Math 252 Homework 7 Written Part **Name:** KEY

Write legibly. Show your work. Graph neatly. Use a ruler for all straight lines.

An example of integration by substitution with an indefinite integral -- notice the steps!

$\int x \cdot \cos(x^2)\,dx$

let $u = x^2$ ← let u equal the "inner" function,

$du = 2x\,dx$ ← take the differential of each side

[taking the differential ≅ do implicit differentiation to both sides, with an invisible/implied independent variable]

$\frac{1}{2}du = x\,dx$ ← rearrange so you're ready to substitute.

$= \frac{1}{2}\int \cos(u)\,du$ ← substitute! x^2 became u, and $x\,dx$ became $\frac{1}{2}du$. The $\frac{1}{2}$ was a constant multiplier, so it's ok to move it outside the integral.

$= \frac{1}{2}\sin(u) + C$ ← find the anti-derivative.

$\boxed{= \frac{1}{2}\sin(x^2) + C}$ ← change back to the original variable.

Showing your work neatly, completely, and correctly, find each integral:

(1) $\int e^{\cos(x)}\sin(x)\,dx$

let $u = \cos(x)$ ✓

$du = -\sin(x)\,dx$

$-du = \sin(x)\,dx$ ✓

$= -\int e^u\,du$ ✓

$= -e^u + C$

$\boxed{= -e^{\cos(x)} + C}$ ✓

(2) $\int \frac{x^3}{\sqrt{1+x^4}}\,dx$

let $u = 1 + x^4$ ✓

$du = 4x^3\,dx$

$\frac{1}{4}du = x^3\,dx$ ✓

$= \frac{1}{4}\int \frac{1}{\sqrt{u}}\,du$ ✓

$= \frac{1}{4}\int u^{-\frac{1}{2}}\,du$

$= \frac{1}{4}\left[2u^{\frac{1}{2}}\right] + C$

$\boxed{= \frac{1}{2}\sqrt{1+x^4} + C}$ ✓

An example of integration by substitution for a definite integral:

$$\int_0^1 \sqrt[3]{1+7x}\, dx$$

let $u = 1+7x$

$du = 7\, dx$

$\frac{1}{7} du = dx$

start: $x=0 \Rightarrow u = 1+7(0) = 1$

end: $x = 1 \Rightarrow u = 1+7(1) = 8$

this is now — convert endpoints too

$$= \frac{1}{7}\int_1^8 u^{\frac{1}{3}}\, du$$

← substitute everything, including the endpoints

$$= \frac{1}{7}\left[\frac{3u^{\frac{4}{3}}}{4}\right]_1^8$$

$$= \frac{1}{7}\left[\frac{3}{4}(8)^{\frac{4}{3}} - \frac{3}{4}(1)^{\frac{4}{3}}\right]$$

$$= \frac{1}{7}\left[\frac{3}{4}(16) - \frac{3}{4}\right]$$

$$= \frac{1}{7}\left[\frac{45}{4}\right] = \boxed{\frac{45}{28}}$$

Showing your work neatly, completely, and correctly, find the integral:

(3) $\displaystyle\int_0^{\frac{\pi}{6}} \frac{\sin(t)}{\cos^2(t)}\, dt$

let $u = \cos(t)$

$du = -\sin(t)\, dt$

$-du = \sin(t)\, dt$

start: $t = 0 \Rightarrow u = \cos(0) = 1$

end: $t = \frac{\pi}{6} \Rightarrow u = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$$= -\int_1^{\sqrt{3}/2} \frac{1}{u^2}\, du$$

$$= -\int_1^{\sqrt{3}/2} u^{-2}\, du$$

$$= -\left[\frac{u^{-1}}{-1}\right]_1^{\sqrt{3}/2}$$

$$= \frac{1}{u}\Big]_1^{\frac{\sqrt{3}}{2}}$$

$$= \boxed{\frac{2}{\sqrt{3}} - 1}$$

OR

$$\frac{2\sqrt{3}}{3} - 1$$

OR

$$\frac{2\sqrt{3} - 3}{3}$$