

Optimization Model for SCRUM Project Management

Gemini

September 5, 2025

Contents

1	Sets (Entities)	1
2	Indices	1
3	Decision Variables	2
4	Goals (Objective Functions)	2
5	Conditions (Constraints)	3

1 Sets (Entities)

This section defines the fundamental sets used in the model, derived from the domain entities.

- **P**: The set of all projects. (Project)
- **T**: The set of all development teams. (Team)
- **W**: The set of all individual workers. (Worker)
- **F**: The set of all features. (Feature)
- **S**: The set of all skills. (Skill)
- **R**: The set of all roles. (Role)
- **PO**: The set of all Product Owners. (ProductOwner)
- **SM**: The set of all Scrum Masters. (ScrumMaster)
- **PB**: The set of all Product Backlogs. (ProductBacklog)
- **SP**: The set of all sprints. (Sprint)
- **SBL**: The set of all Sprint Backlogs. (SprintBacklog)
- **SG**: The set of all Sprint Goals. (SprintGoal)
- **E**: The set of all epics. (Epic)
- **US**: The set of all user stories. (UserStory)
- **TSK**: The set of all tasks. (Task)
- **BL**: The set of all blockers. (Blocker)
- **SH**: The set of all stakeholders. (Stakeholder)
- **REP**: The set of all release plans. (ReleasePlan)

2 Indices

This section defines the indices used to iterate over the sets.

- $p \in P$: Index for a project.
- $t \in T$: Index for a team.
- $w \in W$: Index for a worker.
- $f \in F$: Index for a feature.
- $s \in S$: Index for a skill.
- $r \in R$: Index for a role.
- $po \in PO$: Index for a Product Owner.
- $sm \in SM$: Index for a Scrum Master.
- $sp \in SP$: Index for a sprint.

- $us \in US$: Index for a user story.
- $tsk \in TSK$: Index for a task.
- $bl \in BL$: Index for a blocker.
- $rep \in REP$: Index for a release plan.

3 Decision Variables

This section defines the variables that the optimization model will determine.

- **(DV0) assign_worker_to_team**: $X_{w,t} \in \{0,1\}$. Binary variable, 1 if worker w is assigned to team t , 0 otherwise.
- **(DV1) assign_us_to_sprint**: $Y_{us,sp} \in \{0,1\}$. Binary variable, 1 if user story us is assigned to sprint sp , 0 otherwise.
- **(DV2) assign_task_to_worker**: $Z_{tsk,w} \in \{0,1\}$. Binary variable, 1 if task tsk is assigned to worker w , 0 otherwise.
- **(DV3) select_feature_for_release**: $K_{f,rep} \in \{0,1\}$. Binary variable, 1 if feature f is included in release plan rep , 0 otherwise.
- **(DV4) set_sprint_start_date**: S_{sp}^{start} . Continuous variable representing the start date of sprint sp .
- **(DV5) set_team_size**: N_t . Integer variable representing the number of workers in team t .
- **(DV9) assign_po_to_project**: $U_{po,p} \in \{0,1\}$. Binary variable, 1 if Product Owner po is assigned to project p , 0 otherwise.
- **(DV10) assign_sm_to_team**: $V_{sm,t} \in \{0,1\}$. Binary variable, 1 if Scrum Master sm is assigned to team t , 0 otherwise.

4 Goals (Objective Functions)

This section formulates the objectives to be maximized or minimized. The overall objective function is a weighted sum of these individual goals. Parameters are denoted with Greek letters (e.g., σ_{us} for story points).

- **(G0) maximize_sprint_story_points**: Maximize the total story points completed in a sprint.

$$\max \sum_{us \in US} \sum_{sp \in SP} \sigma_{us} \cdot Y_{us,sp}$$

where σ_{us} is the story points for user story us .

- **(G3) maximize_implemented_feature_priority**: Maximize the priority of implemented features.

$$\max \sum_{f \in F} \sum_{rep \in REP} \pi_f \cdot K_{f,rep}$$

where π_f is the priority value for feature f .

- **(G4) minimize_total_task_effort:** Minimize the effort for all assigned tasks.

$$\min \sum_{tsk \in TSK} \sum_{w \in W} \epsilon_{tsk} \cdot Z_{tsk,w}$$

where ϵ_{tsk} is the effort for task tsk .

- **(G6) minimize_blocker_severity:** Minimize the severity of unresolved blockers. Let $B_{tsk,bl} = 1$ if task tsk is affected by blocker bl .

$$\min \sum_{tsk \in TSK} \sum_{bl \in BL} \delta_{bl} \cdot B_{tsk,bl} \cdot (1 - \text{resolved}_{bl})$$

where δ_{bl} is the severity of blocker bl and resolved_{bl} is a binary parameter.

- **(G10) maximize_assigned_worker_skill_level:** Maximize the skill level of workers assigned to tasks. Let $\text{ReqSkill}_{tsk,s} = 1$ if task tsk requires skill s .

$$\max \sum_{tsk \in TSK} \sum_{w \in W} \sum_{s \in S} (\lambda_{w,s} \cdot \text{ReqSkill}_{tsk,s}) \cdot Z_{tsk,w}$$

where $\lambda_{w,s}$ is the level of skill s for worker w .

5 Conditions (Constraints)

This section formulates the constraints that the solution must satisfy.

- **(C0) worker_availability_constraint:** A task can only be assigned to an available worker.

$$\forall tsk \in TSK, \forall w \in W : Z_{tsk,w} \leq \alpha_w$$

where α_w is a binary parameter indicating worker w 's availability (1 if available, 0 otherwise).

- **(C1) team_size_limit:** The number of workers assigned to a team must be within a given range.

$$\forall t \in T : 3 \leq \sum_{w \in W} X_{w,t} \leq 9$$

- **(C3) project_budget_constraint:** The total cost (e.g., from worker assignments) must not exceed the project budget.

$$\sum_{w \in W} \sum_{t \in T} (\gamma_w \cdot X_{w,t}) \leq B_p$$

where γ_w is the cost/salary of worker w and B_p is the budget for project p .

- **(C4) task_skill_requirement:** A worker assigned to a task must possess the required skill above a minimum level. Let $\text{ReqSkill}_{tsk,s} = 1$ if task tsk requires skill s , and Λ_{min} be the minimum required skill level.

$$Z_{tsk,w} \cdot \text{ReqSkill}_{tsk,s} \cdot (\Lambda_{min} - \lambda_{w,s}) \leq 0 \quad \forall tsk, w, s$$

- **(C7) one_scrum_master_per_team:** Each team must have exactly one Scrum Master.

$$\forall t \in T : \sum_{sm \in SM} V_{sm,t} = 1$$

- **(C11) sprint_backlog_effort_limit:** The total effort of user stories in a sprint must not exceed the team's velocity. Let $Effort_{us}$ be the effort of a user story and Vel_t be the velocity of the team assigned to sprint sp .

$$\sum_{us \in US} Effort_{us} \cdot Y_{us,sp} \leq Vel_t \quad \forall sp \in SP$$

- **(C12) user_story_in_one_sprint:** A user story can be assigned to at most one sprint.

$$\forall us \in US : \quad \sum_{sp \in SP} Y_{us,sp} \leq 1$$