ABV-Indian Institute of Information Technology and Management
Gwalior

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Interest Point Detector

(Harris Detector)

(ITIT-9507)

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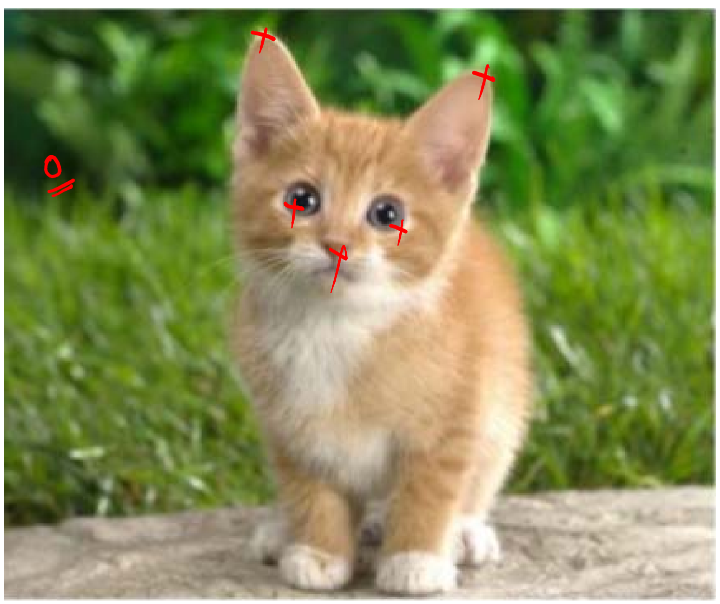
What is an Interest Point?

- What are "Interest Points" and Why do we need them?
  - Interest points are set of pixels in an image:
    - ✓ Discriminative and should be well-localized in both spatial and frequency domain
    - ✓ Invariants to affine transformations and viewpoint
    - ✓ Robust to noise and change in illuminations
    - ✓ ….so on.



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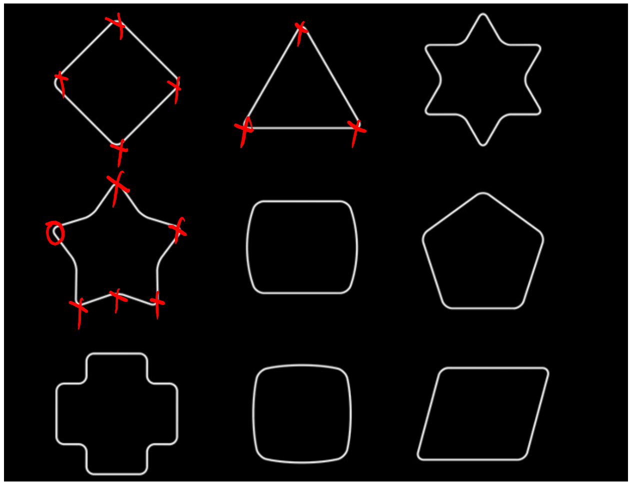
Interest Points

- Mostly, "Interest points" are:
  - The points where there are large change in intensity
    - ✓ Edges
  - Intersection of edge segments
    - ✓ Corners
  - Large curvature (rate of change of gradient), and so on



Uses of Interest Points

- Image correspondence
- Object tracking
- Object recognition
- Point matching for computing disparity
- 3D reconstruction
- Image retrieval and indexing
- Robot navigation
- Computing camera parameters (Stereo calibration)
- Image stitching
- … and so on.

Uses of Interest Points : Image Matching

- Problem – Given two or more images of a scene captured at different angles, the goal is to match points from one image to the corresponding points in the images of other views.

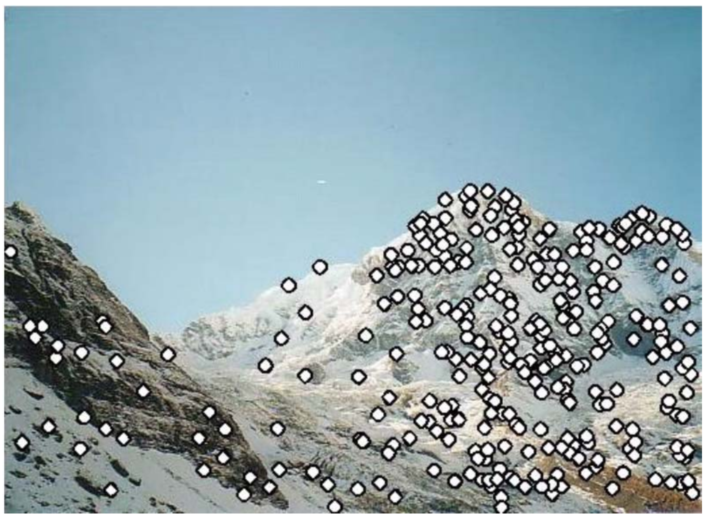
View-1

View-2

Brown, Matthew, and David G. Lowe. "Automatic panoramic image stitching using invariant features." *International journal of computer vision* 74.1 (2007): 59-73.

Uses of Interest Points : Image Matching

- **Step – 1 :** Identify the "Interest Points"
  - Require an "Interest Point Detector"



Interest Point Detection

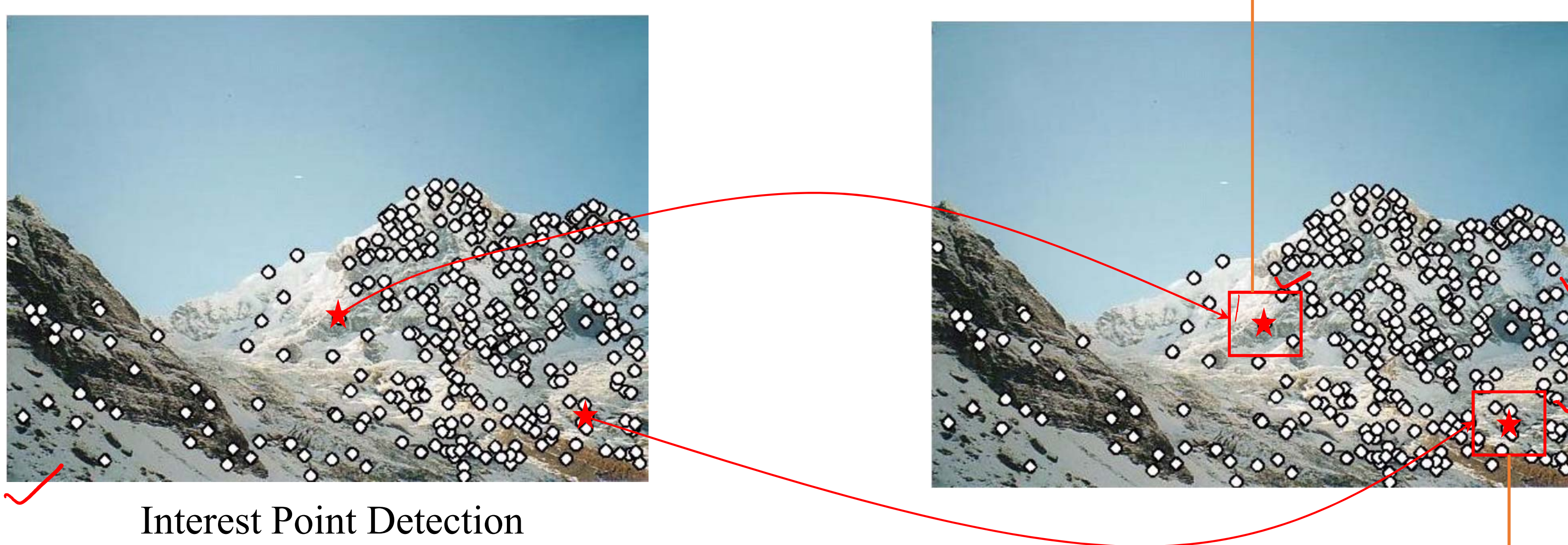
- Properties of "Interest Point Detector"
  - It should able to detect all true interest point
  - Robust to noise and illumination variations
  - Well-localized
  - Invariant to affine transformations

Image Credit : David G. Lowe

Uses of Interest Points : Image Matching

- **Step – 2 :** "Extract Local Feature Descriptor" for each of the "Interest Points"

$$\mathbf{x}_{\mathrm{m}} = [x_{\mathrm{m1}}, x_{\mathrm{m2}}, x_{\mathrm{m3}}, \cdots, x_{\mathrm{mn}}]'$$



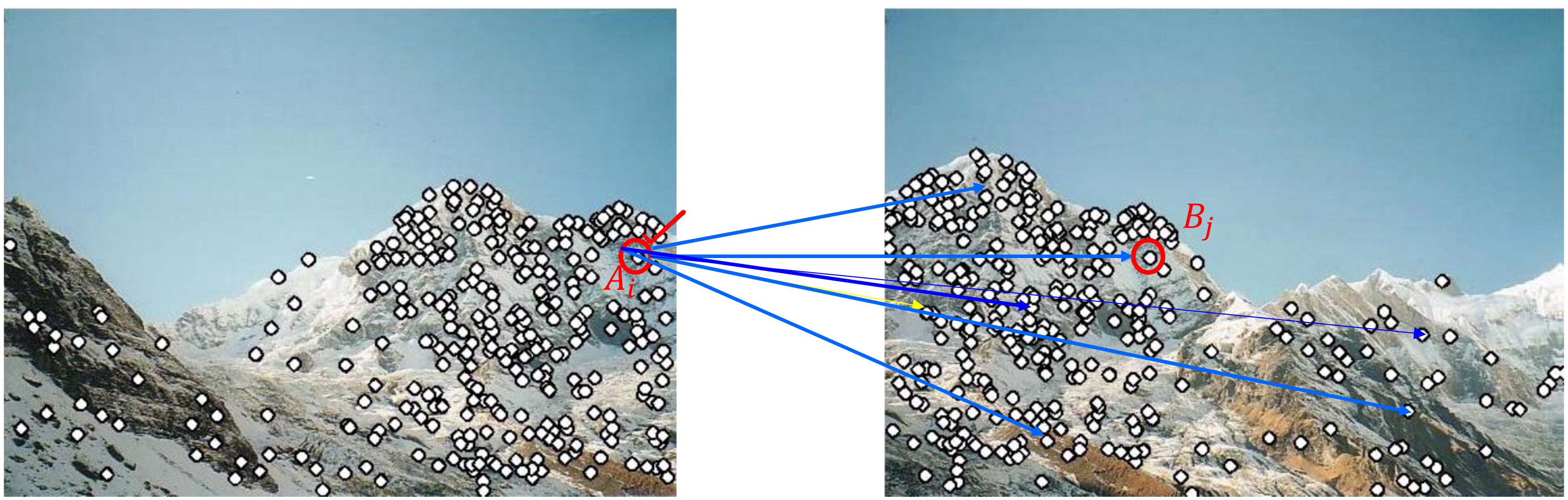
Interest Point Detection

$$\mathbf{x}_{\mathrm{k}} = [x_{\mathrm{k1}}, x_{\mathrm{k2}}, x_{\mathrm{k3}}, \cdots, x_{\mathrm{kn}}]'$$

Image Credit : David G. Lowe

Uses of Interest Points : Image Matching

- **Step – 3 :** Matching – determine correspondence between descriptors in both the images which says which point in "Image – A" correspondence to which point in "Image-B"
- Compute cost (distance) for each point in A with all other points in B
- Correspondence points = $\min_{B_j} J(A_i, B_j) \;\; \forall i$

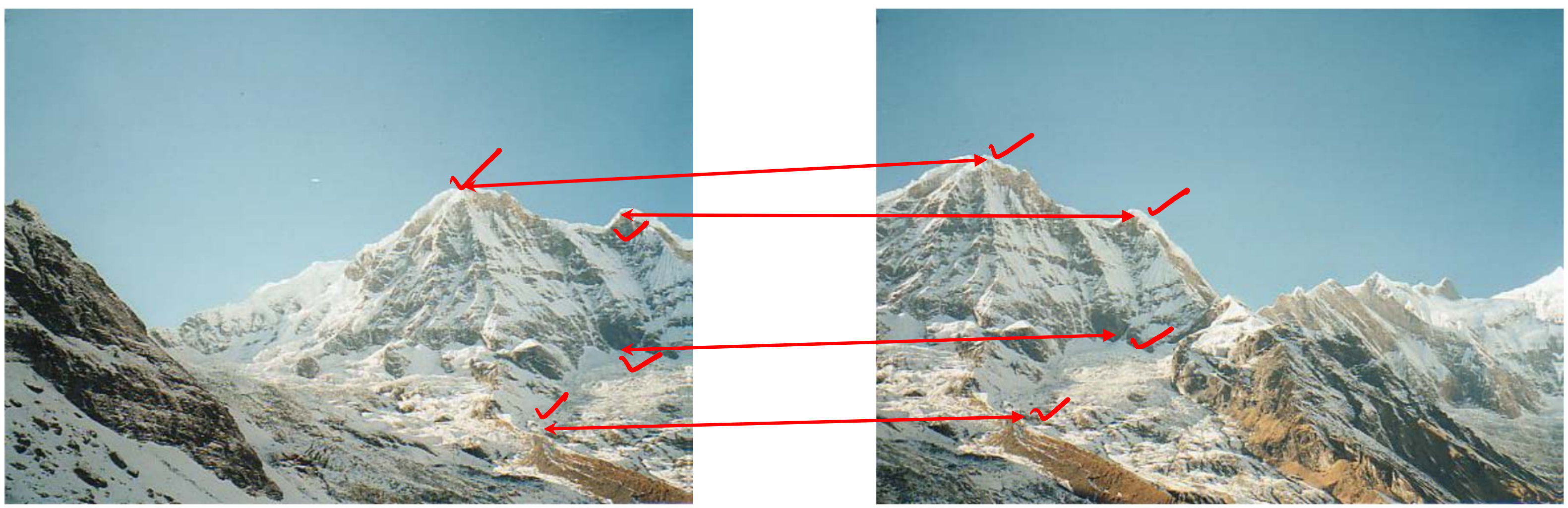


View-1 : Image – A **View-2 : Image – B**

Image Credit : David G. Lowe

Uses of Interest Points : Image Matching

❑ **Step – 3 :** Correspondence – example-1



View-1 **View-2**

Image Credit : David G. Lowe

Application – 2 : Image Stitching

- **Step – 3 :** Example-2 - Correspondence among several images

1 2 3 4 5 6 7 8

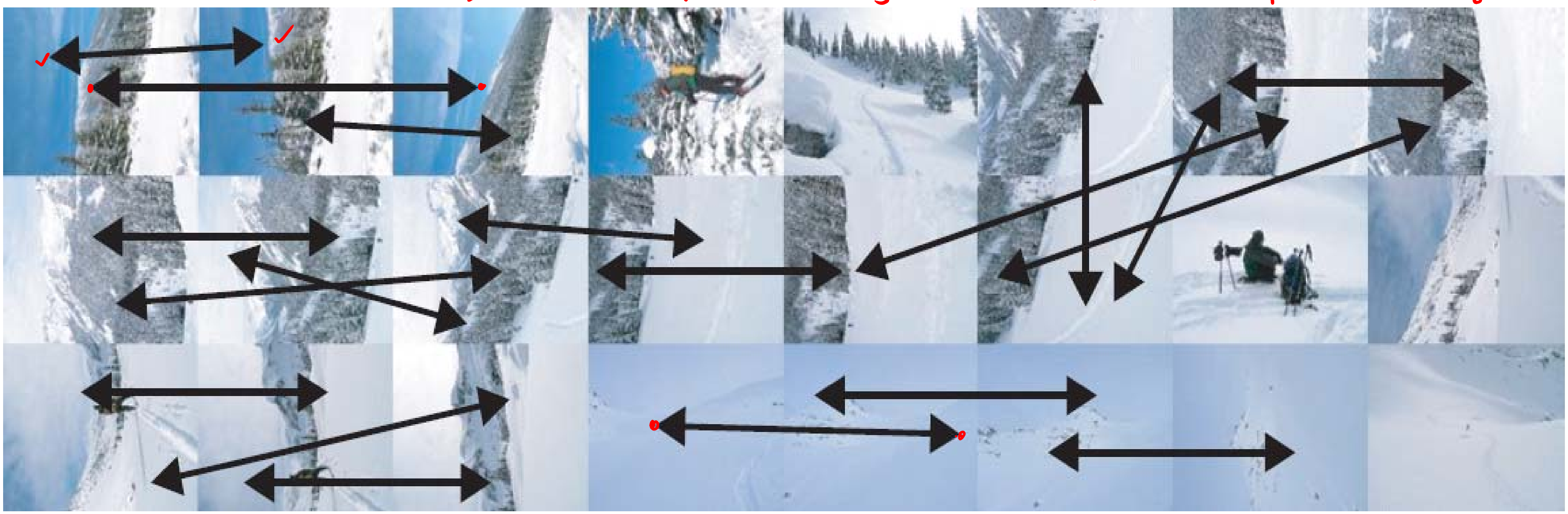
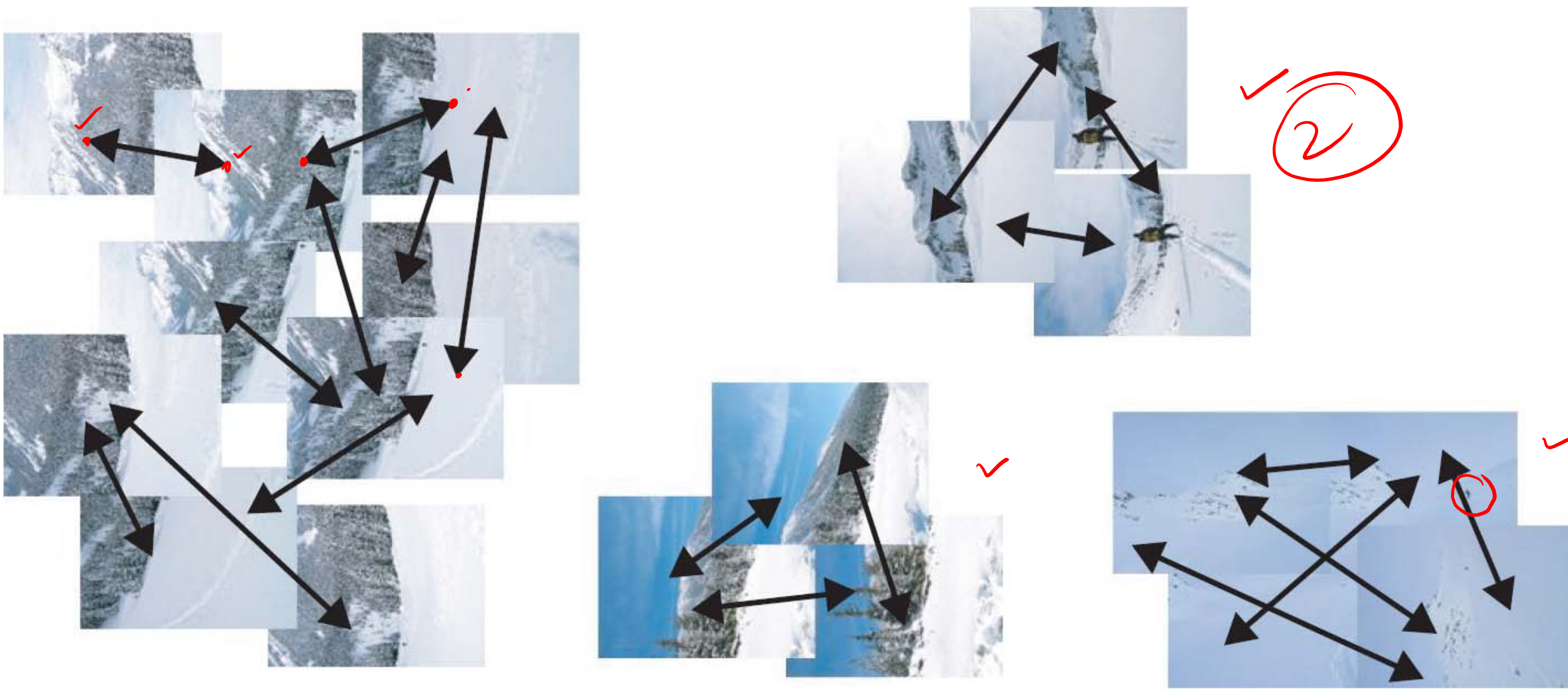


Image Credit : David G. Lowe

Application – 2 : Image Stitching

❑ **Step – 3 :** Example – 2 : Set of connected images (components)



2

Image Credit : David G. Lowe

Application – 2 : Image Stitching

❑ **Step – 3 :** Panoramas : wide-angle view or representation of a physical space

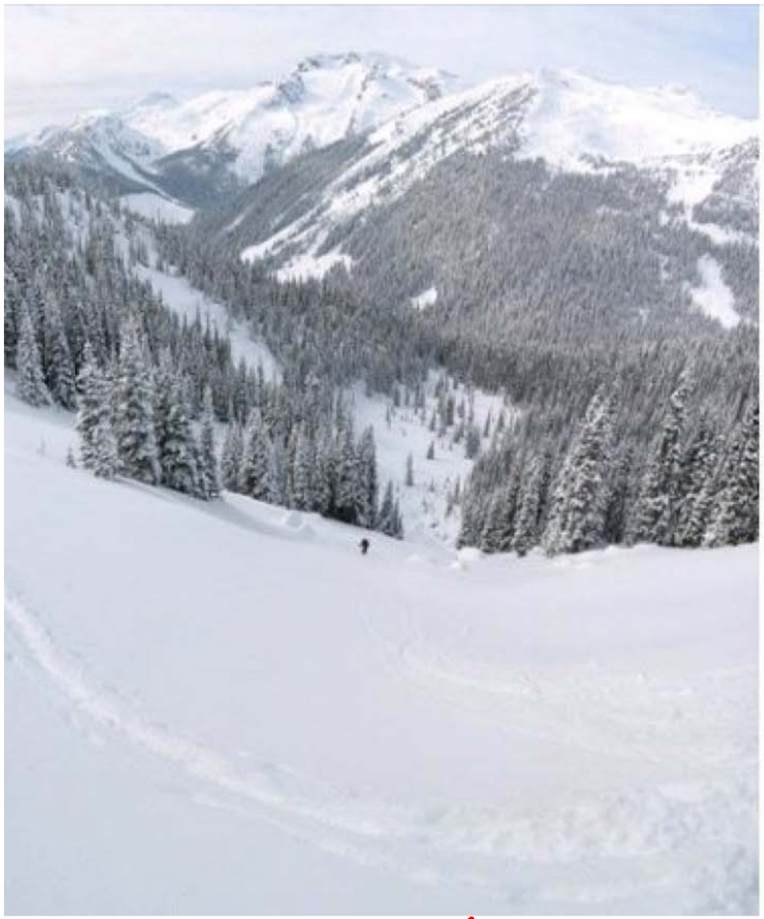
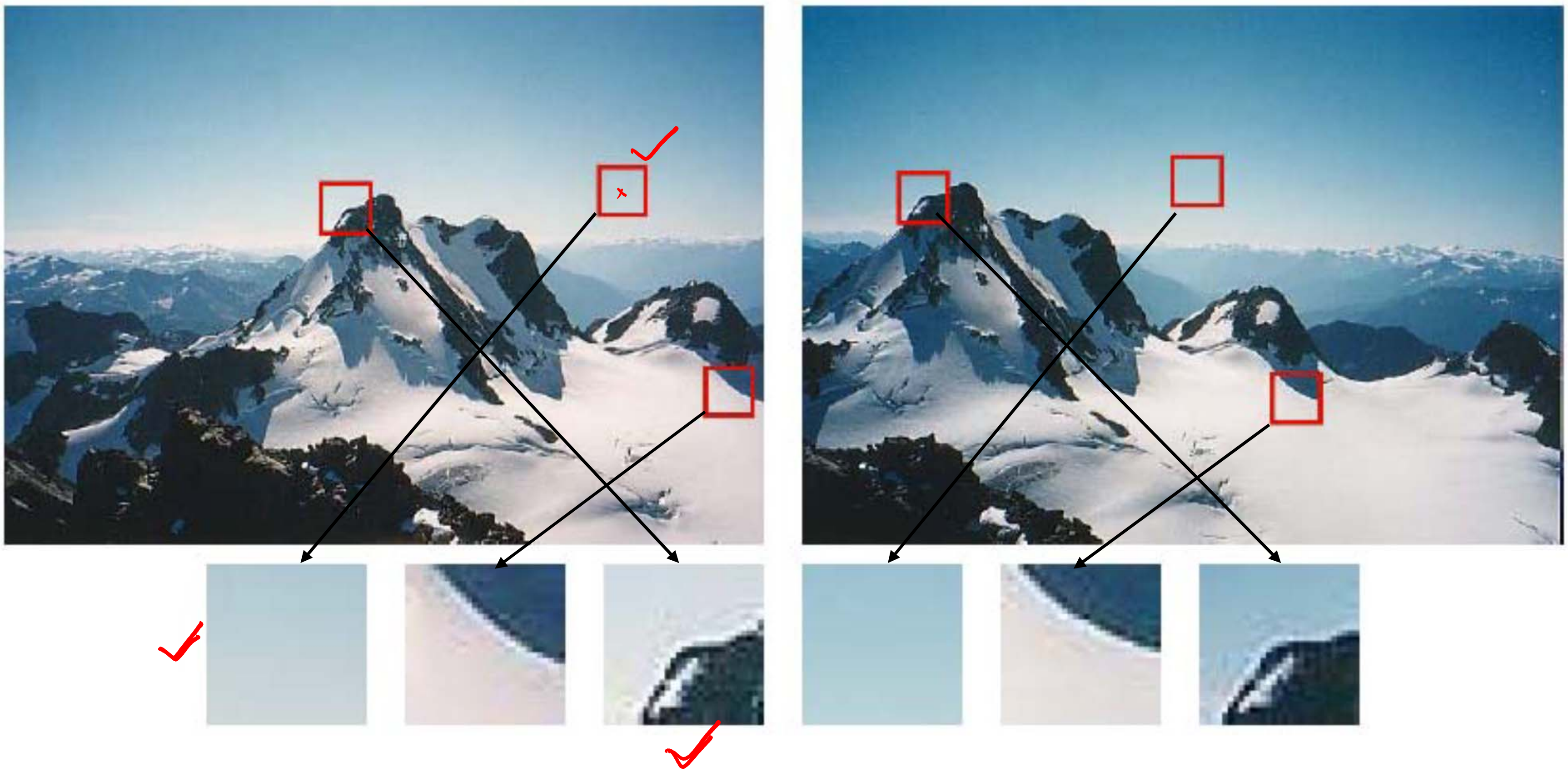


Image Credit : David G. Lowe

Descriptors around Interest Points



Slide Credit – Mubarak Shah & Kristen Grauman

Requirements

Robust "Interest Point Detector" and robust "Descriptor" around each interest point.

Descriptor

- Constructed based on local features around each interest point.
- Properties of a Descriptor:
  - ✓ Repeatability
    - ❖ The same feature can be found in several images despite geometric and photometric transformations
  - ✓ Saliency
    - ❖ Each feature has a distinctive description
  - ✓ Compactness and efficiency
    - ❖ Many fewer features than image pixels
  - ✓ Locality
    - ❖ A feature occupies a relatively small area of the image; robust to clutter and occlusion

Slide Credit – Mubarak Shah & Kristen Grauman

What are the Interest Points here?

THINK ABOUT : CORNERS AND/OR EDEGES?

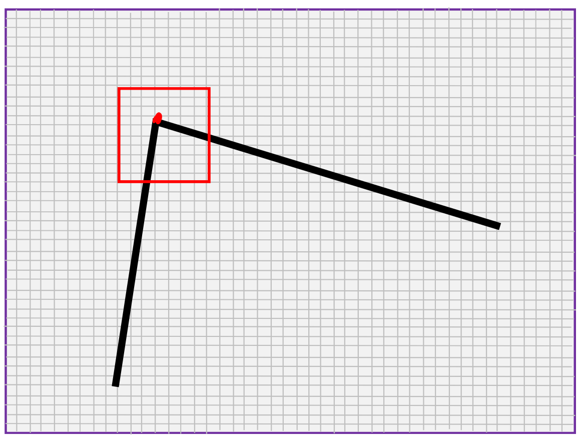
Slide Credit – Kristen Grauman

Corners as distinctive Interest Points

Slide Credit – Kristen Grauman

Harris Corner Detectors

- Corners can be easily identified by looking a small neighbourhood (or window) of that point.
- Shift the window in any direction gives a large change in intensity value.



(a) Synthetic corner

(b) Corner in real image

C.Harris, M.Stephens. "A Combined Corner and Edge Detector". 1988

Harris Corner Detectors

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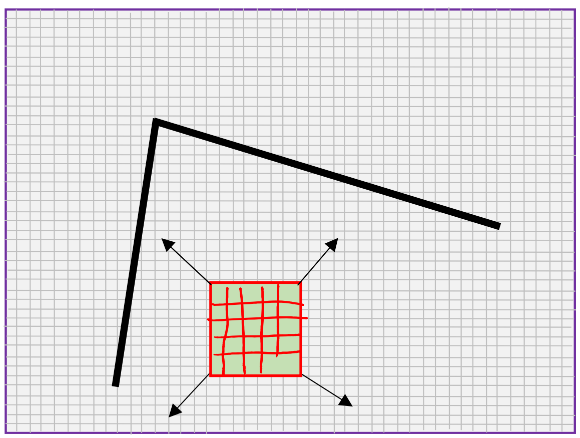
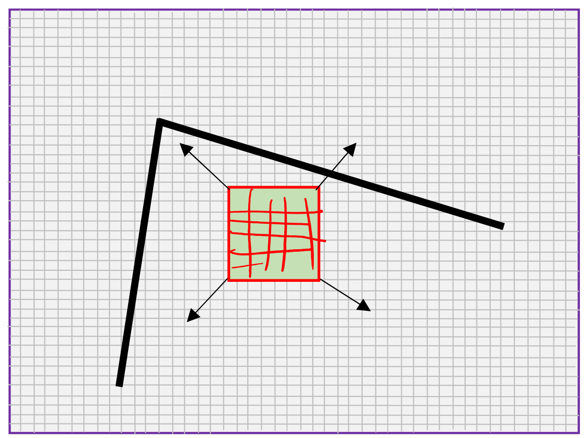


1



2

(a) "Flat" region – No change in intensity

C.Harris, M.Stephens. "A Combined Corner and Edge Detector". 1988

Harris Corner Detectors

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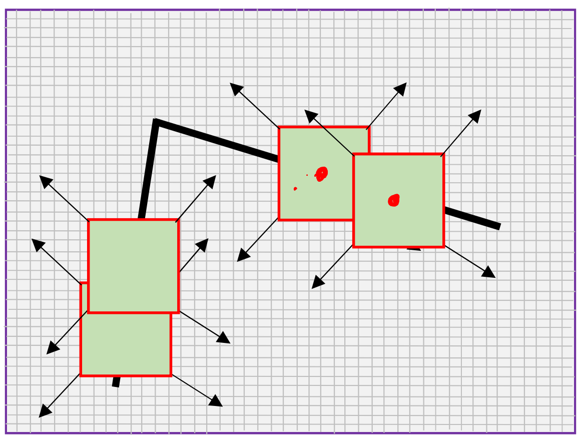
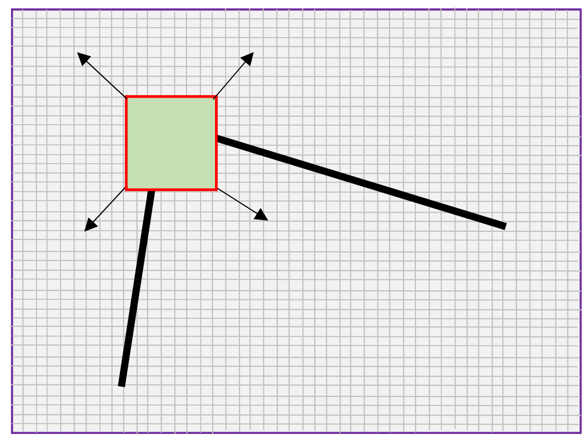
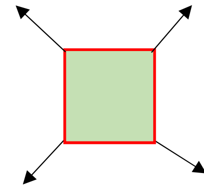


(b) "Along EDGE" –
Very small change in intensity



(c) "At Corner" –
Significant change in intensity in all directions

Harris Corner Detectors



- Corners can be easily identified by looking a small neighbourhood (or window) of that point.
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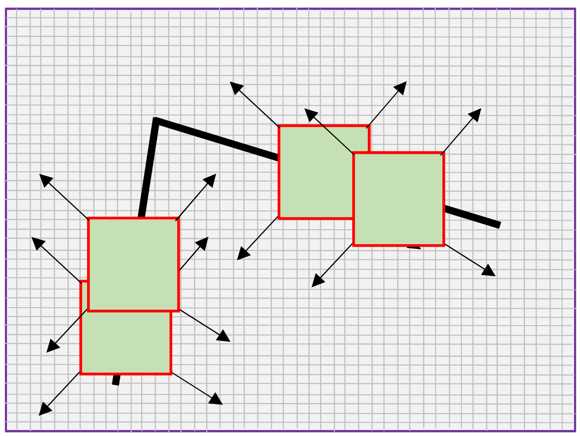

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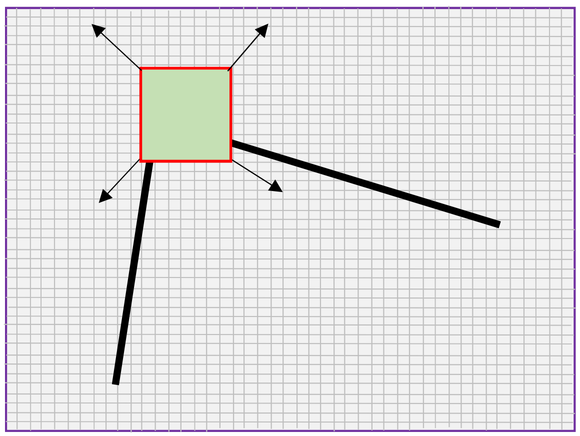


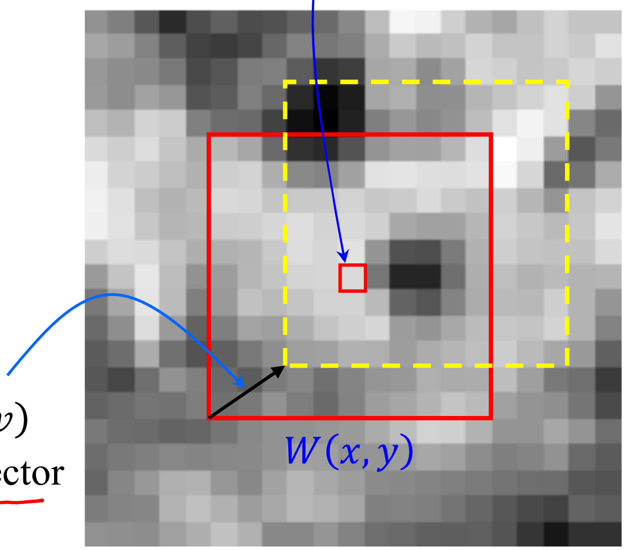
(c) "At Corner" –
Significant change in intensity in all directions

Harris Corner Detectors

- Core Idea :

Pixel under consideration i.e, find whether this pixel is corner pixel or not?

u = 3
v = 2



$$\underbrace{E(u,v)}_{\text{SSD}} = \sum_x \sum_y [\underbrace{I(x+u, y+v)}_{\text{Shifted block}} - \underbrace{I(x,y)}_{\text{Image block}}]^2$$

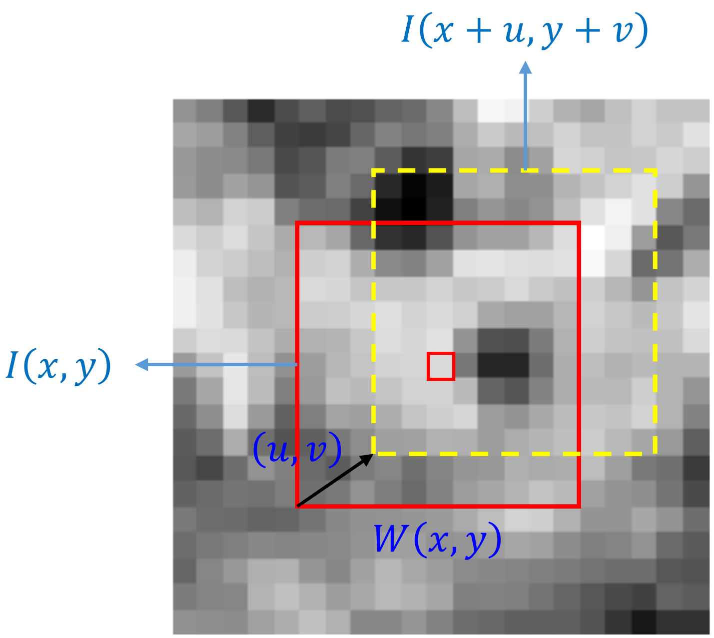
✓ If $E(u,v) > \text{Threshold}$ then pixel at (x, y) is a "corner" else NOT.

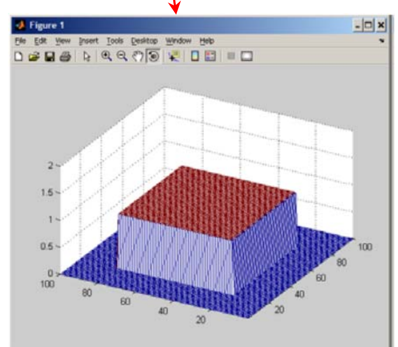
✓ Here, shift vector : $(u, v) = (3,2)$, and so $E(3,2)$ can be computed and compared with the threshold.

Harris Corner Detectors

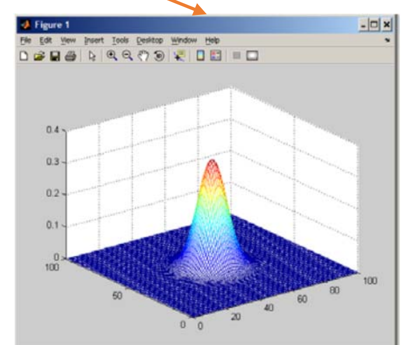
- Core Idea : A better way of SSD error function:

$$\underbrace{E(u,v)}_{\text{SSD}} = \sum_x \sum_y \underbrace{W(x,y)}_{\text{Weight block}} [\underbrace{I(x+u,y+v)}_{\text{Shifted block}} - \underbrace{I(x,y)}_{\text{Image block}}]^2$$





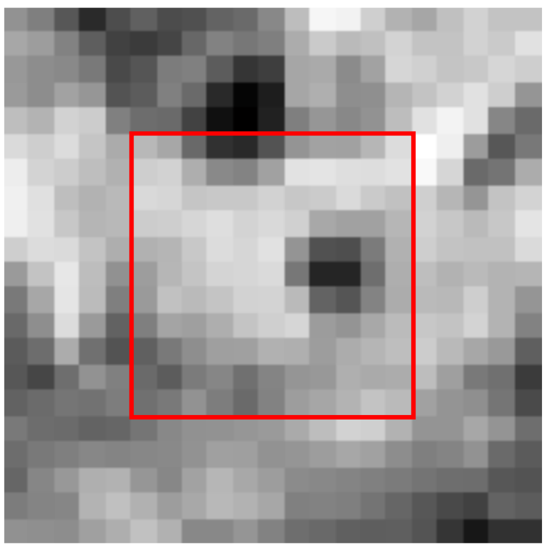
Uniform weight



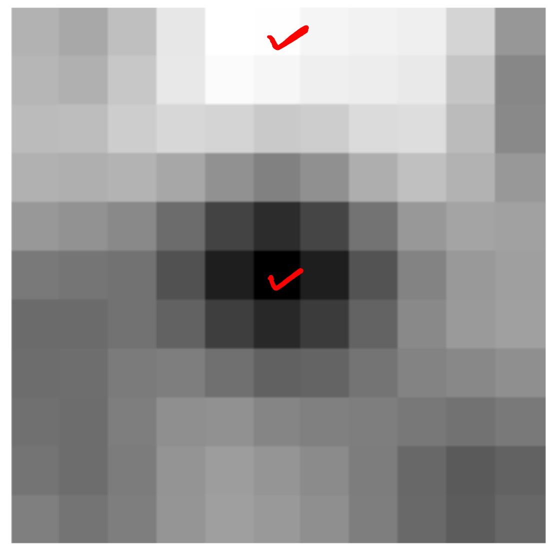
Gaussian weight

Harris Corner Detectors

$$\underbrace{E(u,v)}_{\text{SSD}} = \sum_{x}\sum_{y} \underbrace{W(x,y)}_{\text{Weight block}}[\underbrace{I(x+u,y+v)}_{\text{Shifted block}} - \underbrace{I(x,y)}_{\text{Image block}}]^2$$



$I(x,y)$



$SSD : E(u,v)$

❑ Goal : How this “Sum of Squared Error” (SSD) $E(u,v)$ behaves for a small shift (u,v).

Mathematical Aspect of Harris Corner Detectors

- Consider : $E(u,v) = \sum_x \sum_y W(x,y)[I(x+u,y+v) - I(x,y)]^2$

$$\Rightarrow E(u,v) = \sum_x \sum_y W(x,y)\left[\cancel{I(x,y)} + uI_x + vI_y - \cancel{I(x,y)}\right]^2$$

- Taylor series of $f(x)$ *about* $(x = \mathrm{a})$:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$$

- Taylor series of $f(x,y)$ *about* (u,v) :

$$f(x,y) = f(\mathrm{u},\mathrm{v}) + \frac{\delta f}{\delta x}(x-u) + \frac{\delta f}{\delta y}(\mathrm{y}-v) + \cdots$$

Mathematical Aspect of Harris Corner Detectors

- Taylor series of $I(x+u, y+v)\ about\ (x, y)$:

$$I(x+u, y+v) = I(x, y) + \frac{\delta I}{\delta x}(x+u-x) + \frac{\delta I}{\delta y}(y+v-y)$$

$$= I(x, y) + I_x u + I_y v$$

- so, :

$$E(u, v) = \sum_x \sum_y W(x, y)\left[uI_x + vI_y\ \right]^2$$

$$\Rightarrow E(u, v) = \sum_x \sum_y W(x, y)\left(\begin{bmatrix} I_x & I_y \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}\right)^2$$

$$\Rightarrow E(u, v) = \sum_x \sum_y W(x, y)\left(\begin{bmatrix} u & v \end{bmatrix}\begin{bmatrix} I_x \\ I_y \end{bmatrix}\begin{bmatrix} I_x & I_y \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}\right)$$

$(a^T b)^2$

$(a^T b)^T (a^T b)$

Mathematical Aspect of Harris Corner Detectors

- Now : $\Rightarrow E(u,v) = \sum_x \sum_y W(x,y) \left(\begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} I_x \\ I_y \end{bmatrix} \begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \right)$

$$\Rightarrow E(u,v) = \sum_x \sum_y W(x,y) \left(\begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} I_x I_x & I_x I_y \\ I_y I_x & I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \right)$$

$$\Rightarrow E(u,v) = \begin{bmatrix} u & v \end{bmatrix} \left(\sum_x \sum_y W(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_y I_x & I_y I_y \end{bmatrix} \right) \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\Rightarrow E(u,v) = \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

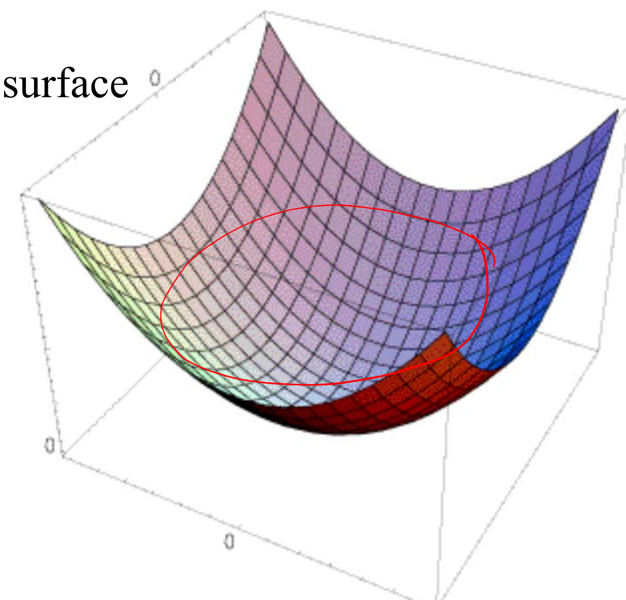
Where, $M = \sum_x \sum_y W(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_y I_x & I_y I_y \end{bmatrix}$

$[u \ v] \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$

Geometrical Interpretation of $E(u,v)$

- Geometrically : $E(u,v) = [u \quad v]\left(\sum_x \sum_y W(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_y I_x & I_y I_y \end{bmatrix}\right) \begin{bmatrix} u \\ v \end{bmatrix}$

Quadratic surface

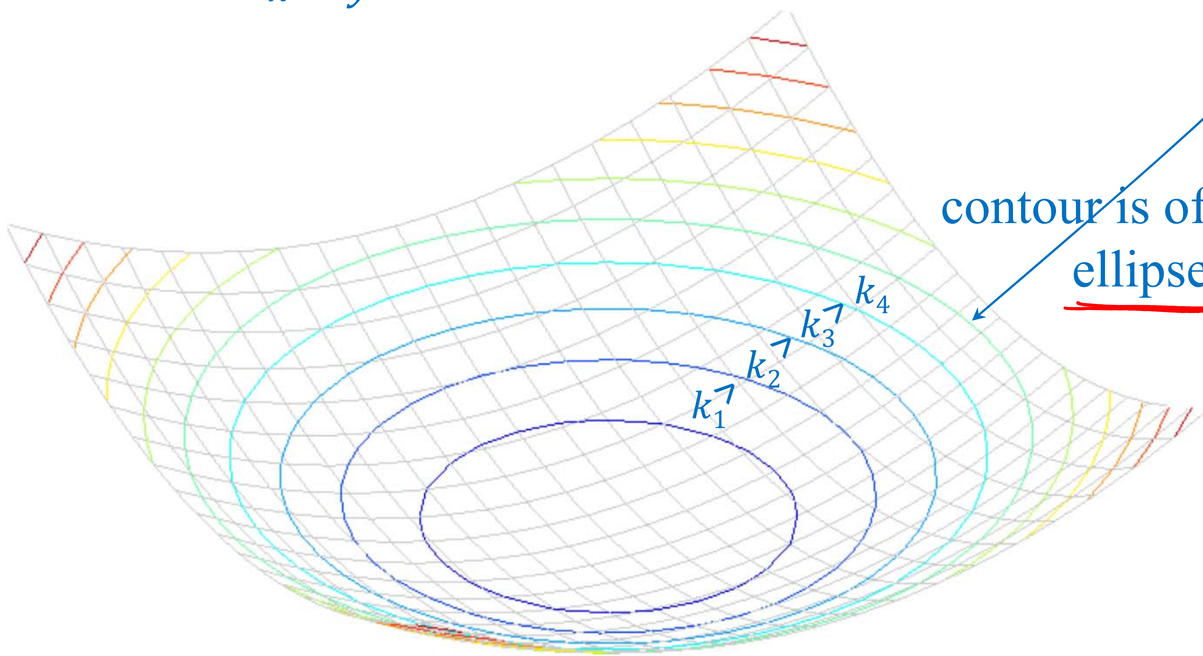


- Characterize by eigenvalues of $M_{2X2} = \sum_x \sum_y W(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_y I_x & I_y I_y \end{bmatrix}$ matrix

Geometrical Interpretation of $E(u,v)$

- Contour of $E(u,v) = k = \text{const}$

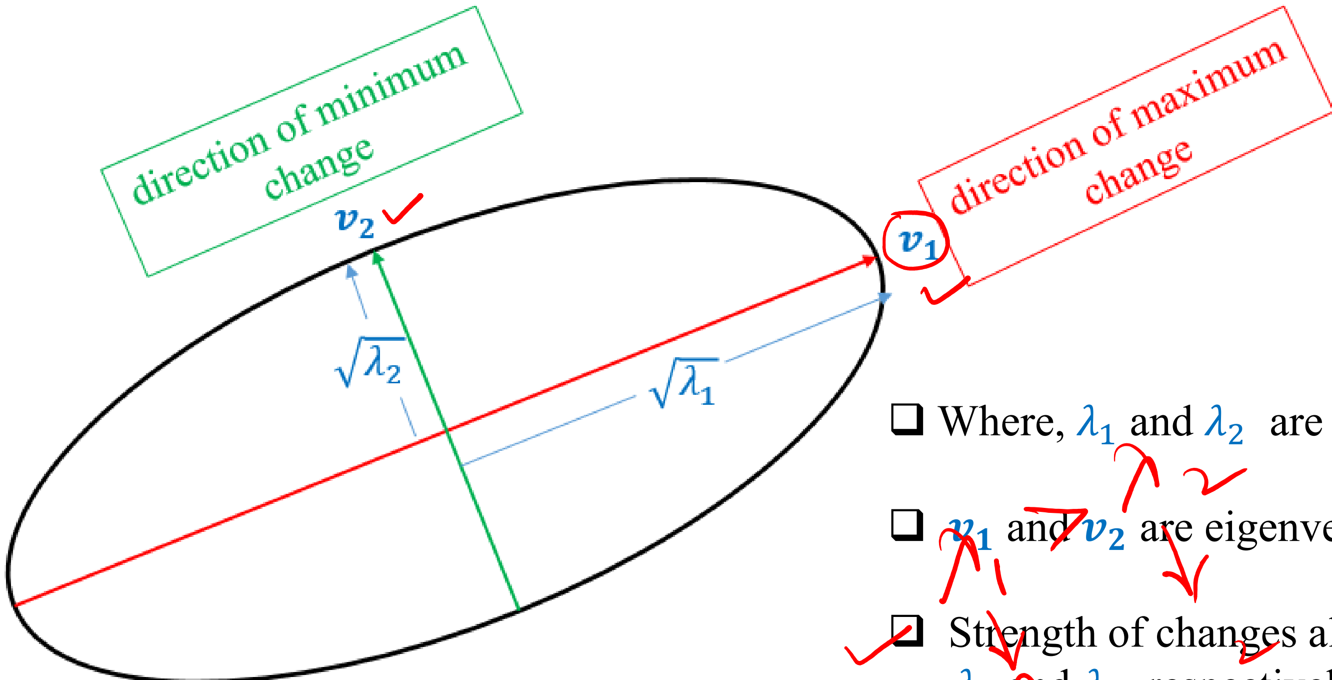
$$E(u,v) = [u \quad v]\left(\sum_x \sum_y W(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_y I_x & I_y I_y \end{bmatrix}\right) \begin{bmatrix} u \\ v \end{bmatrix} = k$$



contour is of shape <u>ellipse</u>

Geometrical Interpretation of $E(u,v)$

- For given (u,v) : $M_{2X2} = \sum_{x}\sum_{y} W(x,y)\begin{bmatrix} I_x I_x & I_x I_y \\ I_y I_x & I_y I_y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$



- Where, λ_1 and λ_2 are eigenvalues of M_{2X2} matrix
- $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ are eigenvectors of M_{2X2} matrix
- Strength of changes along $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ is defined by λ_1 and λ_2, respectively.

Geometrical Interpretation of $E(u, v)$

- Eigenvalues and Eigenvectors of M_{2X2} matrix :
- Let $M_{2X2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
- Eigenvalue equation : $Mx = \lambda x$
- Characteristic equation : $(M - \lambda I)x = 0$

$$\Rightarrow \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a - \lambda & b \\ c & d - \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Homogeneous Eqn in two variables

- Trivial solution : $x_1 = 0 \; and \; x_2 = 0$

$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Geometrical Interpretation of $E(u,v)$

- Eigenvalues and Eigenvectors of M_{2X2} matrix :

- For, Non-trivial solution : $|M - \lambda I| = \left| \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix} \right| = 0$

$$\Rightarrow (a-\lambda)(d-\lambda) - bc = 0$$

$$\Rightarrow \lambda^2 - (a+d)\lambda + (ad-bc) = 0$$

$$\Rightarrow \lambda = \frac{(a+d) \pm \sqrt{(a+d)^2 - 4(ad-bc)}}{2} = \{\lambda_1, \lambda_2\}$$

Where,

$$\lambda_1 = \frac{(a+d) + \sqrt{(a+d)^2 - 4(ad-bc)}}{2}, \qquad \lambda_2 = \frac{(a+d) - \sqrt{(a+d)^2 - 4(ad-bc)}}{2}$$

- Let : $\lambda_1 > \lambda_2$

Geometrical Interpretation of $E(u, v)$

- ❑ Eigenvectors of M_{2X2} :
- ❑ For each λ_i, there will a corresponding eigenvector v_i
- ❑ So, there are two eigenvectors : v_1 and v_2 corresponding to λ_1 and λ_2, respectively.
- ❑ Calculation of v_i for λ_i:

$$(M - \lambda I)v_i = 0$$

$$\Rightarrow \begin{bmatrix} a - \lambda_i & b \\ c & d - \lambda_i \end{bmatrix} \begin{bmatrix} v_{i1} \\ v_{i2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Let : $v_{i1} = k_{i1}$ and $v_{i2} = k_{i2}$

So, $:v_i = \begin{bmatrix} k_{i1} \\ k_{i2} \end{bmatrix}$

Geometrical Interpretation of $E(u, v)$

- Eigenvalues and Eigenvectors Example $-$ 1:
- Let : $M_{2\times 2} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$
- Computing Eigenvalues :

$$|M - \lambda I| = \left| \begin{bmatrix} -\lambda & 1 \\ -2 & -3-\lambda \end{bmatrix} \right| = 0$$

$$\Rightarrow \lambda^2 + 3\lambda + 2 = 0 \Rightarrow \lambda_1 = -1, \text{ and } \lambda_2 = -2$$

- Computing eigenvector for $\lambda_1 = -1$:

$$(M - \lambda I)v_1 = 0$$

$$\Rightarrow \begin{bmatrix} -\lambda_1 & 1 \\ -2 & -3-\lambda_1 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} v_{11} + v_{12} = 0 \\ -2v_{11} - 2v_{12} = 0 \end{matrix}$$

$\Rightarrow v_{11} = -v_{12}$

$\Rightarrow$ Assume $v_{11} = 1$, and so, $v_{12} = -1$

$$v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow (v_1)_{normalize} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Geometrical Interpretation of $E(u, v)$

- Eigenvalues and Eigenvectors Example : -1
- Let : $M_{2\times2} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$
- Computing Eigenvalues :

$$|M - \lambda I| = \left| \begin{bmatrix} -\lambda & 1 \\ -2 & -3-\lambda \end{bmatrix} \right| = 0$$

$\Rightarrow \lambda^2 + 3\lambda + 2 = 0 \Rightarrow \lambda_1 = -1,$ and $\lambda_2 = -2$

- Computing eigenvector for $\lambda_2 = -2$:

$$(M - \lambda I)v_1 = 0$$

$$\Rightarrow \begin{bmatrix} -\lambda_2 & 1 \\ -2 & -3-\lambda_2 \end{bmatrix} \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{array}{r} 2v_{21} + v_{22} = 0 \\ -2v_{21} - v_{22} = 0 \end{array}$$

$$\Rightarrow 2v_{11} = -v_{12}$$

$\Rightarrow$ Assume: $v_{21} = 1,$ and so $v_{22} = -2$

$$\Rightarrow v_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \Rightarrow (v_2)_{normalize} = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Geometrical Interpretation of $E(u, v)$

- Eigenvalues and Eigenvectors Example $-$ 2:

- Let : $M_{2\times 2} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

- Eigenvectors : $v = \begin{bmatrix} -0.8507 & -0.5257 \\ -0.5257 & 0.8507 \end{bmatrix} = \begin{array}{c} [v_1 \quad v_2] \\ \downarrow \quad\quad \downarrow \\ \begin{bmatrix} v_{11} & v_{21} \\ v_{12} & v_{21} \end{bmatrix} \end{array}$

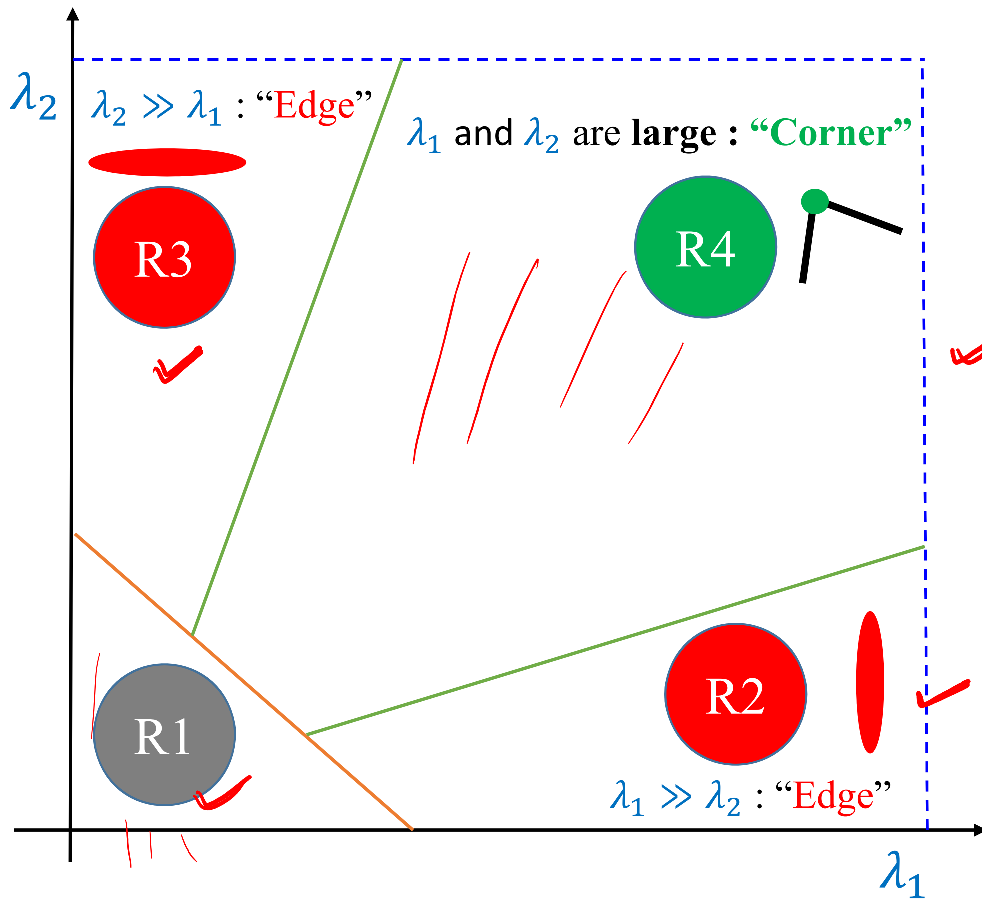
- Eigenvectors : $\begin{array}{c} \lambda_1 \qquad\qquad \\ \begin{bmatrix} \boxed{0.3820} & 0 \\ 0 & \boxed{0.8507} \end{bmatrix} \\ \qquad\qquad \lambda_2 \end{array}$

<u>In Matlab</u>

[Evec Eval] = eig(A)

Classification of Image Points Using Eigenvalues of $M_{2\times 2}$

- Let λ_1 and λ_2 are eigenvalues of $M_{2\times 2}$
- Case -1 : λ_1 and λ_2 are very small
  - R1 → "Flat Region"
- Case -2 : $\lambda_1 \gg \lambda_2$: "Edge"
  - R2 → "Edge"
- Case -3 : $\lambda_2 \gg \lambda_1$: "Edge"
  - R3 → "Edge"
- Case -4 : λ_1 and λ_2 are **large**
  - R4 → "Corner"



Classification of Image Points Using $R = \lambda_1\lambda_2 - k(\lambda_1 + \lambda_2)^2$

- Let λ_1 and λ_2 are eigenvalues of $M_{2\times 2}$
- Case -1 : $|R|$ is very small

  R1 → **“Flat”**

- Case -2 : $R < 0$: “Edge”

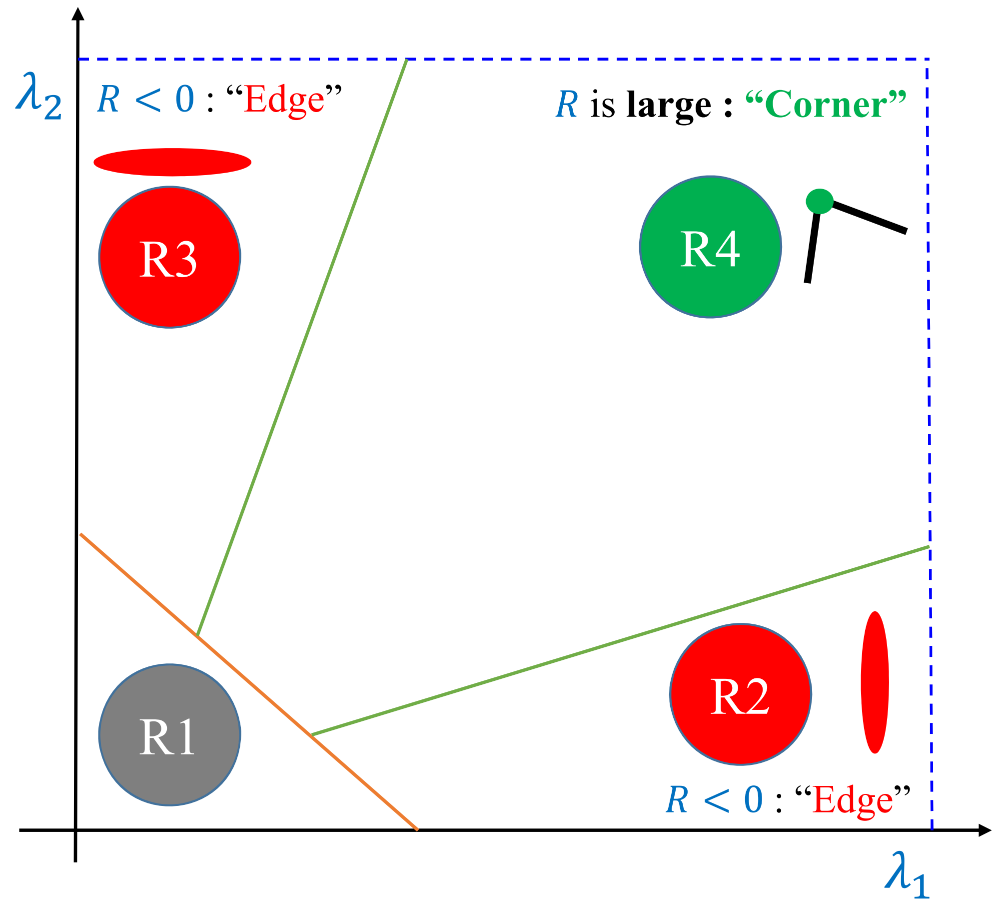
  R2 → **“Edge”**

- Case -3 : $R < 0$: “Edge”

  R3 → **“Edge”**

- Case -4 : R is **large : “Corner”**

  R4 → **“Corner”**

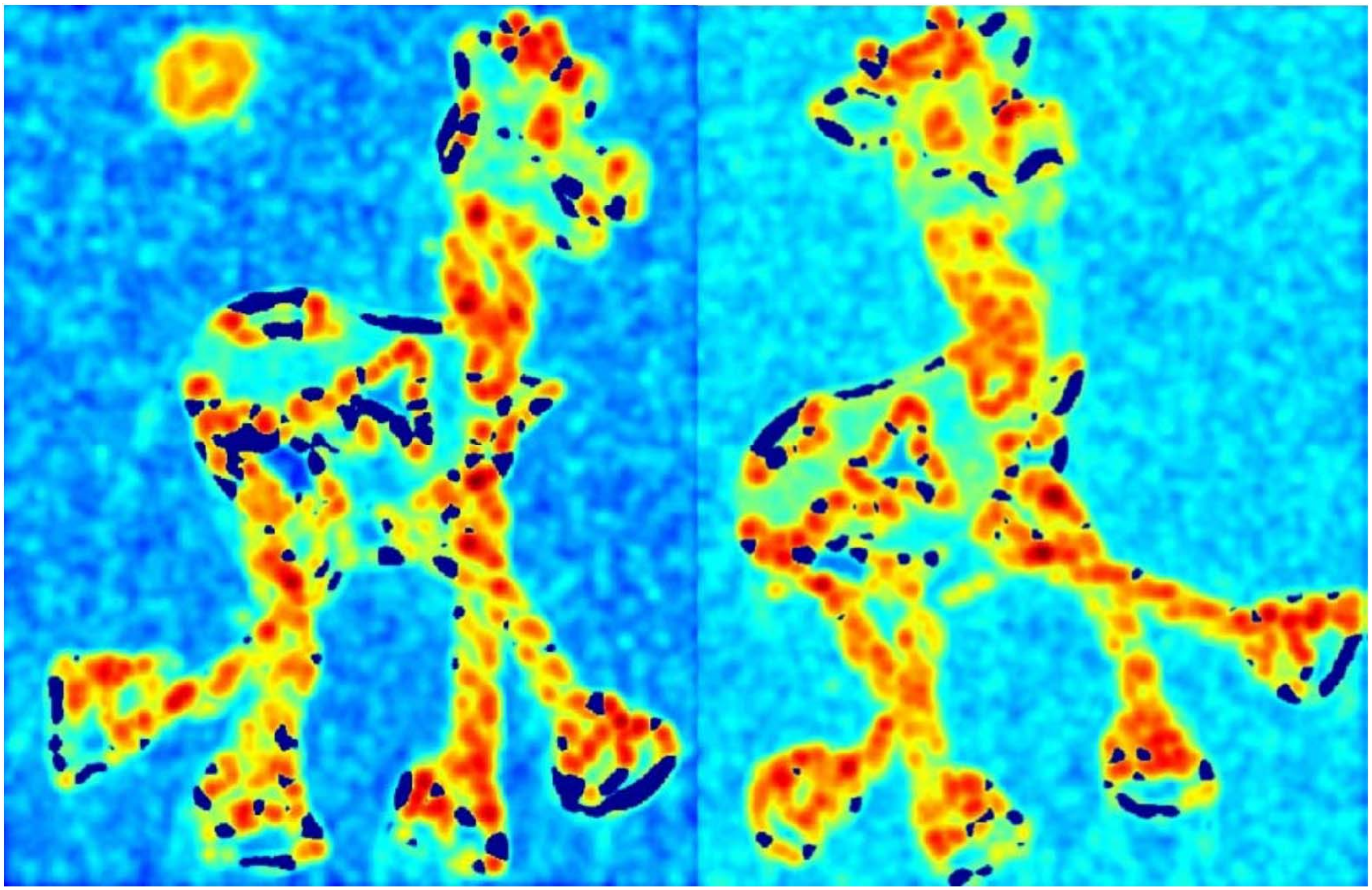


Harris Corner Detectors : Results

Input Image

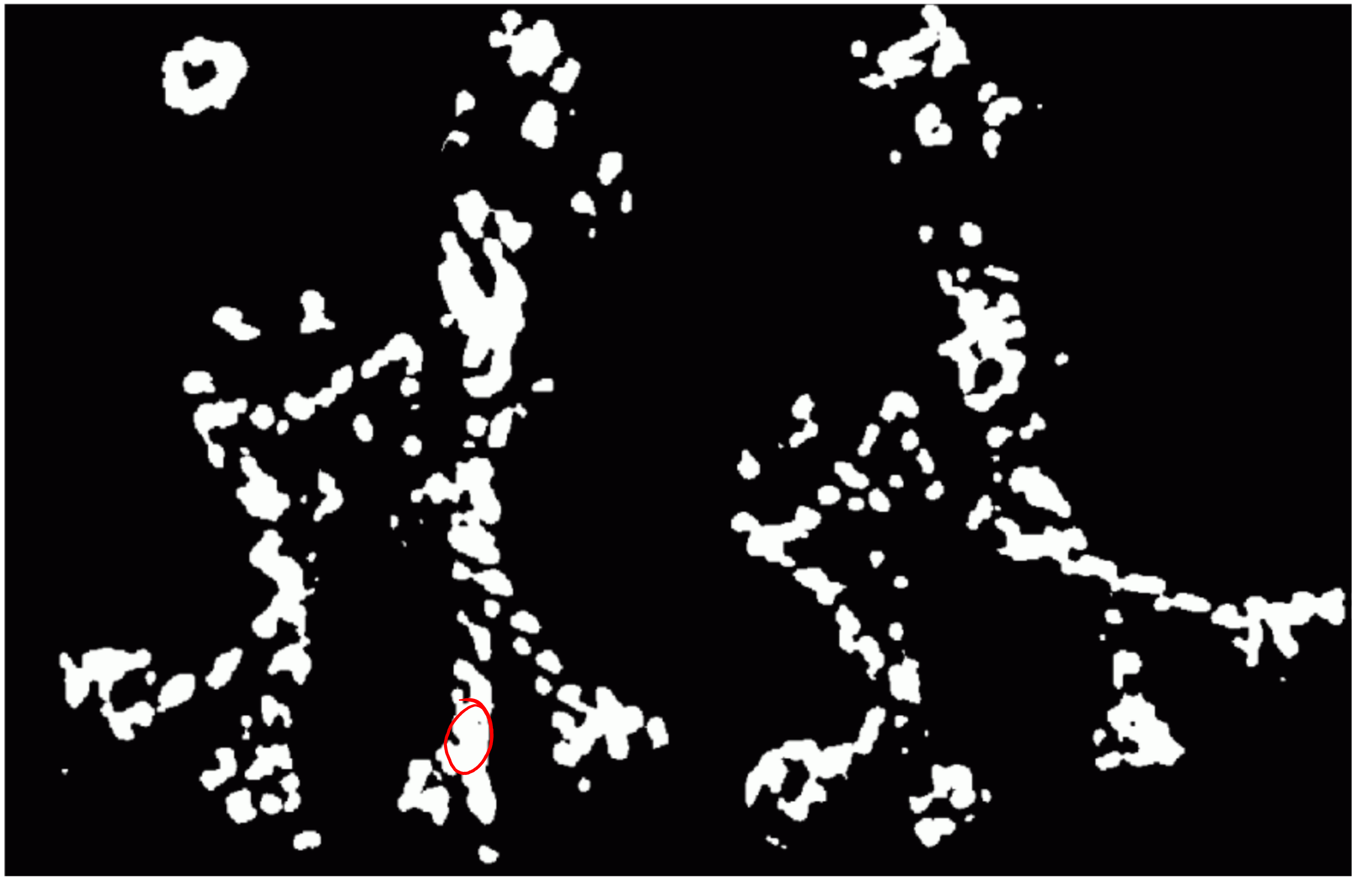
Harris Corner Detectors : Results

Corner Response 'R'



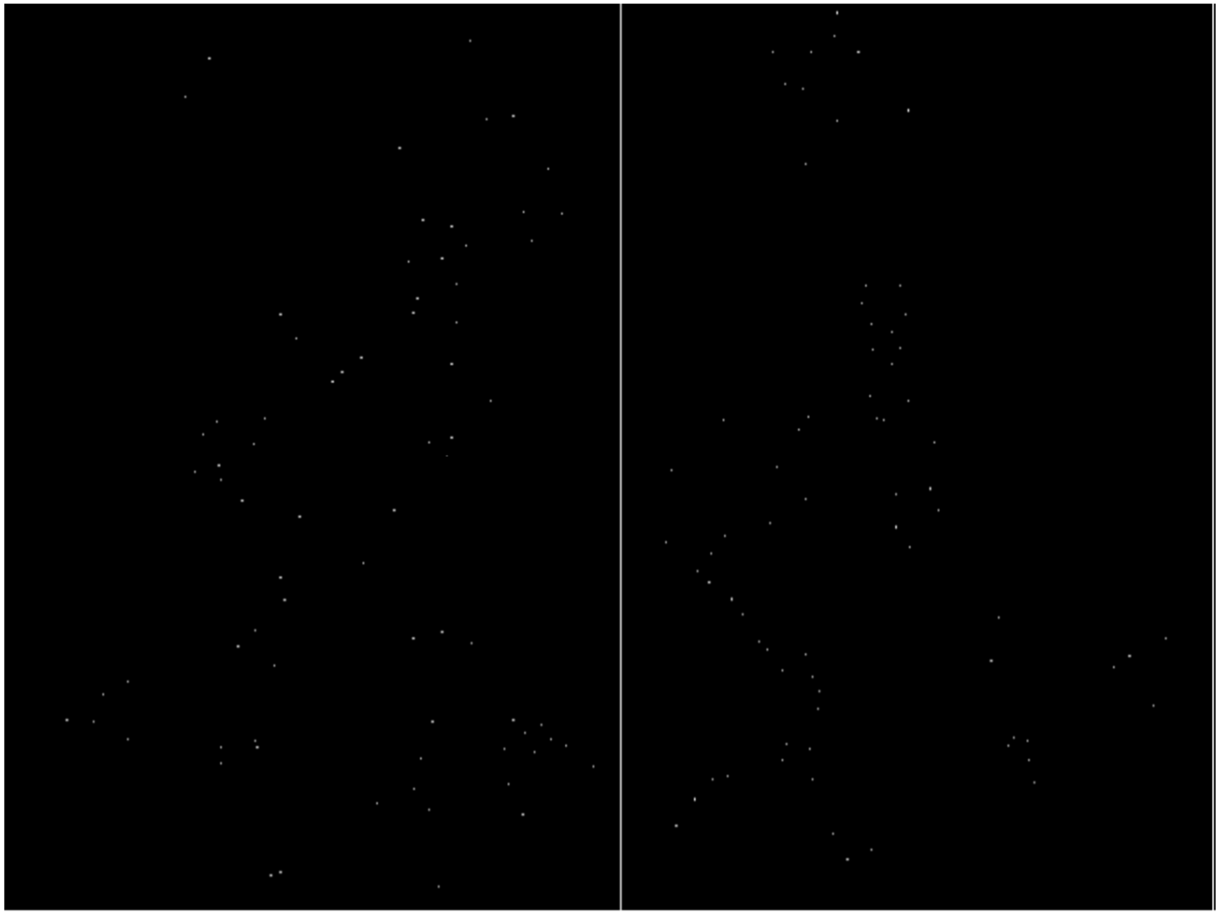
Harris Corner Detectors : Results

Corner Response " R > Threshold" ✓



Harris Corner Detectors : Results

After Applying Non-maxima Suppression on “R”



- The point which is locally maxima will only be treated as “Corner” (or interest point) point else the point will be neglected.

Harris Corner Detectors : Results

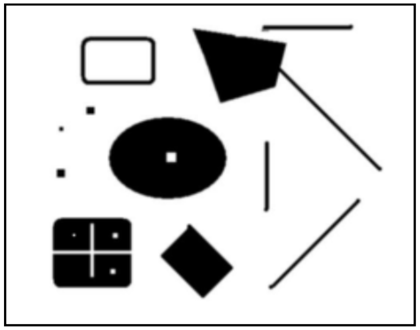
Harris Corner Detectors : Algorithm

- Given an image : I
  - Step-1 : Find I_x and I_y
  - Step-2 : Find $I_x I_x$ (image-1), $I_y I_y$ (image-2), and $I_x I_y$ (image-3)
  - Step-3 : Compute : $g(\sigma) * (I_x I_x)$, $g(\sigma) * (I_y I_y)$, and $g(\sigma) * (I_x I_y)$

    where, $g(\sigma) = e^{-\frac{x^2+y^2}{2\sigma^2}}$

    i.e., in Step-3, convolve all the three images with Gaussian filter
  - Step-4 : Compute $M_{2\times 2}$ (use suitable neighbourhood operation). Example - 3×3, 5×5, or 7×7
  - Step-5 : Find eigenvalues (λ_1 and λ_2) of $M_{2\times 2}$
  - Step-6 : Compute $R = \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2$ (measure of cornerness) and compare it some threshold Th in order to decide whether the pixel is corner or NOT
  - Step-7 : Non-maxima suppression
  - Repeat Step – 4 to Step – 7 for each of the pixels in the image.

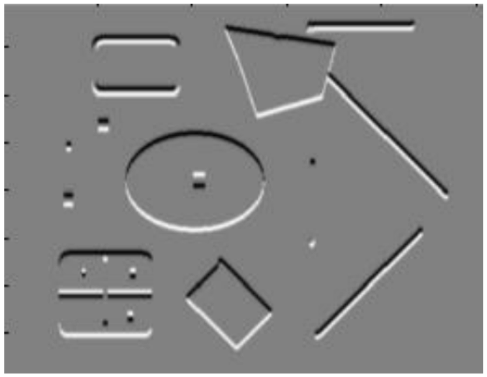
Harris Corner Detectors : Step-wise Implementation



Input image : I

- Step-1

I_x : Derivative image



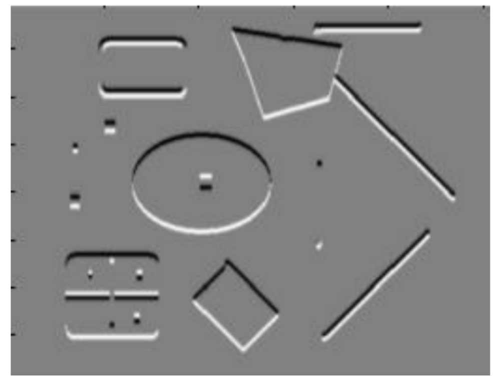
I_y : Derivative image

Source: James Hays

Harris Corner Detectors : Step-wise Implementation

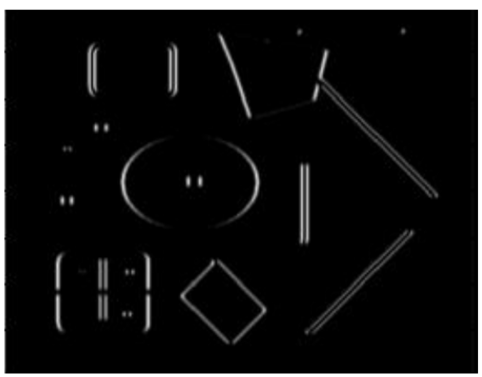
- Step-1 :

I_x : Derivative image

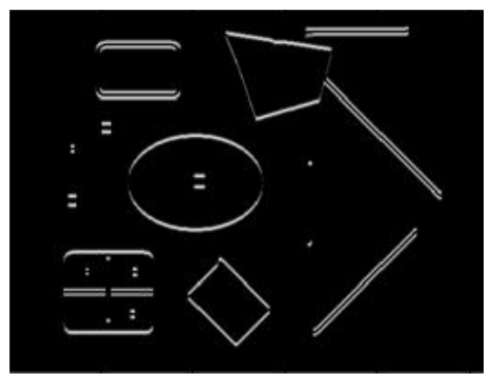


I_y : Derivative image

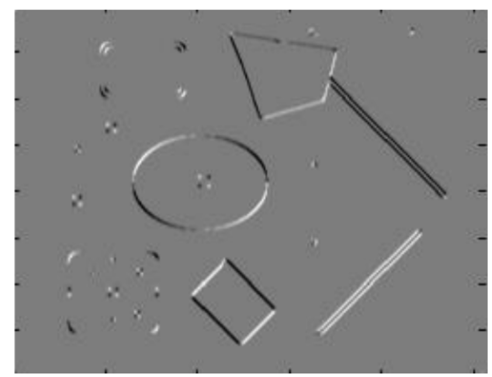
- Step-2 :



$I_x I_x$ (image-1)



$I_y I_y$ (image-2)

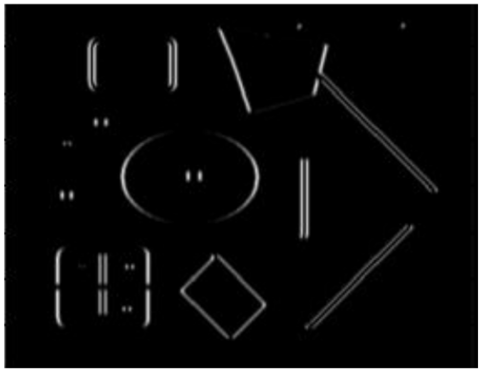


$I_x I_y$ (image-3)

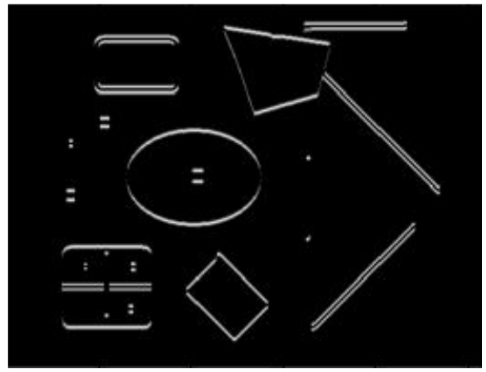
Source: James Hays

Harris Corner Detectors : Step-wise Implementation

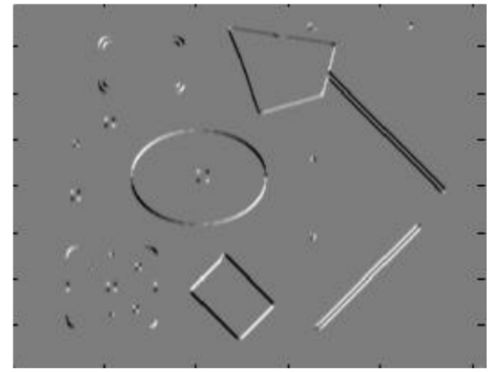
- ❑ Step-2 :



I_xI_x (image-1)



I_yI_y (image-2)



I_xI_y (image-3)

- ❑ Step-3 :

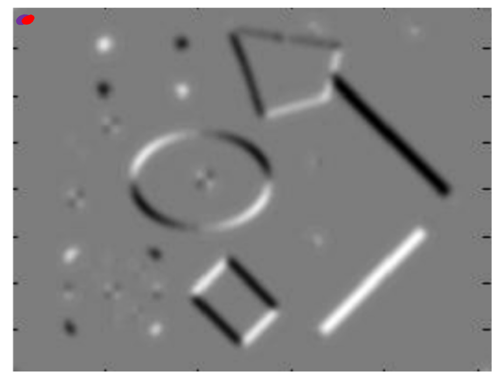
3

$g(\sigma) * (I_xI_x)$

✓

$g(\sigma) * (I_yI_y)$

10



$g(\sigma) * (I_xI_y)$

✓ $M = \begin{bmatrix} 3 & 10 \\ 10 & 2 \end{bmatrix}$

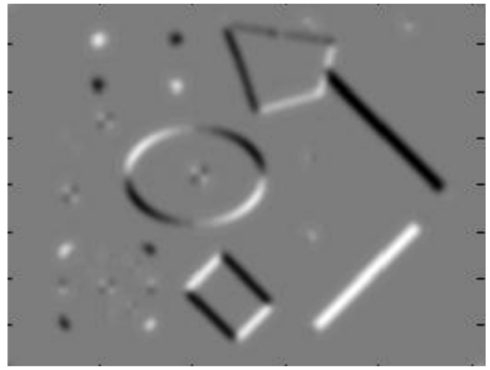
Source: James Hays

Harris Corner Detectors : Step-wise Implementation

❑ Step-4 :

$g(\sigma) * (I_x I_x)$

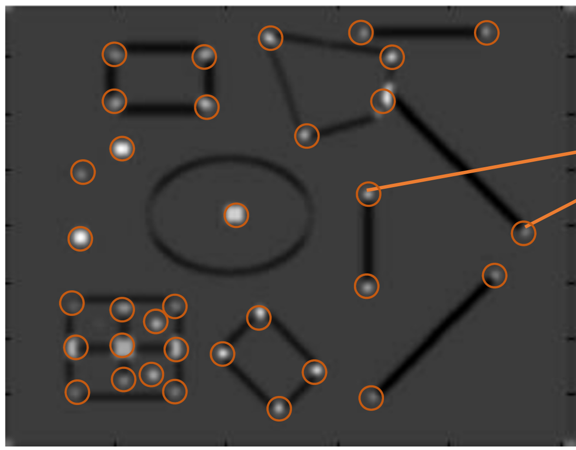
$g(\sigma) * (I_y I_y)$



$g(\sigma) * (I_x I_y)$

❑ Step-4 to Step-7 :

✓ $R = \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2$



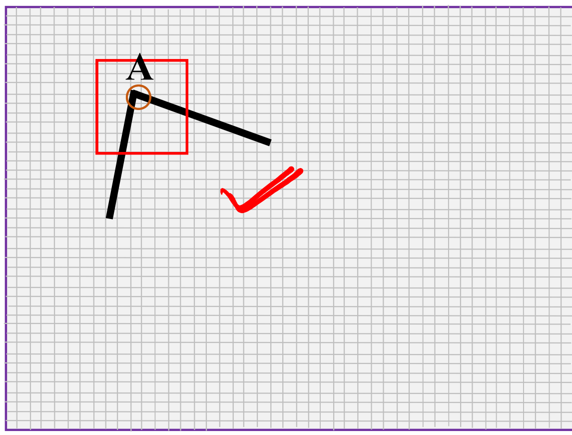
Potential "CORNER" pixels

These pixels can treated as ○ "Interest Points"

Source: James Hays

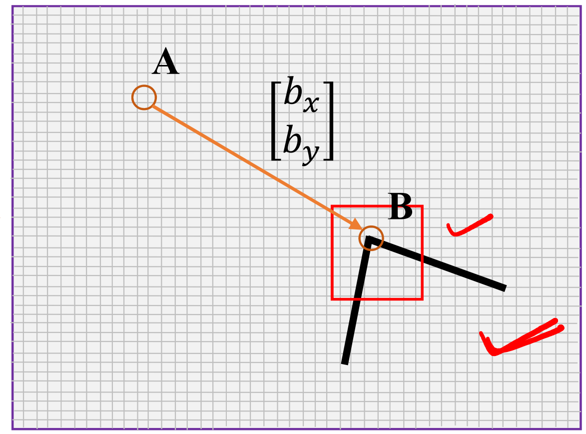
Invariance of Harris Corner Detectors

- ❑ Translation Invariance:



$$I_t = I + b$$

$$\boxed{\begin{aligned} x' &= x + b_x \\ y' &= y + b_y \end{aligned}}$$



Synthetic corner : I

Synthetic corner : I_t

- ❑ Harris Corner Detector (HCD) uses "derivatives" only and "derivative" are invariant to shift, and so HCD is invariant to translation.

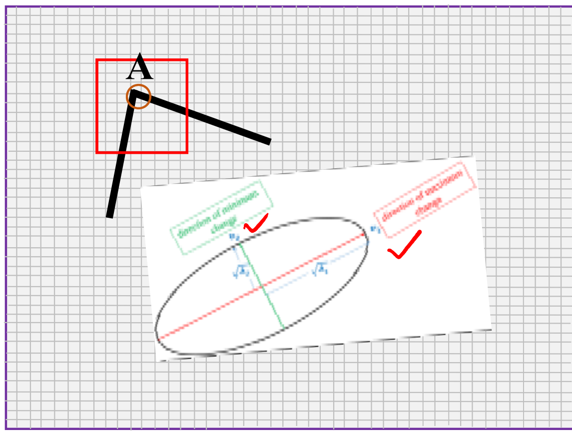
- ❑ In this case: $(I_t)' = (I)'$

$d(I_t) = d(I+b)$

$= d(I)$

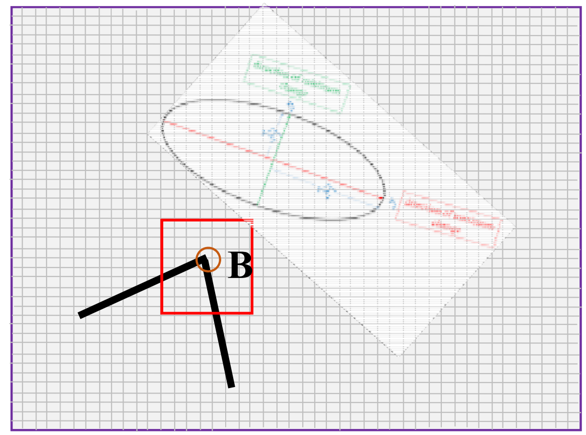
Invariance of Harris Corner Detectors

❑ Rotation Invariance:



Synthetic Image : I

$$I_\theta = R_\theta(I)$$

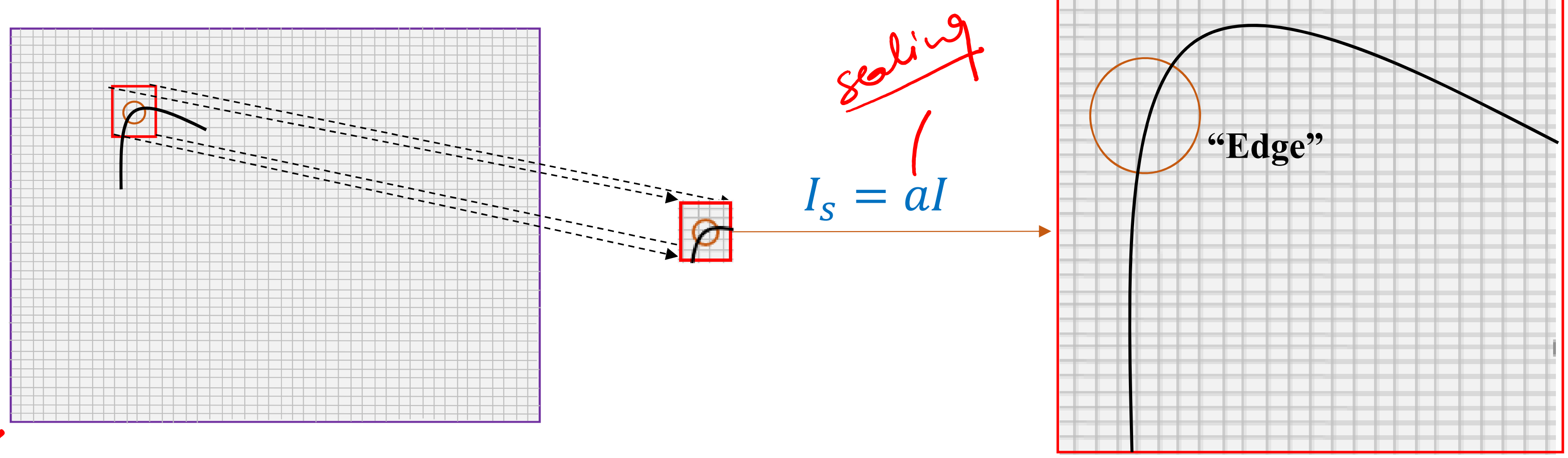


Rotated Image : I_θ

❑ The eigenvectors of $M_{2\times 2}$ still able to capture direction along maximum variation, and hence, Harris Corner Detector (HCD) is invariant to even "Rotation".

Invariance of Harris Corner Detectors

- Scale Invariance:



Synthetic Image : I

Scaled Image : I_s

- In scaled image, the corner is no longer remains a corner. It may converted to an edge. So, HCD may classify such points to an "Edge" rather than a "Corner".

- So, HDR is not scale invariant.

What to do?

So, there is a need of an Interest Point Detector that is Scale invariant as well.

SIFT
Detector-cum-Descriptor

Reference

❖ Richard Szeliski, Computer Vision: Algorithms and Applications, Springer, 2010 (online draft),

❖ Mubarak Shah, “Fundamentals of Computer Vision” (Online available)

❖ Ian Goodfellow, Yoshua Bengio and Aaron Courville, “Deep Learning” (Online available)

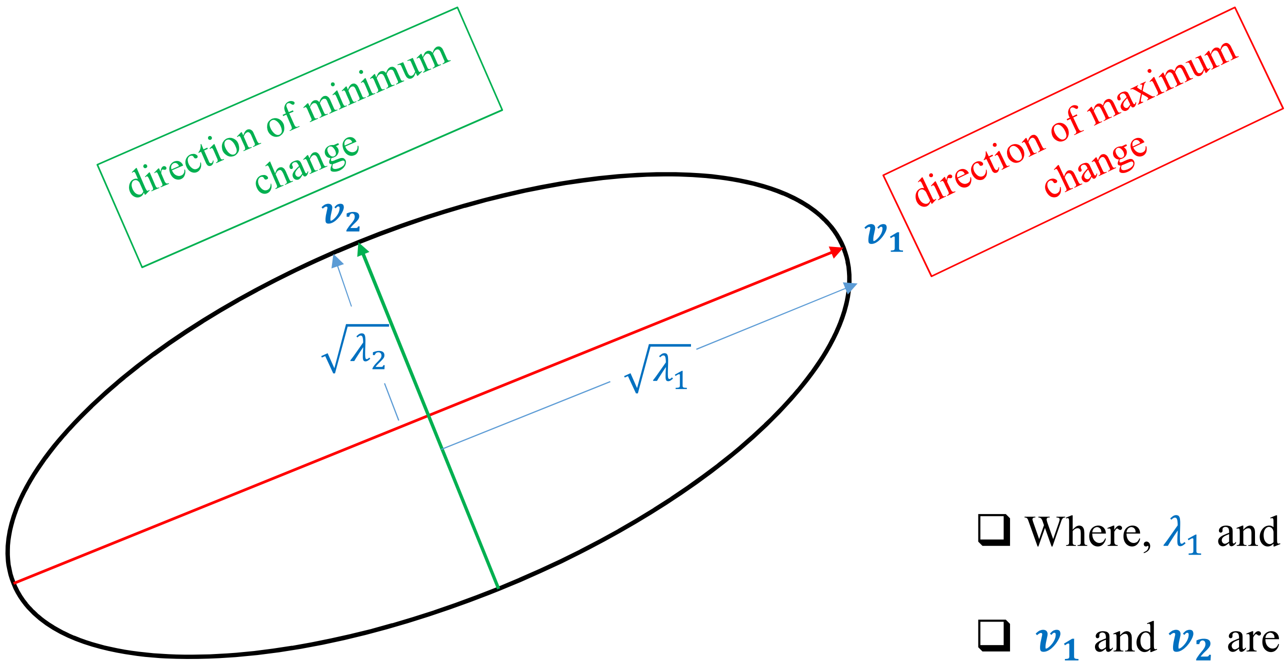
Acknowledgement!

Sources for this lecture include materials from works by Szeliski, Abhijit Mahalanobis, Sedat Ozer, Ulas Bagci, Mubarak Shah, Antonio Torralba, D. Hoiem, Justin Liang, and others. References are given for the source image contents.

Queries!

Geometrical Interpretation of $E(u,v)$

- For given (u,v) : $M_{2X2} = \sum_x \sum_y W(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_y I_x & I_y I_y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$



- Where, λ_1 and λ_2 are eigenvalues of M_{2X2} matrix
- $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ are eigenvectors of M_{2X2} matrix