Optimization Model for Scrum-based Software Development

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September 5, 2025

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1 Sets (Entities)

These sets are derived from the domain model entities.

- P: Set of all Projects
- T: Set of all Teams
- W: Set of all Workers
- F: Set of all Features
- S: Set of all Skills
- R: Set of all Roles
- PO: Set of all Product Owners
- SM: Set of all Scrum Masters
- PB: Set of all Product Backlogs
- SP: Set of all Sprints
- SBL: Set of all Sprint Backlogs
- SG: Set of all Sprint Goals
- E: Set of all Epics
- US: Set of all User Stories
- TSK: Set of all Tasks
- BL: Set of all Blockers
- SH: Set of all Stakeholders
- VEL: Set of all Velocity records
- REP: Set of all Release Plans

2 Indices

These indices are used to refer to individual elements within the sets.

- $p \in P$: an individual Project
- $t \in T$: an individual Team
- $w \in W$: an individual Worker
- $f \in F$: an individual Feature
- $s \in S$: an individual Skill
- $r \in R$: an individual Role
- $us \in US$: an individual User Story
- $tsk \in TSK$: an individual Task

- $sp \in SP$: an individual Sprint
- $bl \in BL$: an individual Blocker
- $sh \in SH$: an individual Stakeholder
- $rep \in REP$: an individual Release Plan

3 Decision Variables

These are the variables the optimization model will determine.

- $X_{wt} \in \{0, 1\}$: 1 if Worker w is assigned to Team t. (DV0)
- $Y_{us,sp} \in \{0, 1\}$: 1 if User Story us is assigned to Sprint sp. (DV1)
- $Z_{tsk,w} \in \{0, 1\}$: 1 if Task tsk is assigned to Worker w. (DV2)
- $A_{f,rep} \in \{0, 1\}$: 1 if Feature f is selected for Release Plan rep. (DV3)
- $B_{t,p} \in \{0, 1\}$: 1 if Team t is assigned to Project p. (DV9)
- $SP_{start,sp}$: Start date of Sprint sp. (DV4)
- $SP_{end,sp}$: End date of Sprint sp. (DV5)

4 Goals (Objective Function)

The primary objective is to maximize a weighted sum of various business and operational goals. The overall objective function is:

$$\text{Maximize} \sum_{i=0}^{11} \omega_i \cdot G_i$$

where ω_i is the weight for goal G_i.

G0 **maximize_feature_priority**:

$$G_0 = \sum_{f \in F} \sum_{rep \in REP} \text{priority}_f \cdot A_{f,rep}$$

G1 **maximize_sprint_story_points**:

$$G_1 = \sum_{us \in US} \sum_{sp \in SP} \text{story\_points}_{us} \cdot Y_{us,sp}$$

G4 **minimize_unresolved_blockers**: (Represented as maximizing resolved blockers)

$$G_4 = \sum_{bl \in BL} (1 - \text{is\_active}_{bl})$$

G6 **minimize_project_duration**: (Represented as maximizing the negative duration)

$$G_6 = -(\max_{p \in P}(\text{project\_end}_p) - \min_{p \in P}(\text{project\_start}_p))$$

G11 **maximize_feature_completion_rate**:

$$G_{11} = \sum_{f \in F} \text{is\_done}_f \quad (\text{where is\_done}_f = 1 \text{ if status is 'done'})$$

5 Conditions (Constraints)

These are the constraints that the solution must satisfy.

C0 **respect_project_budget**: Project costs must not exceed the budget. Let C_w be the cost of worker w and C_t be the fixed cost of team t.

$$\sum_{w \in W} \sum_{t \in T} C_w \cdot X_{wt} \cdot B_{t,p} + \sum_{t \in T} C_t \cdot B_{t,p} \leq \text{budget}_p \quad \forall p \in P$$

C1 **fixed_sprint_duration**: Sprints must adhere to a maximum duration, e.g., 28 days.

$$SP_{end,sp} - SP_{start,sp} \leq 28 \quad \forall sp \in SP$$

C2 **team_size_constraints**: Each team must be within a defined size range [min_size, max_size].

$$\text{min\_size}_t \leq \sum_{w \in W} X_{wt} \leq \text{max\_size}_t \quad \forall t \in T$$

C3 **respect_worker_availability**: Total effort of tasks assigned to a worker must not exceed their capacity.

$$\sum_{tsk \in TSK} \text{effort}_{tsk} \cdot Z_{tsk,w} \leq \text{availability}_w \quad \forall w \in W$$

C4 **story_points_less_than_velocity**: Story points in a sprint cannot exceed team velocity.

$$\sum_{us \in US} \text{story\_points}_{us} \cdot Y_{us,sp} \leq \text{velocity}_t \quad \forall sp \in SP, t \in T$$

(Requires linking sprints to teams, which we assume is given).

C5 **one_team_per_worker**: Each worker can be assigned to at most one team.

$$\sum_{t \in T} X_{wt} \leq 1 \quad \forall w \in W$$

C11 **skill_must_exist_for_task**: A task requiring skill s can only be assigned to a worker w who has that skill. Let $Req_{tsk,s} = 1$ if task tsk requires skill s, and $Has_{w,s} = 1$ if worker w has skill s.

$$Z_{tsk,w} \cdot Req_{tsk,s} \leq Has_{w,s} \quad \forall tsk \in TSK, w \in W, s \in S$$