Mathematical Optimization Model for a SCRUM-Based Software Development Company

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Introduction

This document presents a formal mathematical optimization model for resource allocation, scheduling, and process management in a software development company utilizing the SCRUM framework. The model is derived from a defined domain structure of entities, their relationships, and a set of business goals and constraints.

1 Sets (Entities)

- Project = $\{p|p$ is a project$\}$, with attributes id, name, project_start, project_end, description, bud
- Team = $\{t|t$ is a team$\}$, with attributes id, name, team_size, team_start, team_status, location, te
- Worker = $\{w|w$ is a worker$\}$, with attributes id, name, first_name, email, start_date, status, availa
- Feature = $\{f|f$ is a feature$\}$, with attributes id, title, description, status, priority, estimated_effort
- Skill = $\{s|s$ is a skill$\}$, with attributes id, label, description, level, certified, category
- Role = $\{r|r$ is a role$\}$, with attributes id, role_name, description, area_of_responsibility
- ProductOwner = $\{po|po$ is a product owner$\}$, with attributes id, name, email, availability
- ScrumMaster = $\{sm|sm$ is a scrum master$\}$, with attributes id, name, email, experience
- ProductBacklog = $\{pb|pb$ is a product backlog$\}$, with attributes id, created_on, last_updated, nu
- Sprint = $\{sp|sp$ is a sprint$\}$, with attributes id, sprint_number, start_date, end_date, status, achi
- SprintBacklog = $\{sbl|sbl$ is a sprint backlog$\}$, with attributes id, number_of_tasks, last_updated
- SprintGoal = $\{sg|sg$ is a sprint goal$\}$, with attributes id, objective_description, achievement_stat
- Epic = $\{e|e$ is an epic$\}$, with attributes id, title, description, priority, status, estimated_effort
- UserStory = $\{us|us$ is a user story$\}$, with attributes id, title, description, acceptance_criteria, prio
- Task = $\{tsk|tsk$ is a task$\}$, with attributes id, title, description, status, effort, type
- Blocker = $\{bl|bl$ is a blocker$\}$, with attributes id, title, description, severity, status, detected_on, re
- Velocity = $\{vel|vel$ is a velocity record$\}$, with attributes id, number_of_sprints_used, avg. story_

2 Indices

- $p, p' \in$ Project
- $t, t' \in$ Team
- $w, w' \in$ Worker
- $f, f' \in$ Feature
- $sp, sp' \in$ Sprint
- $sbl, sbl' \in$ SprintBacklog
- $us, us' \in$ UserStory
- $tsk, tsk' \in$ Task
- $bl, bl' \in$ Blocker

3 Goals

- **G0: maximize_team_utilization** - Maximize the overall availability of the team members.
  Mathematical Form: Maximize $Z_0 = \sum_{w \in \text{Worker}} \text{availability}(w)$
- **G1: minimize_project_duration** - Minimize the total duration of the project.
  Mathematical Form: Minimize $Z_1 = \text{project\_end}(p) - \text{project\_start}(p)$
- **G2: maximize_feature_delivery** - Maximize the number of high-priority features delivered.
  Mathematical Form: Maximize $Z_2 = \sum_{f \in \text{Feature}} \text{priority}(f) \cdot x_f$ where $x_f = 1$ if feature f is delivered, 0 otherwise.
- **G3: minimize_blocker_impact** - Minimize the severity and duration of blockers.
  Mathematical Form: Minimize $Z_3 = \sum_{bl \in \text{Blocker}} \text{severity}(bl) \cdot (\text{resolved\_on}(bl) - \text{detected\_on}(bl))$
- **G4: maximize_sprint_goal_achievement** - Maximize the rate of successfully achieved sprint goals.
  Mathematical Form: Maximize $Z_4 = \sum_{sg \in \text{SprintGoal}} \mathbb{I}(\text{achievement\_status}(sg) = \text{Done})$

- **G5: minimize_task_effort** - Minimize the total effort spent on tasks in a sprint.
  Mathematical Form: Minimize $Z_5 = \sum_{tsk \in \text{Task}} \text{effort}(tsk)$

- **G6: maximize_team_velocity** - Maximize the average velocity of a team.
  Mathematical Form: Maximize Z_6 = avg. story_points(vel_t) for a given team t.

- **G7: minimize_budget_variance** - Minimize the difference between planned and actual budget.
  Mathematical Form: Minimize $Z_7 = |\text{planned\_budget}(p) - \text{actual\_budget}(p)|$

4 Conditions

- **C0: team_must_be_assigned** - Ensure every team is assigned to exactly one project.
  Logical Form: $\forall t \in \text{Team}, \exists! p \in \text{Project}$ such that is_assigned_to_project(t, p)

- **C1: worker_belongs_to_team** - Ensure every worker is assigned to a team.
  Logical Form: $\forall w \in \text{Worker}, \exists t \in \text{Team}$ such that belongs_to_team(w, t)

- **C2: sprint_backlog_must_have_sprint** - Every Sprint Backlog must be linked to one Sprint.
  Logical Form: $\forall sbl \in \text{SprintBacklog}, \exists! sp \in \text{Sprint}$ such that belongs_to_sprint(sbl, sp)

- **C3: feature_must_have_backlog** - Every Feature must be part of the Product Backlog.
  Logical Form: $\forall f \in \text{Feature}, \exists pb \in \text{ProductBacklog}$ such that contains_feature(pb, f)

- **C4: user_story_must_have_epic** - Every User Story must be part of an Epic.
  Logical Form: $\forall us \in \text{UserStory}, \exists e \in \text{Epic}$ such that contains_user_story(e, us)

- **C5: task_must_have_story** - Every Task must be part of a User Story.
  Logical Form: $\forall tsk \in \text{Task}, \exists us \in \text{UserStory}$ such that consists_of_tasks(us, tsk)

- **C6: scrum_master_supports_team** - Ensure every team has a Scrum Master.
  Logical Form: $\forall t \in \text{Team}, \exists sm \in \text{ScrumMaster}$ such that is_supported_by(t, sm)

- **C7: po_manages_backlog** - Ensure the Product Backlog is managed by a Product Owner.
  Logical Form: $\forall pb \in \text{ProductBacklog}, \exists po \in \text{ProductOwner}$ such that manages_backlog(po, pb)
- **C8: sprint_has_one_goal** - Each Sprint must have one defined goal.
  Logical Form: $\forall sp \in \text{Sprint}, \exists! sg \in \text{SprintGoal}$ such that pursues_goal(sp, sg)
- **C9: feature_doc_exists** - Critical features must have documentation.
  Logical Form: $\forall f \in \text{Feature}$ where priority$(f) \geq \text{High}, \exists fed \in \text{FeatureDocumentation}$ such that d

5 Decision Variables

- **DV0**: $x_{w,t} \in \{0, 1\}$ - Binary variable indicating if worker w is assigned to team t.
- **DV1**: $y_{f,sp} \in \{0, 1\}$ - Binary variable indicating if feature f is selected for sprint sp.
- **DV2**: $pts_{us} \in \mathbb{Z}^+$ - Integer variable for the story points of user story us, with $1 \leq pts_{us} \leq 20$.
- **DV3**: $size_t \in \mathbb{Z}^+$ - Integer variable for the size of team t, with $3 \leq size_t \leq 9$.
- **DV4**: $dur_{sp} \in \mathbb{Z}^+$ - Integer variable for the duration of sprint sp in days, with $7 \leq dur_{sp} \leq 30$.
- **DV5**: $avail_w \in \mathbb{R}$ - Continuous variable for the availability percentage of worker w, with $0.0 \leq avail_w \leq 1.0$.
- **DV6**: $seq_{tsk} \in \mathbb{Z}^+$ - Integer variable representing the sequence/order of task tsk.
- **DV7**: $cost_f \in \mathbb{R}^+$ - Continuous variable for the budget allocated to feature f, with $0 \leq cost_f$.
- **DV8**: $prio_f \in \mathbb{Z}^+$ - Integer variable for the priority level of feature f, with $1 \leq prio_f \leq 5$.
- **DV9**: $lvl_{tsk,s} \in \mathbb{Z}^+$ - Integer variable for the required skill level s for task tsk.