Optimization Model for SCRUM Project Management

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Contents

1 Sets (Entities)

These sets are derived from the domain model entities. The letters in parentheses are used to denote the sets in mathematical formulations.

- **Project (P)**: The product or initiative to be developed.

- **Team (T)**: Self-organized, cross-functional development team.

- **Worker (W)**: Individual team member working on the project.

- **Feature (F)**: Mid-sized functionality.

- **Skill (S)**: Professional or social competence of a worker.

- **Role (R)**: Defined responsibilities within the Scrum team.

- **ProductOwner (PO)**: Responsible for product vision and Product Backlog.

- **ScrumMaster (SM)**: Supports the team in applying Scrum.

- **ProductBacklog (PB)**: Ordered list of all requirements.

- **Sprint (SP)**: Fixed time period for creating an increment.

- **UserStory (US)**: Requirement from the perspective of a user.

- **Task (TSK)**: Smallest unit of work within a sprint.

- **Blocker (BL)**: Obstacle hindering progress.

- **Stakeholder (SH)**: Interested party in the product.

- **Velocity (VEL)**: Average amount of work per sprint.

- **ReleasePlan (REP)**: Plan for releasing specific features.

- **SprintGoal (SG)**: Objective to be achieved within the sprint.

2 Indices

These indices are used to iterate over the sets defined in the previous section.

- $p \in P$: Index for a project.

- $t \in T$: Index for a team.

- $w \in W$: Index for a worker.

- $f \in F$: Index for a feature.

- $s \in S$: Index for a skill.

- $r \in R$: Index for a role.

- $po \in PO$: Index for a Product Owner.

- $sm \in SM$: Index for a Scrum Master.

- $sp \in SP$: Index for a sprint.

- $us \in US$: Index for a user story.

- $tsk \in TSK$: Index for a task.

- $bl \in BL$: Index for a blocker.

- $rep \in REP$: Index for a release plan.

- $sg \in SG$: Index for a sprint goal.

3 Decision Variables

These are the variables the model will determine to achieve the optimal solution.

- **DV0 (assign_story_to_sprint)**: $x_{us,sp} \in \{0,1\}$. Binary variable, 1 if user story us is assigned to sprint sp, 0 otherwise.

- **DV1 (assign_task_to_worker)**: $y_{tsk,w} \in \{0,1\}$. Binary variable, 1 if task tsk is assigned to worker w, 0 otherwise.

- **DV2 (select_feature_for_release)**: $z_{f,rep} \in \{0,1\}$. Binary variable, 1 if feature f is selected for release plan rep, 0 otherwise.

- **DV3 (set_user_story_priority)**: $prio_{us} \in \mathbb{Z}^+$. Integer variable for the priority of user story us.

- **DV4 (estimate_story_points)**: $spoints_{us} \in \{1, 2, 3, 5, 8, ...\}$. Integer variable for the story points of user story us.

- **DV5 (order_backlog_item)**: $rank_{us} \in \mathbb{Z}^+$. Integer variable for the rank of user story us in the backlog.

- **DV6 (assign_team_to_project)**: $a_{t,p} \in \{0,1\}$. Binary variable, 1 if team t is assigned to project p, 0 otherwise.

- **DV10 (set_task_effort_hours)**: $effort_{tsk} \in \mathbb{R}^+$. Continuous variable for the effort in hours for task tsk.

4 Goals (Objective Function)

The objective function is a weighted sum of the following individual goals.

- **G0 (maximize_completed_story_points)**: Maximize the total story points of user stories assigned to sprints.

$$\text{maximize} \sum_{us \in US} \sum_{sp \in SP} spoints_{us} \cdot x_{us,sp}$$

- **G1 (maximize_team_velocity)**: Maximize the average story points completed per sprint for each team.

$$\text{maximize} \sum_{t \in T} \frac{\sum_{sp \in SP_t} \sum_{us \in US} spoints_{us} \cdot x_{us,sp}}{|SP_t|}$$

- **G2 (minimize_task_effort)**: Minimize the sum of effort for all defined tasks.

$$\text{minimize} \sum_{tsk \in TSK} effort_{tsk}$$

- **G3 (maximize_feature_priority)**: Maximize the priority of features selected for release. ($priority_f$ is a given parameter).

$$\text{maximize} \sum_{f \in F} \sum_{rep \in REP} priority_f \cdot z_{f,rep}$$

- **G5 (maximize_sprint_goal_achievement)**: Maximize the achievement of sprint goals. Let ach_{sg} be a parameter representing potential achievement.

$$\text{maximize} \sum_{sg \in SG} ach_{sg}$$

- **G6 (minimize_project_timeline)**: Minimize the planned end date of the project. (end_date_p is a variable).

$$\text{minimize } end\_date_p$$

- **G8 (maximize_stakeholder_relevance)**: Maximize the relevance of selected features to stakeholders. ($relevance_{f,sh}$ is a parameter).

$$\text{maximize} \sum_{f \in F} \sum_{rep \in REP} \sum_{sh \in SH} relevance_{f,sh} \cdot z_{f,rep}$$

5 Conditions (Constraints)

These are the constraints that the solution must satisfy.

- **C0 (enforce_max_team_size)**: The number of workers assigned to any team must not exceed a maximum value (e.g., 9). Let $b_{w,t}$ be a decision variable for assigning worker w to team t.

$$\sum_{w \in W} b_{w,t} \leq 9 \quad \forall t \in T$$

- **C1 (enforce_min_team_size)**: The number of workers assigned to any team must be at least a minimum value (e.g., 3).

$$\sum_{w \in W} b_{w,t} \geq 3 \quad \forall t \in T$$

- **C2 (sprint_goal_is_defined)**: The description of a sprint goal must exist. This is a data integrity constraint, expressed logically.

$$\text{description}_{sg} \neq \text{NULL} \quad \forall sg \in SG$$

- **C3 (worker_is_available_for_assignment)**: A task can only be assigned to a worker if their status parameter is 'available'.

$$y_{tsk,w} = 0 \quad \forall tsk \in TSK, \forall w \in W \text{ where } status_w \neq \text{'available'}$$

- **C4 (task_effort_is_positive)**: The assigned effort for any task must be greater than zero.
$$effort_{tsk} \geq \epsilon \quad \forall tsk \in TSK \quad (\text{where } \epsilon \text{ is a small positive constant})$$

- **C5 (story_should_have_acceptance_criteria)**: This is a soft constraint. In a model, it could be a penalty in the objective function if a story is selected without criteria.

$$\text{penalty} = \sum_{us \in US} \sum_{sp \in SP} (1 - \mathbb{I}(criteria_{us} \neq \text{NULL})) \cdot x_{us,sp}$$

- **C6 (budget_must_not_be_exceeded)**: The sum of costs associated with tasks and resources must not exceed the project budget.

$$\sum_{tsk \in TSK} cost(effort_{tsk}) + \sum_{w \in W} salary_w \leq budget_p \quad \forall p \in P$$

- **C7 (no_work_on_unapproved_stories)**: A user story cannot be assigned to a sprint if its status parameter is not 'Approved'.

$$x_{us,sp} = 0 \quad \forall us \in US, \forall sp \in SP \text{ where } status_{us} \neq \text{'Approved'}$$

- **C10 (task_in_progress_not_blocked)**: A task cannot be assigned if it is linked to an active blocker.

$$y_{tsk,w} = 0 \quad \forall tsk \in TSK, \forall w \in W \text{ if } \exists bl \in BL \text{ where } blocker\_link_{tsk,bl} = 1 \wedge status_{bl} = \text{'active'}$$