MATH 231 - WEEK 1

Summary of Logical Notation (Notation of Symbolic Logic)

Order of operations: not, and, or, conditional, biconditional.

Negation $\sim p$ "not"

p	$\sim p$
T	F
F	T

Conjunction $p \wedge q$ "and"

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction $p \vee q$ "or"

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive Or $p \oplus q$ "xor"

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional $p \rightarrow q$ "implies" "if p, then q"

p	q	$p \rightarrow q$ or $p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Negation of a Conditional Statement

$$\sim(p \rightarrow q) \equiv p \wedge \sim q$$

p	q	$p \rightarrow q$	$\sim(p \rightarrow q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

The biconditional "iff"

p if and only if q.

$$(p \rightarrow q) \wedge (q \rightarrow p)$$

$$p \leftrightarrow q$$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Contrapositive of a Conditional Statement

$$p \rightarrow q \equiv \sim q \rightarrow \sim p$$

Converse of a Conditional Statement

The converse of $p \rightarrow q$ is $q \rightarrow p$. They are not logically equivalent

Inverse of a Conditional Statement

The inverse of $p \rightarrow q$ is $\sim p \rightarrow \sim q$. They are not logically equivalent.

Also see p. 14, Thm 1.1.1 as needed.

More about the conditional

p = "I'm in Bend." q = "I'm in Oregon."

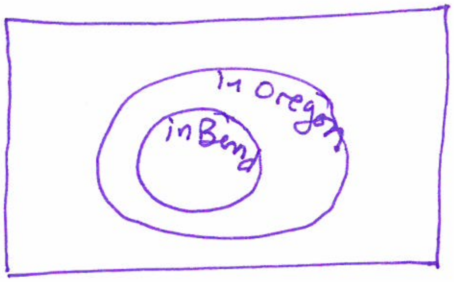
So... $p \rightarrow q$ means: "If I'm in Bend, then I'm in Oregon."

OR

"I'm in Bend, therefore I'm in Oregon."

OR

"I'm in Oregon if I'm in Bend."



Conditional

$p \rightarrow q$

"If I'm in Bend, then I'm in Oregon."

Converse

$q \rightarrow p$

"If I'm in Oregon, then I'm in Bend."

logically equivalent

logically equivalent

Contrapositive

$\sim q \rightarrow \sim p$

"If I'm not in Oregon, then, I'm not in Bend.

Inverse

$\sim p \rightarrow \sim q$

"If I'm not in Bend, then I'm not in Oregon."

The two sides are NOT logically equivalent!

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p	q	$p \rightarrow q$	
T	T	T	If I'm in Bend, then I'm in Oregon. True!
T	F	F	If I'm in Bend, then I'm not in Oregon. False!
F	T	T	If I'm not in Bend, I could still be in Oregon. True!
F	F	T	If I'm not in Bend, I might not be in Oregon. True!