April 18, 2022 (Due: 08:00 April 25, 2022)

1. Show that the n-point Gauss–Chebyshev quadrature rule reads

$$\int_{-1}^{1} \frac{f(x)}{\sqrt{1-x^2}}\,\mathrm{d}x \approx \frac{\pi}{n}\sum_{k=1}^{n} f\Bigl(\cos\frac{(2k-1)\pi}{2n}\Bigr).$$

2. Use a cubic spline function $s(x)$ to approximate $f(x) = \mathrm{e}^x + \ln x$ over $[1, 4]$. Plot the first and second derivatives, as well as the errors. You are encouraged to try different step sizes and observe the behavior of the error with respect to the step size.

3. Show that the DFT matrix F_n diagonalizes

$$J_n = \begin{bmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ & & & 0 & 1 \\ 1 & & & & 0 \end{bmatrix},$$

i.e., $F_n^{-1} J_n F_n$ is diagonal.

4. Plot the discrete Fourier transform of 1024 equispaced sampling points of the function $f(x) = \sin(3x)$ over $[0, 2\pi)$. Where are the peaks? Can you explain what you have observed?

Repeat the experiment for $f(x) = \sin x + \sin(\sqrt[12]{128}\,x)$ over $[0, 12\pi)$ and explain your observation.

5. (optional) You are provided with the data regarding the ongoing COVID outbreak in Shanghai. Use the techniques you have learned from the lectures to model the data. You are encouraged to establish various models for different time periods. Explain what you have observed.

The data can be downloaded from the elearning system. Read some news if you have trouble understanding the data. You are also encouraged to collect more detailed data.