Multi-Objective Optimization Model for a Scrum-Based Software Development Company

${\bf Truely Most Wanted}$

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Notation preface

Relations from the data are enforced as structural constraints. Where relationships used the label *Employee*, it refers to entity **Worker**. Where a relation used *Sprint Review*, it refers to entity **SprintReview**. Attributes from entities are treated as parameters (possibly normalized or binarized) when numeric optimization is required.

Decision Variables (subset; full list in Section 5):

```
x_{t,p} \in \{0,1\}
                             team t assigned to project p (R1)
      y_{w,t} \in \{0,1\}
                             worker w belongs to team t (R2)
  smt_{t.sm} \in \{0,1\}
                             Scrum Master sm supports team t (R6)
   po_{po,pb} \in \{0,1\}
                             Product Owner po manages Product Backlog pb (R5)
    bf_{pb,f} \in \{0,1\}
                             feature f in Product Backlog pb (R7)
   be_{pb,ep} \in \{0,1\}
                             epic ep in Product Backlog pb (R8)
   ue_{ep,us} \in \{0,1\}
                             user story us in epic ep (R9)
   tu_{us,ts} \in \{0,1\}
                             task ts in user story us (R10)
usbl_{us,sbl} \in \{0,1\}
                             user story us in sprint backlog sbl (R11)
sblsp_{sbl,sp} \in \{0,1\}
                             sprint backlog sbl belongs to sprint sp (R12)
   tb_{scb,ts} \in \{0,1\}
                             task ts on scrum board scb (R14)
   rf_{rep,f} \in \{0,1\}
                             feature f in release plan rep (R20)
   pg_{sp,sq} \in \{0,1\}
                             sprint sp pursues sprint goal sq (R13)
```

Key Parameters (from entity attributes):

```
SP_{us} := UserStory.story\_points of us
EFF_{ts} := Task.effort of ts
SEV_{bl} := Blocker.severity of bl
AOG_{sp} := Sprint.achievement\_of\_goal of sp
\overline{V}_t := Velocity.avg\_story\_points for team t (via R19)
NE_{pb} := ProductBacklog.number\_of\_entries of pb
BUD_p := Project.budget of p
PRI_f := Feature.priority of f
INF_{sh} := Stakeholder.influence\_level of sh
SAT_{sre} := SprintRetrospective.team\_satisfaction of sre
TPR_{dev} := DevelopmentSnapshot.test\_status mapped to \{0, 1\}
ATT_{sr} := SprintReview.attendees\_count of sr
CAP_{sbl} := SprintBacklog.total\_effort capacity of sbl
```

Further relational parameters include $B_{ts,bl} \in \{0,1\}$ (R16: task ts is blocked by blocker bl).

1 1. Sets (Entities)

- P Projects
- T Teams
- W Workers

- \bullet F Features
- \bullet S Skills
- \bullet R Roles
- PO ProductOwners
- SM ScrumMasters
- PB ProductBacklogs
- \bullet SP Sprints
- \bullet SPP SprintPlannings
- \bullet DS DailyScrums
- \bullet SR SprintReviews
- SRE SprintRetrospectives
- \bullet SBL SprintBacklogs
- SG SprintGoals
- EPC Epics
- US UserStories
- \bullet TSK Tasks
- \bullet DEV DevelopmentSnapshots
- \bullet BL Blockers
- \bullet SH Stakeholders
- VEL Velocity records
- REP ReleasePlans
- \bullet RM Roadmaps
- \bullet SCB ScrumBoards
- *FED* FeatureDocumentations

2 2. Indices

- $p \in P$, $t \in T$, $w \in W$, $f \in F$, $sk \in S$, $r \in R$
- $po \in PO$, $sm \in SM$, $pb \in PB$, $sp \in SP$, $sbl \in SBL$, $sg \in SG$
- $ep \in EPC$, $us \in US$, $ts \in TSK$, $dev \in DEV$, $bl \in BL$
- $sh \in SH$, $v \in VEL$, $rep \in REP$, $rm \in RM$, $scb \in SCB$, $fed \in FED$

3 3. Goals

Multi-objective model; we can solve via weighted sum $\max \sum_k \omega_k \mathcal{O}_k$ (for max goals) and $\min \sum_k \omega_k \mathcal{O}_k$ (for min goals), or via ϵ -constraints. Below each goal lists its ID, name, and mathematical form.

• [G0] maximize_delivered_story_points:

$$\max \mathcal{O}_0 = \sum_{us \in US} \sum_{sbl \in SBL} SP_{us} \cdot usbl_{us,sbl}.$$

• [G1] minimize_total_task_effort:

$$\min \mathcal{O}_1 = \sum_{ts \in TSK} \sum_{scb \in SCB} \text{EFF}_{ts} \cdot tb_{scb,ts}.$$

• [G2] minimize_blocker_impact:

min
$$\mathcal{O}_2 = \sum_{bl \in BL} \text{SEV}_{bl} \left(\sum_{ts \in TSK} \sum_{scb \in SCB} B_{ts,bl} \cdot tb_{scb,ts} \right).$$

• [G3] maximize_sprint_goal_achievement:

$$\max \mathcal{O}_3 = \sum_{sp \in SP} \sum_{sq \in SG} AOG_{sp} \cdot pg_{sp,sg}.$$

• [G4] maximize_team_velocity:

$$\max \mathcal{O}_4 = \sum_{t \in T} \overline{V}_t \left(\sum_{p \in P} x_{t,p} \right).$$

• [G5] minimize_product_backlog_size:

min
$$\mathcal{O}_5 = \sum_{pb \in PB} \left(\sum_{f \in F} bf_{pb,f} + \sum_{ep \in EPC} be_{pb,ep} \right).$$

• [G6] minimize_project_budget_consumption (proxy):

$$\min \mathcal{O}_6 = \sum_{p \in P} \sum_{t \in T} BUD_p \cdot x_{t,p}.$$

• [G7] maximize_priority_of_released_features:

$$\max \mathcal{O}_7 = \sum_{rep \in REP} \sum_{f \in F} PRI_f \cdot rf_{rep,f}.$$

• [G8] maximize_stakeholder_participation_impact: Let $part_{sh,sr} \in \{0,1\}$ denote stakeholder participation in Sprint Reviews (R17). Then

$$\max \mathcal{O}_8 = \sum_{sr \in SR} \sum_{sh \in SH} INF_{sh} \cdot part_{sh,sr}.$$

• [G9] maximize_team_satisfaction:

$$\max \ \mathcal{O}_9 = \sum_{\mathit{sre} \in \mathit{SRE}} \mathit{SAT}_{\mathit{sre}}.$$

• [G10] minimize_unfinished_tasks_on_board: Let $done_{ts} \in \{0,1\}$ from Task.status. Then

$$\min \mathcal{O}_{10} = \sum_{ts \in TSK} \sum_{scb \in SCB} (1 - done_{ts}) \cdot tb_{scb,ts}.$$

• [G11] maximize_test_pass_rate: Let $pass_{dev} \in \{0,1\}$ mapped from DevelopmentSnapshot.test_status. Then

$$\max \mathcal{O}_{11} = \sum_{dev \in DEV} pass_{dev}.$$

4 4. Conditions

Each condition encodes logical and relational constraints derived from the relationships and attributes.

• [C0] team_assigned_to_exactly_one_project (R1):

$$\sum_{p \in P} x_{t,p} = 1 \quad \forall t \in T.$$

• [C1] worker_belongs_to_exactly_one_team (R2):

$$\sum_{t \in T} y_{w,t} = 1 \quad \forall w \in W.$$

• [C2] scrummaster_supports_one_team (R6):

$$\sum_{sm \in SM} smt_{t,sm} = 1 \quad \forall t \in T.$$

• [C3] product_owner_manages_one_backlog (R5):

$$\sum_{po \in PO} po_{po,pb} = 1 \quad \forall pb \in PB.$$

• [C4] feature_in_at_most_one_backlog (R7):

$$\sum_{pb \in PB} bf_{pb,f} \le 1 \quad \forall f \in F.$$

• [C5] epic_in_at_most_one_backlog (R8):

$$\sum_{pb \in PB} be_{pb,ep} \le 1 \quad \forall ep \in EPC.$$

• [C6] user_story_in_exactly_one_epic (R9):

$$\sum_{ep \in EPC} ue_{ep,us} = 1 \quad \forall us \in US.$$

• [C7] task_in_exactly_one_user_story (R10):

$$\sum_{us \in US} tu_{us,ts} = 1 \quad \forall ts \in TSK.$$

• [C8] user_story_in_at_most_one_sprint_backlog (R11):

$$\sum_{sbl \in SBL} usbl_{us,sbl} \le 1 \quad \forall us \in US.$$

• [C9] sprint_backlog_belongs_to_one_sprint (R12):

$$\sum_{sp \in SP} sblsp_{sbl,sp} = 1 \quad \forall sbl \in SBL.$$

• [C10] sprint_backlog_capacity_respected:

$$\sum_{us \in US} \sum_{ts \in TSK} \text{EFF}_{ts} \cdot tu_{us,ts} \cdot usbl_{us,sbl} \leq \text{CAP}_{sbl} \quad \forall sbl \in SBL.$$

(This bilinear form can be linearized using standard techniques, e.g., auxiliary variables and McCormick envelopes.)

• [C11] release_only_features_in_backlog (R20 & R7):

$$rf_{rep,f} \leq \sum_{pb \in PB} bf_{pb,f} \quad \forall rep \in REP, \ \forall f \in F.$$

• [C12] sprint_pursues_exactly_one_goal (R13):

$$\sum_{sq \in SG} pg_{sp,sg} = 1 \quad \forall sp \in SP.$$

5 5. DecisionVariables

• DV0: $x_{t,p} \in \{0,1\}$ — team t assigned to project p.

• DV1: $y_{w,t} \in \{0,1\}$ — worker w belongs to team t.

• **DV2:** $smt_{t,sm} \in \{0,1\}$ — Scrum Master sm supports team t.

• DV3: $po_{po,pb} \in \{0,1\}$ — Product Owner po manages Product Backlog pb.

• DV4: $bf_{pb,f} \in \{0,1\}$ — feature f in Product Backlog pb.

• **DV5:** $be_{pb,ep} \in \{0,1\}$ — epic ep in Product Backlog pb.

• **DV6:** $ue_{ep,us} \in \{0,1\}$ — user story us in epic ep.

• DV7: $tu_{us,ts} \in \{0,1\}$ — task ts in user story us.

• DV8: $usbl_{us,sbl} \in \{0,1\}$ — user story us in sprint backlog sbl.

• **DV9:** $sblsp_{sbl,sp} \in \{0,1\}$ — sprint backlog sbl belongs to sprint sp.

• DV10: $tb_{scb,ts} \in \{0,1\}$ — task ts on scrum board scb.

• DV11: $rf_{rep,f} \in \{0,1\}$ — feature f in release plan rep.

• DV12: $pg_{sp,sg} \in \{0,1\}$ — sprint sp pursues sprint goal sg.