

Kinds of Proofs (part 2)PROOF BY DIVISION INTO CASESTheorem  $\forall x, P(x) \text{ or } Q(x) \Rightarrow R(x)$ Proof "Either  $P(x)$  or  $Q(x)$  is true."Case 1

- Assume  $P(x)$  is true.
- Clearly and carefully state  $P(x)$ .
  - Step-by-step, show that...
  - $R(x)$  is true.

(might have more than 2 cases!)

Case 2

- Assume  $Q(x)$  is true.
- Clearly and carefully state  $Q(x)$ .
  - Step-by-step, show that...
  - $R(x)$  is true.

□

PROOF BY CONTRAPOSITIONTheorem  $\forall x, P(x) \Rightarrow Q(x)$ .Proof "It is sufficient to show that  $\sim Q(x) \Rightarrow \sim P(x)$ "

- Assume  $Q(x)$  is false.
- Clearly and carefully state that  $Q(x)$  is false.
- Step-by-step, show that...
- $P(x)$  is false.
- Therefore  $P(x) \Rightarrow Q(x)$  □.

PROOF BY CONTRADICTIONTheorem  $\forall x, P(x)$  is true.Proof "Suppose not."

- Clearly and carefully state that  $P(x)$  is false.
- Step-by-step, show that...
- this leads to a contradiction.
- Therefore,  $P(x)$  is true. □.

Cool symbol: $\Rightarrow \Leftarrow$ 

means

contradiction!

## Chapter 5 Summary: More About Sets

Review:  $\{x \in \mathbb{Z} \mid 1 \leq x < 5\} = \{1, 2, 3, 4\}$   
the set of all  $x$  element of the integers such that

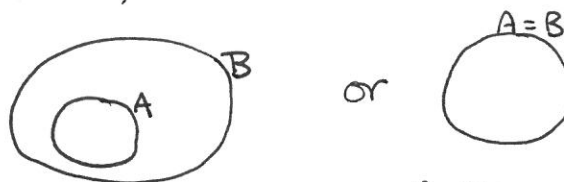
Subsets  $A \subseteq B$  means "A is a subset of B."

A is completely contained in B — every element of A is an element of B.

$\forall x$ , if  $x \in A$ , then  $x \in B$ .

Equality  
 $A = B$  means  
A and B have  
exactly the  
same elements.

Diagram:



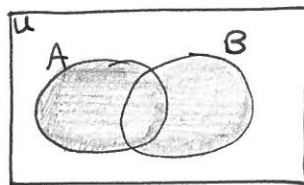
Proper Subset A is a subset of B, and  
there is some element of B that is  
not in A. ( $A \subset B$ .)

The Empty Set (The Null Set)

$\emptyset = \{ \} =$  the set with nothing in it!

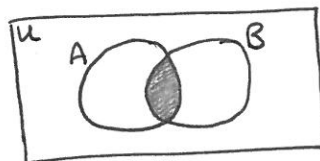
Union  $A \cup B =$  the set of all elements in A and/or  
in B.

Diagram:

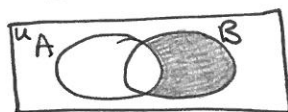


Intersection  $A \cap B =$  the set of all elements in A  
and in B.

Diagram:



Difference  $B - A =$  the set of all elements in B  
but not in A.



Complement  $A^c$  (or  $\bar{A}$ )

The set of all elements of the  
universe that are not in A.

