ABV-Indian Institute of Information Technology and Management Gwalior

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Lucas & Kanade for Optical Flow

(Estimating Velocities of Brightness Pattern in a Video)

(ITIT-9507)

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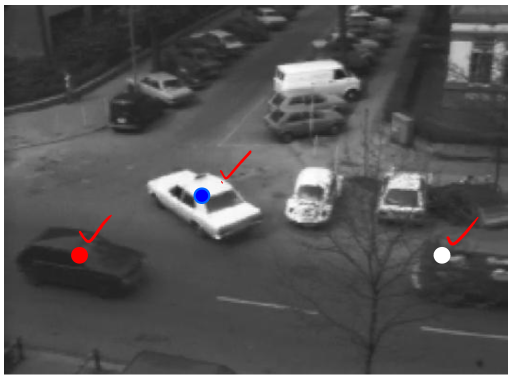
Optical Flow : An Introduction

Hamburg Taxi Sequence

Observe the motion of brightness pattern (intensity values) in each of the frames of a video

Optical Flow : An Introduction

Hamburg Taxi Sequence

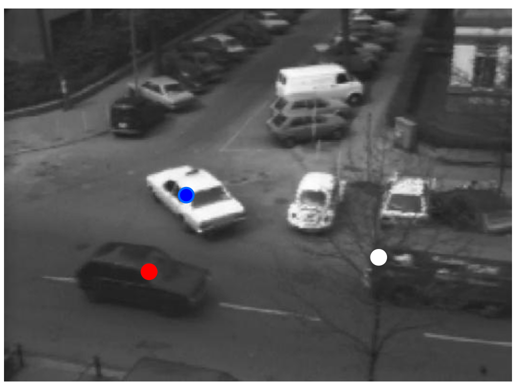


Frame at $t = 2\ sec$

Frame at $t = 4\ sec$

Frame at $t = 6\ sec$

Frame at $t = 8\ sec$



Frame at $t = 10\ sec$

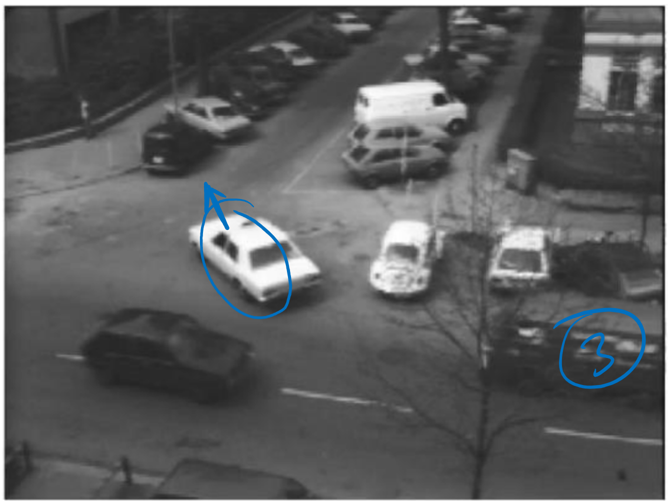
Frame at $t = 12$ sec

Frame at $t = 14$ sec

Frame at $t = 16\ sec$

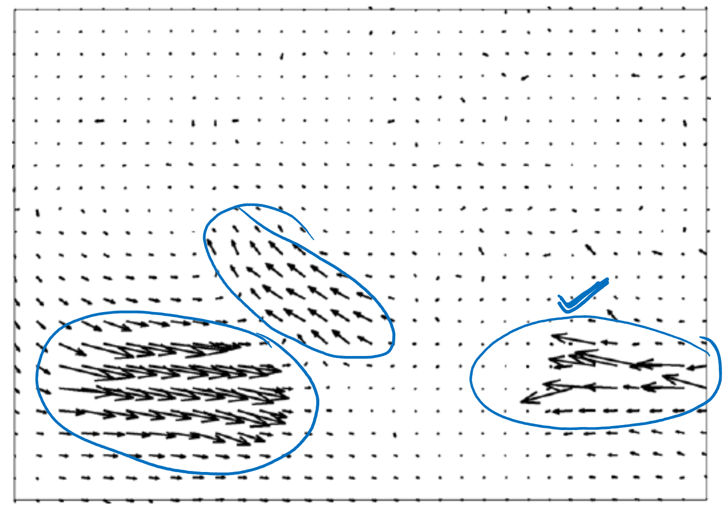
Optical Flow : An Introduction

Consider two frames at time "t" and "$t + dt$"



Frame at t

Frame at $t + dt$

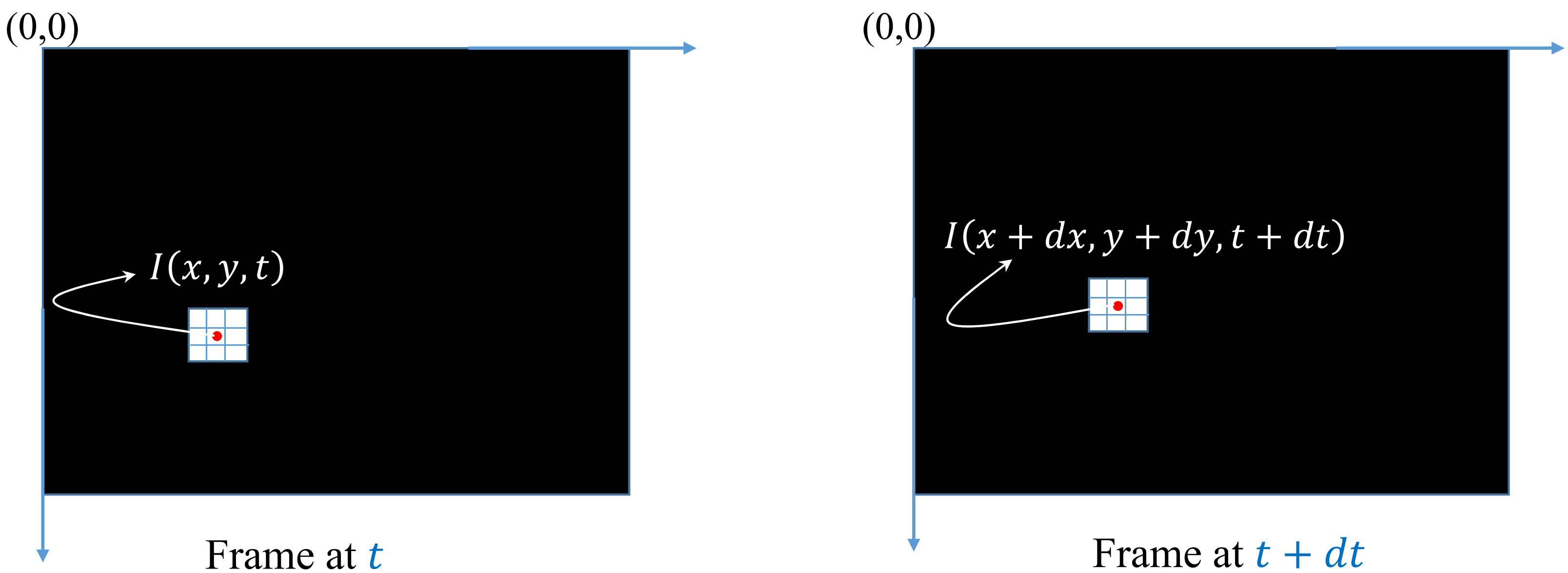


Optical flow of intensities

- **Optical flow :** Rate of change of displacement of brightness pattern (intensity values) in an image at time t w.r.t image at time "$t + dt$"

Optical Flow : An Introduction

- **Optical flow :** Rate of change of displacement of brightness pattern (intensity values) in an image at time t w.r.t image at time "$t + dt$"



Optical flow : $(u, v)|_{I(x,y,t)} = \left(\frac{dx}{dt}, \frac{dy}{dt}\right)$

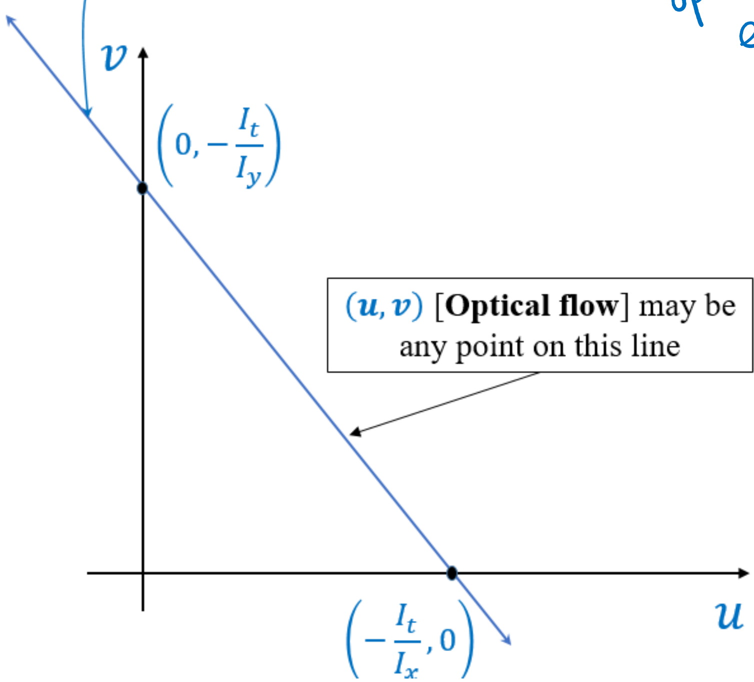
Geometrical Interpretation of Optical Flow Eqn.

Brightness Constancy Assumption

$$\boxed{I(x+dx, y+dy, t+dt) = I(x, y, t)}$$

$$\boxed{I_x u + I_y v + I_t = 0}$$

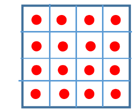
optical flow eqn

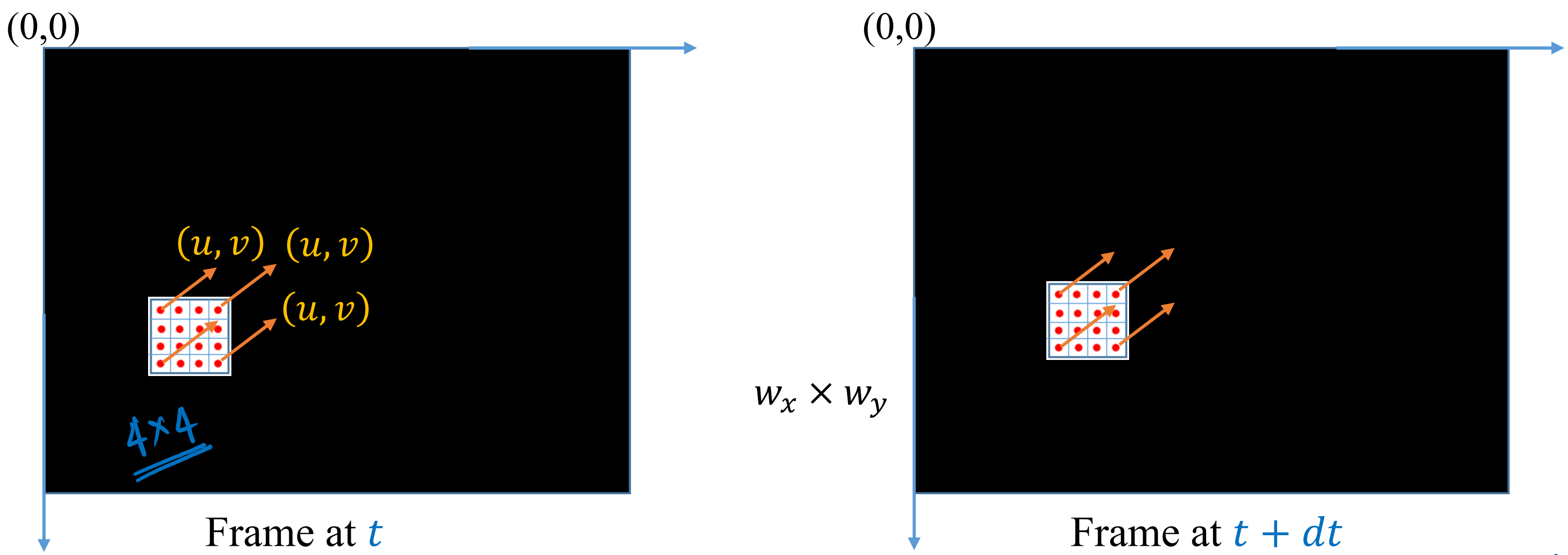


❑ So, if every point of the brightness pattern move independently, there is little hope to get the accurate optical flow $(\boldsymbol{u}, \boldsymbol{v})$.

❖ So, we need some other constraints in order to find the exact optical flow $(\boldsymbol{u}, \boldsymbol{v})$

Lucas-Kanade Optical Flow Assumption



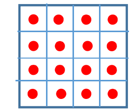


$(u,v) \equiv \begin{bmatrix} u \\ v \end{bmatrix}$

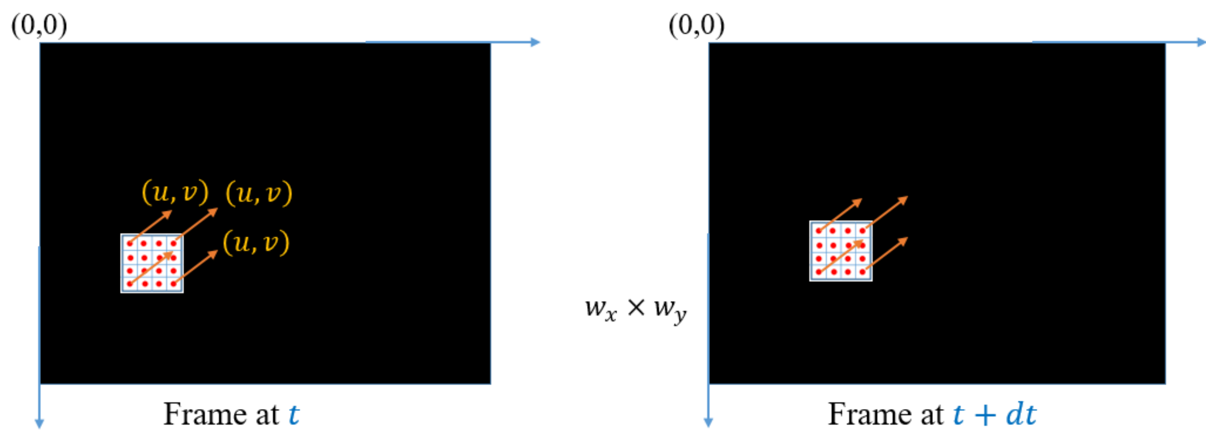
- ❑ Lucas and Kanade [1] proposed to assume that the unknown optic flow vector is constant within some neighbourhood of size $w_x \times w_y$

[1] B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proc. Seventh International Joint Conference on Artificial Intelligence*, pages 674–679, Vancouver, Canada, Aug. 1981.

Lucas-Kanade Optical Flow Equations



- Each pixel of the neighbourhood will give one equation, and so there will be $w_x \times w_y = n$ number of equations.



$I_{x1}u + I_{y1}v + I_{t1} = 0 \longrightarrow (1)$

$I_{x2}u + I_{y2}v + I_{t2} = 0 \longrightarrow (2)$

$\vdots$

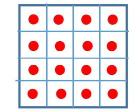
$I_{xn}u + I_{yn}v + I_{tn} = 0 \longrightarrow (n)$

System of optical flow equations in matrix form

$$
\left.
\begin{aligned}
(1) \Rightarrow \quad & I_{x1} u + I_{y1} v = -I_{t1} \\
& I_{x2} u + I_{y2} v = -I_{t2} \\
& \vdots \\
& I_{xn} u + I_{yn} v = -I_{tn}
\end{aligned}
\right\} \Rightarrow
$$

$$
\Rightarrow \begin{bmatrix} I_{x1} & I_{y1} \\ I_{x2} & I_{y2} \\ \vdots & \vdots \\ I_{xn} & I_{yn} \end{bmatrix}_{n\times 2} \begin{bmatrix} u \\ v \end{bmatrix}_{2\times 1} = \begin{bmatrix} -I_{t1} \\ -I_{t2} \\ \vdots \\ -I_{tn} \end{bmatrix}_{n\times 1}
$$

Lucas-Kanade Optical Flow Equations



$$\Rightarrow \begin{bmatrix} I_{x1} & I_{y1} \\ I_{x2} & I_{y2} \\ \vdots & \vdots \\ I_{xn} & I_{yn} \end{bmatrix}_{\underline{n\times 2}} \begin{bmatrix} u \\ v \end{bmatrix}_{2\times 1} = \begin{bmatrix} -I_{t1} \\ -I_{t2} \\ \vdots \\ -I_{tn} \end{bmatrix}_{n\times 1}$$

✓ Overconstrained system of linear equations

$$\Rightarrow \mathbf{Ax} = \mathbf{y} \longrightarrow x = A^{-1}y \quad \times$$

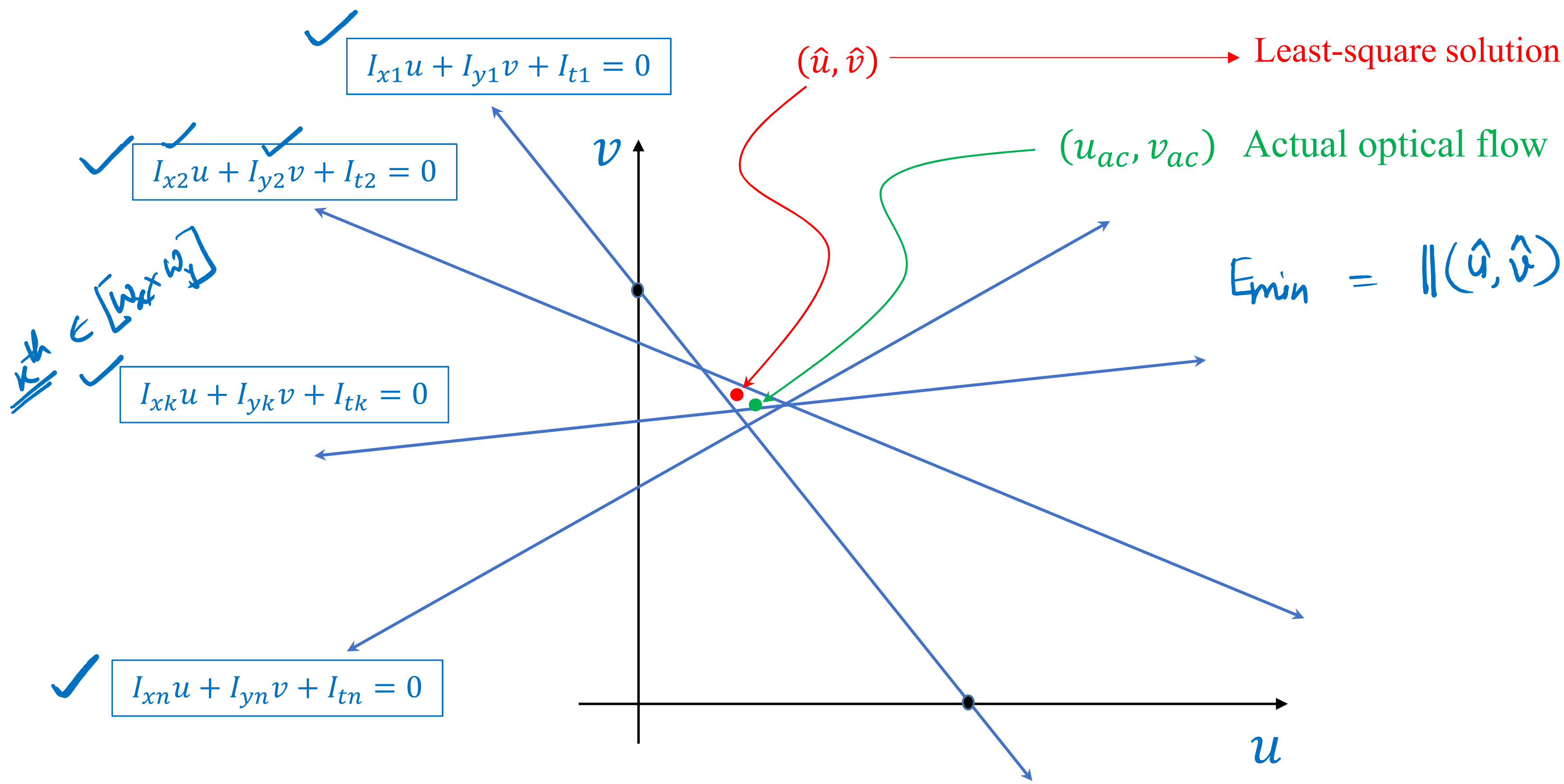
$$\boxed{A' = A^T}$$

$$\Rightarrow \underline{\mathbf{A}'}\ \mathbf{Ax} = \mathbf{A}'\mathbf{y}$$

$$\Rightarrow \mathbf{x} = \underbrace{(\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}'\mathbf{y}} \quad \equiv$$

$$\Rightarrow x = \begin{bmatrix} u \\ v \end{bmatrix}$$

Geometrical Interpretation of Lucas-Kanade Optical Flow Solution



$$E_{min} = \|(\hat{u}, \hat{v}) - (u_{ac}, v_{ac})\|_2$$

Least-Square Method for Lucas-Kanade Optical Flow Vector

$$I_x u_{ac} + I_y v_{ac} + I_t = 0$$

For a pixel at (x, y, t)

- Let the estimated optical flow vector is $(u, v) \neq (u_{ac}, v_{ac})$

$$I_{x1} u + I_{y1} v + I_{t1} = \varepsilon_1$$
$$I_{x2} u + I_{y2} v + I_{t2} = \varepsilon_2$$
$$\vdots$$
$$I_{xn} u + I_{yn} v + I_{tn} = \varepsilon_n$$

$\varepsilon_k = (I_{xk} u + I_{yk} v + I_{tk})$

- **Cost/Objective** function -

$$J(u, v) = \sum_{k=1}^{n} \varepsilon_k^2 = \sum_{k=1}^{n} \left(\underline{I_{xk} u + I_{yk} v + I_{tk}} \right)^2$$

Least-Square Method for Lucas-Kanade Optical Flow Vector

- **Cost/Objective** function -

$$J(u,v) = \sum_{k=1}^{n} \varepsilon_k^2 = \sum_{k=1}^{n} \left(I_{xk}u + I_{yk}v + I_{tk}\right)^2$$

$$\Rightarrow \frac{\partial J}{\partial u} = \sum \frac{\partial}{\partial u}\left(I_{xk}u + I_{yk}v + I_{tk}\right)^2$$

$$= \sum 2\left(I_{xk}u + I_{yk}v + I_{tk}\right) \times I_{xk}$$

$$= 0$$

- To find u and v such that $J(u,v)$ is minimized :

$$\Rightarrow \frac{\partial J(u,v)}{\partial u} = 0 \quad \text{and} \quad \frac{\partial J(u,v)}{\partial v} = 0$$

$$\Rightarrow \sum_{k=1}^{n} \left(I_{xk}u + I_{yk}v + I_{tk}\right) I_{xk} = 0 \qquad \Rightarrow \sum_{k=1}^{n} \left(I_{xk}u + I_{yk}v + I_{tk}\right) I_{yk} = 0$$

$$\Rightarrow \sum_{k=1}^{n} I_{xk}^2\, u + \sum_{k=1}^{n} I_{yk}I_{xk}\, v = -\sum_{k=1}^{n} I_{tk}I_{xk} \quad \rightarrow (1)$$

$$\Rightarrow \sum_{k=1}^{n} I_{xk}I_{yk}\, u + \sum_{k=1}^{n} I_{yk}^2\, v = -\sum_{k=1}^{n} I_{tk}I_{yk} \quad \rightarrow (2)$$

Least-Square Method for Lucas-Kanade Optical Flow Vector

❑ **Lucas-Kandane Optical Flow Vector -**

$$\Rightarrow \sum_{k=1}^{n} I_{xk}^2\, u + \sum_{k=1}^{n} I_{yk} I_{xk}\, v = -\sum_{k=1}^{n} I_{tk} I_{xk} \longrightarrow (1)$$

$$\Rightarrow \sum_{k=1}^{n} I_{xk} I_{yk}\, u + \sum_{k=1}^{n} I_{yk}^2\, v = -\sum_{k=1}^{n} I_{tk} I_{yk} \longrightarrow (2)$$

$$\Rightarrow \begin{bmatrix} \sum_{k=1}^{n} I_{xk}^2 & \sum_{k=1}^{n} I_{yk} I_{xk} \\ \sum_{k=1}^{n} I_{xk} I_{yk} & \sum_{k=1}^{n} I_{yk}^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -\sum_{k=1}^{n} I_{tk} I_{xk} \\ -\sum_{k=1}^{n} I_{tk} I_{yk} \end{bmatrix}$$

Least-Square Method for Lucas-Kanade Optical Flow Vector

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

- **Lucas-Kandane Optical Flow Vector -**

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{n} I_{xk}^2 & \sum_{k=1}^{n} I_{yk} I_{xk} \\ \sum_{k=1}^{n} I_{xk} I_{yk} & \sum_{k=1}^{n} I_{yk}^2 \end{bmatrix}^{-1} \begin{bmatrix} -\sum_{k=1}^{n} I_{tk} I_{xk} \\ -\sum_{k=1}^{n} I_{tk} I_{yk} \end{bmatrix}$$

2x2

✓

$$\begin{bmatrix} u \\ v \end{bmatrix} = \underbrace{\frac{1}{\sum_{k=1}^{n} I_{xk}^2 \sum_{k=1}^{n} I_{yk}^2 - \sum_{k=1}^{n} I_{yk} I_{xk} \sum_{k=1}^{n} I_{xk} I_{yk}} \begin{bmatrix} \sum_{k=1}^{n} I_{yk}^2 & -\sum_{k=1}^{n} I_{yk} I_{xk} \\ -\sum_{k=1}^{n} I_{xk} I_{yk} & \sum_{k=1}^{n} I_{xk}^2 \end{bmatrix}} \begin{bmatrix} -\sum_{k=1}^{n} I_{tk} I_{xk} \\ -\sum_{k=1}^{n} I_{tk} I_{yk} \end{bmatrix}$$

Lucas-Kanade Vs Horn-Shunck Optical Flow

❑ **Lucas & Kandane Optical Flow Vector**

❖ $$J(u,v) = \sum_{k=1}^{n} \varepsilon_k^2 = \sum_{k=1}^{n} \left(I_{xk}u + I_{yk}v + I_{tk}\right)^2$$

❖ More robust to noise and occlusion

❖ Good to find optical flow vectors for some of the selected interest points (sparse optical flow)

❑ **Horn & Shunck Optical Flow Vector**

❖ $$J(u,v) = \iint_{x,y} \left[\left(I_x u + I_y v + I_t\right)^2 + \lambda\left(u_x^2 + u_y^2 + v_x^2 + v_x^2\right)\right] dxdy$$

❖ Less robust to noise and occlusion

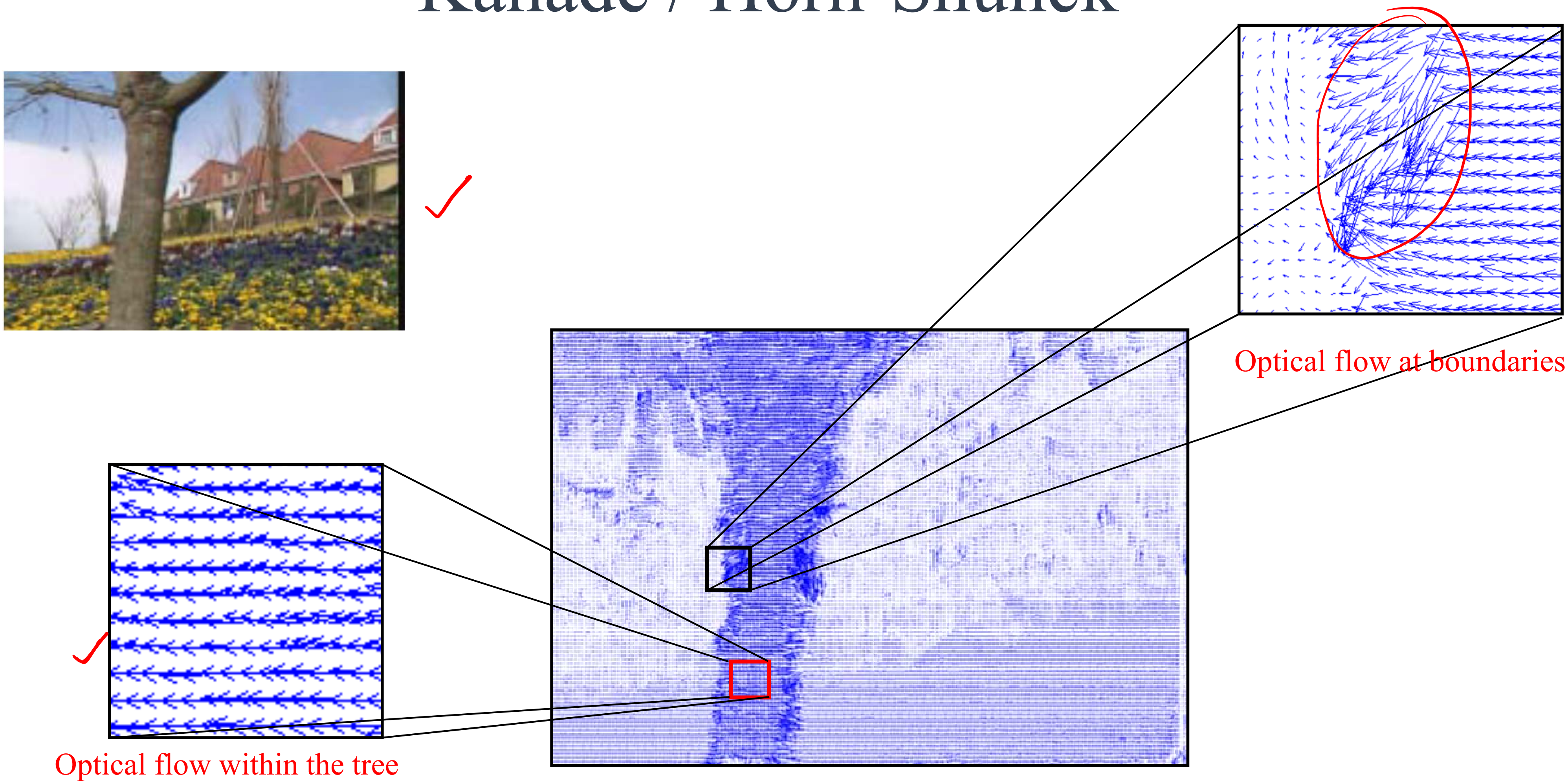
❖ Give dense optical flow (velocities of all the pixels in an image)

Downside of Computing Optical Flow using Lucas-Kanade / Horn-Shunck

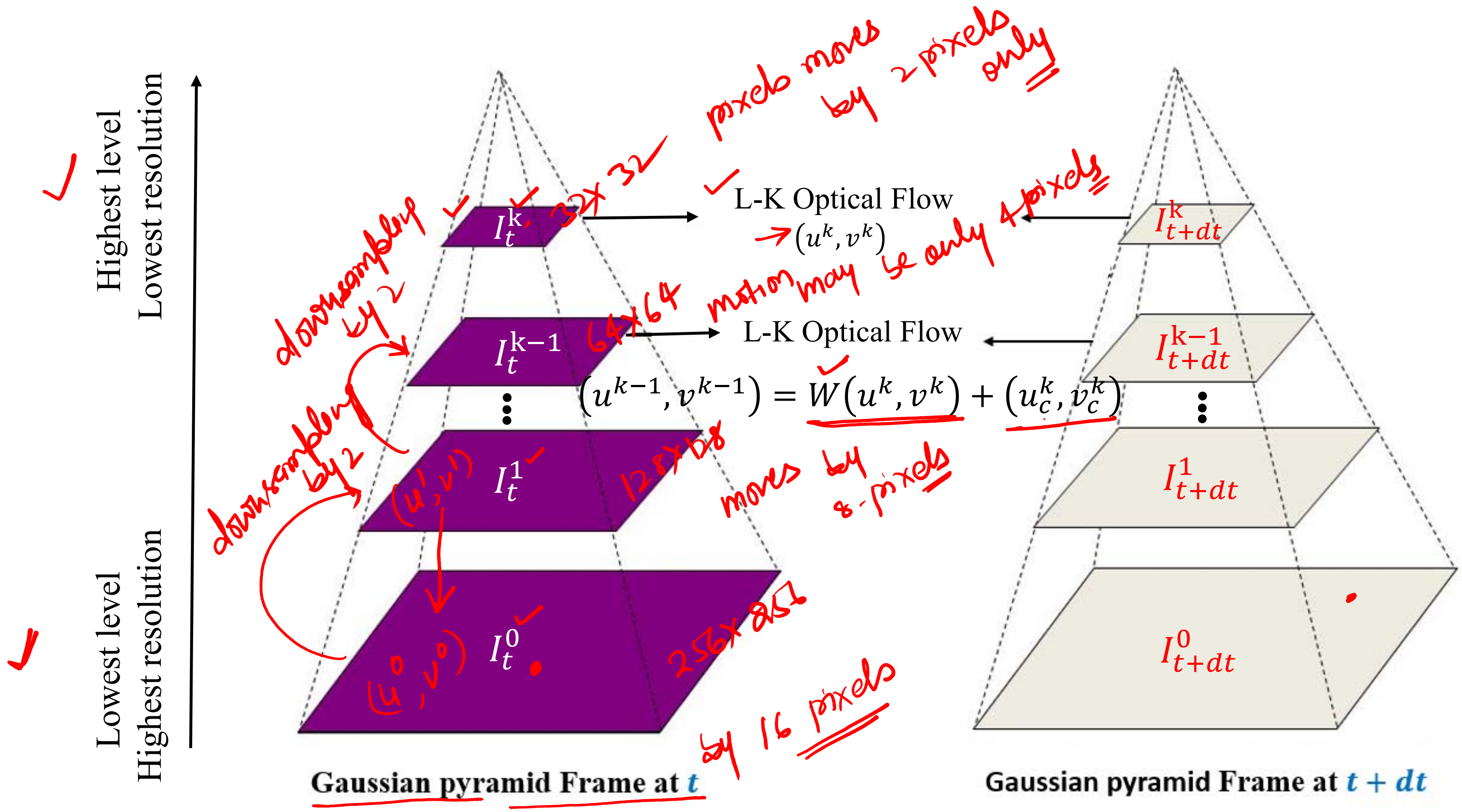
- ❑ Both the methods have their own advantages and disadvantages
  - ❑ Combine advantages of both and come up with efficient optical flow detector [1]
- ❑ Suitable for slow motion
- ❑ For large motion and/or at discontinuities, the optical flow obtained using above methods may not correct.

[1] Bruhn, Andrés, Joachim Weickert, and Christoph Schnörr. "Combining the advantages of local and global optic flow methods." *Joint Pattern Recognition Symposium*. Springer, Berlin, Heidelberg, 2002.

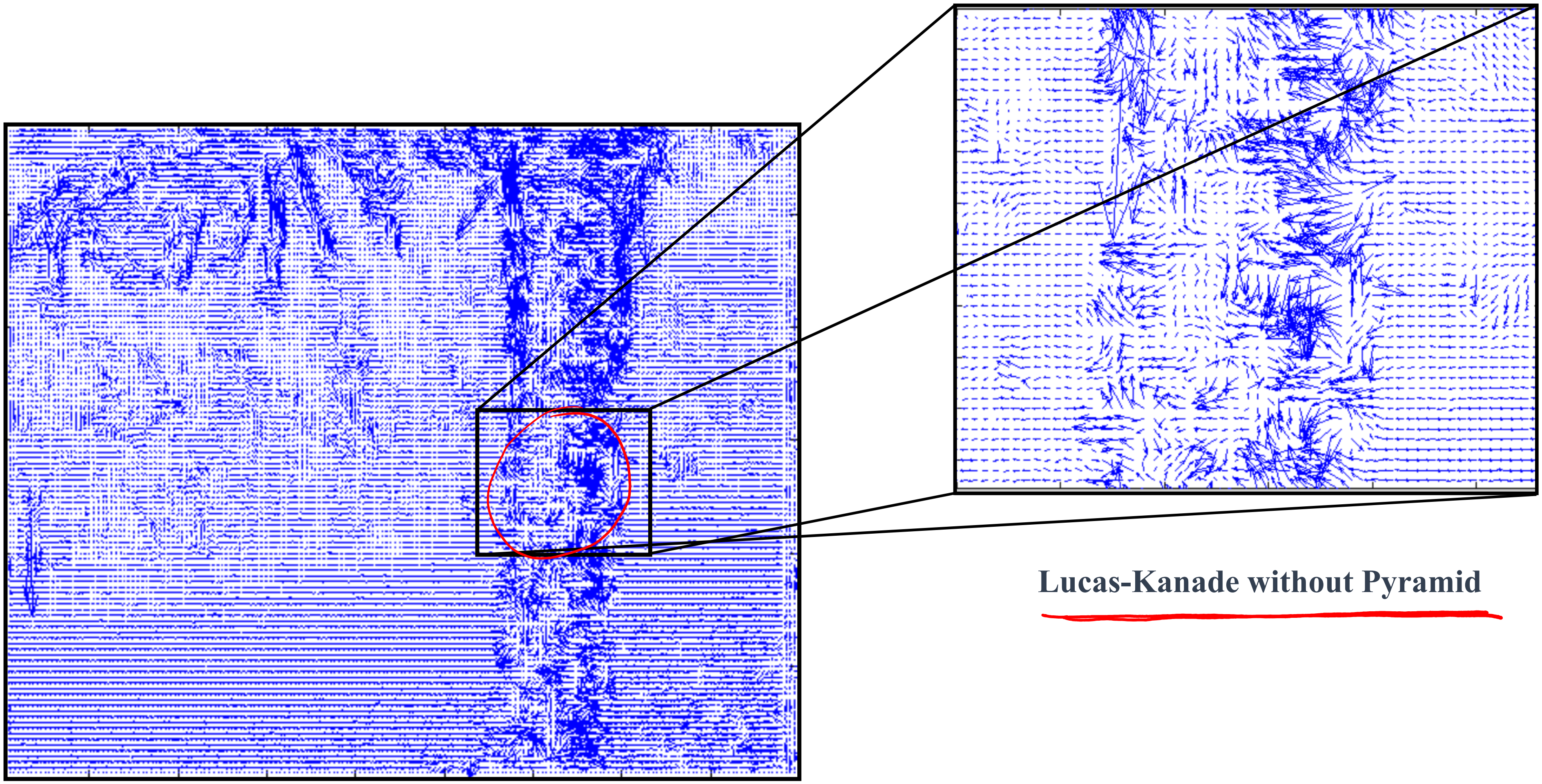
Downside of Computing Optical Flow using Lucas-Kanade / Horn-Shunck



Lucas-Kanade with Pyramid

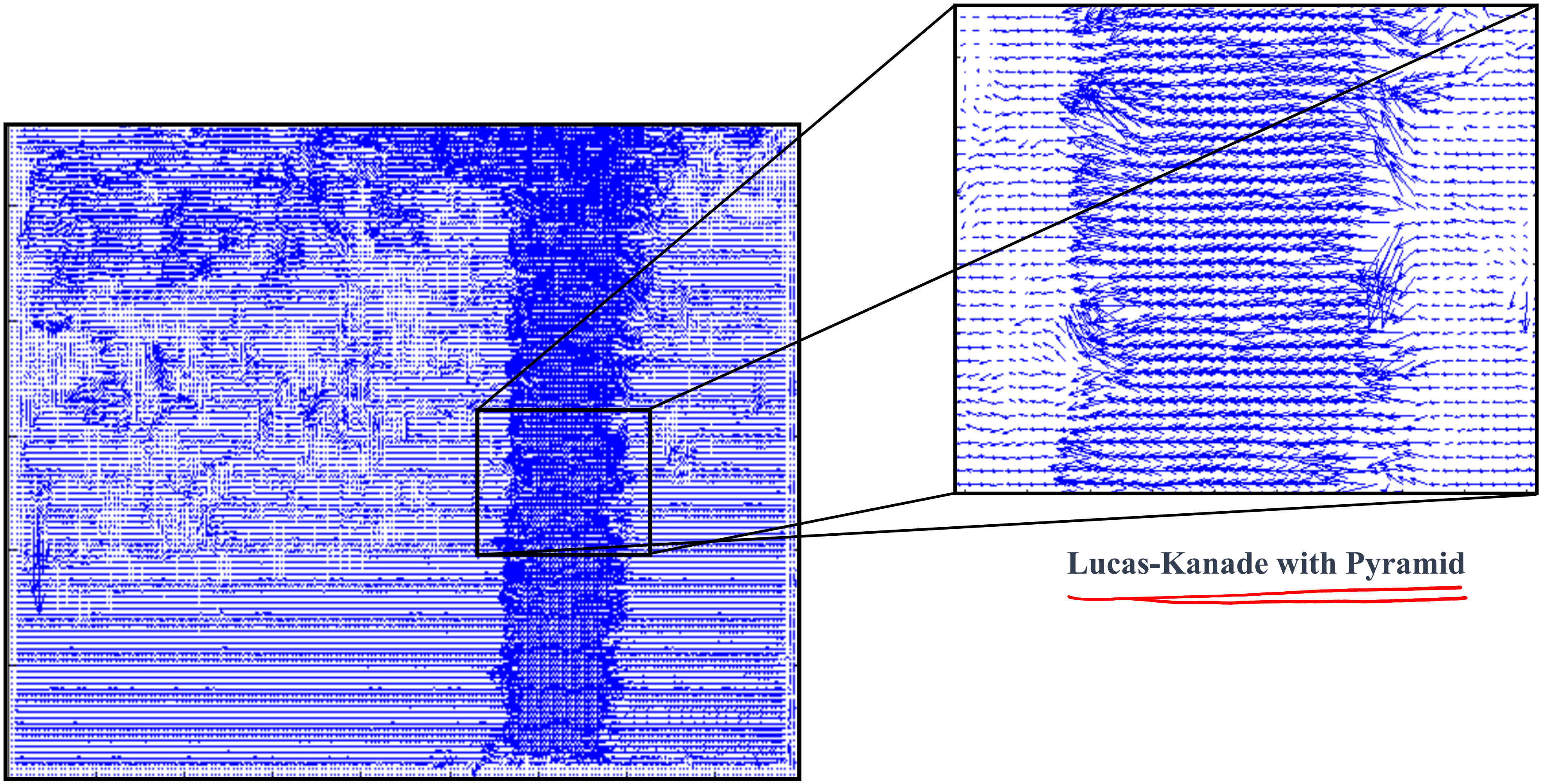


Results of Lucas-Kanade with/without Pyramid



Lucas-Kanade without Pyramid

Results of Lucas-Kanade with/without Pyramid



Lucas-Kanade with Pyramid

Assignment

Pyramidal Implementation of the Affine Lucas Kanade Feature Tracker Description of the algorithm

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Microprocessor Research Labs
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Reference

- Richard Szeliski, Computer Vision: Algorithms and Applications, Springer, 2010 (online draft),
- Mubarak Shah, "Fundamentals of Computer Vision" (Online available)
- Ian Goodfellow, Yoshua Bengio and Aaron Courville, "Deep Learning" (Online available)

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Sources for this lecture include materials from works by Szeliski, Abhijit Mahalanobis, Sedat Ozer, Ulas Bagci, Mubarak Shah, Antonio Torralba, D. Hoiem, Justin Liang, and others. References are given for the source image contents.

Queries!