Mathematical Formulation of a SCRUM Optimization Model

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September 5, 2025

Contents

1 Sets (Entities)

This section defines the fundamental sets used in the model, derived from the entities in the domain.

- P: The set of all projects.
- T: The set of all teams.
- W: The set of all workers.
- F: The set of all features.
- S: The set of all skills.
- R: The set of all roles.
- PO: The set of all product owners.
- SM: The set of all scrum masters.
- PB: The set of all product backlogs.
- SP: The set of all sprints.
- SPP: The set of all sprint planning meetings.
- DS: The set of all daily scrums.
- SR: The set of all sprint reviews.
- SRE: The set of all sprint retrospectives.
- SBL: The set of all sprint backlogs.
- SG: The set of all sprint goals.
- E: The set of all epics.
- US: The set of all user stories.
- TSK: The set of all tasks.
- DEV: The set of all development snapshots.
- BL: The set of all blockers.
- SH: The set of all stakeholders.
- VEL: The set of all velocity measurements.
- REP: The set of all release plans.
- RM: The set of all roadmaps.
- SCB: The set of all scrum boards.
- FED: The set of all feature documentations.

2 Indices

This section defines the indices used to iterate over the sets defined in Section 1.

- $p \in P$: An index representing a specific project.
- $t \in T$: An index representing a specific team.
- $w \in W$: An index representing a specific worker.
- $f \in F$: An index representing a specific feature.
- $s \in S$: An index representing a specific skill.
- $r \in R$: An index representing a specific role.
- $po \in PO$: An index representing a specific product owner.
- $sm \in SM$: An index representing a specific scrum master.
- $pb \in PB$: An index representing a specific product backlog.
- $sp \in SP$: An index representing a specific sprint.
- $spp \in SPP$: An index representing a specific sprint planning meeting.
- $ds \in DS$: An index representing a specific daily scrum.
- $sr \in SR$: An index representing a specific sprint review.
- $sre \in SRE$: An index representing a specific sprint retrospective.
- $sbl \in SBL$: An index representing a specific sprint backlog.
- $sg \in SG$: An index representing a specific sprint goal.
- $e \in E$: An index representing a specific epic.
- $us \in US$: An index representing a specific user story.
- $tsk \in TSK$: An index representing a specific task.
- $dev \in DEV$: An index representing a specific development snapshot.
- $bl \in BL$: An index representing a specific blocker.
- $sh \in SH$: An index representing a specific stakeholder.
- $vel \in VEL$: An index representing a specific velocity measurement.
- $rep \in REP$: An index representing a specific release plan.
- $rm \in RM$: An index representing a specific roadmap.
- $scb \in SCB$: An index representing a specific scrum board.
- $fed \in FED$: An index representing a specific feature documentation.

3 Decision Variables

These are the variables that the optimization model can change to achieve the goals. Parameters (attributes of entities) are denoted with text, e.g., story_points$_{us}$.

- $X_{us,sbl} \in \{0,1\}$: Binary variable; 1 if User Story us is assigned to Sprint Backlog sbl, 0 otherwise.

- $A_{w,t} \in \{0,1\}$: Binary variable; 1 if Worker w is assigned to Team t, 0 otherwise.

- $B_{tsk,w} \in \{0,1\}$: Binary variable; 1 if Task tsk is assigned to Worker w, 0 otherwise.

- $I_{f,rep} \in \{0,1\}$: Binary variable; 1 if Feature f is included in Release Plan rep, 0 otherwise.

- $J_{w,r} \in \{0,1\}$: Binary variable; 1 if Worker w takes on Role r, 0 otherwise.

- $P_f \in \mathbb{Z}^+$: Integer variable for the priority assigned to Feature f.

- $S_{us} \in \mathbb{Z}^+$: Integer variable for the story points assigned to User Story us.

- $E_{tsk} \in \mathbb{R}^+$: Real variable for the effort assigned to Task tsk.

- Status$_{tsk} \in \{$todo, in_progress, done$\}$: Categorical variable for the status of Task tsk.

- Budget$_p \in \mathbb{R}^+$: Real variable for the budget allocated to Project p.

- Severity$_{bl} \in \mathbb{Z}^+$: Integer variable for the severity assigned to Blocker bl.

4 Goals

This section describes the objectives of the model. The overall objective function is a weighted sum of these individual goals, where W_{Gi} is the weight for goal i.

- **G0: maximize_story_points**
  Maximize the total story points from user stories assigned to sprint backlogs.

$$\text{Maximize} \quad W_{G0} \sum_{us \in US} \sum_{sbl \in SBL} S_{us} \cdot X_{us,sbl}$$

- **G2: maximize_feature_priority**
  Maximize the value from features included in release plans, based on their priority.

$$\text{Maximize} \quad W_{G2} \sum_{f \in F} \sum_{rep \in REP} P_f \cdot I_{f,rep}$$

- **G3: maximize_team_satisfaction**
  Maximize the team satisfaction recorded in sprint retrospectives.

$$\text{Maximize} \quad W_{G3} \sum_{sre \in SRE} \text{team\_satisfaction}_{sre}$$

- **G4: minimize_blocker_severity**
  Minimize the sum of severities of all blockers.

$$\text{Minimize} \quad W_{G4} \sum_{bl \in BL} \text{Severity}_{bl}$$

- **G7: minimize_task_effort**
  Minimize the total effort planned for all tasks.

$$\text{Minimize} \quad W_{G7} \sum_{tsk \in TSK} E_{tsk}$$

- **G10: maximize_worker_availability**
  Maximize the total availability of all workers.

$$\text{Maximize} \quad W_{G10} \sum_{w \in W} \text{availability}_w$$

5 Conditions

This section lists the constraints that the solution must satisfy.

- **C0: constrain_team_size_upper**
  The size of any given team must not exceed 9 members. Let WorkersOf(t) be the set of workers assigned to team t.
  $$\forall t \in T: \quad \sum_{w \in W} A_{w,t} \leq 9$$

- **C1: constrain_team_size_lower**
  The size of any given team must be at least 3 members.
  $$\forall t \in T: \quad \sum_{w \in W} A_{w,t} \geq 3$$

- **C4: constrain_sprint_effort_by_velocity**
  The total story points of user stories in a sprint backlog must not exceed the associated team's velocity. Let VelocityOf(t) be the velocity for team t.
  $$\forall sbl \in SBL, \forall t \in T: \quad \sum_{us \in US} S_{us} \cdot X_{us,sbl} \leq \text{VelocityOf}(t)$$

- **C6: task_must_be_done_for_story**
  If a User Story's status is 'done', all its constituent tasks must also be 'done'. Let TasksOf(us) be the set of tasks for a user story.
  $$\forall us \in US: (\text{status}_{us} = \text{'done'}) \implies (\forall tsk \in \text{TasksOf}(us), \text{Status}_{tsk} = \text{'done'})$$

- **C7: constrain_project_by_budget**
  The project's incurred costs must not exceed its allocated budget.
  $$\forall p \in P: \quad \text{IncurredCost}(p) \leq \text{Budget}_p$$

- **C8: require_feature_priority**
  Every feature must have a priority greater than zero.
  $$\forall f \in F: \quad P_f \geq 1$$

- **C10: worker_must_be_available**
  A task can only be assigned to a worker if their status is 'available'.
  $$\forall tsk \in TSK, \forall w \in W: \quad B_{tsk,w} = 1 \implies \text{status}_w = \text{'available'}$$