

Epp 2nd Ed. 2.1 1, 3 - 7, 9 - 12

2.2 1, 2, 5 - 8 (only part a), 11 - 14 (only part a), 31 - 34

2.1 (1) A menagerie consists of seven brown dogs, two black dogs, six gray cats, ten black cats, five blue birds, six yellow birds, and one black bird. Determine which of the following statements are true and which are false.

- (a) There is an animal in the menagerie that is red.
- (b) Every animal in the menagerie is a bird or mammal.
- (c) Every animal in the menagerie is brown or gray or black.
- (d) There is an animal in the menagerie that is neither a cat nor a dog.
- (e) No animal in the menagerie is blue.
- (f) There are a dog, a cat, and a bird in the menagerie that all have the same color.

2.1 (3) Let \mathbb{R} be the domain of the predicates " $x > 1$," " $x > 2$," " $|x| > 2$," and " $x^2 > 4$." Which of the following are true, and which are false?

- (a) $x > 2 \Rightarrow x > 1$
- (b) $x > 2 \Rightarrow x^2 > 4$
- (c) $x^2 > 4 \Rightarrow x > 2$
- (d) $x^2 > 4 \Leftrightarrow |x| > 2$

Find and explain counterexamples to show that the statements in problems 4 - 7 are false.

2.1 (4) $\forall x \in \mathbb{R}, x > \frac{1}{x}$.

2.1 (5) $\forall a \in \mathbb{Z}, \frac{a-1}{a}$ is not an integer.

2.1 (6) \forall positive integers m and n , $m \cdot n \geq m + n$.

2.1 (7) \forall real numbers x and y , $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$.

2.1 (9) Consider the following statement: $\exists x \in \mathbb{R}$ such that $x^2=2$.

Which of the following are equivalent ways of expressing the statement?

- (a) The square of each real number is 2.
- (b) Some real numbers have square 2.
- (c) The number x has square 2, for some real number x .
- (d) If x is a real number, then $x^2=2$.
- (e) Some real number has square 2.
- (f) There is at least one real number whose square is 2.

2.1 (10) Rewrite the following statements informally in at least two different ways, without using variables or the symbols \forall or \exists :

- (a) \forall squares x , x is a rectangle.
- (b) \exists a set A such that A has 16 subsets.

2.1 (11) Rewrite each of the following statements in the form " \forall ____ x , ____."

- (a) All dinosaurs are extinct.
- (b) Every real number is positive, negative, or zero.
- (c) No irrational numbers are integers.
- (d) No logicians are lazy.

2.1 (12) Rewrite each of the following statements in the form " \exists ____ x such that ____."

- (a) Some exercises have answers.
- (b) Some real numbers are rational.

2.2 (1) The following statement is true: " \forall non-zero real numbers x , \exists a real number y such that $x \cdot y = 1$." For each x given below, find a y to make the predicate " $x \cdot y = 1$ " true.

(a) $x = 2$.

(b) $x = -1$.

(c) $x = \frac{3}{4}$.

2.2 (2) The following statement is true: " \forall real numbers x , \exists an integer n such that $n > x$." For each x given below, find n to make the predicate " $n > x$ " true.

(a) $x = 15.83$

(b) $x = 10^8$

(c) $x = 10^{10^{10}}$

Rewrite in English as simply as possible. Do not use variables or \forall or \exists .

2.2 (5a) \forall odd integers n , \exists an integer k such that $n = 2k + 1$.

2.2 (6a) $\forall r \in \mathbb{Q}$, \exists integers a and b such that $r = \frac{a}{b}$.

2.2 (7a) $\forall x \in \mathbb{R}$, \exists a real number y such that $x + y = 0$.

2.2 (8a) $\exists x \in \mathbb{R}$ such that for all real numbers y , $x + y = 0$.

Rewrite formally using quantifiers (\forall and/or \exists) and variables.

2.2 (11a) Everybody trusts somebody.

2.2 (12a) Somebody trusts everybody.

2.2 (13a) Any even integer equals twice some other integer.

2.2 (14a) The number of rows in any truth table equals 2^n for some integer n .

Rewrite each in if-then form:

2.2 (31) Earning a grade of C- in this course is a sufficient condition for it to count toward graduation.

If _____,
then _____.

2.2 (32) Being divisible by 6 is a sufficient condition for being divisible by 3.

If _____,
then _____.

2.2 (33) Being on time each day is a necessary condition for keeping this job.

If _____,
then _____.

2.2 (34) A grade point average of at least 3.7 is a necessary condition for graduating with honors.

If _____,
then _____.