Problem Set #3

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BLG 354 SIGNAL AND SYSTEMS Spring 2019-2020

Istanbul Technical University

May 29th

Question 1

Question 1.a

$$H(e^{jw}) = 1 + e^{-jw} + e^{-2jw} + e^{-3jw} + e^{-4jw}$$

$$H(e^{jw}) = e^{-2jw}(e^{2jw} + e^{jw} + 1 + e^{-jw} + e^{-2jw})$$

$$H(e^{jw}) = e^{-2jw}(2cos2w + 2cosw + 1)$$

Question 1.b

Since it is a LTI system it will always have the period of 2π. It can be seen from the equation below.

$$H(e^{j(w+2\pi)}) = \sum_{0}^{M} b_k e^{-jk(w+2\pi)}$$

$$H(e^{j(w+2\pi)}) = \sum_{0}^{M} b_k e^{-jwk} e^{-j2\pi k} = H(e^{jw})$$

Question 1.c

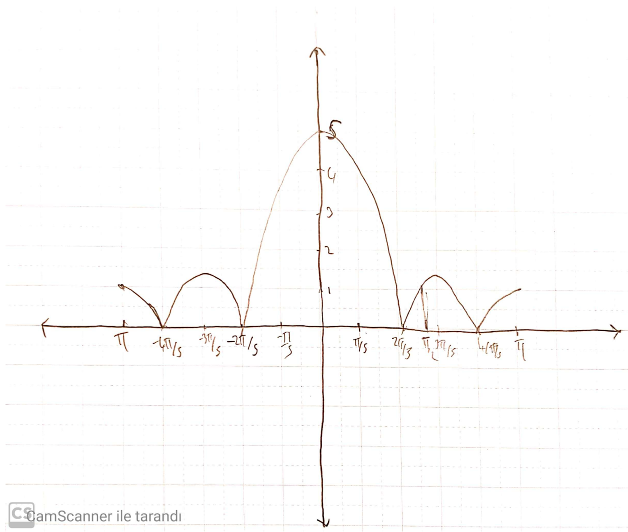

Figure 1: Magnitude

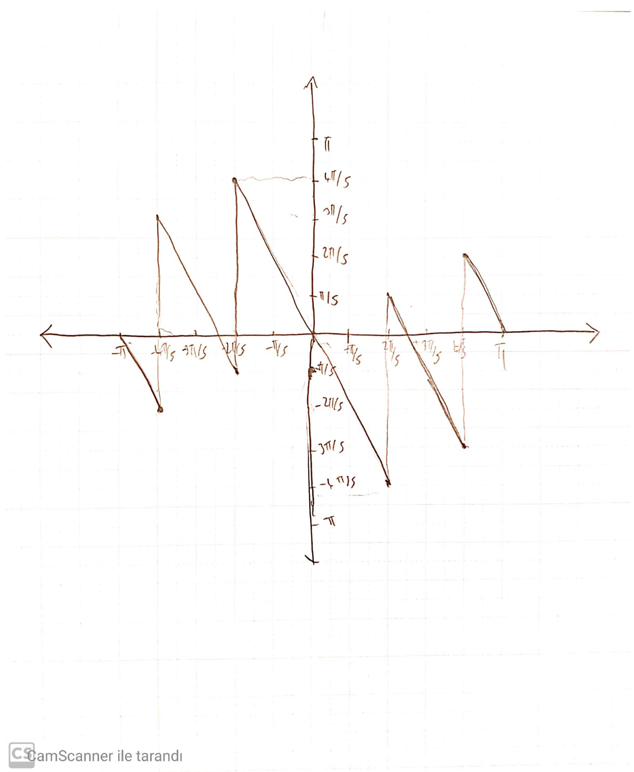

Figure 2: Phase

Question 1.d

$$h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5]$$

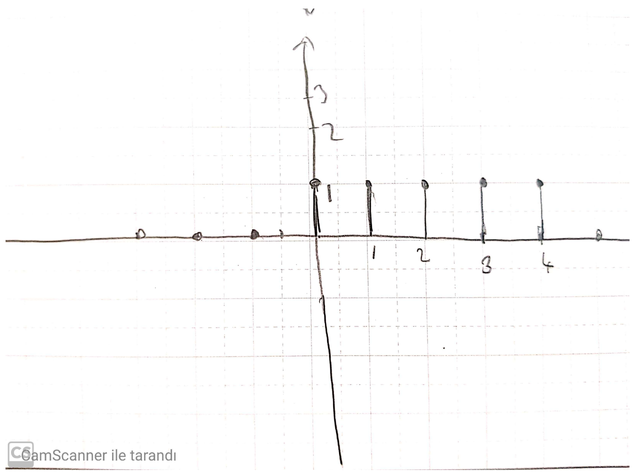

Figure 3: Impulse response

Question 1.e

$$H(e^{jw}) = e^{-2jw}(2cos2w + 2cosw + 1)$$

$$H(e^{j0}) = 5$$

$$H(e^{j0.5\pi}) = (-2 + 0 + 1)e^{j-j\pi}$$

$$H(e^{j0.3\pi}) = (-0,61 + 1,16 + 1)e^{-0.6j\pi}$$

$$y[n] = 20 + 2cos[0,5\pi n - 1.5\pi] - 4.65cos[0.3\pi n - 0.6\pi]$$

Question 1.f

This filter amplifies the low frequencies and fade out the high frequencies therefore it is a low pass filter. Applying this filter to an image will blur the image because it will fade the sharp edges and amplify the smooth gradients.

Question 2

Question 2.a

$$h[n] = h_1[n] * h_2[n] \text{ where}$$

$$y_1[n] = x[n] + x[n-1] \text{as low-pass filter } h_1[n] = \delta[n] + \delta[n-1]$$

$$y_2[n] = x[n] - x[n-1] \text{ as high-pass filter } h_2[n] = \delta[n] - \delta[n-1]$$

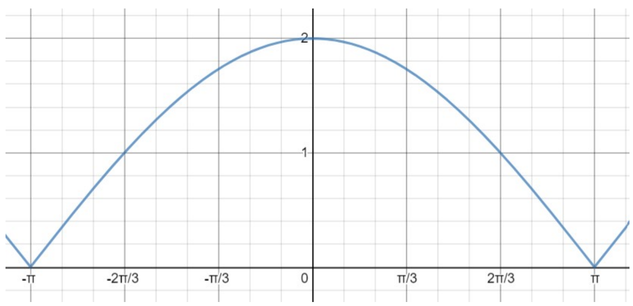

Figure 4: Low-pass filter

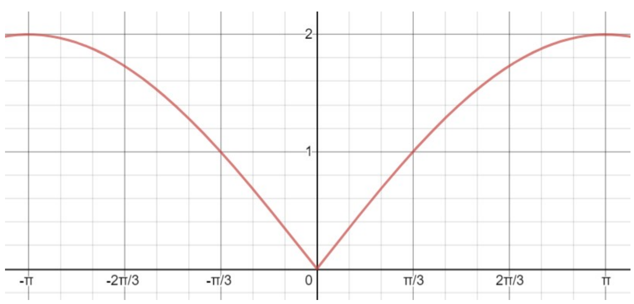

Figure 5: High-pass filter

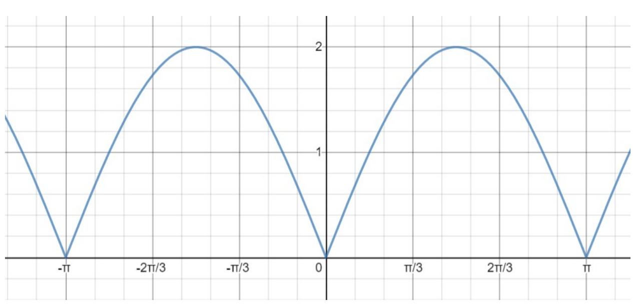

Figure 6: Final filter

Question 2.b

Impulse response

$$h[n] = \delta[n] + \delta[n-1] - \delta[n-1] - \delta[n-2]$$

$$h[n] = \delta[n] - \delta[n-2]$$

Frequency response

$$H_1(e^{jw}) = 1 + e^{-jw}$$

$$H_2(e^{jw}) = 1 - e^{-jw}$$

$$H(e^{jw}) = H_1(e^{jw})H_2(e^{jw}) = 1 - e^{-2jw}$$

Frequency domain $1 - e^{-2jw} = \delta[n] - \delta[n-2]$ in time domain .

Question 2.c

Combining a low-pass and a high-pass filter gave a band-pass filter, however ; not all combinings of lpf and hpf will yield a band pass filter.Since filters I used are not ideal lpf and hpf, middle frequencies are boosted and the others are faded.As filters get closer to the ideal filters final filter will be shaped depending on cut of frequencies .Band-pass filters commonly used in wireless transmitters and receivers in order to have the bandwidth of the allocated signal so that interfering with other stations are avoided .As a daily life example, if you are an rhythm guitarist you usually play around middle frequencies .By applying this filter you can get a clean rhythm guitar tone from your amplifier.

Question 3

Question 3.a

$$X(e^{jw}) = \sum_{-\infty}^{\infty}(n3^{-n}u[n] + e^{j(0.3\pi n+\pi/4)})e^{-jwn}$$

$$X(e^{jw}) = \sum_{-\infty}^{\infty} n3^{-}nu[n]e^{-jwn} + \sum_{-\infty}^{\infty} e^{j(0.3\pi n+\pi/4)}e^{-jwn} \text{ using linearity}$$

$\sum_{-\infty}^{\infty} e^{j(0.3\pi n+\pi/4)} e^{-jwn} = e^{j\pi/4} 2\pi\delta(w-0.3\pi)$

By using frequency differentiation $nx[n] = j\frac{dX(w)}{dw}$

$X(e^{jw}) = \frac{1}{1-\frac{1}{3}e^{-jw}}$ where $x[n] = 3^{-n}u[n]$

$nx[n] \xrightarrow{DTFT} j\frac{dX(w)}{dw} = -j\dfrac{3j\mathrm{e}^{jx}}{(3\mathrm{e}^{jx}-1)^2}$

Final equation will be $X(e^{jw}) = -j\dfrac{3j\mathrm{e}^{jx}}{(3\mathrm{e}^{jx}-1)^2} + e^{j\pi/4}2\pi\delta(w-0.3\pi)$

Question 3.b

Since it is a periodic function, we can calculate DTFT by sampling over a period because by using uniqness of the DTFT we can reconstruct the original signal.In order to make calculations easy DTFT performed on the interval [-3,3]

$$\sum_{-3}^{3} x[n].e^{-jwn} = 0 + e^{-j2w} + 2e^{-jw} + 3 + 2e^{jw} + e^{j2w} + 0$$

By using euler's formula

$$\sum_{-3}^{3} x[n].e^{-jwn} = 3 + 2cos2w + 4cosw$$

Question 3.c

By using linearity DTFT can be calculated as 2 sinc functions.

$X(e^{jw}) = \sum_{-\infty}^{\infty}(\frac{sin0.8\pi n}{\pi n} - \frac{sin0.5\pi n}{\pi n})e^{-jwn}$

$X(e^{jw}) = \sum_{-\infty}^{\infty}\frac{sin0.8\pi n}{\pi n}e^{-jwn} - \sum_{-\infty}^{\infty}\frac{sin0.5\pi n}{\pi n}e^{-jwn}$

$X(e^{jw}) = sinc_1(w) - sinc_2(w)$

$$sinc_1(w) = \begin{cases} 1 & |w| \leq 0.8\pi \\ 0 & 0.8\pi < |w| \leq \pi \end{cases}$$

$$sinc_2(w) = \begin{cases} 1 & |w| \leq 0.5\pi \\ 0 & 0.5\pi < |w| \leq \pi \end{cases}$$

$$X(e^{jw}) = \begin{cases} 1 & 0.5\pi \leq |w| < 0.8\pi \\ 0 & |w| \leq 0.5\pi \text{ and } 0.8\pi \leq |w| < \pi \end{cases}$$

Question 4

Question 4.a

$$a = e^{-jw}$$

$$X(e^{jw}) = \frac{2 + a/4}{-a^2/8 + a/4 + 1}$$

$$X(e^{jw}) = \frac{2 + a/4}{(1 - a/4)(1 + a/2)} = \frac{1}{1 + a/2} + \frac{1}{1 - a/4}$$

$$X(e^{jw}) = \frac{1}{1 + e^{-jw}/2} + \frac{1}{1 - e^{-jw}/4}$$

$$x[n] = 1/4^n u[n] + (-1/2)^n u[n]$$

Question 4.b

Inverse DTFT can be calculated for this function by manipulating sinc functions and using uniqueness theorem. By looking at the angle restrictions it is clear that, this function is a subtraction of two sinc functions.

$$X(e^{jw}) \xrightarrow{IDTFT} x[n] = \frac{sin(0.75\pi n)}{\pi n} - \frac{sin(0.25\pi n)}{\pi n}$$

Question 5

Question 5.a

```
import numpy as np

def dtft_magnitude_plotter(signal):
    ft = np.fft.fft(signal/max(abs(signal)))
    freq = np.fft.fftfreq(len(signal))
    plt.xlim(-0.2,0.2)
    plt.plot(freq,(ft.real**2 + ft.imag**2)**0.5)
```

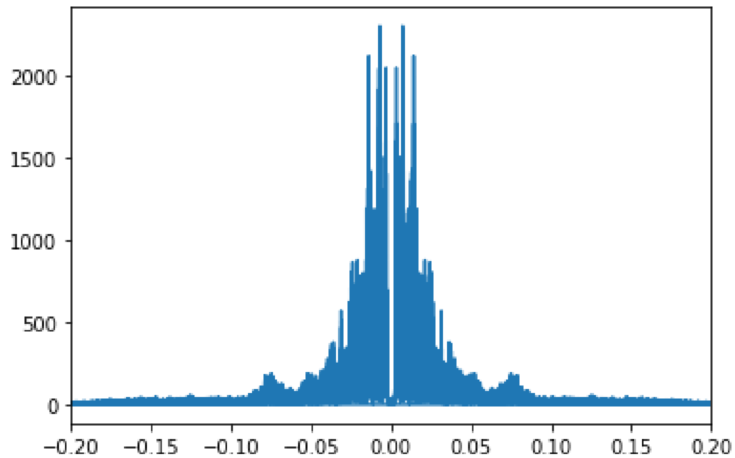

Figure 7: ESG DTFT

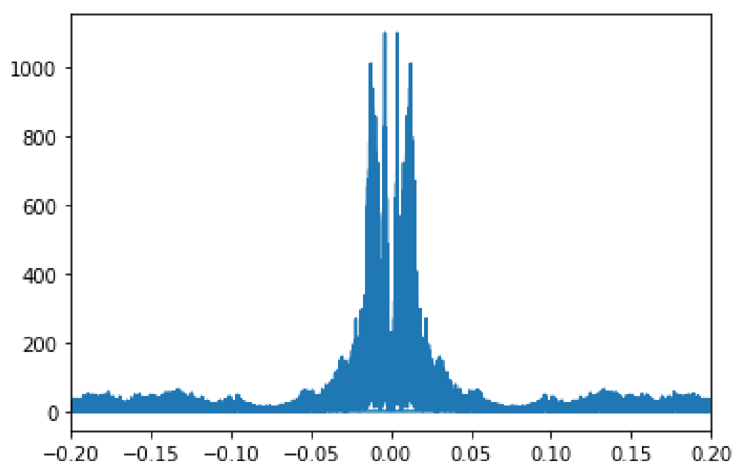

Figure 8: İlber DTFT

Question 5.b

In order to solve this problem exploratory data analysis is made on the training data.First, short time Fourier transform is applied to the signals then mean of the each signal is calculated.By comparing mean results of the signals, it is found that ESG is usually higher than 15000 and İlber is usually lower than 10000. After this analysis, just an if and else block used in order to classify the audio.Stft is used because temporal data such as human voice can be better analyzed with it , however ; same results can be achieved by using dtft. The reason why this problem is easy to solve is that ,ESG has a distinctively higher level of speech tone than İlber .

```python
# exploratory data analysis of ESG

path = os.getcwd()
esg_dir=glob.glob(path+"/data/esg/*.wav")
esg_mean =0
for i,file in enumerate(esg_dir):
    fs, esg = read(file)
    esg_1 = esg[:,0]
    n_fft = 2048
    hop_length = 512
    D = np.abs(librosa.stft(esg_1/1.0, n_fft=n_fft, hop_length=hop_length))
    print(D.mean())
```

```python
# Interference part

path = os.getcwd()
test_dir=glob.glob(path+"/data/test/*.wav")
predicts=[]
for i,file in enumerate(test_dir):#prediction part
    fs, data = read(file)
    data = data[:,0]
    n_fft = 2048
    hop_length = 512
    D = np.abs(librosa.stft(data/1.0, n_fft=n_fft, hop_length=hop_length))
    if(D.mean()>13000):
        predicts.append([file.split("/")[-1],"esg"])
    else:
        predicts.append([file.split("/")[-1],"ilber"])

score=0
for i in range(len(predicts)):#score calculating
    if(predicts[i][0].split("_")[0]==predicts[i][1]):
        score+=1
print("Test accuracy is % {}".format(100*score/len(predicts)))
```