Optimization Model for a SCRUM-Based Software Development Company

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Contents

Introduction

This document presents a formal mathematical optimization model for resource allocation, scheduling, and priority management in a software development company using the SCRUM framework. The model is derived from a defined domain model consisting of Entities, Relationships, Goals, and Constraints.

1 Sets (Entities)

- Project = $\{p|p$ is a Project$\}$, with attributes id, name, project_start, project_end, description, buc
- Team = $\{t|t$ is a Team$\}$, with attributes id, name, team_size, team_start, team_status, location, t
- Worker = $\{w|w$ is a Worker$\}$, with attributes id, name, first_name, email, start_date, status, availa
- Feature = $\{f|f$ is a Feature$\}$, with attributes id, title, description, status, priority, estimated_effor
- Blocker = $\{b|b$ is a Blocker$\}$, with attributes id, title, description, severity, status, detected_on, reso
- SprintGoal = $\{g|g$ is a SprintGoal$\}$, with attributes id, objective_description, achievement_status
- Task = $\{k|k$ is a Task$\}$, with attributes id, title, description, status, effort, type
- Stakeholder = $\{s|s$ is a Stakeholder$\}$, with attributes id, name, organization, role, email, area_of_i
- Velocity = $\{v|v$ is a Velocity$\}$, with attributes id, number_of_sprints_used, avg_story_points, m
- SprintRetrospective = $\{r|r$ is a SprintRetrospective$\}$, with attributes id, date, duration, improvement_actions, team_satisfaction, moderation

2 Indices

- $p, p' \in$ Project
- $t, t' \in$ Team
- $w, w' \in$ Worker
- $f, f' \in$ Feature
- $b, b' \in$ Blocker
- $g, g' \in$ SprintGoal

- $k, k' \in \text{Task}$
- $s, s' \in \text{Stakeholder}$
- $v, v' \in \text{Velocity}$
- $r, r' \in \text{SprintRetrospective}$
- $i, i' \in \text{Sprint}$

3 Goals

- **G0: maximize_team_utilization** - Maximize the average availability of all team workers.

$$\text{Maximize } Z_0 = \sum_{w \in \text{Worker}} \text{availability}(w)$$

- **G1: minimize_project_duration** - Minimize the total duration of the project in days.

$$\text{Minimize } Z_1 = \text{project\_end}(p_0) - \text{project\_start}(p_0) \quad \text{for main project } p_0$$

- **G2: maximize_feature_delivery** - Maximize the number of high-priority features delivered.

$$\text{Maximize } Z_2 = \sum_{f \in \text{Feature}} \mathbb{I}(\text{priority}(f) \geq \text{P}_{\text{high}}) \cdot \text{priority}(f)$$

- **G3: minimize_blocker_impact** - Minimize the number of high-severity blockers.

$$\text{Minimize } Z_3 = \sum_{b \in \text{Blocker}} \mathbb{I}(\text{severity}(b) \geq \text{S}_{\text{critical}}) \cdot \text{severity}(b)$$

- **G4: maximize_sprint_goal_achievement** - Maximize the rate of successfully achieved sprint goals.

$$\text{Maximize } Z_4 = \sum_{g \in \text{SprintGoal}} \mathbb{I}(\text{achievement\_status}(g) = \text{Done})$$

- **G5: minimize_task_effort_variance** - Minimize the variance between estimated and actual task effort.

$$\text{Minimize } Z_5 = \sum_{k \in \text{Task}} (\text{effort}_{\text{actual}}(k) - \text{effort}_{\text{estimated}}(k))^2$$

- **G6: maximize_stakeholder_satisfaction** - Maximize the average satisfaction level of stakeholders.

$$\text{Maximize } Z_6 = \sum_{s \in \text{Stakeholder}} \text{influence\_level}(s)$$

- **G7: minimize_budget_overrun** - Minimize the total project budget overrun.

$$\text{Minimize } Z_7 = \max(0, \text{actual\_cost}(p_0) - \text{budget}(p_0))$$

- **G8: maximize_team_velocity** - Maximize the average velocity of the team.

$$\text{Maximize } Z_8 = \text{avg\_story\_points}(v_t) \quad \text{for team } t\text{'s velocity } v_t$$

- **G9: minimize_retrospective_actions** - Minimize the number of improvement actions needed.

$$\text{Minimize } Z_9 = \sum_{r \in \text{SprintRetrospective}} \text{improvement\_actions}(r)$$

4 Conditions

- **C0: team_must_be_cross_functional** - The team must have a minimum number of distinct skills.

$$\text{count}_{\text{distinct}}(\text{skill}(w) \forall w \in t) \geq \theta_{\text{skills}} \quad \forall t \in \text{Team}$$

- **C1: worker_availability_threshold** - An individual worker's availability must be above a minimum threshold.

$$\text{availability}(w) \geq A_{\min} \quad \forall w \in \text{Worker}$$

- **C2: feature_priority_must_match** - Features with 'Critical' priority must be developed first.

$$\text{start\_date}(f) < \text{start\_date}(f') \quad \forall f, f' | \text{priority}(f) > \text{priority}(f')$$

- **C3: blocker_severity_tolerance** - No blocker with 'Critical' severity is allowed to remain unresolved for more than X days.

  $$(\text{resolved\_on}(b) - \text{detected\_on}(b)) \leq T_{\max} \quad \forall b \in \text{Blocker} | \text{severity}(b) = \text{Critical}$$

- **C4: sprint_goal_must_be_defined** - Every sprint must have a defined goal.

  $$\text{objective\_description}(g_i) \neq \emptyset \quad \forall \text{sprint } i$$

- **C5: task_effort_estimation_required** - Every task must have an effort estimation greater than zero.

  $$\text{effort}_{\text{estimated}}(k) > 0 \quad \forall k \in \text{Task}$$

- **C6: stakeholder_influence_recognition** - Key stakeholders (high influence) must be identified and managed.

  $$\text{influence\_level}(s) \geq I_{\text{key}} \implies \text{status}(s) = \text{Managed} \quad \forall s \in \text{Stakeholder}$$

- **C7: project_budget_hard_limit** - The total project cost must not exceed the allocated budget.

  $$\sum \text{cost}(\text{resources}) \leq \text{budget}(p) \quad \forall p \in \text{Project}$$

- **C8: velocity_consistency** - Team velocity should not drop below a certain stability threshold.

  $$\text{min\_velocity}(v_t) \geq V_{\text{stable}} \quad \forall t \in \text{Team}$$

- **C9: sprint_duration_fixed** - Sprint duration must be fixed and cannot be changed.

  $$(\text{end\_date}(i) - \text{start\_date}(i)) = L_{\text{sprint}} \quad \forall i \in \text{Sprint}$$

5 Decision Variables

- $x_{\text{team\_size}}(t) \in \{5, 6, \ldots, 10\}$: Number of workers assigned to team t.
- $x_{\text{sprint\_length}}(i) \in \{10, 12, 14\}$: The duration of sprint i in days.

- $x_{\text{worker\_assign}}(w, i) \in \{0, 1\}$: 1 if worker w is assigned to sprint i, 0 otherwise.
- $x_{\text{feature\_priority}}(f) \in \{1, 2, 3, 4, 5\}$: Priority level assigned to feature f.
- $x_{\text{story\_points}}(us) \in \{1, 2, 3, 5, 8, 13\}$: Number of story points assigned to user story us.
- $x_{\text{task\_sequence}}(k) \in \mathbb{Z}^+$: The processing order of task k.
- $x_{\text{resource\_budget}}(f) \in \mathbb{R}^+$: Budget allocated to feature f.
- $x_{\text{skill\_req}}(k) \in \{1, 2, 3, 4, 5\}$: Minimum skill level required for task k.
- $x_{\text{contingency}}(k) \in \{0, 1, 2, 3, 4, 5\}$: Buffer days added to task k.
- $x_{\text{concurrent\_tasks}}(w) \in \{1, 2, 3\}$: Max number of tasks worker w can handle.