March 21, 2022 (Due: 08:00 March 28, 2022)

1. When performing interpolation with a complete cubic spline, the choice of derivatives on the boundary is important. Suppose that Bob wants to interpolate the sine function $f(x) = \sin x$ at nine equispaced nodes over $[0, 2\pi]$, with $f'(0) = f'(2\pi) = 1$. Unfortunately, he made a typo on $f'(0)$ in his program and observed some strange results. Try to reproduce Bob's result with a few different values of $f'(0)$. For instance, $f'(0) = 0$, $f'(0) = -1$, etc.

2. The temperature in the human body is not a constant, but rather follows a daily rhythm driven by an internal biological clock. The following table lists 20 averaged values of temperature measurements taken from 70 English sailors in an experiment done in 1971.

Temperature in human body

Time (hour) t	Temperature (°C) $T(t)$
1	36.37
3	36.23
5	36.21
7	36.26
8	36.38
9	36.49
10	36.60
11	36.63
12	36.66
13	36.68
14	36.69
15	36.73
16	36.74
17	36.78
18	36.82
19	36.84
20	36.87
21	36.86
22	36.77
23	36.59

Interpolate the data with a periodic cubic spline and plot your solution for a two-day-period.

3. Interpolate the following data set and visualize your solution on $[-1, 1] \times [-1, 1]$.

x_i	y_i	z_i
−1.0000	−1.0000	1.6389
−1.0000	1.0000	0.5403
1.0000	−1.0000	−0.9900
1.0000	1.0000	0.1086
−0.7313	0.6949	0.9573
0.5275	−0.4899	0.8270
−0.0091	−0.1010	1.6936
0.3031	0.5774	1.3670

You are free to choose any 2D interpolation strategy learned from the lecture. (If Delaunay triangularization is used, you can make use of the MATLAB/Octave function `delaunay`.) Note that different interpolation strategies will lead to different results.

4. Suppose that there is a bimodal function of the form

$$y = \alpha_1 g\big(\beta_1(x - \gamma_1)\big) + \alpha_2 g\big(\beta_2(x - \gamma_2)\big),$$

where $g(x) = \exp(-x^2)$. Can you find out the parameters from the following *noisy* data set sampled from this model?

x_i	y_i
−4.00000	0.00001
−3.50000	0.00726
−3.00000	0.25811
−2.50000	1.87629
−2.00000	1.55654
−1.50000	0.17209
−1.00000	0.00899
−0.50000	0.05511
0.00000	0.24564
0.50000	0.60455
1.00000	0.89370
1.50000	1.03315
2.00000	0.51633
2.50000	0.18032
3.00000	0.04287
3.50000	0.00360
4.00000	0.00045

5. Shanghai is currently at a difficult period. The number of positive cases (including asymptomatic cases) in the COVID-19 testing is growing rapidly as shown in the following table.

Day t	#{Positive Cases} N
1	1 + 1
2	3 + 5
3	2 + 14
4	3 + 16
5	0 + 28
6	3 + 45
7	4 + 51
8	3 + 62
9	4 + 76
10	11 + 64
11	5 + 78
12	1 + 64
13	41 + 128
14	9 + 130
15	5 + 197
16	8 + 150
17	57 + 203
18	8 + 366
19	17 + 492
20	24 + 734

You are asked to approximate the data using a model of the form $N = \alpha \cdot \beta^t$. Try to find the parameters (α, β)
(1) by fitting a linearized model;
(2) by directly fitting the nonlinear model.
Visualize your results.