Mathematical Optimization Model for SCRUM Project Management

AI Analyst

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Contents

Introduction

This document formally defines a multi-objective optimization model for resource allocation, scheduling, and prioritization within a software development company using the SCRUM framework. The model is derived from a defined domain structure (Entities and Relationships) and a set of Goals, Conditions, and Decision Variables.

1 Sets (Entities)

- Project = $\{p|p$ is a Project$\}$
- Team = $\{t|t$ is a Team$\}$
- Worker = $\{w|w$ is a Worker$\}$
- Feature = $\{f|f$ is a Feature$\}$
- Skill = $\{s|s$ is a Skill$\}$
- Role = $\{r|r$ is a Role$\}$
- ProductOwner = $\{po|po$ is a ProductOwner$\}$
- ScrumMaster = $\{sm|sm$ is a ScrumMaster$\}$
- ProductBacklog = $\{pb|pb$ is a ProductBacklog$\}$
- Sprint = $\{sp|sp$ is a Sprint$\}$
- SprintGoal = $\{sg|sg$ is a SprintGoal$\}$
- Blocker = $\{bl|bl$ is a Blocker$\}$
- Stakeholder = $\{sh|sh$ is a Stakeholder$\}$
- Velocity = $\{vel|vel$ is a Velocity$\}$

2 Indices

- $p \in$ Project
- $t \in$ Team
- $w \in$ Worker

- $f \in$ Feature
- $s \in$ Skill
- $sp \in$ Sprint
- $bl \in$ Blocker
- $sh \in$ Stakeholder

3 Goals

- **G0: maximize_team_utilization** - Maximize the average availability of all team members.

$$\text{Maximize } Z_0 = \sum_{w \in \text{Worker}} \text{availability}(w)$$

- **G1: minimize_project_duration** - Minimize the total duration of the project.

$$\text{Minimize } Z_1 = \text{project\_end}(p_0) - \text{project\_start}(p_0)$$

- **G2: maximize_feature_delivery** - Maximize the number of high-priority features delivered.

$$\text{Maximize } Z_2 = \sum_{f \in \text{Feature}} \text{priority}(f) \cdot x_f^{delivered} \quad \text{where } x_f^{delivered} \in \{0, 1\}$$

- **G3: minimize_blocker_impact** - Minimize the severity and duration of blockers.

$$\text{Minimize } Z_3 = \sum_{bl \in \text{Blocker}} \text{severity}(bl) \cdot (\text{resolved\_on}(bl) - \text{detected\_on}(bl))$$

- **G4: maximize_sprint_goal_achievement** - Maximize the rate of successfully achieved sprint goals.

$$\text{Maximize } Z_4 = \sum_{sg \in \text{SprintGoal}} \text{achievement\_status}(sg)$$

- **G5: minimize_task_effort_variance** - Minimize the variance between estimated and actual task effort.

$$\text{Minimize } Z_5 = \sum_{\text{tasks}} (\text{actual\_effort} - \text{estimated\_effort})^2$$

- **G6: maximize_stakeholder_satisfaction** - Maximize the average influence level of satisfied stakeholders.

$$\text{Maximize } Z_6 = \sum_{sh \in \text{Stakeholder}} \text{influence\_level}(sh) \cdot s(sh) \quad \text{where } s(sh) \text{ is a satisfaction measure}$$

- **G7: minimize_budget_overrun** - Minimize the project cost against its allocated budget.

$$\text{Minimize } Z_7 = \max(0, \text{total\_cost} - \text{budget}(p_0))$$

- **G8: maximize_team_velocity** - Maximize the average velocity of the team.

$$\text{Maximize } Z_8 = \text{avg\_story\_points}(vel_t)$$

- **G9: minimize_low_skill_gaps** - Minimize the number of workers with low skill levels.

$$\text{Minimize } Z_9 = \sum_{w \in \text{Worker}} \sum_{s \in \text{Skill}} I[\text{level}(s, w) < \text{threshold}]$$

- **G10: maximize_cross_functionality** - Maximize the number of certified skills within the team.

$$\text{Maximize } Z_{10} = \sum_{w \in \text{Worker}} \sum_{s \in \text{Skill}} \text{certified}(s, w)$$

4 Conditions

- **C0: condition_team_member_availability** - Ensure no team member is below a critical availability threshold.

$$\text{availability}(w) \geq \alpha_{\text{crit}} \quad \forall w \in \text{Worker}$$

- **C1: condition_project_not_delayed** - Ensure the project end date does not exceed its planned deadline.

$$\text{project\_end}(p) \leq \text{project\_deadline}(p)$$

- **C2: condition_high_severity_blockers_resolved** - Ensure no high-severity blockers remain unresolved.

$$\text{status}(bl) = \text{'resolved'} \quad \forall bl \in \text{Blocker where severity}(bl) \geq \text{'high'}$$

- **C3: condition_sprint_goal_met** - Ensure the primary sprint goal is at least partially achieved.

$$\text{achievement\_status}(sg_{sp}) \geq \text{'partial'} \quad \forall sp \in \text{Sprint}$$

- **C4: condition_budget_not_exceeded** - Ensure the project does not exceed its total budget.

$$\text{total\_cost} \leq \text{budget}(p_0)$$

- **C5: condition_critical_skills_covered** - Ensure all skills marked as critical for the project are present.

$$\sum_{w \in \text{Worker}} I[\text{has\_skill}(w, s) \wedge \text{level}(w, s) \geq l_{\text{req}}] \geq 1 \quad \forall s \in S_{\text{critical}}$$

- **C6: condition_product_owner_available** - Ensure the Product Owner has sufficient availability.

$$\text{availability}(po) \geq \beta_{\min}$$

- **C7: condition_minimum_team_size** - Ensure the team has the minimum required number of members.

$$\text{team\_size}(t) \geq n_{\min} \quad \forall t \in \text{Team}$$

- **C8: condition_feature_documented** - Ensure all high-priority features have documentation.

$$\text{has\_documentation}(f) = \text{True} \quad \forall f \in \text{Feature where priority}(f) \geq \text{'high'}$$

- **C9: condition_stakeholder_representation** - Ensure key high-influence stakeholders participate in reviews.

$$\text{participates}(sh, \text{SprintReview}) = \text{True} \quad \forall sh \in \text{Stakeholder where influence\_level}(sh) \geq \gamma_{\text{key}}$$

5 Decision Variables

- **DV0:** $x_{w,t}^{assign} \in \{0,1\}$ - Binary decision to assign worker w to team t.
- **DV1:** $x_{f,sp}^{sprint} \in \{0,1\}$ - Binary decision to include feature f in sprint sp.
- **DV2:** $b_f^{alloc} \in R, [0, 1000000]$ - Budget allocated to feature f.
- **DV3:** $d_{sp}^{sprint} \in Z, [7, 30]$ - Duration of sprint sp in days.
- **DV4:** $x_{tsk,w}^{task} \in \{0,1\}$ - Binary decision to assign task tsk to worker w.
- **DV5:** $pr_f^{backlog} \in Z, [1, 100]$ - Priority rank of feature f in the backlog.
- **DV6:** $a_w^{avail} \in R, [0.0, 1.0]$ - Percentage of time worker w is allocated.
- **DV7:** $date_{rp}^{release} \in \text{Date}, [2023-01-01, 2025-12-31]$ - Planned release date for release plan rp.
- **DV8:** $n_t^{size} \in Z, [3, 12]$ - Number of workers in team t.
- **DV9:** $e_{tsk}^{est} \in R, [0.5, 40]$ - Estimated effort for task tsk in person-hours.
- **DV10:** $pts_{us}^{story} \in Z, [1, 20]$ - Story points assigned to user story us.