Chapter 1: Functions and Models Logarithmic Functions

Book Title: Calculus: Early Transcendentals Printed By: Troy Jeffery (tradozprime@gmail.com) © 2018 Cengage Learning, Cengage Learning

Logarithmic Functions

If b>0 and $b\neq 1$, the exponential function $f(x)=b^x$ is either increasing or decreasing and so it is one-to-one by the Horizontal Line Test. It therefore has an inverse function f^{-1} , which is called the **logarithmic function with base** b and is denoted by \log_b . If we use the formulation of an inverse function given by (3),

$$f^{-1}(x) = y \Leftrightarrow f(y) = x$$

then we have

$$\log_b x = y \quad \Leftrightarrow \quad b^y = x$$

Thus, if x > 0, then $\log_b x$ is the exponent to which the base b must be raised to give x. For example, $\log_{10} 0.001 = -3$ because $10^{-3} = 0.001$.

The cancellation equations (4), when applied to the functions $f(x) = b^x$ and $f^{-1}(x) = \log_b x$, become

$$\log_b(b^x) \ = \ x \qquad ext{for every } x \in \mathbb{R}$$
 $b^{\log_b x} \ = \ x \qquad ext{for every } x > 0$

The logarithmic function \log_b has domain $(0, \infty)$ and range \mathbb{R} . Its graph is the reflection of the graph of $y = b^x$ about the line y = x.

Figure 11 shows the case where b>1. (The most important logarithmic functions have base b>1.) The fact that $y=b^x$ is a very rapidly increasing function for x>0 is reflected in the fact that $y=\log_b x$ is a very slowly increasing function for x>1.

Figure 11

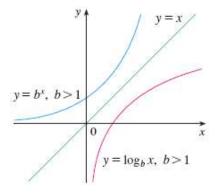
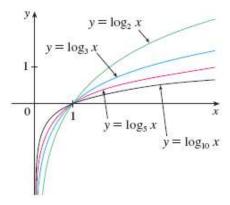


Figure 12 shows the graphs of $y = \log_b x$ with various values of the base b > 1. Since $\log_b 1 = 0$, the graphs of all logarithmic functions pass through the point (1, 0).

Figure 12



The following properties of logarithmic functions follow from the corresponding properties of exponential functions given in Section 1.4.

Laws of Logarithms

If x and y are positive numbers, then

1.
$$\log_b(xy) = \log_b x + \log_b y$$

$$2. \log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$$

3.
$$\log_b(x^r) = r \log_b x$$
 (where r is any real number)

Example 6

Use the laws of logarithms to evaluate $\log_2 80 - \log_2 5$.

Solution Using Law 2, we have

$$\log_2 80 - \log_2 5 = \log_2 \left(\frac{80}{5}\right) = \log_2 16 = 4$$

because $2^4 = 16$.

Chapter 1: Functions and Models Logarithmic Functions

Book Title: Calculus: Early Transcendentals Printed By: Troy Jeffery (tradozprime@gmail.com) © 2018 Cengage Learning, Cengage Learning

© 2019 Cengage Learning Inc. All rights reserved. No part of this work may by reproduced or used in any form or by any means - graphic, electronic, or mechanical, or in any other manner - without the written permission of the copyright holder.