Math 231 – HW 15 **Name:** KEY

Epp 2nd Ed. 11.2 36
11.5 1, 3, 7 - 14, 32 - 36

11.2 (36) A traveler in Europe wants to visit each of the cities shown on the map exactly once, starting and ending in Brussels. The distance (in kilometeres) between each pair of cities is given in the table. Find a Hamiltonian circuit that minimizes the total distance travelled. (Use the map to narrow down the possible circuits to just a few. Then use the table to find the total distance for each of those. Which route has the minimum distance?)

	Berlin	Brussels	Dusseldorf	Luxembourg	Munich
Brussels	783				
Dusseldorf	564	223			
Luxembourg	764	219	224		
Munich	585	771	613	517	
Paris	1,057	308	497	375	832

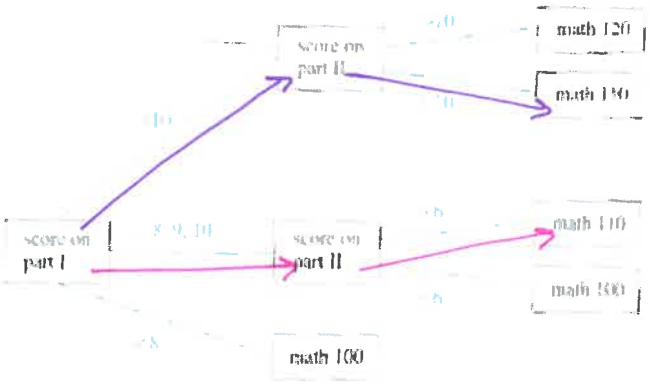
Here's my answer:

Brussels → Düsseldorf: 223
Düsseldorf → Berlin : 564
→ Munich : 585
→ Luxembourg: 517
→ Paris : 375
→ Brussels: 308

total: 2572 km

Can you do better?

11.5 (1) Read the tree from example 11.5.2 from left to right to decide:



(a) what course a student who scored 12 on part I and 4 on part II should take.

Take Math 110

(b) what course a student who scored 8 on part I and 9 on part II should take.

Take Math 110

11.5 (3) What is the total degree of a tree with n vertices? Why?

a tree with n vertices has n-1 edges,

so has degree $= 2(n-1) = 2n-2$

make sure you explain!

11.5 (7a)

Find all terminal vertices for the tree:

V_1, V_5, V_7

Find all internal vertices for the tree:

V_2, V_4, V_3, V_6

11.5 (7b)

Find all terminal vertices for the tree:

V_1, V_2, V_5, V_6, V_8

Find all internal vertices for the tree:

V_3, V_4, V_7

In each of 8 - 14, either draw a graph with the given specifications or explain why no such graph exists:

11.5 (8) tree, nine vertices, nine edges

Not possible. A tree with 9 vertices should have 8 edges.

11.5 (9) graph, connected, nine vertices, nine edges

For example:

11.5 (10) graph, circuit-free, nine vertices, six edges

For example:

11.5 (11) tree, six vertices, total degree 14

Not possible. A tree with 6 vertices should have 5 edges, so its degree would be 10.

11.5 (12) tree, five vertices, total degree 8

For example:

11.5 (13) graph, connected, six vertices, five edges, has a nontrivial circuit

Not possible. A connected graph with 6 vertices and 5 edges is a tree, and therefore has no non-trivial circuits.

11.5 (14) graph, two vertices, one edge, not a tree

For example: 

11.5 (32) Consider the tree shown below with root a.

(a) What is the level of n? 3

(b) What is the level of a? 0

(c) What is the height of this rooted tree? 5

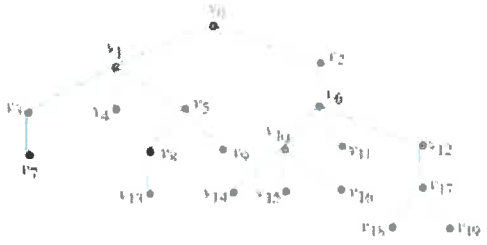
(d) What are the children of n? u, v

(e) What is the parent of g? d

(f) What are the siblings of j? k, l

(g) What are the descendants of f? m, s, t, x, y

11.5 (33)



(a) What is the level of v_8? 3

(b) What is the level of v_0? 0

(c) What is the height of this rooted tree? 5

(d) What are the children of v_{10}? v_{14}, v_{15}, v_{16}

(e) What is the parent of v_5? v_1

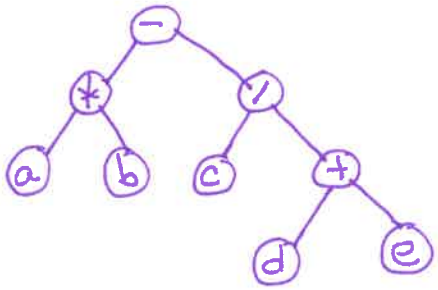
(f) What are the siblings of v_1? v_2

(g) What are the descendants of v_{12}?

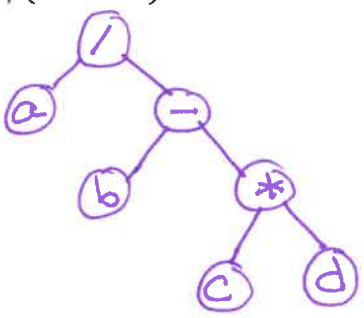
v_{17}, v_{18}, v_{19}

11.5 (34) Draw binary trees to represent the following expressions:

(a) $a \cdot b - (c/(d+e))$



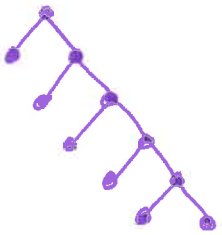
(b) $a/(b - c \cdot d)$



In each of 35 and 36, either draw a graph with the given specifications, or explain why no such graph exists.

11.5 (35) full binary tree, five internal vertices

There's LOTS of answers.



OR

OR

OR

OR

OR ...

11.5 (36) full binary tree, five internal vertices, seven terminal vertices

Not possible. 5 internal vertices implies 6 terminal vertices.