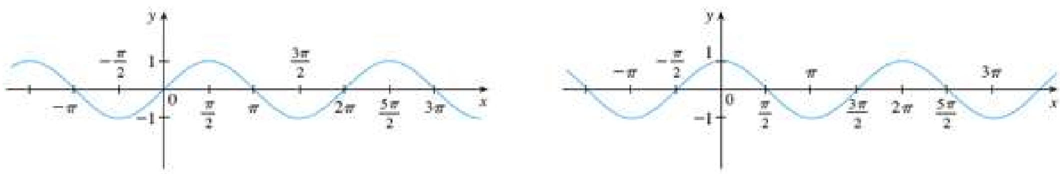
# Trigonometric Functions

Trigonometry and the trigonometric functions are reviewed on Reference Page 2 and also in Appendix D. In calculus the convention is that radian measure is always used (except when otherwise indicated). For example, when we use the function $f(x) = \sin x$, it is understood that $\sin x$ means the sine of the angle whose radian measure is $x$. Thus the graphs of the sine and cosine functions are as shown in Figure 18.

**Figure 18**




(a) $f(x) = \sin x$ (b) $g(x) = \cos x$

> Note
>
> The Reference Pages are located in Formula Cards.

Notice that for both the sine and cosine functions the domain is $(-\infty, \infty)$ and the range is the closed interval $[-1, 1]$. Thus, for all values of $x$, we have

$$-1 \leqslant \sin x \leqslant 1 \qquad -1 \leqslant \cos x \leqslant 1$$

or, in terms of absolute values,

$$|\sin x| \leqslant 1 \qquad |\cos x| \leqslant 1$$

Also, the zeros of the sine function occur at the integer multiples of $\pi$; that is,

$$\sin x = 0 \qquad \text{when} \qquad x = n\pi \quad n \text{ an integer}$$

An important property of the sine and cosine functions is that they are periodic functions and have period $2\pi$. This means that, for all values of $x$,

$$\sin(x + 2\pi) = \sin x \qquad \cos(x + 2\pi) = \cos x$$

The periodic nature of these functions makes them suitable for modeling repetitive phenomena such as tides, vibrating springs, and sound waves. For instance, in Example 1.3.4 we will see that a reasonable model for the number of hours of daylight in Philadelphia $t$ days after January 1 is given by the function

$$L(t) = 12 + 2.8 \sin\left[\frac{2\pi}{365}(t - 80)\right]$$

Example 5

What is the domain of the function $f(x) = \dfrac{1}{1 - 2\cos x}$?

Solution This function is defined for all values of $x$ except for those that make the denominator $0$. But

$$1 - 2\cos x = 0 \quad \Leftrightarrow \quad \cos x = \frac{1}{2} \quad \Leftrightarrow \quad x = \frac{\pi}{3} + 2n\pi$$

or

$$x = \frac{5\pi}{3} + 2n\pi$$

where $n$ is any integer (because the cosine function has period $2\pi$). So the domain of $f$ is the set of all real numbers except for the ones noted above.

The tangent function is related to the sine and cosine functions by the equation

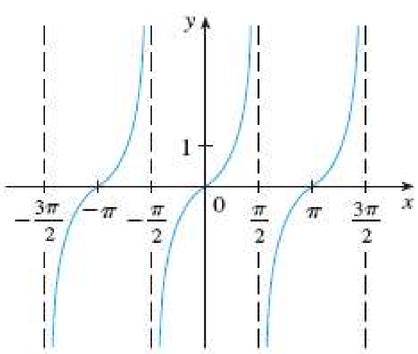
$$\tan x = \frac{\sin x}{\cos x}$$

and its graph is shown in Figure 19. It is undefined whenever $\cos x = 0$, that is, when $x = \pm\pi/2, \pm 3\pi/2, \ldots$. Its range is $(-\infty, \infty)$. Notice that the tangent function has period $\pi$:

$$\tan(x + \pi) = \tan x \qquad \text{for all } x$$

**Figure 19**

$y = \tan x$




The remaining three trigonometric functions (cosecant, secant, and cotangent) are the reciprocals of the sine, cosine, and tangent functions. Their graphs are shown in Appendix D.