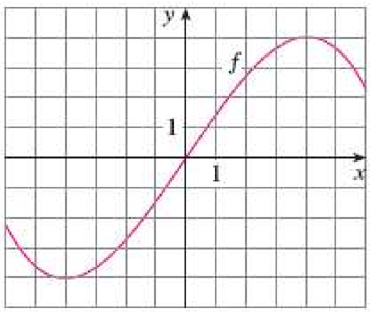
Chapter Review

Exercises

1. Let f be the function whose graph is given.



   a. Estimate the value of $f(2)$.

   b. Estimate the values of x such that $f(x) = 3$.

   c. State the domain of f.

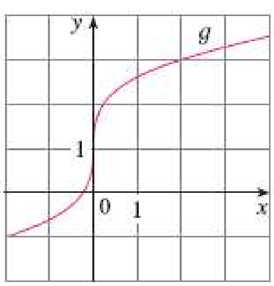
   d. State the range of f.

   e. On what interval is f increasing?

   f. Is f one-to-one? Explain.

   g. Is f even, odd, or neither even nor odd? Explain.

2. The graph of g is given.



   a. State the value of $g(2)$.

   b. Why is g one-to-one?

   c. Estimate the value of $g^{-1}(2)$.

   d. Estimate the domain of g^{-1}.

   e. Sketch the graph of g^{-1}.

3. If $f(x) = x^2 - 2x + 3$, evaluate the difference quotient

$$\frac{f(a+h) - f(a)}{h}$$

4. Sketch a rough graph of the yield of a crop as a function of the amount of fertilizer used.

   5, 6, 7 and 8 Find the domain and range of the function. Write your answer in interval notation.

5. $f(x) = 2/(3x - 1)$

6. $g(x) = \sqrt{16 - x^4}$

7. $h(x) = \ln(x + 6)$

8. $F(t) = 3 + \cos 2t$

9. Suppose that the graph of f is given. Describe how the graphs of the following functions can be obtained from the graph of f.

   a. $y = f(x) + 8$

   b. $y = f(x + 8)$

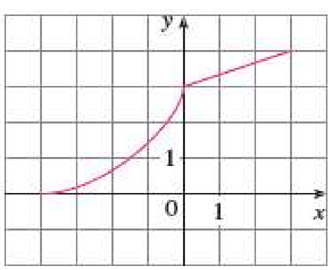
   c. $y = 1 + 2f(x)$

   d. $y = f(x - 2) - 2$

   e. $y = -f(x)$

   f. $y = f^{-1}(x)$

10. The graph of f is given. Draw the graphs of the following functions.



   a. $y = f(x - 8)$

   b. $y = -f(x)$

   c. $y = 2 - f(x)$

   d. $y = \frac{1}{2}f(x) - 1$

   e. $y = f^{-1}(x)$

f. $y = f^{-1}(x+3)$

11, 12, 13, 14, 15 and 16 Use transformations to sketch the graph of the function.

11. $y = (x-2)^3$

12. $y = 2\sqrt{x}$

13. $y = x^2 - 2x + 2$

14. $y = \ln(x+1)$

15. $f(x) = -\cos 2x$

16. $f(x) = \begin{cases} -x & \text{if } x < 0 \\ e^x - 1 & \text{if } x \geqslant 0 \end{cases}$

17. Determine whether f is even, odd, or neither even nor odd.

a. $f(x) = 2x^5 - 3x^2 + 2$

b. $f(x) = x^3 - x^7$

c. $f(x) = e^{-x^2}$

d. $f(x) = 1 + \sin x$

18. Find an expression for the function whose graph consists of the line segment from the point $(-2, 2)$ to the point $(-1, 0)$ together with the top half of the circle with center the origin and radius 1.

19. If $f(x) = \ln x$ and $g(x) = x^2 - 9$, find the functions

a. $f \circ g$,

b. $g \circ f$,

c. $f \circ f$,

d. $g \circ g$,

and their domains.

20. Express the function $F(x) = 1/\sqrt{x + \sqrt{x}}$ as a composition of three functions.

21. Life expectancy improved dramatically in the 20th century. The table gives the life expectancy at birth (in years) of males born in the United States. Use a scatter plot to choose an appropriate type of model. Use your model to predict the life span of a male born in the year 2010.

Birth year	**Life expectancy**
1900	48.3
1910	51.1
1920	55.2
1930	57.4
1940	62.5
1950	65.6
1960	66.6
1970	67.1
1980	70.0
1990	71.8
2000	73.0

22. A small-appliance manufacturer finds that it costs $\$9000$ to produce 1000 toaster ovens a week and $\$12{,}000$ to produce 1500 toaster ovens a week.

    a. Express the cost as a function of the number of toaster ovens produced, assuming that it is linear. Then sketch the graph.

    b. What is the slope of the graph and what does it represent?

    c. What is the y-intercept of the graph and what does it represent?

23. If $f(x) = 2x + \ln x$, find $f^{-1}(2)$.

24. Find the inverse function of $f(x) = \dfrac{x+1}{2x+1}$.

25. Find the exact value of each expression.

    a. $e^{2 \ln 3}$

    b. $\log_{10} 25 + \log_{10} 4$

    c. $\tan\left(\arcsin \frac{1}{2}\right)$

    d. $\sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right)$

26. Solve each equation for x.

    a. $e^{x} = 5$

    b. $\ln x = 2$

c. $e^{e^x} = 2$

d. $\tan^{-1} x = 1$

27. The half-life of palladium-100, $^{100}\mathrm{Pd}$, is four days. (So half of any given quantity of $^{100}\mathrm{Pd}$ will disintegrate in four days.) The initial mass of a sample is one gram.

a. Find the mass that remains after 16 days.

b. Find the mass $m(t)$ that remains after t days.

c. Find the inverse of this function and explain its meaning.

d. When will the mass be reduced to 0.01 g?

28. The population of a certain species in a limited environment with initial population 100 and carrying capacity 1000 is

$$P(t) = \frac{100{,}000}{100 + 900e^{-t}}$$

where t is measured in years.

a.  Graph this function and estimate how long it takes for the population to reach 900.

b. Find the inverse of this function and explain its meaning.

c. Use the inverse function to find the time required for the population to reach 900. Compare with the result of part (a).