$+\frac{10}{10}$

**Math 252 Homework 12 Written Part**   Name: ___KEY___

Write legibly.   Show your work.   Graph neatly.   Use a ruler for all straight lines.

Practice with volumes:
1. Graph the relevant area (before rotation).   Draw and label the axis of rotation.
2. Draw the representative rectangle.
3. Label or list the measurements you will need to find the volume.
4. Find the volume of the representative disk, washer, or cylinder.
5. Set up the integral to find the volume of the whole shape.
6. Solve the integral, expressing your answer in <u>exact</u> (not decimal) form.

(1)   Start with the area bounded by $y=\sqrt{x}$, $y=0$, and $x=4$, then rotate around the line $y=-2$. Use cylindrical shells.

graph, with rect.

thick = dy

$x=y^2$
$y=\sqrt{x}$

height = $4-y^2$

Radius = Top − bot = $y-(-2)$
$=y+2$
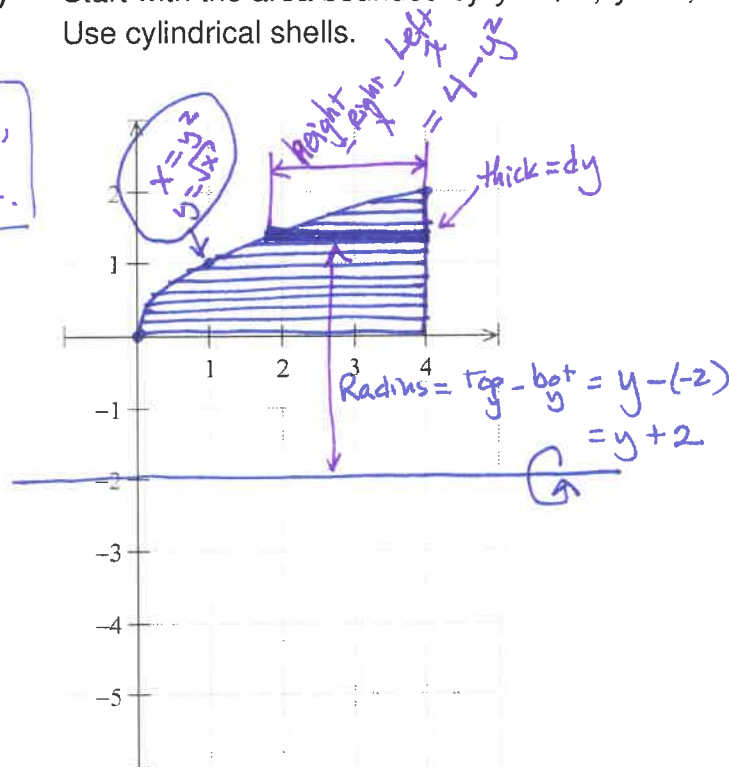
measurements:
thick = dy
radius = $y+2$
height = $4-y^2$

Vol one shell:
$V = 2\pi \cdot$ radius $\cdot$ height $\cdot$ Thick
$= 2\pi(y+2)(4-y^2)\,dy$

$V_{shell} = 2\pi(-y^3-2y^2+4y+8)\,dy$

Total Volume:
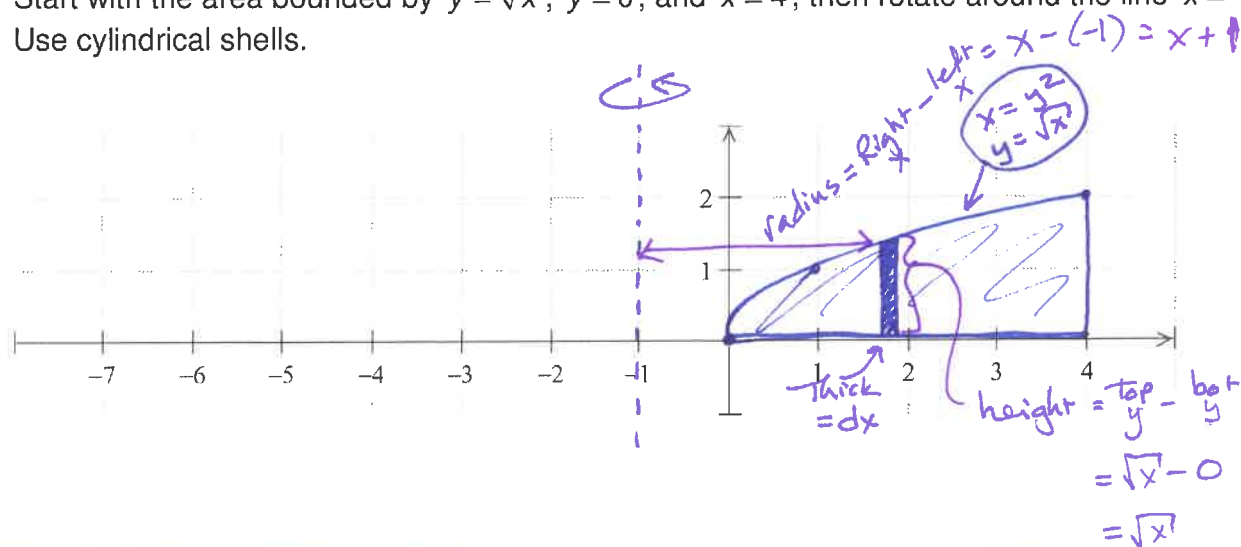$V_{total} = 2\pi\int_0^2 -y^3-2y^2+4y+8\,dy$

$= 2\pi\left[-\frac{1}{4}y^4-\frac{2}{3}y^3+2y^2+8y\right]_0^2$

$= 2\pi\left[-4-\frac{16}{3}+8+16\right]-2\pi[0]$

$= \frac{88\pi}{3}$

(2) Start with the area bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$, then rotate around the line $x = -1$. Use cylindrical shells.

CS

$r = x - (-1) = x + 1$

radius = Right − left = x

$x = y^2$
$y = \sqrt{x}$

thick = dx

height = top − bot = y

$= \sqrt{x} - 0$

$= \sqrt{x}$

graph, with rect

measurements:

$$\text{thick} = dx$$
$$\text{height} = \sqrt{x}$$
$$\text{radius} = x + 1$$

$\text{Vol } \underset{\text{shell}}{\text{one}} = 2\pi \cdot \text{radius} \cdot \text{height} \cdot \text{thick}$

$$= 2\pi(x+1)(\sqrt{x})\,dx$$

$$V_{shell} = 2\pi\left(x^{\frac{3}{2}} + x^{\frac{1}{2}}\right)dx$$

Total Volume:

$$V_{total} = 2\pi \int_0^4 x^{\frac{3}{2}} + x^{\frac{1}{2}}\,dx$$

$$= 2\pi\left[\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}}\right]_0^4$$

$$= 2\pi\left[\frac{2}{5}(\sqrt{4})^5 + \frac{2}{3}(\sqrt{4})^3\right] - 2\pi[0]$$

$$= 2\pi\left[\frac{64}{5} + \frac{16}{3}\right]$$

$$= \boxed{\frac{544\,\pi}{15}}$$