

Write legibly. Show your work. Graph neatly. Use a ruler for all straight lines.

**Integration by Parts:**

- (1) Show how to integrate by filling in the blanks:

$$\begin{aligned}
 & \int x \cdot e^x dx \\
 &= \int \underbrace{x}_u \cdot \underbrace{e^x}_{dv} dx \\
 & \quad \begin{array}{l|l} u = x & dv = e^x dx \\ \hline du = dx & v = e^x \end{array} \\
 &= \overbrace{x}^u \cdot \overbrace{e^x}^v - \int \overbrace{e^x}^v \cdot \overbrace{dx}^{du} \\
 &= x \cdot e^x - e^x + C \\
 & \quad \text{(don't forget + C)}
 \end{aligned}$$

- (2) Explain why we choose
- $u = x$
- , instead of
- $u = e^x$
- .

If we chose  $u = e^x$ , then  $dv = x dx$ .  
 Then  $v = \frac{1}{2}x^2$ , which got more complicated rather than simpler.

- (3) Show how to check your answer from problem 1 by taking a derivative:

$$\begin{aligned}
 & \frac{d}{dx} \left( \underbrace{x \cdot e^x - e^x + C}_{\text{your answer from problem 1}} \right) \\
 &= x \cdot e^x + e^x \cdot 1 - e^x + 0 \\
 &= x \cdot e^x + \cancel{e^x} - \cancel{e^x} \\
 &= x \cdot e^x \quad \leftarrow \text{Make sure it works out!}
 \end{aligned}$$

(4) Show how to integrate:

$$\int x \cdot \sin(x) dx$$

$$\begin{array}{l|l} u = x & dv = \sin(x) dx \\ \hline du = dx & v = -\cos(x) \end{array}$$
$$= \overbrace{x}^u \cdot \overbrace{-\cos(x)}^v - \int \overbrace{-\cos(x)}^v \cdot \overbrace{dx}^{du}$$
$$= -x \cdot \cos(x) + \sin(x) + C$$

(don't forget + C)

(5) Show how to integrate:

$$\int x \cdot \sin(x^2) dx$$

$$\left[ \begin{array}{l} \text{let } u = x^2 \\ du = 2x dx \\ \frac{1}{2} du = x dx \end{array} \right]$$

$$= \int \frac{1}{2} \sin(u) du$$

$$= -\frac{1}{2} \cos(u) + C$$

$$= -\frac{1}{2} \cos(x^2) + C$$

(6) Explain how we know that we shouldn't do problem 5 using integration by parts.

Reason 1 If we can do a problem by substitution, that's easier.

Reason 2 Let's try parts:

$$\begin{array}{l|l} u = x & dv = \sin(x^2) dx \\ \hline du = dx & \end{array}$$

oops! we don't know how to integrate  $\sin(x^2)$ !

Stuck