THEOREM 1.1.1 Logical Equivalences

Given any statement variables p, q, and r, a tautology t and a contradiction c, the following logical equivalences hold:

1. Commutative laws:	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. Associative laws:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive laws:	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity laws:	$p \wedge t \equiv p$	$p \vee c \equiv p$
5. Negation laws:	$p \vee {\sim}p \equiv t$	$p \wedge {\sim}p \equiv c$
6. Double negative law:	${\sim}({\sim}p) \equiv p$	
7. Idempotent laws:	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. De Morgan's laws:	${\sim}(p \wedge q) \equiv {\sim}p \vee {\sim}q$	${\sim}(p \vee q) \equiv {\sim}p \wedge {\sim}q$
9. Universal bound laws:	$p \vee t \equiv t$	$p \wedge c \equiv c$
10. Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. Negations of t and c:	${\sim}t \equiv c$	${\sim}c \equiv t$

EXAMPLE 1.1.14 **Simplifying Statement Forms**

Use Theorem 1.1.1 to verify the logical equivalence

$$ {\sim}({\sim}p \wedge q) \wedge (p \vee q) \equiv p. $$

Solution Use the laws of Theorem 1.1.1 to replace sections of the statement form on the left by logically equivalent expressions. Each time you do this, you obtain a logically equivalent statement form. Continue making replacements until you obtain the statement form on the right.

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\begin{aligned}
{\sim}({\sim}p \wedge q) \wedge (p \vee q) &\equiv ({\sim}({\sim}p) \vee {\sim}q) \wedge (p \vee q) && \text{by De Morgan's laws} \\
&\equiv (p \vee {\sim}q) \wedge (p \vee q) && \text{by the double negative law} \\
&\equiv p \vee ({\sim}q \wedge q) && \text{by the distributive law} \\
&\equiv p \vee (q \wedge {\sim}q) && \text{by the commutative law for } \wedge \\
&\equiv p \vee c && \text{by the negation law} \\
&\equiv p && \text{by the identity law}
\end{aligned}
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Skill in simplifying statement forms is useful in constructing logically efficient computer programs and in designing digital logic circuits.

EXERCISE SET 1.1

Appendix B contains either full or partial solutions to all exercises with blue numbers. When the solution is not complete, the exercise number has an *H* next to it. A ◆ next to an exercise number signals that the exercise is more challenging than usual. Be careful not to get into the habit of turning too quickly to the solutions. Make every effort to work exercises on your own before checking your answers.