Mathematical Formulation of a SCRUM Optimization Model

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Contents

1 Sets (Entities)

The following sets and their corresponding symbols are derived from the `Entities.csv` file. They represent the foundational elements of the SCRUM domain model.

- P: The set of all Projects.
- T: The set of all Teams.
- W: The set of all Workers.
- F: The set of all Features.
- S: The set of all Skills.
- R: The set of all Roles.
- PO: The set of all Product Owners.
- SM: The set of all Scrum Masters.
- PB: The set of all Product Backlogs.
- SP: The set of all Sprints.
- SPP: The set of all Sprint Plannings.
- DS: The set of all Daily Scrums.
- SR: The set of all Sprint Reviews.
- SRE: The set of all Sprint Retrospectives.
- SBL: The set of all Sprint Backlogs.
- SG: The set of all Sprint Goals.
- E: The set of all Epics.
- US: The set of all User Stories.
- TSK: The set of all Tasks.
- DEV: The set of all Development Snapshots.
- BL: The set of all Blockers.
- SH: The set of all Stakeholders.
- VEL: The set of all Velocity metrics.
- REP: The set of all Release Plans.
- RM: The set of all Roadmaps.
- SCB: The set of all Scrum Boards.
- FED: The set of all Feature Documentations.

2 Indices

For each set defined above, a corresponding index is used to refer to a specific element within that set.

- $p \in P$: Index for a Project.
- $t \in T$: Index for a Team.
- $w \in W$: Index for a Worker.
- $f \in F$: Index for a Feature.
- $s \in S$: Index for a Skill.
- $r \in R$: Index for a Role.
- $po \in PO$: Index for a Product Owner.
- $sm \in SM$: Index for a Scrum Master.
- $pb \in PB$: Index for a Product Backlog.
- $sp \in SP$: Index for a Sprint.
- $spp \in SPP$: Index for a Sprint Planning.
- $ds \in DS$: Index for a Daily Scrum.
- $sr \in SR$: Index for a Sprint Review.
- $sre \in SRE$: Index for a Sprint Retrospective.
- $sbl \in SBL$: Index for a Sprint Backlog.
- $sg \in SG$: Index for a Sprint Goal.
- $e \in E$: Index for an Epic.
- $us \in US$: Index for a User Story.
- $tsk \in TSK$: Index for a Task.
- $dev \in DEV$: Index for a Development Snapshot.
- $bl \in BL$: Index for a Blocker.
- $sh \in SH$: Index for a Stakeholder.
- $vel \in VEL$: Index for a Velocity metric.
- $rep \in REP$: Index for a Release Plan.
- $rm \in RM$: Index for a Roadmap.
- $scb \in SCB$: Index for a Scrum Board.
- $fed \in FED$: Index for a Feature Documentation.

Note: Parameters extracted from the model are denoted as $Attribute_{index}$*, e.g.,* $StoryPoints_{us}$ *for the story points of user story us.*

3 Goals

The objective functions of the optimization model are defined below. These are goals to be maximized or minimized, derived from `Goals.csv`.

G0 **maximize_story_points_per_sprint**: Maximize the sum of story points from user stories assigned to a given sprint. Let $x_{us,sp}$ be 1 if user story us is in sprint sp, 0 otherwise.

$$\max \sum_{us \in US} \text{StoryPoints}_{us} \cdot x_{us,sp}$$

G1 **minimize_project_budget**: Minimize the total budget consumed across all projects. This assumes a variable cost c_p for each project.

$$\min \sum_{p \in P} c_p$$

G2 **maximize_high_priority_feature_completion**: Maximize the sum of priorities for all completed features. Let y_f be 1 if feature f is completed, 0 otherwise.

$$\max \sum_{f \in F} \text{Priority}_f \cdot y_f$$

G3 **minimize_unresolved_blockers**: Minimize the count of blockers that are not in a 'resolved' state. Let z_{bl} be 1 if blocker bl is resolved, 0 otherwise.

$$\min \sum_{bl \in BL} (1 - z_{bl})$$

G4 **maximize_team_velocity**: Maximize the average story points for a given team t.

$$\max \text{AvgStoryPoints}_{vel_t}$$

G5 **maximize_sprint_goal_achievement**: Maximize the count of sprints that achieve their goal.

$$\max \sum_{sp \in SP} \text{AchievementOfGoal}_{sp}$$

G6 **maximize_team_satisfaction**: Maximize the satisfaction level reported by a team in a retrospective.

$$\max \text{TeamSatisfaction}_{sre_t}$$

G7 **maximize_high_priority_story_completion**: Maximize the sum of priorities for all completed user stories. Let v_{us} be 1 if user story us is completed, 0 otherwise.

$$\max \sum_{us \in US} \text{Priority}_{us} \cdot v_{us}$$

G8 **minimize_total_task_effort**: Minimize the total effort for all tasks assigned to a sprint. Let $a_{tsk,sp}$ be 1 if task tsk is in sprint sp.

$$\min \sum_{tsk \in TSK} \text{Effort}_{tsk} \cdot a_{tsk,sp}$$

G9 **maximize_stakeholder_influence**: Maximize the influence level of stakeholders engaged in the project.

$$\max \sum_{sh \in SH} \text{InfluenceLevel}_{sh}$$

G10 **minimize_epic_effort**: Minimize the total estimated effort for all planned epics.

$$\min \sum_{e \in E} \text{EstimatedEffort}_{e}$$

G11 **maximize_project_priority**: Maximize the priority of the projects being worked on.

$$\max \sum_{p \in P} \text{Priority}_{p}$$

4 Conditions

The constraints of the optimization model are defined below. These are rules and limits that must be adhered to, derived from `Conditions.csv`.

C0 **project_budget_limit**: The total cost of a project must not exceed its allocated budget. Let c_p be the calculated cost for project p.

$$c_p \leq \text{Budget}_p, \quad \forall p \in P$$

C1 **team_size_upper_limit**: The size of any team must not be greater than 9.

$$\text{TeamSize}_t \leq 9, \quad \forall t \in T$$

C2 **sprint_end_date_is_fixed**: The completion date of any task must be on or before the sprint's end date. Let d_{tsk} be the completion date of task tsk. Let task tsk be in sprint sp.

$$d_{tsk} \leq \text{EndDate}_{sp}, \quad \forall tsk \in TSK, \forall sp \in SP$$

C3 **worker_availability_limit**: The total effort assigned to a worker must not exceed their availability. Let $e_{w,tsk}$ be the effort of worker w on task tsk.

$$\sum_{tsk \in TSK} e_{w,tsk} \leq \text{Availability}_w, \quad \forall w \in W$$

C4 **critical_task_skill_requirement**: A worker assigned to a critical task must have a minimum skill level. Let $a_{w,tsk}$ be 1 if worker w is assigned to task tsk. Let $L_{w,s}$ be the level of worker w in skill s.

$$a_{w,tsk} = 1 \implies L_{w,s} \geq \text{MIN\_SKILL\_LEVEL}, \quad \text{for required skill } s$$

C5 **feature_priority_is_set**: The priority of any feature must be greater than zero.

$$\text{Priority}_f \geq 1, \quad \forall f \in F$$

C6 **story_points_are_positive**: The story points for any user story must be greater than zero.

$$\text{StoryPoints}_{us} > 0, \quad \forall us \in US$$

C7 **prefer_certified_skills**: A soft constraint, represented here as a condition that for a key task, the assigned worker should be certified. Let $C_{w,s}$ be 1 if worker w is certified in skill s.

$$a_{w,tsk} = 1 \implies C_{w,s} = 1, \quad \text{for key tasks } tsk$$

C8 **assigned_team_is_active**: A team assigned to a project must have an 'active' status.

$$\text{TeamStatus}_t = \text{'active'}, \quad \forall t \in T \text{ assigned to a project}$$

C9 **task_effort_is_estimated**: The effort for every task must be a positive number.

$$\text{Effort}_{tsk} > 0, \quad \forall tsk \in TSK$$

C11 **team_size_lower_limit**: The size of any team must be at least 3.

$$\text{TeamSize}_t \geq 3, \quad \forall t \in T$$

5 Decision Variables

The variables that the optimization model can manipulate to achieve the goals are defined below, derived from `DecisionVariables.csv`.

DV0 **assign_story_to_sprint**: Binary variable indicating if a User Story is assigned to a Sprint.

$$x_{us,sp} \in \{0, 1\}, \quad \forall us \in US, \forall sp \in SP$$

DV1 **assign_worker_to_task**: Binary variable indicating if a Worker is assigned to a Task.

$$a_{w,tsk} \in \{0, 1\}, \quad \forall w \in W, \forall tsk \in TSK$$

DV2 **set_feature_priority**: Integer variable for setting the priority of a Feature.

$$p_f \in \{1, 2, 3, 4, 5\}, \quad \forall f \in F$$

DV3 **set_user_story_points**: Integer variable for setting the story points for a User Story.

$$sp_{us} \in \{1, 2, 3, 5, 8, 13, 21, 40\}, \quad \forall us \in US$$

DV4 **select_team_for_project**: Categorical variable to assign a team to a project. Let $a_{t,p}$ be 1 if team t is assigned to project p.

$$a_{t,p} \in \{0, 1\}, \quad \forall t \in T, \forall p \in P$$

DV5 **set_sprint_duration_weeks**: Integer variable setting the duration of a sprint in weeks.

$$d_{sp} \in \{1, 2, 3, 4\}, \quad \forall sp \in SP$$

DV6 **include_feature_in_release**: Binary variable to include a Feature in a Release Plan.

$$i_{f,rep} \in \{0, 1\}, \quad \forall f \in F, \forall rep \in REP$$

DV7 **set_task_effort_hours**: Integer variable for the estimated effort of a Task in hours.

$$e_{tsk} \in \mathbb{Z}^+, \quad 1 \leq e_{tsk} \leq 16, \quad \forall tsk \in TSK$$

DV8 **prioritize_blocker_resolution**: Binary variable to decide if a blocker is actively being resolved.

$$r_{bl} \in \{0, 1\}, \quad \forall bl \in BL$$

DV9 **assign_worker_to_team**: Binary variable to assign a Worker to a Team.

$$m_{w,t} \in \{0, 1\}, \quad \forall w \in W, \forall t \in T$$

DV10 **set_blocker_severity**: Categorical variable for the severity of a Blocker.

$$s_{bl} \in \{\text{low, medium, high, critical}\}, \quad \forall bl \in BL$$

DV11 **set_project_budget**: Continuous variable for the budget of a Project.

$$b_p \in \mathbb{R}^+, \quad 50000 \leq b_p \leq 5000000, \quad \forall p \in P$$

DV12 **define_task_type**: Categorical variable for the type of a Task.

$$t_{tsk} \in \{\text{bug, new\_development, research, testing}\}, \quad \forall tsk \in TSK$$