Mathematical Formulation of a Scrum Project Optimization Model

Gemini AI

September 5, 2025

Contents

1 Sets (Entities)

The fundamental sets of the model are derived from the entities in the Scrum domain. Each set represents a collection of a specific type of object.

- P: Project - The product or initiative to be developed
- T: Team - Self-organized, cross-functional development team
- W: Worker - Individual team member working on the project
- F: Feature - Mid-sized functionality
- S: Skill - Professional or social competence of a worker
- R: Role - Defined responsibilities within the Scrum team
- PO: ProductOwner - Responsible for product vision and Product Backlog
- SM: ScrumMaster - Supports the team in applying Scrum
- PB: ProductBacklog - Ordered list of all requirements
- SP: Sprint - Fixed time period for creating an increment
- SPP: SprintPlanning - Kick-off meeting for Sprint preparation
- DS: DailyScrum - Daily 15-minute team meeting
- SR: SprintReview - Presentation and acceptance of results
- SRE: SprintRetrospective - Retrospective for process improvement
- SBL: SprintBacklog - Selected backlog items + implementation plan
- SG: SprintGoal - Objective to be achieved within the sprint
- E: Epic - Large requirement that can be split into stories
- US: UserStory - Requirement from the perspective of a user
- TSK: Task - Smallest unit of work within a sprint
- DEV: DevelopmentSnapshot - Product at the end of a sprint
- BL: Blocker - Obstacle hindering progress
- SH: Stakeholder - Interested party in the product (internal/external)
- VEL: Velocity - Average amount of work per sprint
- REP: ReleasePlan - Plan for releasing specific features
- RM: Roadmap - Long-term planning across releases
- SCB: ScrumBoard - Visual representation of tasks during the sprint
- FED: FeatureDocumentation - Documentation for a specific feature

2 Indices

Indices are used to refer to specific elements within the sets defined in Section 1.

- $p \in P$: Index for a project
- $t \in T$: Index for a team
- $w \in W$: Index for a worker
- $f \in F$: Index for a feature
- $s \in S$: Index for a skill
- $r \in R$: Index for a role
- $po \in PO$: Index for a Product Owner
- $sm \in SM$: Index for a Scrum Master
- $pb \in PB$: Index for a Product Backlog
- $sp \in SP$: Index for a sprint
- $spp \in SPP$: Index for a Sprint Planning meeting
- $ds \in DS$: Index for a Daily Scrum meeting
- $sr \in SR$: Index for a Sprint Review
- $sre \in SRE$: Index for a Sprint Retrospective
- $sbl \in SBL$: Index for a Sprint Backlog
- $sg \in SG$: Index for a Sprint Goal
- $e \in E$: Index for an epic
- $us \in US$: Index for a user story
- $tsk \in TSK$: Index for a task
- $dev \in DEV$: Index for a development snapshot
- $bl \in BL$: Index for a blocker
- $sh \in SH$: Index for a stakeholder
- $vel \in VEL$: Index for a velocity metric
- $rep \in REP$: Index for a release plan
- $rm \in RM$: Index for a roadmap
- $scb \in SCB$: Index for a Scrum Board
- $fed \in FED$: Index for a feature documentation

3 Decision Variables

Decision variables represent the choices to be made by the optimization model. For any entity instance x, its attributes are denoted as parameters (e.g., Priority_f for the priority of feature f).

DV0: *assign_worker_to_team*

$$X_{w,t} \in \{0,1\} \quad \forall w \in W, t \in T$$

where $X_{w,t} = 1$ if worker w is assigned to team t, and 0 otherwise.

DV1: *assign_story_to_sprint*

$$Y_{us,sp} \in \{0,1\} \quad \forall us \in US, sp \in SP$$

where $Y_{us,sp} = 1$ if user story us is assigned to sprint sp, and 0 otherwise.

DV2: *select_team_size*

$$\text{TeamSize}_t \in \{3,4,...,9\} \quad \forall t \in T$$

An integer variable representing the number of members in team t.

DV3: *set_sprint_duration_days*

$$\text{SprintDuration}_{sp} \in \{7,14,21,28\} \quad \forall sp \in SP$$

A discrete variable for the duration of sprint sp.

DV4: *allocate_feature_budget*

$$\text{FeatureBudget}_f \geq 0 \quad \forall f \in F$$

A continuous variable for the budget allocated to feature f.

DV5: *set_user_story_priority*

$$\text{StoryPriority}_{us} \in \{1,2,3,4,5\} \quad \forall us \in US$$

An integer variable setting the priority for user story us.

DV6: *assign_task_to_worker*

$$Z_{tsk,w} \in \{0,1\} \quad \forall tsk \in TSK, w \in W$$

where $Z_{tsk,w} = 1$ if task tsk is assigned to worker w, and 0 otherwise.

DV7: *set_worker_availability_percent*

$$\text{Availability}_w \in [0.0, 1.0] \quad \forall w \in W$$

A continuous variable for the availability of worker w.

DV8: *include_feature_in_release*

$$U_{f,rep} \in \{0,1\} \quad \forall f \in F, rep \in REP$$

where $U_{f,rep} = 1$ if feature f is included in release plan rep, and 0 otherwise.

4 Goals (Objective Functions)

Goals represent the optimization objectives. They are functions of parameters and decision variables to be maximized or minimized. Each goal is weighted by a factor W_g. The overall objective is typically a weighted sum of these individual goals.

G0: *maximize_total_story_points*

$$\max \left(W_{G0} \cdot \sum_{us \in US} \sum_{sp \in SP} \text{StoryPoints}_{us} \cdot Y_{us,sp} \right)$$

Maximizes the total story points of user stories assigned to sprints.

G2: *maximize_priority_of_features*

$$\max \left(W_{G2} \cdot \sum_{f \in F} \sum_{rep \in REP} \text{Priority}_f \cdot U_{f,rep} \right)$$

Maximizes the value delivered, measured by the sum of priorities of features included in releases.

G3: *minimize_total_task_effort*

$$\min \left(W_{G3} \cdot \sum_{tsk \in TSK} \sum_{w \in W} \text{Effort}_{tsk} \cdot Z_{tsk,w} \right)$$

Minimizes the total effort of all assigned tasks.

G4: *maximize_team_velocity*

$$\max \left(W_{G4} \cdot \sum_{t \in T} \frac{\sum_{us \in US, sp \in SP} \text{StoryPoints}_{us} \cdot Y_{us,sp(t)}}{\text{NumSprints}_t} \right)$$

Maximizes the average velocity across all teams. $sp(t)$ denotes sprints assigned to team t.

G7: *minimize_blocker_severity_sum*

$$\min \left(W_{G7} \cdot \sum_{bl \in BL} \text{Severity}_{bl} \cdot (1 - \text{IsResolved}_{bl}) \right)$$

Minimizes the sum of severities for all unresolved blockers. IsResolved_{bl} is a binary parameter.

5 Conditions (Constraints)

Conditions are constraints that the solution must satisfy. They can be hard constraints (must-match) or soft constraints that can be violated at a penalty.

C0: *enforce_project_budget_limit*

$$\sum_{f \in F} \text{FeatureBudget}_f \leq \text{TotalBudget}_p \quad \forall p \in P$$

The sum of budgets allocated to all features must not exceed the total project budget.

C1: *enforce_scrum_team_size*

$$\sum_{w \in W} X_{w,t} = \text{TeamSize}_t \quad \forall t \in T$$

$$3 \leq \text{TeamSize}_t \leq 9 \quad \forall t \in T$$

The number of workers assigned to a team defines its size, which must be within Scrum limits.

C2: *assign_only_active_workers*

$$X_{w,t} \leq \text{IsActive}_w \quad \forall w \in W, t \in T$$

A worker w can only be assigned to a team t if their status is active ($\text{IsActive}_w = 1$).

C5: *require_story_point_estimation*

$$Y_{us,sp} \cdot \text{StoryPoints}_{us} > 0 \quad \forall us \in US, sp \in SP$$

A user story us can only be assigned to a sprint sp if its story points are greater than zero.

C6: *worker_availability_is_positive*

$$\sum_{tsk \in TSK} Z_{tsk,w} \cdot \text{Effort}_{tsk} \leq \text{Availability}_w \cdot \text{SprintDuration}_{sp} \quad \forall w \in W, sp \in SP$$

The total effort of tasks assigned to a worker w in a sprint sp cannot exceed their available capacity.

C8: *task_effort_must_be_set*

$$Z_{tsk,w} \cdot \text{Effort}_{tsk} > 0 \quad \forall tsk \in TSK, w \in W$$

A task tsk can only be assigned to a worker w if its effort is estimated.

C10: *prefer_certified_skills* (Soft Constraint) This condition is added to the objective function, typically as a maximization goal:

$$\max \left(W_{C10} \cdot \sum_{tsk \in TSK} \sum_{w \in W} \sum_{s \in S} Z_{tsk,w} \cdot \text{HasSkill}_{w,s} \cdot \text{RequiresSkill}_{tsk,s} \cdot \text{IsCertified}_{w,s} \right)$$

It rewards the assignment of tasks to workers who have the required skill and are certified in it.