Write legibly.   Show your work.   Graph neatly.   Use a ruler for all straight lines.

Practice with volumes:
1. Sketch the relevant area (before rotation), marking your scale.
2. Draw the representative rectangle.
3. Label or list the measurements you will need to find the volume.
4. Find the volume of the representative disk, washer, or cylinder.
5. Set up the integral to find the volume of the whole shape.
6. Solve the integral, expressing your answer in <u>exact</u> (not decimal) form.

(1)   Start with the area bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$, then rotate around the line $y = -2$.
Use washers.

graph w/ rect
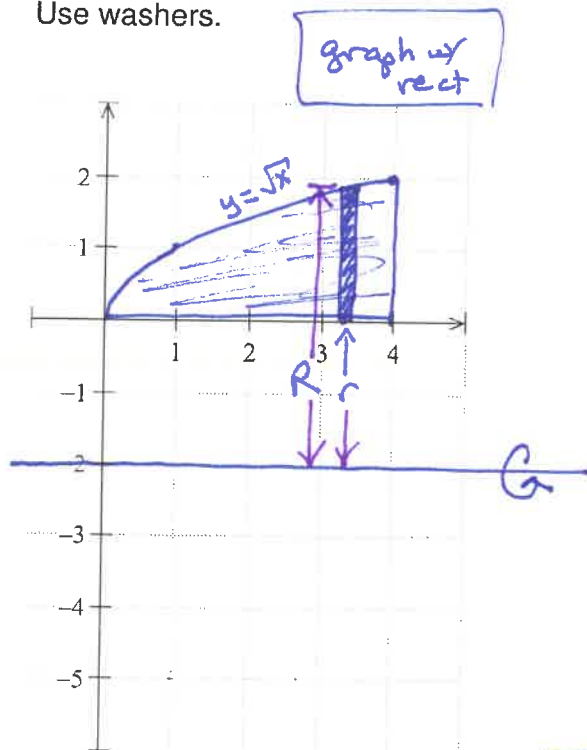
measurements:

thickness $= dx$

$r = 0 - (-2) = 2$

$R = \sqrt{x} - (-2) = \sqrt{x} + 2$

Vol one washer:

$V_{washer} = \pi (R^2 - r^2)\ \text{thick}$

$= \pi ((\sqrt{x} + 2)^2 - (2)^2)\ dx$

$= \pi (x + 4\sqrt{x} + \cancel{4} - \cancel{4})\ dx$

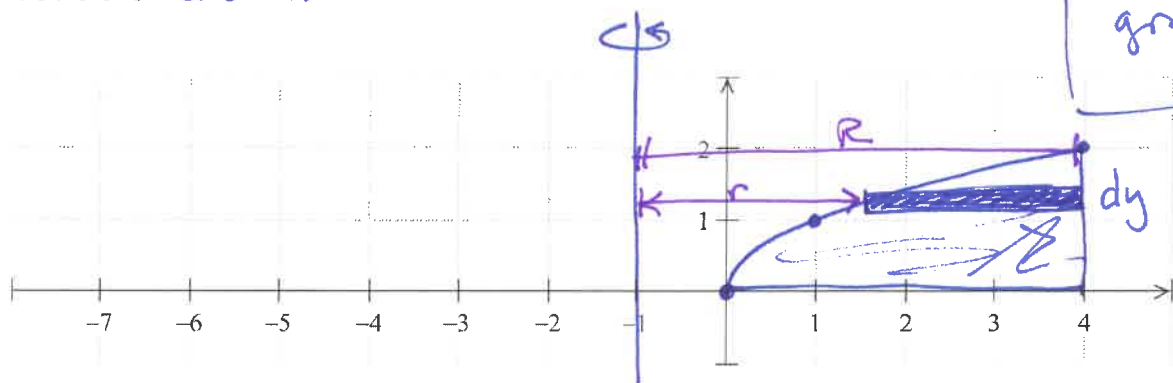$\boxed{V_{wash} = \pi (x + 4\sqrt{x})\ dx}$

Total volume

$\boxed{V_{total} = \pi \int_0^4 x + 4\sqrt{x}\ dx}$

$= \pi \left[\tfrac{1}{2}x^2 + 4 \left(\tfrac{2}{3}x^{\frac{3}{2}} \right) \right]_0^4$

$= \pi \left[\tfrac{16}{2} + \tfrac{8(8)}{3} \right] - \pi \left[0 \right]$

$= \boxed{\dfrac{88\pi}{3}}$

(2) Start with the area bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$, then rotate around the line $x = -1$. Use ~~disks.~~ washers.

graph w/ rect

measurements:

$$\text{thick} = dy$$

$$r = y^2 - (-1) = y^2 + 1$$

$$R = 4 - (-1) = 5$$

Volume one washer:

$$V_{wash} = \pi(R^2 - r^2)\,\text{thick}$$

$$= \pi\left(5^2 - (y^2+1)^2\right)dy$$

$$= \pi\left(25 - (y^4 + 2y^2 + 1)\right)dy$$

$$\boxed{V_{wash} = \pi(24 - y^4 - 2y^2)dy}$$

Total volume:

$$\boxed{V_{total} = \pi\int_0^2 24 - y^4 - 2y^2\,dy}$$

$$= \pi\left[24y - \tfrac{1}{5}y^5 - \tfrac{2}{3}y^3\right]_0^2$$

$$= \pi\left[48 - \tfrac{32}{5} - \tfrac{16}{3}\right] - \pi[0]$$

$$\boxed{= \frac{544\pi}{15}}$$