

Integrated Optimization Model for a SCRUM-based Software Development Organization

(based on provided Entities, Relationships, Goals, Conditions, Decision Variables)

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1 1. Sets (Entities)

- \mathcal{P} (Projects; Entity: **Project**, Index p)
- \mathcal{T} (Teams; Entity: **Team**, Index t)
- \mathcal{W} (Workers; Entity: **Worker**, Index w)
- \mathcal{F} (Features; Entity: **Feature**, Index f)
- \mathcal{S} (Skills; Entity: **Skill**, Index s)
- \mathcal{R} (Roles; Entity: **Role**, Index r)
- \mathcal{PO} (Product Owners; Entity: **ProductOwner**, Index po)
- \mathcal{SM} (Scrum Masters; Entity: **ScrumMaster**, Index sm)
- \mathcal{PB} (Product Backlogs; Entity: **ProductBacklog**, Index pb)
- \mathcal{SP} (Sprints; Entity: **Sprint**, Index sp)
- \mathcal{SPP} (Sprint Plannings; Entity: **SprintPlanning**, Index spp)
- \mathcal{DS} (Daily Scrums; Entity: **DailyScrum**, Index ds)
- \mathcal{SR} (Sprint Reviews; Entity: **SprintReview**, Index sr)
- \mathcal{SRE} (Sprint Retrospectives; Entity: **SprintRetrospective**, Index sre)
- \mathcal{SBL} (Sprint Backlogs; Entity: **SprintBacklog**, Index sbl)
- \mathcal{SG} (Sprint Goals; Entity: **SprintGoal**, Index sg)
- \mathcal{E} (Epics; Entity: **Epic**, Index e)
- \mathcal{US} (User Stories; Entity: **UserStory**, Index us)
- \mathcal{TSK} (Tasks; Entity: **Task**, Index tsk)
- \mathcal{DEV} (Development Snapshots; Entity: **DevelopmentSnapshot**, Index dev)
- \mathcal{BL} (Blockers; Entity: **Blocker**, Index bl)
- \mathcal{SH} (Stakeholders; Entity: **Stakeholder**, Index sh)
- \mathcal{VEL} (Velocities; Entity: **Velocity**, Index vel)
- \mathcal{REP} (Release Plans; Entity: **ReleasePlan**, Index rep)
- \mathcal{RM} (Roadmaps; Entity: **Roadmap**, Index rm)
- \mathcal{SCB} (Scrum Boards; Entity: **ScrumBoard**, Index scb)
- \mathcal{FED} (Feature Documentations; Entity: **FeatureDocumentation**, Index fed)

2 2. Indices

- $p \in \mathcal{P}$, $t \in \mathcal{T}$, $w \in \mathcal{W}$, $f \in \mathcal{F}$, $s \in \mathcal{S}$, $r \in \mathcal{R}$.
- $po \in \mathcal{PO}$, $sm \in \mathcal{SM}$, $pb \in \mathcal{PB}$, $sp \in \mathcal{SP}$, $spp \in \mathcal{SPP}$, $ds \in \mathcal{DS}$.
- $sr \in \mathcal{SR}$, $sre \in \mathcal{SRE}$, $sbl \in \mathcal{SBL}$, $sg \in \mathcal{SG}$, $e \in \mathcal{E}$, $us \in \mathcal{US}$.
- $tsk \in \mathcal{TSK}$, $dev \in \mathcal{DEV}$, $bl \in \mathcal{BL}$, $sh \in \mathcal{SH}$, $vel \in \mathcal{VEL}$.
- $rep \in \mathcal{REP}$, $rm \in \mathcal{RM}$, $scb \in \mathcal{SCB}$, $fed \in \mathcal{FED}$.

Relationship-induced subsets (from Relationships.csv)

- $\mathcal{F}(pb) \subseteq \mathcal{F}$: features contained in product backlog pb (R7).
- $\mathcal{E}(pb) \subseteq \mathcal{E}$: epics contained in product backlog pb (R8).
- $\mathcal{US}(e) \subseteq \mathcal{US}$: user stories in epic e (R9).
- $\mathcal{TSK}(us) \subseteq \mathcal{TSK}$: tasks of user story us (R10).
- $\mathcal{US}(sbl) \subseteq \mathcal{US}$: user stories assigned to sprint backlog sbl (R11).
- $sbl(sp) \in \mathcal{SBL}$: unique sprint backlog of sprint sp (R12).
- $sg(sp) \in \mathcal{SG}$: goal pursued by sprint sp (R13).
- $\mathcal{TSK}(scb) \subseteq \mathcal{TSK}$: tasks represented on board scb (R14).
- $\mathcal{FED}(f)$: documentation linked to feature f (R15).
- $\mathcal{BL}(tsk) \subseteq \mathcal{BL}$: blockers that block task tsk (R16).
- $\mathcal{F}(rep) \subseteq \mathcal{F}$: features planned in release plan rep (R20).
- $rm(rep) \in \mathcal{RM}$: roadmap to which release plan rep belongs (R21).

Parameters (from Entities' attributes)

- eff_f^F = `estimated_effort` of feature f ; prio_{us}^{US} = `priority` of user story us ; sp_{us}^{US} = `story_points`.
- eff_{tsk}^{TSK} = `effort` of task tsk ; sev_{bl}^{BL} = `severity`.
- cards_{scb}^{SCB} = `number_of_cards`; $\text{nTasks}_{sbl}^{SBL}$ = `number_of_tasks`; $\text{totEff}_{sbl}^{SBL}$ = `total_effort`.
- fresh_{pb}^{PB} = `freshness proxy from last_updated`; entries_{pb}^{PB} = `number_of_entries`.
- sat_{sre}^{SRE} = `team_satisfaction`; dur_{sr}^{SR} = `duration`; dur_{ds}^{DS} = `duration`.
- ach_{sp}^{SP} = `achievement_of_goal`; mil_{rm}^{RM} = `count proxy from milestones`.
- maxVel_t^{VEL} , sp_t^{VEL} = `max_velocity`, `avg_story_points` referring to team t (R19).
- avail_{po}^{PO} = `availability`; exp_{sm}^{SM} = `experience`.

3. Goals

- **G0 maximize_velocity** (ID: G0)
Maximize historical team throughput (Velocity.max_velocity).

$$\max Z_0 = \sum_{t \in \mathcal{T}} 1.0 \cdot \max \text{Vel}_t^{VEL}$$

- **G1 maximize_average_story_points** (ID: G1)
Maximize average story points completed (Velocity.avg_story_points).

$$\max Z_1 = \sum_{t \in \mathcal{T}} 1.0 \cdot \overline{\text{sp}}_t^{VEL}$$

- **G2 minimize_total_sprint_effort** (ID: G2)
Minimize total planned effort in sprint backlogs.

$$\min Z_2 = \sum_{sp \in \mathcal{SP}} \sum_{tsk \in \mathcal{TSK}} \text{eff}_{tsk}^{TSK} \cdot x_{tsk,sp}^{\text{assign}}$$

- **G3 minimize_open_tasks** (ID: G3)
Minimize number of cards (open tasks) on Scrum Boards.

$$\min Z_3 = \sum_{scb \in \mathcal{SCB}} n_{scb}^{\text{cards}} \quad \text{with } n_{scb}^{\text{cards}} = \sum_{tsk \in \mathcal{TSK}(scb)} x_{tsk,scb}^{\text{onBoard}}$$

- **G4 minimize_blocker_severity** (ID: G4)
Minimize cumulative blocker severity affecting selected tasks.

$$\min Z_4 = \sum_{tsk \in \mathcal{TSK}} \sum_{bl \in \mathcal{BL}(tsk)} \text{sev}_{bl}^{BL} \cdot x_{tsk,.}^{\text{assign}}$$

- **G5 maximize_team_satisfaction** (ID: G5)
Maximize team satisfaction (from Retrospective).

$$\max Z_5 = \sum_{sre \in \mathcal{SRE}} 0.8 \cdot \text{sat}_{sre}^{SRE}$$

- **G6 minimize_feature_effort** (ID: G6)
Minimize total estimated feature effort selected for release.

$$\min Z_6 = \sum_{f \in \mathcal{F}} \text{eff}_f^F \cdot y_f^{\text{feat}}$$

- **G7 maximize_story_priority_value** (ID: G7)
Maximize sum of priorities of selected user stories.

$$\max Z_7 = \sum_{us \in \mathcal{US}} 0.6 \cdot \text{prio}_{us}^{US} \cdot x_{us}^{\text{story}}$$

- **G8 minimize_task_effort** (ID: G8)
Prefer smaller tasks to improve flow.

$$\min Z_8 = \sum_{tsk \in \mathcal{TSK}} 0.9 \cdot \text{eff}_{tsk}^{TSK} \cdot x_{tsk,.}^{\text{assign}}$$

- **G9 maximize_roadmap_objectives** (ID: G9)

Maximize milestones proxy on roadmaps.

$$\max Z_9 = \sum_{rm \in \mathcal{RM}} 0.5 \cdot \text{mil}_{rm}^{RM}$$

- **G10 maximize_product_backlog_freshness** (ID: G10)

Favor recently updated backlogs (freshness proxy).

$$\max Z_{10} = \sum_{pb \in \mathcal{PB}} 0.4 \cdot \text{fresh}_{pb}^{PB}$$

- **G11 minimize_cycle_instability** (ID: G11)

Minimize missed-goal proxy (1 - achievement_of_goal).

$$\min Z_{11} = \sum_{sp \in \mathcal{SP}} 0.3 \cdot (1 - \text{ach}_{sp}^{SP})$$

Multi-objective aggregation (optional) One may combine the above via weighted sum:

$$\max Z = \sum_{g \in \{0,1,5,7,9,10\}} Z_g - \sum_{h \in \{2,3,4,6,8,11\}} Z_h$$

with all Z . as defined above.

4. Conditions

- **C0 max_tasks_per_sprint** (ID: C0)

Constrain total number of tasks planned per sprint backlog.

$$\sum_{tsk \in \mathcal{TSK}} x_{tsk,sp}^{\text{assign}} \leq N_{\text{tasks}}^{\max}, \quad \forall sp \in \mathcal{SP}$$

- **C1 max_features_per_release** (ID: C1)

Limit features in any release plan.

$$\sum_{f \in \mathcal{F}(rep)} y_f^{\text{feat}} \leq N_{\text{feat}}^{\max}, \quad \forall rep \in \mathcal{REP}$$

- **C2 min_team_availability** (ID: C2)

Minimum Product Owner availability reserved for backlog work.

$$a^{PO} \geq A_{PO}^{\min}$$

- **C3 min_scrum_master_experience** (ID: C3)

Scrum Master experience threshold for complex sprints.

$$\exp_{sm}^{SM} \geq E^{\min}, \quad \forall sm \in \mathcal{SM}$$

- **C4 max_blocker_severity** (ID: C4)

Cumulative blocker severity affecting planned tasks bounded.

$$\sum_{tsk \in \mathcal{TSK}} \sum_{bl \in \mathcal{BL}(tsk)} \text{sev}_{bl}^{BL} \cdot x_{tsk,\cdot}^{\text{assign}} \leq S^{\max}$$

- **C5 max_total_feature_effort** (ID: C5)
Cap on total effort of selected features for a release.

$$\sum_{f \in \mathcal{F}} \text{eff}_f^F \cdot y_f^{\text{feat}} \leq E_{\text{feat}}^{\max}$$

- **C6 max_task_effort** (ID: C6)
Selected task effort cannot exceed ceiling.

$$\text{eff}_{tsk}^{TSK} \cdot x_{tsk, \cdot}^{\text{assign}} \leq E_{\text{task}}^{\max}, \quad \forall tsk \in \mathcal{TSK}$$

- **C7 min_story_points_mix** (ID: C7)
Maintain minimum average story points of selected stories.

$$\frac{\sum_{us \in \mathcal{US}} \text{sp}_{us}^{US} \cdot x_{us}^{\text{story}}}{\sum_{us \in \mathcal{US}} x_{us}^{\text{story}}} \geq \bar{S}^{\min}$$

- **C8 max_backlog_size** (ID: C8)
Bound the number of entries in Product Backlog(s).

$$\text{entries}_{pb}^{PB} \leq N_{\text{backlog}}^{\max}, \quad \forall pb \in \mathcal{PB}$$

- **C9 max_review_duration** (ID: C9)
Sprint Review timeboxed.

$$d^{SR} \leq D_{SR}^{\max} \quad (\text{and } d^{SR} \leq 240 \text{ minutes})$$

- **C10 max_daily_scrum_duration** (ID: C10)
Daily Scrum ≤ 15 minutes.

$$d^{DS} \leq 15$$

- **C11 min_team_size** (ID: C11)
Ensure minimum team size for cross-functionality.

$$\text{team_size}_t \geq N_{\text{team}}^{\min}, \quad \forall t \in \mathcal{T}$$

- **C12 max_tasks_on_board** (ID: C12)
WIP limit on Scrum Board.

$$\sum_{tsk \in \mathcal{TSK}} x_{tsk, scb}^{\text{onBoard}} \leq L^{\text{WIP}}, \quad \forall scb \in \mathcal{SCB}$$

5. Decision Variables

- $y_f^{\text{feat}} \in \{0, 1\}$ (DV0 `select_feature_f`): include feature f in release.
- $x_{us}^{\text{story}} \in \{0, 1\}$ (DV1 `select_user_story_us`): include user story us in next sprint.
- $x_{tsk, sp}^{\text{assign}} \in \{0, 1\}$ (DV2 `assign_task_to_sprint`): assign task tsk to sprint sp (active).
- $r_{us}^{US} \in \{1, \dots, 100\}$ (DV3 `prioritize_user_story_us`): priority rank for us .
- $h^{\text{cap}} \geq 0$ (DV4 `allocate_team_capacity_hours`): total capacity (hours) allocated to sprint.

- $sp_{us}^{\text{set}} \in \{0, \dots, 100\}$ (DV5 `set_story_points_us`): assigned story points for us .
- $e_{tsk}^{\text{set}} \in \{0, \dots, 100\}$ (DV6 `set_task_effort_t`): assigned effort for tsk .
- $d^{SR} \in \{0, \dots, 240\}$ (DV7 `schedule_review_duration`): minutes for Sprint Review.
- $L^{\text{WIP}} \in \{1, \dots, 200\}$ (DV8 `limit_board.wip`): WIP limit for Scrum Board(s).
- $z_{rep}^{\text{rep}} \in \{0, 1\}$ (DV9 `select_release_plan_rep`): activate release plan rep .
- $d^{\text{target}} \in \{\text{dev}, \text{staging}, \text{production}\}$ (DV10 `choose_deployment_target`): deployment target for snapshot(s).
- $\alpha^{SG} \in \{0, 1\}$ (DV11 `set_sprint_goal_achievement_target`): target indicator for goal achievement.
- $a^{PO} \in \{0, \dots, 100\}$ (DV12 `assign_product_owner_availability`): PO availability reserved (%).

Integrating Relationships with Decision Variables

- Task–Story consistency: $x_{tsk,sp}^{\text{assign}} \leq \sum_{us:tsk \in \mathcal{TSK}(us)} x_{us}^{\text{story}}$.
- Release feature selection consistent with active plan: $y_f^{\text{feat}} \leq \sum_{rep:f \in \mathcal{F}(rep)} z_{rep}^{\text{rep}}$.
- Board cards proxy: $n_{scb}^{\text{cards}} = \sum_{tsk \in \mathcal{TSK}(scb)} x_{tsk,scb}^{\text{onBoard}}$, with $x_{tsk,scb}^{\text{onBoard}} \in \{0, 1\}$.
- Sprint capacity: $\sum_{tsk} e_{tsk}^{\text{set}} \cdot x_{tsk,sp}^{\text{assign}} \leq h^{\text{cap}}$.