QUANTUM FOURIER TRANSFORM

Tensor Networks Approach

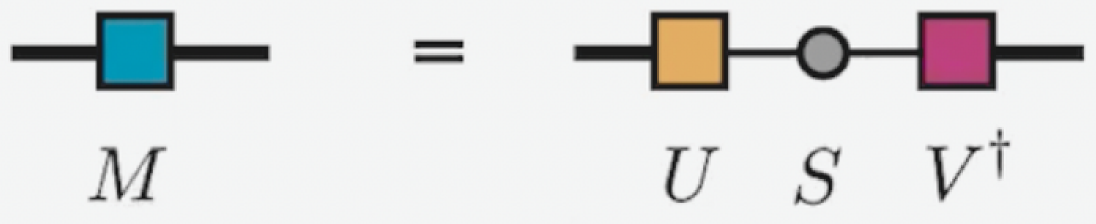
EDARA YASWANTH BALAJI

Singular value decomposition (SVD)

$$M = \begin{bmatrix} 0.435839 & 0.223707 & 0.10 \\ 0.435839 & 0.223707 & -0.10 \\ 0.223707 & 0.435839 & 0.10 \\ 0.223707 & 0.435839 & -0.10 \end{bmatrix}$$

Can factorize as

$$\begin{bmatrix} 1/2 & -1/2 & 1/2 \\ 1/2 & -1/2 & -1/2 \\ 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 0.933 & 0 & 0 \\ 0 & 0.300 & 0 \\ 0 & 0 & 0.200 \end{bmatrix} \begin{bmatrix} 0.707107 & 0.707107 & 0 \\ -0.707107 & 0.707107 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



We put a cutoff limit and throw away some values

$$\underset{U}{\begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}} \quad \underset{S}{\begin{bmatrix} 0.933 \end{bmatrix}} \quad \underset{V^T}{\begin{bmatrix} 0.707107 & 0.707107 & 0 \end{bmatrix}}$$

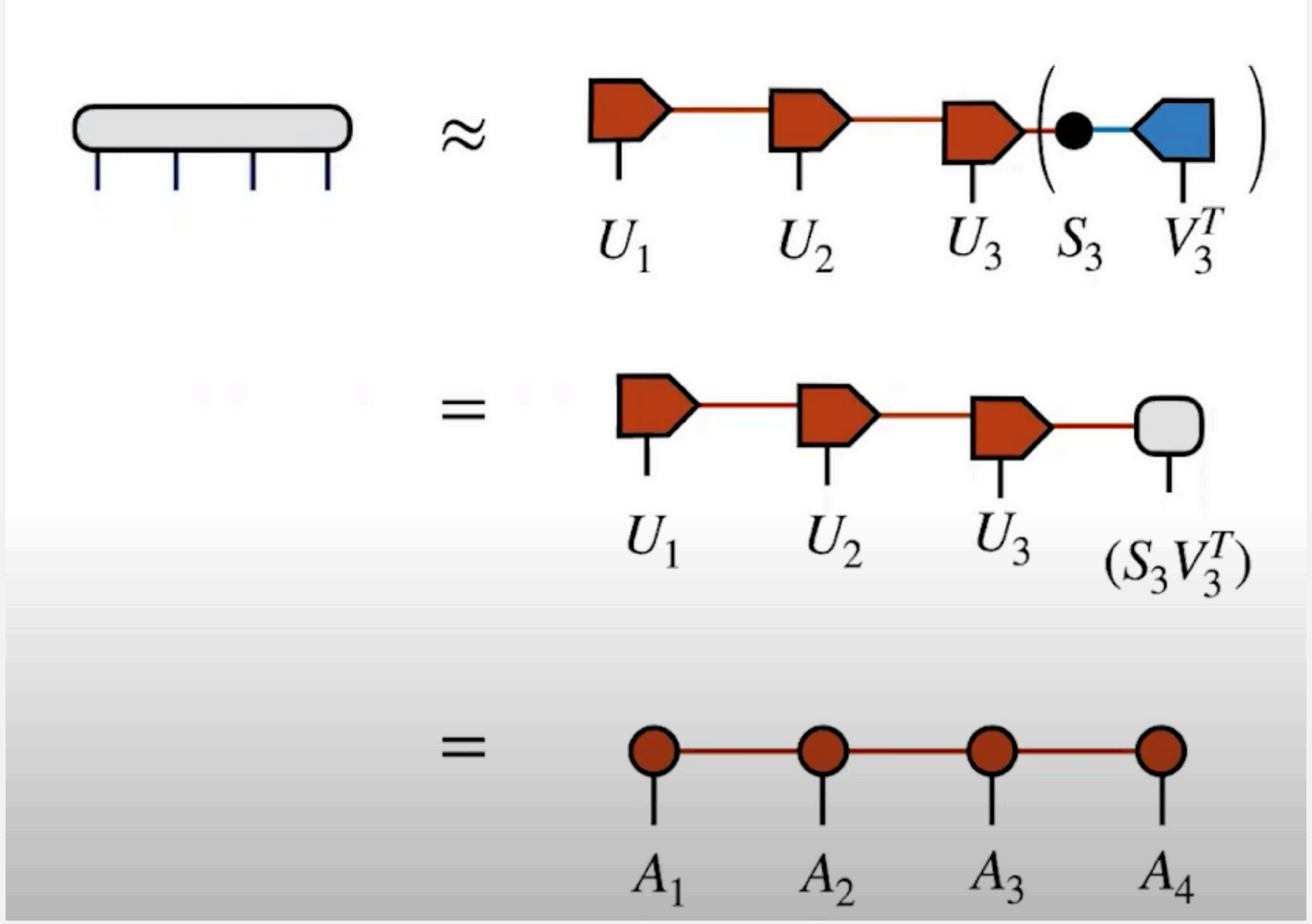
$$M_3 = \begin{bmatrix} 0.329773 & 0.329773 & 0 \\ 0.329773 & 0.329773 & 0 \\ 0.329773 & 0.329773 & 0 \\ 0.329773 & 0.329773 & 0 \end{bmatrix}$$

Truncating SVD =

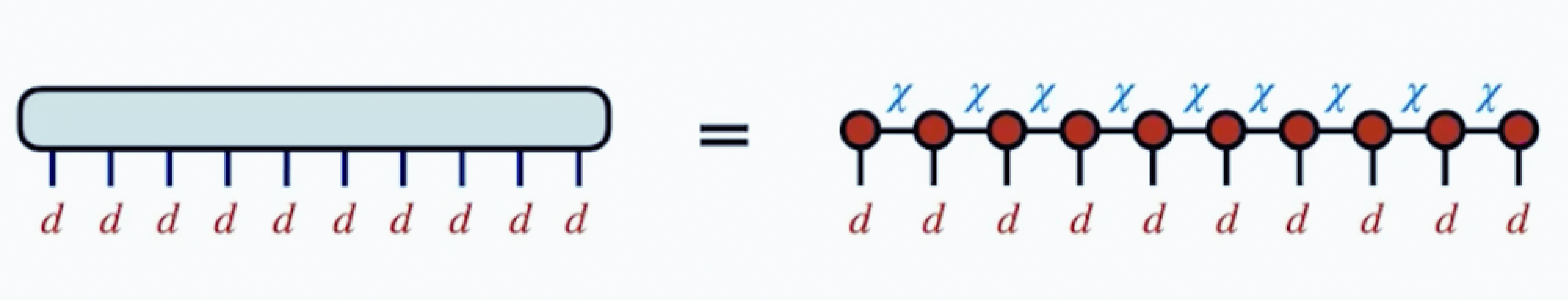
Controlled approximation for M

$$||M_3 - M||^2 = 0.13 = (0.3)^2 + (0.2)^2$$

Error



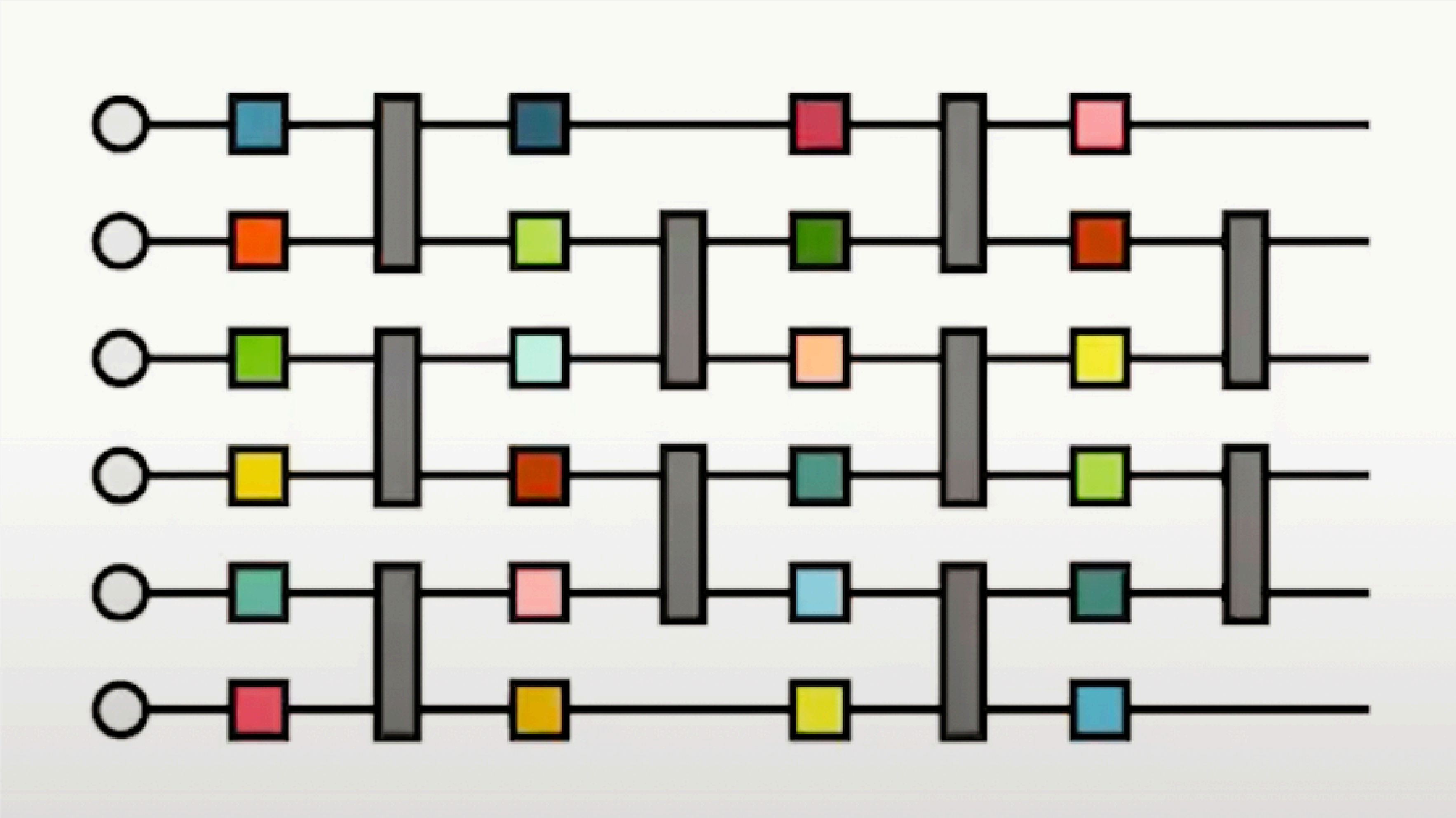
We keep repeating SVD till we get this result



$$d^N \longrightarrow N d \chi^2$$

The new state is called ‘Matrix Product State’ (MPS)

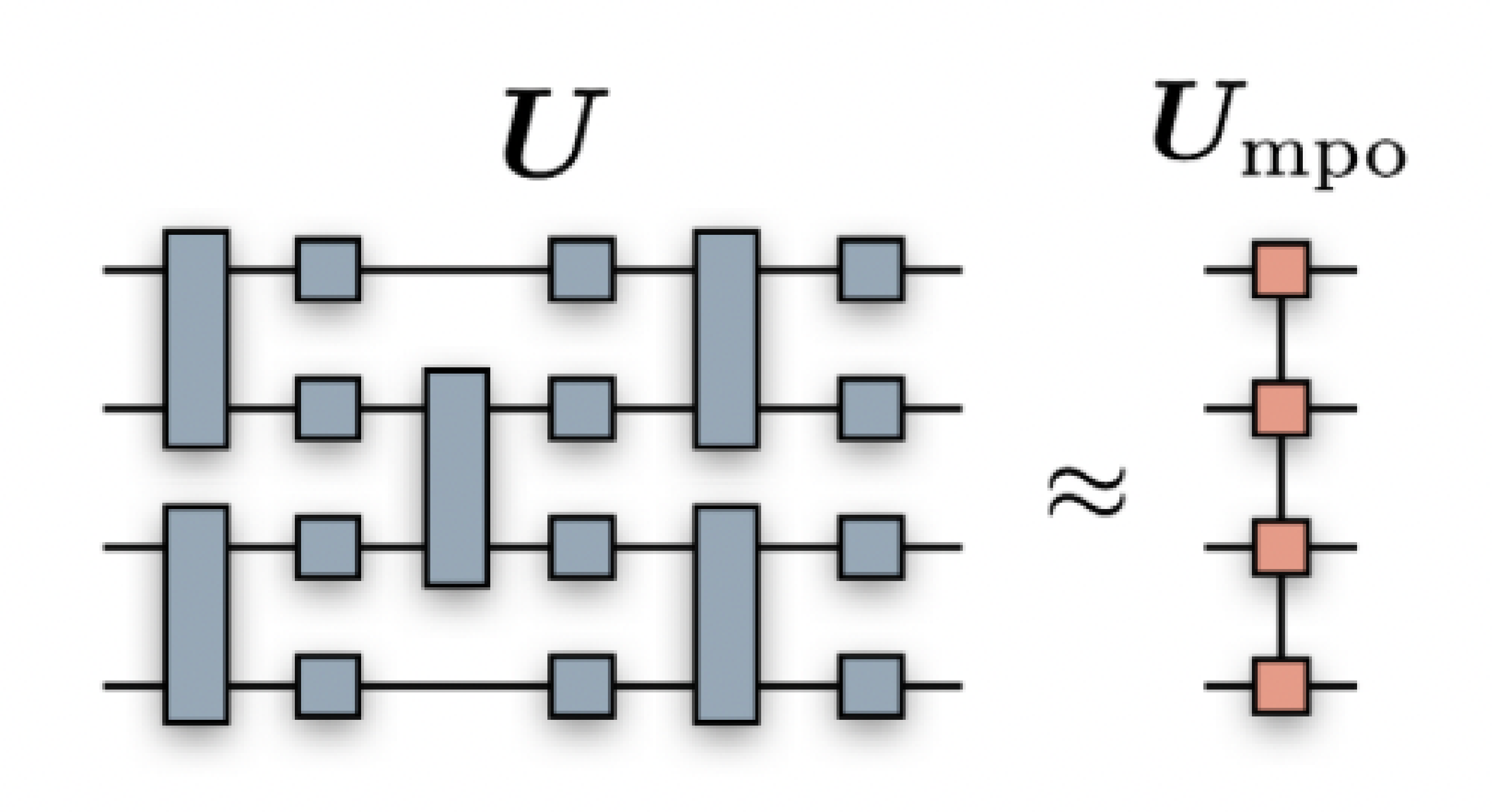
Quantum Circuit

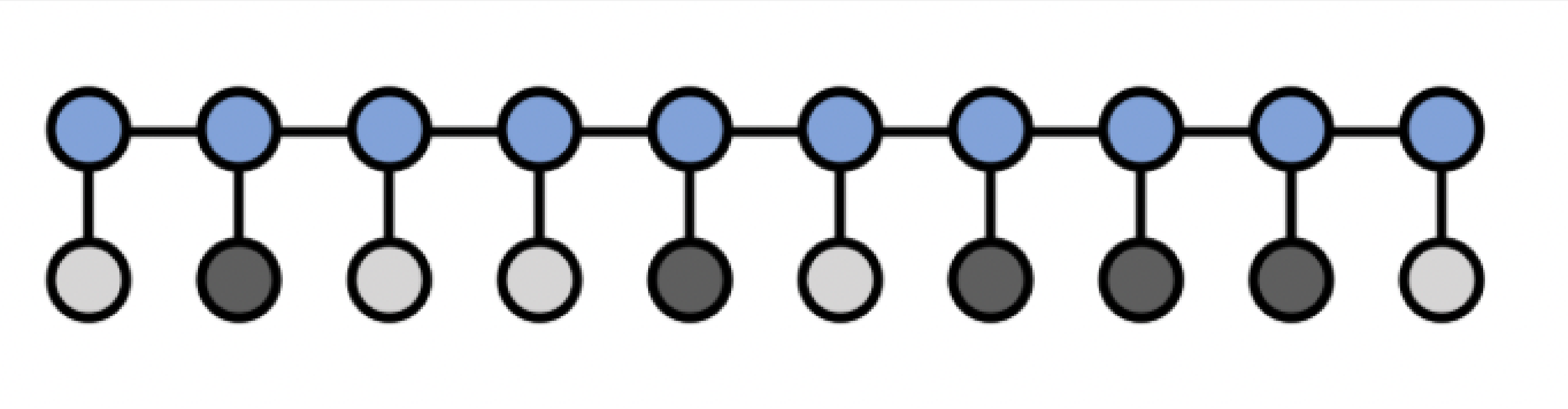


Now we don’t need to store the whole matrix and multiply, we can apply indivudual gates (small matrices) directly to the required qubit.

MATRIX PRODUCT OPERATOR (MPO)

- The full circuit can be stores in a MPO format and then directly multiplied to the MPS after taking the input. This reduces time by a lot as we don't need to apply gates anymore.

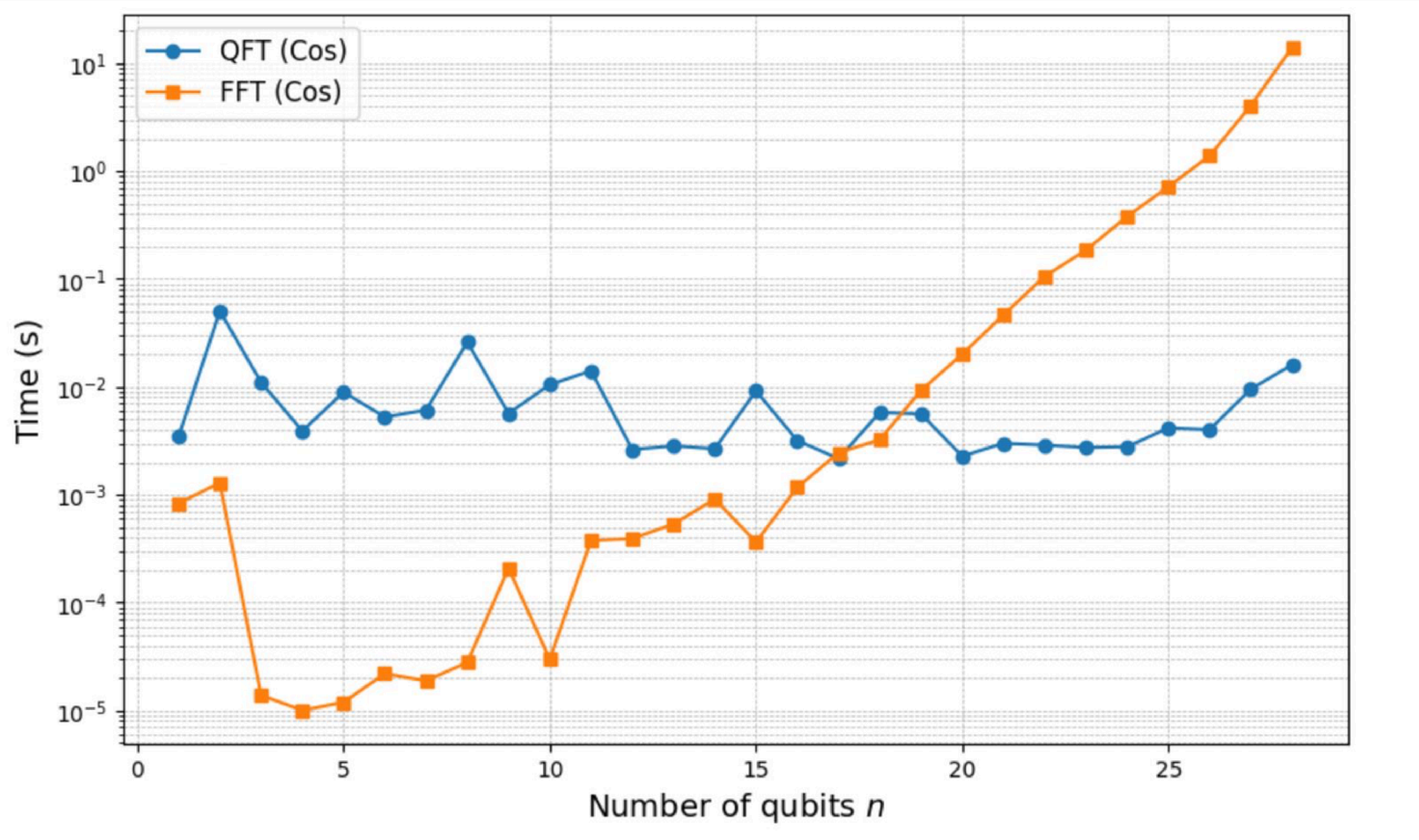




Directly multiply MPO and MPS

Low entanglement -> Low bond order

COMPARISION WITH FFT

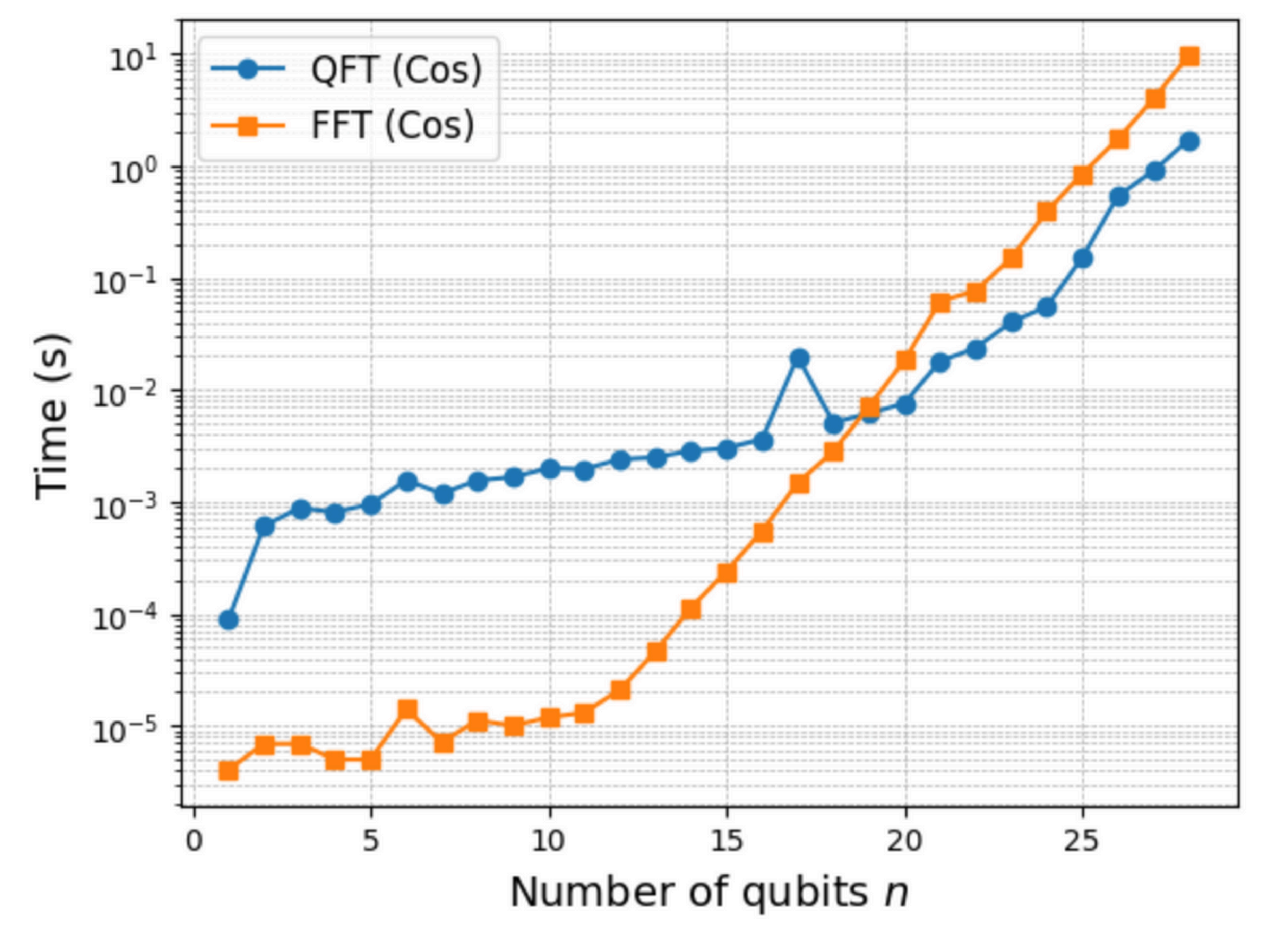


Time to convert the array to MPS is not included in this graph

QFT time = time to multiplying MPO and MPS

AS the number of qubits increase, we can see the advantage

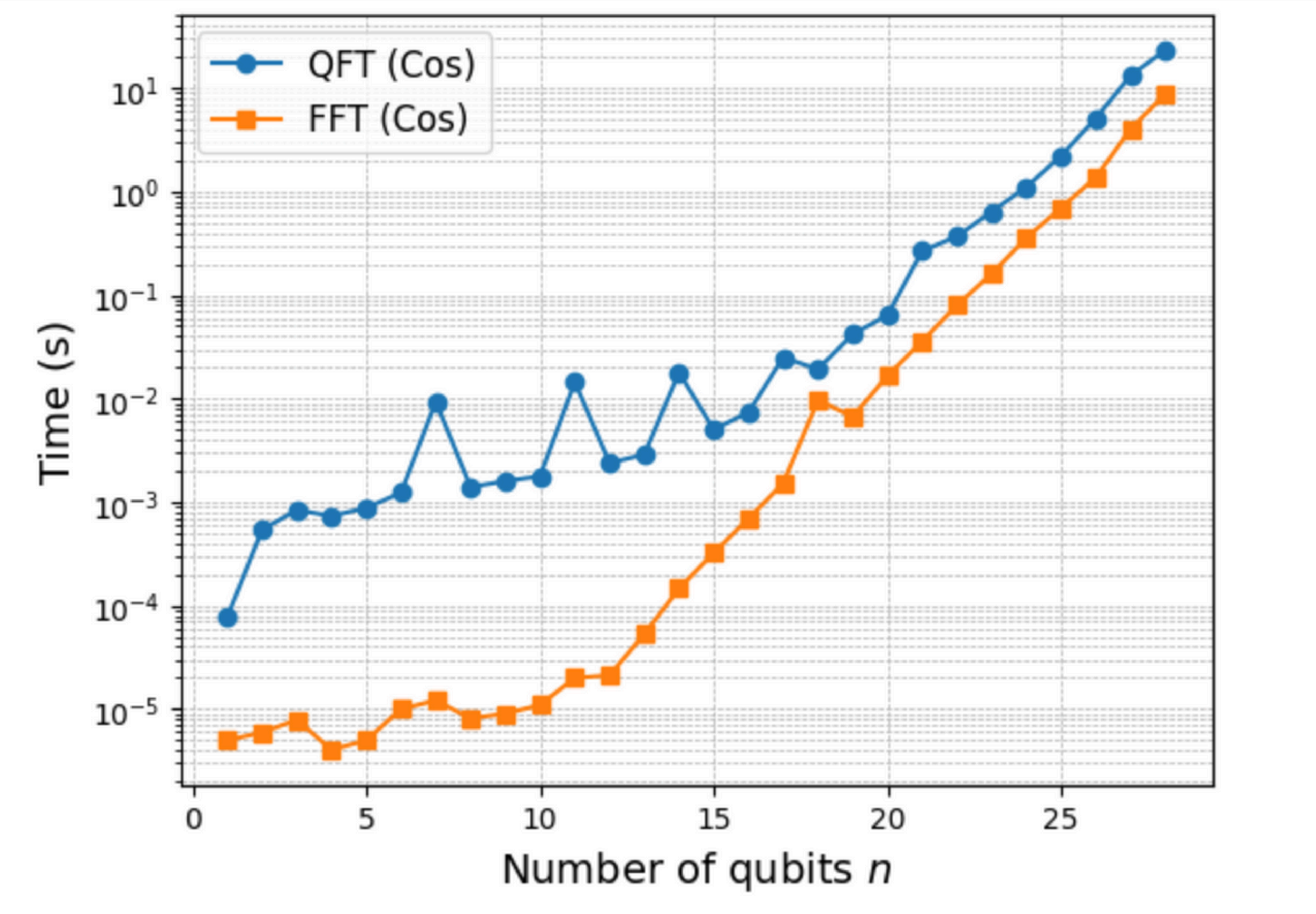
GRAPH



Time to convert the array to MPS is included in this graph

Cutoff Value set : 10^(-12)

GRAPH

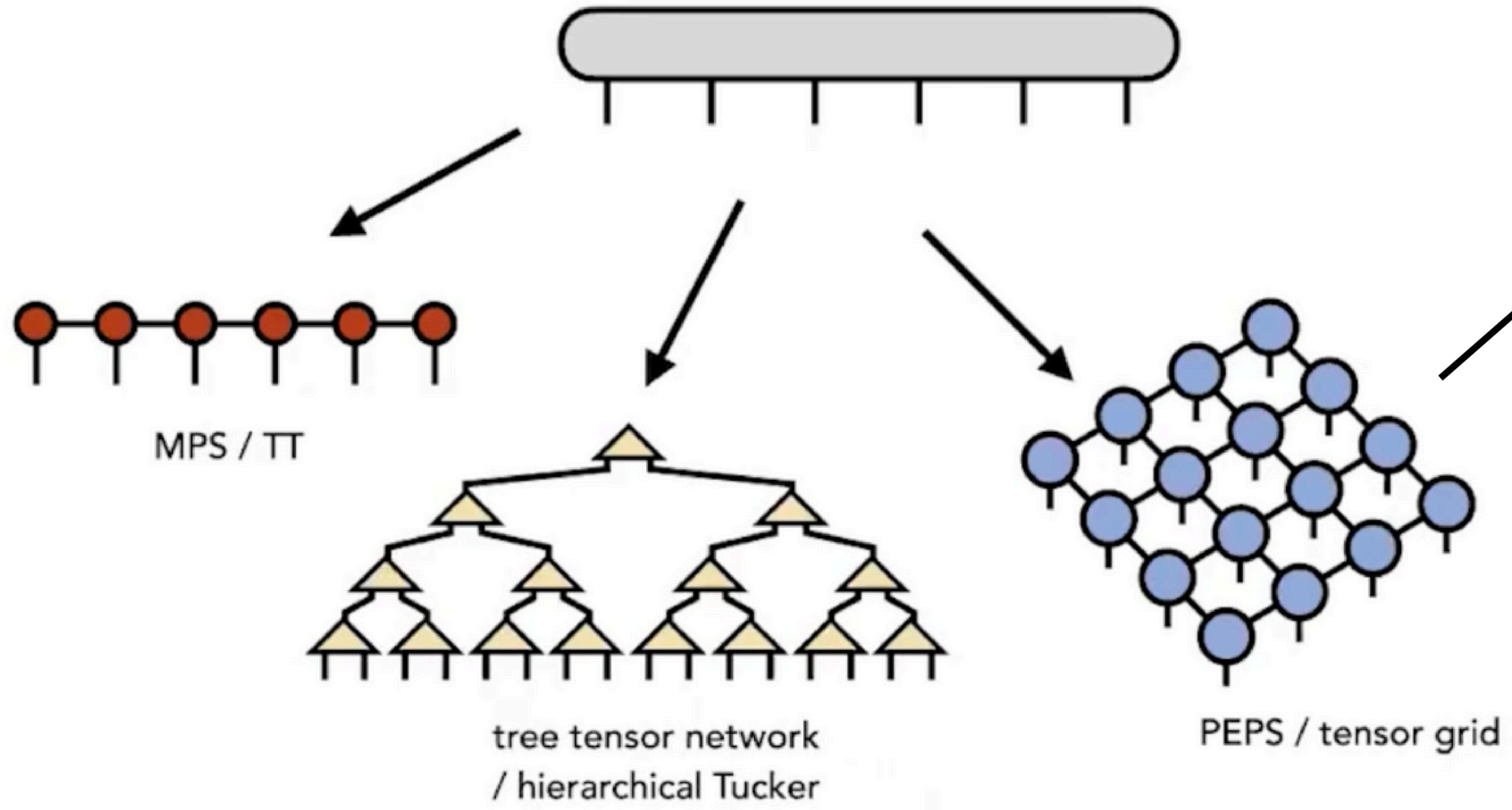


Time to convert the array to MPS is included in this graph

Cutoff Value set : 10^(-18)

Given that QFT has low entanglement, are there alternative classical compression techniques (besides MPS) that could achieve similar efficiency with lower computational overhead?

Alternate Techniques



Tensor grid can represent the entanglement structure efficiently, but at the cost of heavier contraction algorithms. It required good initialization

Currently MPS might be the best possible way of simulating quantum algorithms

The optimal tree structure is not always obvious, and efficient algorithms for conversion from a dense array to a TTN are an active area of research

Can QFT-based simulations be extended to other quantum algorithms beyond Fourier transforms?

YES

(Bonus if the quantum algorithm has low entanglement)

How feasible is implementing a hybrid approach where classical FFT is combined with quantum-inspired MPS techniques?

It is very feasible but there are a few limitations

- Data conversion to MPS and back to an array is taking time
- Come up with an algorithm that works on data in MPS format from beginning to the end
- Find a faster way to convert data to MPS

Advance mathematical techniques for quantum many-body systems

Thank You