Optimization Model for Scrum-based Software Development

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Contents

1 Sets (Entities)

We define the following sets based on the entities in the domain model. Each capital letter represents a set of instances of that entity.

- P: Set of all Projects
- T: Set of all Teams
- W: Set of all Workers
- F: Set of all Features
- S: Set of all Skills
- R: Set of all Roles
- PO: Set of all Product Owners
- SM: Set of all Scrum Masters
- PB: Set of all Product Backlogs
- SP: Set of all Sprints
- SBL: Set of all Sprint Backlogs
- SG: Set of all Sprint Goals
- E: Set of all Epics
- US: Set of all User Stories
- TSK: Set of all Tasks
- BL: Set of all Blockers
- SH: Set of all Stakeholders
- VEL: Set of all Team Velocities
- REP: Set of all Release Plans

Let $A(e, attr)$ denote the value of attribute $attr$ for an entity instance e.

2 Indices

The following indices are used to iterate over the sets defined above.

- $p \in P$: index for projects
- $t \in T$: index for teams
- $w \in W$: index for workers
- $f \in F$: index for features
- $s \in S$: index for skills
- $us \in US$: index for user stories
- $tsk \in TSK$: index for tasks
- $sp \in SP$: index for sprints
- $bl \in BL$: index for blockers

3 Decision Variables

These variables represent the decisions to be optimized by the model.

- $X_{w,tsk} \in \{0, 1\}$: 1 if worker w is assigned to task tsk, 0 otherwise. (DV0)
- $Y_{us,sp} \in \{0, 1\}$: 1 if user story us is assigned to sprint sp, 0 otherwise. (DV1)
- $Z_{f,rep} \in \{0, 1\}$: 1 if feature f is selected for release plan rep, 0 otherwise. (DV8)
- $P_f \in \{1, ..., 5\}$: Priority assigned to feature f. (DV3)
- $S_{tsk} \in \{$'todo', 'in_progress', 'done'$\}$: Status of task tsk. (DV5)
- D_{sp}^{start}: Start date of sprint sp. (DV4)

4 Goals (Objective Functions)

The primary objectives of the optimization model. The final objective function is a weighted sum of these individual goals.

- **G0 (maximize_feature_priority):** Maximize the priority of implemented features.

$$\max \sum_{f \in F} \sum_{rep \in REP} P_f \cdot Z_{f,rep}$$

- **G2 (maximize_team_velocity):** Maximize the average story points completed.

$$\max \frac{\sum_{us \in US} \sum_{sp \in SP} A(us, \text{story\_points}) \cdot Y_{us,sp}}{|SP|}$$

- **G3 (minimize_task_effort):** Minimize the total effort of scheduled tasks.

$$\min \sum_{tsk \in TSK} \sum_{w \in W} A(tsk, \text{effort}) \cdot X_{w,tsk}$$

- **G5 (minimize_number_of_blockers):** Minimize the number of unresolved blockers.

$$\min \sum_{bl \in BL} [A(bl, \text{status}) \neq \text{'resolved'}]$$

  where $[\cdot]$ is the Iverson bracket.

- **G7 (minimize_project_end_date):** Minimize the final project delivery date.

$$\min \max_{p \in P} A(p, \text{project\_end})$$

5 Conditions (Constraints)

These are the rules and limitations that the solution must adhere to.

- **C0 (budget_limit):** The total cost of tasks must not exceed the project budget. Let C_w be the cost per effort unit for worker w.

$$\sum_{w \in W} \sum_{tsk \in TSK} A(tsk, \text{effort}) \cdot C_w \cdot X_{w,tsk} \leq A(p, \text{budget}) \quad \forall p \in P$$

- **C1 (team_size_constraint):** Each team must have a size between 5 and 9.

$$5 \leq A(t, \text{team\_size}) \leq 9 \quad \forall t \in T$$

- **C2 (sprint_effort_capacity):** Total effort of user stories in a sprint must not exceed the team's velocity. Let $T(sp)$ be the team for sprint sp.

$$\sum_{us \in US} A(us, \text{story\_points}) \cdot Y_{us,sp} \leq A(VEL_{T(sp)}, \text{avg.\_story\_points}) \quad \forall sp \in SP$$

- **C3 (skill_requirement):** A worker assigned to a task must have the required skill. Let S_{tsk} be the set of required skills for task tsk, and S_w be the set of skills for worker w.

$$X_{w,tsk} = 0 \quad \text{if } S_{tsk} \not\subseteq S_w, \quad \forall w \in W, \forall tsk \in TSK$$

- **C5 (blocker_active):** A task cannot be 'done' if it has an active blocker. Let B_{tsk} be the set of blockers for task tsk.

$$S_{tsk} \neq \text{'done'} \quad \text{if } \exists bl \in B_{tsk} \text{ such that } A(bl, \text{status}) \neq \text{'resolved'}$$

- **C7 (sprint_date_fixed):** Sprints for a given team cannot overlap. Let D_{sp}^{end} be the end date.

$$D_{sp_i}^{end} \leq D_{sp_j}^{start} \vee D_{sp_j}^{end} \leq D_{sp_i}^{start} \quad \forall sp_i, sp_j \in SP, i \neq j, \text{ for same team}$$