# Bits, Bytes and Words

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Based on slides by Randal E. Bryant and David R. O'Hallaron

# Agenda

Representing information as bits

Bit-level manipulation

Integers
- Representation: unsigned and signed
- Conversion, casting
- Expanding, truncating

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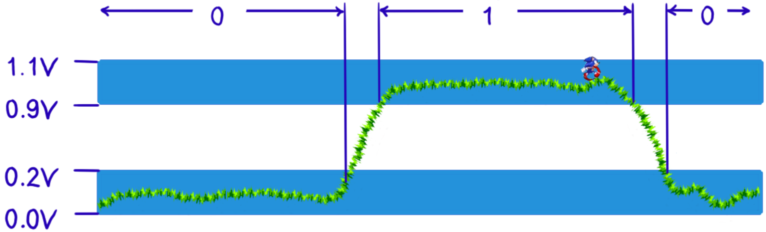
# Everything is bits

- Each bit is 0 or 1
- By interpreting sets of bits in various ways...
  - ...computers determine what to do.
  - ...represent and manipulate numbers, sets, strings—*data*.

**Why bits? Why not decimals? Could it have been some some other way?**

# Everything is bits

- **Why bits? Electronic implementation.**
  - Easy to store with bistable elements.
  - Reliably transmitted on noisy and inaccurate wires via *error correction*.




- **... But there exist models that do not use bits.**
  - The Setun computer developed in the Soviet Union used ternary *trits*.
  - Quantum computers use *qubits* that are in a superposition of the two states.
    - ...error correction is the main challenge here.

# Everything is bit vectors

## A sequence of bits is called a *bit vector*

$$\langle x_{w-1}, \ldots, x_0 \rangle$$

- Number bits from 0 to $w - 1$.
- Bit $x_0$ typically called *least significant* and $x_{w-1}$ *most significant*.
  - Due to how bit vectors can be interpreted as binary numbers.
- **Bit vectors are not numbers.**
  - Can represent many kinds of objects.
  - ...but we will mostly focus on number representations.

# Binary numbers

- **Base 2 numbers.**
  - Represent $15213_{10}$ as $11101101101101_2$
    - $\langle 0011\ 1011\ 0110\ 1101 \rangle$ (with $w = 16$)
  - Represent $\frac{15_{10}}{213_{10}}$ as $\frac{1111_2}{11010101_2}$
    - $\langle 0000\ 0000\ 0000\ 1111\ 0000\ 0000\ 1101\ 0101 \rangle$ ($w = 32$)
    - 16 bits for each of numerator and denominator.
    - (This is not how we actually represent rational numbers in a computer−we'll see how next week.)
- **Machine numbers are of some finite size.**
  - If we use $w$ bits to represent a number, only $2^w$ distinct values are possible.
  - How we interpret those bits can vary.
  - **Why do we use finite-sized numbers?**
    - A "$w$-bit machine" handles numbers of up to $w$ bits "natively" (meaning fast).
    - A bit vector of some natively supported size is called a *word*.

# Encoding byte values

**Byte = 8 bit word**

- (Machine-specific, but is true for all mainstream machines.)
- 256 different values.
- Binary $00000000_2$ to $11111111_2$.
- Decimal $0_{10}$ to $255_{10}$.
- Hexadecimal $00_{16}$ to $FF_{16}$.
  - Base 16 number representation.
  - Uses characters 0—9 and A—F.
  - In C we write $FA1D37B_{16}$ as
    - `0xFA1D37B`
    - `0xfa1d37b` (case does not matter)

| Hex | Dec | Bits |
|---|---|---|
| 0 | 0 | $\langle 0000 \rangle$ |
| 1 | 1 | $\langle 0001 \rangle$ |
| 2 | 2 | $\langle 0010 \rangle$ |
| 3 | 3 | $\langle 0011 \rangle$ |
| 4 | 4 | $\langle 0100 \rangle$ |
| 5 | 5 | $\langle 0101 \rangle$ |
| 6 | 6 | $\langle 0110 \rangle$ |
| 7 | 7 | $\langle 0111 \rangle$ |
| 8 | 8 | $\langle 1000 \rangle$ |
| 9 | 9 | $\langle 1001 \rangle$ |
| A | 10 | $\langle 1010 \rangle$ |
| B | 11 | $\langle 1011 \rangle$ |
| C | 12 | $\langle 1100 \rangle$ |
| D | 13 | $\langle 1101 \rangle$ |
| E | 14 | $\langle 1110 \rangle$ |
| F | 15 | $\langle 1111 \rangle$ |

# Let’s play a game

`http://topps.diku.dk/compsys/integers.html`

# Example data representations

| C data type | Typical 16-bit | Typical 32-bit | Typical 64-bit | x86-64 |
|---|---|---|---|---|
| `char` | 1 | 1 | 1 | 1 |
| `short` | 1 | 2 | 2 | 2 |
| `int` | 2 | 4 | 4 | 4 |
| `long` | 4 | 4 | 8 | 8 |
| `int32_t` | 4 | 4 | 4 | 4 |
| `int64_t` | 8 | 8 | 8 | 8 |
| `float` | 4 | 4 | 4 | 4 |
| `double` | 8 | 8 | 8 | 8 |
| `pointer` | 2 | 4 | 8 | 8 |

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Bit-level manipulation

Integers

- Representation: unsigned and signed
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# Boolean algebra

**Developed by George Boole in 19th century**

- Algebraic representation of logic (“truth values”).
- Encode *true* as 1 and *false* as 0.

**And**

| $\wedge$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

**Or**

| $\vee$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 1 |

**Not**

| $\neg$ | |
|---|---|
| 0 | 1 |
| 1 | 0 |

**Exclusive-or**

| $\oplus$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

- These operations can be implemented with tiny electronic *gates*.

# Boolean algebra on bit vectors

- The truth tables generalise to *bit vectors*, applied elementwise.

$$\begin{array}{cc} & \langle 01101001\rangle \\ \wedge & \langle 01010101\rangle \\ \hline & \langle 01000001\rangle \end{array} \quad \begin{array}{cc} & \langle 01101001\rangle \\ \vee & \langle 01010101\rangle \\ \hline & \langle 01111101\rangle \end{array} \quad \begin{array}{cc} & \langle 01101001\rangle \\ \oplus & \langle 01010101\rangle \\ \hline & \langle 00111100\rangle \end{array} \quad \begin{array}{cc} \neg & \langle 01101001\rangle \\ \hline & \langle 10010110\rangle \end{array}$$

- This is the form they take when available in programming languages such as C.
- ...although C uses different symbols.

# Bit-level operations in C

**Operators `&` ($\wedge$), `|` ($\vee$), `^` ($\oplus$), `~` ($\neg$) available in C.**

- Apply to any integral type.
  - E.g. `long`, `int`, `short`, `char`...
- Interpret operands as bit vectors.
- Applied element-wise.

**Examples**

- `~0x41 = 0xBE`
  - $\neg\langle 01000001\rangle = \langle 10111110\rangle$
- `~0x00 = 0xFF`
  - $\langle 00000000\rangle = \langle 11111111\rangle$
- `0x69 & 0x55 = 0x41`
  - $\langle 01101001\rangle \wedge \langle 01010101\rangle = \langle 01000001\rangle$
- `0x69 & 0x55 = 0x7D`
  - $\langle 01101001\rangle \wedge \langle 01010101\rangle = \langle 01111101\rangle$

# Contrast: logical operators in C

The logical operators interpret numbers as *single boolean values*, not as bit vectors!

- **`&&`, `||`, `!`**
  - View 0 as false.
  - Anything nonzero as true.
  - Always produce 0 or 1.
  - **Early termination:** `1 || (0/0)` is safe.
- **Examples**
  - `!0x41 = 0x00`
  - `!0x00 = 0x01`
  - `!!0x41 = 0x01`
  - `0x69 && 0x55 = 0x01`
  - `0x69 || 0x55 = 0x01`
- **Do not confuse the logical and bitwise operators!**

# Shift operations

- **Left shift `x << y`**
  - Shift bit-vector `x` left by `y` positions.
    - Throws away excess bits on the left.
    - Fills with zeroes on right.
- **Right shift `x >> y`**
  - Shift bit-vector `x` right by `y` positions.
    - Throws away excess bits on the left.
  - Logical shift: Fill with 0s on left.
  - Arithmetic shift: Replicate most significant bit on left.
- **Undefined behaviour in C**
  - Shifting a negative amount or by the vector size or more.

| $x$ | $\langle 01100010 \rangle$ |
|---|---|
| $x << 3$ | $\langle 00010000 \rangle$ |
| $x >> 2$ | $\langle 00011000 \rangle$ |
| $x >>^{a} 2$ | $\langle 00011000 \rangle$ |

| $x$ | $\langle 10100010 \rangle$ |
|---|---|
| $x << 3$ | $\langle 00010000 \rangle$ |
| $x >> 2$ | $\langle 00101000 \rangle$ |
| $x >>^{a} 2$ | $\langle 11101000 \rangle$ |

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# Encoding integers

Suppose $x_i$ is the $i$th bit of a $w$-bit word (with $x_0$ being the least significant bit).

**Unsigned**

$$\mathrm{Bits2N}(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

**Two's complement (AKA *signed*)**

$$\mathrm{TC2Int}(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

```
int16_t x =  15213;
int16_t y = -15213;
```

| | Decimal | Hex | Bits |
|---|---|---|---|
| x | 15213 | 3 B 5 D | $\langle$0011 1011 0110 1101$\rangle$ |
| y | -15213 | C 4 9 3 | $\langle$1100 0100 1001 0011$\rangle$ |

**Sign bit**

- For 2's complement, most significant bit ($x_{w-1}$) indicates sign.
  - 0 for non-negative.
  - 1 for negative.

# Two’s complement encoding example

```
int16_t x =  15213; // 0011 1011 0110 1101
int16_t y = -15213; // 1100 0100 1001 0011
```

| Weight | 15213 | | -15213 | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 0 | 0 | 1 | 2 |
| 4 | 1 | 4 | 0 | 0 |
| 8 | 1 | 8 | 0 | 0 |
| 16 | 0 | 0 | 1 | 16 |
| 32 | 1 | 32 | 0 | 0 |
| 64 | 1 | 64 | 0 | 0 |
| 128 | 0 | 0 | 1 | 128 |
| 256 | 1 | 256 | 0 | 0 |
| 512 | 1 | 512 | 0 | 0 |
| 1024 | 0 | 0 | 1 | 1024 |
| 2048 | 1 | 2047 | 0 | 0 |
| 4096 | 1 | 4096 | 0 | 0 |
| 8192 | 1 | 8192 | 0 | 0 |
| 16384 | 0 | 0 | 1 | 16384 |
| -32768 | 0 | 0 | 1 | -32768 |
| **Sum** | | **15213** | | **-15213** |

# Numeric ranges

**Unsigned**

$$\begin{aligned} \text{UMin} &= \quad 0 = 0\ldots0_2 \\ \text{UMax} &= \quad 2^w - 1 = 1\ldots1_2 \end{aligned}$$

**Two's complement**

$$\begin{aligned} \text{SMin} &= \quad -2^{w-1} = 10\ldots0_2 \\ \text{SMax} &= \quad 2^{w-1} - 1 = 01\ldots1_2 \\ -1 & \quad\quad = 1\ldots1_2 \end{aligned}$$

**Values for** $w = 16$**:**

| | Decimal | Hex | Bits |
|---|---|---|---|
| UMax | 65535 | F F F F | ⟨1111 1111 1111 1111⟩ |
| SMax | 32767 | 7 F F F | ⟨0111 1111 1111 1111⟩ |
| SMin | -32768 | 8 0 0 0 | ⟨1000 0000 0000 0000⟩ |
| -1 | -1 | F F F F | ⟨1111 1111 1111 1111⟩ |
| 0 | 0 | 0 0 0 0 | ⟨0000 0000 0000 0000⟩ |

# Values for different word sizes

| | w | | | |
|---|---|---|---|---|
| | **8** | **16** | **32** | **64** |
| **UMax** | 255 | 65,535 | 4,294,967,295 | 18,446,744,073,709,551,615 |
| **SMax** | 127 | 32,767 | 2,147,483,647 | 9,223,372,036,854,775,807 |
| **SMin** | -128 | -32,768 | -2,147,483,648 | -9,223,372,036,854,775,808 |

**Observations**

$$|\text{SMin}| = \text{SMax} + 1$$
$$|\text{UMax}| = 2 \cdot \text{SMax} + 1$$

**Note the assymetric range.**

**C Programming**

- `#include <limits.h>`
- Declares constants, e.g:
  - `ULONG_MAX`
  - `LONG_MAX`
  - `LONG_MIN`
- Values are platform-specific.

# Unsigned and signed numeric values (here $w = 4$)

| $x$ | $\text{Bits2N}(x)$ | $\text{TC2Int}(x)$ |
|---|---|---|
| $\langle 0000 \rangle$ | 0 | 0 |
| $\langle 0001 \rangle$ | 1 | 1 |
| $\langle 0010 \rangle$ | 2 | 2 |
| $\langle 0011 \rangle$ | 3 | 3 |
| $\langle 0100 \rangle$ | 4 | 4 |
| $\langle 0101 \rangle$ | 5 | 5 |
| $\langle 0110 \rangle$ | 6 | 6 |
| $\langle 0111 \rangle$ | 7 | 7 |
| $\langle 1000 \rangle$ | 8 | -8 |
| $\langle 1001 \rangle$ | 9 | -7 |
| $\langle 1010 \rangle$ | 10 | -6 |
| $\langle 1011 \rangle$ | 11 | -5 |
| $\langle 1100 \rangle$ | 12 | -4 |
| $\langle 1101 \rangle$ | 13 | -3 |
| $\langle 1110 \rangle$ | 14 | -2 |
| $\langle 1111 \rangle$ | 15 | -1 |

- **Equivalence**
  - Same encoding for non-negative values.
- **Uniqueness**
  - Every bit vector represents distinct integer value.
  - Each representable integer has unique bit encoding.
  - The representation is **bijective**.
- **Can invert mappings**
  - $\text{N2Bits}(x) = \text{Bits2N}^{-1}(x)$
    - Bit vector for unsigned integer in range.
  - $\text{Int2TC}(x) = \text{TC2Int}^{-1}(x)$
    - Bit vector for Two's Complement integer in range.

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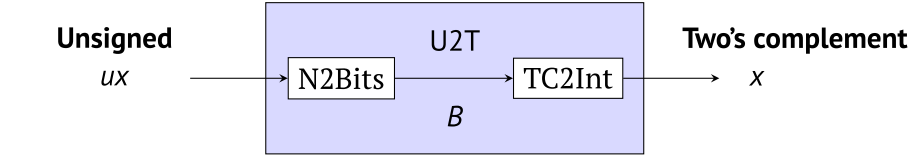
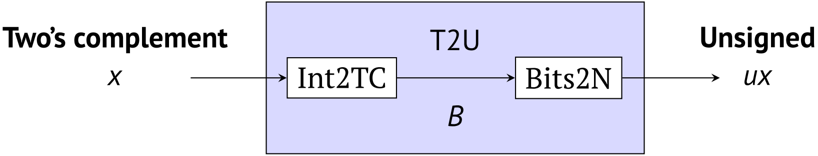
# Mapping between signed and unsigned




Maintain same bit pattern




Mapping between unsigned and two's complement numbers:

**Keep bit representations and reinterpret.**

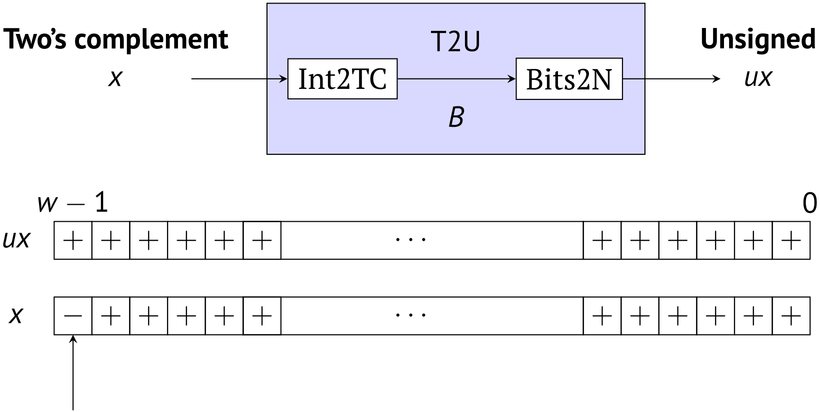
# Mapping signed $\Leftrightarrow$ unsigned

| Bits | Signed | Unsigned |
|---|---|---|
| $\langle 0000 \rangle$ | 0 | 0 |
| $\langle 0001 \rangle$ | 1 | 1 |
| $\langle 0010 \rangle$ | 2 | 2 |
| $\langle 0011 \rangle$ | 3 | 3 |
| $\langle 0100 \rangle$ | 4 | 4 |
| $\langle 0101 \rangle$ | 5 | 5 |
| $\langle 0110 \rangle$ | 6 | 6 |
| $\langle 0111 \rangle$ | 7 | 7 |
| $\langle 1000 \rangle$ | -8 | 8 |
| $\langle 1001 \rangle$ | -7 | 9 |
| $\langle 1010 \rangle$ | -6 | 10 |
| $\langle 1011 \rangle$ | -5 | 11 |
| $\langle 1100 \rangle$ | -4 | 12 |
| $\langle 1101 \rangle$ | -3 | 13 |
| $\langle 1110 \rangle$ | -2 | 14 |
| $\langle 1111 \rangle$ | -1 | 15 |

$\longrightarrow$ $\boxed{\text{T2U}}$ $\longrightarrow$

$\longleftarrow$ $\boxed{\text{U2T}}$ $\longleftarrow$

# Relation between signed and unsigned

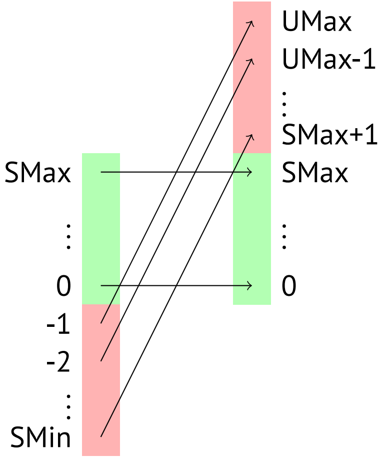



**Large negative weight** becomes **large positive weight.**

# Conversion (that is, *reinterpretation*) visualized

**Two's complement to unsigned**

- Ordering inversion.
- Negative numbers become large positive numbers.



# Signed versus unsigned in C

**C makes working with this more error-prone than it should be.**

**Types**
- Signedness part of type: `unsigned int`, `int32_t`, `uint32_t`.

**Constants**
- By default are considered signed integers.
- Unsigned with `U` suffix: `0U, 4294967259U`

**Casting**
- Explicit casting between signed and unsigned:

```
int tx, ty;
unsigned int ux, uy;
tx = (int) ux;
uy = (unsigned int) ty;
```

- Implicit casting due to assignments and other expressions:

```
tx = ux;
uy = ty;
```

# Casting surprises

**Evaluation**

- If there is a mix of unsigned and signed in single expression, *signed values implicitly cast to unsigned*.
- Including comparison operations $<$, $>$, $==$, $<=$, $>=$.
- Examples for $w = 32$: $SMin = -2,147,483,648$, $SMax = 2,147,483,647$:

| Const LHS | Relation | Const RHS | Evaluation |
|---|---|---|---|
| 0 | $==$ | 0U | |

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| Const LHS | Relation | Const RHS | Evaluation |
|---|---|---|---|
| 0 | $==$ | 0U | unsigned |
| -1 | $<$ | 0 | |

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| 2147483647 | $>$ | -2147483647-1 | |

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|---|---|---|---|
| 0 | $==$ | 0U | unsigned |
| -1 | $<$ | 0 | signed |
| -1 | $>$ | 0U | unsigned |
| 2147483647 | $>$ | -2147483647-1 | signed |
| 2147483647U | $<$ | -2147483647-1 | |

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| -1 | < | 0 | signed |
| -1 | > | 0U | unsigned |
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| -1 | < | 0 | signed |
| -1 | > | 0U | unsigned |
| 2147483647 | > | -2147483647-1 | signed |
| 2147483647U | < | -2147483647-1 | unsigned |
| -1 | > | -2 | |

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| -1 | $>$ | -2 | signed |
| (unsigned int)-1 | $>$ | $-2$ | |

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| 2147483647 | $>$ | -2147483647-1 | signed |
| 2147483647U | $<$ | -2147483647-1 | unsigned |
| -1 | $>$ | -2 | signed |
| (unsigned int)-1 | $>$ | $-2$ | unsigned |
| 2147483647 | $<$ | 2147483648U | |

# Casting surprises

**Evaluation**

- If there is a mix of unsigned and signed in single expression, *signed values implicitly cast to unsigned*.
- Including comparison operations $<, >, ==, <=, >=$.
- Examples for $w = 32$: $SMin = -2,147,483,648$, $SMax = 2,147,483,647$:

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- Bit representation is maintained.
- ...but reinterpreted.
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- Expression containing signed and unsigned int:
  - `int` is cast to `unsigned int`!
  - **When can this go bad?**

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**Advice:** Avoid arithmetic on unsigned types—only use them for bit operations.

**But:** Some C operators (`sizeof`) and many functions return unsigned types (e.g. `size_t`). C is always ready to stab you in the back.

Representing information as bits

Bit-level manipulation

Integers

- Representation: unsigned and signed
- Conversion, casting
- Expanding, truncating

# Truncation

**Task**
- Given $k + w$-bit signed integer $x$.
- Convert it to $w$-bit integer $x'$ with same value i possible.

**Approach**
- Remove the $k$ most significant bits.
- Equivalent to computing $x' = x \bmod 2^w$.
- Numerical change if number has no representation in $w$ bits.
- Otherwise safe.

| $w$ | $x$ | $TC2Int(x)$ |
|---|---|---|
| 8 | $\langle 11111111 \rangle$ | $-1$ |
| 4 | $\langle 1111 \rangle$ | $-1$ |
| 8 | $\langle 10000000 \rangle$ | $-128$ |
| 4 | $\langle 0000 \rangle$ | $0$ |

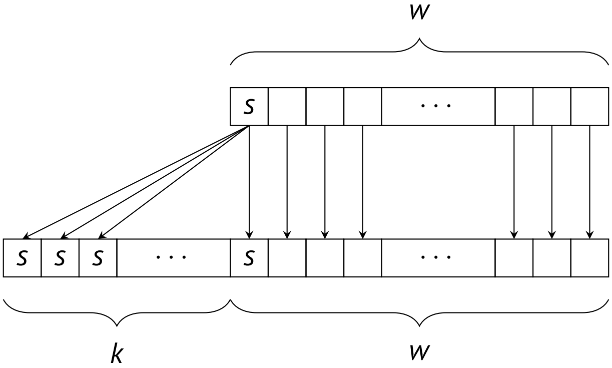
# Sign extension

**Task**

- Given $w$-bit signed integer $x$.
- Convert it to $w + k$-bit integer $x'$ with same value.

**Approach**

- Make $k$ copies of sign bit (most significant bit):
- $x' = \langle \underbrace{x_{w-1}, \ldots, x_{w-1}}_{k \text{ copies of sign bit.}} x_{w-1} \cdots x_0 \rangle$



# Sign extension example

```
short int x = 15213;
int       ix = (int) x;
short int y = -15213;
int       iy = (int) y;
```

| | Decimal | Hex | Bits |
|---|---|---|---|
| x | $15213_{10}$ | 3B 6D | $\langle$0011 1011 0110 1101$\rangle$ |
| ix | $15213_{10}$ | 00 00 3B 6D | $\langle$0000 0000 0000 0000 0011 1011 0110 1101$\rangle$ |
| y | $-15213_{10}$ | C4 93 | $\langle$1100 0100 1001 0011$\rangle$ |
| iy | $-15213_{10}$ | FF FF C4 93 | $\langle$1111 1111 1111 1111 1100 0100 1001 0011$\rangle$ |

# Summary: basic rules for expanding and truncating

**Expanding (e.g. `short` to `int`)**

- Unsigned: zeros added.
- Signed: sign extension.
- Both yield expected result.

**Truncating (e.g. `unsigned int` to `unsigned short`)**

- Bits are truncated.
- Result reinterpreted.
- Unsigned: modulo operation.
- Signed: similar to a modulo operation.
- For small numbers yield expected behaviour.