

SCRUM Domain Model – Mathematical Optimization Formulation

Generated for TrulyMostWanted

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Contents

1	1. Sets (Entities)	2
2	2. Indices	3
3	3. Goals	4
4	4. Conditions	6
5	5. DecisionVariables	8

1 1. Sets (Entities)

- \mathcal{P} : Projects (**Project**)
- \mathcal{T} : Teams (**Team**)
- \mathcal{W} : Workers (**Worker**)
- \mathcal{F} : Features (**Feature**)
- \mathcal{K} : Skills (**Skill**)
- \mathcal{R} : Roles (**Role**)
- \mathcal{PO} : Product Owners (**ProductOwner**)
- \mathcal{SM} : Scrum Masters (**ScrumMaster**)
- \mathcal{PB} : Product Backlogs (**ProductBacklog**)
- \mathcal{SP} : Sprints (**Sprint**)
- \mathcal{SPP} : Sprint Plannings (**SprintPlanning**)
- \mathcal{DS} : Daily Scrums (**DailyScrum**)
- \mathcal{SR} : Sprint Reviews (**SprintReview**)
- \mathcal{SRE} : Sprint Retrospectives (**SprintRetrospective**)
- \mathcal{SBL} : Sprint Backlogs (**SprintBacklog**)
- \mathcal{SG} : Sprint Goals (**SprintGoal**)
- \mathcal{E} : Epics (**Epic**)
- \mathcal{US} : User Stories (**UserStory**)
- \mathcal{TSK} : Tasks (**Task**)
- \mathcal{DEV} : Development Snapshots (**DevelopmentSnapshot**)
- \mathcal{BL} : Blockers (**Blocker**)
- \mathcal{SH} : Stakeholders (**Stakeholder**)
- \mathcal{VEL} : Velocities (**Velocity**)
- \mathcal{REP} : Release Plans (**ReleasePlan**)
- \mathcal{RM} : Roadmaps (**Roadmap**)
- \mathcal{SCB} : Scrum Boards (**ScrumBoard**)
- \mathcal{FED} : Feature Documentations (**FeatureDocumentation**)

2 2. Indices

- $p \in \mathcal{P}$ (from P)
- $t \in \mathcal{T}$ (from T)
- $w \in \mathcal{W}$ (from W)
- $f \in$ (from F)
- $s \in \mathcal{SP}$ (from SP)
- $r \in \mathcal{R}$ (from R)
- $po \in \mathcal{PO}$ (from PO)
- $sm \in \mathcal{SM}$ (from SM)
- $pb \in \mathcal{PB}$ (from PB)
- $spp \in \mathcal{SPP}$ (from SPP)
- $ds \in \mathcal{DS}$ (from DS)
- $sr \in \mathcal{SR}$ (from SR)
- $sre \in \mathcal{SRE}$ (from SRE)
- $sbl \in \mathcal{SBL}$ (from SBL)
- $sg \in \mathcal{SG}$ (from SG)
- $e \in \mathcal{E}$ (from E)
- $us \in \mathcal{US}$ (from US)
- $tsk \in \mathcal{TSK}$ (from TSK)
- $dev \in \mathcal{DEV}$ (from DEV)
- $bl \in \mathcal{BL}$ (from BL)
- $sh \in \mathcal{SH}$ (from SH)
- $vel \in \mathcal{VEL}$ (from VEL)
- $rep \in \mathcal{REP}$ (from REP)
- $rm \in \mathcal{RM}$ (from RM)
- $scb \in \mathcal{SCB}$ (from SCB)
- $fed \in \mathcal{FED}$ (from FED)

3 3. Goals

Notation (parameters used by goals). When needed, we refer to attributes from `Entities.csv` as parameters:

- $v_{vel}^{avg}, v_{vel}^{max}, v_{vel}^{min}$ from `Velocity` (`avg_story_points`, `max_velocity`, `min_velocity`).
- B_p from `Project` (`budget`); B_p^{alloc} is decision (see Sec. 5).
- eff_{tsk}^{base} baseline task effort; \hat{eff}_{tsk} decision refinement (Sec. 5).
- sp_{us} story points of a user story.
- g_{sg}^{ach} achievement status (numeric score) of a sprint goal.
- $avail_w$ availability of a worker.
- π_f^F priority of a feature.
- sev_{bl} severity of a blocker.
- att_{sr} attendees count of a sprint review.
- $TotEff_{sbl}$ total effort recorded for a sprint backlog.
- Binary incidence parameters to reflect `Relationships.csv`: $\alpha_{us,sbl} \in \{0, 1\}$ for *R11* (*is_in_sprint_backlog*), $\beta_{sbl,s} \in \{0, 1\}$ for *R12*, $\gamma_{tsk,bl} \in \{0, 1\}$ for *R16*, $\delta_{vel,t} \in \{0, 1\}$ for *R19*, $\rho_{f,rep} \in \{0, 1\}$ for *R20*, $\eta_{e,pb} \in \{0, 1\}$ for *R8*, $\zeta_{us,e} \in \{0, 1\}$ for *R9*, $\kappa_{us,tsk} \in \{0, 1\}$ for *R10*.

Decision variables used by goals (see Sec. 5 for domains).

$x_{t,p}$ (DV0), $y_{us,s}$ (DV1), $z_{tsk,w}$ (DV2), $r_{f,rep}$ (DV3), $b_{e,pb}$ (DV4), a_s (DV5), π_{us} (DV6), B_p^{alloc} (DV7), C_s (DV8)

G0 maximize_velocity_avg_story_points

Logical: Prefer teams/sprints with higher historical average velocity.

Mathematical (KPI-driven, linked to active sprints via a_s and team via $\delta_{vel,t}$ and assignment

$x_{t,p}$):

$$\max \sum_{vel \in \mathcal{VEL}} v_{vel}^{avg} \left(\sum_{t \in \mathcal{T}} \delta_{vel,t} \sum_{p \in \mathcal{P}} x_{t,p} \right) \left(\sum_{s \in \mathcal{SP}} a_s \right)$$

G1 maximize_velocity_max_velocity

Logical: Push historical peak velocity upward.

Mathematical:

$$\max \sum_{vel \in \mathcal{VEL}} v_{vel}^{max} \left(\sum_t \delta_{vel,t} \sum_p x_{t,p} \right)$$

G2 minimize_velocity_min_velocity

Logical: Reduce low-end velocity dips.

Mathematical:

$$\min \sum_{vel \in \mathcal{VEL}} v_{vel}^{min} \left(\sum_t \delta_{vel,t} \sum_p x_{t,p} \right)$$

G3 minimize_project_budget

Logical: Keep total allocated project budget low.

Mathematical:

$$\min \sum_{p \in \mathcal{P}} B_p^{alloc}$$

G4 minimize_task_effort

Logical: Minimize refined implementation effort for selected scope.

Mathematical (only tasks actually assigned are counted):

$$\min \sum_{tsk \in \mathcal{TSK}} \hat{\text{eff}}_{tsk} \left(\sum_{w \in \mathcal{W}} z_{tsk,w} \right)$$

G5 maximize_user_story_story_points

Logical: Maximize delivered story points by scheduling stories.

Mathematical:

$$\max \sum_{us \in \mathcal{US}} \text{sp}_{us} \left(\sum_{s \in \mathcal{SP}} y_{us,s} \right)$$

G6 maximize_sprint_goal_achievement

Logical: Prefer plans where sprint goals are achieved (proxy by activating sprints).

Mathematical:

$$\max \sum_{sg \in \mathcal{SG}} g_{sg}^{\text{ach}} \left(\sum_{s \in \mathcal{SP}} a_s \right)$$

G7 maximize_worker_availability

Logical: Prefer assigning tasks to more available workers.

Mathematical:

$$\max \sum_{w \in \mathcal{W}} \text{avail}_w \left(\sum_{tsk \in \mathcal{TSK}} z_{tsk,w} \right)$$

G8 maximize_feature_priority

Logical: Prefer including higher-priority features in release plans.

Mathematical:

$$\max \sum_{rep \in \mathcal{REP}} \sum_{f \in} \pi_f^F r_{f,rep}$$

G9 minimize_blocker_severity

Logical: Reduce overall severity exposure of blocked tasks.

Mathematical:

$$\min \sum_{tsk \in \mathcal{TSK}} \sum_{bl \in \mathcal{BL}} \gamma_{tsk,bl} \text{sev}_{bl} \left(\sum_w z_{tsk,w} \right)$$

G10 maximize_sprint_review_attendees

Logical: Increase stakeholder engagement in reviews (proxy via invitations weighted by relevance if desired).

Mathematical (two variants; choose one):

$$\max \sum_{sr \in \mathcal{SR}} \text{att}_{sr} \quad \text{or} \quad \max \sum_{sr \in \mathcal{SR}} \sum_{sh \in \mathcal{SH}} i_{sh,sr}$$

G11 minimize_sprint_total_effort

Logical: Keep sprint workload manageable.

Mathematical:

$$\min \sum_{sbl \in \mathcal{SBL}} \text{TotEff}_{sbl}$$

4 4. Conditions

Notation (parameters used by conditions).

- $\text{ready}_{us} \in \{0, 1\}$ indicates acceptable **status** for a user story (C0).
- $\text{effEst}_f \geq 0$ indicates presence of **estimated_effort** for a feature (C1).
- $\text{blocked}_{tsk} \in \{0, 1\}$ from **status** or $\exists bl : \gamma_{tsk,bl} = 1$ (C2).
- dur_{ds} in minutes for **DailyScrum** (C4).
- $\text{hasGoal}_s \in \{0, 1\}$ indicates that sprint s has an attached goal (C5 via R13).
- $\text{recent}_{pb} \in \{0, 1\}$ indicates **last_updated** within policy window (C6).
- $\text{active}_{sbl} \in \{0, 1\}$ allowed values of **status** (C7).
- B_p^{\max} budget cap (from **Project.budget**) (C8).
- $\text{rel}_{sh} \in \{0, 1, \dots\}$ stakeholder relevance score (C9).
- $\text{start}_s < \text{end}_s$ dates for sprint s (C10).
- $\text{fbDoc}_{sr} \in \{0, 1\}$ whether review has **feedback_documentation** (C11).
- Capacity feasibility linking: for each s , capacity C_s (DV8) and story assignment $y_{us,s}$ with sp_{us} .

Feasibility/linking constraints from Relationships.csv.

$$(R1) \text{ Team} \rightarrow \text{Project: } \sum_{p \in \mathcal{P}} x_{t,p} \leq 1 \quad \forall t \in \mathcal{T} \quad (1)$$

$$(R11\&R12) \text{ US} \rightarrow \text{SBL} \rightarrow \text{Sprint: } y_{us,s} \leq \sum_{sbl} \alpha_{us,sbl} \beta_{sbl,s} \quad \forall us, s \quad (2)$$

$$(R10) \text{ US consists of Tasks: } \sum_{tsk} \kappa_{us,tsk} \geq 1 \quad \forall us \quad (3)$$

$$(R20) \text{ ReleasePlan includes Features: } r_{f,rep} \leq 1 \quad \forall f, rep \quad (4)$$

$$(R19) \text{ Velocity refers to Team (activation requires a team on a project): } \sum_t \delta_{vel,t} \sum_p x_{t,p} \geq a_s \quad \forall vel, \forall s \quad (5)$$

C0 **require_user_story_ready_status**

Logical: Only ready stories may be scheduled.

Mathematical:

$$y_{us,s} \leq \text{ready}_{us} \quad \forall us \in \mathcal{US}, \forall s \in \mathcal{SP}$$

C1 **require_feature_defined_effort**

Logical: Features must have an estimated effort to be selected for a release.

Mathematical:

$$r_{f,rep} \leq \mathbf{1}[\text{effEst}_f > 0] \quad \forall f \in \mathcal{F}, \forall rep \in \mathcal{REP}$$

C2 **disallow_blocked_tasks**

Logical: Exclude currently blocked tasks from commitment.

Mathematical:

$$\sum_w z_{tsk,w} \leq 1 - \text{blocked}_{tsk} \quad \forall tsk \in \mathcal{TSK}$$

C3 prefer_high_team_availability

Logical: Favor higher availability in assignment (soft constraint).

Mathematical (modeled as upper bound on load vs. availability):

$$\sum_{tsk} z_{tsk,w} \hat{\text{eff}}_{tsk} \leq M \cdot \text{avail}_w \quad \forall w \in \mathcal{W}$$

with a sufficiently large M or calibrated capacity units.

C4 limit_daily_scrum_duration

Logical: Daily must respect 15-minute timebox.

Mathematical:

$$\text{dur}_{ds} \leq 15 \quad \forall ds \in \mathcal{DS}$$

C5 ensure_sprint_with_goal

Logical: Each active sprint has a defined goal.

Mathematical:

$$a_s \leq \text{hasGoal}_s \quad \forall s \in \mathcal{SP}$$

C6 require_backlog_currency

Logical: Backlog must be recently updated before planning.

Mathematical (activation of any story from pb implies recency):

$$\sum_{us,s} y_{us,s} \leq U \cdot \sum_{pb} \text{recent}_{pb} \quad (U \text{ big-M})$$

C7 limit_sprint_backlog_status

Logical: Only active/planned sprint backlogs may be used.

Mathematical:

$$\sum_{us} y_{us,s} \leq U \cdot \sum_{sbl} \text{active}_{sbl} \beta_{sbl,s} \quad \forall s$$

C8 guard_budget_threshold

Logical: Allocation must not exceed project budget cap.

Mathematical:

$$B_p^{\text{alloc}} \leq B_p^{\text{max}} \quad \forall p \in \mathcal{P}$$

C9 require_stakeholder_relevance

Logical: Invite only stakeholders relevant to presented features/reviews.

Mathematical (threshold τ):

$$i_{sh,sr} \leq \mathbf{1}[\text{rel}_{sh} \geq \tau] \quad \forall sh \in \mathcal{SH}, \forall sr \in \mathcal{SR}$$

C10 ensure_sprint_dates_valid

Logical: Sprint start must precede end date.

Mathematical (data validation):

$$\text{start}_s < \text{end}_s \quad \forall s \in \mathcal{SP}$$

C11 prefer_review_feedback_captured

Logical: Favor reviews with documented feedback (soft feasibility).

Mathematical (link invitations to documentation):

$$\sum_{sh} i_{sh,sr} \leq U \cdot \text{fbDoc}_{sr} \quad \forall sr \in \mathcal{SR}$$

5 5. Decision Variables

- DV0 **assign_team_to_project** $x_{t,p} \in \{0, 1\}$
Whether team t is assigned to project p (maps to $R1$).
- DV1 **assign_user_story_to_sprint** $y_{us,s} \in \{0, 1\}$
Whether user story us is scheduled in sprint s (uses $R11$, $R12$).
- DV2 **assign_task_to_worker** $z_{tsk,w} \in \{0, 1\}$
Whether task tsk is assigned to worker w .
- DV3 **select_feature_for_release** $r_{f,rep} \in \{0, 1\}$
Whether feature f is included in release plan rep (uses $R20$ and $R21$ for roadmap context).
- DV4 **select_epic_for_backlog** $b_{e,pb} \in \{0, 1\}$
Whether epic e is included in product backlog pb (uses $R8$).
- DV5 **activate_sprint** $a_s \in \{0, 1\}$
Whether sprint s is active in the plan (links to $R13$ and $R22$).
- DV6 **prioritize_user_story_rank** $\pi_{us} \in \mathbb{Z}$, $1 \leq \pi_{us} \leq 100$
Rank assigned to a user story us (use with ordering constraints if desired).
- DV7 **allocate_budget_to_project** $B_p^{\text{alloc}} \in \mathbb{R}_{\geq 0}$
Allocated budget amount to project p with $0 \leq B_p^{\text{alloc}} \leq 10^8$.
- DV8 **set_sprint_capacity_points** $C_s \in \mathbb{Z}_{\geq 0}$
Capacity (story points) planned for sprint s , $0 \leq C_s \leq 1000$.
- DV9 **set_task_effort_refinement** $\hat{\text{eff}}_{tsk} \in \mathbb{Z}_{\geq 0}$
Refined effort estimate for task tsk , $0 \leq \hat{\text{eff}}_{tsk} \leq 100$.
- DV10 **choose_scrum_master_for_team** $m_{t,sm} \in \{0, 1\}$
Whether Scrum Master sm is linked to team t (uses $R6$ and $R18$).
- DV11 **invite_stakeholder_to_review** $i_{sh,sr} \in \{0, 1\}$
Whether stakeholder sh is invited to Sprint Review sr (uses $R17$).

Typical capacity coupling (illustrative, optional):

$$\sum_{us \in \mathcal{US}} \text{sp}_{us} y_{us,s} \leq C_s \quad \forall s \in \mathcal{SP}$$