Mathematical Optimization Model for Scrum-based Software Development

Formulated by Gemini

September 4, 2025

Contents

1 Sets (Entities)

This section defines the primary sets used in the optimization model, derived from the domain entities.

- P: The set of all Projects.
- T: The set of all Teams.
- W: The set of all Workers.
- F: The set of all Features.
- S: The set of all Skills.
- R: The set of all Roles.
- PO: The set of all Product Owners.
- SM: The set of all Scrum Masters.
- PB: The set of all Product Backlogs.
- SP: The set of all Sprints.
- SPP: The set of all Sprint Plannings.
- DS: The set of all Daily Scrums.
- SR: The set of all Sprint Reviews.
- SRE: The set of all Sprint Retrospectives.
- SBL: The set of all Sprint Backlogs.
- SG: The set of all Sprint Goals.
- E: The set of all Epics.
- US: The set of all User Stories.
- TSK: The set of all Tasks.
- DEV: The set of all Development Snapshots.
- BL: The set of all Blockers.
- SH: The set of all Stakeholders.
- VEL: The set of all Velocity metrics.
- REP: The set of all Release Plans.
- RM: The set of all Roadmaps.
- SCB: The set of all Scrum Boards.
- FED: The set of all Feature Documentations.

2 Indices

This section defines the indices used to refer to specific elements within the sets.

- $p \in P$: An index for a specific project.
- $t \in T$: An index for a specific team.
- $w \in W$: An index for a specific worker.
- $f \in F$: An index for a specific feature.
- $s \in S$: An index for a specific skill.
- $r \in R$: An index for a specific role.
- $po \in PO$: An index for a specific product owner.
- $sm \in SM$: An index for a specific scrum master.
- $pb \in PB$: An index for a specific product backlog.
- $sp \in SP$: An index for a specific sprint.
- $us \in US$: An index for a specific user story.
- $tsk \in TSK$: An index for a specific task.
- $bl \in BL$: An index for a specific blocker.
- $sh \in SH$: An index for a specific stakeholder.

3 Goals

This section outlines the objectives of the optimization model. The final objective function is a weighted sum of these individual goals. Let $\delta_{i,\text{status}}$ be a binary parameter that is 1 if entity i has the specified status, and 0 otherwise.

G0: maximize_completed_story_points
Maximize the sum of story points from completed user stories.

$$\max \sum_{us \in US} \text{story\_points}_{us} \cdot \delta_{us,\text{done}}$$

G1: minimize_project_costs
Minimize the total budget consumed by a project.

$$\min \text{budget}_p$$

G2: maximize_feature_priority_value
Maximize the sum of priorities of completed features.

$$\max \sum_{f \in F} \text{priority}_f \cdot \delta_{f,\text{completed}}$$

G3: minimize_blocker_severity
Minimize the cumulative severity of all active blockers.

$$\min \sum_{bl \in BL} \text{severity}_{bl} \cdot \delta_{bl,\text{active}}$$

G4: maximize_team_velocity
Maximize the average velocity for a team.

$$\max \text{ avg\_story\_points}_{t}$$

G5: maximize_sprint_goal_achievement
Maximize the number of successfully achieved sprint goals.

$$\max \sum_{sg \in SG} \text{achievement\_status}_{sg}$$

G6: minimize_task_effort
Minimize the total effort for all tasks.

$$\min \sum_{tsk \in TSK} \text{effort}_{tsk}$$

G7: maximize_stakeholder_satisfaction
Maximize the influence level of stakeholders whose features are addressed.

$$\max \sum_{sh \in SH} \text{influence\_level}_{sh} \cdot \delta_{sh,\text{feature\_addressed}}$$

G8: minimize_time_to_resolve_blockers
Minimize the time difference between blocker resolution and detection.

$$\min \sum_{bl \in BL} (\text{resolved\_on}_{bl} - \text{detected\_on}_{bl})$$

G9: maximize_team_satisfaction
Maximize the reported team satisfaction from retrospectives.

$$\max \text{ team\_satisfaction}_{sre}$$

4 Conditions

This section outlines the constraints that the solution to the optimization model must satisfy. Let x, y, z represent various binary decision variables defined in the next section.

C0: sprint_backlog_must_not_exceed_velocity
The sum of story points for user stories in a sprint cannot exceed the team's velocity.

$$\sum_{us \in US} \text{story\_points}_{us} \cdot x_{us,sp} \leq \text{velocity}_{t} \quad \forall sp \in SP, \forall t \in T$$

C1: worker_availability_must_be_respected
The total effort of tasks assigned to a worker cannot exceed their availability.

$$\sum_{tsk \in TSK} \text{effort}_{tsk} \cdot y_{w,tsk} \leq \text{availability}_{w} \quad \forall w \in W$$

C2: project_budget_must_not_be_exceeded
The total cost of the project cannot exceed its budget.

$$\text{TotalCost}(p) \leq \text{budget}_p \quad \forall p \in P$$

C3: team_size_must_be_within_scrum_limits
The size of any given team must be within a specific range [min, max].

$$3 \leq \text{team\_size}_t \leq 9 \quad \forall t \in T$$

C4: task_must_have_required_skills
A worker assigned to a task must have the required skill. Let S_w be the set of skills for worker w and S_{tsk} be the required skills for task tsk.

$$y_{w,tsk} = 1 \implies S_{tsk} \subseteq S_w \quad \forall w \in W, \forall tsk \in TSK$$

C5: sprint_must_have_a_goal
Every sprint must be associated with at least one sprint goal.

$$\forall sp \in SP, \exists sg \in SG : \text{is\_goal\_for\_sprint}(sg, sp) = 1$$

C6: epic_must_be_broken_down
An epic cannot be assigned to a sprint.

$$\sum_{sp \in SP} \text{assign\_epic\_to\_sprint}_{e,sp} = 0 \quad \forall e \in E$$

C7: a_sprint_has_a_fixed_duration
The duration of a sprint is constant.

$$\text{end\_date}_{sp} - \text{start\_date}_{sp} = \text{Constant} \quad \forall sp \in SP$$

C8: a_worker_belongs_to_one_team
Each worker must be assigned to exactly one team.

$$\sum_{t \in T} z_{w,t} = 1 \quad \forall w \in W$$

C9: a_product_owner_manages_the_backlog
Each product backlog is managed by exactly one product owner.

$$\sum_{po \in PO} \text{manages}_{po,pb} = 1 \quad \forall pb \in PB$$

5 Decision Variables

This section defines the variables that the optimization model will determine.

DV0: assign_user_story_to_sprint
Binary variable: 1 if User Story us is assigned to Sprint sp, 0 otherwise.

$$x_{us,sp} \in \{0, 1\} \quad \forall us \in US, \forall sp \in SP$$

DV1: assign_worker_to_task
Binary variable: 1 if Worker w is assigned to Task tsk, 0 otherwise.

$$y_{w,tsk} \in \{0,1\} \quad \forall w \in W, \forall tsk \in TSK$$

DV2: assign_worker_to_team
Binary variable: 1 if Worker w is assigned to Team t, 0 otherwise.

$$z_{w,t} \in \{0,1\} \quad \forall w \in W, \forall t \in T$$

DV3: select_feature_for_release
Binary variable: 1 if Feature f is included in Release Plan rep, 0 otherwise.

$$\alpha_{f,rep} \in \{0,1\} \quad \forall f \in F, \forall rep \in REP$$

DV4: start_date_of_sprint
Integer variable representing the start date of Sprint sp.

$$\text{start\_date}_{sp} \in \mathbb{Z}^+ \quad \forall sp \in SP$$

DV5: task_sequence_order
Integer variable defining the execution order of Task tsk.

$$\text{order}_{tsk} \in \mathbb{Z}^+ \quad \forall tsk \in TSK$$

DV6: team_size_for_project
Integer variable for the number of workers in Team t.

$$\text{team\_size}_t \in \{3,4,...,10\} \quad \forall t \in T$$

DV7: effort_allocated_to_task
Continuous variable for the hours allocated to Task tsk.

$$\text{effort}_{tsk} \in \mathbb{R}^+ \quad \forall tsk \in TSK$$

DV8: sprint_count_for_release
Integer variable for the number of sprints in Release Plan rep.

$$\text{sprint\_count}_{rep} \in \mathbb{Z}^+ \quad \forall rep \in REP$$

DV9: budget_allocated_to_feature
Continuous variable for the budget allocated to Feature f.

$$\text{budget}_f \in \mathbb{R}^+ \quad \forall f \in F$$

DV10: worker_availability_percentage
Continuous variable representing the availability of Worker w.

$$\text{availability}_w \in [0,1] \quad \forall w \in W$$