}{c}$$

Length of open belt $\quad L_o = 2c+\pi(r_1+r_2)+\frac{(r_1-r_2)^2}{c} \qquad \text{.....(2)}$

Belt length for cross belt drive

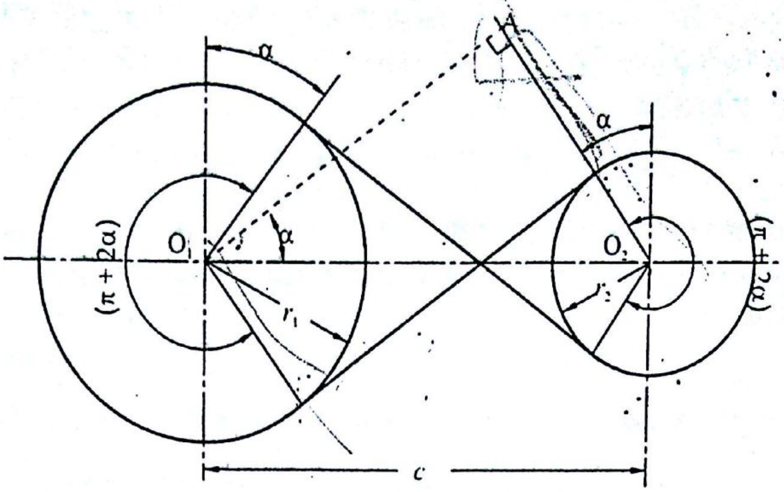


Fig. 16.3 Cross belt drive

From the geometry of the fig. 16.3,

Length of belt in contact with larger pulley $l_1 = (\pi + 2\alpha) r_1$

Length of belt in contact with smaller pulley $l_2 = (\pi + 2\alpha) r_2$

From O_1 draw a line O_1A parallel to the belt, which is not in contact with either of the pulleys.

From triangle O_1O_2A

$$O_1A = [O_1O_2^2 - O_2A^2]^{1/2} = [c^2 - (r_1+r_2)^2]^{1/2} = c\left[1 - \left(\frac{r_1+r_2}{c}\right)^2\right]^{1/2}$$

Expanding the term within the brackets by binomial theorem and neglecting the higher power terms being small, we get

$$\left[1 - \left(\frac{r_1+r_2}{c}\right)^2\right]^{1/2} = 1 - \frac{1}{2}\left(\frac{r_1+r_2}{c}\right)^2$$

$$\therefore \quad O_1A = c\left[1 - \frac{1}{2}\left(\frac{r_1+r_2}{c}\right)^2\right]$$

Length of belt not in contact with either of the pulleys $l_3 = 2\,O_1A = 2c\left[1 - \frac{1}{2}\left(\frac{r_1+r_2}{c}\right)^2\right]$

$$= 2c - \frac{(r_1+r_2)^2}{c}$$

Length of cross belt $L_c = l_1 + l_2 + l_3$

i.e., $$L_c = (\pi+2\alpha)r_1+(\pi+2\alpha)r_2+2c-\frac{(r_1+r_2)^2}{c}$$

$$= \pi(r_1+r_2)+2\alpha(r_1+r_2)+2c-\frac{(r_1+r_2)^2}{c} \quad .....(1)$$

Also from triangle $O_1 O_2 A$

$$\sin\alpha = \frac{O_2A}{O_1O_2} = \frac{r_1+r_2}{c}$$

For small values of α, $\sin\alpha = \alpha$

$$\therefore \alpha = \frac{r_1+r_2}{c}$$

Substituting the value of α in the equation (1) we get,

$$L_c = \pi(r_1+r_2)+\frac{2(r_1+r_2)^2}{c}+2c-\frac{(r_1+r_2)^2}{c}$$

Length of cross belt $$L_c = 2c+\pi(r_1+r_2)+\frac{(r_1+r_2)^2}{c} \quad .....(2)$$

Angle of contact or lap

For open belt drive,

Angle of contact on smaller pulley $$\theta_S = \pi-2\alpha=\pi-2\sin^{-1}\left(\frac{r_1-r_2}{c}\right)$$

or $$\theta_S = \pi-2\sin^{-1}\left(\frac{d_1-d_2}{2c}\right)$$

Angle of contact on larger pulley $$\theta_L = \pi+2\alpha=\pi+2\sin^{-1}\left(\frac{r_1-r_2}{c}\right)$$

or $$\theta_S = \pi+2\sin^{-1}\left(\frac{d_1-d_2}{2c}\right)$$

In the above formulae, the absolute difference of $(r_1 - r_2)$ or $(d_1 - d_2)$ must be used, while calculating the angle of lap.

If the belt is used to transmit power between the two pulleys of unequal diameters, the belt will slip first on the pulley having smaller angle of contact, i.e., on the smaller pulley. Therefore it is necessary to take θ_S into account while designing the belt.

For equal diameter pulleys, $d_1 = d_2$

and angle of contact $\theta_S = \theta_L = \theta = \pi$ rad

For crossed belt derive, the angle of contact on both the pulleys are the same.

i.e., $$\theta_S = \theta_L=\theta=\pi+2\alpha = \pi+2\sin^{-1}\left(\frac{r_1+r_2}{c}\right)$$

or $$\theta = \pi+2\sin^{-1}\left(\frac{d_1+d_2}{2c}\right)$$ where θ is in radians.

Ratio of belt tensions

Consider a flat belt partly wound around a pulley so that the angle of lap is θ and let T_1 and T_2 is the tight and slack side tensions in the belt, when it is about to slip in the direction shown. Now consider the forces acting on an elemental piece of belt subtending an angle $\delta\theta$ at the center of the pulley. These forces are T and $(T + \delta T)$, the tensions at the two extremities of the belt, the normal reaction R acting radially, and the force of friction μR opposing the slip and acting perpendicular to R as shown in fig. 16.4.

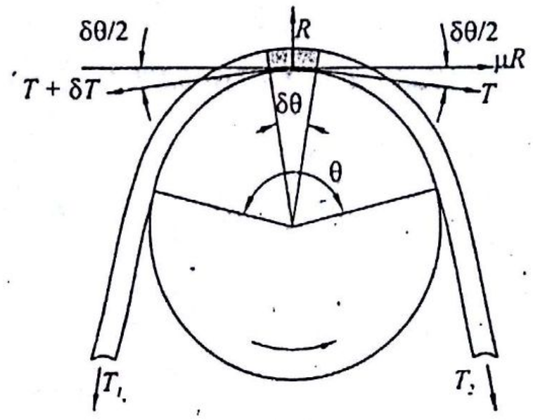


Fig. 16.4 Ratio of belt tensions

Resolving the forces radially,

$$R = (T+\delta T)\sin\left(\frac{\delta\theta}{2}\right) + T\sin\left(\frac{\delta\theta}{2}\right) = 2T\sin\left(\frac{\delta\theta}{2}\right) + \delta T\sin\left(\frac{\delta\theta}{2}\right)$$

Since the angle $\frac{\delta\theta}{2}$ is very small, $\sin\left(\frac{\delta\theta}{2}\right) = \frac{\delta\theta}{2}$

$\therefore$ the above equation becomes,

$$R = 2T\,\frac{\delta\theta}{2} + \delta T\,\frac{\delta\theta}{2}$$

Neglecting $\delta T \cdot \frac{\delta\theta}{2}$ being small, then

$$R = T\delta\theta \qquad \text{.....(1)}$$

Now resolving the forces tangentially,

$$\mu R + T\cos\left(\frac{\delta\theta}{2}\right) = (T+\delta T)\cos\left(\frac{\delta\theta}{2}\right)$$

or $$\mu R = \delta T\cos\left(\frac{\delta\theta}{2}\right)$$

Since $\frac{\delta\theta}{2}$ is small, $\cos\left(\frac{\delta\theta}{2}\right) = 1$

$$\therefore \quad \mu R = \delta T$$

or
$$R = \frac{\delta T}{\mu} \qquad .....(2)$$

Equating the equations (1) and (2) we get,

$$T\delta\theta = \frac{\delta T}{\mu}$$

$$\frac{\delta T}{T} = \mu\,\delta\theta$$

Integrating between their respective limits, we get

$$\int_{T_2}^{T_1} \frac{dT}{T} = \int_0^{\theta} \mu\, d\theta$$

i.e.,
$$\log_e\left(\frac{T_1}{T_2}\right) = \mu\theta$$

Ratio of tensions
$$\frac{T_1}{T_2} = e^{\mu\theta} \qquad .....(3)$$

Power transmitted by belts

The power transmitted by any belt depends on the arc of contact, difference in belt tensions, coefficient of friction and center distance. The pulley having the lower value of $\mu\theta$ (usually the smaller pulley) governs the power transmission. If T_1 and T_2 are the tight and slack side tensions respectively in Newton and if v is speed of the belt in m/sec. then,

Power
$$P = \frac{(T_1 - T_2)\, v}{1000} \text{ kW}$$

Initial tension: The belt is assembled with an initial tension T_o. When power is being transmitted, the tension in the tight side increases from T_o to T_1 and on the slack side decreases from T_o to T_2. If the belt is assumed to obey Hooke's law and its length to remain constant, then the increase in length of the tight side is equal to decrease in length of the slack side.

i.e.,
$$T_1 - T_0 = T_0 - T_2$$

Since the length and cross-sectional area of the belt are the same on each side, hence

$$\therefore \text{ Initial tension } T_0 = \frac{T_1 + T_2}{2}$$

Creep

The phenomenon of creep of belt arises through the difference in tensions on the two sides of a belt. Since the belt is made of elastic material, the stretch in the belt due to different tensions on two sides of the pulley will be different. The part of the belt leaving the follower and approaching the driver is the *tight side* and is stretched more than the part of the belt leaving the driver and approaching the follower or the *slack side*. These uneven extensions and contractions of the belt due to varying tension in it, causes a relative motion of the belt on the pulley. This relative motion is known as *creep* in the belt.

Stepped pulleys

A stepped pulley or *cone pulley* drive as shown in fig. 16.5 is used for changing the speed of the driven shaft while the driving shaft runs at constant speed. Stepped pulleys on the two shafts are so arranged that the smallest step of one pulley be opposite to the largest step of the other. By properly proportioning the diameters of the different pairs of steps, it is possible to get any desired series of speeds for the driven shaft by shifting the belt from one pair of the steps to another. In designing such a pair of pulleys, two factors must be taken into account. They are;

i) The ratio of the diameters of successive pairs of steps must be such as to give the desired speed ratios.
ii) The sum of the diameters of any pair of steps must be such as to maintain the proper tightness of the belt for all positions.

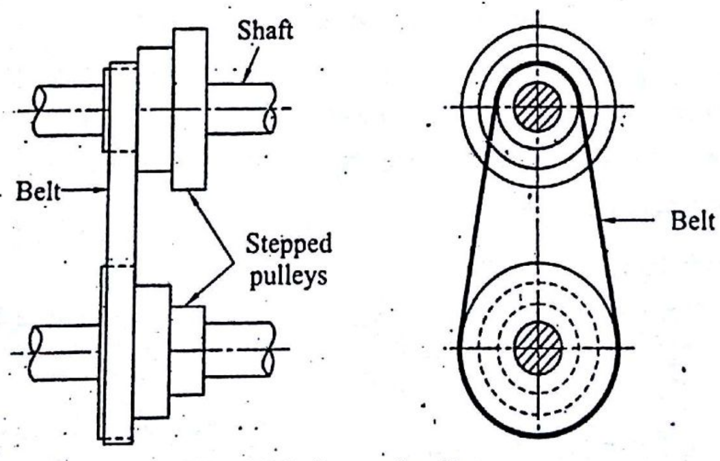


Fig. 16.5 Stepped pulley

Tight and loose pulleys

A tight and loose pulleys drive is used when the driven shaft is to be started or stopped whenever desired without stopping the belt. They consist of two pulleys placed side by side upon the driven shaft as shown in fig. 16.6. The tight pulley also known as *fast pulley*, is keyed to the shaft, whereas the loose pulley turns loose upon the driven shaft and is kept in place by the hub of the tight pulley and a collar. The driving shaft carries a pulley, whose width is the sum of the width of the tight and loose pulleys as shown in fig. 16.6b. The belt, when in motion, can be moved by means of a shipper that guides its advancing side, on to either the tight or the loose pulley. The driver pulley has a flat face, because the belt must occupy different position upon it, whereas the tight and loose pulleys have crowned faces. The crown facing will allow the shifting of the belt and will retain it in position when shifted upon them.

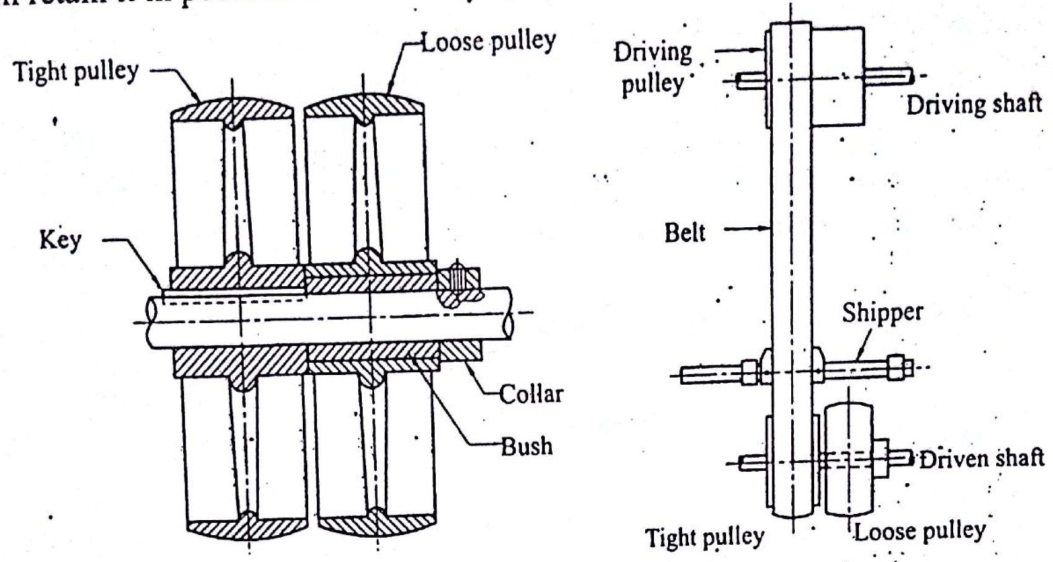


Fig. 16.6 Tight and loose pulleys

Jockey pulley or idler pulley

High speed ratios and short center distances decreases the arc of contact on the smaller pulley and thereby the power transmitting capacity of the drive is seriously reduced. The proper arc of contact may be obtained by the use of spring loaded or gravity idlers. Idlers increase the angle of wrap and thereby reduce the belt tensions required for a given power. In practice, the idler is usually located near the smallest diameter pulley and on the slack side as shown in fig. 16.7.

The disadvantages of this drive are:

1. It is non reversible.
2. The bending stress developed in the belt reduces the belt life.

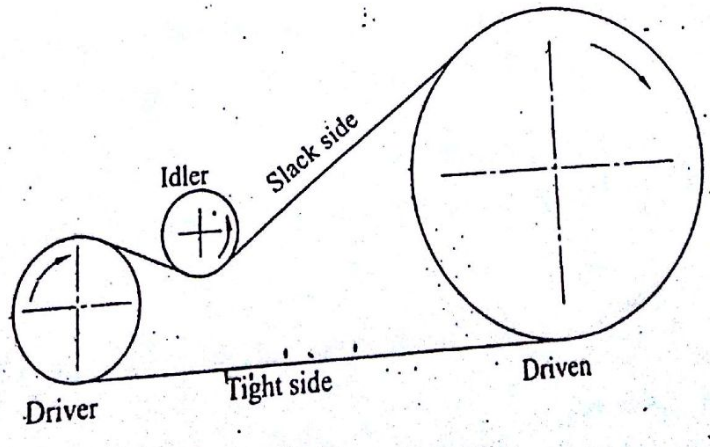


Fig. 16.7 Jockey pulley

V-BELTS

V-Belts are used to transmit power between the shafts whose center distance is short. V-belts are usually endless and trapezoidal in cross section as shown in fig. 16.8. It consists of a central layer of cord of fabric to carry the load and molded in rubber or rubber like compounds to transmit the pressure of the cords to the side walls of the belt. Both rubber and cords are generally enclosed in an elastic wearing cover. The belt rests on the two sides of the groove on a grooved pulley called the *sheave* as shown in fig. 16.9. As the belt tension increases, belt wedges, thus reducing the tendency to slip. This permits the drive to operate with reduced angles of contact and low initial tension. Multiple V-belt drives are used when the power transmitted is too great for a single belt.

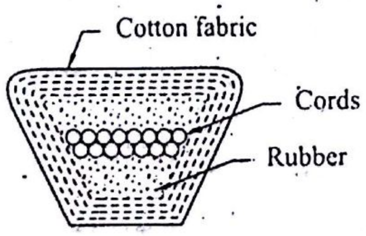


Fig. 16.8 V-belt

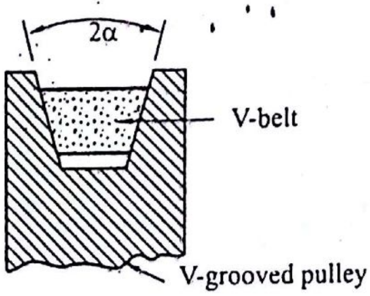


Fig. 16.9 V-belt wedge

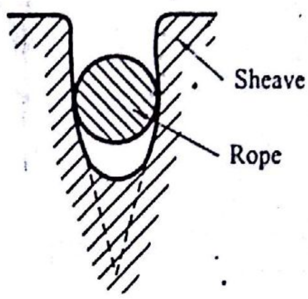


Fig. 16.10 Rope drive

Advantages and disadvantages of V-belt drive over flat belt drive

Advantages:

1. Due to wedging action, V-belts transmit more power without slip.
2. V-belts are used for short center distance drives.
3. High velocity ratio and high efficiency.
4. Ability to absorb shock and compactness.
5. V-belt drive is not affected by changes in direction of rotation.
6. As the V-belts are made endless, there is no joint trouble.
7. The drive is smooth and maintenance is low.

Disadvantages:

1. V-belts cannot be used in places where it is difficult to put on an endless belt.
3. V-belts are not so durable as flat ones and they cannot be used with larger center distances.
3. V-belt drives are not economical.
4. Improper belt tensioning and mismatching of belt length can reduce service life.

ROPE DRIVES

Rope drives are used to transmit power over longer distances. The rope runs over pulleys, called *sheaves*, having grooved surfaces. Ropes are made of cotton, hemp, manila or steel wires. One of the main advantages of rope drives is that number of separate drives may be taken from one driving pulley. In the case of multiple rope system, even if one rope breaks, the continuity of power is maintained. When the drive is horizontal, the slack side is kept on the top so that any sag tends to wedge the rope in the groove, thereby increasing the angle of lap. The grooves on the pulleys are V-shaped as shown in fig. 16.10; the angle between the two faces being from 40° to 50°. The rope rests on the two sides as shown and not on the bottom of the groove.

Ratio of tensions in a V-belt or rope drive: Let 2α be the angle of groove and let the reaction between the belt on either side be R_n and R is the equivalent reaction.

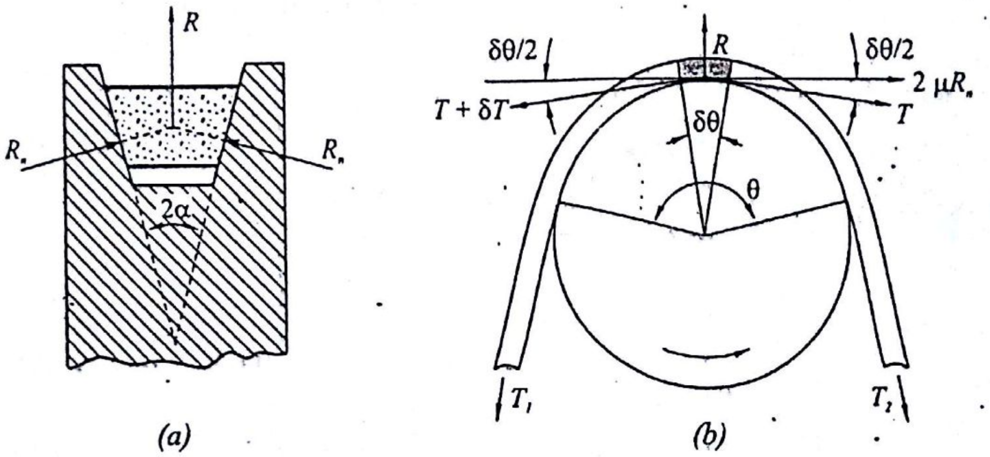


Fig. 16.11 Ratio of V-belt tensions

Resolving the forces radially (refer fig. 16.11a) we get,

$$R = R_n \sin\alpha + R_n \sin\alpha = 2\,R_n \sin\alpha \qquad \text{..... (1)}$$

The equivalent coplanar forces acting on an elemental piece of rope subtending an angle $\delta\theta$ at the center of the pulley as shown in fig. 16.11b are therefore $T+\delta T$, T, R and μR_n. Since two sides of the belt are in contact with the grooved pulley, the total friction force on the belt is $2\,\mu R_n$.

Resolving these forces tangentially,

$$2\mu R_n + T\cos\left(\frac{\delta\theta}{2}\right) = (T+\delta T)\cos\left(\frac{\delta\theta}{2}\right)$$

or

$$2\,\mu R_n = \delta T\cos\left(\frac{\delta\theta}{2}\right) \qquad \text{..... (2)}$$

Since $\left(\frac{\delta\theta}{2}\right)$ is small, $\cos\left(\frac{\delta\theta}{2}\right) = 1$

therefore equation (2) becomes,

$$2\,\mu R_n = \delta T$$

or

$$R_n = \frac{\delta T}{2\mu} \qquad \text{..... (3)}$$

Substituting the value of R_n in equation (1) we get,

$$R = \frac{2\delta T}{2\mu}\sin\alpha = \frac{\delta T}{\mu}\sin\alpha \quad \text{.....(4)}$$

Now resolving the forces radially,

$$R = T\sin\left(\frac{\delta\theta}{2}\right) + (T+\delta T)\sin\left(\frac{\delta\theta}{2}\right)$$

$$= 2T\sin\left(\frac{\delta\theta}{2}\right) + \delta T\sin\left(\frac{\delta\theta}{2}\right)$$

Since $\left(\frac{\delta\theta}{2}\right)$ is small, $\sin\left(\frac{\delta\theta}{2}\right) = \left(\frac{\delta\theta}{2}\right)$

$$\therefore \quad R = 2T\left(\frac{\delta\theta}{2}\right) + \delta T\left(\frac{\delta\theta}{2}\right)$$

Neglecting the term $\delta T\left(\frac{\delta\theta}{2}\right)$ being small, we get

$$R = T\delta\theta \quad \text{.....(5)}$$

Equating the equations (4) and (5) we have,

$$T\delta\theta = \frac{\delta T}{\mu}\sin\alpha$$

or

$$\frac{\delta T}{T} = \frac{\mu\delta\theta}{\sin\alpha}$$

Integrating between their respective limits, we get

$$\int_{T_1}^{T_2}\frac{dT}{T} = \int_0^{\theta}\frac{\mu d\theta}{\sin\alpha}$$

i.e., $$\log_e\left(\frac{T_1}{T_2}\right) = \frac{\mu\theta}{\sin\alpha}$$

$$\boxed{\therefore \text{ Ratio of tensions } \frac{T_1}{T_2} = e^{\mu\theta/\sin\alpha}} \quad \text{.....(6)}$$

In the above equation, the angle of lap θ is in radian and the semi groove angle α is in degrees.

GEAR DRIVES

Gears are toothed wheels used to transmit power from one shaft to another when a constant speed ratio is desired and the distance between the shafts is relatively small.

Advantages and disadvantages of gear drives

Advantages:

1. It is a positive drive and is used to connect closely spaced shafts.
2. High efficiency, compactness, reliable service, more life, simple operation and low maintenance.
3. It can transmit heavier loads than other drives and can be used where precise timing is desired.

Disadvantages:

1. They are not suitable for large center distance because the drive becomes bulky.
2. High production cost.
3. Due to errors and inaccuracies in their manufacture, the drive may become noisy accompanied by vibrations at high speeds.

Classification of gears

The gears may be classified according to the relative position of the axes of the shafts on which the gears are mounted (refer fig. 16.12).

Parallel axes	***Intersecting axes***	***Non intersection and non-parallel axes***
1. Spur gears 2. Helical gears 3. Double helical gears 4. Herringbone gears	Bevel gears	Worm gears

Law of gearing

The law of gearing states that in order to transmit motion at a constant angular velocity ratio, the pitch point (i.e., the point where the line of centers intersects the pitch circles) must remain fixed.

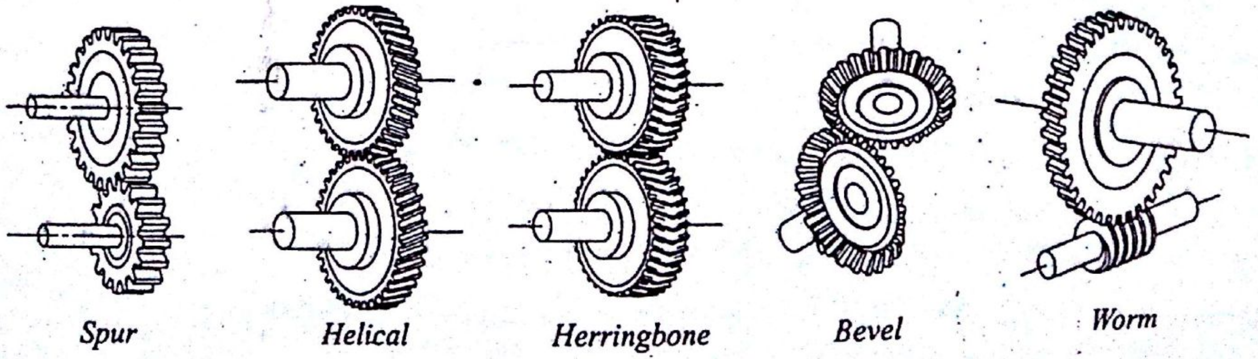


Fig. 16.12 Types of gears

Gear tooth profiles

The commonly used forms of teeth that satisfy the law of gearing are:

1. Involute, and 2. Cycloidal.

An *involute* may be defined as the locus of a point on a straight line, which rolls without slipping on the circumference of a circle. The characteristic of an involute is that any normal to an involute is tangent to the base circle from which the involute is generated.

A *cycloid* is the locus of a point on the circumference of a circle, which rolls, without slipping along a straight line. The faces of the teeth are epicycloid generated on the pitch circle and the flanks are hypocycloid generated inside the pitch circles.

Comparison between involute and cycloidal tooth forms

Feature	*Involute*	*Cycloidal*
Basic form	Constant for all gears	Variable depending upon the gear ratio
Manufacturing process	Easy	Difficult
Strength	Good	Weaker than the involute tooth
Angular velocity at varying center distances	Constant	Variable
Wear	More	Less
Interference	May exist accordingly	No interference

Spur gear

This is the simplest form of gears for transmitting power between two parallel shafts. The teeth are straight and parallel to the axis. Spur gear imposes only radial loads on the bearings.

Spur gear nomenclature

The nomenclature for a spur gear is shown in fig. 16.13 and the following terms are defined.

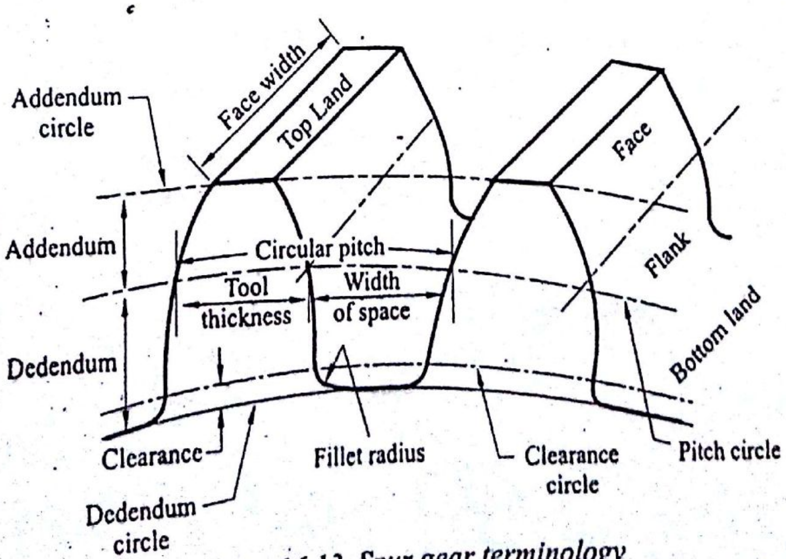


Fig. 16.13 Spur gear terminology

Pitch circle is a theoretical circle upon which all computations are made. The pitch circles of a pair of mating gears are tangent to each other.

Pitch circle diameter D is the diameter of the pitch circle.

Circular pitch p is the distance from a point on one tooth to the corresponding point on the next tooth measured along the pitch circle.

Diametral pitch P is the ratio of the number of teeth on a gear to the pitch circle diameter in inches. It is used with English units.

Module m is the ratio of the pitch circle diameter of a gear in millimeter to the number of teeth, i.e., $m = D/z$.

Tooth thickness is the thickness of tooth measured along the pitch circle.

Width of tooth space is the width of the space between teeth measured along the pitch circle. Thus the circular pitch is equal to the sum of the tooth thickness and the width of space.

Top land is the surface of the top of the tooth.

Bottom land is the surface of the bottom of tooth space.

Addendum circle is the circle passing through the outer ends of the teeth of a gear.

Dedendum circle or the *root circle* is the circle passing through the bottom of the spaces.

Addendum is the radial distance from the pitch circle to addendum circle.

Dedendum is the radial distance from the pitch circle to the dedendum circle.

The *face* of the tooth is the surface of the tooth between the pitch cylinder and the addendum cylinder.

The *flank* is the surface of the tooth between the pitch cylinder and dedendum cylinder.

Face width is the length of the gear tooth measured along an element of the pitch surface.

The *whole depth* is the full depth of a gear tooth and is the sum of its addendum and dedendum.

The *working depth* is the depth of engagement of a pair of gears; i.e., it is the sum of their addendums.

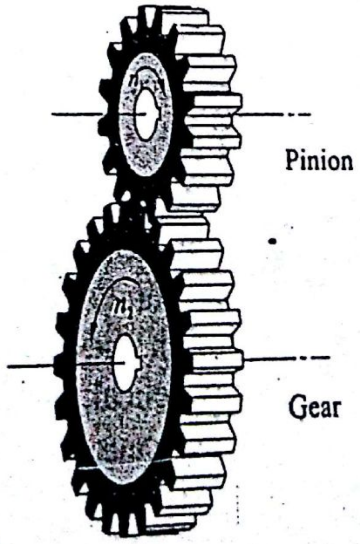


Fig. 6.14 Spur gear drive

The *pinion* is the smaller of the two gears in mesh.

The *gear* is the larger of the two gears in mesh

Pressure angle is the angle, which the common normal to two teeth at the point of contact makes with the common tangent to the two pitch circles at the pitch point

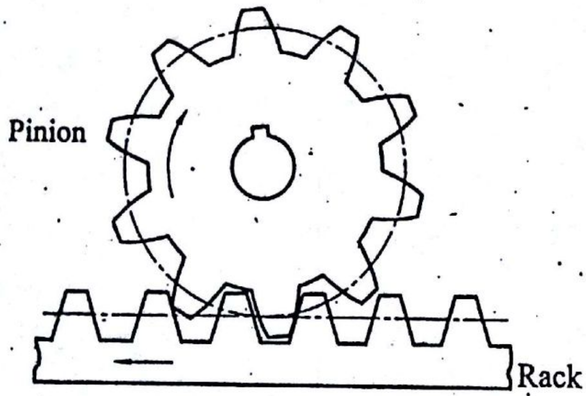
Angular velocity ratio : The angular velocity ratio for a pair of spur gear is inversely proportional to their pitch circle radii or their number of teeth.

i.e., Velocity ratio $$i = \frac{\omega_1}{\omega_2} = \frac{n_1}{n_2} = \frac{d_2}{d_1} = \frac{z_2}{z_1} \qquad \text{..... (1)}$$

Where
ω_1 = Angular velocity of the driver (usually pinion)
ω_2 = Angular velocity of the follower (gear)
d_1 = Pitch circle diameter of the driver
d_2 = Pitch circle diameter of the follower
z_1 = Number of teeth on the driver
z_2 = Number of teeth on the follower

Pinion and rack

A *rack* is a gear of infinite diameter. Its pitch circle is a straight line. As a pinion drives the rack, it moves in a straight line as shown in fig. 16.15. The side of an involute rack tooth must be a plane.



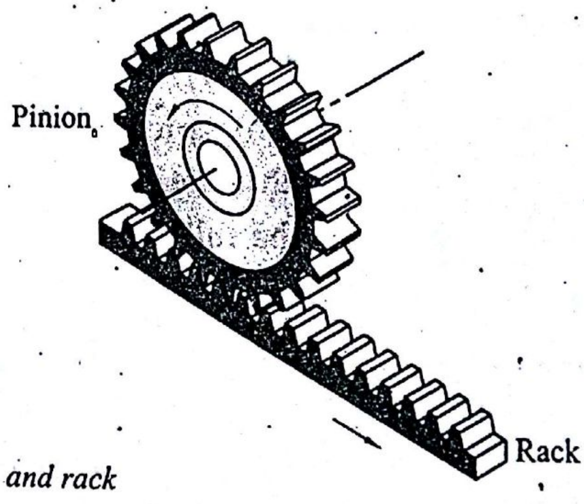


Fig. 16.15 Pinion and rack

Helical gears

Helical gears differ from spur gears in that they have teeth that are cut in the form of helix on their pitch cylinders instead of parallel to the axis of rotation. In order for two helical gears on parallel shafts to be mesh, they must have the same helix angle and be of opposite hands. The initial contact of helical gear is a point, which becomes a line of increasing length as contact continues. Because of gradual engagement of the teeth, helical gears run more quietly than spur gears and can be operated at high pitch line velocities. A disadvantage of helical gears is that the helix angle results in a thrust load in addition to the usual tangential and separating loads.

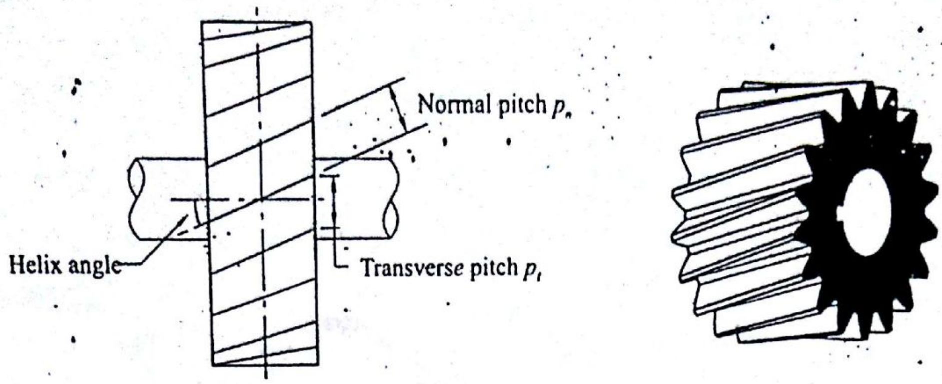


Fig. 16.16 Parallel helical gear

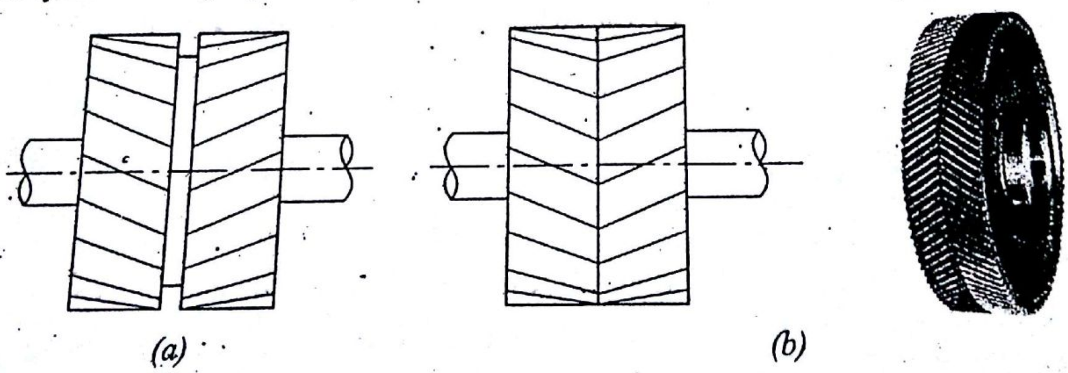
Helical gear nomenclature

Transverse circular pitch p_t is the distance between corresponding points on adjacent tooth measured in the diametral plane.

Normal circular pitch p_n is the distance between corresponding points on adjacent tooth measured in a direction normal to the teeth.

Helix angle is the angle between an element of the helical tooth and the axis of rotation of the gear.

Double helical gear: A double helical gear is shown in fig. 16.17*a*. These gears have two sets of opposed helical teeth. One having a right-hand helix and the other left-hand helix. The combination of right-hand and left-hand helices absorbs the axial thrust within the gear itself and eliminates the thrust on the bearings. It is used to transmit heavy loads at high speeds.



(a) (b)

Fig. 16.17(a) Double helical gear, (b) Herringbone gear

Herringbone gear: It is essentially the same as the double helical gears but in this gear there is no space separating the two opposed sets of helical teeth as shown in fig. 16.17*b*.

Bevel gears

Bevel gears are the most commonly used gears for transmitting power between intersecting shafts. The pitch surfaces are rolling cones. The point of intersection of the

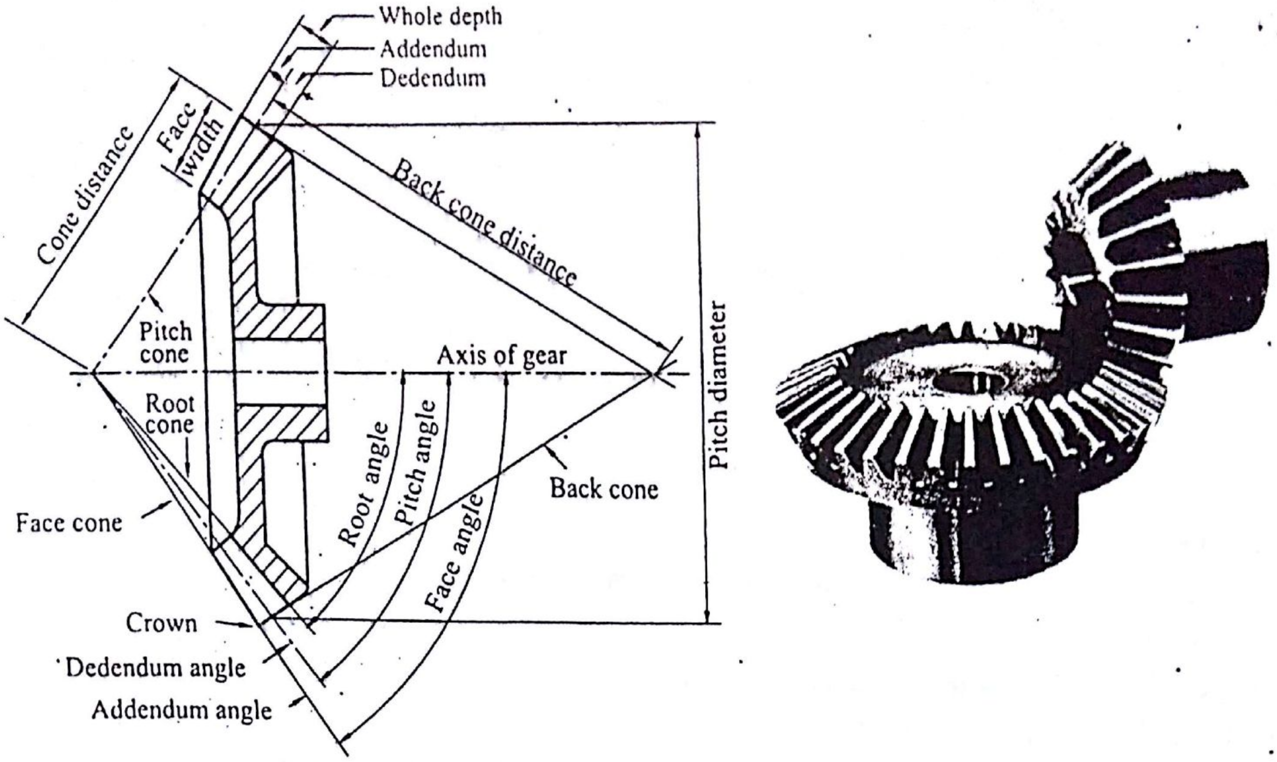
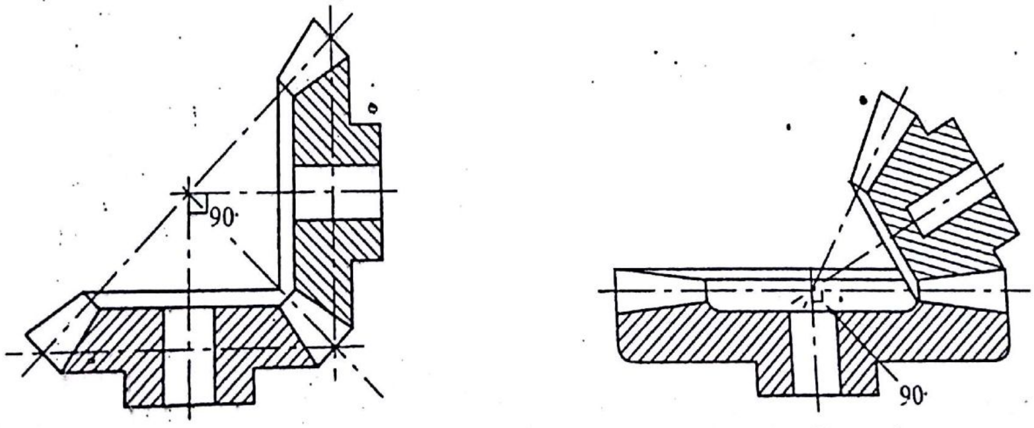


Fig. 16.18 Bevel gear

two shafts is usually the apex of both pitch cones. The simplest form of the bevel gear is the straight bevel gear. The tooth section becomes gradually smaller as the apex of the cone is approached. Straight bevel gears impose thrust and radial loads on their support bearings. The bevel gear terminology is shown in fig. 16.18.

Miter gear : A pair of bevel gears of the same size on shafts intersecting at right angles is called *miter gears* and is shown in fig. 16.19 (a).



(a) (b)

Fig. 16.19 (a) Miter gear, (b) Crown gear

Crown gear : If the pitch angle of the bevel gear becomes 90° as shown in fig. 16.19(b), the gear is called *crown gear*. The crown gear in bevel gearing corresponds to the rack in spur gearing.

Worm gears

Worm gearing is a type of screw gearing. This type of gearing is used for transmitting power between non-intersecting and non-parallel shafts. The usual shaft angle is 90°. A simple worm gear combination consists of a screw meshing with a helical gear as shown in fig. 16.20. The screw is called the *worm* and the gear the *worm gear* or *worm wheel*. Worm gearing is used to provide high angular velocity reduction.

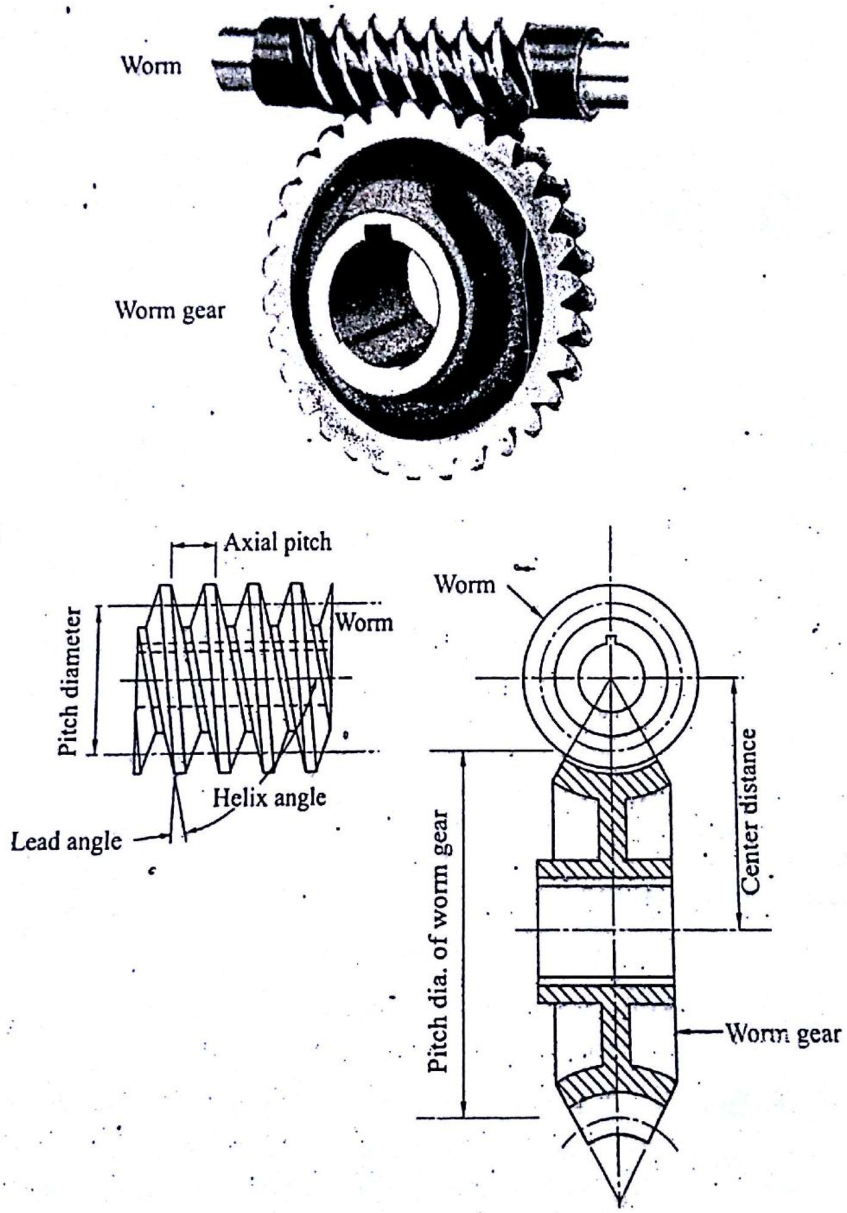


Fig. 16.20 Worm gear

The hand of the helix is the same for both mating members. The static capacity is curtailed by the high generation of heat produced by sliding across the face of the teeth. Worm gears are some times used where irreversibility in a mechanism is required.

Velocity ratio of worm gearing $= \frac{n_w}{n_g} = \frac{z_g}{z_w}$

where n_w and n_g are the speed of the worm and worm gear, z_w is the number of threads (starts) on the worm and z_g is the number of teeth on the worm gear.

Advantages and disadvantages of worm gear

Advantages:

1. High speed ratios with comparatively small dimensions of the drive.
2. Noiseless operation.
3. Irreversibility (self locking property).

Disadvantages:

1. High sliding velocities across the teeth.
2. High power losses due to friction.
3. More cost and less efficiency.

GEAR TRAINS

A gear train is composed of two or more gears in mesh for the purpose of transmitting motion or power from one shaft to another.

Train value

Train value is the ratio of the angular velocity of the driven gear to that of the driving gear. The train value is the reciprocal of the velocity ratio.

Direction of rotation: The direction of the gear's rotation is conventionally specified as clockwise or counter-clockwise. Clockwise rotation will be regarded as positive and counter clockwise as negative. When gears mesh externally they rotate in opposite directions and when the gears mesh internally they rotate in the same direction.

Types of gear trains

The gear train may be classified broadly into the following:

1. Simple gear train, 2. Compound gear train
3. Reverted gear train and 4. Epicyclic gear train

Simple gear train: A simple gear train is one in which each shaft carries only one gear as shown in fig. 16.21. If gear 1 is the driver and gear 3 is the driven, then the motion is transmitted from 1 to 3 through the intermediate gear 2, known as *idler*. An idler does not affect the train value; it serves only to fill up space and reverses the direction of rotation. It should be noted that, if an odd number of idlers are used, the first and the last shafts rotate in the same direction; if an even number or no idlers are used the first and last shafts rotate in the opposite directions.

Referring to the fig. 16.21 and using n_1 and z_1, n_2 and z_2, n_3 and z_3 to represent the speeds and number of teeth of gears 1, 2 and 3 respectively. For each pair of meshing gears, the angular velocities vary inversely as the radii and, therefore as their numbers of teeth.

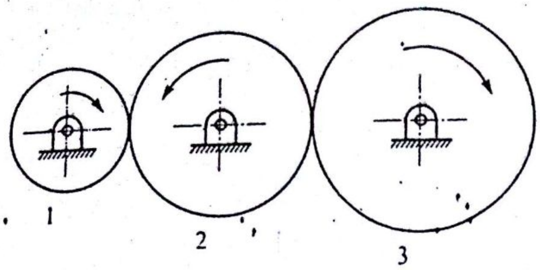


Fig. 16.21 Simple gear train

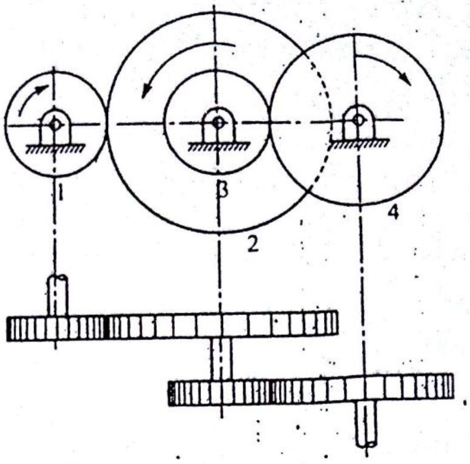


Fig. 16.22 Compound gear train

From gears 1 and 2

$$\frac{n_1}{n_2} = \frac{z_2}{z_1} \qquad \text{.....(1)}$$

Similarly from gears 2 and 3

$$\frac{n_2}{n_3} = \frac{z_3}{z_2} \qquad \text{.....(2)}$$

Multiplying (1) and (2) we get

$$\frac{n_1}{n_2} \times \frac{n_2}{n_3} = \frac{z_2}{z_1} \times \frac{z_3}{z_2} \qquad \text{.....(3)}$$

$\therefore$ Speed ratio $\quad \dfrac{n_1}{n_3} = \dfrac{z_3}{z_1}$

Train value $\quad \dfrac{n_3}{n_1} = \dfrac{z_1}{z_3}$

i.e., $\quad$ Train value $= \dfrac{\text{Speed of the last shaft}}{\text{Speed of the first shaft}} = \dfrac{\text{Number of teeth on driver}}{\text{Number of teeth on driven}}$

Compound gear train: A compound gear train is one in which each shaft caries two or more gears and are keyed to it. Fig. 16.22 represents a compound gear train in which gears 2 and 3 constitute a compound gear.

From gears 1 and 2

$$\frac{n_1}{n_2} = \frac{z_2}{z_1} \qquad \text{.....(1)}$$

Similarly from gears 3 and 4

$$\frac{n_3}{n_4} = \frac{z_4}{z_3} \qquad \text{.....(2)}$$

Multiplying (1) and (2) we get

$$\frac{n_1}{n_2} \times \frac{n_3}{n_4} = \frac{z_2}{z_1} \times \frac{z_4}{z_3} \qquad \text{.....(3)}$$

As gears 2 and 3 are compound gear, $n_2 = n_3$. Therefore the equation (3) becomes,

Speed ratio $$\frac{n_1}{n_4} = \frac{z_2}{z_1} \times \frac{z_4}{z_3}$$

Train value $$\frac{n_4}{n_1} = \frac{z_1}{z_2} \times \frac{z_3}{z_4}$$

i.e., $$\text{Train value} = \frac{\text{Speed of the last shaft}}{\text{Speed of the first shaft}} = \frac{\text{Product of teeth on driver}}{\text{Product of teeth on driven}}$$

The advantage of compound gear train is the larger velocity ratio in limited space.

Reverted gear train: A reverted gear train is one in which the first and last gears are on the same axis as shown in fig. 16.23. In reverted gear trains the center distances of the two pairs of gears must be the same, since the diametral pitch is the same for both pairs.

i.e., $$z_1 + z_2 = z_3 + z_4$$

Train value $$\frac{n_4}{n_1} = \frac{z_1}{z_2} \times \frac{z_3}{z_4} = \frac{\text{Product of teeth on driver}}{\text{Product of teeth on driven}}$$

Reverted gear trains are used in automotive transmissions, lathe back gears and in clocks.

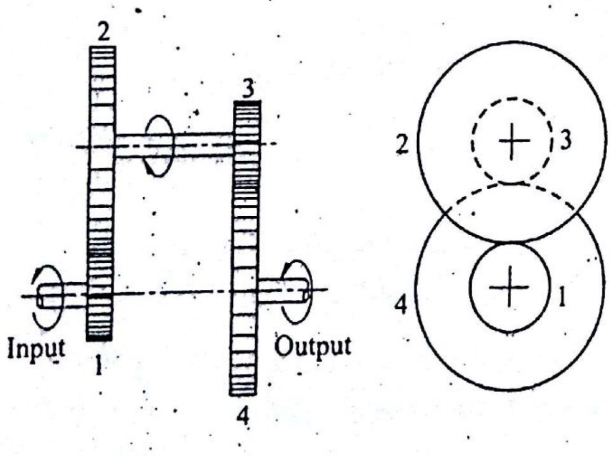


Fig. 16.23 Reverted gear train

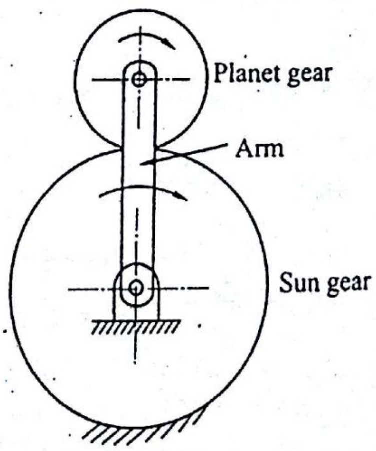


Fig. 16.24 Epicyclic gear train

Epicyclic gear train: An epicyclic gear train is one in which the axis of one or more gears moves relative to the frame. In other words, the arm instead of being fixed is turning about the axis of one of the gears of the train. Large speed reductions may be obtained with epicyclic train. If the fixed gear is an annular a compact unit results. It is used in speed reduction units and automobile differential.

In epicyclic gear train, the gear at the center is called the *sun*, and the gears whose axes move are called *planets*. In the arrangement shown in fig. 16.24 the sun gear is fixed to the frame. The planet gear carried by a revolving arm rotates not only about its own center but also about the center of the fixed gear.

Example 1 : A counter shaft has a pulley 1200 mm diameter keyed to it and it is to have a speed of 200 rpm. It is to be driven by an electric motor which has a speed of 1000 rpm. What diameter pulley should be fitted to the electric motor? Find the velocity ratio and the speed of the belt.

Data: $d_2 = 1200$ mm, $n_2 = 200$ rpm, $n_1 = 1000$ rpm

Solution:

$$\text{Velocity ratio} = \frac{n_1}{n_2} = \frac{d_2}{d_1}$$

$$\text{i.e.,} \quad \frac{1000}{200} = \frac{1200}{d_1}$$

$\therefore$ Diameter of the motor pulley $d_1 = 240$ mm

$$\text{Velocity ratio} = \frac{n_1}{n_2} = \frac{1000}{200} = \frac{5}{1} = 5:1$$

$$\text{Speed of belt} = \frac{\pi d_1 n_1}{60 \times 1000} = \frac{\pi d_2 n_2}{60 \times 1000}$$

$$= \frac{\pi \times 240 \times 1000}{60 \times 1000} = 12.57 \text{ m/s}$$

Example 2 : In a belt drive, the velocity ratio is 1/2. The driving pulley runs at 800 rpm and the diameter of the driven pulley is 0.4 m. Find the speed of the driven pulley and the diameter of the driving pulley.

Data: $\frac{n_1}{n_2} = \frac{1}{2}$, $n_1 = 800$ rpm, $d_2 = 0.4$ m $= 400$ mm

Solution:

$$\text{Velocity ratio} = \frac{n_1}{n_2}$$

$$\text{i.e.,} \quad \frac{1}{2} = \frac{800}{n_2}$$

$\therefore$ Speed of driven pulley $n_2 = 1600$ rpm

$$\text{also,} \quad \frac{n_1}{n_2} = \frac{d_2}{d_1}$$

Scanned by CamScanner

i.e., $\frac{1}{2} = \frac{400}{d_1}$

$\therefore$ Diameter of driver pulley d_1 = **800 mm**

Example 3 : The drive between an electric motor and the spindle is shown in fig. 16.25. The pulleys B and C are keyed to the same shaft. If the motor pulley revolves at 2000 rpm, find the speed of the machine spindle pulley.

Data: $d_A = 200$ mm, $d_B = 400$ mm, $d_C = 200$ mm, $d_D = 50$ mm, $n_A = 2000$ rpm

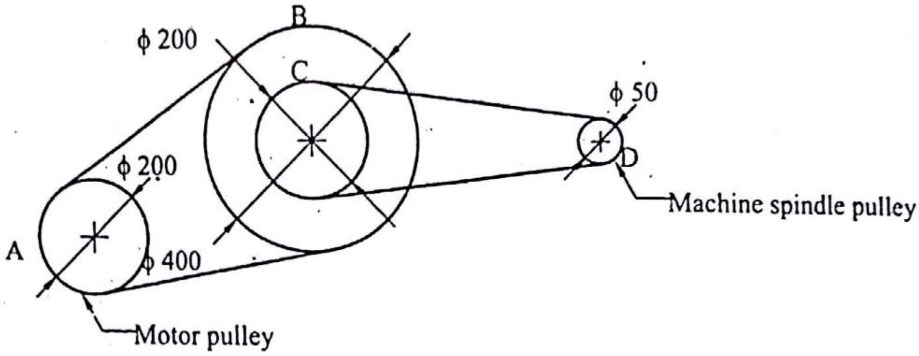


Fig. 16.25

Solution:

From pulleys A and B, velocity ratio is.

$$\frac{n_A}{n_B} = \frac{d_B}{d_A}$$

i.e., $$\frac{2000}{n_B} = \frac{400}{200}$$

$\therefore$ Speed of the pulley B is, $n_B = 1000$ rpm

Since pulleys B and C are keyed to the same shaft, $n_B = n_C$

$\therefore$ Speed of the pulley C is, $n_C = 1000$ rpm

From pulleys C and D, velocity ratio is $= \frac{n_C}{n_D} \quad \frac{d_D}{d_C}$

i.e., $$\frac{1000}{n_D} = \frac{50}{200}$$

$\therefore$ Speed of machine spindle n_D = **4000 rpm**

Example 4 : The sum of the diameters of two pulleys connected by a flat belt is 600 mm. If they run at 1400 rpm and 2100 rpm, determine the diameter of each pulley. Also find the length of open belt if the center distance between the two pulleys is 3 m. (VTU, June 2010)

14 LUBRICATION

Friction is the resistance to relative motion between the two surfaces in contact. Any substance placed between any two rubbing surfaces, which reduces friction is called *lubricant.*

The important functions of lubricant in bearings are:

1. To reduce friction between the sliding surfaces by separating them with thin film of oil.
2. To reduce wear and thereby increasing the life of bearing.
3. To remove the frictional heat from the bearing.
4. To provide protection against corrosion.

Types of lubricants

Lubricants are classified into the following three groups:

(i) Liquid, (ii) Semi-liquid, and (iii) Solid.

The liquid lubricants generally used in bearings are mineral oils, synthetic oils or animal and vegetable oils. The mineral oils are the most commonly used because of their lower cost and stability. Liquid lubricants are usually preferred where they may be retained.

Grease is a semi-liquid lubricant having higher viscosity than oils. Grease is employed where slow speed and heavy pressure exists and where oil drip from the bearings is undesirable.

Solid lubricants are useful in reducing friction where oil films cannot be maintained because of pressure or temperature. They should be softer than materials being lubricated. Graphite is the most common solid lubricant. Other solid lubricants are soapstone, talc, wax, mica, French chalk, etc.

Properties of lubricants

The following are the important properties of a lubricant.

Viscosity: Viscosity is a measure of internal resistance of fluid to shear and indicates its relative resistance to flow. When the oil is used as a lubricant, its viscosity is important because the load carrying capacity is proportional to the viscosity. High viscosity oil can support heavier loads and has more internal friction. Viscosity decreases with an increase in oil temperature. It is desirable that the change of viscosity with temperature be kept to a minimum.

Absolute or dynamic viscosity or coefficient of viscosity η is defined as the force required to move a fat surface of unit area at unit velocity when separated by an oil film of unit thickness. In SI system of measurements the unit of viscosity is N-s/m^2 (Pa-s).

The *kinematic viscosity* υ is the absolute viscosity η divided by mass density ρ i.e., $\upsilon = \eta / \rho$. The SI unit is m^2/s.

Flash point: Flash point is the minimum temperature at which an oil gives off sufficient vapour to ignite momentarily on introduction of a flame. A good lubricant should have the flash point above the operating temperature.

Fire point: Fire point is the lowest temperature at which an oil gives off sufficient vapour to burn continuously for at least five seconds on the introduction of a flame.

Pour point: Pour point is the lowest temperature at which an oil ceases to flow when cooled.

Cloud point: Cloud point is the temperature at which an oil becomes cloudy in appearance when cooled.

Oiliness: Oiliness is the ability of an oil to maintain an unbroken lubricating film between the rubbing surfaces.

Viscosity index: Viscosity index is used to denote the degree of variation of viscosity with temperature.

Requirements of a good lubricant

1. It must have sufficient viscosity to build up the necessary pressure to keep the solid surfaces apart.
2. Minimum film strength.
3. High flash point and fire point.
4. Non-volatile.
5. Free from the corrosive acids.
6. It should have physical stability with regard to temperature and pressure.
7. Chemical stability against oxidation.
8. Proper fluidity at low temperatures.
9. Resistance to emulsion.

Lubricator

A *lubricator* is a device used to supply lubricant continuously and at a regulated rate. Some of the important types of lubricators are discussed below:

Drop-feed oiler: Fig. 14.1 shows a drop feed oiler. It consists of a glass container or reservoir with a metal base, which has a drip hole at the center. The rate of feed is adjusted by means of a screw which slightly raises or lowers the needle, and can be seen through a glass window. Drop feed oilers are commonly used on high grade machinery and give good service, if the reservoirs are not allowed to run dry. They have the objectionable feature that the rate of oil delivery varies with the head of oil in the reservoir and with the oil temperature.

Wick feed oiler (Syphon wick lubricator): A wick feed lubricator is shown in fig. 14.2. It works on the principle of the capillary action of some absorbent material in carrying oil to the bearing. It consists of a glass container or reservoir with a central

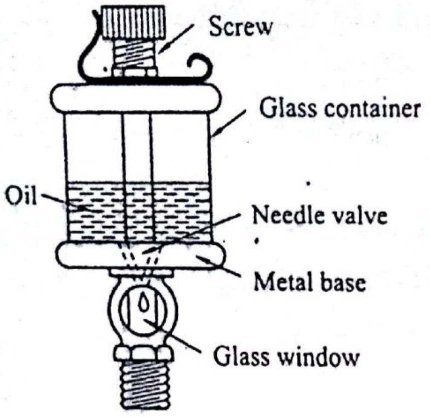


Fig. 14.1 Drop feed oiler

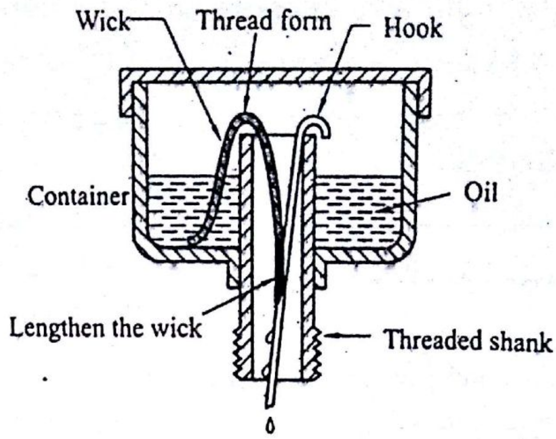


Fig. 14.2 Wick feed oiler

pipe and a shank, which can be screwed into the oil hole. Oil is drawn from the reservoir and down the bearing through a wick of wool yarn, the rate of flow being regulated by the number of strands of the yarn. For uniform rate of feed, the discharge end of the wick should be 50 mm below the lowest level of oil in the reservoir. Wick feed devices tend to feed at all times, even when the equipment is not running. To stop the feed it is necessary to lift the wick out of the pipe by means of a suspension hook.

Bottle oiler or needle lubricator: A bottle oiler is shown in fig. 14.3. It consists of an inverted glass or plastic container with a needle passing from the oil reservoir through a wooden stopper to the bearing. The needle rests on the journal and is loosely fitted to the stopper. When the journal rotates, the needle is shaken by the irregularities on journal surface and oil passes from the oil reservoir, through the gap between the needle and the stopper to the bearing. When the journal is stationary, there is no oil feed to the bearing.

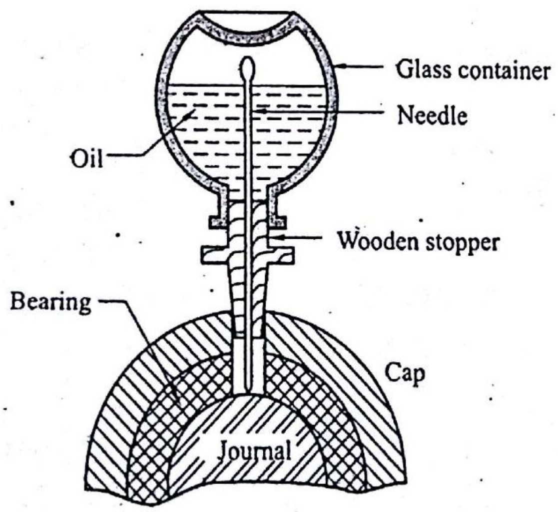


Fig. 14.3 Bottle oiler

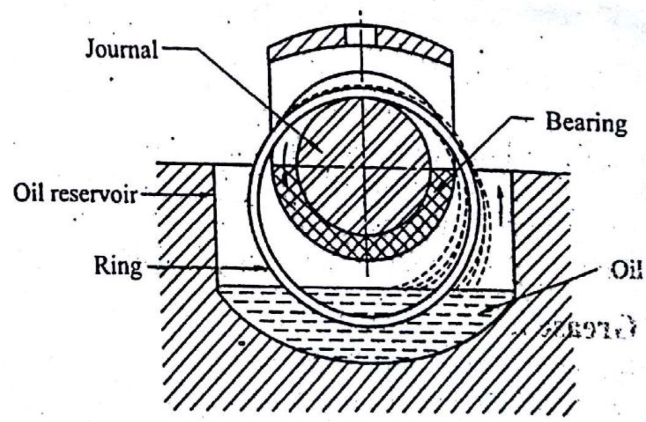


Fig. 14.4 Ring oiler

Ring oiler: Ring oiled bearing illustrated in fig. 14.4 employs one or more rings which hang loosely over the journal and revolve with it through friction. The lower part of the ring dipping in the oil carries it to the top of the journal where it is distributed through oil grooves to the bearing surfaces. The ring oiling method is considered as one of the most reliable methods, and is extensively used for line shafting and horizontal machine bearings. The ring oiler is not very satisfactory at high speed as the oil may be thrown off due to centrifugal force and the rings may slip very much from the journal. The advantages of ring oiling are uniform lubrication, greater oil economy, cleanliness and less attention required.

Splash lubrication: This method of lubrication is employed in machines that have cranks or gears enclosed in a housing, which acts as a reservoir for oil. As the moving parts dip in the oil, a spray is formed which reaches the cylinder walls, piston rings, crank pin and crankshaft bearings as shown in fig. 14.5. With oil level properly maintained, a constant supply of oil is delivered to the bearings.

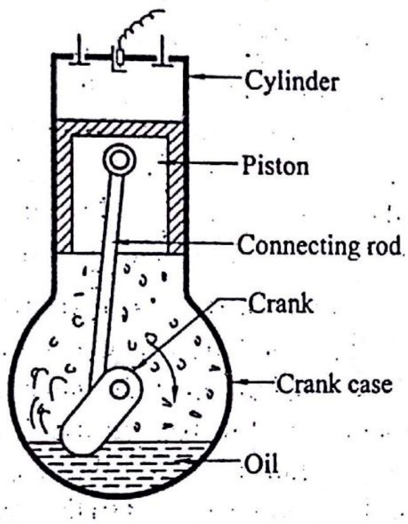


Fig. 14.5 Splash lubrication

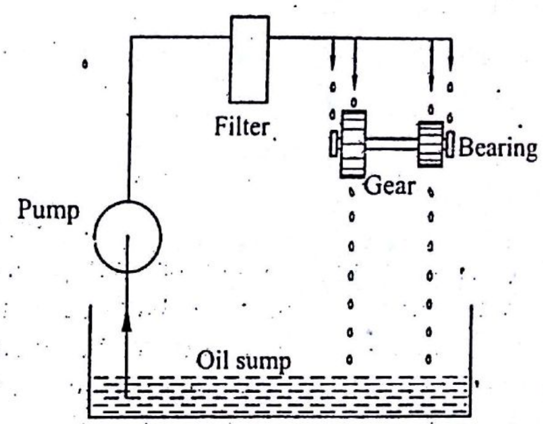


Fig. 14.6 Pressure lubrication

Pressure lubrication: Bearing may be positively lubricated by introducing the lubricant under pressure at proper points on the bearing. Oil is sucked by a pump from the oil sump, delivered under pressure to the lubricating points through a filter as shown in fig. 14.6. From the bearing, oil returns by gravity to the sump and is re-circulated by the pump. The flow of oil may be in such quantities that most of the heat generated in the bearing will be carried away by the lubricant itself, so that the viscosity of the lubricant may thus be controlled. This is necessary in heavily loaded bearings.

Grease lubrication: Grease is used as a lubricant where dripping or spattering of oil is not permissible as in food or chemical processing equipment. Grease is not a mobile lubricant, hence it must be applied directly to the surface to be lubricated under pressure. Grease is used where the parts are loaded heavily and where the fluid film lubrication is not possible due to slow speeds.

Fig. 14.7 shows a simple grease lubricator manually operated by turning the cap on the cup and forcing out greases to the surface to be lubricated. It is also called *screw cap lubricator.*

Telltale lubricator: Fig. 14.8 shows a telltale lubricator. It consists of a container that carries a spring loaded piston. The pressure on the grease due to the spring force forces out the grease to the surface to be lubricated. The aperture through which the grease passed can be varied by means of slotted screw. The movement of the piston rod end indicates whether the lubrication is in progress or not. This lubricator is best suitable where a continuous supply of grease flow is to be maintained.

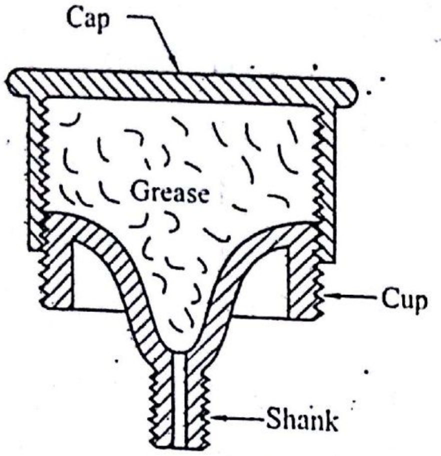


Fig. 14.7 Screw cap lubricator

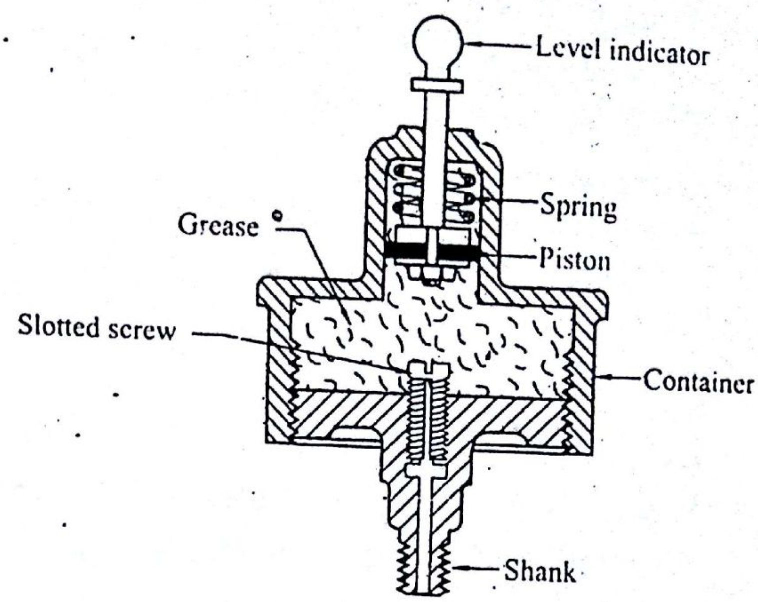


Fig. 14.8 Telltale lubricator

Choose the correct answer:

1. The shear resistance of an oil is called
   (a) Viscosity (b) Oiliness (c) Lubricant (d) Porosity
2. Viscosity of an oil increases with decrease in
   (a) Pressure (b) Temperature (c) Both (a) and (b) (d) None of the above
3. The unit of absolute viscosity is
   (a) Pa-s (b) m^2/s (c) Nm/s (d) MPa
4. Kinematic viscosity ν is
   (a) η/ρ (b) η/g (c) ρ/η (d) η
   where η = dynamic viscosity, ρ = mass density, g = acceleration due to gravity
5. Solid lubricant is
   (a) Grease (b) Graphite (c) Mineral oil (d) Synthetic oil
6. Grease is
   (a) Liquid lubricant (b) Solid lubricant
   (c) Semi-liquid lubricant (d) Semi-solid lubricant