GRAPHS - A REFERENCE SHEET (Ch 11)

A simple graph has no loops or parallel edges.

A complete graph on n vertices (K_n) is a simple graph where every vertex is connected to every other vertex.

A graph is a subgraph of a "larger" graph if every vertex and every edge of the subgraph is in the "larger" graph.

The degree of a vertex is the number of ends of edges connected to that vertex.

The total degree of a graph is the sum of all the degrees of all the vertices.

FACT: The total degree of any graph is even.

A walk is a finite sequence of adjacent vertices and edges. It starts and ends with a vertex.

A path is a walk without repeated edges.

A simple path has no repeated edges or vertices

A closed walk starts and ends at the same vertex.

A circuit is a closed walk with no repeated edges.

A simple circuit is a circuit with no repeated edges or vertices (except start = end)

Two vertices are connected iff there is a walk from one to the other.

A graph is connected iff every two vertices are connected.

An Euler circuit contains every edge exactly once, and every vertex (perhaps more than once).

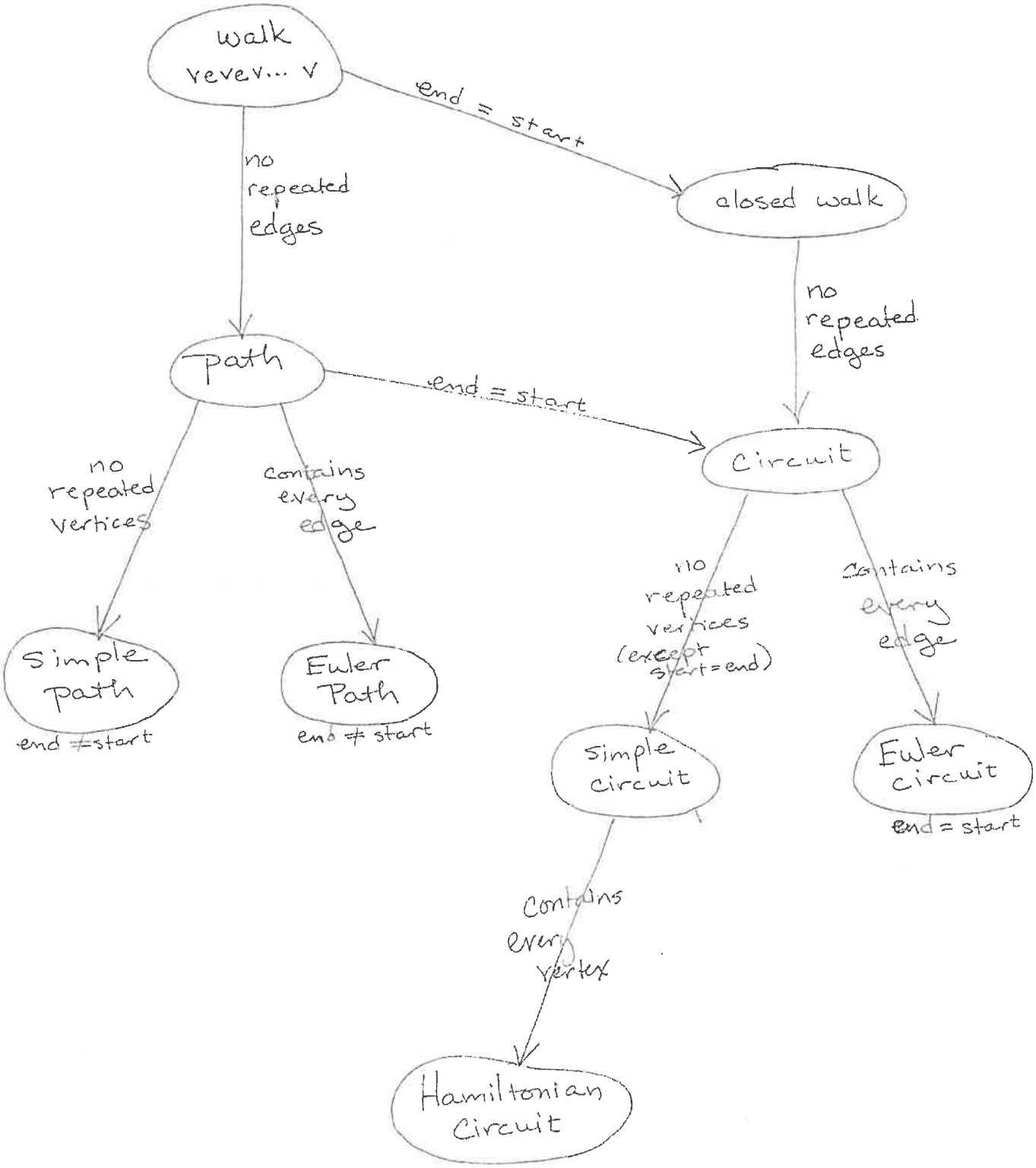
> FACT: If a graph is connected and every vertex is of even degree, then it has an Euler circuit.

An Euler path contains every edge exactly once, and every vertex (perhaps more than once), and ends not where it began.

> FACT: An Euler path exists iff the start and end vertices have odd degree, every other vertex has even degree, and the graph is connected.

A Hamiltonian circuit is a circuit that contains every vertex exactly once (except start = end)

Think travelling salesman!


walk
vevev... v
end = start
no repeated edges
closed walk
no repeated edges
Path
end = start
Circuit
no repeated vertices
contains every edge
no repeated vertices (except start=end)
contains every edge
Simple Path
end ≠ start
Euler Path
end ≠ start
Simple circuit
Euler circuit
end = start
contains every vertex
Hamiltonian Circuit

TREES

A tree is a connected graph with no circuits.

ex: 

trivial tree: • ← one vertex, no edge!

A forest is one or more trees.

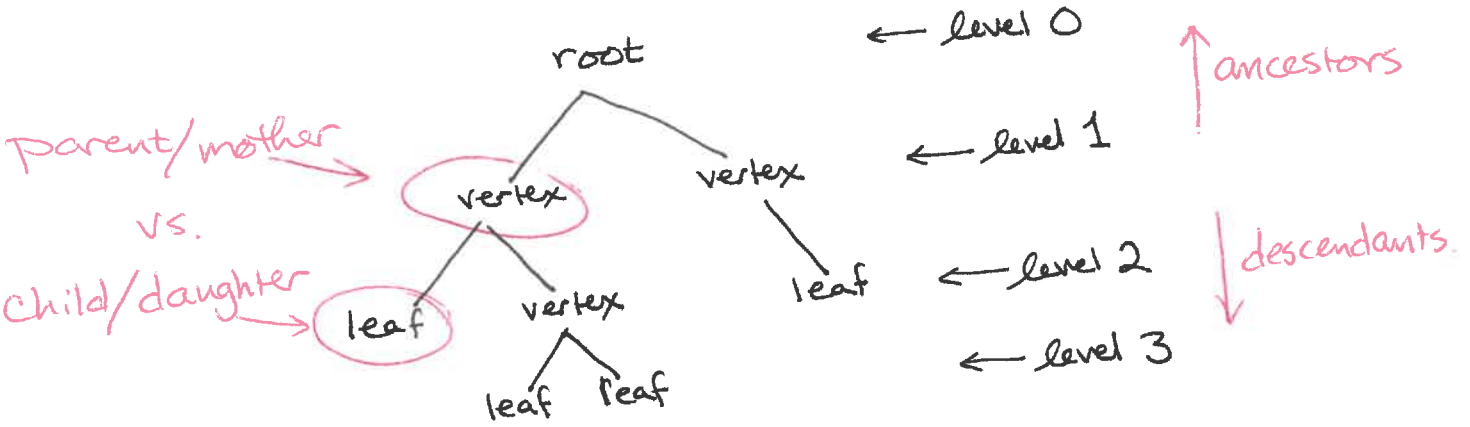
A leaf vertex (a terminal vertex) is of degree 1.

A branch vertex (an internal vertex) is of degree > 1.

> FACT: A tree with n vertices has n-1 edges.

> FACT: Any connected graph with n vertices and n-1 edges is a tree.

Rooted Trees specify a root - the start vertex.



In a Binary Tree, each internal vertex has at most 2 children.

In a Full Binary Tree, each internal vertex has exactly 2 children.