

(24)

$$K(A) = \|A\|_2 \|A^{-1}\|_2.$$

$$A = U \Sigma V^*, \|A\|_2 = \|\Sigma V^*\|_2 = \|V \Sigma^*\|_2 = \|\Sigma\|_2 = \sigma_{\max} = 100.$$

$$\|A\|_F = \|\Sigma\|_F = \sqrt{\sum \sigma_i^2} = 101 \Rightarrow \sigma_{\min} \in 101^2 - 100^2$$

$$\left\{ \begin{aligned} K(A) &= \|A\|_2 \|A^{-1}\|_2 = 100 \|A^{-1}\|_2. \\ \|A^{-1}\|_2 &= \|V \Sigma^{-1} U^*\|_2 = \|\Sigma^{-1}\|_2 = \frac{1}{\sigma_{\min}} \end{aligned} \right\} \Rightarrow K(A) = \frac{100}{\sigma_{\min}} = \frac{\sigma_{\max}}{\sigma_{\min}}.$$

$$K(A) \geq \frac{100}{\sup(\sigma_{\min})}$$

$$= \frac{100}{1}$$

$$= 100$$

$$\sigma_1^2 + \dots + \sigma_{m-1}^2 + 100^2 = 101^2.$$

$$\sigma_1^2 + \dots + \sigma_{m-1}^2 = 201.$$

$$\sigma_1^2 = 201 - (\sigma_2^2 + \dots + \sigma_{m-1}^2)$$

$$\sigma_1^2 \leq 201 - (m-2+1) \cdot \sigma_1^2.$$

$$= 201 - 200 \sigma_1^2.$$

$$\Rightarrow \sigma_1^2 \leq 1.$$