Womanium 2025 - Qubiteers

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I. Introduction

In this work, we try to implement the quantum code for the Galton Board. Following the requirements of the assignment, we try to implement the circuits for Binomial, Exponential and hadamard quantum walk distributions. We do this by changing the probability distribution for each peg on the board by considering different quantum gates.

The circuit proposed for a Quantum Galton Board (QGB) [1] with one peg can be seen in Fig 1. Here, the qubits $q1$, $q2$ and $q3$ represent the positions of the ball which starts at the qubit $q2$. This is represented by the Pauli-X gate which is applied on $q2$. The Hadamard gate on the $q0$ wire acts as the control gate put in equal superposition for the ball to go either in the right or the left direction. $q1$ and $q3$ represent these two directions. This qubit-to-direction mapping is done by the Controlled-SWAP and Controlled-NOT gates on these 4 qubit wires.

We modify this circuit using different gates and controlled-gate configurations to obtain different distributions such as the Exponential Distirbution and the Hadamard Quantum-Walk Distribution.

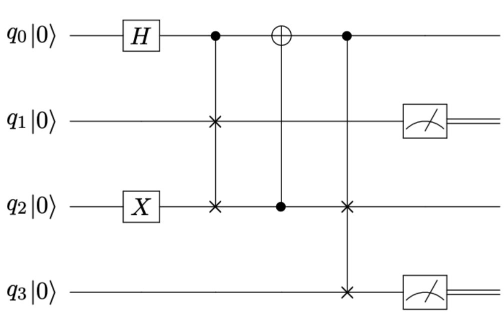

Fig. 1. Circuit for QGB with one peg. *q0* is the control qubit and *q2* represents the starting position of an active ball.

II. Results

A. Without Noise

We implement the proposed circuit for the binomial distribution first. The result can be seen in Fig 3. The results of this distribution when run as a simulation, are ideal. The mean of this distribution is 5.9995, ideal being 6.0000 and the variance being 2.9884, with the ideal value being 3.0000.

We also implement the circuits for the Exponential and hadamard quantum walk distributions. The results can be seen in Figures **??** and 9.

For the exponential distribution, the achieved mean is 2.2440, with the ideal value being 2.2474. The achieved variance for this distribution is 6.2462 with the ideal value being 6.2562.

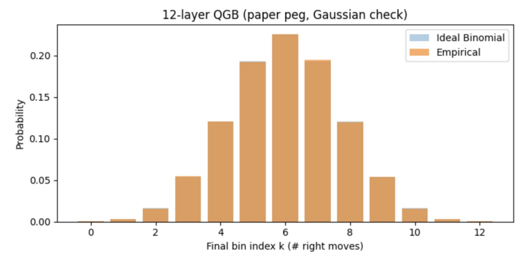

Fig. 2. Binomial Distribution - Probability distribution for a 12-layer quantum Gaussian ball (QGB) simulation without noise, showing close agreement between the ideal binomial distribution and the simulated results.

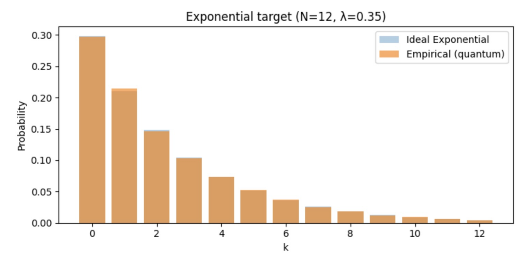

Fig. 3. Exponential Distribution - Probability distribution for a quantum circuit targeting an exponential distribution under ideal (non-noisy) simulation, showing near-perfect alignment with the theoretical exponential curve.

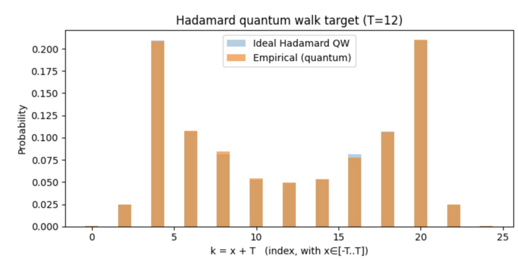

Fig. 4. Hadamard Quantum Walk - Probability distribution for a Hadamard quantum walk under ideal conditions, demonstrating the distinctive interference pattern characteristic of quantum walks.

For the hadamard quantum walk distribution, the achieved mean is 11.9756, with the ideal value being 12.0000. The achieved variance for this distribution is 42.5002 with the ideal value being 42.5859.

The detailed code and the circuit can be found in the **Quantum Galton box (no noise).ipynb** code file in the GitHub repository.

B. *With Noise*

We also simulate these circuits with a noisy model. We do simulations on the noisy model for each of the distributions, with the original and the optimized circuits. The distributions obtained for the binomial distribution circuits for the original and optimized circuits can be seen in Figures 5 and 6. For the exponential distribution, the outputs can be seen in Figures 7 and 8. For the hadamard quantum walk distribution, the outputs can be seen in Figures 9 and 10. The cumulated results can be seen in Table I. The detailed code and the circuit can be found in the **QGB (noise).ipynb** code file in the GitHub repository.

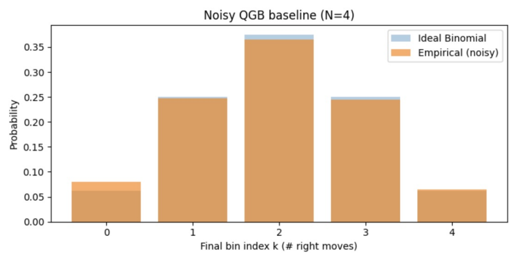

Fig. 5. Noisy QGB baseline – Probability distribution of final bin index for noisy QGB, compared with the ideal binomial distribution, showing deviation due to noise.

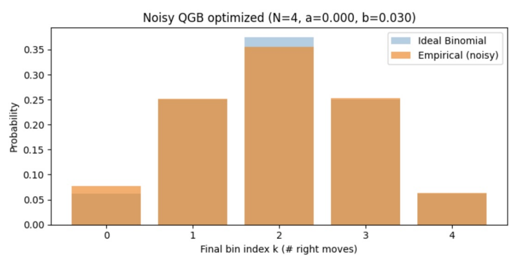

Fig. 6. Noisy QGB optimized – Optimized noisy QGB distribution achieving slightly reduced TVD compared to baseline.

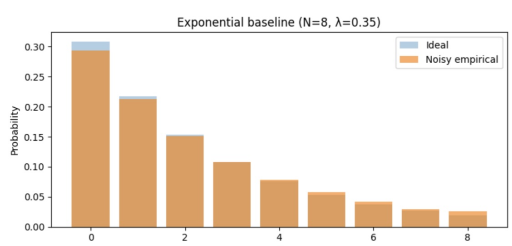

Fig. 7. Exponential baseline – Noisy empirical exponential distribution vs. ideal, highlighting mismatch caused by hardware noise.

References

[1] Carney, Mark, and Ben Varcoe. "Universal Statistical Simulator." arXiv preprint arXiv:2202.01735 (2022).

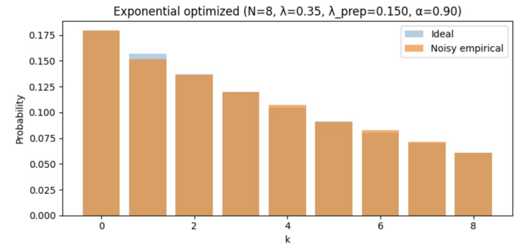

Fig. 8. Exponential optimized – Optimized exponential distribution with significantly lower TVD than baseline.

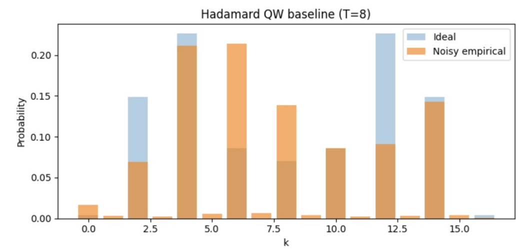

Fig. 9. Hadamard QW baseline – Noisy quantum walk distribution showing large deviation from the ideal due to interference effects.

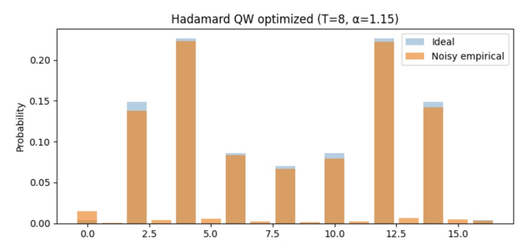

Fig. 10. Hadamard QW optimized – Optimized quantum walk distribution with improved alignment to the ideal and lower TVD.

TABLE I
SUMMARY OF OBTAINED DISTRIBUTIONS FOR BASELINE AND OPTIMIZED CASES.

Case	Mean (ideal)	Var (ideal)
Noisy QGB baseline	1.9687 (2.0000)	1.0673 (1.0000)
Noisy QGB optimized	1.9744 (2.0000)	1.0646 (1.0000)
Exponential baseline	2.1126 (1.9833)	4.6715 (4.2917)
Exponential optimized	3.1476 (3.1217)	6.2120 (6.1963)
Hadamard QW baseline	7.5736 (8.0000)	15.1166 (19.1250)
Hadamard QW optimized	7.9403 (8.0000)	19.5743 (19.1250)