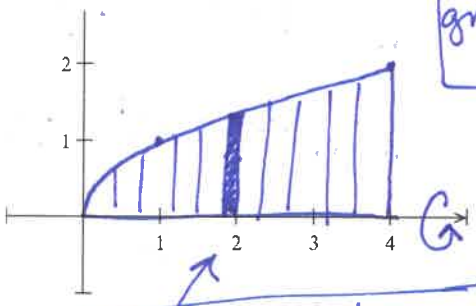
Math 252 Homework 10 Written Part **Name:** KEY

Write legibly. Show your work. Graph neatly. Use a ruler for all straight lines.

Practice with volumes:

1. Sketch the relevant area (before rotation), marking your scale.
2. Draw the representative rectangle.
3. Label or list the measurements you will need to find the volume.
4. Find the volume of the representative disk, washer, or cylinder.
5. Set up the integral to find the volume of the whole shape.
6. Solve the integral, expressing your answer in exact (not decimal) form.

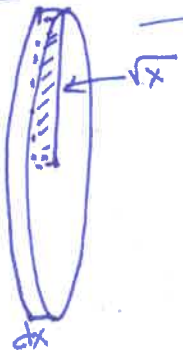
(1) Start with the area bounded by $y=\sqrt{x}$, $y=0$, and $x=4$, then rotate around the x-axis. Use disks.



graph w/ rect.

thickness = dx

radius = $y=\sqrt{x}$



Volume of one disk $= \pi(\sqrt{x})^2\,dx$

Volume of one disk $= \pi \cdot x\,dx$

Total Volume $= \int_0^4 \pi x\,dx$

$= \pi \cdot \frac{x^2}{2}\Big]_0^4$

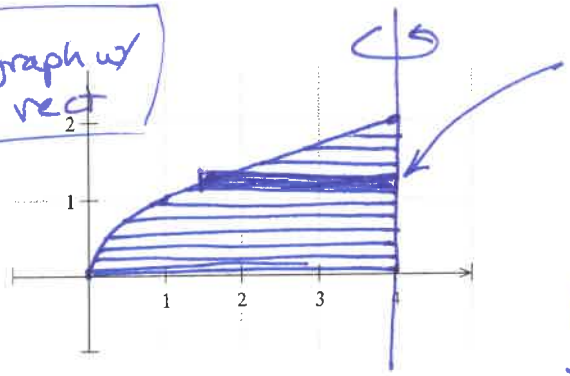
$= \pi\left[\frac{16}{2} - \frac{0}{2}\right]$

Vol. $= 8\pi$

aka $x = y^2$

(2) Start with the area bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$, then rotate around the line $x = 4$. Use disks.

graph w/ rect



← thick = dy

Radius = Right x − left x

measurements

radius = $4 - y^2$

$\Rightarrow$ Volume of one disk = π (radius)^2 thick

$= \pi (4 - y^2)^2 \, dy$

Vol of one disk = $\pi (16 - 8y^2 + y^4) \, dy$

$\Rightarrow$ Total Volume = $\pi \int_0^2 16 - 8y^2 + y^4 \, dy$

$= \pi \left[16y - \frac{8}{3} y^3 + \frac{1}{5} y^5 \right]_0^2$

$= \pi \left[32 - \frac{64}{3} + \frac{32}{5} \right] - \pi [0]$

Total Vol. = $\frac{256\pi}{15}$