Some symbols from Chapter 2

$\forall$ = for any, for all, for every, ...

$\exists$ = there exists, there is a, there is a least one, ...
there are some...

statements (or propositions): P ; $6 > 5$; John Q. Smith is a student

lowercase

uppercase

predicates: $P(x)$; $x > 5$; He is a student.

any predicate has a __domain__, every possible input.

any predicate has a __truth set__, every input that makes it true.

Set notations:

Set brackets: $\{ \dots \}$

__ex__ $\{1, 2, 4, 8\} = \{ x \in \mathbb{Z} \mid x \text{ is a factor of } 8 \}$

the set of

all x

element of

the integers

such that (the line is used for such that __only__ in set notation. Everywhere else, use s.t. or write the words)

$x \in A$ x __is__ __an__ __element__ __of__ the set A

$x \notin A$ x __is__ __not__ __an__ __element__ __of__ the set A.

$\mathbb{R}$ = the real numbers

$\mathbb{Z}$ = the integers   "Zahlen" $\mathbb{Z}^+$, $\mathbb{Z}^-$, etc... $\mathbb{Z}^{even}$, $\mathbb{Z}^{odd}$, ...

$\mathbb{Q}$ = the rational numbers

$\mathbb{N}$ = the natural numbers $= \mathbb{Z}^{nonneg.}$

__Subsets__: A is a subset of B iff every element of A is in B.

So! $\mathbb{N}$ is a subset of $\mathbb{Z}$, $\mathbb{Z}$ is a subset of $\mathbb{Q}$, $\mathbb{Q}$ is a subset of $\mathbb{R}$!

Key points of 2.2

Conditional

$p \to q$

or $\forall x \in D, P(x) \Rightarrow Q(x)$

Contrapositive (go backwards and use nots)

$\sim q \to \sim p$

"q if p" " p only if q "

" If p, then q. "

" p is sufficient to imply q "

" if you know p is true, then you know q is true "

Converse (go backwards)

$q \to p$

inverse (use nots)

$\sim p \to \sim q$

" p if q "

" p is necessary to imply q "

" if you know p is false, then you know q is false "

Biconditional

$p \leftrightarrow q$

" p if and only if q "

" p iff q "

" p is necessary and sufficient to imply q "

" p and q are both true or both false "

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converse error

$p \rightarrow q$

q

$\therefore p$

All cats have 4 legs.
I have 4 legs.
∴ I am a cat.

(The converse error assumes that the converse of a conditional is true.)

inverse error

$p \rightarrow q$

$\sim p$

$\therefore \sim q$

All cats have 4 legs.
I am not a cat.
∴ I don't have 4 legs.

(The inverse error assumes that the inverse of a conditional is true.)

How do these relate to this diagram?

Quantified Statements

$\forall x \in D$, $P(x)$ is true.

"for all x in the domain D, the statement is true."

$\exists x \in D$ s.t. $Q(x)$ is true.

Notice! $\exists$ is always followed by a "such that."

"there exists an x in the domain D such that the statement is true."

Multiply Quantified Statements

$\forall x$, $\exists y$ s.t. $P(x,y)$ is true.

"For all x, there exists a y such that the statement is true."

$\exists x$ s.t. $\forall y$ $P(x,y)$ is true.

"There exists an x such that for all y the statement is true."

Negations of Quantified Statements

$\forall x$, $P(x)$ is true $\xrightarrow{\text{negation}}$ $\exists x$ s.t. $P(x)$ is not true.

$$\sim(\forall, P) \equiv \exists, \sim P$$

$\exists x$ s.t. $P(x)$ is true $\xrightarrow{\text{negation}}$ $\forall x$, $P(x)$ is not true.

$$\sim(\exists, P) \equiv \forall, \sim P$$