Optimization Model for a SCRUM-based Software Development Domain

Prepared for: SCRUM Optimization Modeling

August 11, 2025

Contents

1 1. Sets (Entities)

- $\mathcal{P}$ (Projects) [Index: p]
- $\mathcal{T}$ (Teams) [Index: t]
- $\mathcal{W}$ (Workers) [Index: w]
- $\mathcal{F}$ (Features) [Index: f]
- $\mathcal{S}$ (Skills) [Index: s]
- $\mathcal{R}$ (Roles) [Index: r]
- $\mathcal{PO}$ (Product Owners) [Index: po]
- $\mathcal{SM}$ (Scrum Masters) [Index: sm]
- $\mathcal{PB}$ (Product Backlogs) [Index: pb]
- $\mathcal{SP}$ (Sprints) [Index: sp]
- $\mathcal{SPP}$ (Sprint Plannings) [Index: spp]
- $\mathcal{DS}$ (Daily Scrums) [Index: ds]
- $\mathcal{SR}$ (Sprint Reviews) [Index: sr]
- $\mathcal{SRE}$ (Sprint Retrospectives) [Index: sre]
- $\mathcal{SBL}$ (Sprint Backlogs) [Index: sbl]
- $\mathcal{SG}$ (Sprint Goals) [Index: sg]
- $\mathcal{E}$ (Epics) [Index: e]
- $\mathcal{US}$ (User Stories) [Index: u]
- $\mathcal{TSK}$ (Tasks) [Index: τ]
- $\mathcal{DEV}$ (Development Snapshots) [Index: d]
- $\mathcal{BL}$ (Blockers) [Index: b]
- $\mathcal{SH}$ (Stakeholders) [Index: h]
- $\mathcal{VEL}$ (Velocities) [Index: v]
- $\mathcal{REP}$ (Release Plans) [Index: rpl]
- $\mathcal{RM}$ (Roadmaps) [Index: rm]
- $\mathcal{SCB}$ (Scrum Boards) [Index: scb]
- $\mathcal{FED}$ (Feature Documentations) [Index: fd]

Relationship sets (from `Relationships.csv`). (We denote the relation names as subsets of Cartesian products; the original CSV used "Employee" which we consistently treat as *Worker*.)

- AssignedTeamProject $\subseteq \mathcal{T} \times \mathcal{P}$ (R1)
- BelongsToTeam $\subseteq \mathcal{W} \times \mathcal{T}$ (R2)
- HasSkill $\subseteq \mathcal{W} \times \mathcal{S}$ (R3)
- TakesOnRole $\subseteq \mathcal{W} \times \mathcal{R}$ (R4)
- ManagesBacklog $\subseteq \mathcal{PO} \times \mathcal{PB}$ (R5)
- IsSupportedBy $\subseteq \mathcal{T} \times \mathcal{SM}$ (R6)
- PBcontainsFeature $\subseteq \mathcal{PB} \times \mathcal{F}$ (R7)
- PBcontainsEpic $\subseteq \mathcal{PB} \times \mathcal{E}$ (R8)
- EpicContainsStory $\subseteq \mathcal{E} \times \mathcal{US}$ (R9)
- StoryHasTasks $\subseteq \mathcal{US} \times \mathcal{TSK}$ (R10)
- StoryInSBL $\subseteq \mathcal{US} \times \mathcal{SBL}$ (R11)
- SBLofSprint $\subseteq \mathcal{SBL} \times \mathcal{SP}$ (R12)
- SprintHasGoal $\subseteq \mathcal{SP} \times \mathcal{SG}$ (R13)
- SCBcontainsTask $\subseteq \mathcal{SCB} \times \mathcal{TSK}$ (R14)
- DocumentsFeature $\subseteq \mathcal{FED} \times \mathcal{F}$ (R15)
- TaskBlockedBy $\subseteq \mathcal{TSK} \times \mathcal{BL}$ (R16)
- StakeholderInReview $\subseteq \mathcal{SH} \times \mathcal{SR}$ (R17)
- ModeratesRetro $\subseteq \mathcal{SM} \times \mathcal{SRE}$ (R18)
- VelocityOfTeam $\subseteq \mathcal{VEL} \times \mathcal{T}$ (R19)
- ReleasePlansFeature $\subseteq \mathcal{REP} \times \mathcal{F}$ (R20)
- ReleaseInRoadmap $\subseteq \mathcal{REP} \times \mathcal{RM}$ (R21)
- SprintGeneratesDev $\subseteq \mathcal{SP} \times \mathcal{DEV}$ (R22)

2 2. Indices

- $p \in \mathcal{P},\ t \in \mathcal{T},\ w \in \mathcal{W},\ f \in \mathcal{F},\ s \in \mathcal{S},\ r \in \mathcal{R},\ po \in \mathcal{PO},\ sm \in \mathcal{SM}$
- $pb \in \mathcal{PB},\ sp \in \mathcal{SP},\ spp \in \mathcal{SPP},\ ds \in \mathcal{DS},\ sr \in \mathcal{SR},\ sre \in \mathcal{SRE},\ sbl \in \mathcal{SBL},\ sg \in \mathcal{SG}$
- $e \in \mathcal{E},\ u \in \mathcal{US},\ \tau \in \mathcal{TSK},\ d \in \mathcal{DEV},\ b \in \mathcal{BL},\ h \in \mathcal{SH}$
- $v \in \mathcal{VEL},\ rpl \in \mathcal{REP},\ rm \in \mathcal{RM},\ scb \in \mathcal{SCB},\ fd \in \mathcal{FED}$

Parameters (from entity attributes). We reference entity attributes as parameters (read from data):
budget(p), estimatedEffort(f), severity(b), attendees(sr), teamSat(sre), entries(pb), avgSP(v), benefit(sg), effortEpic(e), avail(w), achieve(sp), etc.
We also use binary status indicators such as ready(f), openSprint(sp), activePB(pb), deployable(rpl), docPresent(d), moderationSet(sre), planningDoc(spp), activeWorker(w), relevant(h), and certified(s).

3 3. Goals

We use a weighted scalarization of individual goal terms ϕ_g consistent with `Goals.csv`. For a goal with `GoalType` = `max` we add $+\,\text{Weight} \cdot \phi_g$; for `min` we add $-\,\text{Weight} \cdot \phi_g$.

Objective

$$\max \Big(1.0\cdot\phi_{\text{G0}} - 1.0\cdot\phi_{\text{G1}} - 0.9\cdot\phi_{\text{G2}} - 1.0\cdot\phi_{\text{G3}} + 0.8\cdot\phi_{\text{G4}} + 0.6\cdot\phi_{\text{G5}} - 0.5\cdot\phi_{\text{G6}} + 0.4\cdot\phi_{\text{G7}} + 0.5\cdot\phi_{\text{G8}} - 0.8\cdot\phi_{\text{G9}} + 0.5\cdot\phi_{\text{G10}} - 0.7$$

Decision variables (used below).

$x_{u,sp} \in \{0,1\}$	(assign_user_story_to_sprint)
$y_{f,rpl} \in \{0,1\}$	(select_feature_for_release)
$a_{w,\tau} \in \{0,1\}$	(allocate_worker_to_task)
$z_e \in \{0,1\}$	(choose_epic_for_planning)
$h_\tau \in [0,1000]$	(set_task_effort)
$\text{pr}_u \in \{1,2,3,4,5\}$	(prioritize_user_story)
$gTarget_{sp} \in [0,100]$	(set_sprint_goal_target)
$b_{pb} \in \{0,1\}$	(enable_backlog_item)
$n_t^{\text{team}} \in \{3,4,\ldots,12\}$	(staff_team_size)
$n_{sbl}^{\text{tasks}} \in \{0,1,\ldots,500\}$	(plan_number_of_tasks)
$b_t^{\text{sm}} \in \{0,1\}$	(assign_scrum_master)
$\iota_{h,sr} \in \{0,1\}$	(select_stakeholder_for_review)
$\ell_s^{\text{skill}} \in \{1,2,3,4,5\}$	(set_skill_level_target)

Per-goal definitions

- **G0 maximize_velocity_points**:

$$\phi_{\text{G0}} = \sum_{v \in \mathcal{VEL}} \text{avgSP}(v)$$

- **G1 minimize_project_budget**:

$$\phi_{\text{G1}} = \sum_{p \in \mathcal{P}} \text{budget}(p)$$

- **G2 minimize_feature_effort**:

$$\phi_{\text{G2}} = \sum_{f \in \mathcal{F}} \text{estimatedEffort}(f) \cdot \Big(\sum_{rpl \in \mathcal{REP}} y_{f,rpl} \Big)$$

- **G3 minimize_task_effort**:

$$\phi_{\mathrm{G3}} = \sum_{\tau \in \mathcal{TSK}} h_{\tau}$$

- **G4 maximize_sprint_goal_achievement**:

$$\phi_{\mathrm{G4}} = \sum_{sp \in \mathcal{SP}} \mathrm{achieve}(sp)$$

- **G5 maximize_team_satisfaction**:

$$\phi_{\mathrm{G5}} = \sum_{sre \in \mathcal{SRE}} \mathrm{teamSat}(sre)$$

- **G6 minimize_backlog_size**:

$$\phi_{\mathrm{G6}} = \sum_{pb \in \mathcal{PB}} \mathrm{entries}(pb) \cdot b_{pb}$$

- **G7 maximize_review_attendance**:

$$\phi_{\mathrm{G7}} = \sum_{sr \in \mathcal{SR}} \mathrm{attendees}(sr) + \sum_{sr \in \mathcal{SR}} \sum_{h \in \mathcal{SH}} \iota_{h,sr}$$

- **G8 maximize_skill_levels**:

$$\phi_{\mathrm{G8}} = \sum_{s \in \mathcal{S}} \ell_s^{\mathrm{skill}}$$

- **G9 minimize_blocker_severity**:

$$\phi_{\mathrm{G9}} = \sum_{b \in \mathcal{BL}} \mathrm{severity}(b)$$

- **G10 maximize_sprint_benefit**:

$$\phi_{\mathrm{G10}} = \sum_{sg \in \mathcal{SG}} \mathrm{benefit}(sg)$$

- **G11 minimize_epic_effort**:

$$\phi_{\mathrm{G11}} = \sum_{e \in \mathcal{E}} \mathrm{effortEpic}(e) \cdot z_e$$

- **G12 maximize_worker_availability**:

$$\phi_{\mathrm{G12}} = \sum_{w \in \mathcal{W}} \mathrm{avail}(w)$$

4 4. Conditions

Each condition is stated with its ID, snake_case name, and a logical/mathematical constraint consistent with `Conditions.csv`.

- **C0 team_size_within_bounds** (IsSum=False, GoalType=min):

$$3 \leq n_t^{\text{team}} \leq 12 \quad \forall t \in \mathcal{T}.$$

- **C1 only_active_workers_assigned**:

$$a_{w,\tau} \leq \text{activeWorker}(w) \quad \forall (w,\tau) \in \mathcal{W} \times \mathcal{TSK}.$$

- **C2 user_story_priority_allowed**:

$$\text{pr}_u \in \{1,2,3,4,5\}_{\text{allowed}} \quad \forall u \in \mathcal{US}.$$

  (Implementable via binaries or bounds depending on policy.)

- **C3 task_status_permitted**:

$$\sum_{w \in \mathcal{W}} a_{w,\tau} \leq M \cdot \text{permittedTask}(\tau) \quad \forall \tau \in \mathcal{TSK}.$$

- **C4 feature_status_ready_only**:

$$y_{f,rpl} \leq \text{ready}(f) \quad \forall (f,rpl) \in \mathcal{F} \times \mathcal{REP}.$$

- **C5 sprint_status_open_only**:

$$x_{u,sp} \leq \text{openSprint}(sp) \quad \forall (u,sp) \in \mathcal{US} \times \mathcal{SP}.$$

- **C6 backlog_status_active**:

$$b_{pb} \leq \text{activePB}(pb) \quad \forall pb \in \mathcal{PB}.$$

- **C7 stakeholder_relevance_required**:

$$\iota_{h,sr} \leq \text{relevant}(h) \quad \forall (h,sr) \in \mathcal{SH} \times \mathcal{SR}.$$

- **C8 skill_certified_when_required**:

$$\ell_s^{\text{skill}} \geq \text{reqLevel}(s), \quad \text{certified}(s) \geq \text{reqCert}(s) \quad \forall s \in \mathcal{S}.$$

- **C9 release_status_permitted**:

$$\sum_{f \in \mathcal{F}} y_{f,rpl} \leq M \cdot \text{deployable}(rpl) \quad \forall rpl \in \mathcal{REP}.$$

- **C10 documentation_present_for_snapshot**:

$$\text{docPresent}(d) = 1 \quad \forall d \in \mathcal{DEV} \text{ generated (via R22)}.$$

- **C11 retrospective_moderation_set**:

$$\text{moderationSet}(sre) = 1 \quad \forall sre \in \mathcal{SRE}.$$

- **C12 planning_outcome_documented**:

$$\text{planningDoc}(spp) = 1 \quad \forall spp \in \mathcal{SPP}.$$

Canonical linking constraints (examples).

(Stories in an SBL must belong to its sprint) $\sum_{sbl:(sbl,sp)\in\text{SBLofSprint}} \mathbf{1}\{(u, sbl) \in \text{StoryInSBL}\} = x_{u,sp} \quad \forall u \in \mathcal{U}$

(Tasks planned per SBL) $\sum_{\tau:(u,\tau)\in\text{StoryHasTasks}} 1 = n_{sbl}^{\text{tasks}} \quad \forall u \in \mathcal{U}$

5 5. DecisionVariables

- **DV0 assign_user_story_to_sprint**: $x_{u,sp} \in \{0, 1\}$ (domain $\{0, 1\}$, min 0, max 1)
- **DV1 select_feature_for_release**: $y_{f,rpl} \in \{0, 1\}$ (domain $\{0, 1\}$, min 0, max 1)
- **DV2 allocate_worker_to_task**: $a_{w,\tau} \in \{0, 1\}$ (domain $\{0, 1\}$, min 0, max 1)
- **DV3 choose_epic_for_planning**: $z_e \in \{0, 1\}$ (domain $\{0, 1\}$, min 0, max 1)
- **DV4 set_task_effort**: $h_\tau \in [0, 1000]$ (real, min 0, max 1000)
- **DV5 prioritize_user_story**: $\text{pr}_u \in \{1, 2, 3, 4, 5\}$ (integer, min 1, max 5)
- **DV6 set_sprint_goal_target**: $gTarget_{sp} \in [0, 100]$ (real, min 0, max 100)
- **DV7 enable_backlog_item**: $b_{pb} \in \{0, 1\}$ (binary, min 0, max 1)
- **DV8 staff_team_size**: $n_t^{\text{team}} \in \{3, 4, \ldots, 12\}$ (integer, min 3, max 12)
- **DV9 plan_number_of_tasks**: $n_{sbl}^{\text{tasks}} \in \{0, 1, \ldots, 500\}$ (integer, min 0, max 500)
- **DV10 assign_scrum_master**: $b_t^{\text{sm}} \in \{0, 1\}$ (binary, min 0, max 1)
- **DV11 select_stakeholder_for_review**: $\iota_{h,sr} \in \{0, 1\}$ (binary, min 0, max 1)
- **DV12 set_skill_level_target**: $\ell_s^{\text{skill}} \in \{1, 2, 3, 4, 5\}$ (integer, min 1, max 5)