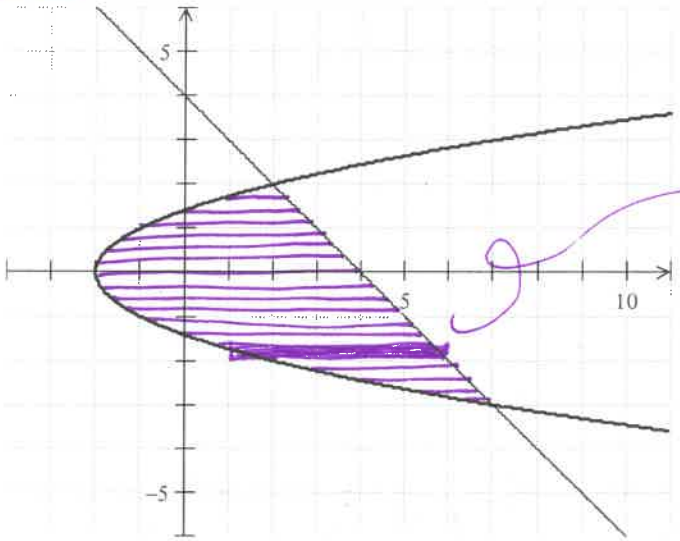
Math 252 Homework 9 Written Part **Name:** KEY

Write legibly. Show your work. Graph neatly. Use a ruler for all straight lines.

(1) Set up an integral that would find the area between the two curves $x = y^2 - 2$ (the bold line) and $x + y = 4$ (the thin line). Follow the steps as given below.

So? $x = 4 - y$



Right x − Left x

$= (4 - y) - (y^2 - 2)$

$= (-y^2 - y + 6)$

a. Show (with algebra!) how to find the intersections of the two functions.

$x = y^2 - 2$ → $x + y = 4$

$y^2 + y - 2 = 4$

$y^2 + y - 6 = 0$

$(y + 3)(y - 2) = 0$

$y = -3, 2$

$y = -3$	$y = 2$
$x = 9 - 2$	$x = 4 - 2$
$x = 7$	$x = 2$
$(7, -3)$	$(2, 2)$

b. On the graph above, draw and label your representative rectangle. Explain why you chose either vertical or horizontal rectangles.

I chose horizontal rectangles so that I would always have the same functions on left and right of each rectangle. Also, solving $x = y^2 - 2$ for y would be a pain.

c. Set up the integral that will find the area between the curves. Simplify.

$$\int_{-3}^{2} (-y^2 - y + 6)\, dy$$

over!

d. Finally, show how to solve the integral. Compare to the graph to see if your final answer is reasonable.

$$\int_{-3}^{2} -y^2 - y + 6 \, dy$$

$$= -\frac{y^3}{3} - \frac{y^2}{2} + 6y \Big]_{-3}^{2}$$

$$= \left[-\frac{8}{3} - 2 + 12\right] - \left[-\frac{(-3)^3}{3} - \frac{(-3)^2}{2} - 18\right]$$

$$= \left[-\frac{8}{3} + 10\right] - \left[9 - \frac{9}{2} - 18\right]$$

$$= -\frac{8 \cdot 2}{3 \cdot 2} + \frac{9 \cdot 3}{2 \cdot 3} + 10 + 9$$

$$= \frac{-16}{6} + \frac{27}{6} + \frac{114}{6}$$

$$= \boxed{\frac{125}{6}}$$