Chapter 3 Summary (part 1)

Correct Format for Proofs

Theorem Clearly state the thing you're planning to prove

Proof Step-by-step,
in order, line-by-line,
Carefully explain your reasoning.
end with your conclusion.
$\square$.

Kinds of Proofs (part 1)

PROOF OF EXISTENCE

Theorem $\exists\, x$ s.t. x is blah.

Proof You only need to find one example that works! $\square$.

PROOF BY EXHAUSTION

Theorem blah is true for this list of things.

Proof Go through the list of things one by one, and show that blah is true for each. $\square$.

DIRECT PROOF

Theorem $\forall\, x,\ P(x) \Rightarrow Q(x)$.

Proof
- clearly and carefully state $P(x)$ is true.
- step-by-step, use algebra, arithmetic or other reasoning to get to:
- $Q(x)$ is true. $\square$.

Notation conventions

use n, m, k, etc... for integers

use x, y, z, etc... for real numbers

$\emptyset$ does not mean zero!

Things you get to assume in Chapter 3

1. The sum, difference, and product of integers is always an integer. (But not true for quotients!)
2. The product of non-zero numbers is non-zero.
3. n is even $\iff \exists k \in \mathbb{Z}$ s.t. $n = 2k$
4. n is odd $\iff \exists k \in \mathbb{Z}$ s.t. $n = 2k+1$
5. n is prime $\iff \forall r, s \in \mathbb{Z}^+$, if $n = r \cdot s$, then $r = 1$ or $s = 1$ (also, $n > 1$)
6. What are the first few primes?
   2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, ...
7. r is rational $\iff \exists a, b \in \mathbb{Z}$ s.t. $r = \frac{a}{b}$ and $b \neq 0$.

Divisibility

n is divisible by d iff $n = d \cdot k$ for some integer k.

Notation $d \mid n$ "d divides evenly into n"
"d divides n"
"d is a factor of n"

$$\boxed{d \mid n \iff \exists k \in \mathbb{Z} \text{ s.t. } n = k \cdot d}$$

Disproof

Theorem $\forall$ blah, blah is true

Disproof Find a counter example. Only need to find one example where the theorem is not true.