An Optimization Model for a SCRUM-Oriented Software Development Domain

(Author)

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Contents

1 1. Sets (Entities)

- $\mathcal{P}$ (Projects; **Project**): products/initiatives; attributes: id, name, project_start, project_end, description, budget, status, target_audience, priority.
- $\mathcal{T}$ (Teams; **Team**): self-organized, cross-functional teams; attributes: id, name, team_size, team_start, team_status, location, team_type.
- $\mathcal{W}$ (Workers; **Worker**): team members; attributes: id, name, first_name, email, start_date, status, availability.
- $\mathcal{F}$ (Features; **Feature**): mid-sized functionality; attributes: id, title, description, status, priority, estimated_effort.
- $\mathcal{S}$ (Skills; **Skill**): professional/social competence; attributes: id, label, description, level, certified, category.
- $\mathcal{R}$ (Roles; **Role**): responsibilities; attributes: id, role_name, description, area_of_responsibility.
- $\mathcal{PO}$ (**ProductOwner**): attributes: id, name, email, availability.
- $\mathcal{SM}$ (**ScrumMaster**): attributes: id, name, email, experience.
- $\mathcal{PB}$ (**ProductBacklog**): attributes: id, created_on, last_updated, number_of_entries, status.
- $\mathcal{SP}$ (**Sprint**): attributes: id, sprint_number, start_date, end_date, status, achievement_of_goal.
- $\mathcal{SPP}$ (**SprintPlanning**): attributes: id, date, duration_(min), moderation, outcome_documentation.
- $\mathcal{DS}$ (**DailyScrum**): attributes: id, date, time, duration, moderation.
- $\mathcal{SR}$ (**SprintReview**): attributes: id, date, duration, feedback_documentation, attendees_count.
- $\mathcal{SRE}$ (**SprintRetrospective**): attributes: id, date, duration, improvement_actions, team_satisfaction, moderation.
- $\mathcal{SBL}$ (**SprintBacklog**): attributes: id, number_of_tasks, last_updated, status, total_effort.
- $\mathcal{SG}$ (**SprintGoal**): attributes: id, objective_description, achievement_status, benefit.
- $\mathcal{E}$ (**Epic**): attributes: id, title, description, priority, status, estimated_effort.
- $\mathcal{US}$ (**UserStory**): attributes: id, title, description, acceptance_criteria, priority, story_points, status.
- $\mathcal{TSK}$ (**Task**): attributes: id, title, description, status, effort, type.
- $\mathcal{DEV}$ (**DevelopmentSnapshot**): attributes: id, version_number, creation_date, test_status, deployment_target, documentation.
- $\mathcal{BL}$ (**Blocker**): attributes: id, title, description, severity, status, detected_on, resolved_on.
- $\mathcal{SH}$ (**Stakeholder**): attributes: id, name, organization, role, email, area_of_interest, influence_level, relevance_to_feature.
- $\mathcal{VEL}$ (**Velocity**): attributes: id, number_of_sprints_used, avg._story_points, max_velocity, min_velocity, trend.

- $\mathcal{REP}$ (**ReleasePlan**): attributes: id, version, planned_date, included_features, status.
- $\mathcal{RM}$ (**Roadmap**): attributes: id, start_date, end_date, milestones, objectives, versions.
- $\mathcal{SCB}$ (**ScrumBoard**): attributes: id, board_type, columns_(todo/done...), number_of_cards, last_updated.
- $\mathcal{FED}$ (**FeatureDocumentation**): attributes: id, title, description, creation_date, change_log, linked_requirements, author.

Relationship-induced incidence sets (from Relationships.csv).

- $A^{\text{team-proj}} \subseteq \mathcal{T} \times \mathcal{P}$ (`R1 is_assigned_to_project`)
- $A^{\text{work-team}} \subseteq \mathcal{W} \times \mathcal{T}$ (`R2 belongs_to_team`)
- $A^{\text{work-skill}} \subseteq \mathcal{W} \times \mathcal{S}$ (`R3 has_skill`)
- $A^{\text{work-role}} \subseteq \mathcal{W} \times \mathcal{R}$ (`R4 takes_on_role`)
- $A^{\text{po-pb}} \subseteq \mathcal{PO} \times \mathcal{PB}$ (`R5 manages_backlog`)
- $A^{\text{team-sm}} \subseteq \mathcal{T} \times \mathcal{SM}$ (`R6 is_supported_by`)
- $A^{\text{pb-feat}} \subseteq \mathcal{PB} \times \mathcal{F}$ (`R7 contains_feature`)
- $A^{\text{pb-epic}} \subseteq \mathcal{PB} \times \mathcal{E}$ (`R8 contains_epic`)
- $A^{\text{epic-us}} \subseteq \mathcal{E} \times \mathcal{US}$ (`R9 contains_user_story`)
- $A^{\text{us-task}} \subseteq \mathcal{US} \times \mathcal{TSK}$ (`R10 consists_of_tasks`)
- $A^{\text{us-sbl}} \subseteq \mathcal{US} \times \mathcal{SBL}$ (`R11 is_in_sprint_backlog`)
- $A^{\text{sbl-sp}} \subseteq \mathcal{SBL} \times \mathcal{SP}$ (`R12 belongs_to_sprint`)
- $A^{\text{sp-sg}} \subseteq \mathcal{SP} \times \mathcal{SG}$ (`R13 pursues_goal`)
- $A^{\text{scb-task}} \subseteq \mathcal{SCB} \times \mathcal{TSK}$ (`R14 contains_tasks`)
- $A^{\text{fed-feat}} \subseteq \mathcal{FED} \times \mathcal{F}$ (`R15 documents_feature`)
- $A^{\text{task-bl}} \subseteq \mathcal{TSK} \times \mathcal{BL}$ (`R16 is_blocked_by`)
- $A^{\text{sh-sr}} \subseteq \mathcal{SH} \times \mathcal{SR}$ (`R17 participates_in`)
- $A^{\text{sm-sre}} \subseteq \mathcal{SM} \times \mathcal{SRE}$ (`R18 moderates_retrospective`)
- $A^{\text{vel-team}} \subseteq \mathcal{VEL} \times \mathcal{T}$ (`R19 refers_to_team`)
- $A^{\text{rep-feat}} \subseteq \mathcal{REP} \times \mathcal{F}$ (`R20 plans_release`)
- $A^{\text{rep-rm}} \subseteq \mathcal{REP} \times \mathcal{RM}$ (`R21 is_part_of_roadmap`)
- $A^{\text{sp-dev}} \subseteq \mathcal{SP} \times \mathcal{DEV}$ (`R22 generates_snapshot`)

2 2. Indices

- $p \in \mathcal{P}$, $t \in \mathcal{T}$, $w \in \mathcal{W}$, $f \in \mathcal{F}$, $s \in \mathcal{S}$, $r \in \mathcal{R}$, $po \in \mathcal{PO}$, $sm \in \mathcal{SM}$, $pb \in \mathcal{PB}$, $sp \in \mathcal{SP}$, $spp \in \mathcal{SPP}$, $ds \in \mathcal{DS}$, $sr \in \mathcal{SR}$, $sre \in \mathcal{SRE}$, $sbl \in \mathcal{SBL}$, $sg \in \mathcal{SG}$, $e \in \mathcal{E}$, $us \in \mathcal{US}$, $tsk \in \mathcal{TSK}$, $dev \in \mathcal{DEV}$, $bl \in \mathcal{BL}$, $sh \in \mathcal{SH}$, $vel \in \mathcal{VEL}$, $rep \in \mathcal{REP}$, $rm \in \mathcal{RM}$, $scb \in \mathcal{SCB}$, $fed \in \mathcal{FED}$.

Parameters induced by attributes (examples).

- budget_p, priority^F_f, estEffort^F_f, storyPts_{us}, effort_{tsk}, sev_{bl}, $\text{achGoal}_{sp} \in [0,1]$, $\text{teamSat}_{sre} \in [1,5]$, $\text{status}_{\cdot} \in \{0,1,\dots\}$, avgSP_{vel}, etc.

3 3. Goals

Each goal G_i is stated with its ID/Name and an objective expression.

- **G0 maximize_average_story_points**
  $\max \sum_{(vel,t)\in A^{\text{vel-team}}} \omega_{G0}\, \text{avgSP}_{vel} \Big(\sum_{p:(t,p)\in A^{\text{team-proj}}} x^{\text{team,proj}}_{t,p} \Big)$.
- **G1 minimize_sprint_backlog_effort**
  $\min\ \omega_{G1} \sum_{(us,sbl)\in A^{\text{us-sbl}}} \sum_{sp:(sbl,sp)\in A^{\text{sbl-sp}}} \text{storyPts}_{us}\, x^{\text{us,sp}}_{us,sp}$.
- **G2 minimize_blocker_severity**
  $\min\ \omega_{G2} \sum_{(tsk,bl)\in A^{\text{task-bl}}} \text{sev}_{bl}\,(1 - x^{\text{bl}}_{bl})$ (resolve to reduce severity impact).
- **G3 maximize_sprint_goal_achievement**
  $\max\ \omega_{G3} \sum_{(sp,sg)\in A^{\text{sp-sg}}} \text{achGoal}_{sp}$.
- **G4 minimize_project_budget_usage**
  $\min\ \omega_{G4} \sum_{p\in\mathcal{P}} \text{budget}_p \Big(\sum_{t:(t,p)\in A^{\text{team-proj}}} x^{\text{team,proj}}_{t,p} \Big)$.
- **G5 maximize_feature_priority_delivered**
  $\max\ \omega_{G5} \sum_{(rep,f)\in A^{\text{rep-feat}}} \text{priority}^F_f\, x^{\text{feat,rep}}_{f,rep}$.
- **G6 minimize_task_effort**
  $\min\ \omega_{G6} \sum_{tsk\in\mathcal{TSK}} \text{effort}_{tsk}\, x^{\text{task}}_{tsk}$.
- **G7 maximize_team_satisfaction**
  $\max\ \omega_{G7} \sum_{sre\in\mathcal{SRE}} \text{teamSat}_{sre}$.
- **G8 minimize_daily_scrum_duration**
  $\min\ \omega_{G8} \sum_{ds\in\mathcal{DS}} y^{\text{dsDur}}_{ds}$.
- **G9 maximize_stakeholder_relevance**
  $\max\ \omega_{G9} \sum_{sh\in\mathcal{SH}} \text{relevance}_{sh}$ (weighted participation in reviews optional).
- **G10 minimize_open_blockers**
  $\min\ \omega_{G10} \sum_{bl\in\mathcal{BL}} (1 - x^{\text{bl}}_{bl})$.
- **G11 maximize_release_readiness**
  $\max\ \omega_{G11} \sum_{rep\in\mathcal{REP}} \text{status}^{REP}_{rep} \Big/ \big|\mathcal{REP}\big|$ (proxy via release statuses).
- **G12 minimize_planning_time**
  $\min\ \omega_{G12} \sum_{spp\in\mathcal{SPP}} y^{\text{planDur}}_{spp}$.

4 4. Conditions

Each condition C_j is stated with ID/Name, logic, and the corresponding constraint(s).

- **C0 must_match_active_product_backlog**
  Logic: Product Backlog must be active. Math: $\text{status}_{pb}^{PB} \geq 1 \quad \forall pb \in \mathcal{PB}$.
- **C1 must_match_worker_availability**
  Logic: Assigned workers must be available.
  Math: $\sum_{tsk \in \mathcal{TSK}} x_{w,tsk}^{\text{work,task}} \, \text{effort}_{tsk} \leq \text{availability}_w \quad \forall w \in \mathcal{W}$.
- **C2 cannot_match_task_done_outside_sprint**
  Logic: Tasks marked done must belong to a sprint backlog.
  Math: $x_{tsk}^{\text{task}} \leq \sum_{(us,tsk) \in A^{\text{us-task}}} \sum_{(us,sbl) \in A^{\text{us-sbl}}} \sum_{(sbl,sp) \in A^{\text{sbl-sp}}} 1 \quad \forall tsk$.
- **C3 must_match_ci_test_passed**
  Logic: Only increments with passed tests are releasable.
  Math: For all $(sp, dev) \in A^{\text{sp-dev}}$: $\text{testStatus}_{dev} = 1 \Rightarrow \text{releaseEligible}_{sp} = 1$; linearization: $\text{releaseEligible}_{sp} \leq \text{testStatus}_{dev}$.
- **C4 must_match_sprint_goal_defined**
  Logic: Each sprint must have a defined goal.
  Math: $\sum_{(sp,sg) \in A^{\text{sp-sg}}} 1 \geq 1 \quad \forall sp \in \mathcal{SP}$.
- **C5 may_match_high_influence_stakeholders**
  Logic (soft): Prefer high-influence stakeholders in reviews.
  Math (penalty/reward term): add $+\lambda_{C5} \sum_{(sh,sr) \in A^{\text{sh-sr}}} \text{influence}_{sh}$ to objective (or constraint with slack).
- **C6 must_match_feature_effort_cap**
  Logic: Effort of selected features per sprint $\leq$ cap.
  Math: $\sum_{(us,sbl) \in A^{\text{us-sbl}}} \sum_{(us,tsk) \in A^{\text{us-task}}} \text{effort}_{tsk} \; x_{tsk}^{\text{task}} \leq \text{CapEffort}_{sp} \quad \forall sp$.
- **C7 must_match_story_points_capacity**
  Logic: Sum of story points respect capacity.
  Math: $\sum_{us \in \mathcal{US}} \text{storyPts}_{us} \, x_{us,sp}^{\text{us,sp}} \leq y_{sp}^{\text{capSP}} \quad \forall sp$, with $y_{sp}^{\text{capSP}} \leq \overline{CSP}$.
- **C8 must_match_sprint_backlog_committed**
  Logic: Sprint backlog must be committed before start.
  Math: $\text{status}_{sbl}^{SBL} \geq 1 \quad \forall (sbl, sp) \in A^{\text{sbl-sp}}$.
- **C9 may_match_positive_retrospective_trend**
  Logic (soft): prefer improving team satisfaction.
  Math: add $+\lambda_{C9} \sum_{sre} \text{teamSat}_{sre}$ to objective (or enforce $\text{teamSat}_{sre} \geq \underline{S}$).
- **C10 must_match_team_size_bounds**
  Logic: Team size within bounds.
  Math: $\underline{n} \leq \text{team\_size}_t \leq \overline{n} \quad \forall t \in \mathcal{T}$.
- **C11 cannot_match_open_blockers_at_release**
  Logic: No open blockers at release.
  Math: $\sum_{bl \in \mathcal{BL}} (1 - x_{bl}^{\text{bl}}) = 0$ for releases rep (or $\leq \epsilon$).
- **C12 may_match_review_feedback_documented**
  Logic (soft): documented feedback preferred.
  Math: add $+\lambda_{C12} \sum_{sr \in \mathcal{SR}} \text{hasFeedback}_{sr}$ to objective.

5 5. DecisionVariables

- **Binary assignment**
  $x^{\text{team,proj}}_{t,p} \in \{0,1\}$ (`DV0`): team t assigned to project p; only allowed if $(t,p) \in A^{\text{team-proj}}$.
  $x^{\text{us,sp}}_{us,sp} \in \{0,1\}$ (`DV1`): user story us selected in sprint sp (through its sprint backlog).
  $x^{\text{task}}_{tsk} \in \{0,1\}$ (`DV2`): task tsk selected/committed.
  $x^{\text{work,task}}_{w,tsk} \in \{0,1\}$ (`DV3`): worker w assigned to task tsk.
  $x^{\text{feat,rep}}_{f,rep} \in \{0,1\}$ (`DV4`): feature f included in release plan rep.
  $x^{\text{bl}}_{bl} \in \{0,1\}$ (`DV5`): blocker bl is resolved (1) or open (0).

- **Integer/real planning variables**
  $y^{\text{capSP}}_{sp} \in \mathbb{Z}_{\geq 0}$ (`DV6`): sprint capacity in story points; bounds $0 \leq y^{\text{capSP}}_{sp} \leq 500$.
  $y^{\text{dsDur}}_{ds} \in \mathbb{Z}_{\geq 0}$ (`DV7`): Daily Scrum duration (min); bounds $0 \leq y^{\text{dsDur}}_{ds} \leq 30$.
  $y^{\text{planDur}}_{spp} \in \mathbb{Z}_{\geq 0}$ (`DV8`): Sprint Planning duration (min); bounds $0 \leq y^{\text{planDur}}_{spp} \leq 480$.
  $y^{\text{budget}}_{p} \in \mathbb{R}_{\geq 0}$ (`DV9`): allocated project budget; bounds $0 \leq y^{\text{budget}}_{p} \leq 10^6$.
  $y^{\text{teamSize}}_{t} \in \mathbb{Z}_{\geq 0}$ (`DV10`): configured team size; bounds $3 \leq y^{\text{teamSize}}_{t} \leq 15$.
  $y^{\text{prioW}} \in \mathbb{R}_{\geq 0}$ (`DV11`): global priority weight; bounds $0 \leq y^{\text{prioW}} \leq 10$.
  $y^{\text{WIP}}_{t} \in \mathbb{Z}_{\geq 0}$ (`DV12`): WIP limit for team t; bounds $0 \leq y^{\text{WIP}}_{t} \leq 100$.

Standard linking and capacity constraints (illustrative).

- If $x^{\text{us,sp}}_{us,sp} = 1$ then at least one task of us must be selected: $\sum_{tsk:(us,tsk)\in A^{\text{us-task}}} x^{\text{task}}_{tsk} \geq x^{\text{us,sp}}_{us,sp}$.
- Worker assignment implies task selection: $x^{\text{work,task}}_{w,tsk} \leq x^{\text{task}}_{tsk}$.
- WIP per team (via board) bounded: $\sum_{(scb,tsk)\in A^{\text{scb-task}}} x^{\text{task}}_{tsk} \leq y^{\text{WIP}}_{t}$ for tasks routed to team t.

Multi-objective aggregation (optional). If a single scalar objective is preferred, combine all goals using weights ω_{Gi} and signs per `GoalType`:

$$\max \sum_{i\in\{G0,\ldots,G12\}} \sigma_i\, \omega_{Gi}\, \Phi_i(x,y) \quad \text{with } \sigma_i = +1 \text{ for “maximize” and } \sigma_i = -1 \text{ for “minimize”.}$$