Mathematical Optimization Model for Scrum-Based Software Development

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Contents

1 Sets (Entities)

- Project = $\{p|p$ is a project$\}$, with attributes id, name, project_start, project_end, description, budget, stat
- Team = $\{t|t$ is a team$\}$, with attributes id, name, team_size, team_start, team_status, location, team_typ
- Worker = $\{w|w$ is a worker$\}$, with attributes id, name, first_name, email, start_date, status, availability
- Feature = $\{f|f$ is a feature$\}$, with attributes id, title, description, status, priority, estimated_effort
- Skill = $\{s|s$ is a skill$\}$, with attributes id, label, description, level, certified, category
- Role = $\{r|r$ is a role$\}$, with attributes id, role_name, description, area_of_responsibility
- ProductOwner = $\{po|po$ is a product owner$\}$, with attributes id, name, email, availability
- ScrumMaster = $\{sm|sm$ is a scrum master$\}$, with attributes id, name, email, experience
- ProductBacklog = $\{pb|pb$ is a product backlog$\}$, with attributes id, created_on, last_updated, number_of
- Sprint = $\{sp|sp$ is a sprint$\}$, with attributes id, sprint_number, start_date, end_date, status, achievement
- SprintPlanning = $\{spp|spp$ is a sprint planning$\}$, with attributes id, date, duration_(min), moderation, ou
- DailyScrum = $\{ds|ds$ is a daily scrum$\}$, with attributes id, date, time, duration, moderation
- SprintReview = $\{sr|sr$ is a sprint review$\}$, with attributes id, date, duration, feedback_documentation, att
- SprintRetrospective = $\{sre|sre$ is a sprint retrospective$\}$, with attributes id, date, duration, improvement_
- SprintBacklog = $\{sbl|sbl$ is a sprint backlog$\}$, with attributes id, number_of_tasks, last_updated, status,
- SprintGoal = $\{sg|sg$ is a sprint goal$\}$, with attributes id, objective_description, achievement_status, bene

- Epic = $\{e | e$ is an epic$\}$, with attributes id, title, description, priority, status, estimated_effort
- UserStory = $\{us | us$ is a user story$\}$, with attributes id, title, description, acceptance_criteria, priority, sto
- Task = $\{tsk | tsk$ is a task$\}$, with attributes id, title, description, status, effort, type
- DevelopmentSnapshot = $\{dev | dev$ is a development snapshot$\}$, with attributes id, version_number, creati
- Blocker = $\{bl | bl$ is a blocker$\}$, with attributes id, title, description, severity, status, detected_on, resolved_
- Stakeholder = $\{sh | sh$ is a stakeholder$\}$, with attributes id, name, organization, role, email, area_of_interes
- Velocity = $\{vel | vel$ is a velocity$\}$, with attributes id, number_of_sprints_used, avg._story_points, max_
- ReleasePlan = $\{rep | rep$ is a release plan$\}$, with attributes id, version, planned_date, included_features, st
- Roadmap = $\{rm | rm$ is a roadmap$\}$, with attributes id, start_date, end_date, milestones, objectives, versi
- ScrumBoard = $\{scb | scb$ is a scrum board$\}$, with attributes id, board_type, columns_(todo/done...), numb
- FeatureDocumentation = $\{fed | fed$ is a feature documentation$\}$, with attributes id, title, description, creat

2 Indices

- $p, t, w, f, s, r, po, sm, pb, sp, spp, ds, sr, sre, sbl, sg, e, us, tsk, dev, bl, sh, vel, rep, rm, scb, fed$ (index over their respective sets)
- i, j, k (general indices)

3 Goals

- **G0: maximize_team_velocity** - Maximize the average story points completed per sprint.

$$\text{maximize} \sum_{vel \in \text{Velocity}} \omega_{G0} \cdot \text{avg.\_story\_points}_{vel}$$

- **G1: minimize_sprint_overhead** - Minimize time spent in meetings to maximize development time.

$$\text{minimize} \sum_{spp \in \text{SprintPlanning}} \omega_{G1} \cdot \text{duration}_{spp} + \sum_{sr \in \text{SprintReview}} \omega_{G1} \cdot \text{duration}_{sr} + \sum_{sre \in \text{SprintRetrospective}} \omega_{G1} \cdot \text{duratio}$$

- **G2: minimize_blocker_severity** - Minimize the impact and severity of blockers that occur.

$$\text{minimize} \sum_{bl \in \text{Blocker}} \omega_{G2} \cdot \text{severity}_{bl}$$

- **G3: maximize_feature_priority_alignment** - Maximize the sum of priorities for features selected in a release.

$$\text{maximize} \sum_{f \in \text{Feature}} \omega_{G3} \cdot \text{priority}_f \cdot x_f^{\text{release}} \quad \text{where } x_f^{\text{release}} \in \{0, 1\}$$

- **G4: minimize_budget_variance** - Minimize the difference between planned and actual project budget.

$$\text{minimize } \omega_{G4} \cdot |\text{budget}_{\text{planned}} - \text{budget}_{\text{actual}}|$$

- **G5: maximize_team_stability** - Maximize the number of workers with high availability status.

$$\text{maximize} \sum_{w \in \text{Worker}} \omega_{G5} \cdot \text{availability}_w$$

- **G6: minimize_story_effort_variance** - Minimize the difference between estimated and actual story effort.

$$\text{minimize} \sum_{us \in \text{UserStory}} \omega_{G6} \cdot |\text{story\_points}_{us}^{\text{est}} - \text{story\_points}_{us}^{\text{actual}}|$$

- **G7: maximize_sprint_goal_achievement** - Maximize the rate at which sprint goals are successfully met.

$$\text{maximize} \sum_{sg \in \text{SprintGoal}} \omega_{G7} \cdot \mathbb{I}(\text{achievement\_status}_{sg} = \text{Done})$$

- **G8: minimize_context_switching** - Minimize the number of different tasks a worker is assigned to in a sprint.

$$\text{minimize} \sum_{w \in \text{Worker}} \omega_{G8} \cdot \left(\sum_{tsk \in \text{Task}} x_{w,tsk}^{\text{assign}} \right)$$

- **G9: maximize_skill_utilization** - Maximize the use of certified skills within the team for assigned tasks.

$$\text{maximize} \sum_{s \in \text{Skill}} \sum_{tsk \in \text{Task}} \omega_{G9} \cdot \text{certified}_s \cdot x_{s,tsk}^{\text{req}} \cdot x_{w(s),tsk}^{\text{assign}}$$

4 Conditions

- **C0: team_has_scrum_master** - A team must be supported by exactly one Scrum Master.

$$\sum_{sm \in \text{ScrumMaster}} \mathbb{I}(\text{supported\_team}_{sm} = t) = 1 \quad \forall t \in \text{Team}$$

- **C1: sprint_has_goal** - Every sprint must have exactly one defined sprint goal.

$$\sum_{sg \in \text{SprintGoal}} \mathbb{I}(\text{parent\_sprint}_{sg} = sp) = 1 \quad \forall sp \in \text{Sprint}$$

- **C2: user_story_has_acceptance_criteria** - Every user story must have defined acceptance criteria before entering a sprint.

$$\text{acceptance\_criteria}_{us} \neq \emptyset \quad \forall us \in \text{UserStory where status}_{us} \neq \text{Backlog}$$

- **C3: task_belongs_to_only_one_story** - A task can only be associated with a single user story.

$$\sum_{us \in \text{UserStory}} x_{tsk,us}^{\text{part\_of}} \leq 1 \quad \forall tsk \in \text{Task}$$

- **C4: product_owner_manages_backlog** - The product backlog must be managed by exactly one Product Owner.

$$\sum_{po \in \text{ProductOwner}} \mathbb{I}(\text{manages\_backlog}_{po} = pb) = 1 \quad \forall pb \in \text{ProductBacklog}$$

- **C5: worker_availability_not_zero** - A worker cannot be assigned tasks if their availability is zero.

$$\text{availability}_w = 0 \implies \sum_{tsk \in \text{Task}} x^{\text{assign}}_{w,tsk} = 0 \quad \forall w \in \text{Worker}$$

- **C6: sprint_duration_fixed** - The duration of a sprint must be a fixed value (e.g., 14 days).

$$\text{end\_date}_{sp} - \text{start\_date}_{sp} = \text{DURATION}_{\text{FIXED}} \quad \forall sp \in \text{Sprint}$$

- **C7: blocker_must_have_severity** - Every blocker must have a severity level assigned when detected.

$$\text{severity}_{bl} \in \{1, 2, 3, 4, 5\} \quad \forall bl \in \text{Blocker where status}_{bl} = \text{Active}$$

- **C8: feature_in_release_planned** - A feature can only be included in a release if its status is 'planned' or 'done'.

$$x^{\text{release}}_f = 1 \implies \text{status}_f \in \{\text{Planned}, \text{Done}\} \quad \forall f \in \text{Feature}$$

- **C9: budget_must_be_positive** - The project budget must be a positive value.

$$\text{budget}_p > 0 \quad \forall p \in \text{Project}$$

- **C10: story_points_positive** - Story points for a user story must be a positive integer.

$$\text{story\_points}_{us} \in \mathbb{Z}^+ \quad \forall us \in \text{UserStory}$$

- **C11: team_has_minimum_size** - A team must have at least 3 members to be viable.

$$\text{team\_size}_t \geq 3 \quad \forall t \in \text{Team}$$

5 Decision Variables

- **DV0:** $x^{\text{assign}}_{w,tsk} \in \{0, 1\}$ - Binary assignment of worker w to task tsk.
- **DV1:** $x^{\text{sprint}}_{us} \in \{0, 1\}$ - Binary selection of user story us for the sprint backlog.
- **DV2:** $x^{\text{release}}_f \in \{0, 1\}$ - Binary selection of feature f for a release plan.
- **DV3:** $\text{story\_points}^{\text{est}}_{us} \in \mathbb{Z}^+$, where $1 \leq \text{story\_points}^{\text{est}}_{us} \leq 20$ - Story points estimation.
- **DV4:** $\text{availability}_w \in \mathbb{R}$, where $0.0 \leq \text{availability}_w \leq 1.0$ - Worker availability percentage.
- **DV5:** $\text{DURATION}_{\text{FIXED}} \in \mathbb{Z}^+$, where $7 \leq \text{DURATION}_{\text{FIXED}} \leq 21$ - Sprint duration in days.
- **DV6:** $\text{priority}_f \in \mathbb{Z}^+$, where $1 \leq \text{priority}_f \leq 10$ - Feature priority level.
- **DV7:** $\text{severity}_{bl} \in \mathbb{Z}^+$, where $1 \leq \text{severity}_{bl} \leq 5$ - Blocker severity level.
- **DV8:** $\text{team\_size}_t \in \mathbb{Z}^+$, where $3 \leq \text{team\_size}_t \leq 9$ - Team size.
- **DV9:** $\text{budget}_p \in \mathbb{R}^+$, where $0.0 \leq \text{budget}_p \leq 1,000,000.0$ - Project budget.
- **DV10:** $x^{\text{req}}_{s,tsk} \in \{0, 1\}$ - Binary requirement of skill s for task tsk.