1.4 Exercises

1, 2, 3 and 4 Use the Law of Exponents to rewrite and simplify the expression.

1.

a. $\dfrac{4^{-3}}{2^{-8}}$

b. $\dfrac{1}{\sqrt[3]{x^4}}$

2.

a. $8^{4/3}$

b. $x(3x^2)^3$

3.

a. $b^8(2b)^4$

b. $\dfrac{(6y^3)^4}{2y^5}$

4.

a. $\dfrac{x^{2n} \cdot x^{3n-1}}{x^{n+2}}$

b. $\dfrac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}}$

5.

a. Write an equation that defines the exponential function with base $b > 0$.

b. What is the domain of this function?

c. If $b \neq 1$, what is the range of this function?

d. Sketch the general shape of the graph of the exponential function for each of the following cases.

i. $b > 1$

ii. $b = 1$

iii. $0 < b < 1$

6.

a. How is the number e defined?

b. What is an approximate value for e?

c. What is the natural exponential function?

 7, 8, 9 and 10 Graph the given functions on a common screen. How are these graphs related?

7. $y = 2^x$, $y = e^x$, $y = 5^x$, $y = 20^x$

8. $y = e^x$, $y = e^{-x}$, $y = 8^x$, $y = 8^{-x}$

9. $y = 3^x$, $y = 10^x$, $y = \left(\frac{1}{3}\right)^x$, $y = \left(\frac{1}{10}\right)^x$

10. $y = 0.9^x$, $y = 0.6^x$, $y = 0.3^x$, $y = 0.1^x$

11, 12, 13, 14, 15 and 16 Make a rough sketch of the graph of the function. Do not use a calculator. Just use the graphs given in Figures 3 and 13 and, if necessary, the transformations of Section 1.3.

11. $y = 4^x - 1$

12. $y = (0.5)^{x-1}$

13. $y = -2^{-x}$

14. $y = e^{|x|}$

15. $y = 1 - \frac{1}{2}e^{-x}$

16. $y = 2\left(1 - e^x\right)$

17. Starting with the graph of $y = e^x$, write the equation of the graph that results from

a. shifting 2 units downward.

b. shifting 2 units to the right.

c. reflecting about the x-axis.

d. reflecting about the y-axis.

e. reflecting about the x-axis and then about the y-axis.

18. Starting with the graph of $y = e^x$, find the equation of the graph that results from

a. reflecting about the line $y = 4$.

b. reflecting about the line $x = 2$.

19–20 Find the domain of each function.

19.

a. $f(x) = \dfrac{1 - e^{x^2}}{1 - e^{1-x^2}}$

b. $f(x) = \dfrac{1 + x}{e^{\cos x}}$

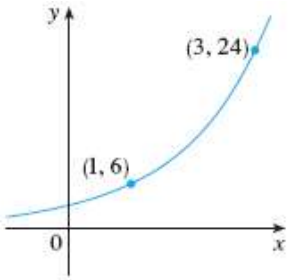
20.

a. $g(t) = \sqrt{10^t - 100}$

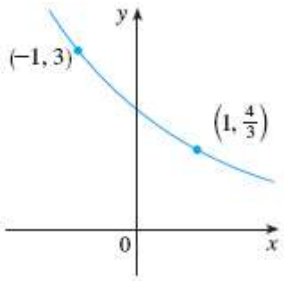
b. $g(t) = \sin(e^t - 1)$

21–22 Find the exponential function $f(x) = Cb^x$ whose graph is given.

21.



22.



23. If $f(x) = 5^x$, show that

$$\frac{f(x+h) - f(x)}{h} = 5^x \left(\frac{5^h - 1}{h} \right)$$

24. Suppose you are offered a job that lasts one month. Which of the following methods of payment do you prefer?

I. One million dollars at the end of the month.

II. One cent on the first day of the month, two cents on the second day, four cents on the third day, and, in general, 2^{n-1} cents on the nth day.

25. Suppose the graphs of $f(x) = x^2$ and $g(x) = 2^x$ are drawn on a coordinate grid where the unit of measurement is 1 inch. Show that, at a distance 2 ft to the right of the origin, the height of the graph of f is 48 ft but the height of the graph of g is about 265 mi.

26. Compare the functions $f(x) = x^5$ and $g(x) = 5^x$ by graphing both functions in several viewing rectangles. Find all points of intersection of the graphs correct to one decimal place. Which function grows more rapidly when x is large?

27. Compare the functions $f(x) = x^{10}$ and $g(x) = e^x$ by graphing both f and g in several viewing rectangles. When does the graph of g finally surpass the graph of f?

28. Use a graph to estimate the values of x such that $e^x > 1{,}000{,}000{,}000$.

29. A researcher is trying to determine the doubling time for a population of the bacterium *Giardia lamblia*. He starts a culture in a nutrient solution and estimates the bacteria count every four hours. His data are shown in the table.

Time (hours)	0	4	8	12	16	20	24
Bacteria count (CFU/mL)	37	47	63	78	105	130	173

a. Make a scatter plot of the data.

b. Use a graphing calculator to find an exponential curve $f(t) = a \cdot b^t$ that models the bacteria population t hours later.

c. Graph the model from part (b) together with the scatter plot in part (a). Use the TRACE feature to determine how long it takes for the bacteria count to double.

G. lamblia

30. A bacteria culture starts with 500 bacteria and doubles in size every half hour.

    a. How many bacteria are there after 3 hours?

    b. How many bacteria are there after t hours?

    c. How many bacteria are there after 40 minutes?

    d. Graph the population function and estimate the time for the population to reach $100{,}000$.

31. The half-life of bismuth-210, $^{210}\mathrm{Bi}$, is 5 days.

    a. If a sample has a mass of 200 mg, find the amount remaining after 15 days.

    b. Find the amount remaining after t days.

    c. Estimate the amount remaining after 3 weeks.

    d. Use a graph to estimate the time required for the mass to be reduced to 1 mg.

32. An isotope of sodium, $^{24}\mathrm{Na}$, has a half-life of 15 hours. A sample of this isotope has mass 2 g.

    a. Find the amount remaining after 60 hours.

    b. Find the amount remaining after t hours.

    c. Estimate the amount remaining after 4 days.

    d. Use a graph to estimate the time required for the mass to be reduced to 0.01 g.

33. Use the graph of V in Figure 11 to estimate the half-life of the viral load of patient 303 during the first month of treatment.

34. After alcohol is fully absorbed into the body, it is metabolized with a half-life of about 1.5 hours. Suppose you have had three alcoholic drinks and an hour later, at midnight, your blood alcohol concentration (BAC) is $0.6\ \mathrm{mg/mL}$.

    a. Find an exponential decay model for your BAC t hours after midnight.

    b. Graph your BAC and use the graph to determine when your BAC is $0.08\ \mathrm{mg/mL}$.

    Source: Adapted from P. Wilkinson et al., “Pharmacokinetics of Ethanol after Oral Administration in the Fasting State,” *Journal of Pharmacokinetics and Biopharmaceutics* 5 (1977): 207–24.

35. Use a graphing calculator with exponential regression capability to model the population of the world with the data from 1950 to 2010 in Table 1. Use the model to estimate the population in 1993 and to predict the population in the year 2020.

36. The table gives the population of the United States, in millions, for the years 1900–2010. Use a graphing calculator with exponential regression capability to model the US population since 1900. Use the model to estimate the population in 1925 and to predict the population in the year 2020.

Year	Population
1900	76
1910	92
1920	106
1930	123
1940	131
1950	150
1960	179
1970	203
1980	227
1990	250
2000	281
2010	310

37. If you graph the function

$$f(x) = \frac{1 - e^{1/x}}{1 + e^{1/x}}$$

you'll see that f appears to be an odd function. Prove it.

38. Graph several members of the family of functions

$$f(x) = \frac{1}{1 + ae^{bx}}$$

where $a > 0$. How does the graph change when b changes? How does it change when a changes?