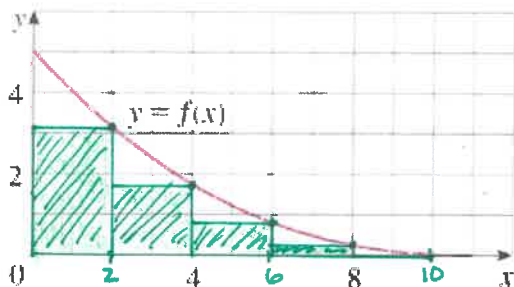


Write legibly. Show your work. Graph neatly. Use a ruler for all straight lines.

For the following limit forms, either give the answer or label it as indeterminate. Be ready to discuss/explain your answers next class.

- (1) By reading values from the given graph of f , use five and then ten rectangles to find a lower estimate and an upper estimate for the area under the given graph from $x=0$ to $x=10$. In each case, carefully draw the rectangles that you use.

a. five rectangles, lower estimate:

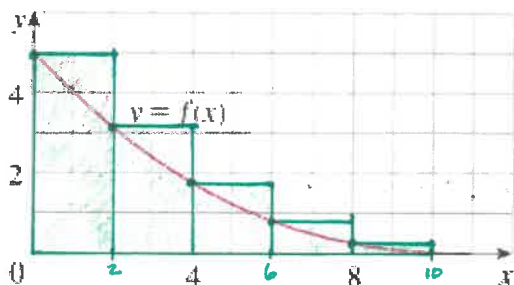


$$R_5 = 2(3.1 + 1.8 + 0.8 + 0.2 + 0)$$

$$= 11.8$$

approximate is ok!

b. five rectangles, upper estimate:

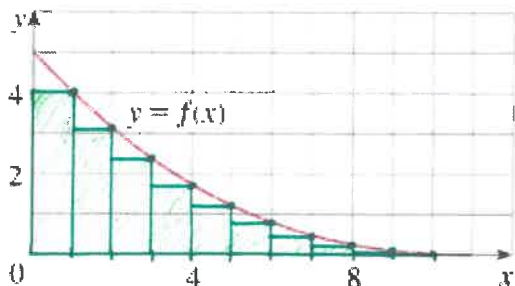


$$L_5 = 2(5 + 3.1 + 1.8 + 0.8 + 0.2)$$

$$= 21.8$$

approx ok!

c. ten rectangles, lower estimate:

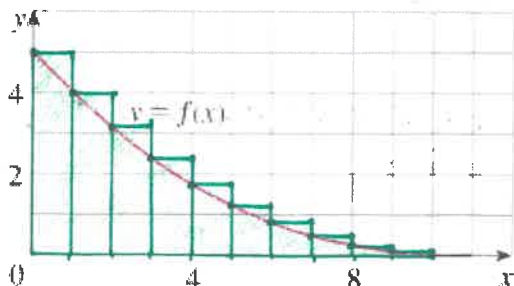


$$R_{10} = 1(4 + 3.1 + 2.3 + 1.8 + 1.2 + 0.8 + 0.4 + 0.2 + 0.1 + 0)$$

$$= 13.9$$

approx ok!

d. ten rectangles, upper estimate:



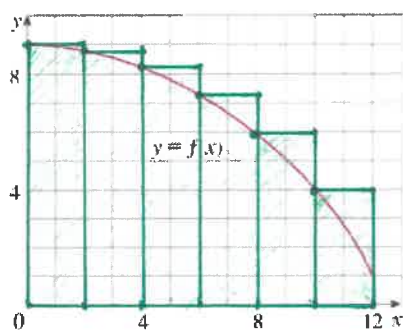
$$L_{10} = 1(5 + 4 + 3.1 + 2.3 + 1.8 + 1.2 + 0.8 + 0.4 + 0.2 + 0.1)$$

$$= 18.9$$

approx ok!

- (2) Use six rectangles to find estimates of each type for the area under the given graph of f from $x=0$ to $x=12$. DRAW THE RECTANGLES.

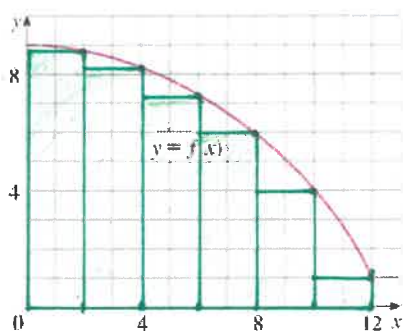
a. L_6 (left endpoints)



$$L_6 = 2(9 + 8.8 + 8.2 + 7.2 + 6 + 4)$$

$$L_6 = 86.4$$

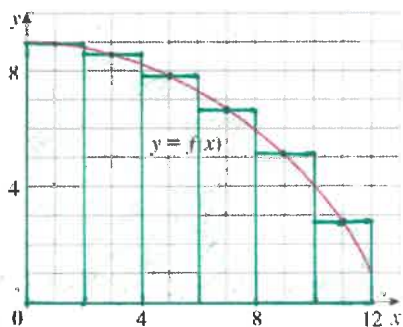
b. R_6 (right endpoints)



$$R_6 = 2(8.8 + 8.2 + 7.2 + 6 + 4 + 1)$$

$$R_6 = 70.4$$

c. M_6 (midpoints)



$$M_6 = 2(9 + 8.6 + 7.9 + 6.7 + 5.1 + 2.8)$$

$$M_6 = 80.2$$

d. Which of the three is a clear underestimate of the area under the curve?

R_6

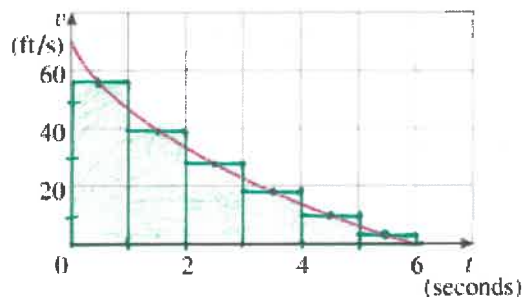
e. Which of the three is a clear overestimate of the area under the curve?

L_6

f. Which of the three is the best estimate of the area under the curve?

M_6

- (3) The velocity graph of a braking car is shown. Use it to estimate the distance travelled by the car while the brakes are being applied. (Use M_6 .)



$$M_6 = 1 \text{ sec} (57 + 40 + 28 + 19 + 9 + 2 \text{ ft/sec})$$

$$= 155 \text{ feet}$$

no unit? $\left(-\frac{1}{2}\right)$