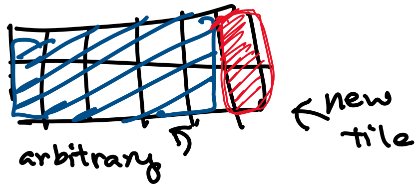
LC790 Domino & Trinominal Tiling :)

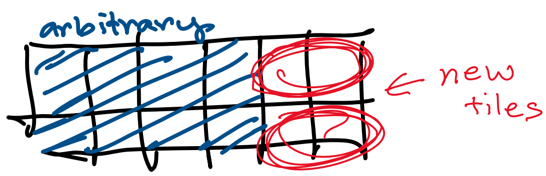
Monday, August 14, 2023 4:06 PM

① Define dp[i] as # ways to form an 2×i complete rectangle.

A: dp[i-1] can transition to dp[i] using vertical dominoes



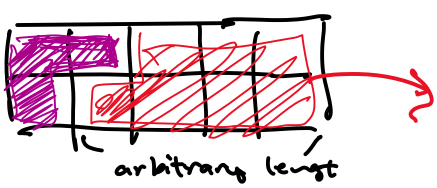
B: dp[i-2] can transition to dp[i] using horizontal dominoes



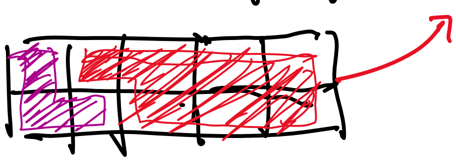
C: Trinomials are more difficult.

① Say we already placed dp[x] and we are placing a trinomial. There are 2 types of placing.

①



②



realize these 2 shapes are exactly the same,

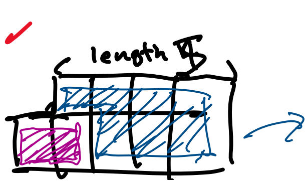
So # ways using trinominal = 2 · number of ways to create

↳ DP[i].

② How to create

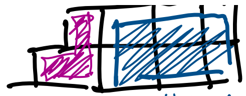
Case 1

Case 2

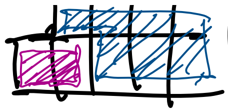


note that this is length 4

Case 1



Case 2



note that this is length 4

We can develop a separate dp reoccurence for making  of length i. $\Rightarrow$ this is $DP[i]$ as opposed to $dp[i]$,

$$DP[i] = \underbrace{dp[i-2]}_{\text{case 1}} + \underbrace{DP[i-1]}_{\text{case 2}}$$

③ Original dp reccurence relation using trinomial,

Any $2 \times N$ rectangle can use trinomial where $N \geq 3$.

So final dp recurrence is

$$dp[i] = dp[i-1] + dp[i-2] + 2 \times DP[i-1]$$

$$\because \quad DP[i-1] = dp[i-3] + DP[i-2] \qquad ①$$

$$\therefore \quad dp[i] = dp[i-1] + dp[i-2] + 2(dp[i-3] + DP[i-2])$$

$$dp[i-1] = dp[i-2] + dp[i-3] + 2DP[i-2] \qquad ②$$

① − ② !

$$dp[i] - dp[i-1] = dp[i-1] + dp[i-3]$$

$$dp[i] - dp[i-1] = dp[i-1] + dp[i-3]$$

$$\Rightarrow \quad dp[i] = 2\,dp[i-1] + dp[i-3].$$