Math 231 – HW 5 **Name:** KEY

Epp 2nd Ed. 3.1 1ab, 2ab, 3, 6, 8, 9, 13, 14, 15, 16, 25, 27
Remember -- FORMAT is as important as CONTENT – get them both right!

3.1 (1) Assume that m and n are particular integers. Justify your answers to each of the following questions:

(a) Is 6m+8n even? Yes! $6m+8n = 2(3m+4n)$

$= 2(\text{integer})$ because products and sums of integers are integers.

$= \text{even}.$

(b) Is 10mn + 7 odd? Yes! $10mn+7 = 10mn+6+1$

$= 2(5mn+3)+1$

$= 2(\text{integer})+1$ Because products and sums of integers are integers.

$= \text{odd}.$

3.1 (2) Assume that r and s are particular integers. Justify your answers to each of the following questions:

(a) Is 4rs even? Yes! $4rs = 2(2rs)$

$= 2(\text{integer})$ Because products and sums of integers are integers.

$= \text{even}$

(b) Is $6r+4s^2+3$ odd? Yes! $6r+4s^2+3 = 6r+4s^2+2+1$

$= 2(3r+2s^2+1)+1$

$= 2(\text{integer})+1$ Because products and sums of integers are integers.

$= \text{odd}.$

Prove the statements in problems 3 and 6:

3.1 (3) There is an integer n>5 such that 2^n-1 is prime.

check? $n=6$? $2^6-1=63$ … not prime.

$\boxed{n=7?\quad 2^7-1=127, \text{ which is prime!}}$

3.1 (6) There is a real number x so that $2^x > x^{10}$.

Yes! For example: let $x=0$: $2^0 > 0^{10}$, $1 > 0$ ✓ OR let $x=1$: $2^1 > 1^{10}$, $2 > 1$ ✓ OR let $x=59$: $2^{59} > 59^{10}$, $5.76\ldots \times 10^{17} > 5.11\ldots \times 10^{17}$

Prove the statements in 8 and 9 by the method of exhaustion:

3.1 (8) Every positive even! integer less than 26 can be expressed as a sum of three or fewer perfect squares. (For instance, $10 = 1^2 + 3^2$, and $16 = 4^2$.)

Theorem: $\forall n \in \mathbb{Z}^{\text{even}}$ s.t. $0 < n < 26$, n can be expressed as a sum of 3 or fewer perfect squares.

Proof: (by exhaustion)

$2 = 1^2 + 1^2$

$4 = 2^2$

$6 = 2^2 + 1^2 + 1^2$

$8 = 2^2 + 2^2$

$10 = 3^2 + 1^2$

$12 = 2^2 + 2^2 + 2^2$

$14 = 3^2 + 2^2 + 1$

$16 = 4^2$

$18 = 4^2 + 1^2 + 1^2$ (or $= 3^2 + 3^2$)

$20 = 4^2 + 2^2$

$22 = 3^2 + 3^2 + 2^2$

$24 = 4^2 + 2^2 + 2^2$

□.

3.1 (9) For each integer n such that $1 \le n \le 10$, $n^2 - n + 11$ is a prime number.

Theorem: $\forall n \in \mathbb{Z}$ s.t. $1 \le n \le 10$, $n^2 - n + 11$ is prime.

Proof: (by exhaustion)

n	n^2-n+11
1	11
2	13
3	17
4	23
5	31
6	41
7	53
8	67

n	n^2-n+11
9	83
10	101

all of which are prime!

□.

Prove the statements in problems 13 and 14. Follow the directions for writing proofs of universal statements given in this section.

3.1 (13) If n is any even integer, then $(-1)^n = 1$.

Theorem: $\forall n \in \mathbb{Z}^{even}, (-1)^n = 1.$

Proof: Since n is even, $n = 2k$ for some integer k.

then, $(-1)^n = (-1)^{2k}$

$= ((-1)^2)^k$

$= (1)^k$

$= 1$ $\square$.

3.1 (14) If n is any odd integer, then $(-1)^n = -1$.

Theorem: $\forall n \in \mathbb{Z}^{odd}, (-1)^n = -1$

Proof: Since n is odd, $n = 2k+1$ for some integer k.

then, $(-1)^n = (-1)^{2k+1}$

$= (-1)^{2k} \cdot (-1)^1$

$= ((-1)^2)^k \cdot (-1)$

$= (1)^k \cdot (-1)$

$= -1$. $\square$.

Disprove the statements in problems 15 and 16 by giving a counterexample. Answer with a complete sentence!

3.1 (15) For all positive integers n, if n is prime, then n is odd.

False! 2 is a positive, prime integer, but not odd.

3.1 (16) For all real numbers a and b, if a < b, then $a^2 < b^2$.

False! For example, $-2 < 1$,

but $(-2)^2 < (1)^2$

or $4 < 1$ is <u>not</u> true.

Prove the statements that are true, and give counterexamples to disprove the statements that are false:

3.1 (25) The product of any two odd integers is odd.

Theorem $\forall\, m, n \in \mathbb{Z}^{odd}$, $m \cdot n$ is odd.

Proof Since m is odd, $m = 2k+1$ for some integer k.
Since n is odd, $n = 2k'+1$ for some integer k'.
Then
$$m \cdot n = (2k+1)(2k'+1)$$
$$= 4kk' + 2k' + 2k + 1$$
$$= 2(2kk' + k' + k) + 1$$
$$= 2(\text{integer}) + 1$$
Because products and sums of integers are integers.
$$= \text{odd}.$$
$\square$.

3.1 (27) The difference of any two odd integers is odd.

False! For instance $5 - 1 = 4$, which is even.

(In fact, the statement that would be true is that the difference of any two odd integers is always even. Could you prove it?)