

This is a Very Important Title!

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This abstract is abstract.

I. INTRODUCTION

II. CONCLUSION

Assumptions: const V , P , N .

REFERENCES

- Reference 1
- Reference 2

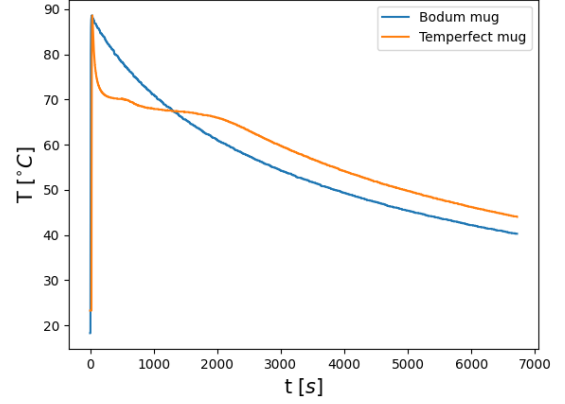


Figure 1. Temperature against time for the two mugs.

APPENDIX I.

In figure 1 we can see the temperature of the Temperfect mug and the Bodum mug plotted against the time. Since the water was not poured at the same temperature this has been adjusted in the plot as seen in figure 2.

We can see from figure 1 that the Temperfect mug absorbs a lot of heat very quickly. This is because it goes through a phase change, from solid to liquid. It then slowly goes back to solid again and gives heat back to the liquid in the mug. That makes it so that the temperature of the liquid drops more slowly. When all of the material in the mug is solid, the liquid will drop in temperature as if it was a regular mug, like the Bodum.

The reason this is beneficial is that the heat flux is related to the ∇T , so by reducing the temperature quickly, and storing it as latent heat, you can re-introduce it to the liquid later, at a lower temperature and therefore reduce the heat flux.

Assume constant pressure P and volume V . This means that the work $W = 0$, so that the change in internal energy $dU = dQ$. We also assume a constant number of particles N . In this condition we can use the definition of temperature (given by the total differential of the entropy S), so that the temperature T is given by

$$T^{-1} = \left(\frac{\partial S}{\partial U} \right)_{N,V},$$

Since we only have that S is a function of T we get that

$$T^{-1} = \frac{dS}{dU} = \frac{dS}{dQ} = \frac{dS}{dT} \frac{dT}{dQ}.$$

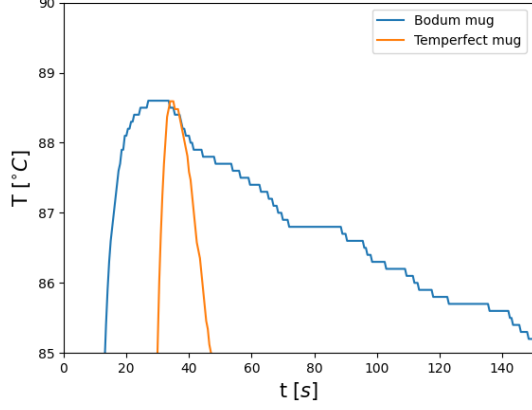


Figure 2. Closeup of the starting points where the hot water is poured in.

$$T^{-1} \frac{dQ}{dT} = \frac{dS}{dT}.$$

Under constant volume the heat capacity $C_V = (\partial U / \partial T)_{V,N}$, that becomes $C_V = (dQ/dT)$ in our case. We then get that

$$\frac{dS}{dT} = \frac{C_V}{T}. \quad (1)$$

In figure 3 we see the slope of the entropy dS/dT against the temperature T , calculated by this method in H₂O. The heat capacity of H₂O is listed in I. We use this to calculate the entropy. In figure 4 we see the entropy against the temperature.

Table I. Table of the heat capacity C of H₂O in kJ/kgK .

Solid	2.108
Liquid	4.184
Gas	1.996

The feature of the graph that is relevant is the part where the phase changes are happening. That is from ca. 0s to 2000s.

APPENDIX II.

A.

The multiplicity of an Einstein crystal is given by

$$\Omega(N, q) \approx \frac{(q + N)!}{q!N!},$$

where q is the number of energy units $\epsilon = hf$, where f is the frequency and h is Planck's constant, and N is the

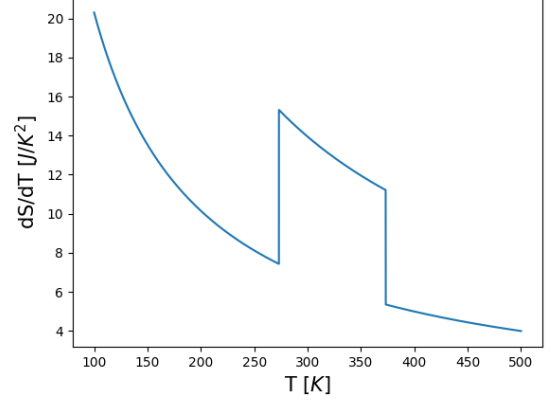


Figure 3. Figure of the slope of entropy dS/dT against the temperature T .

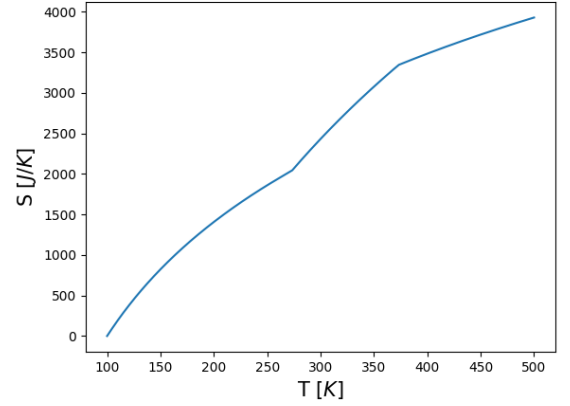


Figure 4. Figure of the entropy S against the temperature T .

number of particles in the crystal. We use this to find the entropy

$$S = k \ln \Omega,$$

where k is Boltzmann's constant. We have that the internal energy of the crystal is

$$U = q\epsilon + \frac{N}{2}\epsilon,$$

if we include the ground-states. This of course gives us that

$$dU = \epsilon dq,$$

since we vary q .

The thermodynamic identity says that

$$dU = TdS - PdV + \mu dN,$$

where μ is the chemical potential, P is the pressure and V is the volume. We assume that the volume, pressure

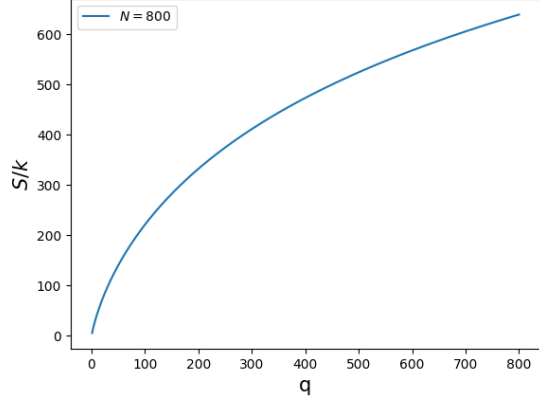


Figure 5. Figure of numerically calculated entropy S/k against the number of energy units q .

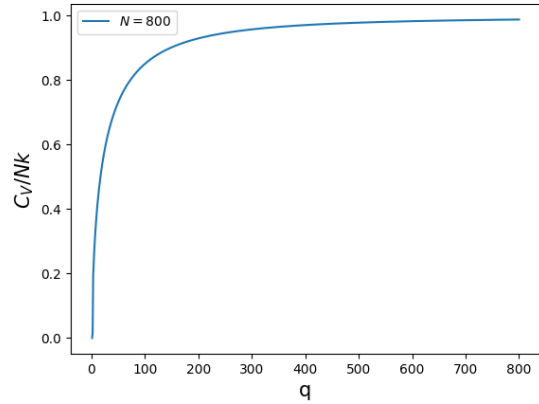


Figure 6. Figure of numerically calculated heat capacity C_V against the number of energy units q .

and number of particles stay constant. We then get that

$$T = \frac{dU}{dS} = \epsilon \frac{dq}{dS} \Leftrightarrow \frac{T}{\epsilon} = \frac{dq}{dS}.$$

Numerically we find this as

$$\tilde{T}_i = \frac{T_i}{\epsilon} = \frac{q_i - q_{i-1}}{S_i - S_{i-1}}.$$

We have that the heat capacity

$$C_V = \frac{dU}{dT} = \epsilon \frac{dq}{dT} = \frac{dq}{dT/\epsilon} = \frac{dq}{d\tilde{T}}.$$

We find this numerically as

$$C_{V,i} = \frac{q_i - q_{i-1}}{\tilde{T}_i - \tilde{T}_{i-1}}.$$

We see S/k as a function of q in figure 5 and $\frac{C_V}{Nk}$ as a function of q in figure 6.

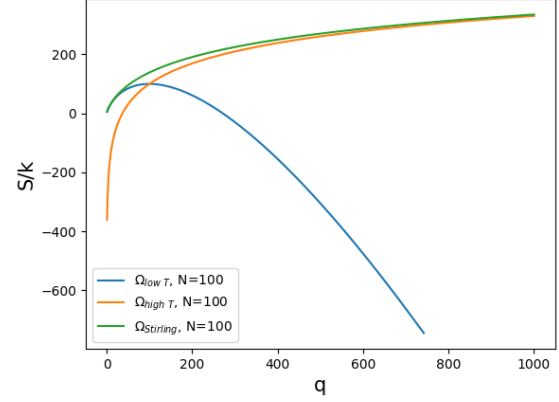


Figure 7. Figure of the low temperature approximation of the multiplicity $\Omega_{low\ T}$ vs. the high temperature approximation $\Omega_{high\ T}$ vs. the Stirling approximation $\Omega_{Stirling}$ against the number of energy units q .

B.

We have that the multiplicity of an Einstein solid is

$$\Omega(N, q) = \frac{(q + N - 1)!}{q!(N - 1)!},$$

where N is the number of particles and q is the number of energy units $\hbar f$. If we use $N \gg 1$, we can see that

$$\Omega(N, q) \approx \frac{(q + N)!}{q!N!}.$$

If also have $q \gg 1$ we can Stirling approximate this expression. That means that the temperature is $T > 0K$. This gives us that

$$\Omega = \frac{(q + N)!}{q!N!} \approx \frac{(q + N)^{q+N} e^{-(q+N)}}{q^q e^{-q} N^N e^{-N}} = \left(\frac{q + N}{q}\right)^q \left(\frac{q + N}{N}\right)^N$$

From the lectures we have that the multiplicity for the low temperature limit is

$$\Omega_{low\ T}(q, N) \approx \left(\frac{Ne}{q}\right)^q.$$

And the high temperature limit is

$$\Omega_{high\ T} \approx \left(\frac{qe}{N}\right)^N.$$

We see the comparison of the different Ω 's against each other in figure 7. From the figure we see that the low temperature approximation fits the Stirling approximation at low q and the high temperature approximation at high q .

We had that

$$dS = \frac{1}{U/Nk} dU = \frac{Nk}{U} dU = \frac{2Nk}{2q + N} dq.$$

This gives us that the entropy is

$$S(U, N) = Nk \int_{U_0}^U \frac{1}{U} dU = Nk \ln \frac{U}{U_0},$$

where $U_0 = (1/2)N\epsilon$ is the ground state energy.

We have that the temperature

$$T(U, N) = \left(\frac{\partial S}{\partial U}\right)^{-1} = \left(Nk \frac{1}{U}\right)^{-1} = \frac{U}{Nk}.$$

And that the heat capacity

$$C_V(T, N) = \frac{dU}{dT} = \frac{d}{dT}(NkT) = Nk.$$

The specific heat capacity per. mole is

$$c = \frac{C}{n} = \frac{Nk}{n},$$

where $n = N/N_A$ is the number of moles and N_A is Avogadro's number. We then get that

$$c = \frac{Nk}{N/N_A} = N_A k = R,$$

where R is the universal gas constant.

C.

We see from figure 6 that C_V/Nk goes to one. This means that our analytical and numerical results corresponds more and more as q gets higher.

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APPENDIX III.

Since we have that

$$U = NkT = \epsilon q + \frac{N}{2}\epsilon$$

for the Einstein crystal, we can see that

$$dU = Nk dT = \epsilon dq \Leftrightarrow dT = \frac{\epsilon}{Nk} dq.$$

We can use this to model how q changes in the mug, changes the temperature of the mug material. But the Einstein solid is not valid for liquids. The Temperfect mug has a material that will melt when you put hot liquid in the mug and therefore the Einstein model is not valid at that point. After a period of time the mug material will go back into solid and here the Einstein model might be valid. As energy units is transferred from the mug material to the liquid in the mug.