Optimization Model for SCRUM-based Software Development

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Contents

1 Sets (Entities)

The following sets are defined based on the entities in the domain model. The capitalized letter represents the set, while the lowercase letter in parentheses is the index used to denote an element of that set.

- $P(p)$: The set of all Projects.
- $T(t)$: The set of all Teams.
- $W(w)$: The set of all Workers.
- $F(f)$: The set of all Features.
- $S(s)$: The set of all Skills.
- $R(r)$: The set of all Roles.
- $PO(po)$: The set of all Product Owners.
- $SM(sm)$: The set of all Scrum Masters.
- $PB(pb)$: The set of all Product Backlogs.
- $SP(sp)$: The set of all Sprints.
- $SPP(spp)$: The set of all Sprint Plannings.
- $DS(ds)$: The set of all Daily Scrums.
- $SR(sr)$: The set of all Sprint Reviews.
- $SRE(sre)$: The set of all Sprint Retrospectives.
- $SBL(sbl)$: The set of all Sprint Backlogs.
- $SG(sg)$: The set of all Sprint Goals.
- $E(e)$: The set of all Epics.
- $US(us)$: The set of all User Stories.
- $TSK(tsk)$: The set of all Tasks.
- $DEV(dev)$: The set of all Development Snapshots.
- $BL(bl)$: The set of all Blockers.
- $SH(sh)$: The set of all Stakeholders.
- $VEL(vel)$: The set of all Velocity metrics.
- $REP(rep)$: The set of all Release Plans.
- $RM(rm)$: The set of all Roadmaps.
- $SCB(scb)$: The set of all Scrum Boards.
- $FED(fed)$: The set of all Feature Documentations.

2 Indices

The following indices are used throughout the mathematical formulation to refer to individual elements within the sets defined above.

- $p \in P$: index for a Project.
- $t \in T$: index for a Team.
- $w \in W$: index for a Worker.
- $f \in F$: index for a Feature.
- $s \in S$: index for a Skill.
- $r \in R$: index for a Role.
- $po \in PO$: index for a Product Owner.
- $sm \in SM$: index for a Scrum Master.
- $pb \in PB$: index for a Product Backlog.
- $sp \in SP$: index for a Sprint.
- $spp \in SPP$: index for a Sprint Planning meeting.
- $ds \in DS$: index for a Daily Scrum meeting.
- $sr \in SR$: index for a Sprint Review.
- $sre \in SRE$: index for a Sprint Retrospective.
- $sbl \in SBL$: index for a Sprint Backlog.
- $sg \in SG$: index for a Sprint Goal.
- $e \in E$: index for an Epic.
- $us \in US$: index for a User Story.
- $tsk \in TSK$: index for a Task.
- $dev \in DEV$: index for a Development Snapshot.
- $bl \in BL$: index for a Blocker.
- $sh \in SH$: index for a Stakeholder.
- $vel \in VEL$: index for a Velocity metric.
- $rep \in REP$: index for a Release Plan.
- $rm \in RM$: index for a Roadmap.
- $scb \in SCB$: index for a Scrum Board.
- $fed \in FED$: index for a Feature Documentation.

3 Goals (Objective Functions)

The primary objectives of the optimization model are listed below. These can be combined into a multi-objective function, often using the provided weights.

eted_story_points Sum of story points for all completed user stories. Let S_{us}^{done} be a binary variable that is 1 if the status of user story us is 'done'.

$$\text{Maximize} \quad \sum_{us \in US} \text{story\_points}_{us} \cdot S_{us}^{\mathrm{done}}$$

t_budget_overrun Minimize the difference between actual cost and planned budget. Let C_p be the actual cost of project p.

$$\text{Minimize} \quad \sum_{p \in P} (C_p - \text{budget}_p)$$

re_business_value Maximize the sum of priority scores for all implemented features. Let S_f^{done} be 1 if feature f is done.

$$\text{Maximize} \quad \sum_{f \in F} \text{priority}_f \cdot S_f^{\mathrm{done}}$$

ize_team_velocity Maximize the average story points a team completes per sprint.

$$\text{Maximize} \quad \frac{1}{|T|} \sum_{t \in T} \text{avg\_story\_points}_{\mathrm{vel}(t)}$$

blockers_severity Minimize the sum of severity levels for all unresolved blockers. Let S_{bl}^{open} be 1 if blocker bl is open.

$$\text{Minimize} \quad \sum_{bl \in BL} \text{severity}_{bl} \cdot S_{bl}^{\mathrm{open}}$$

team_satisfaction Maximize the average team satisfaction score from retrospectives.

$$\text{Maximize} \quad \frac{1}{|SRE|} \sum_{sre \in SRE} \text{team\_satisfaction}_{sre}$$

e_total_task_effort Minimize the total estimated effort for all tasks.

$$\text{Minimize} \quad \sum_{tsk \in TSK} \text{effort}_{tsk}$$

igh_priority_epics Maximize the number of completed epics with the highest priority. Let P_{max} be the max priority level.

$$\text{Maximize} \quad \sum_{e \in E | \text{priority}_e = \mathrm{P}_{max}} S_e^{\mathrm{done}}$$

print_goal_failures Minimize the number of sprints where the goal was not achieved. Let A_{sp} be 1 if sprint goal is not achieved.

$$\text{Minimize} \quad \sum_{sp \in SP} A_{sp}$$

interest_coverage Maximize delivery weighted by stakeholder influence. Let $D_{f,sh}$ be 1 if feature f is relevant to stakeholder sh.

$$\text{Maximize} \quad \sum_{f \in F} \sum_{sh \in SH} \text{influence\_level}_{sh} \cdot S_f^{\mathrm{done}} \cdot D_{f,sh}$$

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_resolve_blockers Minimize the time between detection and resolution of blockers.

$$\text{Minimize} \quad \sum_{bl \in BL} (\text{resolved\_on}_{bl} - \text{detected\_on}_{bl})$$

orker_availability Maximize the utilization of workers based on their availability percentage.

$$\text{Maximize} \quad \sum_{w \in W} \text{availability}_w$$

4 Conditions (Constraints)

The model is subject to the following constraints, ensuring that solutions are feasible and adhere to SCRUM principles.

_le_team_velocity The total effort in a sprint backlog must not exceed the velocity of the assigned team. Let $T(sbl)$ be the team for sprint backlog sbl.

$$\text{total\_effort}_{sbl} \leq \text{avg\_story\_points}_{\text{vel}(T(sbl))} \quad \forall sbl \in SBL$$

er_must_be_active A worker assigned to a task must have an 'active' status.

$$\text{status}_w = \text{'active'} \quad \forall w \text{ assigned to any } tsk \in TSK$$

team_size_gt_min A team must have a minimum number of members (e.g., 3).

$$\text{team\_size}_t \geq 3 \quad \forall t \in T$$

team_size_le_max A team must not exceed a maximum number of members (e.g., 9).

$$\text{team\_size}_t \leq 9 \quad \forall t \in T$$

must_have_points Any user story in a sprint must have assigned story points greater than zero.

$$\text{story\_points}_{us} > 0 \quad \forall us \text{ in any } sbl \in SBL$$

uration_le_15_min Daily scrums must not exceed 15 minutes.

$$\text{duration}_{ds} \leq 15 \quad \forall ds \in DS$$

sprint_has_a_goal Every sprint must have a non-empty objective description.

$$\text{objective\_description}_{sg} \neq \text{NULL} \quad \forall sg \in SG$$

must_be_in_sprint A task being worked on must belong to the current sprint backlog.

$$\text{status}_{tsk} \in \{\text{'in\_progress'}, \text{'done'}\} \implies tsk \in \text{current } sbl$$

es_documentation A completed feature should have linked documentation.

$$\text{status}_f = \text{'done'} \implies \exists fed \in FED \text{ linked to } f$$

ect_has_start_date Every project must have a defined start date.

$$\text{project\_start}_p \neq \text{NULL} \quad \forall p \in P$$

level_requirement A worker on a specialized task must have a required skill level.

$$\text{level}_{s(w)} \geq \text{required\_level}_{s(tsk)} \quad \forall w, tsk, s$$

must_be_addressed A blocker with high severity cannot have a status of 'ignored'.

$$\text{severity}_{bl} = \text{'high'} \implies \text{status}_{bl} \neq \text{'ignored'} \quad \forall bl \in BL$$

cklog_is_managed A product backlog must be updated regularly.

$$\text{last\_updated}_{pb} \geq (\text{current\_date} - \Delta t) \quad \forall pb \in PB$$

5 Decision Variables

The following variables represent the decisions to be made by the optimization model.

to_sprint_backlog Assign a User Story to a Sprint Backlog.

$$X_{us,sbl} \in \{0, 1\} \quad \forall us \in US, sbl \in SBL$$

gn_worker_to_task Assign a Worker to a Task.

$$Y_{w,tsk} \in \{0, 1\} \quad \forall w \in W, tsk \in TSK$$

ats_for_user_story Estimate effort for a User Story.

$$SP_{us} \in \{1, 2, 3, 5, 8, 13, 21\} \quad \forall us \in US$$

riority_for_feature Set the business priority for a Feature.

$$P_f \in \{1, 2, \ldots, 10\} \quad \forall f \in F$$

e_for_release_plan Include a Feature in a Release Plan.

$$Z_{f,rep} \in \{0, 1\} \quad \forall f \in F, rep \in REP$$

int_duration_days Define the length of a Sprint in days.

$$\text{Duration}_{sp} \in \{7, 14, 21, 30\} \quad \forall sp \in SP$$

e_team_to_project Assign a Team to a Project.

$$A_{t,p} \in \{0, 1\} \quad \forall t \in T, p \in P$$

task_effort_hours Estimate the effort in hours for a Task.

$$E_{tsk} \in \mathbb{R}^+ \quad \text{where } 0.5 \leq E_{tsk} \leq 16.0 \quad \forall tsk \in TSK$$

t_budget_amount Allocate a budget to a Project.

$$B_p \in \mathbb{R}^+ \quad \text{where } 10000 \leq B_p \leq 1000000 \quad \forall p \in P$$

hoose_worker_role Assign a Role to a Worker.

$$R_{w,r} \in \{0, 1\} \quad \forall w \in W, r \in R \text{ s.t. } \sum_r R_{w,r} \geq 1$$

pdate_task_status Set the status of a Task.

$$S_{tsk} \in \{\text{ToDo, InProgress, Done}\} \quad \forall tsk \in TSK$$

11: set_team_size Define the number of members in a team.

$$\text{Size}_t \in \{3, 4, \ldots, 9\} \quad \forall t \in T$$

_blocker_for_task Flag a Task with a new Blocker.

$$C_{bl,tsk} \in \{0, 1\} \quad \forall bl \in BL, tsk \in TSK$$