

# SCRUM Domain Optimization Model

Generated by GPT-5 Thinking

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## 1 1. Sets (Entities)

- $\mathcal{P}$  (P): set of Projects.
- $\mathcal{T}$  (T): set of Teams.
- $\mathcal{W}$  (W): set of Workers.
- $\mathcal{F}$  (F): set of Features.
- $\mathcal{SK}$  (S): set of Skills.
- $\mathcal{R}$  (R): set of Roles.
- $\mathcal{PO}$  (PO): set of Product Owners.
- $\mathcal{SM}$  (SM): set of Scrum Masters.
- $\mathcal{PB}$  (PB): set of Product Backlogs.
- $\mathcal{SP}$  (SP): set of Sprints.
- $\mathcal{SPP}$  (SPP): set of Sprint Planning events.
- $\mathcal{DS}$  (DS): set of Daily Scrums.
- $\mathcal{SR}$  (SR): set of Sprint Reviews.
- $\mathcal{SRE}$  (SRE): set of Sprint Retrospectives.
- $\mathcal{SBL}$  (SBL): set of Sprint Backlogs.
- $\mathcal{SG}$  (SG): set of Sprint Goals.
- $\mathcal{E}$  (E): set of Epics.
- $\mathcal{US}$  (US): set of User Stories.
- $\mathcal{TSK}$  (TSK): set of Tasks.
- $\mathcal{DEV}$  (DEV): set of Development Snapshots.
- $\mathcal{BL}$  (BL): set of Blockers.
- $\mathcal{SH}$  (SH): set of Stakeholders.
- $\mathcal{VEL}$  (VEL): set of Velocity measurements.
- $\mathcal{REP}$  (REP): set of Release Plans.
- $\mathcal{RM}$  (RM): set of Roadmaps.
- $\mathcal{SCB}$  (SCB): set of Scrum Boards.
- $\mathcal{FED}$  (FED): set of Feature Documents.

## 2 2. Indices

- $p \in \mathcal{P}, t \in \mathcal{T}, w \in \mathcal{W}, f \in \mathcal{F}, e \in \mathcal{E}, u \in \mathcal{US}, k \in \mathcal{TSK}, s \in \mathcal{SP}, b \in \mathcal{BL}, v \in \mathcal{VEL}, d \in \mathcal{DEV}, r \in \mathcal{REP}.$

### Attribute-derived parameters (examples; sanitized names):

- $\text{eff}_f^F \in \mathbb{R}_{\geq 0}$ : Feature estimated\_effort.
- $\text{prio}_f^F \in \mathbb{R}_{\geq 0}$ : Feature priority.
- $\text{sp}_u \in \mathbb{Z}_{\geq 0}$ : UserStory story\_points.
- $\text{prio}_u^{US} \in \mathbb{R}_{\geq 0}$ : UserStory priority.
- $\text{hasAC}_u \in \{0, 1\}$ : 1 if acceptance\_criteria present.
- $\text{eff}_k^{TSK} \in \mathbb{R}_{\geq 0}$ : Task effort.
- $\text{avail}_w \in [0, 1]$ : Worker availability.
- $\text{attend}_s^{SR} \in \mathbb{Z}_{\geq 0}$ : SprintReview attendees\_count (for sprint  $s$ ).
- $\overline{\text{sp}}_v \in \mathbb{R}_{\geq 0}$ ,  $\text{minVel}_v$ ,  $\text{maxVel}_v$ : Velocity avg\_story\_points, min\_velocity, max\_velocity.
- $\text{cap}_t \in \mathbb{R}_{\geq 0}$ : team capacity proxy (derived from team\_size and availability).
- $\text{cap}_s^{SP} \in \mathbb{R}_{\geq 0}$ : sprint capacity (e.g., person-days).
- $\text{isAllowedTarget}_{d,\tau} \in \{0, 1\}$ : whether deployment target  $\tau \in \{\text{staging}, \text{production}\}$  allowed for snapshot  $d$ .
- $\text{isActivePB} \in \{0, 1\}$ : ProductBacklog status active flag.

### Relationship-derived sets (examples):

- $\text{US}(e) \subseteq \mathcal{US}$ : user stories in Epic  $e$  (R9).
- $\text{TSK}(u) \subseteq \mathcal{TSK}$ : tasks in UserStory  $u$  (R10).
- $\text{BLK}(k) \subseteq \mathcal{BL}$ : blockers of Task  $k$  (R16).
- $\text{F}(PB) \subseteq \mathcal{F}$ : features in Product Backlog (R7).

## 3 3. Goals

### • Decision variables used by goals (subset):

- $x_f \in \{0, 1\}$ : select\_feature (DV0) for release.
- $y_{u,s} \in \{0, 1\}$ : assign\_user\_story\_to\_sprint (DV1).
- $z_{k,w} \in \{0, 1\}$ : allocate\_task\_to\_worker (DV2).
- $q \in \{3, \dots, 10\}$ : review\_attendees\_quota (DV8).
- $L \in \{3, \dots, 12\}$ : limit\_sprint\_wip (DV10).
- $d_d^{\text{stag}}, d_d^{\text{prod}} \in \{0, 1\}$ ,  $d_d^{\text{stag}} + d_d^{\text{prod}} = 1$ : choose\_deployment\_target (DV4).

### • G0 (maximize\_velocity\_points):

$$\max \sum_{s \in \mathcal{SP}} \sum_{u \in \mathcal{US}} \text{sp}_u y_{u,s}$$

(proxy for maximizing average story points delivered per sprint).

- **G1** (maximize\_goal\_achievement\_rate):

$$\max \sum_{s \in \mathcal{SP}} \gamma_s \quad \text{with } \gamma_s \in \{0, 1\} \text{ indicating Sprint } s \text{ goal achieved}$$

- **G2** (minimize\_open\_blockers):

$$\min \sum_{k \in \mathcal{TSK}} \sum_{b \in \text{BLK}(k)} \beta_{k,b}$$

where  $\beta_{k,b} \in \{0, 1\}$  flags blocker  $b$  unresolved for task  $k$ .

- **G3** (minimize\_feature\_effort):

$$\min \sum_{f \in \mathcal{F}} \text{eff}_f^F x_f$$

- **G4** (maximize\_story\_points\_completed):

$$\max \sum_{s \in \mathcal{SP}} \sum_{u \in \mathcal{US}} \text{sp}_u y_{u,s}$$

- **G5** (minimize\_task\_effort):

$$\min \sum_{k \in \mathcal{TSK}} \text{eff}_k^{TSK} \left( \sum_{w \in \mathcal{W}} z_{k,w} \right)$$

- **G6** (maximize\_team\_utilization):

$$\max \sum_{w \in \mathcal{W}} \text{avail}_w \left( \sum_{k \in \mathcal{TSK}} z_{k,w} \right)$$

- **G7** (minimize\_least\_velocity\_gap):

$$\min \sum_{s \in \mathcal{SP}} \left| \sum_u \text{sp}_u y_{u,s} - \min_{v \in \mathcal{VEL}} \text{minVel}_v \right|$$

- **G8** (maximize\_priority\_delivery):

$$\max \sum_{f \in \mathcal{F}} \text{prio}_f^F x_f$$

- **G9** (minimize\_backlog\_wip):

$$\min \sum_{s \in \mathcal{SP}} \sum_{u \in \mathcal{US}} \omega_{u,s} \quad \text{with } \omega_{u,s} \in \{0, 1\} \text{ indicating "in-progress"}$$

- **G10** (maximize\_review\_attendance):

$$\max \sum_{s \in \mathcal{SP}} \text{attend}_s^{SR} \quad \text{subject to } \text{attend}_s^{SR} \geq q$$

- **G11** (maximize\_test\_status\_pass):

$$\max \sum_{d \in \mathcal{DEV}} \tau_d \quad \text{with } \tau_d \in \{0, 1\} \text{ indicating snapshot tests passing}$$

## 4 4. Conditions

- **C0** (`sprint_within_dates`) (Must-Match): For all  $s \in \mathcal{SP}$ ,  $\text{start}(s) \leq \text{end}(s)$ .
- **C1** (`sprint_end_after_start`) (Must-Match): For all  $s$ ,  $\text{end}(s) - \text{start}(s) \geq 1$  day.
- **C2** (`stories_have_acceptance_criteria`) (Must-Match):

$$y_{u,s} \leq \text{hasAC}_u \quad \forall u \in \mathcal{US}, \forall s \in \mathcal{SP}.$$

- **C3** (`stories_within_priority_bounds`) (Must-Match):

$$y_{u,s} = 1 \Rightarrow \underline{P} \leq \text{prio}_u^{US} \leq \overline{P} \quad \forall u, s.$$

- **C4** (`tasks_have_type`) (Must-Match): Each scheduled task has a valid type:

$$\sum_w z_{k,w} \leq \text{hasType}_k \quad \forall k \in \mathcal{TSK}, \text{hasType}_k \in \{0, 1\}.$$

- **C5** (`blockers_resolved_before_done`) (Cannot-Match if blocked):

$$\sum_w z_{k,w} \leq 1 - \max_{b \in \text{BLK}(k)} \beta_{k,b} \quad \forall k \in \mathcal{TSK}.$$

- **C6** (`team_size_capacity_check`) (May-Match):

$$\sum_k \text{eff}_k^{TSK} \left( \sum_w z_{k,w} \right) \leq \sum_{t \in \mathcal{T}} \text{cap}_t \cdot (1 + \theta),$$

where  $\theta$  is an optional buffer (DV7).

- **C7** (`sprint_backlog_effort_limit`) (Must-Match):

$$\sum_u \text{sp}_u y_{u,s} \leq \text{cap}_s^{SP} \quad \forall s \in \mathcal{SP}.$$

- **C8** (`release_plan_has_status`) (Must-Match):  $\text{status}(r) \in \{\text{planned}, \text{approved}\}$  for all  $r \in \mathcal{REP}$ .
- **C9** (`deployment_target_allowed`) (Must-Match):

$$d_d^{\text{stag}} \leq \text{isAllowedTarget}_{d, \text{staging}}, \quad d_d^{\text{prod}} \leq \text{isAllowedTarget}_{d, \text{production}} \quad \forall d \in \mathcal{DEV}.$$

- **C10** (`velocity_reference_team`) (Must-Match):

$$\sum_{v \in \mathcal{VEL}: v \leftrightarrow t} 1 = 1 \quad \forall t \in \mathcal{T} \quad (\text{R19}).$$

- **C11** (`product_backlog_is_active`) (Must-Match):  $\text{isActivePB} = 1$ .

## 5 5. Decision Variables

- **DV0** `select_feature`:  $x_f \in \{0, 1\}$  for  $f \in \mathcal{F}$  (Domain  $\{0, 1\}$ ). Min=0, Max=1.
- **DV1** `assign_user_story_to_sprint`:  $y_{u,s} \in \{0, 1\}$  for  $u \in \mathcal{US}, s \in \mathcal{SP}$ . Min=0, Max=1.
- **DV2** `allocate_task_to_worker`:  $z_{k,w} \in \{0, 1\}$  for  $k \in \mathcal{TSK}, w \in \mathcal{W}$ . Min=0, Max=1.
- **DV3** `set_task_priority_weight`:  $h_k \in \{0, 1, 2, 3, 4, 5\}$  for  $k \in \mathcal{TSK}$ . Min=0, Max=5.
- **DV4** `choose_deployment_target`:  $d_d^{\text{stag}}, d_d^{\text{prod}} \in \{0, 1\}, d_d^{\text{stag}} + d_d^{\text{prod}} = 1$  (encode categorical). Min=0, Max=1.
- **DV5** `enable_blocker_resolution`:  $\rho_b \in \{0, 1\}$  for  $b \in \mathcal{BL}$ . Min=0, Max=1.
- **DV6** `select_epic_for_breakdown`:  $\xi_e \in \{0, 1\}$  for  $e \in \mathcal{E}$ . Min=0, Max=1.
- **DV7** `team_capacity_buffer_percent`: choose  $\theta \in \{0, 0.05, 0.10, 0.15, 0.20\}$  (encodable via binaries). Min=0, Max=0.20.
- **DV8** `choose_review_attendees_quota`:  $q \in \{3, 4, 5, 6, 7, 8, 9, 10\}$ . Min=3, Max=10.
- **DV9** `select_tests_for_snapshot`:  $g_d \in \{0, 1\}$  for  $d \in \mathcal{DEV}$  (gate enabled). Min=0, Max=1.
- **DV10** `limit_sprint_wip`:  $L \in \{3, \dots, 12\}$ . Min=3, Max=12.
- **DV11** `choose_release_version`:  $v^* \in \{1, 2, 3, 4, 5\}$ . Min=1, Max=5.

### Canonical scheduling/consistency constraints (linking DVs to data):

- Each task assigned to at most one worker:  $\sum_w z_{k,w} \leq 1 \quad \forall k$ .
- WIP limit per sprint backlog (board-level):  $\sum_u \omega_{u,s} \leq L \quad \forall s$ .
- Capacity coupling:  $\sum_k \text{eff}_k^{\text{TSK}} (\sum_w z_{k,w}) \leq (1 + \theta) \sum_t \text{cap}_t$ .
- Story-task linkage:  $y_{u,s} = 1 \Rightarrow \sum_{k \in \text{TSK}(u)} \sum_w z_{k,w} \geq 1$ .
- Epic selection implies at least one story scheduled:  $\xi_e = 1 \Rightarrow \sum_{u \in \text{US}(e)} \sum_s y_{u,s} \geq 1$ .