Math 231 – HW 2 Name: ___KEY___

Rewrite the statements in if-then form:

(2) I am on time for work if I catch the 8:05 bus.

If __I catch the 8:05 bus__ , then __I am on time for work.__

(3) Freeze or I'll shoot.

see 13a for why!

If __you do not freeze__ , then __I will shoot.__

OR: If I did not shoot you, then you froze.

Write truth tables for the statement forms:

(5) $\sim p \vee q \rightarrow \sim q$ Notice order of operations here: $((\sim p) \vee q) \rightarrow (\sim q)$

p	q	$\sim p$	$\sim p \vee q$	$\sim q$	$(\sim p \vee q) \rightarrow (\sim q)$
T	T	F	T	F	F
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

(6) $p \vee (\sim p \wedge q) \rightarrow q$ Notice order of ops: $(p \vee (\sim p \wedge q)) \rightarrow (q)$

p	q	$\sim p$	$\sim p \wedge q$	$p \vee (\sim p \wedge q)$	q	$(p \vee (\sim p \wedge q)) \rightarrow q$
T	T	F	F	T	T	T
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	F	T	F	F	F	T

(12) *Use the logical equivalence* $p \wedge q \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$ *to rewrite the following statement:*

If __x > 2__ or __x < −2__ then __$x^2 > 4$.__ Becomes: $(p \rightarrow r) \wedge (q \rightarrow r)$
__(p__ ∨ __q)__ → r

If x > 2, then $x^2 > 4$, and if x < −2, then $x^2 > 4$.

Use truth tables to verify that:

(13)(a) $p \rightarrow q \equiv \sim p \vee q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

same values!

p	q	$\sim p$	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

So,
$(p \rightarrow q) \equiv (\sim p \vee q)$

(13)(b) $\sim(p \rightarrow q) \equiv p \wedge \sim q$

p	q	$p \rightarrow q$	$\sim(p \rightarrow q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

p	q	$\sim q$	$p \wedge \sim q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

same values!

Use truth tables to establish the truth of each statement

(20) A conditional statement is not logically equivalent to its converse.

conditional

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

converse

p	q	$q \rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	T

NOT the same!

(21) A conditional statement is not logically equivalent to its inverse.

conditional

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

inverse

p	q	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

NOT the same!

(22) A conditional statement and its contrapositive are logically equivalent.

conditional

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

contrapositive

p	q	$\sim p$	$\sim q$	$\sim q \rightarrow \sim p$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

the same!

(23) The converse and the inverse of a conditional statement are logically equivalent to each other.

converse

p	q	$q \rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	T

inverse

p	q	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

the same!

Use the contrapositive to rewrite the statements in if-then form in two ways.   Assume that "only if" has its formal, logical meaning.

(25) The Cubs will win the pennant only if they win tomorrow's game.

$p \rightarrow q$ If __the Cubs win the pennant__ , then __they will have won tomorrow's game.__

$\sim q \rightarrow \sim p$ If __the Cubs don't win tomorrow's game__ , then __they will not win the pennant__

(26) Sam will be allowed on Signe's racing boat only if he is an expert sailor.

$p \to q$ If _Sam is allowed on Signe's boat_, then _he is an expert sailor._

$\sim q \to \sim p$ If _Sam is not an expert sailor_, then _he will not be allowed on Signe's boat._

(27) Taking the long view on your education, you go to the Prestige Corporation and ask what you should do to be hired when you graduate. The Personnel Director replies that you will be hired only if you major in mathematics or computer science, get a B average or better, and take accounting. You do, in fact, become a math major, get a B+ average, and take accounting. You return to Presitige Corporation, make a formal application, and are turned down. Did the Personnel Director lie to you? Explain!

No, they did not lie to you — they said that if you were hired, then all three things would be true. That's not the same as saying that if all 3 things were true that you would get hired.

$$ p \to q \neq q \to p \ ! $$

Section 1.3:

Use modus ponens (the conditional) or modus tollens (the contrapositive) to fill in the blanks in the arguments of 1 - 5 so as to produce valid inferences.

(1) If $\sqrt{2}$ is rational, then $\sqrt{2} = \dfrac{a}{b}$ for some integers a and b.

It is not true that $\sqrt{2} = \dfrac{a}{b}$ for some integers a and b.

∴ _$\sqrt{2}$ is not rational_

contrapositive (modus tollens)

(2) If this is a **while** loop, then the body of the loop may never be executed.

This is a while loop.

∴ The body of this loop may never be executed.

conditional. (modus ponens)

(3) If logic is easy, then I am a monkey's uncle.

I am not a monkey's uncle.

∴ _Logic is not easy._

contrapositive (modus tollens)

(4) If this polygon is a triangle, then the sum of its interior angles is 180°.

The sum of the interior angles of this polygon is not 180°.

∴ _The polygon is not a triangle._

contrapositive (modus tollens)

(5) If they were unsure of the adress, then they would have telephoned.

They did not telephone.

contrapositive (modus tollens)

∴ They were sure of the address.

(36) Put the clues in order to find the treasure (see book for the clues, fill in the step-by-step conclusions):

(c) __The house is next to the lake.__

(a) __If the house is next to the lake, then the treasure is not in the kitchen.__

∴ __The treasure is not in the kitchen.__

(b) __If the tree in the front yard is an elm, then the treasure is in the kitchen.__

∴ __The tree in the front yard is not an elm__   (contrapositive!)

(d) __The tree in the front yard is an elm, or the treasure is buried under the flagpole.__

∴ __The treasure is buried under the flagpole!__

Hint: You don't need all five clues!

(38) Put the clues in order to find the murderer (see book for the clues, fill in the step-by-step conclusions):

(a) __Lord H. was killed by a blow to the head with a brass candlestick.__

(b) __If the cook was in the kitchen, then the butler poisoned Lord H.__

∴ __The cook was not in the kitchen__   (contrapositive)

(e) __If the cook was not in the kitchen, then Sara was not in the dining room.__

∴ __Sara was not in the dining room.__

(b) __Either Lady H or Sara was in the dining room.__

∴ __Lady H was in the dining room.__

(d) __If Lady H was in the dining room, then the chauffer killed Lord H.__

∴ The chauffer killed Lord H.

and you don't need the last clue!