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Math 231 – HW 1 **Name:** KEY

Epp 2nd Ed. 1.1 5abcd, 7, 8, 13, 15, 18, 20, 39, 40, 42

(5) *Indicate which of the following sentences are statements. If it's not a statement, say why.*

(a) 1024 is the smallest 4-digit number that is a perfect square. YES

(b) She is a math major. No. who's she?

(c) $128 = 2^6$ Yes.

(d) $x = 2^6$ No. What's x?

For problems 7 and 8, write the statement in symbolic form using the symbols $\sim$, $\wedge$, and $\vee$ and the indicated letter to represent component statements:

(7) Juan is a math major but not a computer science major.
(m = "Juan is a math major", c = "Juan is a computer science major")

$m \wedge \sim c$

(8) h = "John is healthy", w = "John is wealthy", s = "John is wise"

(a) John is healthy and wealthy but not wise.

$h \wedge w \wedge \sim s$ OR $(h \wedge w) \wedge \sim s$

(b) John is not wealthy but he is healthy and wise.

$\sim w \wedge h \wedge s$ OR $\sim w \wedge (h \wedge s)$

(c) John is neither healthy, wealthy, nor wise.

$\sim h \wedge \sim w \wedge \sim s$ OR $\sim (h \vee w \vee s)$

Write truth tables for the statement forms in problems 13 and 15:

(13) $(p \wedge q) \vee \sim(p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$\sim(p \vee q)$	$(p \wedge q) \vee \sim(p \vee q)$
T	T	T	T	F	T
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	T

(15) $\sim p \wedge (q \vee \sim r)$

p	q	r	$\sim p$	$\sim r$	$q \vee \sim r$	$\sim p \wedge (q \vee \sim r)$
T	T	T	F	F	T	F
T	T	F	F	T	T	F
T	F	T	F	F	F	F
T	F	F	F	T	T	F
F	T	T	T	F	T	T
F	T	F	T	T	T	T
F	F	T	T	F	F	F
F	F	F	T	T	T	T

Determine which of the pairs of statement forms in 18 and 20 are logically equivalent. Justify your answers using truth tables. Read t to be a tautology and c to be a contradiction.

(18) $\sim(p \vee q)$ *and* $\sim p \wedge \sim q$

p	q	$p \vee q$	$\sim(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

They're the same!

(20) $p \vee c$ and p

p	c	$p \vee c$
T	F	T
F	F	F

They're the same!

Use truth tables to establish which of the statements forms in problems 39 and 40 are tautologies and which are contradictions:

(39) $((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$

oops!

$\sim q$	p	q	r	$\sim p$	$\sim p \wedge q$	$q \wedge r$	$(\sim p \wedge q) \wedge (q \wedge r)$	$((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$
F	T	T	T	F	F	T	F	F
F	T	T	F	F	F	F	F	F
T	T	F	T	F	F	F	F	F
T	T	F	F	F	F	F	F	F
F	F	T	T	T	T	T	T	F
F	F	T	F	T	T	F	F	F
T	F	F	T	T	F	F	F	F
T	F	F	F	T	F	F	F	F

Contradiction

(40) $(\sim p \vee q) \vee (p \wedge \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$p \wedge \sim q$	$(\sim p \vee q) \vee (p \wedge \sim q)$
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

tautology

In problem 42, a logical equivalence is derived from Theorem 1.1.1. Supply a reason for each step:

(42)

$(p \vee \sim q) \wedge (\sim p \vee \sim q)$

$\equiv (\sim q \vee p) \wedge (\sim q \vee \sim p)$ by (1) commutativity

$\equiv \sim q \vee (p \wedge \sim p)$ by (3) distributitive laws

$\equiv \sim q \vee c$ by (5) negation laws

$\equiv \sim q$ by (4) identity laws

Therefore, $(p \vee \sim q) \wedge (\sim p \vee \sim q) \equiv \sim q$.