ABV-Indian Institute of Information Technology and Management Gwalior

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Optical Flow

(Estimating Velocities of Brightness Pattern in a Video)

(ITIT-9507)

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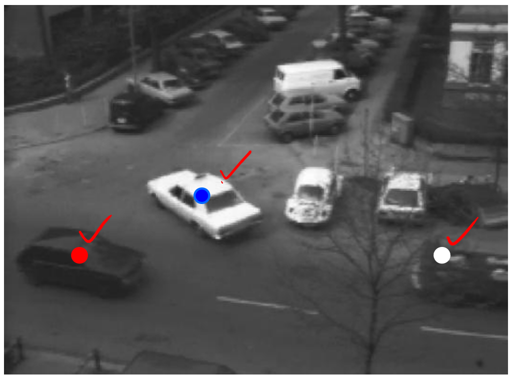
Optical Flow : An Introduction

Hamburg Taxi Sequence

Observe the motion of brightness pattern (intensity values) in each of the frames of a video

Optical Flow : An Introduction

Hamburg Taxi Sequence

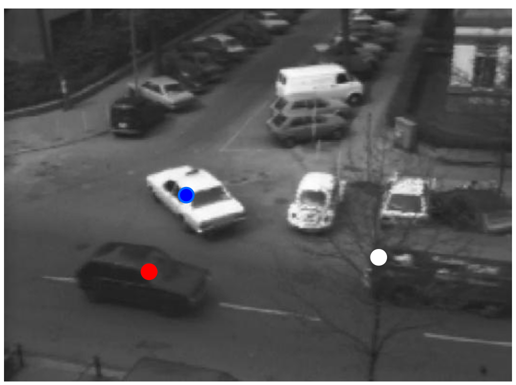


Frame at $t = 2\ sec$

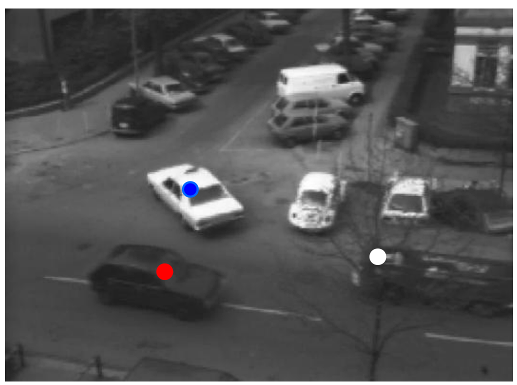
Frame at $t = 4\ sec$

Frame at $t = 6\ sec$

Frame at $t = 8\ sec$



Frame at $t = 10\ sec$



Frame at $t = 12$ sec

Frame at $t = 14$ sec

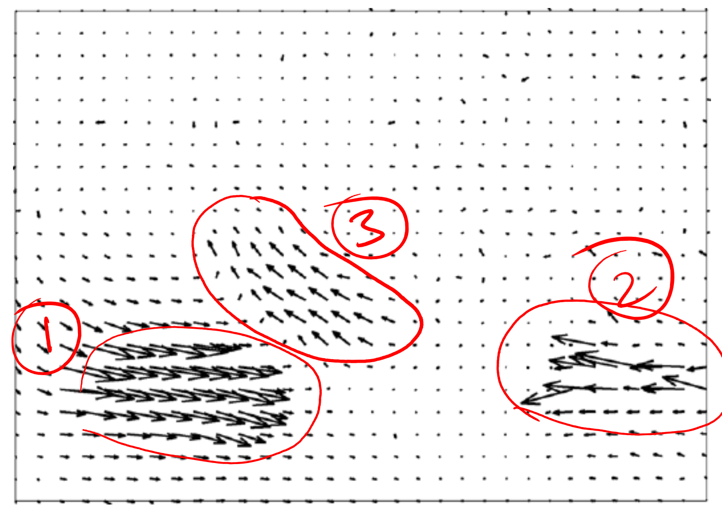
Frame at $t = 16\ sec$

Optical Flow : An Introduction

Consider two frames at time "t" and "$t + dt$"

Frame at t

Frame at $t + dt$

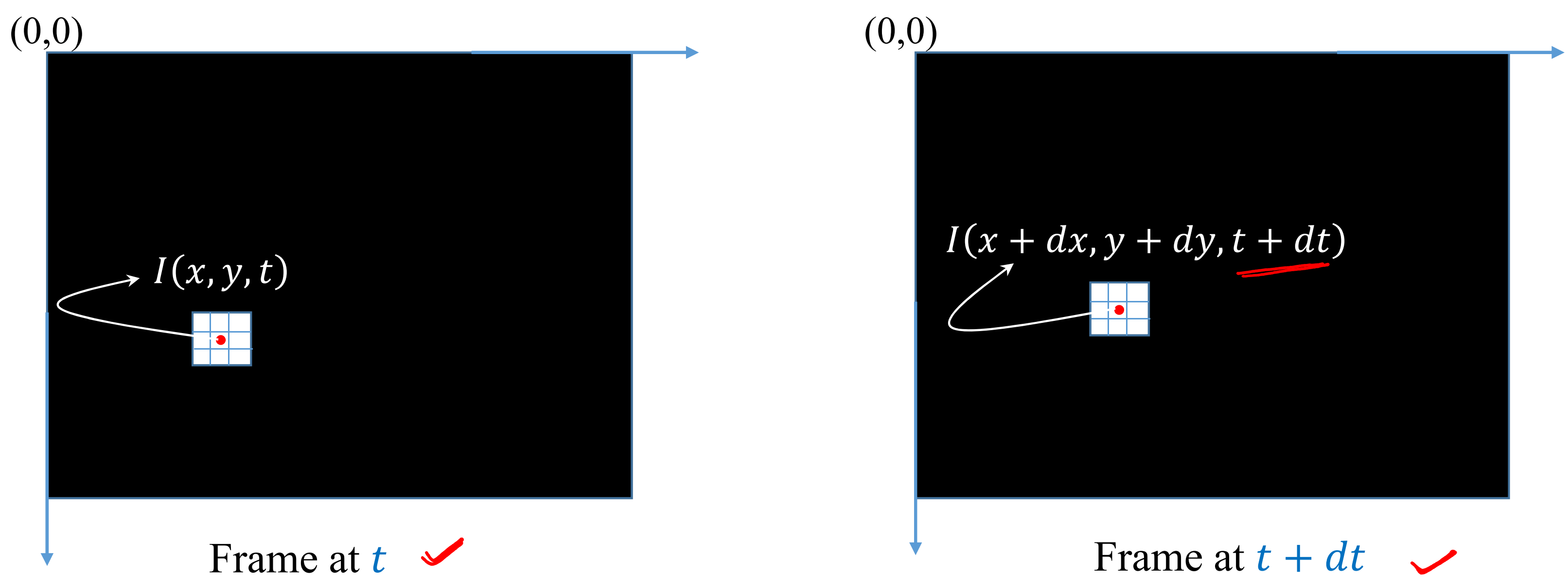


Optical flow of intensities

- ☐ **Optical flow :** Rate of change of displacement of brightness pattern (intensity values) in an image at time t w.r.t image at time "$t + dt$"

Optical Flow : An Introduction

- **Optical flow :** Rate of change of displacement of brightness pattern (intensity values) in an image at time t w.r.t image at time "$t + dt$"

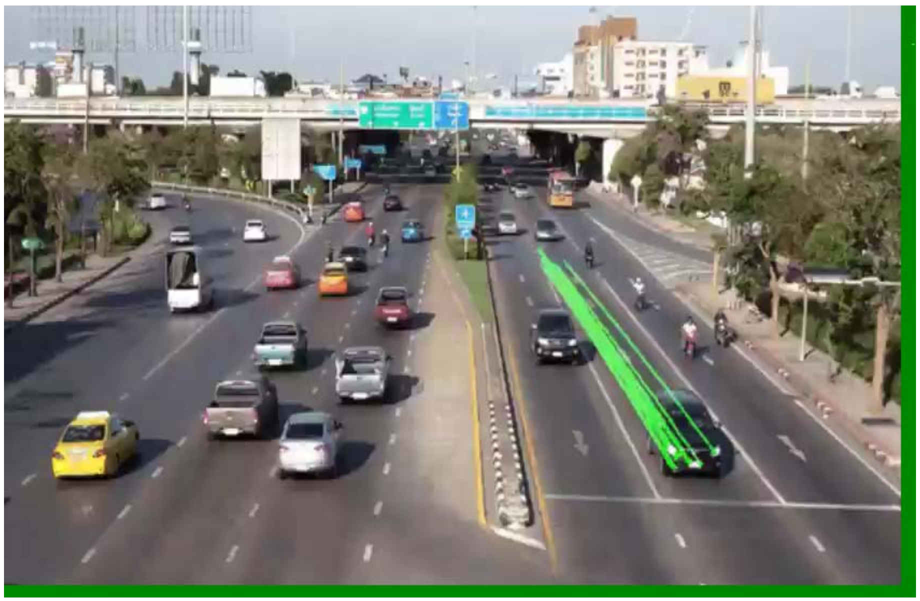


Optical flow : $(u, v)|_{I(x,y,t)} = \left(\frac{dx}{dt}, \frac{dy}{dt}\right)$

Why do we need optical flow?

- Optical flow can be used for many of the computer vision problems. Some of them are listed below:
  - Object tracking
  - Object segmentation from video
  - Object Detection
  - Video stabilization
  - 3D reconstruction using optical from
  - Foreground separation
  - Robot navigation
  - … and so on.

Optical flow for Object Tracking



Source : https://nanonets.com/blog/optical-flow/

Optical flow for Moving Object Segmentation

- Discontinuities in the optical flow can help in segmenting images into regions that correspond to different objects

Fig-1: Motion-based segmentation using optical flow

Source: vorgelegt von

Determining Optical Flow

Berthold K.P. Horn and Brian G. Schunck
Artificial Intelligence Laboratory, Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.

ABSTRACT

Optical flow cannot be computed locally, since only one independent measurement is available from the image sequence at a point, while the flow velocity has two components. A second constraint is needed. A method for finding the optical flow pattern is presented which assumes that the apparent velocity of the brightness pattern varies smoothly almost everywhere in the image. An iterative implementation is shown which successfully computes the optical flow for a number of synthetic image sequences. The algorithm is robust in that it can handle image sequences that are quantized rather coarsely in space and time. It is also insensitive to quantization of brightness levels and additive noise. Examples are included where the assumption of smoothness is violated at singular points or along lines in the image.

Horn & Schunck Optical flow Assumptions

The optical flow cannot be computed at a point in the image independently of neighboring points without introducing additional constraints, because the velocity field at each image point has two components while the change in image brightness at a point in the image plane due to motion yields only one constraint

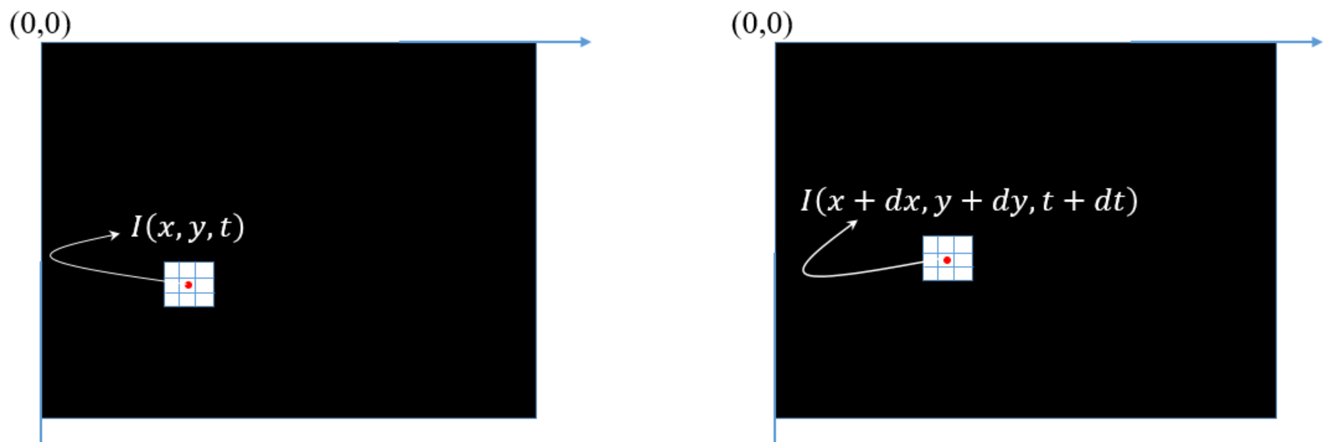
1

Brightness Constancy Assumption

$$I(x+dx, y+dy, t+dt) = I(x, y, t)$$

Intensity at $(x+dx, y+dy)$ in $(t+dt)$ frame is equal to intensity at (x, y) in (t) frame

Optical flow : Brightness Constancy Assumption



Frame at t Frame at $t + dt$

- Brightness Constancy Assumption : :

$$\underline{I(x + dx, y + dy, t + dt)} = I(x, y, t)$$

①

$$\left[f(x+a) = f(x) + \frac{\partial f}{\partial x}(x+a-a) + \frac{\partial^2 f}{\partial x^2}\frac{(x)^2}{2!} + \dots \right]$$

Taylor expansion about (x, y, t)

$$\Rightarrow \cancel{I(x, y, t)} + \frac{\delta I}{\delta x}(\cancel{x} + dx - \cancel{x}) + \frac{\delta I}{\delta y}(\cancel{y} + dy - \cancel{y}) + \frac{\delta I}{\delta t}(t + dt - t) + \underline{higher\ order\ terms} = \cancel{I(x, y, t)}$$

Optical flow : Brightness Constancy Assumption

- Brightness Constancy: $I(x + dx, y + dy, t + dt) = I(x, y, t)$

Taylor expansion about (x, y, t)

Neglect

$$\Rightarrow \cancel{I(x,y,t)} + \frac{\delta I}{\delta x}(\cancel{x} + dx - \cancel{x}) + \frac{\delta I}{\delta y}(\cancel{y} + dy - \cancel{y}) + \frac{\delta I}{\delta t}(\cancel{t} + dt - \cancel{t}) + \cancel{higher\ order\ terms} = \cancel{I(x,y,t)}$$

$$\Rightarrow \frac{\delta I}{\delta x}dx + \frac{\delta I}{\delta y}dy + \frac{\delta I}{\delta t}dt = 0 \quad \Rightarrow \quad \frac{\partial I}{\partial x}\left(\frac{\partial x}{\partial t}\right) + \frac{\partial I}{\partial y}\left(\frac{\partial y}{\partial t}\right) + \frac{\partial I}{\partial t} = 0$$

NOTE

$$\Rightarrow I_x dx + I_y dy + I_t dt = 0$$

$$\Rightarrow I_x \frac{dx}{dt} + I_y \frac{dy}{dt} + I_t = 0$$

$$\Rightarrow I_x u + I_y v + I_t = 0 \qquad (1)$$

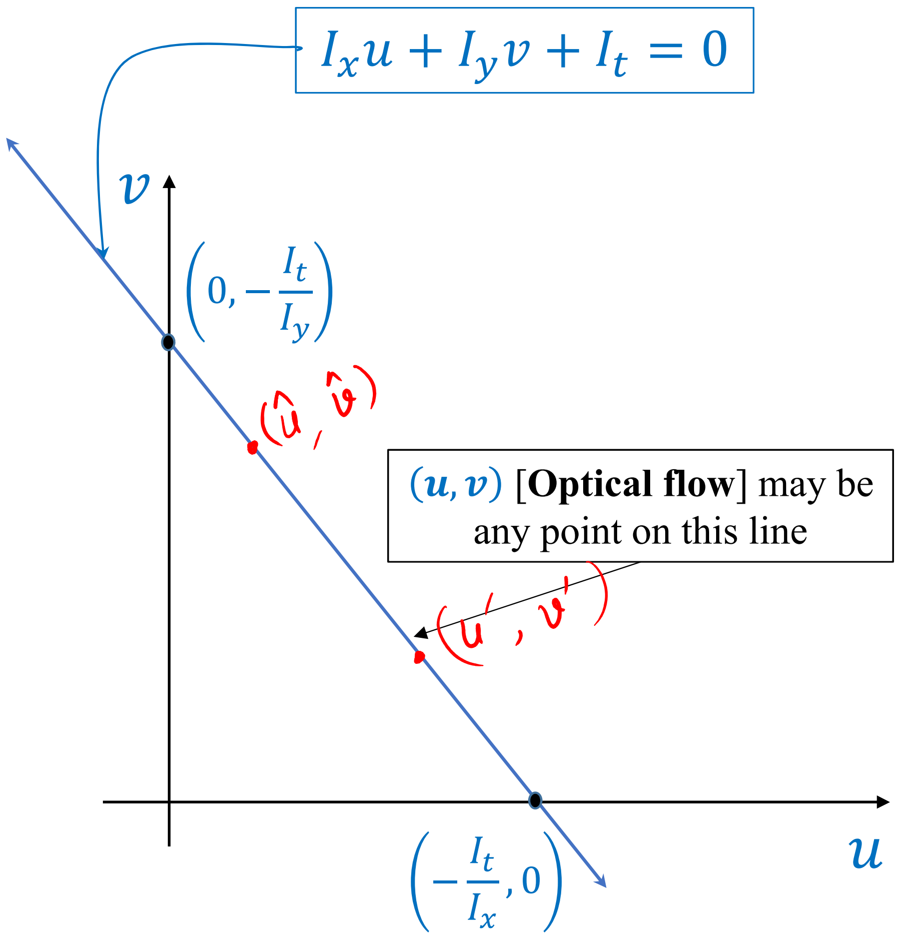
Optical flow for a pixel at (x, y) in frame t i.e., $(u, v)|_{I(x,y,t)} = \left(\frac{dx}{dt}, \frac{dy}{dt}\right)$

$\Rightarrow$ Two unknowns

and (BCA) for a pixel gives only one equation. So, we can not get the unique solution for (u, v).

For unique solution, we require some other constraints for each pixel.

Geometrical Interpretation of Optical Flow Eqn.



- ❑ So, if every point of the brightness pattern move independently, there is little hope to get the accurate optical flow $(\boldsymbol{u}, \boldsymbol{v})$.

- So, we need some other constraints in order to find the exact optical flow $(\boldsymbol{u}, \boldsymbol{v})$

Horn & Schunck Optical flow Smoothness Assumptions

- **Intuition** behind Smoothness Assumption :
  1) Opaque object of finite size undergo rigid motion or deformation.
  2) Neighbouring points on object should have similar velocities, and
  3) Velocity field of the brightness pattern varies smoothly almost everywhere
     - $\frac{du}{dx}, \frac{du}{dy}, \frac{dv}{dx}, \text{ and } \frac{dv}{dy}$ are very small [Velocities are differentiable and their change w.r.t "x" and "y" must be very small]

2

$u(x,y)$

$v(x,y)$

Smoothness Assumption

$$\min_{(u,v)}\left[\left(\frac{du}{dx}\right)^2+\left(\frac{du}{dy}\right)^2+\left(\frac{dv}{dx}\right)^2+\left(\frac{dv}{dy}\right)^2\right]=\min_{(u,v)}\left[u_x^2+u_y^2+v_x^2+v_x^2\right]$$

Cost Function for Optical Flow

- Cost Function over a window:

$$J(u,v) = \iint_{x,y} \left[\left(I_x u + I_y v + I_t\right)^2 + \lambda\left(u_x^2 + u_y^2 + v_x^2 + v_x^2\right) \right] dxdy \longrightarrow *$$

$u_x = \frac{\partial u}{\partial x}$

$u_y = \frac{\partial u}{\partial y}$

$$\Rightarrow J(u,v) = \iint_{x,y} L\left(x, y, u, v, u_x, u_y, v_x, v_y\right) dxdy \longrightarrow (1)$$

Where, $L\left(x, y, u, v, u_x, u_y, v_x, v_y\right) = \left(I_x u + I_y v + I_t\right)^2 + \lambda\left(u_x^2 + u_y^2 + v_x^2 + v_x^2\right)$

$v_x = \frac{\partial v}{\partial x}$

$v_y = \frac{\partial v}{\partial y}$

For maxima/minima of equation of type (1) can be found using lemma's of vibrational calculus

https://en.wikipedia.org/wiki/Euler%E2%80%93Lagrange_equation

Cost Function of Optical Flow & its Solution

$\nabla^2 f = \frac{\partial^2}{\partial x^2} f + \frac{\partial^2}{\partial y^2} f$

- Euler–Lagrange equation:

$$\Rightarrow J(u,v) = \iint_{x,y} L(x,y,u,v,u_x,u_y,v_x,v_y)\,dxdy$$

$$\frac{\partial J}{\partial u} = 0$$

Euler–Lagrange equation:

$\frac{d}{\partial y}(2\lambda u_y)$ $\Rightarrow 2\lambda u_{yy}$

$$\Rightarrow \frac{\partial L}{\partial u} - \frac{d}{dx}\left(\frac{\partial L}{\partial u_x}\right) - \frac{d}{dy}\left(\frac{\partial L}{\partial u_y}\right) = 0$$

$$\Rightarrow 2(I_x u + I_y v + I_t)I_x - 2\lambda u_{xx} - 2\lambda u_{yy} = 0$$

$$\Rightarrow (I_x u + I_y v + I_t)I_x - \lambda \nabla^2 u = 0 \longrightarrow (2)$$

$\nabla^2 u = u_{xx} + u_{yy}$

$$\frac{\partial J}{\partial v} = 0$$

Euler–Lagrange equation:

$$\Rightarrow \frac{\partial L}{\partial v} - \frac{d}{dx}\left(\frac{\partial L}{\partial v_x}\right) - \frac{d}{dy}\left(\frac{\partial L}{\partial v_y}\right) = 0$$

$$\Rightarrow 2(I_x u + I_y v + I_t)I_y - 2\lambda v_{xx} - 2\lambda v_{yy} = 0$$

$$\Rightarrow (I_x u + I_y v + I_t)I_y - \lambda \nabla^2 v = 0 \longrightarrow (3)$$

Cost Function of Optical Flow & its Solution

$$\begin{cases} (I_x u + I_y v + I_t) I_x - \lambda \nabla^2 u = 0 \longrightarrow (2) \\ (I_x u + I_y v + I_t) I_y - \lambda \nabla^2 v = 0 \longrightarrow (3) \end{cases}$$

- ❑ Solve for u and v using (2) and (3)
- ❑ How to find I_x, I_y, I_t, and $\nabla^2 u$

Implementation Aspects

❑ Masks:

	Kernels		
Frame t	$\begin{bmatrix}-1 & 1\\-1 & 1\end{bmatrix}$ k_{x1}	$\begin{bmatrix}-1 & -1\\1 & 1\end{bmatrix}$ k_{y1}	$\begin{bmatrix}-1 & -1\\-1 & -1\end{bmatrix}$ k_{t1}
Frame t + dt	$\begin{bmatrix}-1 & 1\\-1 & 1\end{bmatrix}$ k_{x2}	$\begin{bmatrix}-1 & -1\\1 & 1\end{bmatrix}$ k_{y2}	$\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$ k_{t2}

Frame at t

$I(x, y, t)$

Frame at $t + dt$

$I(x + dx, y + dy, t + dt)$

$$I_x = 0.5[(Frame\ t) * k_{x1} + (Frame\ t + dt) * k_{x2}]$$

$$I_y = 0.5[(Frame\ t) * k_{y1} + (Frame\ t + dt) * k_{y2}]$$

$$I_t = [(Frame\ t) * k_{t1} + (Frame\ t + dt) * k_{t2}]$$

$\begin{bmatrix}-1 & 0 & 1\end{bmatrix}$

$\begin{bmatrix}-1 & 1\end{bmatrix}$

$\frac{1}{3}\begin{bmatrix}-1 & 0 & 1\\-1 & 0 & 1\\-1 & 0 & 1\end{bmatrix}$

Implementation Aspects

❑ Masks:

❖ Laplacian Kernels

0	$-\frac{1}{4}$	0
$-\frac{1}{4}$	1	$-\frac{1}{4}$
0	$-\frac{1}{4}$	0

$u(x, y)$

	u_1	
u_4	u	u_2
	u_3	

Frame at t

$I(x, y, t)$

Frame at $t + dt$

$I(x + dx, y + dy, t + dt)$

$$\nabla^2 u = u_{xx} + u_{yy} = u - \frac{1}{4}(u_1 + u_2 + u_3 + u_4)$$

$$= u - u_{avg}$$

Cost Function of Optical Flow & its Solution

$$\begin{cases} (I_x u + I_y v + I_t)I_x - \lambda \nabla^2 u = 0 \longrightarrow (2) \\ (I_x u + I_y v + I_t)I_y - \lambda \nabla^2 v = 0 \longrightarrow (3) \end{cases}$$

$$\Rightarrow \begin{cases} (I_x u + I_y v + I_t)I_x - \lambda(u - u_{avg}) = 0 \longrightarrow (4) \\ (I_x u + I_y v + I_t)I_y - \lambda(v - v_{avg}) = 0 \longrightarrow (5) \end{cases}$$

Solving (4) and (5):

$$\boxed{u = u_{avg} - I_x \frac{P}{D} \quad \& \quad v = v_{avg} - I_y \frac{P}{D}}$$

Where, $P = I_x u_{avg} + I_y v_{avg} + I_t$ $\qquad D = \lambda + I_x^2 + I_y^2$

Optical Flow Algorithm

- For $k = 0$
- Initialize : u_k and v_k as zero matrices
- Find I_x, I_y, I_t, and thereby compute u and v using eqn. given below:

$$\boxed{u = u_{avg} - I_x \frac{P}{D} \quad \& \quad v = v_{avg} - I_y \frac{P}{D}}$$

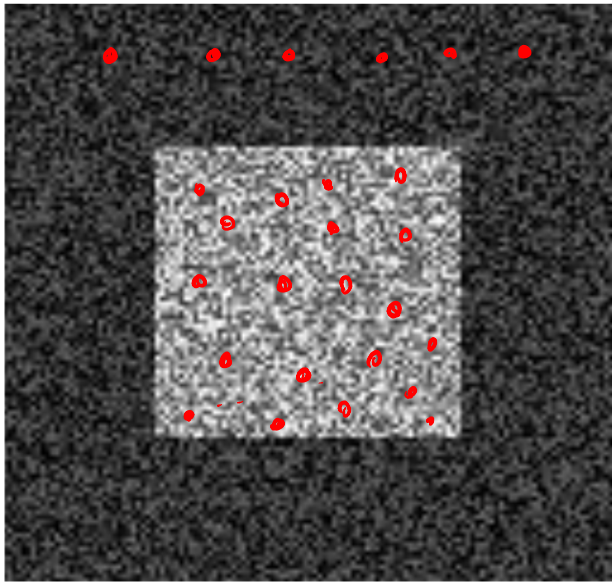
Where, $P = I_x u_{avg} + I_y v_{avg} + I_t$ $\qquad$ $D = \lambda + I_x^2 + I_y^2$

- Repeat until convergence

Optical Flow Result

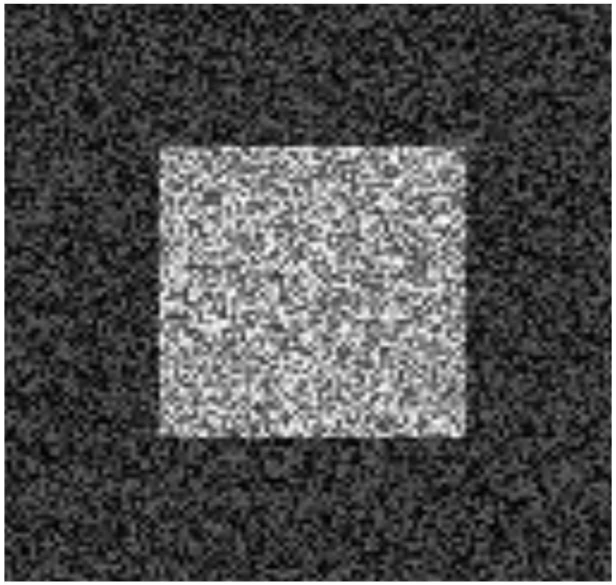
- Synthetic Image -

Frame at t



$I(x, y, t)$

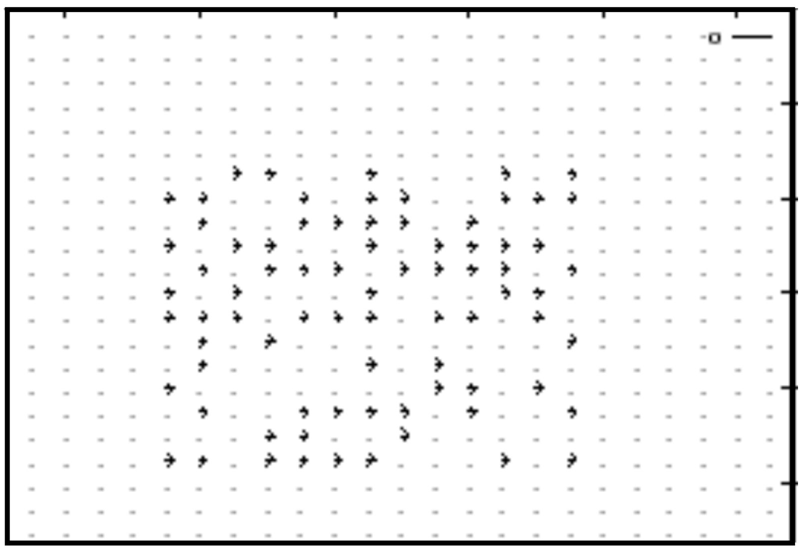
Frame at $t + dt$



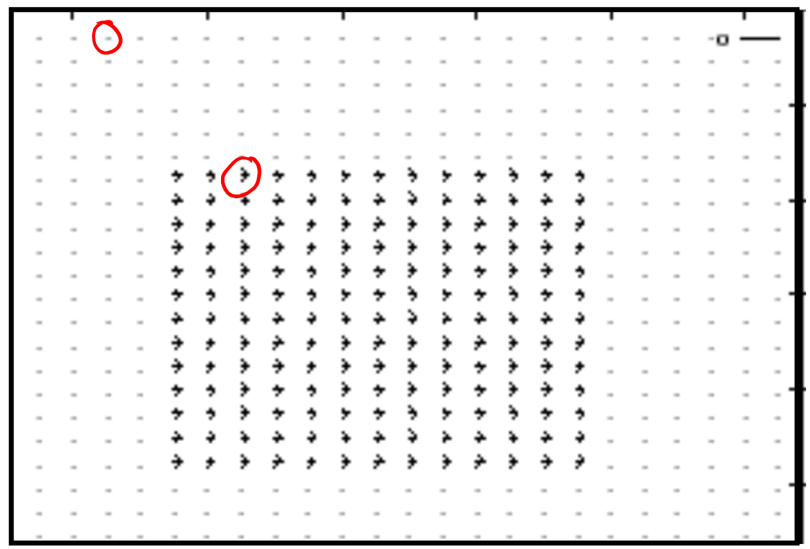
$I(x + dx, y + dy, t + dt)$

Optical Flow Result

- Horn and Schunck-



Optical flow at iter - 1



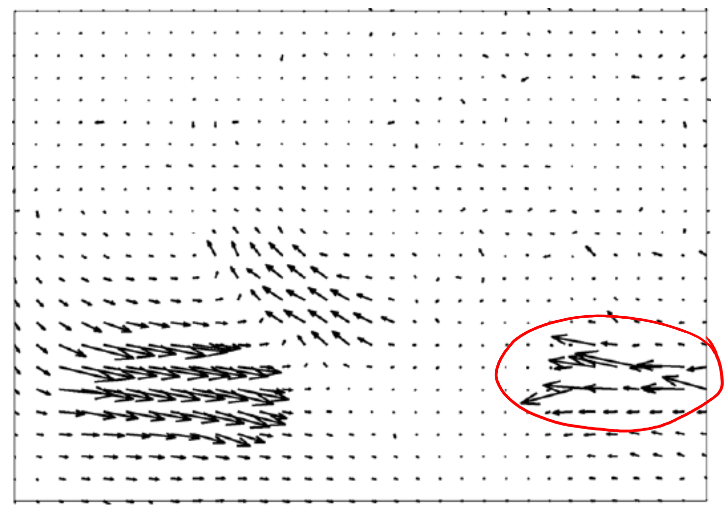
Optical flow at iter - 10

Optical Flow Result

- Horn and Schunck-

Frame at t

Frame at $t + dt$



Optical flow at iter - 20

Reference

- Richard Szeliski, Computer Vision: Algorithms and Applications, Springer, 2010 (online draft),
- Mubarak Shah, "Fundamentals of Computer Vision" (Online available)
- Ian Goodfellow, Yoshua Bengio and Aaron Courville, "Deep Learning" (Online available)

Acknowledgement!

Sources for this lecture include materials from works by Szeliski, Abhijit Mahalanobis, Sedat Ozer, Ulas Bagci, Mubarak Shah, Antonio Torralba, D. Hoiem, Justin Liang, and others. References are given for the source image contents.

Queries!