Mathematical Optimization Model for Scrum Project Planning

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Contents

1 Sets (Entities)

This section defines the fundamental sets used in the model, derived from the entities in the domain.

- P: Set of all Projects
- T: Set of all Teams
- W: Set of all Workers
- F: Set of all Features
- S: Set of all Skills
- R: Set of all Roles
- PO: Set of all Product Owners
- SM: Set of all Scrum Masters
- PB: Set of all Product Backlogs
- SP: Set of all Sprints
- SPP: Set of all Sprint Planning meetings
- DS: Set of all Daily Scrums
- SR: Set of all Sprint Reviews
- SRE: Set of all Sprint Retrospectives
- SBL: Set of all Sprint Backlogs
- SG: Set of all Sprint Goals
- E: Set of all Epics
- US: Set of all User Stories
- TSK: Set of all Tasks
- DEV: Set of all Development Snapshots
- BL: Set of all Blockers
- SH: Set of all Stakeholders
- VEL: Set of all Velocity records
- REP: Set of all Release Plans
- RM: Set of all Roadmaps
- SCB: Set of all Scrum Boards
- FED: Set of all Feature Documentations

2 Indices

This section defines the indices used to iterate over the sets defined in Section 1.

- Let $p \in P$ be an index for the set of Projects.
- Let $t \in T$ be an index for the set of Teams.
- Let $w \in W$ be an index for the set of Workers.
- Let $f \in F$ be an index for the set of Features.
- Let $s \in S$ be an index for the set of Skills.
- Let $r \in R$ be an index for the set of Roles.
- Let $po \in PO$ be an index for the set of Product Owners.
- Let $sm \in SM$ be an index for the set of Scrum Masters.
- Let $pb \in PB$ be an index for the set of Product Backlogs.
- Let $sp \in SP$ be an index for the set of Sprints.
- Let $spp \in SPP$ be an index for the set of Sprint Planning meetings.
- Let $ds \in DS$ be an index for the set of Daily Scrums.
- Let $sr \in SR$ be an index for the set of Sprint Reviews.
- Let $sre \in SRE$ be an index for the set of Sprint Retrospectives.
- Let $sbl \in SBL$ be an index for the set of Sprint Backlogs.
- Let $sg \in SG$ be an index for the set of Sprint Goals.
- Let $e \in E$ be an index for the set of Epics.
- Let $us \in US$ be an index for the set of User Stories.
- Let $tsk \in TSK$ be an index for the set of Tasks.
- Let $dev \in DEV$ be an index for the set of Development Snapshots.
- Let $bl \in BL$ be an index for the set of Blockers.
- Let $sh \in SH$ be an index for the set of Stakeholders.
- Let $vel \in VEL$ be an index for the set of Velocity records.
- Let $rep \in REP$ be an index for the set of Release Plans.
- Let $rm \in RM$ be an index for the set of Roadmaps.
- Let $scb \in SCB$ be an index for the set of Scrum Boards.
- Let $fed \in FED$ be an index for the set of Feature Documentations.

3 Goals

This section defines the objective functions of the optimization model. The model can be configured to optimize for one or more of these goals, often as a weighted sum. Decision variables used here (e.g., $X_{us,sp}, Y_{f,rep}$) are formally defined in Section 5.

- **(G0) maximize_total_priority**:

  Description: Maximize the sum of priorities for all selected User Stories to deliver the highest business value.

  Mathematical Formulation:

$$\text{maximize} \quad Z_0 = \sum_{us \in US} \sum_{sp \in SP} \text{priority}_{us} \cdot X_{us,sp}$$

- **(G1) minimize_total_effort**:

  Description: Minimize the sum of story points for all selected User Stories to achieve goals with the least amount of work.

  Mathematical Formulation:

$$\text{minimize} \quad Z_1 = \sum_{us \in US} \sum_{sp \in SP} \text{story\_points}_{us} \cdot X_{us,sp}$$

- **(G2) maximize_feature_completion**:

  Description: Maximize the number of Features that reach a 'done' status within the planning period. Let S_f be 1 if feature f is completed, 0 otherwise.

  Mathematical Formulation:

$$\text{maximize} \quad Z_2 = \sum_{f \in F} S_f$$

- **(G5) minimize_number_of_open_blockers**:

  Description: Minimize the count of active Blockers with a status other than 'resolved'. Let R_{bl} be 1 if blocker bl is to be resolved.

  Mathematical Formulation:

$$\text{minimize} \quad Z_5 = |BL| - \sum_{bl \in BL} R_{bl}$$

- **(G6) maximize_sprint_goal_achievement**:

  Description: Maximize the number of Sprints where the primary Sprint Goal is successfully achieved. Let G_{sp} be 1 if the goal for sprint sp is achieved.

  Mathematical Formulation:

$$\text{maximize} \quad Z_6 = \sum_{sp \in SP} G_{sp}$$

4 Conditions

This section lists the constraints that the solution to the optimization problem must satisfy.

- **(C0) total_effort_must_not_exceed_velocity**:

  Description: The sum of story points in a Sprint Backlog must not exceed the team's average Velocity. Let T_{sp} be the team assigned to sprint sp.

  Mathematical Formulation:

$$\sum_{us \in US} \text{story\_points}_{us} \cdot X_{us,sp} \leq \text{avg\_story\_points}_{\text{vel}_{T_{sp}}} \quad \forall sp \in SP$$

- **(C1) worker_availability_must_be_respected**:

  Description: The sum of effort for tasks assigned to a Worker must not exceed their specified availability. Let dur_{sp} be the duration of sprint sp.

  Mathematical Formulation:

$$\sum_{tsk \in TSK} \text{effort}_{tsk} \cdot A_{w,tsk} \leq \text{availability}_{w} \cdot \text{dur}_{sp} \quad \forall w \in W, \forall sp \in SP$$

- **(C2) project_must_stay_within_budget**:

  Description: The total cost incurred cannot exceed the allocated project budget. Let cost_{tsk} be the cost of a task.

  Mathematical Formulation:

$$\sum_{w \in W} \sum_{tsk \in TSK} \text{cost}_{tsk} \cdot A_{w,tsk} \leq \text{budget}_{p} \quad \forall p \in P$$

- **(C4) user_story_requires_acceptance_criteria**:

  Description: A User Story cannot be selected for a sprint unless its acceptance_criteria field is filled. Let has_ac_{us} be a binary parameter (1 if criteria exist, 0 otherwise).

  Mathematical Formulation:

$$X_{us,sp} \leq \text{has\_ac}_{us} \quad \forall us \in US, \forall sp \in SP$$

- **(C5) task_must_belong_to_a_user_story**:

  Description: A Task can only be assigned to a worker if its parent User Story is selected for the sprint. Let US_{tsk} be the parent user story of task tsk.

  Mathematical Formulation:

$$A_{w,tsk} \leq X_{US_{tsk},sp} \quad \forall w \in W, \forall tsk \in TSK, \forall sp \in SP$$

5 Decision Variables

This section provides the formal mathematical definition for the variables used in the model's goals and conditions.

- **(DV0) select_user_story_for_sprint**:

Description: Binary decision variable to determine if a User Story (us) is selected for the current sprint (sp).

Mathematical Definition:

$$X_{us,sp} \in \{0,1\} \quad \forall us \in US, \forall sp \in SP$$

- **(DV1) assign_worker_to_task**:

  Description: Variable to assign a specific Worker (w) to a specific Task (tsk).

  Mathematical Definition:

$$A_{w,tsk} \in \{0,1\} \quad \forall w \in W, \forall tsk \in TSK$$

- **(DV2) assign_feature_to_release**:

  Description: Binary decision to include a Feature (f) in a specific Release Plan (rep).

  Mathematical Definition:

$$Y_{f,rep} \in \{0,1\} \quad \forall f \in F, \forall rep \in REP$$

- **(DV3) set_sprint_start_date**:

  Description: Decision on which date a specific Sprint (sp) should begin.

  Mathematical Definition:

$$D_{sp} \geq 0 \quad \forall sp \in SP \quad \text{(Integer or Date type)}$$

- **(DV7) resolve_blocker_in_sprint**:

  Description: Binary decision on whether to allocate resources to resolve a Blocker (bl) in the current sprint (sp).

  Mathematical Definition:

$$R_{bl,sp} \in \{0,1\} \quad \forall bl \in BL, \forall sp \in SP$$