6.1

$$(I-2P)^*(I-2P) = (I-2P)(I-2P) = I-4P+4P^2 = I$$

6.2

$$F\begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} x_m \\ \vdots \\ x_1 \end{pmatrix} \Rightarrow F = \begin{pmatrix} & & 1 \\ & \iddots & \\ 1 & & \end{pmatrix} \quad F^* = F, \quad F^*F = F^2 = I$$

$$E = \frac{1}{2}(I+F) \quad E^2 = \frac{1}{4}(I+F)^2 = \frac{1}{4}(I+2F+F^2) = \frac{1}{4}(2I+2F) = \frac{1}{2}(I+F) = E$$

$\Rightarrow$ E Projector.

$$E^* = \frac{1}{2}(I+F) = E$$

$\Rightarrow$ E orthogonal Projector.

$$\begin{pmatrix} \frac{1}{2} & \cdots & \frac{1}{2} & & \frac{1}{2} & \cdots & \frac{1}{2} \\ & \ddots & & 1 & & & \\ \frac{1}{2} & & \frac{1}{2} & & \frac{1}{2} & & \frac{1}{2} \end{pmatrix}$$

6.5

$P^2 = P \neq 0$.

$$\|P\|_2 = \max_{\|x\|=1} \|Px\|_2 = \max_{\|x\|=1} \|P^2x\|_2 \le \|P\|_2 \max_{\|x\|_2=1} \|Px\|_2 = \|P\|_2^2$$

$\Rightarrow \|P\|_2^2 \ge \|P\|_2$

$\Rightarrow \|P\|_2 \ge 1$.

$P = U\Sigma V^*$.

$$\|P\|_2 = \max_{x\neq 0} \frac{\|U\Sigma V^* x\|_2}{\|x\|_2} = \max_{x\neq 0} \frac{\|\Sigma V^* x\|_2}{\|x\|_2} = \max_{x\neq 0} \frac{\|\Sigma V^* x\|_2}{\|V^* x\|_2} = \|\Sigma\|_2.$$

$\|P\|_2 = 1 \Leftrightarrow \|\Sigma\|_2 = 1$

$\Leftrightarrow \Sigma = I$.

$\Leftrightarrow P = UV^*, \ P^2 = UV^*UV = UV^*$

$\Leftrightarrow V^*U = I$.

$\Leftrightarrow U = V$

$\Leftrightarrow P^* = P$ Orthogonal Projector