Mathematical Optimization Model for a Scrum-Based Software Development Company

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September 5, 2025

Contents

1 Sets (Entities)

The following sets represent the core entities of the domain model.

- P: Set of all Projects (E_0)
- T: Set of all Teams (E_1)
- W: Set of all Workers (E_2)
- F: Set of all Features (E_3)
- S: Set of all Skills (E_4)
- R: Set of all Roles (E_5)
- PO: Set of all Product Owners (E_6)
- SM: Set of all Scrum Masters (E_7)
- PB: Set of all Product Backlogs (E_8)
- SP: Set of all Sprints (E_9)
- US: Set of all User Stories (E_{17})
- TSK: Set of all Tasks (E_{18})
- BL: Set of all Blockers (E_{20})
- SH: Set of all Stakeholders (E_{21})
- VEL: Set of all Velocities (E_{22})
- SRE: Set of all Sprint Retrospectives (E_{13})
- E: Set of all Epics (E_{16})
- SBL: Set of all Sprint Backlogs (E_{14})
- REP: Set of all Release Plans (E_{23})
- ... (and so on for all other entities)

2 Indices

Indices are used to refer to specific instances within the sets defined above.

- $p \in P$: index for a project
- $t \in T$: index for a team
- $w \in W$: index for a worker
- $f \in F$: index for a feature
- $s \in S$: index for a skill
- $r \in R$: index for a role
- $po \in PO$: index for a product owner

- $sm \in SM$: index for a scrum master
- $sp \in SP$: index for a sprint
- $us \in US$: index for a user story
- $tsk \in TSK$: index for a task
- ... (and so on for all other indices)

3 Goals (Objective Functions)

The following objectives aim to optimize the project and process outcomes. The final objective function would be a weighted sum of these individual goals.

G0 **maximize_project_budget**: $\max \sum_{p \in P} \text{budget}_p \cdot W_{G0}$

G1 **maximize_project_priority**: $\max \sum_{p \in P} \text{priority}_p \cdot W_{G1}$

G2 **maximize_feature_priority**: $\max \sum_{f \in F} \text{priority}_f \cdot \text{select\_f}_f \cdot W_{G2}$

G3 **maximize_story_points_per_sprint**: $\max \sum_{sp \in SP} \sum_{us \in US} \text{story\_points}_{us} \cdot \text{assign\_us\_sp}_{us,sp} \cdot W_{G3}$

G4 **minimize_task_effort**: $\min \sum_{tsk \in TSK} \text{effort}_{tsk} \cdot W_{G4}$

G5 **minimize_blocker_severity**: $\min \sum_{bl \in BL} \text{severity}_{bl} \cdot W_{G5}$

G6 **maximize_team_velocity**: $\max \sum_{t \in T} \text{avg\_story\_points}_t \cdot W_{G6}$

4 Conditions (Constraints)

These are the constraints that the solution must satisfy.

C0 **limit_team_size_max**: $\sum_{w \in W} \text{assign\_w\_t}_{w,t} \leq 9, \quad \forall t \in T$

C1 **limit_team_size_min**: $\sum_{w \in W} \text{assign\_w\_t}_{w,t} \geq 3, \quad \forall t \in T$

C2 **sprint_duration_limit**: $\text{duration\_sp}_{sp} \leq 28, \quad \forall sp \in SP$

C4 **sprint_backlog_capacity**: $\sum_{us \in US} \text{story\_points}_{us} \cdot \text{assign\_us\_sp}_{us,sp} \leq \text{velocity}_t, \quad \forall sp \in SP, t \in T$ (where sprint sp is conducted by team t)

C5 **worker_must_have_role**: $\sum_{r \in R} \text{assign\_w\_r}_{w,r} \geq 1, \quad \forall w \in W$

C7 **team_requires_scrum_master**: $\sum_{sm \in SM} \text{assign\_sm\_t}_{sm,t} = 1, \quad \forall t \in T$

C8 **positive_project_budget**: $\text{budget}_p \geq 0, \quad \forall p \in P$

C11 **po_manages_one_backlog**: $\sum_{pb \in PB} \text{assign\_po\_pb}_{po,pb} = 1, \quad \forall po \in PO$

5 Decision Variables

These are the variables that the optimization model will determine.

DV0 **assign_worker_to_team**: $\text{assign\_w\_t}_{w,t} \in \{0,1\}, \quad \forall w \in W, t \in T$

DV1 **assign_team_to_project**: $\text{assign\_t\_p}_{t,p} \in \{0,1\}, \quad \forall t \in T, p \in P$

DV2 **assign_task_to_worker**: $\text{assign\_tsk\_w}_{tsk,w} \in \{0,1\}, \quad \forall tsk \in TSK, w \in W$

DV3 **select_user_story_for_sprint**: $\text{assign\_us\_sp}_{us,sp} \in \{0,1\}, \quad \forall us \in US, sp \in SP$

DV4 **assign_feature_to_release**: $\text{assign\_f\_rep}_{f,rep} \in \{0,1\}, \quad \forall f \in F, rep \in REP$

DV5 **set_feature_priority**: $\text{priority}_f \in \mathbb{Z}^+, \quad \forall f \in F$

DV6 **set_project_budget**: $\text{budget}_p \in \mathbb{R}^+, \quad \forall p \in P$

DV7 **determine_sprint_duration_days**: $\text{duration\_sp}_{sp} \in \{7, 8, ..., 28\}, \quad \forall sp \in SP$

DV8 **assign_worker_to_role**: $\text{assign\_w\_r}_{w,r} \in \{0,1\}, \quad \forall w \in W, r \in R$