1.5 Exercises

1.
   a. What is a one-to-one function?

   b. How can you tell from the graph of a function whether it is one-to-one?

2.
   a. Suppose f is a one-to-one function with domain A and range B. How is the inverse function f^{-1} defined? What is the domain of f^{-1}? What is the range of f^{-1}?

   b. If you are given a formula for f, how do you find a formula for f^{-1}?

   c. If you are given the graph of f, how do you find the graph of f^{-1}?

3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 and 14 A function is given by a table of values, a graph, a formula, or a verbal description. Determine whether it is one-to-one.

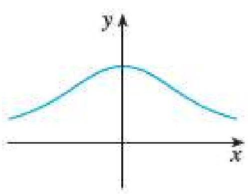
3.

x	1	2	3	4	5	6
$f(x)$	1.5	2.0	3.6	5.3	2.8	2.0

4.

x	1	2	3	4	5	6
$f(x)$	1.0	1.9	2.8	3.5	3.1	2.9

5.



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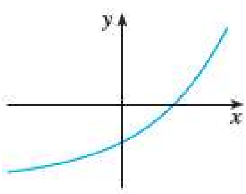

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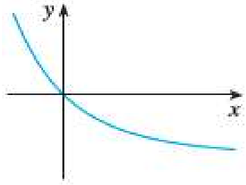

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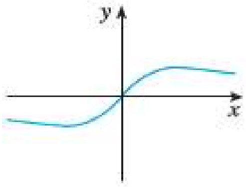


9. $f(x) = 2x - 3$

10. $f(x) = x^4 - 16$

11. $g(x) = 1 - \sin x$

12. $g(x) = \sqrt[3]{x}$

13. $f(t)$ is the height of a football t seconds after kickoff.

14. $f(t)$ is your height at age t.

15. Assume that f is a one-to-one function.

    a. If $f(6) = 17$, what is $f^{-1}(17)$?

    b. If $f^{-1}(3) = 2$, what is $f(2)$?

16. If $f(x) = x^5 + x^3 + x$, find $f^{-1}(3)$ and $f\left(f^{-1}(2)\right)$.

17. If $g(x) = 3 + x + e^x$, find $g^{-1}(4)$.

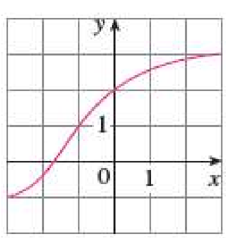
18. The graph of f is given.

    a. Why is f one-to-one?

    b. What are the domain and range of f^{-1}?

    c. What is the value of $f^{-1}(2)$?

    d. Estimate the value of $f^{-1}(0)$.



19. The formula $C = \frac{5}{9}(F - 32)$, where $F \geqslant -459.67$, expresses the Celsius temperature C as a function of the Fahrenheit temperature F. Find a formula for the inverse function and interpret it. What is the domain of the inverse function?

20. In the theory of relativity, the mass of a particle with speed v is

$$m = f(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where m_0 is the rest mass of the particle and c is the speed of light in a vacuum. Find the inverse function of f and explain its meaning.

21, 22, 23, 24, 25 and 26 Find a formula for the inverse of the function.

21. $f(x) = 1 + \sqrt{2 + 3x}$

22. $f(x) = \dfrac{4x - 1}{2x + 3}$

23. $f(x) = e^{2x-1}$

24. $y = x^2 - x, \quad x \geqslant \dfrac{1}{2}$

25. $y = \ln(x + 3)$

26. $y = \dfrac{1 - e^{-x}}{1 + e^{-x}}$

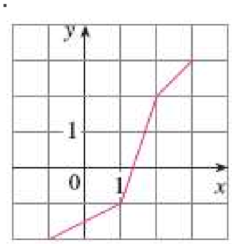
27–28 Find an explicit formula for f^{-1} and use it to graph f^{-1}, f, and the line $y = x$ on the same screen. To check your work, see whether the graphs of f and f^{-1} are reflections about the line.

27. $f(x) = \sqrt{4x + 3}$

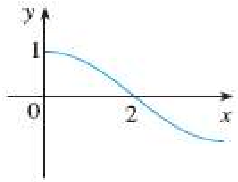
28. $f(x) = 1 + e^{-x}$

29–30 Use the given graph of f to sketch the graph of f^{-1}.

29.



30.



31. Let $f(x)=\sqrt{1-x^2}, 0 \leqslant x \leqslant 1$.

    a. Find f^{-1}. How is it related to f?

    b. Identify the graph of f and explain your answer to part (a).

32. Let $g(x)=\sqrt[3]{1-x^3}$.

    a. Find g^{-1}. How is it related to g?

    b. Graph g. How do you explain your answer to part (a)?

33.

    a. How is the logarithmic function $y=\log_b x$ defined?

    b. What is the domain of this function?

    c. What is the range of this function?

    d. Sketch the general shape of the graph of the function $y=\log_b x$ if $b>1$.

34.

    a. What is the natural logarithm?

    b. What is the common logarithm?

    c. Sketch the graphs of the natural logarithm function and the natural exponential function with a common set of axes.

35, 36, 37 and 38 Find the exact value of each expression.

35.

    a. $\log_2 32$

    b. $\log_8 2$

36.

    a. $\log_5 \dfrac{1}{125}$

    b. $\ln(1/e^2)$

37.

    a. $\log_{10} 40+\log_{10} 2.5$

    b. $\log_8 60-\log_8 3-\log_8 5$

38.

a. $e^{-\ln 2}$

b. $e^{\ln(\ln e^3)}$

39, 40 and 41 Express the given quantity as a single logarithm.

39. $\ln 10 + 2\ln 5$

40. $\ln b + 2\ln c - 3\ln d$

41. $\frac{1}{3}\ln(x+2)^3 + \frac{1}{2}\left[\ln x - \ln\left(x^2+3x+2\right)^2\right]$

42. Use Formula 10 to evaluate each logarithm correct to six decimal places.

a. $\log_5 10$

b. $\log_3 57$

43 and 44 Use Formula 10 to graph the given functions on a common screen. How are these graphs related?

43. $y = \log_{1.5} x, \qquad y = \ln x, \qquad y = \log_{10} x, \qquad y = \log_{50} x$

44. $y = \ln x, \qquad y = \log_{10} x, \qquad y = e^x, \qquad y = 10^x$

45. Suppose that the graph of $y = \log_2 x$ is drawn on a coordinate grid where the unit of measurement is an inch. How many miles to the right of the origin do we have to move before the height of the curve reaches 3 ft?

46. Compare the functions $f(x) = x^{0.1}$ and $g(x) = \ln x$ by graphing both f and g in several viewing rectangles. When does the graph of f finally surpass the graph of g?

47–48 Make a rough sketch of the graph of each function. Do not use a calculator. Just use the graphs given in Figures 12 and 13 and, if necessary, the transformations of Section 1.3.

47.

a. $y = \log_{10}(x+5)$

b. $y = -\ln x$

48.

a. $y = \ln(-x)$

b. $y = \ln |x|$

49–50

a. What are the domain and range of f?

b. What is the x-intercept of the graph of f?

c. Sketch the graph of f.

49. $f(x) = \ln x + 2$

50. $f(x) = \ln(x-1) - 1$

51, 52, 53 and 54 Solve each equation for x.

51.

a. $e^{7-4x} = 6$

b. $\ln(3x-10) = 2$

52.

a. $\ln(x^2-1) = 3$

b. $e^{2x} - 3e^{x} + 2 = 0$

53.

a. $2^{x-5} = 3$

b. $\ln x + \ln(x-1) = 1$

54.

a. $\ln(\ln x) = 1$

b. $e^{ax} = Ce^{bx}$, where $a \neq b$

55–56 Solve each inequality for x.

55.

a. $\ln x < 0$

b. $e^{x} > 5$

56.

a. $1 < e^{3x-1} < 2$

b. $1 - 2\ln x < 3$

57.

a. Find the domain of $f(x) = \ln(e^x - 3)$.

b. Find f^{-1} and its domain.

58.

a. What are the values of $e^{\ln 300}$ and $\ln(e^{300})$?

b. Use your calculator to evaluate $e^{\ln 300}$ and $\ln(e^{300})$. What do you notice? Can you explain why the calculator has trouble?

59. CAS Graph the function $f(x) = \sqrt{x^3 + x^2 + x + 1}$ and explain why it is one-to-one. Then use a computer algebra system to find an explicit expression for $f^{-1}(x)$. (Your CAS will produce three possible expressions. Explain why two of them are irrelevant in this context.)

60. CAS

a. If $g(x) = x^6 + x^4$, $x \geqslant 0$, use a computer algebra system to find an expression for $g^{-1}(x)$.

b. Use the expression in part (a) to graph $y = g(x)$, $y = x$, and $y = g^{-1}(x)$ on the same screen.

61. If a bacteria population starts with 100 bacteria and doubles every three hours, then the number of bacteria after t hours is $n = f(t) = 100 \cdot 2^{t/3}$.

a. Find the inverse of this function and explain its meaning.

b. When will the population reach $50{,}000$?

62. When a camera flash goes off, the batteries immediately begin to recharge the flash's capacitor, which stores electric charge given by

$$Q(t) = Q_0\left(1 - e^{-t/a}\right)$$

(The maximum charge capacity is Q_0 and t is measured in seconds.)

a. Find the inverse of this function and explain its meaning.

b. How long does it take to recharge the capacitor to 90% of capacity if $a = 2$?

63, 64, 65, 66, 67 and 68 Find the exact value of each expression.

63.

a. $\cos^{-1}(-1)$

b. $\sin^{-1}(0.5)$

64.

a. $\tan^{-1}\sqrt{3}$

b. $\arctan(-1)$

65.

a. $\csc^{-1}\sqrt{2}$

b. $\arcsin 1$

66.

a. $\sin^{-1}(-1/\sqrt{2})$

b. $\cos^{-1}(\sqrt{3}/2)$

67.

a. $\cot^{-1}(-\sqrt{3})$

b. $\sec^{-1} 2$

68.

a. $\arcsin(\sin(5\pi/4))$

b. $\cos\left(2\sin^{-1}\left(\frac{5}{13}\right)\right)$

69. Prove that $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$.

70, 71 and 72 Simplify the expression.

70. $\tan(\sin^{-1} x)$

71. $\sin(\tan^{-1} x)$

72. $\sin(2 \arccos x)$

73–74 Graph the given functions on the same screen. How are these graphs related?

73. $y = \sin x,\ -\pi/2 \leqslant x \leqslant \pi/2;\qquad y = \sin^{-1} x;\qquad y = x$

74. $y = \tan x,\ -\pi/2 < x < \pi/2;\qquad y = \tan^{-1} x;\qquad y = x$

75. Find the domain and range of the function

$$g(x) = \sin^{-1}(3x + 1)$$

76.

a. Graph the function $f(x) = \sin(\sin^{-1} x)$ and explain the appearance of the graph.

b. Graph the function $g(x) = \sin^{-1}(\sin x)$. How do you explain the appearance of this graph?

77.

a. If we shift a curve to the left, what happens to its reflection about the line $y = x$? In view of this geometric principle, find an expression for the inverse of $g(x) = f(x + c)$, where f is a one-to-one function.

b. Find an expression for the inverse of $h(x) = f(cx)$, where $c \neq 0$.

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