Mathematical Formulation of a SCRUM Optimization Model

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Contents

This document provides a mathematical and logical formulation of an optimization model for resource and task management within a software development company using the SCRUM framework. The model is derived from a domain model specified by a set of entities, relationships, goals, conditions, and decision variables.

The general form of the optimization model is:

$$\begin{aligned} \text{Optimize} \quad & Z = \sum_{g \in G} w_g \cdot f_g(\mathbf{X}) \\ \text{subject to} \quad & h_c(\mathbf{X}) \leq 0 \quad \forall c \in C_{must} \\ & \mathbf{X} \in \mathcal{D} \end{aligned}$$

where Z is the multi-objective function (a weighted sum of all individual goals g), $\mathbf{X}$ is the vector of decision variables, w_g is the weight for each goal, h_c are the constraint functions for all "must-match" conditions c, and $\mathcal{D}$ is the domain of the decision variables.

1 Sets (Entities)

The fundamental sets of the model are derived from the entities in the domain.

- **Project** (P): The set of all projects.
- **Team** (T): The set of all teams.
- **Worker** (W): The set of all workers.
- **Feature** (F): The set of all features.
- **Skill** (S): The set of all skills.
- **Role** (R): The set of all roles.
- **ProductOwner** (PO): The set of all Product Owners.
- **ScrumMaster** (SM): The set of all Scrum Masters.
- **ProductBacklog** (PB): The set of all Product Backlogs.
- **Sprint** (SP): The set of all sprints.
- **SprintBacklog** (SBL): The set of all Sprint Backlogs.
- **SprintGoal** (SG): The set of all Sprint Goals.
- **Epic** (E): The set of all epics.
- **UserStory** (US): The set of all user stories.
- **Task** (TSK): The set of all tasks.
- **Blocker** (BL): The set of all blockers.
- **Stakeholder** (SH): The set of all stakeholders.
- **Velocity** (VEL): The set of all team velocity records.
- **ReleasePlan** (REP): The set of all release plans.
- **FeatureDocumentation** (FED): The set of all feature documentations.

2 Indices

Indices are used to iterate over the elements of the sets defined above.

- $p \in P$: index for projects.
- $t \in T$: index for teams.
- $w \in W$: index for workers.
- $f \in F$: index for features.
- $s \in S$: index for skills.
- $r \in R$: index for roles.
- $po \in PO$: index for Product Owners.
- $sm \in SM$: index for Scrum Masters.
- $pb \in PB$: index for Product Backlogs.
- $sp \in SP$: index for sprints.
- $sbl \in SBL$: index for Sprint Backlogs.
- $sg \in SG$: index for Sprint Goals.
- $e \in E$: index for epics.
- $us \in US$: index for user stories.
- $tsk \in TSK$: index for tasks.
- $bl \in BL$: index for blockers.
- $sh \in SH$: index for stakeholders.
- $vel \in VEL$: index for velocity records.
- $rep \in REP$: index for release plans.
- $fed \in FED$: index for feature documentations.

3 Goals

The objective functions of the optimization model. The overall goal is a weighted sum of these individual functions. Let $X_{\dots}$ denote relevant binary decision variables (e.g., $X_{f,rep} = 1$ if feature f is in release plan rep).

G0: maximize_team_velocity

$$\max \sum_{t \in T} \text{avg\_story\_points}_t$$

G1: minimize_project_budget

$$\min \sum_{p \in P} \text{budget}_p$$

G2: maximize_feature_priority_sum

$$\max \sum_{f \in F} \sum_{rep \in REP} \text{priority}_f \cdot X_{f,rep}$$

G3: minimize_total_task_effort

$$\min \sum_{tsk \in TSK} \sum_{w \in W} \text{effort}_{tsk} \cdot X_{w,tsk}$$

G4: maximize_sprint_goal_achievement

$$\max \sum_{sg \in SG} \text{achievement\_status}_{sg}$$

G5: minimize_blocker_count

$$\min \sum_{bl \in BL} 1$$

G6: maximize_story_points_in_sprint

$$\max \sum_{us \in US} \sum_{sp \in SP} \text{story\_points}_{us} \cdot X_{us,sp}$$

G7: maximize_team_satisfaction

$$\max \sum_{t \in T} \text{team\_satisfaction}_t$$

G8: minimize_project_end_date

$$\min \max_{p \in P} (\text{project\_end}_p)$$

G9: maximize_stakeholder_engagement

$$\max \sum_{sh \in SH} \sum_{f \in F} \text{influence\_level}_{sh} \cdot \text{relevance}_{sh,f}$$

G10: minimize_blocker_severity_sum

$$\min \sum_{bl \in BL} \text{severity}_{bl}$$

G11: maximize_sprint_review_attendance

$$\max \sum_{sp \in SP} \text{attendees\_count}_{sp}$$

G12: maximize_certified_skills_in_team

$$\max \sum_{w \in W} \sum_{s \in S} \text{certified}_s \cdot X_{w,s}$$

4 Conditions

The constraints that the solution must satisfy. Let parameters be denoted by their attribute names and indexed by the relevant set index.

C0: enforce_max_team_size

$$\text{team\_size}_t \leq 9 \quad \forall t \in T$$

C1: enforce_min_team_size

$$\text{team\_size}_t \geq 3 \quad \forall t \in T$$

C2: enforce_project_budget

$$\sum_{t \in T, \ldots} \text{cost} \leq \text{budget}_p \quad \forall p \in P$$

C3: require_active_worker_status Let $\delta_w = 1$ if status_w = 'active', 0 otherwise.

$$X_{w,tsk} \leq \delta_w \quad \forall w \in W, tsk \in TSK$$

C4: respect_sprint_backlog_capacity

$$\text{total\_effort}_{sbl} \leq \text{avg\_story\_points}_t \quad \forall sbl \in SBL, t \in T \text{ assigned to } sbl$$

C5: require_high_feature_priority Let P_{min} be the minimum priority threshold.

$$\text{priority}_f \geq P_{min} \quad \forall f \in F \text{ selected for development}$$

C6: require_user_story_acceptance_criteria Let $\gamma_{us} = 1$ if $\text{acceptance\_criteria}_{us}$ is not null, 0 otherwise.

$$X_{us,sp} \leq \gamma_{us} \quad \forall us \in US, sp \in SP$$

C7: require_sprint_goal_definition Let $\sigma_{sp} = 1$ if sprint sp has a goal, 0 otherwise.

$$\text{start\_sprint}_{sp} \leq \sigma_{sp} \quad \forall sp \in SP$$

C8: require_project_start_date Let $\lambda_p = 1$ if project_start_p is not null, 0 otherwise. Any assignment to project p must be 0 if $\lambda_p = 0$.

$$X_{t,p} \leq \lambda_p \quad \forall t \in T, p \in P$$

C10: prefer_experienced_scrum_master (Soft Constraint) Could be modeled by adding a penalty to the objective function if $\text{experience}_{sm} < E_{min}$.

C11: require_feature_documentation_link Let $\phi_f = 1$ if feature f has linked documentation, 0 otherwise.

$$\text{status}_f = \text{'Done'} \implies \phi_f = 1 \quad \forall f \in F$$

C12: prevent_work_on_closed_projects Let $\omega_p = 1$ if status_p = 'closed', 0 otherwise.

$$X_{us,sp,p} \cdot \omega_p = 0 \quad \forall us \in US, sp \in SP, p \in P$$

5 Decision Variables

The variables that the optimization model will determine.

DV0: assign_worker_to_task ($X_{w,tsk}$)

$$X_{w,tsk} \in \{0,1\} \quad \forall w \in W, tsk \in TSK$$

DV1: assign_story_to_sprint ($X_{us,sp}$)

$$X_{us,sp} \in \{0,1\} \quad \forall us \in US, sp \in SP$$

DV2: assign_team_to_project ($X_{t,p}$)

$$X_{t,p} \in \{0,1\} \quad \forall t \in T, p \in P$$

DV3: select_feature_for_release ($X_{f,rep}$)

$$X_{f,rep} \in \{0,1\} \quad \forall f \in F, rep \in REP$$

DV4: set_sprint_backlog_effort (E_{sbl})

$$E_{sbl} \in \mathbb{Z}^+ \quad (e.g., 0 \leq E_{sbl} \leq 200) \quad \forall sbl \in SBL$$

DV5: set_team_size (S_t)

$$S_t \in \{3,4,...,9\} \quad \forall t \in T$$

DV6: schedule_release_date (D_{rep})

$$D_{rep} \in \mathbb{Z}^+ \quad \forall rep \in REP$$

DV7: update_task_status ($Y_{tsk,\text{status}}$)

$$Y_{tsk,\text{status}} \in \{0,1\} \quad \forall tsk \in TSK, \text{status} \in \{\text{'ToDo', 'InProgress', ...}\}$$

$$\sum_{\text{status}} Y_{tsk,\text{status}} = 1 \quad \forall tsk \in TSK$$

DV8: prioritize_feature (P_f)

$$P_f \in \{1,2,...,100\} \quad \forall f \in F$$

DV9: assign_worker_to_team ($X_{w,t}$)

$$X_{w,t} \in \{0,1\} \quad \forall w \in W, t \in T$$

DV10: set_project_budget (B_p)

$$B_p \in \mathbb{R}^+ \quad \forall p \in P$$

DV11: approve_sprint_goal_achievement (A_{sg})

$$A_{sg} \in \{0,1\} \quad \forall sg \in SG$$