

We give a map reduction from PCP to $AMBIG_{CFG}$.

For an PCP instance $\{(a_i, b_i)\}_{i=1}^n$ where $a_i, b_i \in \Sigma^*$, We construct a CFG $G = (V, \Sigma', R, S)$ as following:

- $V = \{S, A, B, A', B'\}$.
- $\Sigma' = \Sigma \cap [1, n]$. Here we consider every integer as one symbol.
- R contains the following rules:
 - $S \rightarrow A' | B'$.
 - $A \rightarrow \epsilon, B \rightarrow \epsilon$.
 - $A \rightarrow iAa_i$ for every $1 \leq i \leq n$.
 - $B \rightarrow iBb_i$ for every $1 \leq i \leq n$.
 - $A' \rightarrow iAa_i$ for every $1 \leq i \leq n$.
 - $B' \rightarrow iBb_i$ for every $1 \leq i \leq n$.

G does not accept ϵ . For every string $w = st$ where $s \in [1, n]^+$ and $t \in \Sigma^*$, G accepts w first changing S to A' if and only if $t = a_{s_{|S|}} \cdots a_{s_1}$; G accepts w first changing S to B' if and only if $t = b_{s_{|S|}} \cdots b_{s_1}$. So G accepts a string w with two distinct parse tree if and only if the PCP instance is a YES instance.