Optimization Model for Scrum Project Management

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Contents

1 Sets (Entities)

The following sets represent the core entities in the Scrum domain model.

- P: Set of all projects.
- T: Set of all teams.
- W: Set of all workers.
- F: Set of all features.
- S: Set of all skills.
- R: Set of all roles.
- PO: Set of all product owners.
- SM: Set of all scrum masters.
- PB: Set of all product backlogs.
- SP: Set of all sprints.
- SBL: Set of all sprint backlogs.
- E: Set of all epics.
- US: Set of all user stories.
- TSK: Set of all tasks.
- BL: Set of all blockers.
- SH: Set of all stakeholders.
- VEL: Set of all velocity records.
- REP: Set of all release plans.

2 Indices

The following indices are used to iterate over the sets defined above.

- $p \in P$: Index for projects.
- $t \in T$: Index for teams.
- $w \in W$: Index for workers.
- $f \in F$: Index for features.
- $us \in US$: Index for user stories.
- $sp \in SP$: Index for sprints.
- $tsk \in TSK$: Index for tasks.
- $bl \in BL$: Index for blockers.
- $sh \in SH$: Index for stakeholders.
- $rep \in REP$: Index for release plans.

3 Decision Variables

These are the variables that the optimization model will determine.

- **ID: DV0 (assign_worker_to_team)**: $X_{wt} \in \{0, 1\}$. 1 if worker w is assigned to team t.
- **ID: DV1 (assign_story_to_sprint)**: $Y_{us,sp} \in \{0, 1\}$. 1 if user story us is assigned to sprint sp.
- **ID: DV2 (assign_team_to_project)**: $A_{tp} \in \{0, 1\}$. 1 if team t is assigned to project p.
- **ID: DV3 (select_feature_for_release)**: $B_{f,rep} \in \{0, 1\}$. 1 if feature f is selected for release plan rep.
- **ID: DV8 (assign_task_to_worker)**: $C_{tsk,w} \in \{0, 1\}$. 1 if task tsk is assigned to worker w.

4 Goals (Objective Function)

The objective is to maximize a weighted sum of various business goals. Minimization goals are included by negating their value. Parameters like $priority_p$ or $effort_{tsk}$ are assumed to be input data from the system.

$Maximize \quad Z = \sum_{g \in Goals} w_g \cdot \text{Term}_g$

- **ID: G0 (maximize_project_priority)**: Prioritize work on high-priority projects.

$$w_{G0} \cdot \sum_{p \in P} \sum_{t \in T} priority_p \cdot A_{tp}$$

- **ID: G2 (maximize_feature_priority)**: Prioritize implementing high-priority features.

$$w_{G2} \cdot \sum_{f \in F} \sum_{rep \in REP} priority_f \cdot B_{f,rep}$$

- **ID: G3 (maximize_sprint_story_points)**: Maximize value delivered per sprint.

$$w_{G3} \cdot \sum_{sp \in SP} \sum_{us \in US} story\_points_{us} \cdot Y_{us,sp}$$

- **ID: G4 (minimize_project_budget_usage)**: Minimize cost.

$$-w_{G4} \cdot \sum_{p \in P} budget_p$$

- **ID: G5 (minimize_task_effort)**: Increase efficiency.

$$-w_{G5} \cdot \sum_{tsk \in TSK} \sum_{w \in W} effort_{tsk} \cdot C_{tsk,w}$$

- **ID: G6 (minimize_blocker_severity)**: Reduce risks.

$$-w_{G6} \cdot \sum_{bl \in BL} severity_{bl}$$

- **ID: G11 (maximize_user_story_priority)**: Focus on important user stories.

$$w_{G11} \cdot \sum_{us \in US} \sum_{sp \in SP} priority_{us} \cdot Y_{us,sp}$$

5 Conditions (Constraints)

These are the rules and limitations that the solution must adhere to.

- **ID: C0 (valid_team_size)**: Team size must be within Scrum limits.

$$3 \leq \sum_{w \in W} X_{wt} \leq 9 \quad \forall t \in T$$

- **ID: C1 (valid_sprint_dates)**: Sprint start date must precede end date.

$$start\_date_{sp} < end\_date_{sp} \quad \forall sp \in SP$$

- **ID: C4 (team_assigned_to_project)**: Each team must be assigned to exactly one project.

$$\sum_{p \in P} A_{tp} = 1 \quad \forall t \in T$$

- **ID: C6 (worker_availability_constraint)**: The total effort of tasks assigned to a worker in a sprint cannot exceed their availability.

$$\sum_{tsk \in TSK_{sp}} effort_{tsk} \cdot C_{tsk,w} \leq availability_w \quad \forall w \in W, \forall sp \in SP$$

- **ID: C9 (user_story_has_points)**: A user story can only be assigned to a sprint if it has story points.

$$Y_{us,sp} \cdot story\_points_{us} > 0 \quad \forall us \in US, \forall sp \in SP$$

- **ID: C10 (project_budget_limit)**: Total worker costs for a project must not exceed its budget. Let $cost_w$ be the cost of worker w.

$$\sum_{t \in T} \sum_{w \in W} A_{tp} \cdot X_{wt} \cdot cost_w \leq budget_p \quad \forall p \in P$$

- **ID: C12 (worker_has_one_team)**: Each worker can be assigned to at most one team.

$$\sum_{t \in T} X_{wt} \leq 1 \quad \forall w \in W$$