Mathematical Optimization Model for Scrum-based Software Development

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Contents

1 Sets (Entities)

This section defines the fundamental sets used in the model, based on the entities from `Entities.csv`.

- P: Set of all Projects (E0)
- T: Set of all Teams (E1)
- W: Set of all Workers (E2)
- F: Set of all Features (E3)
- S: Set of all Skills (E4)
- R: Set of all Roles (E5)
- PO: Set of all Product Owners (E6)
- SM: Set of all Scrum Masters (E7)
- PB: Set of all Product Backlogs (E8)
- SP: Set of all Sprints (E9)
- SBL: Set of all Sprint Backlogs (E14)
- SG: Set of all Sprint Goals (E15)
- E: Set of all Epics (E16)
- US: Set of all User Stories (E17)
- TSK: Set of all Tasks (E18)
- BL: Set of all Blockers (E20)
- SH: Set of all Stakeholders (E21)
- VEL: Set of all Velocity measurements (E22)
- REP: Set of all Release Plans (E23)

2 Indices

This section defines the indices used to iterate over the sets defined above.

- $p \in P$: Index for Projects
- $t \in T$: Index for Teams
- $w \in W$: Index for Workers
- $f \in F$: Index for Features

- $s \in S$: Index for Skills
- $r \in R$: Index for Roles
- $sp \in SP$: Index for Sprints
- $us \in US$: Index for User Stories
- $tsk \in TSK$: Index for Tasks
- $bl \in BL$: Index for Blockers
- $rep \in REP$: Index for Release Plans

3 Decision Variables

This section defines the decision variables of the optimization model, based on `DecisionVariables.csv`. These are the outputs of the model.

- **(DV0)** $A_{w,t} \in \{0, 1\}$: Binary variable, 1 if worker w is assigned to team t, 0 otherwise.
- **(DV1)** $B_{us,sp} \in \{0, 1\}$: Binary variable, 1 if user story us is assigned to sprint sp, 0 otherwise.
- **(DV2)** $C_{tsk,w} \in \{0, 1\}$: Binary variable, 1 if task tsk is assigned to worker w, 0 otherwise.
- **(DV3)** $D_{f,rep} \in \{0, 1\}$: Binary variable, 1 if feature f is included in release plan rep, 0 otherwise.
- **(DV4)** $TeamSize_t \in \mathbb{Z}^+$: Integer variable for the number of members in team t.
- **(DV5)** $StartDate_{sp} \in \mathbb{R}^+$: Continuous variable for the start date of sprint sp.
- **(DV6)** $Avail_{w,p} \in [0, 1]$: Float variable for the percentage of worker w's availability allocated to project p.
- **(DV7)** $Rank_{us} \in \mathbb{Z}^+$: Integer variable for the rank of user story us in the product backlog.
- **(DV8)** $Duration_{sp} \in \{1, 2, 3, 4\}$: Integer variable for the duration of sprint sp in weeks.
- **(DV9)** $E_{w,r,t} \in \{0, 1\}$: Binary variable, 1 if worker w takes on role r in team t, 0 otherwise.
- **(DV11)** $StoryPoints_{us} \in \{1, 2, 3, 5, 8, 13, 21\}$: Integer variable for the estimated story points of user story us.

4 Goals (Objective Functions)

This section lists the optimization goals from `Goals.csv`, which can be combined into a multi-objective function, typically using weights.

- **(G0) maximize_sprint_priority**: Maximize the value from high-priority user stories in sprints.

$$\max \sum_{us \in US} \sum_{sp \in SP} \text{priority}(us) \cdot B_{us,sp}$$

- **(G1) maximize_completed_story_points**: Maximize the throughput of the team.

$$\max \sum_{us \in US} \sum_{sp \in SP} \text{StoryPoints}_{us} \cdot B_{us,sp}$$

- **(G2) minimize_release_effort**: Minimize the total effort for a release.

$$\min \sum_{f \in F} \sum_{rep \in REP} \text{estimated\_effort}(f) \cdot D_{f,rep}$$

- **(G4) minimize_project_duration**: Minimize the time-to-market.

$$\min(\text{project\_end}(p) - \text{project\_start}(p))$$

- **(G5) minimize_open_blockers**: Minimize impediments to progress.

$$\min \sum_{bl \in BL} \mathbb{I}(\text{status}(bl) = \text{'open'})$$

  where $\mathbb{I}(\cdot)$ is the indicator function.

- **(G8) minimize_budget_usage**: Ensure the project is cost-effective.

$$\min \sum_{w \in W} \sum_{p \in P} \text{cost}(w) \cdot \text{Avail}_{w,p}$$

5 Conditions (Constraints)

This section lists the constraints from `Conditions.csv` that the solution must adhere to.

- **(C0) sprint_capacity_limit**: The work assigned to a sprint must not exceed the team's velocity. Let V_t be the velocity for team t.

$$\forall sp \in SP, \forall t \in T : \sum_{us \in US} \text{StoryPoints}_{us} \cdot B_{us,sp} \leq V_t$$

- **(C1) project_budget_limit**: Total project costs must not exceed the budget.

$$\forall p \in P : \sum_{w \in W} \text{cost}(w) \cdot \text{Avail}_{w,p} \leq \text{budget}(p)$$

- **(C3) task_skill_requirement**: A worker assigned to a task must have the required skills. Let $\text{HasSkill}_{w,s}$ and $\text{ReqSkill}_{tsk,s}$ be binary parameters.

$$\forall tsk \in TSK, w \in W, s \in S : \text{ReqSkill}_{tsk,s} \cdot C_{tsk,w} \leq \text{HasSkill}_{w,s}$$

- **(C4) agile_team_size**: Team size must be within defined agile limits.

$$\forall t \in T : 3 \leq \sum_{w \in W} A_{w,t} \leq 9$$

- **(C6) blocked_task_halt**: A task cannot be assigned if it is blocked. Let IsBlocked_{tsk} be a binary parameter.

$$\forall tsk \in TSK : \sum_{w \in W} C_{tsk,w} \leq 1 - \text{IsBlocked}_{tsk}$$

- **(C7) team_scrum_master_assignment**: Each team must have one Scrum Master.

$$\forall t \in T : \sum_{w \in W} E_{w,\text{'ScrumMaster'},t} = 1$$

- **(C11) unique_worker_team_assignment**: Each worker can be on at most one team.

$$\forall w \in W : \sum_{t \in T} A_{w,t} \leq 1$$