8.3 Hydrostatic Pressure AND Force.

UNITS "REVIEW:" (metric or SI system)

ft, ft^2, ft^3 $\Leftarrow$ distance, area, volume = m, m^2, m^3

lbs = weight $\Leftarrow$ mass = kilogram = kg

g = acceleration due to gravity (on Earth) = $9.8\ m/sec^2$

lbs $\Leftarrow$ force = Newton = $N = \frac{kg \cdot m}{s^2}$

lbs/ft^2 $\Leftarrow$ pressure = Pascal = $Pa = \frac{N}{m^2} = \frac{kg}{m \cdot sec^2}$

kilopascal = kPa = 1000 Pa.

62.5 lbs/ft^3 $\Leftarrow$ density of water = $1000\ kg/m^3$

ρ = "rho" = density (mass density = mass per volume)

δ = "delta" = weight density $= \frac{m \cdot a}{volume}$ = (force exerted per volume) $\delta = \rho \cdot g$

FORMULAS REVIEW/SUMMARY

FORCE = MASS · ACCELERATION $\Leftrightarrow$ $F = ma$

$$\text{PRESSURE} = \frac{\text{FORCE}}{\text{AREA}}$$

FORCE = PRESSURE · AREA

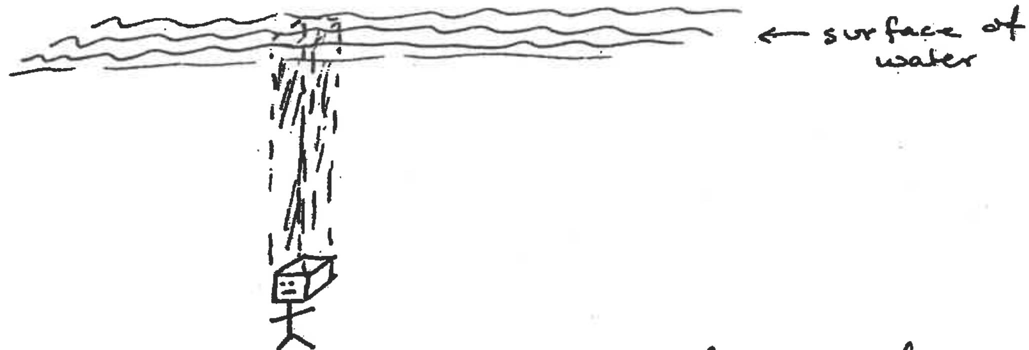
Pressure = Depth · Weight Density $\leftarrow$ PRESSURE = DEPTH · MASS DENSITY · ACCEL OF GRAVITY

How to do your homework

- draw a careful picture for each (label it!)
- draw a representative rectangle for each — label carefully
- write the integral that needs to be solved.
- solve the integral (calculator solutions are acceptable.)

LET'S WORK ON UNDERSTANDING THESE CONCEPTS:

EXAMPLE:



What's the force pushing down on the top of Block's head?

$$\text{Force} = \text{Mass} \cdot \text{Acceleration}$$

$$\text{Force} = \left(\begin{array}{c}\text{Mass of}\\ \text{column}\\ \text{of}\\ \text{water}\end{array}\right)\left(\begin{array}{c}\text{Acceleration}\\ \text{due to}\\ \text{gravity}\end{array}\right)$$

$$\text{Force} = \left(\begin{array}{c}\text{Volume}\\ \text{of}\\ \text{Column}\end{array}\right)\left(\begin{array}{c}\text{mass}\\ \text{Density}\\ \text{of}\\ \text{Water}\end{array}\right)\left(\begin{array}{c}\text{Acceleration}\\ \text{due to}\\ \text{gravity}\end{array}\right)$$

$$\text{Force} = \left(\begin{array}{c}\text{Area of}\\ \text{top of}\\ \text{head}\end{array}\right)\left(\begin{array}{c}\text{Depth}\\ \text{of}\\ \text{Water}\end{array}\right)\left(\begin{array}{c}\text{mass}\\ \text{Density}\\ \text{of}\\ \text{Water}\end{array}\right)\left(\begin{array}{c}\text{Acceleration}\\ \text{due to}\\ \text{Gravity}\end{array}\right)$$

$$\boxed{\text{FORCE} = (\text{AREA})(\text{DEPTH})(\text{DENSITY})(\text{ACCEL. OF GRAVITY})}$$

What's the pressure pushing down on Block's head?

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}, \quad \text{so} \ldots$$

$$\boxed{\text{PRESSURE} = (\text{DEPTH})(\text{DENSITY})(\text{ACCEL OF GRAVITY})}$$

An amazing fact about fluids: Block will feel the same pressure no matter which way he turns — at any particular depth the pressure is constant:

FOR EXAMPLE:

What is the force on the bottom of a 10 m by 30 m swimming pool in which the water is 3 meters deep? (Ignore atmospheric pressure)

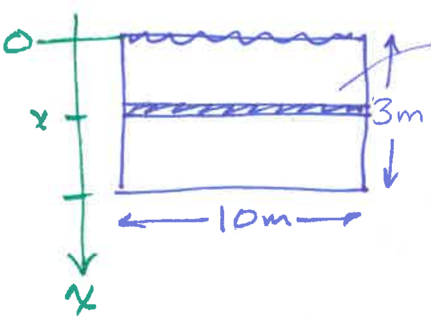
$$\text{Force} = (\text{Area})(\text{Depth})(\text{Density})\left(\begin{array}{c}\text{Acceleration}\\ \text{of Gravity}\end{array}\right)$$

$$= (10\text{m})(30\text{m})(3\text{ m})\left(1000\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)$$

$$= 8{,}820{,}000\ \frac{\text{kg-m}}{\text{s}^2}$$

$$\boxed{= 8{,}820{,}000 \text{ Newtons } \quad (\text{N})}$$

What is the force on one of the narrow ends of the pool? (Ignore atmospheric pressure)



$$\underset{\text{on a slice}}{\text{Force}} = \text{Area} \cdot \text{Depth} \cdot \text{Density} \cdot \text{Accel}$$

$$= (\underset{\text{m}\cdot\text{m}}{10\,dx})(\underset{\text{m}}{x})\left(1000\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)$$

$$= 98{,}000\,x\,dx \quad \frac{\text{kg-m}}{\text{s}^2} \quad (\text{N})$$

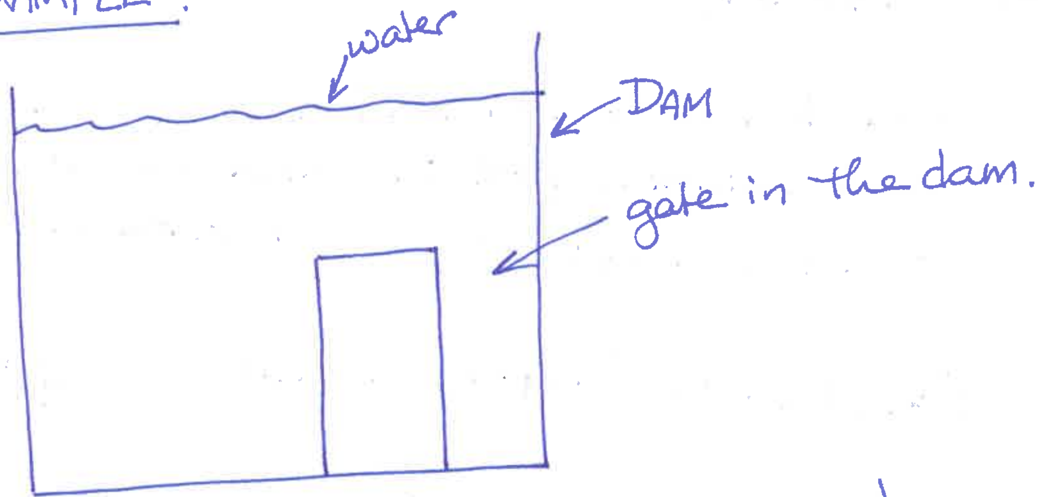
$$\text{Total Force} = \int_0^3 98000\,x\,dx$$

$$= 49000\,x^2\Big]_0^3$$

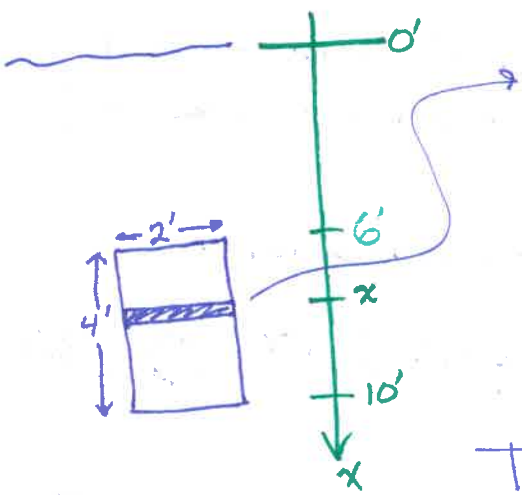
$$= 49000\,(3^2 - 0^2)$$

$$\boxed{= 441{,}000 \text{ Newtons}}$$

For Example:



If the water is 10 feet deep, and the gate is 4 feet tall and 2 feet wide, what is the force on the gate? (Ignore atmospheric pressure.)



$$\text{Force on slice} = (\text{Area})(\text{depth})(\text{weight density})$$

$$= \underset{ft \cdot ft}{(2\,dx)} \cdot \underset{ft}{(x)} \cdot \underset{lbs/ft^3}{(62.5)}$$

$$= 125x\,dx \text{ lbs}$$

$$\text{Total Force} = \int_6^{10} 125x\,dx$$

$$= \frac{125}{2}x^2\Big]_6^{10}$$

$$= \frac{125}{2}(100-36)$$

$$\boxed{= 4000 \text{ lbs of force on the gate}}$$