Optimization Model for a SCRUM-based Software Development Company

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Contents

1 Sets (Entities)

The following sets represent the core entities within the SCRUM domain model.

- P: Set of all Projects ($p \in P$)
- T: Set of all Teams ($t \in T$)
- W: Set of all Workers ($w \in W$)
- F: Set of all Features ($f \in F$)
- S: Set of all Skills ($s \in S$)
- R: Set of all Roles ($r \in R$)
- PO: Set of all Product Owners ($po \in PO$)
- SM: Set of all Scrum Masters ($sm \in SM$)
- PB: Set of all Product Backlogs ($pb \in PB$)
- SP: Set of all Sprints ($sp \in SP$)
- SBL: Set of all Sprint Backlogs ($sbl \in SBL$)
- SG: Set of all Sprint Goals ($sg \in SG$)
- E: Set of all Epics ($e \in E$)
- US: Set of all User Stories ($us \in US$)
- TSK: Set of all Tasks ($tsk \in TSK$)
- BL: Set of all Blockers ($bl \in BL$)
- SH: Set of all Stakeholders ($sh \in SH$)
- VEL: Set of all Velocities ($vel \in VEL$)
- REP: Set of all Release Plans ($rep \in REP$)

2 Indices

The following indices are used to iterate over the sets defined above.

- p: index for a project from the set P
- t: index for a team from the set T
- w: index for a worker from the set W
- f: index for a feature from the set F
- s: index for a skill from the set S
- r: index for a role from the set R
- po: index for a Product Owner from the set PO
- sm: index for a Scrum Master from the set SM
- sp: index for a Sprint from the set SP
- us: index for a User Story from the set US
- tsk: index for a Task from the set TSK
- bl: index for a Blocker from the set BL
- sh: index for a Stakeholder from the set SH
- rep: index for a Release Plan from the set REP

3 Goals (Objective Function)

The primary objective is to solve a multi-objective optimization problem. The function Z is a weighted sum of all individual goals.

$$\max Z = \sum_{i \in \text{Goals}_{\max}} W_i \cdot G_i - \sum_{j \in \text{Goals}_{\min}} W_j \cdot G_j$$

Where W_k is the weight for goal G_k. The individual goals are:

G0 **maximize_project_budget**: Maximize the total allocated budget. Let b_p be the budget for project p.

$$\max \sum_{p \in P} b_p$$

G1 **maximize_feature_priority**: Maximize the value from implementing high-priority features. Let $x_{f,rep}$ be a binary variable to include feature f in release rep, and P_f be its priority.

$$\max \sum_{f \in F} \sum_{rep \in REP} P_f \cdot x_{f,rep}$$

G2 **minimize_task_effort**: Minimize the total effort spent on tasks. Let e_{tsk} be the effort for task tsk.

$$\min \sum_{tsk \in TSK} e_{tsk}$$

G3 **maximize_story_points_per_sprint**: Maximize story points in a sprint. Let $y_{us,sp}$ be a binary variable to include user story us in sprint sp, and SP_{us} be its story points.

$$\max \sum_{sp \in SP} \sum_{us \in US} SP_{us} \cdot y_{us,sp}$$

G5 **minimize_blocker_severity**: Minimize the impact of blockers. Let z_{bl} be a binary variable for resolving blocker bl, and S_{bl} be its severity.

$$\min \sum_{bl \in BL} S_{bl} \cdot (1 - z_{bl})$$

G9 **maximize_worker_availability**: Maximize the use of available workforce. Let A_w be the availability of worker w.

$$\max \sum_{w \in W} A_w$$

4 Conditions (Constraints)

The optimization is subject to the following constraints, which ensure the validity of the solution within the SCRUM framework.

C0 **team_size_limit**: The size of any team t must be between 3 and 9 members. Let a_{wt} be the assignment of worker w to team t.

$$\forall t \in T: \quad 3 \leq \sum_{w \in W} a_{wt} \leq 9$$

C1 **project_budget_limit**: The budget b_p for any project p cannot exceed a predefined maximum $B_{\max}$.

$$\forall p \in P: \quad b_p \leq B_{\max}$$

C3 **feature_priority_threshold**: A feature f can only be included in a release plan if its priority P_f is above a threshold (e.g., 3).

$$\forall f \in F, \forall rep \in REP: \quad x_{f,rep} \cdot (P_f - 3) \geq 0$$

C5 **task_effort_cap**: The effort e_{tsk} for any single task cannot exceed a cap (e.g., 16 hours).

$$\forall tsk \in TSK: \quad e_{tsk} \leq 16$$

C8 **product_owner_must_be_assigned**: Each project must have exactly one product owner. Let $c_{po,p}$ be the assignment of PO po to project p.

$$\forall p \in P: \quad \sum_{po \in PO} c_{po,p} = 1$$

C9 **user_story_must_have_points**: A user story us can only be assigned to a sprint sp if its story points SP_{us} are greater than 0.

$$\forall us \in US, \forall sp \in SP: \quad y_{us,sp} \cdot SP_{us} > 0$$

Structural Constraint (Worker Assignment): A worker can be assigned to at most one team.

$$\forall w \in W: \quad \sum_{t \in T} a_{wt} \leq 1$$

Structural Constraint (Task to Worker): The total effort of tasks assigned to a worker w in a sprint sp cannot exceed their capacity (Availability $\times$ Sprint Duration).

$$\forall w \in W, \forall sp \in SP: \quad \sum_{tsk \in TSK} e_{tsk} \cdot d_{tsk,w} \leq A_w \cdot D_{sp}$$

5 Decision Variables

These are the variables that the optimization model will determine.

DV0 $a_{wt} \in \{0,1\}$: Binary variable, 1 if Worker w is assigned to Team t, 0 otherwise.

DV1 $k_{tp} \in \{0,1\}$: Binary variable, 1 if Team t is assigned to Project p, 0 otherwise.

DV2 $y_{us,sp} \in \{0,1\}$: Binary variable, 1 if User Story us is selected for Sprint sp, 0 otherwise.

DV3 $d_{tsk,w} \in \{0,1\}$: Binary variable, 1 if Task tsk is assigned to Worker w, 0 otherwise.

DV5 $P_f \in \{1,2,3,4,5\}$: Integer variable for the priority of Feature f.

DV6 $b_p \in R^+$: Continuous variable for the budget of Project p.

DV10 $x_{f,rep} \in \{0,1\}$: Binary variable, 1 if Feature f is included in Release Plan rep, 0 otherwise.

DV11 $z_{bl} \in \{0,1\}$: Binary variable, 1 if Blocker bl is set to be resolved, 0 otherwise.