Mathematical Optimization Model for Scrum Project Management

Generated by Gemini

September 4, 2025

Contents

1 Sets (Entities)

These sets are derived from the domain model entities and form the basis of the optimization problem. Each set contains all instances of a particular entity.

- **Project (P)**: The set of all projects.
- **Team (T)**: The set of all teams.
- **Worker (W)**: The set of all workers.
- **Feature (F)**: The set of all features.
- **Skill (S)**: The set of all skills.
- **Role (R)**: The set of all Scrum roles.
- **ProductOwner (PO)**: The set of all product owners.
- **ScrumMaster (SM)**: The set of all scrum masters.
- **ProductBacklog (PB)**: The set of all product backlogs.
- **Sprint (SP)**: The set of all sprints.
- **UserStory (US)**: The set of all user stories.
- **Task (TSK)**: The set of all tasks.
- **Blocker (BL)**: The set of all blockers.
- **Stakeholder (SH)**: The set of all stakeholders.
- **ReleasePlan (REP)**: The set of all release plans.

2 Indices

These indices are used to iterate over the sets defined above in mathematical formulations.

- $p \in P$: Index for a project.
- $t \in T$: Index for a team.
- $w \in W$: Index for a worker.
- $f \in F$: Index for a feature.
- $s \in S$: Index for a skill.
- $r \in R$: Index for a role.
- $po \in PO$: Index for a product owner.
- $sm \in SM$: Index for a scrum master.
- $pb \in PB$: Index for a product backlog.
- $sp \in SP$: Index for a sprint.
- $us \in US$: Index for a user story.

- $tsk \in TSK$: Index for a task.
- $bl \in BL$: Index for a blocker.
- $sh \in SH$: Index for a stakeholder.
- $rep \in REP$: Index for a release plan.

3 Decision Variables

These are the variables that the optimization model will determine. They represent the choices to be made, such as assignments and allocations.

- **DV0: assign_story_to_sprint** ($X_{us,sp} \in \{0,1\}$): 1 if User Story us is assigned to Sprint sp, 0 otherwise.
- **DV1: assign_worker_to_task** ($Y_{w,tsk} \in \{0,1\}$): 1 if Worker w is assigned to Task tsk, 0 otherwise.
- **DV2: select_feature_for_release** ($Z_{f,rep} \in \{0,1\}$): 1 if Feature f is selected for Release Plan rep, 0 otherwise.
- **DV3: assign_worker_to_team** ($A_{w,t} \in \{0,1\}$): 1 if Worker w is assigned to Team t, 0 otherwise.
- **DV4: set_team_size** ($S_t \in Z^+$): The number of workers assigned to Team t.
- **DV5: allocate_worker_availability** ($Avail_{w,sp} \in [0,1]$): The fraction of availability for Worker w in Sprint sp.
- **DV10: choose_blocker_to_resolve** ($B_{bl,sp} \in \{0,1\}$): 1 if Blocker bl is chosen to be resolved in Sprint sp, 0 otherwise.

4 Goals (Objective Function)

The objective function defines the primary goals of the optimization. It is typically a weighted sum of various maximization and minimization goals. The final objective is to Maximize/Minimize a weighted sum of these terms.

- **G0: maximize_story_points_in_sprint**
  Maximize: $1.0 \cdot \sum_{sp \in SP} \sum_{us \in US} \text{story\_points}_{us} \cdot X_{us,sp}$
- **G1: minimize_effort_for_blockers**
  Minimize: $1.5 \cdot \sum_{sp \in SP} \sum_{bl \in BL} \text{effort}_{bl} \cdot B_{bl,sp}$
- **G2: maximize_feature_priority_in_release**
  Maximize: $1.2 \cdot \sum_{rep \in REP} \sum_{f \in F} \text{priority}_f \cdot Z_{f,rep}$
- **G6: maximize_achievement_of_sprint_goal**
  Maximize: $2.0 \cdot \sum_{sp \in SP} \text{achievement\_status}_{sp}$
- **G8: maximize_stakeholder_satisfaction**
  Maximize: $1.3 \cdot \sum_{sh \in SH} \sum_{f \in F} \text{relevance}_{sh,f} \cdot Z_{f,rep}$

5 Conditions (Constraints)

These are the rules and limitations that the solution must adhere to. They define the feasible region for the decision variables.

- **C0: sprint_effort_must_not_exceed_velocity**
  $\forall t \in T, \forall sp \in \text{SprintsOfTeam}(t) : \sum_{us \in US} \text{story\_points}_{us} \cdot X_{us,sp} \leq \text{velocity}_t$
- **C1: team_size_must_be_within_scrum_limits**
  $\forall t \in T : 3 \leq S_t \leq 9$, where $S_t = \sum_{w \in W} A_{w,t}$
- **C2: worker_must_not_exceed_availability**
  $\forall w \in W, \forall sp \in SP : \sum_{tsk \in TSK} \text{effort}_{tsk} \cdot Y_{w,tsk} \leq Avail_{w,sp} \cdot \text{SprintDuration}_{sp}$
- **C4: user_story_must_have_criteria**
  $\forall us \in US, \forall sp \in SP : X_{us,sp} \leq \text{HasAcceptanceCriteria}_{us}$, where $\text{HasAcceptanceCriteria}_{us}$ is 1 if true, 0 if false.
- **C5: team_must_have_scrum_master**
  $\forall t \in T : \exists sm \in SM$ such that relationship $R6(\text{Team}_t, \text{ScrumMaster}_{sm})$ is true.
- **C6: task_cannot_be_done_if_blocked**
  $\forall tsk \in TSK$: $\text{status}_{tsk} \neq$ 'done' if $\exists bl \in BL$ with $R16(\text{Task}_{tsk}, \text{Blocker}_{bl})$ and $\text{status}_{bl} \neq$ 'resolved'.
- **C7: prefer_certified_skills (Soft Constraint)**
  This can be added to the objective function as a maximization term: Maximize $\sum_{w \in W} \sum_{tsk \in TSK} \text{HasSkill}_{w}$ $Y_{w,tsk}$, where HasSkill is a parameter indicating if worker w has the skill required for task tsk.
- **C9: project_must_adhere_to_end_date**
  $\forall sp \in SP : \text{end\_date}_{sp} \leq \text{project\_end}_{\text{project(sp)}}$