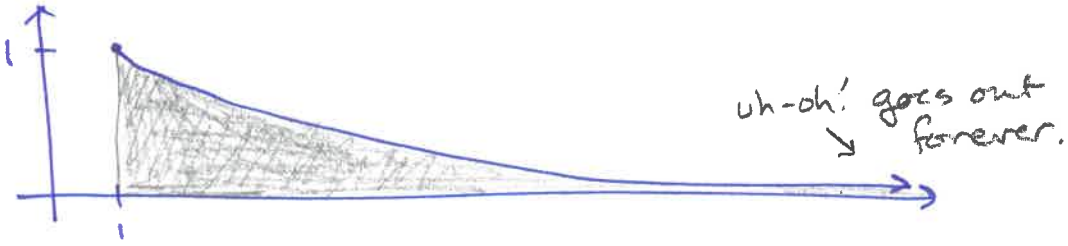
Math 252 Homework 16 Written Part **Name:** KEY

Write legibly. Show your work. Graph neatly. Use a ruler for all straight lines.

(1) Consider the integral: $\int_1^\infty \frac{1}{x^{3/2}}\,dx$.

a. Sketch a graph of the area measured by the integral. Include scale numbers.



b. Which endpoint makes the integral improper? Replace it with t, and rewrite the integral using a limit as t approaches the problem endpoint.

$$= \lim_{t\to\infty} \int_1^t \frac{1}{x^{3/2}}\,dx$$

c. Calculate the integral, showing your work clearly, and always writing complete steps.

$$= \lim_{t\to\infty} \int_1^t x^{-3/2}\,dx$$

$$= \lim_{t\to\infty} \left[-2x^{-1/2}\right]_1^t$$

$$= \lim_{t\to\infty} \left[-\frac{2}{\sqrt{x}}\right]_1^t$$

$$= \lim_{t\to\infty} \left[\frac{-2}{\sqrt{t}} + \frac{2}{\sqrt{1}}\right]$$

$\frac{-2}{\sqrt{t}} \to 0$

$$\boxed{= 2}$$

(2) Consider the integral: $\int_0^4 \frac{1}{\sqrt{x}}\,dx$.

a. Sketch a graph of the area measured by the integral. Include scale numbers.

b. Which endpoint makes the integral improper? Replace it with t, and rewrite the integral using a limit as t approaches the problem endpoint.

$$= \lim_{t\to 0} \int_t^4 \frac{1}{\sqrt{x}}\,dx$$

c. Calculate the integral, showing your work clearly, and always writing complete steps.

$$= \lim_{t\to 0} \int_t^4 x^{-1/2}\,dx$$

$$= \lim_{t\to 0} \left[2x^{1/2} \right]_t^4$$

$$= \lim_{t\to 0} \left[2\sqrt{x} \right]_t^4$$

$$= \lim_{t\to 0} \left[2\sqrt{4} - 2\sqrt{t} \right]$$

$$\boxed{= 4}$$