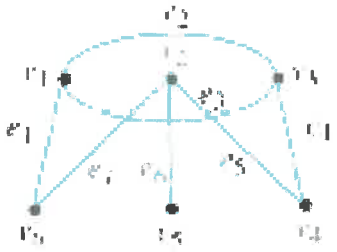
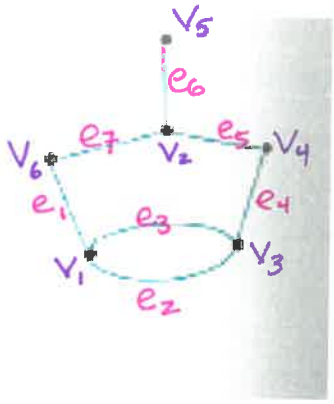
Math 231 – HW 14 **Name:** KEY

Epp 2nd Ed. 11.1 5, 6, 8, 10, 12 (optional), 17, 18, 25, 26
11.2 8, 9, 12 - 14, 16 - 20, 22 - 24

11.1 (5) Show that the two drawings represent the same graph by labelling the vertices and edges of the right hand drawing to correspond to those of the left hand drawing.

5.

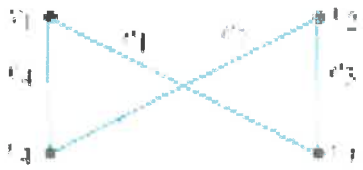


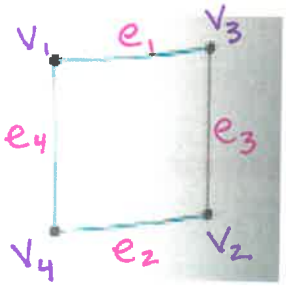


Other answers are possible – make sure your edges connect to the same vertices!

11.1 (6) Show that the two drawings represent the same graph by labelling the vertices and edges of the right hand drawing to correspond to those of the left hand drawing.

6.

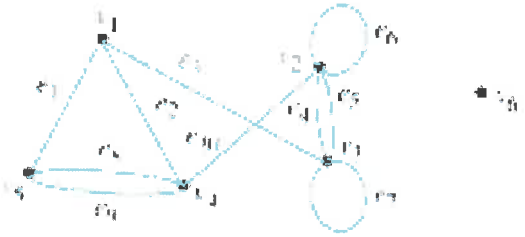




other answers are possible!

11.1 (8)

8.



i. edges incident on v_1: e_1, e_2, e_3

ii. vertices adjacent to v_3: v_1, v_2, v_3

iii. edges adjacent to e_1: e_2, e_3, e_8, e_9

iv. all loops: e_6, e_7

v. all parallel edges: $e_8 \| e_9$, $e_4 \| e_5$

vi. all isolated vertices: v_6

vii. degree of v_3: 5

viii. total degree of the graph: $2(10 \text{ edges}) = 20$

11.1 (10) Use the graph on p. 608 to answer and explain:

(a) Does Sports Illustrated contain printed writing?

Yes. Sports Illustrated is an instance of a sports magazine, which is a periodical, which contains printed writing.

(b) Does Poetry Magazine contain long words?

Yes. Poetry Magazine is an instance of a literary journal, which is a scholarly journal which contains long words.

11.1 (12) (optional) How will the wolf, goat, cabbage, and ferryman get across the river?

Hint: who's the character with the most restrictions?

11.1 (17) Does a graph with exactly 4 vertices of degrees 1, 1, 1, and 4 exist? Explain why not or show a solution.

No, this is not possible. The total degree of any graph must be even, and $1+1+1+4=7$, which is odd.

11.1 (18) Does a graph with exactly 4 vertices of degrees 1, 2, 3, and 4 exist? Explain why not, or show a solution.

Yes. Here's an example:

There's more than one solution!

11.1 (25) (a) Can 15 people each have exactly 3 friends in a group (if friendship is reciprocal)? Explain why not, or show a solution.

No. Explain why!

(b) Can 4 people have exactly 3 friends each? Explain why not, or show a solution.

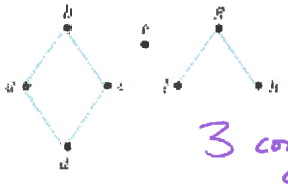
Yes. Here's how:

11.1 (26) Can 25 people each shake hands with exactly 1 other person in the group? Explain why not, or show a solution.

No. Explain why!

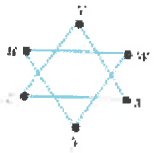
11.2 (8) How many connected components for each graph?

a.



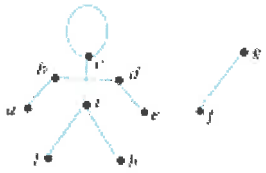
3 connected components. (notice vertex e!)

b.



2 connected components. (the two triangles are separate – not connected by a vertex.)

c.



3 connected comp's. (the arms and body aren't connected)

d.

2 connected comp's. (the two edges are not connected by a vertex)

11.2 (9) Does each graph have an Euler circuit? Explain.

(a) A connected graph with vertices of degree 2, 2, 3, 3, and 4.

No, it does not have an Euler circuit. Explain!

(b) A connected graph with vertices of degree 2, 2, 4, 4, and 6.

Yes, it has an Euler circuit. Every connected graph whose vertices are all even has an Euler circuit.

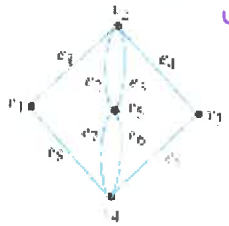
(c) A graph with vertices of degree 2, 2, 4, 4, and 6.

It does not necessarily have an Euler circuit, since it may or may not be connected!

Determine which graphs have Euler circuits. Find Euler circuits for those graphs that have them. If they do not have an Euler circuit, explain why.

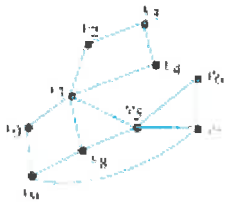
11.2 (12 - 14, 16, 17)

12.



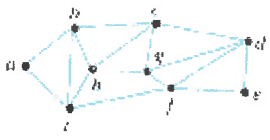
Yes. For example, $e_1, e_8, e_5, e_4, e_2, e_7, e_6, e_3$.

13



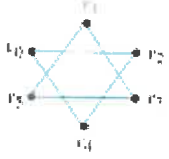
No. four vertices have odd degree: v_1, v_7, v_8, v_9

14.



Yes. For example, abcdefihg fdgchbia.

16.

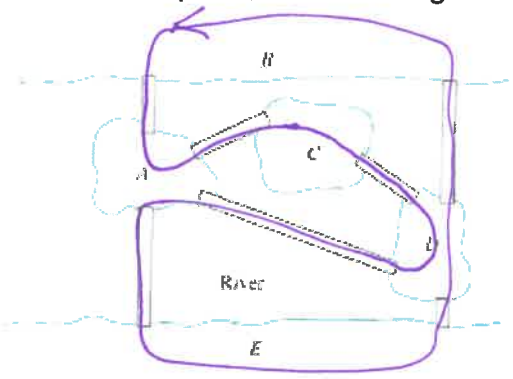


No. The graph isn't even connected.

17

No. C and D are of odd degree.

11.2 (18) Is it possible to take a walk around the city whose map is shown below, starting and ending at the same point, and crossing each bridge exactly once? If so, how can this be done?

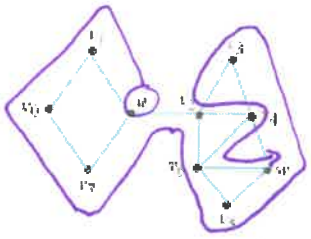


Yes. For example

There are many solutions!

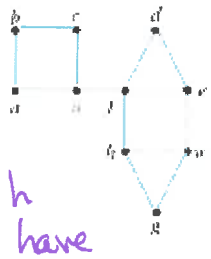
11.2 (19, 20) Determine whether there is an Euler path from u to w. If there is, find it. If there isn't, explain why.

19.



Yes. For example

20

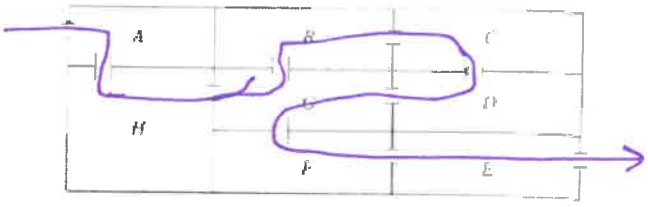


No. e and h also have odd degree.

11.2 (22 - 24)

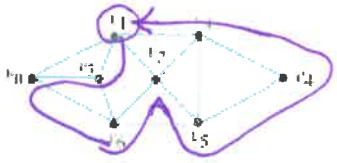
Find Hamiltonian circuits for each of the graphs in 23 and 24. every vertex, no repeated edges.

22. The following is a floor plan of a house. Is it possible to enter the house in room A, travel through every interior doorway of the house exactly once, and exit out of room E? If so, how can this be done?



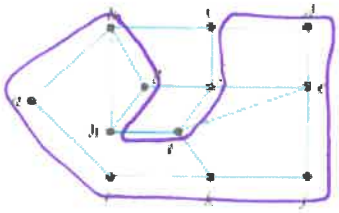
Yes. For ex

23.



For ex.

24.



For ex.