Optimization Model for a SCRUM-based Software Development Company

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Contents

Introduction

We formulate a mixed-integer optimization model that uses the provided `Entities.csv`, `Relationships.csv`, and the generated `Goals.csv`, `Conditions.csv`, and `DecisionVariables.csv`. The model aligns team structure, backlog composition, sprint planning, and delivery outcomes under SCRUM.

1 1. Sets (Entities)

- P — Projects (Project)
- T — Teams (Team)
- W — Workers (Worker)
- F — Features (Feature)
- $\mathcal{S}$ — Skills (Skill)
- R — Roles (Role)
- PO — Product Owners (ProductOwner)
- SM — Scrum Masters (ScrumMaster)
- PB — Product Backlogs (ProductBacklog)
- SP — Sprints (Sprint)
- SPP — Sprint Plannings (SprintPlanning)
- DS — Daily Scrums (DailyScrum)
- SR — Sprint Reviews (SprintReview)
- SRE — Sprint Retrospectives (SprintRetrospective)
- SBL — Sprint Backlogs (SprintBacklog)
- SG — Sprint Goals (SprintGoal)
- E — Epics (Epic)
- US — User Stories (UserStory)
- TK — Tasks (Task)
- DEV — Development Snapshots (DevelopmentSnapshot)
- BL — Blockers (Blocker)
- SH — Stakeholders (Stakeholder)
- VEL — Velocity records (Velocity)
- REP — Release Plans (ReleasePlan)
- RM — Roadmaps (Roadmap)
- SCB — Scrum Boards (ScrumBoard)
- FED — Feature Documentations (FeatureDocumentation)

2 2. Indices

- $p \in P,\ t \in T,\ w \in W,\ f \in F,\ r \in R,\ po \in PO,\ sm \in SM,\ pb \in PB$
- $sp \in SP,\ sbl \in SBL,\ sg \in SG,\ e \in E,\ us \in US,\ tk \in TK,\ dev \in DEV$
- $bl \in BL,\ sh \in SH,\ vel \in VEL,\ rep \in REP,\ rm \in RM,\ scb \in SCB,\ fed \in FED$

Parameters (from entity attributes).

- $budget_p$ (Project.budget), $priority_f$ (Feature.priority), $effort_{tk}$ (Task.effort), $storypts_{us}$ (UserStory.story_points)
- $totalEff_{sbl}$ (SprintBacklog.total_effort), $cards_{scb}$ (ScrumBoard.number_of_cards), $entries_{pb}$ (ProductBacklog.number_of_entries)
- $severity_{bl}$ (Blocker.severity), $achv_{sg}$ (SprintGoal.achievement_status), $attend_{sr}$ (SprintReview.attendees_count)
- sat_{sre} (SprintRetrospective.team_satisfaction), $relev_{sh}$ (Stakeholder.relevance_to_feature)
- $maxvel_{vel}$, $minvel_{vel}$ (Velocity.max_velocity / min_velocity), $teamsize_t$ (Team.team_size)

Relationship incidence parameters (from `Relationships.csv`). Binary constants $A^{(k)}$ capture fixed links when needed; e.g., $A^{R19}_{vel,t} = 1$ if Velocity record vel refers to Team t (R19), otherwise 0. Similar incidence tensors can be defined for R17, R20, R21 if treated as data.

Decision variables (from `DecisionVariables.csv`). All variables are binary unless stated.

$X_{t,p}$	(DV0) Team t assigned to Project p
$Y_{w,t}$	(DV1) Worker w belongs to Team t
$R_{w,r}$	(DV2) Worker w takes Role r
$A_{po,pb}$	(DV3) PO po manages Backlog pb
$S_{t,sm}$	(DV4) SM sm supports Team t
$B_{f,pb}$	(DV5) Feature f in Backlog pb
$E_{e,pb}$	(DV6) Epic e in Backlog pb
$U_{us,e}$	(DV7) User Story us in Epic e
$V_{tk,us}$	(DV8) Task tk belongs to Story us
$PICK_{us,sbl}$	(DV9) Story us in SprintBacklog sbl
$D_{sbl,sp}$	(DV10) SprintBacklog sbl belongs to Sprint sp
$G_{sp,sg}$	(DV11) Sprint sp pursues Goal sg
$C_{scb,tk}$	(DV12) Task tk shown on ScrumBoard scb
$FDoc_{fed,f}$	(DV13) Documentation fed belongs to Feature f
$Q_{sp,dev}$	(DV14) Sprint sp generates DevSnapshot dev

3 3. Goals

Let weights w_g be taken from `Goals.csv` (column `Weight`). The global objective is a weighted sum of the individual goals Z_g, respecting their min/max polarity. For readability we list each goal with its ID, name, and mathematical form.

- **G0 maximize_story_points_delivered**:

$$Z_0 = \sum_{us \in US} storypts_{us} \cdot \left(\sum_{sbl \in SBL} PICK_{us,sbl} \right), \quad \max$$

- **G1 minimize_total_task_effort**:

$$Z_1 = \sum_{tk \in TK} effort_{tk} \cdot \left(\sum_{us \in US} V_{tk,us} \right), \quad \min$$

- **G2 minimize_blocker_severity**:

$$Z_2 = \sum_{bl \in BL} severity_{bl}, \quad \min$$

- **G3 maximize_feature_priority**:

$$Z_3 = \sum_{f \in F} \sum_{pb \in PB} priority_f \, B_{f,pb}, \quad \max$$

- **G4 minimize_project_budget**:

$$Z_4 = \sum_{p \in P} budget_p, \quad \min$$

- **G5 maximize_velocity_ceiling**:

$$Z_5 = \sum_{vel \in VEL} vel, \quad \max$$

- **G6 maximize_stakeholder_relevance**:

$$Z_6 = \sum_{sh \in SH} relev_{sh}, \quad \max$$

- **G7 maximize_sprint_goal_achievement**:

$$Z_7 = \sum_{sp \in SP} \sum_{sg \in SG} achv_{sg} \, G_{sp,sg}, \quad \max$$

- **G8 minimize_scrum_board_wip**:

$$Z_8 = \sum_{scb \in SCB} cards_{scb}, \quad \min$$

- **G9 minimize_product_backlog_size**:

$$Z_9 = \sum_{pb \in PB} entries_{pb}, \quad \min$$

- **G10 maximize_sprint_review_engagement**:

$$Z_{10} = \sum_{sr \in SR} attend_{sr}, \quad \max$$

- **G11 maximize_team_satisfaction**:

$$Z_{11} = \sum_{sre \in SRE} sat_{sre}, \quad \max$$

- **G12 maximize_feature_throughput**:

$$Z_{12} = \sum_{f \in F} \sum_{pb \in PB} \text{estimated\_effort}_f \, B_{f,pb}, \quad \max$$

Composite objective. Let $\mathcal{G}_{\max} = \{0, 3, 5, 6, 7, 10, 11, 12\}$ and $\mathcal{G}_{\min} = \{1, 2, 4, 8, 9\}$. With weights w_g:

$$\max \sum_{g \in \mathcal{G}_{\max}} w_g Z_g \; - \sum_{g \in \mathcal{G}_{\min}} w_g Z_g$$

4 4. Conditions

Below, each condition references the `Conditions.csv` entry and is expressed as linear constraints when applicable.

- **C1 team_assignment_uniqueness (R1)**: Each team has exactly one project.

$$\sum_{p \in P} X_{t,p} = 1 \qquad \forall t \in T$$

- **C2 worker_to_single_team (R2)**: Each worker belongs to exactly one team.

$$\sum_{t \in T} Y_{w,t} = 1 \qquad \forall w \in W$$

- **C3 team_capacity_respects_size**: Team headcount cannot exceed team_size.

$$\sum_{w \in W} Y_{w,t} \leq teamsize_t \qquad \forall t \in T$$

- **C4 user_story_to_one_epic (R9)**:

$$\sum_{e \in E} U_{us,e} = 1 \qquad \forall us \in US$$

- **C5 task_to_one_user_story (R10)**:

$$\sum_{us \in US} V_{tk,us} = 1 \qquad \forall tk \in TK$$

- **C6 user_story_to_max_one_sprint_backlog (R11)**:

$$\sum_{sbl \in SBL} PICK_{us,sbl} \leq 1 \qquad \forall us \in US$$

- **C7 sprint_backlog_belongs_to_one_sprint (R12)**:

$$\sum_{sp \in SP} D_{sbl,sp} = 1 \qquad \forall sbl \in SBL$$

- **C8 sprint_has_one_goal (R13)**:

$$\sum_{sg \in SG} G_{sp,sg} = 1 \qquad \forall sp \in SP$$

- **C9 backlog_entry_count_consistency**:

$$\sum_{f \in F} B_{f,pb} + \sum_{e \in E} E_{e,pb} \leq entries_{pb} \qquad \forall pb \in PB$$

- **C10 sprint_backlog_capacity**:

$$\sum_{us \in US} storypts_{us}\, PICK_{us,sbl} \leq totalEff_{sbl} \qquad \forall sbl \in SBL$$

- **C11 velocity_floor**: Using incidence $A^{R19}_{vel,t}$, enforce team capacity lower bound (planning guideline).

$$\sum_{us \in US} storypts_{us} \sum_{sbl \in SBL} PICK_{us,sbl} \;\geq\; \sum_{t \in T} \left(\sum_{vel \in VEL} A^{R19}_{vel,t}\, minvel_{vel} \right)$$

- **C12 stakeholder_influence_cap** (planning governance over reviews):

$$\sum_{sh \in SH} \text{influence\_level}_{sh} \;\leq\; \Gamma$$

  where Γ is a policy parameter.

- **C0 limit_project_budget** (portfolio guardrail): with unit cost κ per story point,

$$\kappa \sum_{us \in US} storypts_{us} \sum_{sbl \in SBL} PICK_{us,sbl} \;\leq\; \sum_{p \in P} budget_p$$

Further relationship guards (always-on).

$$\sum_{pb \in PB} A_{po,pb} \leq 1 \qquad \forall po \in PO \quad \text{(R5)}$$

$$\sum_{po \in PO} A_{po,pb} = 1 \qquad \forall pb \in PB \quad \text{(R5)}$$

$$\sum_{sm \in SM} S_{t,sm} = 1 \qquad \forall t \in T \quad \text{(R6)}$$

$$\sum_{pb \in PB} B_{f,pb} \leq 1 \qquad \forall f \in F \quad \text{(R7)}$$

$$\sum_{pb \in PB} E_{e,pb} \leq 1 \qquad \forall e \in E \quad \text{(R8)}$$

$$\sum_{f \in F} FDoc_{fed,f} = 1 \qquad \forall fed \in FED \quad \text{(R15)}$$

$$\sum_{dev \in DEV} Q_{sp,dev} = 1 \qquad \forall sp \in SP \quad \text{(R22)}$$

5 5. DecisionVariables

- **DV0 assign_team_to_project** $X_{t,p} \in \{0,1\}$
- **DV1 assign_worker_to_team** $Y_{w,t} \in \{0,1\}$
- **DV2 assign_worker_to_role** $R_{w,r} \in \{0,1\}$
- **DV3 assign_po_to_backlog** $A_{po,pb} \in \{0,1\}$
- **DV4 assign_scrum_master_to_team** $S_{t,sm} \in \{0,1\}$
- **DV5 put_feature_in_backlog** $B_{f,pb} \in \{0,1\}$
- **DV6 put_epic_in_backlog** $E_{e,pb} \in \{0,1\}$
- **DV7 link_userstory_to_epic** $U_{us,e} \in \{0,1\}$
- **DV8 link_task_to_userstory** $V_{tk,us} \in \{0,1\}$
- **DV9 pick_userstory_into_sprint_backlog** $PICK_{us,sbl} \in \{0,1\}$
- **DV10 link_sprint_backlog_to_sprint** $D_{sbl,sp} \in \{0,1\}$
- **DV11 link_sprint_to_goal** $G_{sp,sg} \in \{0,1\}$
- **DV12 show_task_on_scrum_board** $C_{scb,tk} \in \{0,1\}$
- **DV13 link_feature_doc_to_feature** $FDoc_{fed,f} \in \{0,1\}$
- **DV14 link_sprint_to_development_snapshot** $Q_{sp,dev} \in \{0,1\}$