Mathematical Optimization Model for Scrum-based Software Development

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Contents

1 Sets (Entities)

The fundamental sets of the model are derived from the entities in the domain. Each set is a collection of all instances of a particular entity.

- **Projects** (P): The set of all projects to be developed.
- **Teams** (T): The set of all self-organized, cross-functional development teams.
- **Workers** (W): The set of all individual team members.
- **Features** (F): The set of all mid-sized functionalities.
- **Skills** (S): The set of all professional or social competences.
- **Sprints** (SP): The set of all fixed time periods for creating an increment.
- **User Stories** (US): The set of all requirements from a user's perspective.
- **Tasks** (TSK): The set of all smallest units of work within a sprint.
- **Blockers** (BL): The set of all obstacles hindering progress.
- **Stakeholders** (SH): The set of all interested parties in the product.
- **Release Plans** (REP): The set of all plans for releasing specific features.

2 Indices

Indices are used to refer to specific elements within the sets defined above.

- $p \in P$: An index for a specific project.
- $t \in T$: An index for a specific team.
- $w \in W$: An index for a specific worker.
- $f \in F$: An index for a specific feature.
- $s \in S$: An index for a specific skill.
- $sp \in SP$: An index for a specific sprint.
- $us \in US$: An index for a specific user story.
- $tsk \in TSK$: An index for a specific task.
- $bl \in BL$: An index for a specific blocker.
- $sh \in SH$: An index for a specific stakeholder.
- $rep \in REP$: An index for a specific release plan.

3 Decision Variables

These are the variables that the optimization model will determine to achieve the goals while respecting the conditions.

DV0 **assign_story_to_sprint** ($X_{us,sp}$): A binary variable that is 1 if User Story us is assigned to Sprint sp, and 0 otherwise.

$$X_{us,sp} \in \{0,1\} \quad \forall us \in US, sp \in SP$$

DV1 **assign_worker_to_task** ($Y_{w,tsk}$): A binary variable that is 1 if Worker w is assigned to Task tsk, and 0 otherwise.

$$Y_{w,tsk} \in \{0,1\} \quad \forall w \in W, tsk \in TSK$$

DV2 **assign_team_to_project** ($Z_{t,p}$): A binary variable that is 1 if Team t is assigned to Project p, and 0 otherwise.

$$Z_{t,p} \in \{0,1\} \quad \forall t \in T, p \in P$$

DV5 **include_feature_in_release** ($A_{f,rep}$): A binary variable that is 1 if Feature f is included in Release Plan rep, and 0 otherwise.

$$A_{f,rep} \in \{0,1\} \quad \forall f \in F, rep \in REP$$

DV8 **set_team_size** (N_t): An integer variable representing the number of members assigned to Team t.

$$N_t \in \mathbb{Z}^+ \quad \forall t \in T$$

4 Goals (Objective Functions)

The goals of the optimization model are to maximize or minimize certain objectives. The overall objective function could be a weighted sum of these individual goals.

G0 **maximize_story_points_per_sprint**: Maximize the value delivered in each sprint. The weight is $w_{G0} = 1.5$.

$$\text{maximize} \quad w_{G0} \sum_{sp \in SP} \sum_{us \in US} \text{story\_points}_{us} \cdot X_{us,sp}$$

G2 **maximize_feature_priority_value**: Maximize the business value by implementing high-priority features. The weight is $w_{G2} = 1.2$.

$$\text{maximize} \quad w_{G2} \sum_{rep \in REP} \sum_{f \in F} \text{priority}_f \cdot A_{f,rep}$$

G3 **minimize_task_effort**: Minimize the total human effort required. The weight is $w_{G3} = 1.0$.

$$\text{minimize} \quad w_{G3} \sum_{w \in W} \sum_{tsk \in TSK} \text{effort}_{tsk} \cdot Y_{w,tsk}$$

G4 **maximize_team_velocity**: Aims to improve the average throughput of teams, modeled here as maximizing the sum of story points per team. The weight is $w_{G4} = 1.8$.

$$\text{maximize} \quad w_{G4} \sum_{t \in T} \sum_{sp \in SP} \sum_{us \in US} \text{story\_points}_{us} \cdot X_{us,sp} \cdot \text{BelongsToTeam}_{us,t}$$

G5 **minimize_active_blockers**: Minimize the impact of impediments on the project. The weight is $w_{G5} = 2.0$.

$$\text{minimize} \quad w_{G5} \sum_{bl \in BL} \text{is\_active}_{bl}$$

G7 **minimize_project_completion_time**: Deliver projects as quickly as possible. The weight is $w_{G7} = 1.3$.

$$\text{minimize} \quad w_{G7} \sum_{p \in P} (\text{project\_end}_p - \text{project\_start}_p)$$

5 Conditions (Constraints)

These are the rules and limitations that the solution must adhere to. They represent the business rules, resource limitations, and dependencies of the system.

C0 **project_budget_limit**: The total cost incurred for a project cannot exceed its budget. Let Cost(...) be a function calculating costs from assignments.

$$\text{Cost}(Y_{w,tsk}, Z_{t,p}, \dots) \leq \text{budget}_p \quad \forall p \in P$$

C1 **team_size_constraint**: Each team must have a size within the conventional Scrum limits.

$$3 \leq N_t \leq 9 \quad \forall t \in T$$

C2 **worker_skill_requirement**: A worker can only be assigned to a task if they possess the required skill. Let $\text{HasSkill}_{w,s}$ be 1 if worker w has skill s, and $\text{ReqSkill}_{tsk,s}$ be 1 if task tsk requires skill s.

$$Y_{w,tsk} \cdot \text{ReqSkill}_{tsk,s} \leq \text{HasSkill}_{w,s} \quad \forall w \in W, tsk \in TSK, s \in S$$

C4 **worker_availability_limit**: The total effort of tasks assigned to a worker in a sprint cannot exceed their availability for that sprint.

$$\sum_{tsk \in TSK} \text{effort}_{tsk} \cdot Y_{w,tsk} \leq \text{availability}_w \quad \forall w \in W$$

C6 **sprint_backlog_effort_cap**: The total story points of user stories in a sprint backlog should not exceed the team's velocity.

$$\sum_{us \in US} \text{story\_points}_{us} \cdot X_{us,sp} \leq \text{velocity}_t \quad \forall sp \in \text{SprintsOfTeam}_t, \forall t \in T$$

C7 **task_dependency_precedence**: A task cannot be started until all its blocking tasks are completed. Let $P(tsk)$ be the set of tasks that must precede task tsk.

$$\text{finish\_time}_{tsk'} \leq \text{start\_time}_{tsk} \quad \forall tsk \in TSK, \forall tsk' \in P(tsk)$$

C10 **user_story_points_required**: Any user story assigned to a sprint must have a non-zero story point value.

$$X_{us,sp} \cdot \text{story\_points}_{us} > 0 \quad \forall us \in US, sp \in SP$$