Optimization Model for a Scrum-Based Software Development Company

Mathematical Formulation

Domain Modeling and Optimization Team

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Contents

1. Sets (Entities)

$\mathcal{P}$: Set of Projects, where each $p \in \mathcal{P}$ has attributes: id, name, project_start, project_end, description, budget, status, target_audience, priority.

$\mathcal{T}$: Set of Teams, where each $t \in \mathcal{T}$ has attributes: id, name, team_size, team_start, team_status, location, team_type.

$\mathcal{W}$: Set of Workers, where each $w \in \mathcal{W}$ has attributes: id, name, first_name, email, start_date, status, availability.

$\mathcal{F}$: Set of Features, where each $f \in \mathcal{F}$ has attributes: id, title, description, status, priority, estimated_effort.

$\mathcal{S}$: Set of Skills, where each $s \in \mathcal{S}$ has attributes: id, label, description, level, certified, category.

$\mathcal{R}$: Set of Roles, where each $r \in \mathcal{R}$ has attributes: id, role_name, description, area_of_responsibility.

$\mathcal{PO}$: Set of Product Owners, where each $po \in \mathcal{PO}$ has attributes: id, name, email, availability.

$\mathcal{SM}$: Set of Scrum Masters, where each $sm \in \mathcal{SM}$ has attributes: id, name, email, experience.

$\mathcal{PB}$: Set of Product Backlogs, where each $pb \in \mathcal{PB}$ has attributes: id, created_on, last_updated, number_of_entries, status.

$\mathcal{SP}$: Set of Sprints, where each $sp \in \mathcal{SP}$ has attributes: id, sprint_number, start_date, end_date, status, achievement_of_goal.

$\mathcal{SPP}$: Set of Sprint Plannings, where each $spp \in \mathcal{SPP}$ has attributes: id, date, duration_in_min, moderation, outcome_documentation.

$\mathcal{DS}$: Set of Daily Scrums, where each $ds \in \mathcal{DS}$ has attributes: id, date, time, duration, moderation.

$\mathcal{SR}$: Set of Sprint Reviews, where each $sr \in \mathcal{SR}$ has attributes: id, date, duration, feedback_documentation, attendees_count.

$\mathcal{SRE}$: Set of Sprint Retrospectives, where each $sre \in \mathcal{SRE}$ has attributes: id, date, duration, improvement_actions, team_satisfaction, moderation.

$\mathcal{SBL}$: Set of Sprint Backlogs, where each $sbl \in \mathcal{SBL}$ has attributes: id, number_of_tasks, last_updated, status, total_effort.

$\mathcal{SG}$: Set of Sprint Goals, where each $sg \in \mathcal{SG}$ has attributes: id, objective_description, achievement_status, benefit.

$\mathcal{E}$: Set of Epics, where each $e \in \mathcal{E}$ has attributes: id, title, description, priority, status, estimated_effort.

$\mathcal{US}$: Set of User Stories, where each $us \in \mathcal{US}$ has attributes: id, title, description, acceptance_criteria, priority, story_points, status.

$\mathcal{TSK}$: Set of Tasks, where each $tsk \in \mathcal{TSK}$ has attributes: id, title, description, status, effort, type.

$\mathcal{DEV}$: Set of Development Snapshots, where each $dev \in \mathcal{DEV}$ has attributes: id, version_number, creation_date, test_status, deployment_target, documentation.

$\mathcal{BL}$: Set of Blockers, where each $bl \in \mathcal{BL}$ has attributes: id, title, description, severity, status, detected_on, resolved_on.

$\mathcal{SH}$: Set of Stakeholders, where each $sh \in \mathcal{SH}$ has attributes: id, name, organization, role, email, area_of_interest, influence_level, relevance_to_feature.

$\mathcal{VEL}$: Set of Velocity Records, where each $vel \in \mathcal{VEL}$ has attributes: id, number_of_sprints_used, avg._story_points, max_velocity, min_velocity, trend.

$\mathcal{REP}$: Set of Release Plans, where each $rep \in \mathcal{REP}$ has attributes: id, version, planned_date, included_features, status.

$\mathcal{RM}$: Set of Roadmaps, where each $rm \in \mathcal{RM}$ has attributes: id, start_date, end_date, milestones, objectives, versions.

$\mathcal{SCB}$: Set of Scrum Boards, where each $scb \in \mathcal{SCB}$ has attributes: id, board_type, columns_(todo/done...), number_of_cards, last_updated.

$\mathcal{FED}$: Set of Feature Documentations, where each $fed \in \mathcal{FED}$ has attributes: id, title, description, creation_date, change_log, linked_requirements, author.

2. Indices

$p \in \mathcal{P}$: Index for projects.

$t \in \mathcal{T}$: Index for teams.

$w \in \mathcal{W}$: Index for workers.

$f \in \mathcal{F}$: Index for features.

$s \in \mathcal{S}$: Index for skills.

$r \in \mathcal{R}$: Index for roles.

$sp \in \mathcal{SP}$: Index for sprints.

$us \in \mathcal{US}$: Index for user stories.

$tsk \in \mathcal{TSK}$: Index for tasks.

$bl \in \mathcal{BL}$: Index for blockers.

$sh \in \mathcal{SH}$: Index for stakeholders.

$vel \in \mathcal{VEL}$: Index for velocity records.

$rep \in \mathcal{REP}$: Index for release plans.

3. Goals

G0: maximize_project_budget
Maximize total project budget:

$$\max \sum_{p \in \mathcal{P}} \text{budget}(p) \quad \text{with weight } 1.5$$

G1: minimize_project_duration
Minimize total duration across projects:

$$\min \sum_{p \in \mathcal{P}} (\text{project\_end}(p) - \text{project\_start}(p)) \quad \text{with weight } 1.2$$

G2: maximize_team_size
Maximize total team capacity:

$$\max \sum_{t \in \mathcal{T}} \text{team\_size}(t) \quad \text{with weight } 0.8$$

G3: minimize_worker_unavailability
Minimize number of inactive workers:

$$\min \sum_{w \in \mathcal{W}} \mathbb{I}[\text{status}(w) \neq \text{active}] \quad \text{with weight } 1.0$$

G4: maximize_feature_priority
Maximize sum of feature priorities:

$$\max \sum_{f \in \mathcal{F}} \text{priority}(f) \quad \text{with weight } 1.3$$

G5: minimize_task_effort
Minimize total task effort:

$$\min \sum_{tsk \in \mathcal{TSK}} \text{effort}(tsk) \quad \text{with weight } 1.1$$

G6: maximize_story_points
Maximize total story points:

$$\max \sum_{us \in \mathcal{US}} \text{story\_points}(us) \quad \text{with weight } 1.4$$

G7: minimize_sprint_duration
Minimize sprint length:

$$\min \sum_{sp \in \mathcal{SP}} (\text{end\_date}(sp) - \text{start\_date}(sp)) \quad \text{with weight } 0.9$$

G8: maximize_velocity_trend
Maximize velocity trend:

$$\max \sum_{vel \in \mathcal{VEL}} \text{trend}(vel) \quad \text{with weight } 1.6$$

G9: minimize_blocker_severity
Minimize total blocker severity:

$$\min \sum_{bl \in \mathcal{BL}} \text{severity}(bl) \quad \text{with weight } 1.3$$

G10: maximize_stakeholder_influence
Maximize sum of stakeholder influence:

$$\max \sum_{sh \in \mathcal{SH}} \text{influence\_level}(sh) \quad \text{with weight } 1.0$$

G11: minimize_epic_estimated_effort
Minimize total epic effort:

$$\min \sum_{e \in \mathcal{E}} \text{estimated\_effort}(e) \quad \text{with weight } 1.1$$

G12: maximize_sprint_achievement
Maximize average sprint goal achievement:

$$\max \frac{1}{|\mathcal{SP}|} \sum_{sp \in \mathcal{SP}} \text{achievement\_of\_goal}(sp) \quad \text{with weight } 1.5$$

G13: minimize_documentation_age
Minimize age of documentation (inverse of creation date):

$$\min \sum_{fed \in \mathcal{FED}} (T - \text{creation\_date}(fed)) \quad (\text{T} = \text{current date}), \text{ weight } 0.7$$

G14: maximize_release_plan_inclusion
Maximize number of features in release plans:

$$\max \sum_{rep \in \mathcal{REP}} |\text{included\_features}(rep)| \quad \text{with weight } 1.2$$

4. Conditions

C0: require_project_status_active
Only active projects are considered:

$$\forall p \in \mathcal{P} : \text{status}(p) = \text{active}$$

C1: require_team_status_active
Only active teams:

$$\forall t \in \mathcal{T} : \text{team\_status}(t) = \text{active}$$

C2: require_worker_status_active
Only active workers:

$$\forall w \in \mathcal{W} : \text{status}(w) = \text{active}$$

C3: require_feature_status_completed
Only completed features:

$$\forall f \in \mathcal{F} : \text{status}(f) = \text{completed} \Rightarrow \text{include in goals}$$

C4: require_user_story_status_done
Only done user stories:

$$\forall us \in \mathcal{US} : \text{status}(us) = \text{done}$$

C5: require_task_status_not_blocked
Tasks must not be blocked:

$$\forall tsk \in \mathcal{TSK} : \text{status}(tsk) \neq \text{blocked}$$

C6: require_sprint_status_completed
Only completed sprints:

$$\forall sp \in \mathcal{SP} : \text{status}(sp) = \text{completed}$$

C7: require_blocker_status_resolved
All blockers must be resolved:

$$\forall bl \in \mathcal{BL} : \text{status}(bl) = \text{resolved}$$

C8: require_skill_certified
Skills must be certified:

$$\forall s \in \mathcal{S} : \text{certified}(s) = \text{true}$$

C9: require_role_assigned
Each worker must have at least one role:

$$\forall w \in \mathcal{W}, \exists r \in \mathcal{R} : (w, r) \in \text{takes\_on\_role}$$

C10: require_velocity_min_value
Minimum velocity must be positive:

$$\forall vel \in \mathcal{VEL} : \text{min\_velocity}(vel) > 0$$

C11: require_release_status_planned
Only planned releases:

$$\forall rep \in \mathcal{REP} : \text{status}(rep) = \text{planned}$$

C12: require_roadmap_milestones_defined
Roadmaps must have milestones:

$$\forall rm \in \mathcal{RM} : \text{milestones}(rm) \neq \emptyset$$

C13: require_daily_scrum_held
Daily scrum must occur each day of sprint:

$$\forall sp \in \mathcal{SP}, \forall d \in [\text{start\_date}(sp), \text{end\_date}(sp)] : \exists ds \in \mathcal{DS} \mid \text{date}(ds) = d$$

C14: require_sprint_goal_defined
Every sprint must have a goal:

$$\forall sp \in \mathcal{SP}, \exists sg \in \mathcal{SG} : \text{pursues\_goal}(sp, sg) \land \text{objective\_description}(sg) \neq \emptyset$$

5. DecisionVariables

`project_priority_weight`: Weight applied to project priority ranking, $\in [0.0, 10.0]$.

`team_size_capacity`: Maximum number of workers per team, $\in \mathbb{Z}, [1, 20]$.

`worker_availability_hours`: Weekly availability per worker, $\in \mathbb{Z}, [0, 40]$.

`task_effort_estimate`: Estimated effort for a task in hours, $\in \mathbb{Z}, [1, 160]$.

`user_story_points`: Story points assigned to a user story, $\in \{1, 2, 3, 5, 8, 13\}$.

`sprint_duration_days`: Duration of a sprint in days, $\in \{7, 14, 21, 28\}$.

`blocker_severity_level`: Severity level of a blocker, $\in \{1, 2, 3\}$.

`feature_priority_score`: Priority score of a feature, $\in \mathbb{Z}, [1, 10]$.

`velocity_avg_story_points`: Average story points per sprint, $\in \mathbb{R}, [0.0, 100.0]$.

`stakeholder_influence_level`: Influence level of a stakeholder, $\in \mathbb{Z}, [0, 5]$.

`release_version_number`: Version number for a release, $\in \mathbb{R}, [1.0, 10.0]$.

`documentation_change_count`: Number of changes in documentation, $\in \mathbb{Z}, [0, 100]$.

`scrum_board_column_count`: Number of columns on a Scrum Board, $\in \mathbb{Z}, [2, 6]$.

`task_type_category`: Type of task, $\in \{\text{development}, \text{testing}, \text{bug\_fix}, \text{documentation}\}$.

`skill_level_rating`: Skill proficiency level, $\in \{1, 2, 3, 4, 5\}$.