

Multi-Objective Optimization Model for a Scrum-Based Software Development Company

Generated from Entities, Relationships, Goals, Conditions, Decision Variables CSVs

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Modeling Overview

We formulate a mixed-integer, multi-objective model that uses: (i) the **Entities** as sets and attributes, (ii) the **Relationships** as incidence sets between entities, (iii) the provided **Decision Variables**, and (iv) **Goals** and **Conditions** as (soft/hard) objectives and constraints. We scalarize the goals via a weighted sum with signs derived from **GoalType**.

Notation conventions. For each entity attribute used in goals/conditions we introduce parameters named intuitively:

a_w	: Worker availability (from <code>Worker.availability</code>)
b_p	: Project budget ceiling (from <code>Project.budget</code>)
sp_{us}	: UserStory story points (from <code>UserStory.story_points</code>)
ef_{tsk}	: Task effort (from <code>Task.effort</code>)
pr_f	: Feature priority (from <code>Feature.priority</code>)
nt_{sbl}	: SprintBacklog number of tasks (from <code>SprintBacklog.number_of_tasks</code>)
ag_{sp}	: Sprint goal achievement (from <code>Sprint.achievement_of_goal</code>)
ts_{sre}	: Team satisfaction (from <code>SprintRetrospective.team_satisfaction</code>)
sev_{bl}	: Blocker severity (from <code>Blocker.severity</code>)
\overline{sp}_{vel}	: Velocity average story points (from <code>Velocity.avg._story_points</code>)
tr_{vel}	: Velocity trend (from <code>Velocity.trend</code>)
att_{sr}	: SprintReview attendees count (from <code>SprintReview.attendees_count</code>)

For relationships, we use incidence sets, e.g. $(us, tsk) \in \mathcal{E}^{US-TSK}$ iff UserStory *us* consists of Task *tsk* (R10).

1 1. Sets (Entities)

- \mathcal{P} : Projects (**Project**)
- \mathcal{T} : Teams (**Team**)
- \mathcal{W} : Workers (**Worker**)
- \mathcal{F} : Features (**Feature**)
- \mathcal{S} : Skills (**Skill**)
- \mathcal{R} : Roles (**Role**)
- \mathcal{PO} : Product Owners (**ProductOwner**)
- \mathcal{SM} : Scrum Masters (**ScrumMaster**)
- \mathcal{PB} : Product Backlogs (**ProductBacklog**)
- \mathcal{SP} : Sprints (**Sprint**)
- \mathcal{SPP} : Sprint Plannings (**SprintPlanning**)
- \mathcal{DS} : Daily Scrums (**DailyScrum**)
- \mathcal{SR} : Sprint Reviews (**SprintReview**)

- SRE : Sprint Retrospectives (`SprintRetrospective`)
- SBL : Sprint Backlogs (`SprintBacklog`)
- SG : Sprint Goals (`SprintGoal`)
- \mathcal{E} : Epics (`Epic`)
- US : User Stories (`UserStory`)
- TSK : Tasks (`Task`)
- DEV : Development Snapshots (`DevelopmentSnapshot`)
- BL : Blockers (`Blocker`)
- SH : Stakeholders (`Stakeholder`)
- VEL : Velocities (`Velocity`)
- REP : Release Plans (`ReleasePlan`)
- RM : Roadmaps (`Roadmap`)
- SCB : Scrum Boards (`ScrumBoard`)
- FED : Feature Documentations (`FeatureDocumentation`)

Relationship incidence sets (from `Relationships.csv`). We introduce:

- $\mathcal{E}^{PB-F} \subseteq \mathcal{PB} \times \mathcal{F}$ for R7 (*contains_feature*)
- $\mathcal{E}^{PB-E} \subseteq \mathcal{PB} \times \mathcal{E}$ for R8 (*contains_epic*)
- $\mathcal{E}^{E-US} \subseteq \mathcal{E} \times \mathcal{US}$ for R9 (*contains_user_story*)
- $\mathcal{E}^{US-TSK} \subseteq \mathcal{US} \times \mathcal{TSK}$ for R10 (*consists_of_tasks*)
- $\mathcal{E}^{SBL-SP} \subseteq \mathcal{SBL} \times \mathcal{SP}$ for R12 (*belongs_to_sprint*)
- $\mathcal{E}^{SP-SG} \subseteq \mathcal{SP} \times \mathcal{SG}$ for R13 (*pursues_goal*)
- $\mathcal{E}^{SCB-TSK} \subseteq \mathcal{SCB} \times \mathcal{TSK}$ for R14 (*contains_tasks*)
- $\mathcal{E}^{FED-F} \subseteq \mathcal{FED} \times \mathcal{F}$ for R15 (*documents_feature*)
- $\mathcal{E}^{TSK-BL} \subseteq \mathcal{TSK} \times \mathcal{BL}$ for R16 (*is_blocked_by*)
- $\mathcal{E}^{VEL-T} \subseteq \mathcal{VEL} \times \mathcal{T}$ for R19 (*refers_to_team*)
- $\mathcal{E}^{REP-F} \subseteq \mathcal{REP} \times \mathcal{F}$ for R20 (*plans_release*)
- $\mathcal{E}^{REP-RM} \subseteq \mathcal{REP} \times \mathcal{RM}$ for R21 (*is_part_of_roadmap*)
- $\mathcal{E}^{SP-DEV} \subseteq \mathcal{SP} \times \mathcal{DEV}$ for R22 (*generates_snapshot*)

(Other relationships may be handled analogously as metadata or fixed associations.)

2 2. Indices

- $p \in \mathcal{P}, t \in \mathcal{T}, w \in \mathcal{W}, f \in \mathcal{F}, s \in \mathcal{S}, r \in \mathcal{R},$
- $po \in \mathcal{PO}, sm \in \mathcal{SM}, pb \in \mathcal{PB}, sp \in \mathcal{SP}, spp \in \mathcal{SPP}, ds \in \mathcal{DS},$
- $sr \in \mathcal{SR}, sre \in \mathcal{SRE}, sbl \in \mathcal{SBL}, sg \in \mathcal{SG}, e \in \mathcal{E},$
- $us \in \mathcal{US}, tsk \in \mathcal{TSK}, dev \in \mathcal{DEV}, bl \in \mathcal{BL}, sh \in \mathcal{SH},$
- $vel \in \mathcal{VEL}, rep \in \mathcal{REP}, rm \in \mathcal{RM}, scb \in \mathcal{SCB}, fed \in \mathcal{FED}.$

3 3. Goals

We scalarize goals with weights $w_i^{(G)}$ and signs from `GoalType`:

$$\max Z = \sum_{\text{max goals } i} w_i^{(G)} T_i - \sum_{\text{min goals } j} w_j^{(G)} T_j \quad (+ \text{ soft-condition preferences; see Section 4}).$$

Each item lists its ID/Name and the term T it contributes.

- **[G0] maximize_team_availability** (IsSum=True, max, `Worker.availability`, weight = 1.0)

$$T_{G0} = \sum_{w \in \mathcal{W}} a_w.$$

- **[G1] minimize_total_project_budget** (IsSum=True, min, `Project.budget`, weight = 1.0)
Decision uses allocated budget B_p ; limit by b_p (see constraints).

$$T_{G1} = \sum_{p \in \mathcal{P}} B_p.$$

- **[G2] maximize_story_points_delivered** (IsSum=True, max, `UserStory.story_points`, weight = 1.2)

$$T_{G2} = \sum_{us \in \mathcal{US}} sp_{us} x_{us}.$$

- **[G3] minimize_task_effort** (IsSum=True, min, `Task.effort`, weight = 1.0)

$$T_{G3} = \sum_{tsk \in \mathcal{TSK}} ef_{tsk} x_{tsk}.$$

- **[G4] maximize_feature_priority** (IsSum=True, max, `Feature.priority`, weight = 0.8)

$$T_{G4} = \sum_{f \in \mathcal{F}} pr_f x_f.$$

- **[G5] minimize_number_of_tasks_in_sprint_backlog** (IsSum=True, min, `SprintBacklog.number_of_tasks`, weight = 0.7)

$$T_{G5} = \sum_{sbl \in \mathcal{SBL}} nt_{sbl}.$$

- **[G6] maximize_sprint_goal_achievement** (IsSum=True, max, `Sprint.achievement_of_goal`, weight = 1.5)

Uses user-set scalar weight ω on goal attainment if desired.

$$T_{G6} = \sum_{sp \in \mathcal{SP}} \omega ag_{sp}.$$

- **[G7] maximize_team_satisfaction** (IsSum=True, max, SprintRetrospective.team_satisfaction, weight = 1.3)

$$T_{G7} = \sum_{sre \in \mathcal{SR}\mathcal{E}} \text{ts}_{sre}.$$
- **[G8] minimize_blocker_severity** (IsSum=True, min, Blocker.severity, weight = 1.1)

$$T_{G8} = \sum_{bl \in \mathcal{BL}} \text{sev}_{bl}.$$
- **[G9] maximize_average_velocity** (IsSum=True, max, Velocity.avg._story_points, weight = 1.4)

$$T_{G9} = \sum_{vel \in \mathcal{VEL}} \overline{\text{sp}}_{vel}.$$
- **[G10] maximize_velocity_trend** (IsSum=True, max, Velocity.trend, weight = 0.9)

$$T_{G10} = \sum_{vel \in \mathcal{VEL}} \text{tr}_{vel}.$$
- **[G11] maximize_review_attendance** (IsSum=True, max, SprintReview.attendees_count, weight = 0.5)

$$T_{G11} = \sum_{sr \in \mathcal{SR}} \text{att}_{sr}.$$

4 4. Conditions

Hard *Must-Match* (CriteriaType = 2) constraints are enforced strictly. *Cannot-Match* (CriteriaType = 0) exclude items. *May-Match* (CriteriaType = 1) are modeled as soft preferences via bonus terms P_k added to Z with their weights.

- **[C0] must_have_certified_skills** (Must-Match, Skill.certified, weight = 1.0)
 Logical: only certified skills are considered in any skill-dependent assignment.
 Math: define $\mathcal{S}^{cert} = \{s \in \mathcal{S} \mid \text{certified}(s) = 1\}$ and restrict any skill-related sums to \mathcal{S}^{cert} .
- **[C1] prefer_high_feature_status** (May-Match, Feature.status, weight = 0.5)
 Logical: prefer features with advanced/ready status.
 Math (soft): $P_{C1} = \sum_{f \in \mathcal{F}} \mathbf{1}\{\text{status}(f) \in \text{Ready/Approved}\} x_f.$
- **[C2] exclude_blocked_tasks** (Cannot-Match, Task.status, weight = 1.0)
 Logical: tasks that are currently blocked are excluded.
 Math: $x_{tsk} = 0 \quad \forall tsk \in \mathcal{TSK} \text{ with } \text{status}(tsk) = \text{Blocked}.$
- **[C3] must_have_team_status_active** (Must-Match, Team.team_status, weight = 1.0)
 Logical: only active teams can receive workers.
 Math: $\sum_{w \in \mathcal{W}} y_{w,t} = 0 \quad \forall t \in \mathcal{T} \text{ with } \text{team_status}(t) \neq \text{Active}.$
- **[C4] prefer_product_backlog_status_ready** (May-Match, ProductBacklog.status, weight = 0.6)
 Logical: prefer PBs in “Ready” state.
 Math (soft): $P_{C4} = \sum_{pb \in \mathcal{PB}} \mathbf{1}\{\text{status}(pb) = \text{Ready}\}.$
- **[C5] must_have_worker_status_active** (Must-Match, Worker.status, weight = 1.0)
 Logical: only active workers can be assigned.
 Math: $\sum_{t \in \mathcal{T}} y_{w,t} = 0 \quad \forall w \in \mathcal{W} \text{ with } \text{status}(w) \neq \text{Active}.$

- **[C6] prefer_team_type_cross_functional** (May-Match, `Team.team_type`, weight = 0.4)
 Logical: prefer cross-functional teams.
 Math (soft): $P_{C6} = \sum_{t \in \mathcal{T}} \mathbf{1}\{\text{team_type}(t) = \text{CrossFunctional}\} \cdot \min\{1, \sum_w y_{w,t}\}$.
- **[C7] must_have_sprint_status_active** (Must-Match, `Sprint.status`, weight = 1.0)
 Logical: only active sprints are counted in sprint-related calculations.
 Math: restrict sums involving \mathcal{SP} to $\mathcal{SP}^{act} = \{sp \in \mathcal{SP} \mid \text{status}(sp) = \text{Active}\}$.
- **[C8] exclude_low_priority_user_stories** (Cannot-Match, `UserStory.priority`, weight = 1.0)
 Logical: very low priority stories are excluded.
 Math: $x_{us} = 0 \quad \forall us \in \mathcal{US} \text{ with } \text{priority}(us) < \theta$.
- **[C9] prefer_high_benefit_sprint_goal** (May-Match, `SprintGoal.benefit`, weight = 0.7)
 Logical: prefer sprints whose goals promise higher benefit.
 Math (soft): $P_{C9} = \sum_{(sp, sg) \in \mathcal{E}^{SP-SG}} \text{benefit}(sg)$.
- **[C10] must_have_skill_category_required** (Must-Match, `Skill.category`, weight = 1.0)
 Logical: only skills from required categories are eligible.
 Math: restrict skill sums to $\mathcal{S}^{req} = \{s \in \mathcal{S} \mid \text{category}(s) \in \text{RequiredSet}\}$.
- **[C11] prefer_location_remote** (May-Match, `Team.location`, weight = 0.3)
 Logical: prefer teams that operate remotely, if allowed.
 Math (soft & gated): $P_{C11} = \rho \cdot \sum_{t \in \mathcal{T}} \mathbf{1}\{\text{location}(t) = \text{Remote}\} \cdot \min\{1, \sum_w y_{w,t}\}$.

Soft-condition aggregation. Let $w_k^{(C)}$ be the weight of a May-Match condition. We augment the objective:

$$Z' = Z + \sum_{\text{May-Match } k} w_k^{(C)} P_k.$$

5 5. Decision Variables

• Binary selections

- $x_f \in \{0, 1\}$: (DV0) `select_feature` — feature f selected for implementation.
- $x_{us} \in \{0, 1\}$: (DV1) `select_user_story` — user story us included in the sprint.
- $x_{tsk} \in \{0, 1\}$: (DV2) `select_task` — task tsk included in the sprint plan.
- $y_{w,t} \in \{0, 1\}$: (DV3) `assign_worker_to_team` — worker w assigned to team t .
- $\rho \in \{0, 1\}$: (DV10) `allow_remote_team`.

• Integer/scalar choices

- $L \in \{1, 2, 3, 4\}$: (DV4) `sprint_length_weeks`.
- $B_p \in [0, 10,000,000]$: (DV5) `allocate_project_budget` for each $p \in \mathcal{P}$.
- $N_t \in \{1, \dots, 50\}$: (DV6) `team_size` for each $t \in \mathcal{T}$.
- $K \in \{1, \dots, 100\}$: (DV7) `max_parallel_tasks`.
- $V \in [0, 200]$: (DV8) `velocity_target`.
- $\theta \in \{1, 2, 3, 4, 5\}$: (DV9) `priority_threshold`.
- $\omega \in [0, 10]$: (DV11) `sprint_goal_weight`.

Core structural constraints (logic from relationships/attributes).

$$\sum_{w \in \mathcal{W}} y_{w,t} = N_t \quad \forall t \in \mathcal{T} \quad \text{(team size equals a)} \quad (1)$$

$$\sum_{tsk \in \mathcal{TSK}} x_{tsk} \leq K \quad \text{(cap concurrent/planned tasks; DV7)} \quad (2)$$

$$B_p \leq b_p \quad \forall p \in \mathcal{P} \quad \text{(allocated budget v)} \quad (3)$$

$$\sum_{us \in \mathcal{US}} \text{sp}_{us} x_{us} \geq V \quad \text{(meet velocity target)} \quad (4)$$

$$x_{us} \leq \mathbf{1}\{\text{priority}(us) \geq \theta\} \quad \forall us \in \mathcal{US} \quad \text{(priority threshold;)} \quad (5)$$

$$x_f \leq \mathbf{1}\{\text{priority}(f) \geq \theta\} \quad \forall f \in \mathcal{F} \quad \text{(priority threshold)} \quad (6)$$

$$x_{tsk} \leq \sum_{us: (us,tsk) \in \mathcal{E}^{US-TSK}} x_{us} \quad \forall tsk \in \mathcal{TSK} \quad \text{(task only if its use)} \quad (7)$$

$$\sum_{t \in \mathcal{T}} y_{w,t} \leq 1 \quad \forall w \in \mathcal{W} \quad \text{(worker to at most)} \quad (8)$$

$$y_{w,t} = 0 \quad \forall (w,t) : \text{team_status}(t) \neq \text{Active} \text{ or } \text{status}(w) \neq \text{Active} \quad \text{(C3 \& C5 hard gat)} \quad (9)$$

$$x_{tsk} = 0 \quad \forall tsk : \text{status}(tsk) = \text{Blocked} \quad \text{(C2 gating)} \quad (10)$$

Final scalarized objective (including soft preferences). Let $\mathcal{G}^{max} = \{G0, G2, G4, G6, G7, G9, G10, G11\}$, $\mathcal{G}^{min} = \{G1, G3, G5, G8\}$ and T_{Gi} as above. Let $\mathcal{C}^{soft} = \{C1, C4, C6, C9, C11\}$ with preference terms P_{Ck} . Weights come from the CSVs. The model solves:

$$\max Z'' = \sum_{i \in \mathcal{G}^{max}} w_i^{(G)} T_{Gi} - \sum_{j \in \mathcal{G}^{min}} w_j^{(G)} T_{Gj} + \sum_{k \in \mathcal{C}^{soft}} w_k^{(C)} P_{Ck}$$

subject to (1)–(10), domain bounds of Section 5, and entity/relationship gating described in Section 4.