Math 231 – HW 3 **Name:** KEY ~48 too long?

Epp 2^{nd} Ed. 2.1	1, 3 - 7, 9 - 12
2.2	1, 2, 5 - 8 (only part a), 11 - 14 (only part a), 31 - 34

2.1 (1) A menagerie consists of seven brown dogs, two black dogs, six gray cats, ten black cats, five blue birds, six yellow birds, and one black bird. Determine which of the following statements are true and which are false.

(a) There is an animal in the menagerie that is red. FALSE

(b) Every animal in the menagerie is a bird or mammal. TRUE

(c) Every animal in the menagerie is brown or gray or black. FALSE

(d) There is an animal in the menagerie that is neither a cat nor a dog. TRUE

(e) No animal in the menagerie is blue. FALSE

(f) There are a dog, a cat, and a bird in the menagerie that all have the same color. TRUE

2.1 (3) Let $\mathbb{R}$ be the domain of the predicates "x>1," "x>2," "$|x| > 2$," and "$x^2 > 4$." Which of the following are true, and which are false?

(a) $x > 2 \Rightarrow x > 1$ TRUE

(b) $x > 2 \Rightarrow x^2 > 4$ TRUE

(c) $x^2 > 4 \Rightarrow x > 2$ FALSE for example, try $x = -3$.

(d) $x^2 > 4 \Leftrightarrow |x| > 2$ TRUE

Find and explain counterexamples to show that the statements in problems 4 - 7 are false.

2.1 (4) $\forall x \in \mathbb{R}, x > \frac{1}{x}$. False. For instance, if $x = 1$, Then $1 > \frac{1}{1}$ is false.

[or, $x = \frac{1}{2}$, $\frac{1}{2} \not> 2$; or $x = -2$, $-2 \not> -\frac{1}{2}$]

2.1 (5) $\forall a \in \mathbb{Z}, \frac{a-1}{a}$ is not an integer.

False For instance, if $a = 1$, $\frac{a-1}{a} = \frac{0}{1} = 0$, which is an integer.

[Or, if $a = -1$, $\frac{-1-1}{-1} = \frac{-2}{-1} = 2$, which is an integer]

2.1 (6) $\forall$ positive integers m and n, $m \cdot n \geq m + n$.

False. For example, if $m = 1$, then $1 \cdot n \geq 1 + n$ is not true.

2.1 (7) $\forall$ real numbers x and y, $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$.

False. In fact, this is almost never true! Choose any x and y such that $x \neq 0$ and $y \neq 0$.

2.1 (9) Consider the following statement: $\exists x \in \mathbb{R}$ such that $x^2=2$.
Which of the following are equivalent ways of expressing the statement?

(a) The square of each real number is 2. $\boxed{\text{No}}$

(b) Some real numbers have square 2. $\boxed{\text{Yes}}$ This is ok in formal logical language.

(c) The number x has square 2, for some real number x. $\boxed{\text{YES}}$

(d) If x is a real number, then $x^2=2$. $\boxed{\text{NO}}$

(e) Some real number has square 2. $\boxed{\text{YES}}$

(f) There is at least one real number whose square is 2. $\boxed{\text{YES}}$

2.1 (10) Rewrite the following statements informally is at least two different ways, without using variables or the symbols $\forall$ or $\exists$:

(a) $\forall$ squares x, x is a rectangle.

example answers: All squares are rectangles.
Anything that is a square is also a rectangle.

(b) $\exists$ a set A such that A has 16 subsets.

example answers: There is some set that has 16 subsets.
There exists at least one set that has 16 subsets.

2.1 (11) Rewrite each of the following statements in the form "$\forall$___x, ____."

example answers.

(a) All dinosaurs are extinct.

$\forall$ dinosaurs x, x is extinct.

(b) Every real number is positive, negative, or zero.

$\forall$ real numbers x, $x>0$ OR $x<0$ OR $x=0$.

(c) No irrational numbers are integers.

$\forall$ irrational numbers x, $x \notin \mathbb{Z}$.

(d) No logicians are lazy.

$\forall$ logicians x, x is not lazy

2.1 (12) Rewrite each of the following statements in the form "$\exists$___x such that ____."

(a) Some exercises have answers.

$\exists$ exercise(s) x such that x has an answer.

(b) Some real numbers are rational.

$\exists$ real numbers(s) x such that $x \in \mathbb{Q}$.

2.2 (1) The following statement is true: "$\forall$ non-zero real numbers x, $\exists$ a real number y such that $x \cdot y = 1$." For each x given below, find a y to make the predicate "$x \cdot y = 1$" true.

(a) x = 2. $\Rightarrow$ $\boxed{y = \frac{1}{2}}$

(b) x = -1. $\Rightarrow$ $\boxed{y = -1}$

(c) $x = \frac{3}{4}$. $\Rightarrow$ $\boxed{y = \frac{4}{3}}$

2.2 (2) The following statement is true: "$\forall$ real numbers x, $\exists$ an integer n such that n>x." For each x given below, find n to make the predicate "n>x" true.

(a) x=15.83 $\Rightarrow$ $\boxed{n = 16}$ (or more!)

(b) $x=10^8$ $\Rightarrow$ $\boxed{n = 10^8 + 1}$ (or more!)

(c) $x = 10^{10^{10}}$ $\Rightarrow$ $\boxed{n = 10^{10^{10}} + 1}$ (or more!)

Rewrite in English as simply as possible. Do not use variables or $\forall$ or $\exists$. example answers.

2.2 (5a) $\forall$ odd integers n, $\exists$ an integer k such that n=2k+1.

Any odd number equals two times some integer plus one.

2.2 (6a) $\forall r \in \mathbb{Q}$, $\exists$ integers a and b such that $r = \frac{a}{b}$.

Any rational number can be written as a ratio of integers.

2.2 (7a) $\forall x \in \mathbb{R}$, $\exists$ a real number y such that x+y=0.

Any real number can be added to some real number to get zero.

2.2 (8a) $\exists x \in \mathbb{R}$ such that for all real numbers y, x+y=0.

There is some real number that can be added to any real number to get zero. (by the way, this is false!)

Rewrite formally using quantifiers (∀ and/or ∃) and variables.

↙ example answers.

2.2 (11a) Everybody trusts somebody.

∀ people x, ∃ person y s.t. x trusts y.

2.2 (12a) Somebody trusts everybody.

∃ person x s.t. for ∀ people y, x trusts y.

2.2 (13a) Any even integer equals twice some other integer.

$\forall \underbrace{\text{even integer } n}_{\text{OR } n \in \mathbb{Z}^{\text{even}}}, \exists \underbrace{\text{integer } k}_{k \in \mathbb{Z}}$ s.t. $n = 2k$.

2.2 (14a) The number of rows in any truth table equals 2^n for some integer n.

$\forall$ truth table, $\exists \underbrace{\text{integer } n}_{\text{OR } n \in \mathbb{Z}}$ s.t. 2^n = # rows in the table.

Rewrite each in if-then form:

2.2 (31) Earning a grade of C- in this course is a sufficient condition for it to count toward graduation.

If you earn a C- in the course,

then it counts for graduation.

OR: If the course doesn't count, then you didn't get a C-.

2.2 (32) Being divisible by 6 is a sufficient condition for being divisible by 3.

If a number is divisible by 6,

then it is divisible by 3.

OR: If a number is not divisible by 3, then it is not divisible by 6.

2.2 (33) Being on time each day is a necessary condition for keeping this job.

If you keep this job,

then you will have been on time every day.

OR: If you are ever late, you will lose this job.

2.2 (34) A grade point average of at least 3.7 is a necessary condition for graduating with honors.

If you graduate with honors,

then your GPA is at least 3.7.

OR: If you don't have at least a 3.7, you will not graduate with honors.