Mathematical Optimization Model for a SCRUM-Based Software Development Company

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Introduction

This document formalizes the optimization problem for resource allocation, scheduling, and process improvement in a SCRUM-based software development environment. The model is derived from a defined domain model consisting of Entities, Relationships, Goals, and Constraints.

1 Sets (Entities)

- Project = $\{p_1, p_2, ..., p_n\}$: The set of all projects.
- Team = $\{t_1, t_2, ..., t_n\}$: The set of all teams.
- Worker = $\{w_1, w_2, ..., w_n\}$: The set of all workers/employees.
- Feature = $\{f_1, f_2, ..., f_n\}$: The set of all features.
- Skill = $\{s_1, s_2, ..., s_n\}$: The set of all skills.
- Role = $\{r_1, r_2, ..., r_n\}$: The set of all roles.
- ProductBacklog = $\{pb_1, pb_2, ..., pb_n\}$: The set of all product backlogs.
- Sprint = $\{sp_1, sp_2, ..., sp_n\}$: The set of all sprints.
- SprintBacklog = $\{sbl_1, sbl_2, ..., sbl_n\}$: The set of all sprint backlogs.
- UserStory = $\{us_1, us_2, ..., us_n\}$: The set of all user stories.
- Task = $\{tsk_1, tsk_2, ..., tsk_n\}$: The set of all tasks.
- Blocker = $\{bl_1, bl_2, ..., bl_n\}$: The set of all blockers.
- DevelopmentSnapshot = $\{dev_1, dev_2, ..., dev_n\}$: The set of all development snapshots.

2 Indices

- $p, q \in$ Project
- $t, u \in$ Team
- $w, v \in$ Worker
- $f, g \in$ Feature

- $s \in$ Skill
- $r \in$ Role
- $sp \in$ Sprint
- $sbl \in$ SprintBacklog
- $us \in$ UserStory
- $tsk \in$ Task
- $bl \in$ Blocker
- $dev \in$ DevelopmentSnapshot

3 Goals

- **G0: maximize_team_velocity** - Maximize the average velocity of the team.

$$\text{Maximize} \sum_{t \in \text{Team}} \text{velocity}(t)$$

- **G1: minimize_blocker_severity** - Minimize the impact of blockers.

$$\text{Minimize} \sum_{bl \in \text{Blocker}} \text{severity}(bl)$$

- **G2: maximize_feature_completion** - Maximize the number of features completed.

$$\text{Maximize} \sum_{f \in \text{Feature}} \mathbb{I}(\text{status}(f) = \text{'Done'})$$

- **G3: minimize_sprint_overhead** - Minimize the total time spent in meetings.

$$\text{Minimize} \sum_{sp \in \text{Sprint}} \text{meeting\_duration}(sp)$$

- **G4: maximize_stakeholder_satisfaction** - Maximize stakeholder satisfaction.

$$\text{Maximize} \sum_{sh \in \text{Stakeholder}} \text{satisfaction}(sh)$$

- **G5: minimize_task_effort_variance** - Minimize effort estimation variance for a task.

$$\text{Minimize } |\text{estimated\_effort}(tsk) - \text{actual\_effort}(tsk)| \quad \forall\, tsk \in \text{Task}$$

- **G6: maximize_worker_availability** - Maximize worker availability.

$$\text{Maximize} \sum_{w \in \text{Worker}} \text{availability}(w)$$

- **G7: minimize_budget_overrun** - Minimize the total budget overrun.

$$\text{Minimize} \sum_{p \in \text{Project}} \max(0, \text{actual\_cost}(p) - \text{budget}(p))$$

- **G8: maximize_sprint_goal_achievement** - Maximize sprint goal achievement rate.

$$\text{Maximize} \sum_{sg \in \text{SprintGoal}} \mathbb{I}(\text{achievement\_status}(sg) = \text{'Achieved'})$$

- **G9: minimize_technical_debt** - Minimize the level of technical debt.

$$\text{Minimize technical\_debt}(dev) \quad \forall\, dev \in \text{DevelopmentSnapshot}$$

4 Conditions

- **C0: team_has_scrum_master** - A team must be supported by a Scrum Master.

$$\exists\, sm \in \text{ScrumMaster supportedby}(t, sm) \quad \forall\, t \in \text{Team}$$

- **C1: worker_availability_threshold** - A worker's availability must be $\geq 70\%$.

$$\text{availability}(w) \geq 0.7 \quad \forall\, w \in \text{Worker}$$

- **C2: feature_high_priority** - The feature must have a 'High' or 'Critical' priority.

$$\text{priority}(f) \in \{\text{'High'}, \text{'Critical'}\} \quad \forall\, f \in \text{Feature}$$

- **C3: blocker_is_resolved** - The blocker must have a status of 'Resolved'.

$$\text{status}(bl) = \text{'Resolved'} \quad \forall\, bl \in \text{Blocker}$$

- **C4: sprint_is_active** - The sprint must have an 'Active' status.

$$\text{status}(sp) = \text{'Active'} \quad \forall\, sp \in \text{Sprint}$$

- **C5: story_has_acceptance_criteria** - A user story must have defined acceptance criteria.

$$\text{acceptance\_criteria}(us) \neq \emptyset \quad \forall\, us \in \text{UserStory}$$

- **C6: task_is_dev_type** - The task must be of type 'Development'.

$$\text{type}(tsk) = \text{'Development'} \quad \forall\, tsk \in \text{Task}$$

- **C7: skill_is_certified** - The skill must be certified.

$$\text{certified}(s) = \text{True} \quad \forall\, s \in \text{Skill}$$

- **C8: project_is_running** - The project status must be 'In Progress'.

$$\text{status}(p) = \text{'In Progress'} \quad \forall\, p \in \text{Project}$$

- **C9: snapshot_test_passed** - The development snapshot test status must be 'Passed'.

$$\text{test\_status}(dev) = \text{'Passed'} \quad \forall\, dev \in \text{DevelopmentSnapshot}$$

- **C10: review_has_min_attendees** - The sprint review must have ≥ 3 attendees.

$$\text{attendees\_count}(sr) \geq 3 \quad \forall\, sr \in \text{SprintReview}$$

5 Decision Variables

- **DV0:** $x_{w,t} \in \{0, 1\}$ - Binary assignment of worker w to team t.
- **DV1:** $y_{f,sbl} \in \{0, 1\}$ - Binary inclusion of feature f in sprint backlog sbl.

- **DV2:** $z_{us} \in \mathbb{Z}^+$ - Story points for user story us, where $1 \leq z_{us} \leq 20$.
- **DV3:** $d_{sp} \in \mathbb{Z}^+$ - Duration of sprint sp in days, where $7 \leq d_{sp} \leq 30$.
- **DV4:** $a_{tsk,w} \in \{0, 1\}$ - Binary assignment of task tsk to worker w.
- **DV5:** $p_f \in \mathbb{Z}^+$ - Priority level of feature f, where $1 \leq p_f \leq 5$.
- **DV6:** $m_{sp} \in \mathbb{Z}^+$ - Total meeting minutes for sprint sp.
- **DV7:** $av_w \in \mathbb{R}$ - Planned availability of worker w, where $0.0 \leq av_w \leq 1.0$.
- **DV8:** $size_t \in \mathbb{Z}^+$ - Size of team t, where $3 \leq size_t \leq 9$.
- **DV9:** $rd_p \in \mathbb{Z}^+$ - Release day for project p (days from start), where $30 \leq rd_p \leq 365$.
- **DV10:** $budget_f \in \mathbb{R}^+$ - Budget allocated to feature f.