QUANTITATIVE APTITUDE

NUMBER SYSTEM

CONCEPTS

In Hindu–Arabic system we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called digits to represent any number. This is the decimal system where we use the digits 0 to 9. Here 0 is called *insignificant digit* where as 1, ... , 9 are called *significant digits*.

• Classification of Numbers:

Natural Numbers: The numbers 1, 2, 3, 4, 5, 6, ... which we use in counting are known as *natural numbers.* The set of all *natural numbers* can be represented by N = {1, 2, 3, 4, 5, ...}

Whole Numbers: If we include 0 among the natural numbers then the numbers 0, 1, 2, 3, 4, 5, ... are called *whole numbers.* Hence, every natural number is a whole number. The set of *whole numbers* is represented by W.

Integers: All counting numbers and their negatives including zero are known as *integers.*

The set of integers can be represented by Z or I.

Z = {. . . . . . –4, –3, –2, –1, 0, 1, 2, 3, 4, ...}

Every *natural number* is an *integer* but every *integer* is not *natural number.*

Positive Integers: The set I + = {1, 2, 3, 4, ...} is the set of all positive integers. Positive integers and Natural numbers are synonyms.

Negative Integers: The set I – = {. . . , –3, –2, –1} is the set of all negative integers.

0 (zero) is neither positive nor negative.

Non Negative Integers: The set {0, 1, 2, 3, . . . } is the set of all non negative integers.

Rational Numbers: The numbers of the form $\frac{p}{q}$, where p and q are integers, p is not divisible by q and $q \neq 0$, are known as *rational numbers.*

(or) Any number that can be written in fraction form is a *rational number.* This includes *integers*, *terminating decimals*, and *repeating decimals* as well as *fractions.*

e.g.: $\frac{3}{7}, \frac{5}{2}, -\frac{5}{9}, \frac{1}{2}, -\frac{3}{5}$ etc

The set of rational numbers is denoted by *Q*.

Irrational Numbers: Any real number that cannot be written in fraction form is an *irrational number.* Numbers which are both *non-terminating as well as non-repeating decimals* are called irrational numbers.

e.g.: Absolute value of $\frac{10}{3}, \frac{22}{7}, \sqrt{2}, \sqrt{3}, \sqrt{10} \ldots$

Note: A *terminating decimal* will have a finite number of digits after the decimal point.

e.g.: $\frac{3}{4} = 0.75, \frac{5}{4} = 1.25, \frac{25}{16} = 1.5625.$

Repeating Decimals: A decimal number that has digits that repeat forever.

e.g.: $\frac{1}{3} = 0.333\ldots$(here, 3 repeats forever.)

Non–Repeating Decimal: A decimal that neither *terminates* nor *repeats***.**

e.g.: $\sqrt{2} = 1.4142135623\ldots$

Real Numbers: The rational and irrational numbers together are called *real numbers.*

e.g.: $\frac{13}{21}, \frac{2}{5}, \frac{-3}{7}, \frac{+4}{2}$ etc are real numbers.

The set of real numbers is denoted by *R*.

Even Numbers: Any integer that can be divided exactly by 2**.**

e.g.: 2, 6, 0, -8, -10, ... are even numbers.

Odd Numbers: An integer that cannot be divided exactly by 2 is an Odd number.

e.g.: 1, 3, -5, -7, ... are odd numbers.

Prime Numbers: A Prime Number can be divided evenly only by 1, or itself. And it must be a whole number greater than 1.

e.g.: Numbers 2, 3, 5, 7, 11, 13, 17, ... are prime.

All primes which are greater than 3 are of the form $(6n+1)$ or $(6n-1)$.

Note:

• 1 is not a prime number.

• 2 is the least and only even prime number.

• 3 is the least odd prime number.

• Prime numbers up to 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83,89,97.

There are 25 prime numbers up to 100.

Composite Number: Natural numbers greater than 1 which are not prime, are known as *composite numbers.* The number 1 is neither *prime* nor *composite.*

Two numbers which have only 1 as the common factor are called co–primes (or) relatively prime to each other.

e.g.: 3 and 5 are co primes.

Note:

Natural Numbers = 1 + Prime + Composite Numbers.

Whole Numbers = 0 (Zero) + Natural Numbers.

Integers = Negative Integers + 0 + Positive Integers.

Real Numbers = Rational + Irrational Numbers.

• **Test of Divisibility:**

Divisibility by 2: A number is divisible by 2 if the unit's digit is either zero or divisible by 2.

e.g.: Units digit of 76 is 6 which is divisible by 2 hence 76 is divisible by 2.

Units digit of 330 is 0 so it is divisible by 2.

Divisibility by 3: A number is divisible by 3 if sum of all digits in it is divisible by 3.

e.g.: The number 273 is divisible by 3 since 2 + 7 + 3 = 12 which is divisible by 3.

Divisibility by 4: A number is divisible by 4, if the number formed by the last two digits in it is divisible by 4, or last two digits are zeros.

e.g.: The number 5004 is divisible by 4 since last two digits 04 is divisible by 4.

Divisibility by 5: A number is divisible by 5 if the units digit in the number is either 0 or 5.

e.g.: 375 is divisible by 5 as 5 is in the units place.

Divisibility by 6: A number is divisible by 6 if it is even and sum of all digits is divisible by 3.

e.g.: The number 6492 is divisible by 6 as it is even and sum of its digits 6 + 4 + 9 + 2 = 21 is divisible by 3.

Divisibility by 7:

Step-1: Remove unit's digit. And double it.

Step-2: Subtract it from the rest of the number.

Step-3: Check whether the resulted number is divisible by 7 or not.

Step-4: Repeat the above steps until the resulted number is either 0 (zero) or divisible by 7.

e.g.: Consider the number 10717.

Step-1: Removing the unit's digit *i.e.* 7. Double of 7 =14.

Step-2: 1071 – 14 = 1057.

Step-3: Now remove 7 from 1057 and double it *i.e.* 14.

Step-4: 105 – 14 = 91.

Step-5: Now remove 1 and double it *i.e.* 2.

Step-6: 9 – 2 = 7

The final result 7 is divisible by 7. So the given number *i.e.* 10717 is also divisible by 7.

Divisibility by 8: A number is divisible by 8, if the number formed by last 3 digits is divisible by 8.

e.g.: The number 6573392 is divisible by 8 as the last 3 digits '392' is divisible by 8.

Divisibility by 9: A number is divisible by 9 if the sum of its digit is divisible by 9.

e.g.: The number 15606 is divisible by 9 as the sum of the digits 1 + 5 + 6 + 0 + 6 = 18 is divisible by 9.

Divisibility by 10: Last digit should be zero.

e.g.: The last digit of 4470 is zero. So, it is divisible by 10.

Divisibility by 11: A number is divisible by 11 if the difference of the sum of the digits at *odd places* and sum of the digits at the *even places* is either zero or divisible by 11. **(or)** Subtract the first digit from a number made by the other digits. If that number is divisible by 11 then the original number is also divisible by 11.

e.g.: In the number 9823, the sum of the digits at odd places is 9+2=11 and the sum of the digits at even places is 8+3=11. The difference between them is 11 – 11 = 0. Hence, the given number is divisible by 11.

e.g.: 14641

1464 – 1 is 1463

146 – 3 is 143

14 – 3 = 11, which is divisible by 11, so 14641 is also divisible by 11.

• If a number 'N' is divisible by two numbers *'a'* and *'b'*, where *a*, *b* are co primes, then 'N' is divisible by *'ab'*.

Co-prime Numbers: Two numbers are co-prime to each other if they have '*no common factor except* 1'.

Divisibility by 12: A number is divisible by 12 if it is divisible by 3 and 4.

e.g.: The number 1644 is divisible by 12 as it is divisible by 3 and 4. Here 3 and 4 because they are co-prime to each other.

Divisibility by 13: Iteratively add 4 times the last digit to the rest until you get a number divisible by 13 .

e.g.: 7462 ⇒ 746 + (2×4) = 754 ⇒ 75+ (4×4) = 91

91 is divisible by 13. So, 7462 is also divisible by 13.

Divisibility by 14: The number is divisible by 7 and 2.

Divisibility by 15: The number is divisible by 3 and 5.

Divisibility by 16:

With a 3 digit number: Multiply hundreds digit by 4, then add the last two digits.

e.g.: 352 ⇒ (3×4)+52 = 12 + 52 = 64

64 is divisible by 16. So, 352 is also divisible by 16.

With a more than 3 digit number: The last four digits form a number is divisible by 16.

e.g.: 38512 ⇒ Here is 8512 is divisible by 16. So, 38512 is also divisible by 16.

Divisibility by 17:

Subtract 5 times the last digit from the rest.

e.g.: 3961 ⇒ 396 – (1×5) = 391 ⇒ 39 – (1×5) = 34

34 is divisible by 17. So, 3961 is also divisible by 17.

Divisibility by 18: An even number satisfying the divisibility test by 9 is also divisible by 18.

e.g.: The number 80388 is divisible by 18 as it satisfies the divisibility test of 9.

Divisibility by 19: Add twice the last digit to the rest.
e.g.: 10944 ⇒ 1094 + (4 × 2) = 1102
⇒ 110 + (2×2) = 114 ⇒ 11 + (4 × 2) = 11 + 8 = 19.
Divisibility by 20: Last digit is zero & tens digit is even.
e.g.: 980; Last digit is zero. And tens digit is even.
Divisibility by 25: A number is divisible by 25 if the number formed by the last two digits is divisible by 25 or the last two digits are zero.
e.g.: The number 7975 is divisible by 25 as the last two digits are divisible by 25.

• Common Factors:

A common factor of two or more numbers is a number which divides each of them exactly.
e.g.: 3 is a common factor of 6 and 15.

▪ Highest Common Factor (HCF):

Highest common factor of two or more numbers is the greatest number that *divides each of them exactly.*
e.g.: 3, 4, 6, 12 are the factors of 12 and 36. Among them the greatest is 12. Hence the HCF of 12, 36 is 12.
HCF is also called as Greatest common divisor (GCD) or Greatest Common measure (GCM).

Method of Finding HCF: Method of division

▪ HCF of Two Numbers:

Step 1: Greater number is divided by the smaller number.
Step 2: Divisor of step–1 is divided by its remainder.
Step 3: Divisor of step–2 is divided by its remainder. This could be continued until the remainder is 0.
Then HCF = Divisor of the last step.
e.g.: Find the HCF of 96 and 348.
Explanation: Here the divisor of the last step is 12. So, HCF of 96 and 348 is 12.

96)348(3
288
60)96(1
60
36)60(1
36
24)36(1
24
→ 12)24(2
24
0

▪ HCF of More than Two Numbers:

Step 1: Take any two numbers as your wish and find their HCF.
Step 2: Now find the HCF of third number and HCF obtained for the previous two numbers.
Step 3: Now find the HCF of fourth number and HCF obtained in the previous step. Continue the same process till the last number. The final HCF is concluded to be the HCF of all the given numbers.
e.g.: Find the HCF of 120, 246, 100.

120) 246 (2
240
→ 6) 120 (20
120
0

6 is HCF of 120, 246. Now take 3rd number (*i.e.* 100) and HCF obtained in the above step (*i.e.* 6) and find HCF.

6)100(16
96
4)6(1
4
→ 2)4(2
4
0

▪ HCF of Decimals:

e.g.: Find the HCF of 3.2, 4.12, 1.3, 7.
Explanation: First eliminate the influence of decimals by multiplying it either by 10 or 100 or 1000 etc. Here multiply the numbers with 100 to make all the numbers decimal free. *i.e.* 320, 412, 130, 700.
Now, find the HCF of above numbers. We get it as 2. Did you remember we multiplied all the numbers by 100 to eliminate the influence of decimals. Hence, now we divide the answer by 100 to get HCF of the original numbers. The HCF is $\frac{2}{100} = 0.02$

▪ LCM (Least Common Multiple):

Least common multiple of two or more given numbers is the '*least or lowest number*' which is divisible by each of them exactly. In the sense without a non zero remainder.

Method of Finding LCM:

Step-1: Write numbers in a line separated by comma.
Step 2: Divide any two of the given numbers exactly with a least possible prime number then the quotients and the undivided numbers are written in the next line.
Step 3: Repeat the same process till all the numbers in the line are prime to each other *i.e.* no more common factors exist.

Conclusion: The product of all divisors and the numbers in the last line is the LCM of the numbers.
e.g.: Find the LCM of 14, 18, 24, 30.

2	14, 18, 24, 30
3	7, 9, 12, 15
	7, 3, 4, 5

The LCM of 14, 18, 24, 30 = 2×3×7×3×4×5 = 2520.

▪ **LCM of Decimals:** Let us observe an example.
Find the LCM of 1.6, 0.28, 3.2, 4.9.
Explanation: First eliminate the decimals by multiplying with either 10 or 100 or 1000 etc. In this case, multiply all the numbers with 100.
Then numbers will become 160, 28, 320, 490.
Now, find the LCM of the above numbers as earlier.

2	160, 28, 320, 490
2	80, 14, 160, 245
2	40, 7, 80, 245
2	20, 7, 40, 245
2	10, 7, 20, 245
5	5, 7, 10, 245
7	1, 7, 2, 49
	1, 1, 2, 7

LCM 160, 28, 320, 490 = 2×2×2×2×2×5×7×2×7 = 15680.
Did you remember, we have multiplied all the numbers by 100 to eliminate the influence of decimals. Hence, we divide the answer by 100 to get actual LCM of the given numbers. So, the LCM is $\frac{15680}{100} = 156.80$

▪ **Finding LCM and HCF of Fractions:**

$$\text{LCM} = \frac{\text{LCM of the numbers in numerator}}{\text{HCF of the numbers in denominator}}$$

$$\text{HCF} = \frac{\text{HCF of the numbers in numerator}}{\text{LCM of the numbers in denominator}}$$

e.g.: Find the LCM of $\frac{2}{5}, \frac{81}{100}, \frac{125}{302}$.
Explanation: First find the 'LCM of the numerator'.
As there is no common number (prime) which can divide any two of the numbers hence the product itself is the LCM. *i.e.* LCM = 2 × 81 × 125 = 20250.
Now find the 'HCF of the numbers in denominator'.
HCF of 5 and 100 is 5 and HCF of 5 and 302 is 1.
$\therefore$ LCM of the given fractions = $\frac{20250}{1} = 20250$

e.g.: Find the HCF of $\frac{4}{9}, \frac{10}{21}, \frac{20}{63}$
Explanation: HCF of numerators 4, 10 and 20 = 2.
LCM of denominators 9, 21 and 63 = 63.
HCF of the given fractions = $\frac{2}{63}$.

▪ **Key Points on LCM and HCF:**

1) HCF of fractions is always a fraction but LCM of fractions may be a fraction or an integer.
2) The product of any two numbers is equal to product of their LCM and HCF.
e.g.: What is LCM and HCF of 32 and 450 ?
a) 7200, 8 b) 7100, 2 c) 7800, 2 d) 7200, 2
Explanation: Product of 32 and 450 is 14400
The LCM of 32 and 450 is 7200.
The HCF of 32 and 450 is 2.
(or) You can verify from options.
Option-a: $7200 \times 8 \neq 32 \times 450$.
Option-b: $7100 \times 2 \neq 32 \times 450$.
Option-c: $7800 \times 2 \neq 32 \times 450$.
Option-d: $7200 \times 2 = 32 \times 450$.
3) To find the greatest number that will exactly divide x, y and z; Required number = HCF of x, y, z.
4) To find the greatest number that will divide x, y and z leaving remainders a, b and c respectively.
Required number = HCF of $(x–a)$, $(y–b)$ and $(z–c)$.
5) To find the least number which is exactly divisible by x, y and z. Required number = LCM of x, y and z.
6) To find the least number which when divided by x, y, z leaves the remainders a, b, c respectively.
Then it is always observed that,
$(x - a) = (y - b) = (z - c) = \text{K}$ (Assume).
Required number = (LCM of x, y and z) – K.
7) To find the least number which when divided by x, y and z leaves the same remainder 'r' in each case. Required number = (LCM of x, y and z) + r.
8) To find the greatest number that will divide x, y and z leaving the same remainder in each case,
If the value of remainder 'r' is given, then
Required number = HCF of $(x–r)$, $(y–r)$ and $(z–r)$.
If the value of remainder is not given, then
Required number = HCF of $|(x–y)|$, $|(y–z)|$, $|(z–x)|$.

• **Complete Remainder:**
A remainder obtained by dividing a given number by the method of successive division is called complete remainder.

e.g.: A certain number when successively divided by 2, 3 and 5 leave remainders 1, 2 and 4 respectively. What is the complete remainder or remainder when the same number be divided by 30?
Explanation: For example, if a number when divided by 2, leaves remainder 1 would be of form = $2n + 1$.
And a number when divided by 3, leaves remainder 2 would be of form = $3n + 2$.

So, a number when successfully divided by 2, 3, 5 leaves remainder 1, 2, 4 would be of the form = $2[3(5n+4)+2]+1 = 30n+29$.

When $(30n + 29)$ is divisible by 30, the remainder is 29.

1) When there are two divisors d_1, d_2 and two remainders r_1, r_2 the complete remainder is given by $d_1 r_2 + r_1$.

2) When there are three divisors d_1, d_2, d_3 and three remainders r_1, r_2, r_3 the complete remainder is given by $d_1 d_2 r_3 + d_1 r_2 + r_1$.

3) When there are four divisors d_1, d_2, d_3, d_4 and four remainders r_1, r_2, r_3, r_4 the complete remainder is given by $d_1 d_2 d_3 r_4 + d_1 d_2 r_3 + d_1 r_2 + r_1$.

4) In any case if there are no remainders consider them as zeros.

▪ **Fractions and Ordering Fractions:**

1) In the fraction $\frac{3}{4}$; bottom number (denominator) says how many parts the whole is divided into. The top number (the numerator) says how many parts we have.

2) Fraction = $\frac{\text{Numerator}}{\text{Denominator}}$

Such a fraction is known as common fraction.

3) A fraction whose denominator is 10 or 100 or 1000 etc is called a decimal fraction.

4) Fractions whose denominators are same are called like fractions. For example, $\frac{3}{7}, \frac{5}{7}$ are like fractions.

5) Fractions whose denominators are different are called unlike fractions. For example, $\frac{3}{4}, \frac{5}{13}$.

6) When two fractions have the same denominator, the greater fraction is that which has greater numerator.

7) When two fractions have the same numerator, the greater fraction is that which has the smaller denominator.

8) If the identity is not possible, convert the fraction into the convenient form.

e.g.: Arrange $\frac{3}{5}, \frac{13}{16}, \frac{5}{7}, \frac{97}{104}$ in ascending order.

Explanation: LCM of 5, 16, 7, 104 is 7280.

Now multiply the numerator and denominator of the fractions with a number such that the denominator equals 7280.

i.e. $\frac{3\times1456}{5\times1456}, \frac{13\times455}{16\times455}, \frac{5\times1040}{7\times1040}, \frac{97\times70}{104\times70}$

$= \frac{4368}{7280}, \frac{5915}{7280}, \frac{5200}{7280}, \frac{6790}{7280}$.

Now compare the fractions.

$\frac{4368}{7280} < \frac{5200}{7280} < \frac{5915}{7280} < \frac{6790}{7280}$.

The order is $\frac{3}{5} < \frac{5}{7} < \frac{13}{16} < \frac{97}{104}$.

9) To find a fraction that lies between the two fractions $\frac{a}{b}$ and $\frac{x}{y}$, use the formula $\frac{a}{b} < \frac{a+x}{b+y} < \frac{x}{y}$.

e.g.: Find three fractions between $\frac{1}{3}$ and $\frac{4}{5}$.

Explanation: $\frac{1}{3}, \left(\frac{1+4}{3+5}\right), \frac{4}{5} = \frac{1}{3}, \frac{5}{8}, \frac{4}{5}$

Now, find one fraction between $\frac{1}{3}$ and $\frac{5}{8}$ and one fraction between $\frac{5}{8}$ and $\frac{4}{5}$.

$\Rightarrow \frac{1}{3}, \left(\frac{1+5}{3+8}\right), \frac{5}{8}, \left(\frac{5+4}{8+5}\right), \frac{4}{5} \Rightarrow \frac{1}{3}, \frac{6}{11}, \frac{5}{8}, \frac{9}{13}, \frac{4}{5}$

• **Finding the Square Root:**

Division Method: Let us take an example 64516.

```
      2)6 45 16 (254
        4
     45) 245
         225
    50 4)20 16
         2016
            0
```

$\therefore$ Square root of 64516 is 254.

Let us observe the above working rule in words.

Step 1: Group the digits in pairs, starting with the digit in the units place.

Step 2: Think of the largest number whose square is equal to or just less than the first pair. Take this number as the divisor and also as the quotient.

In the given example, largest number whose square is near to 6 is 2 (*i.e.* $2^2 = 4$). So, 2 is the divisor and quotient.

Step 3: Subtract the product of the divisor and the quotient from the first pair and bring down the next pair to the right of the remainder. This becomes the new dividend.

Step 4: Double the quotient and put a blank for a number beside it (*i.e.* 4[?]). Now think of a largest number (for example 5) to fill in the blank in such a way that the product of a new divisor (*i.e.* 45) and this digit (*i.e.* 5) is equal to or less than new dividend (*i.e.* 245).

Step 5: Repeat steps (2), (3) and (4) till all the pairs have been taken up. Now, the quotient so obtained is the required square root of the given number.

Observe another example below.

Square Root of 119716 is 346.

Step 1: Group two digits as pairs. 11, 97, 16.

Step 2: Largest number whose square is near to the 11 is 3. Hence 3 is the divisor and also quotient.

$$3)\ \overline{11}\ \overline{97}\ \overline{16}\ (3$$
$$\underline{9\quad\quad}$$

Step 3: Now 297 is the new dividend.

$$3)\ \overline{11}\ \overline{97}\ \overline{16}\ (3$$
$$\underline{9\quad\quad}$$
$$2\ 97$$

Step 4: Double the quotient 3 *i.e.* 3×2=6 and put a blank for a number beside 6 *i.e.* 6[?]. Now think of a largest number (for example 4) to fill in the blank in such a way that the product of a new divisor (*i.e.* 64) and this digit (*i.e.* 4) is less than or equal to new dividend (*i.e.* 297).

Step 5:

3×2 = 6

$$\begin{array}{r|l} 3) & \overline{11}\ \overline{97}\ \overline{16}\ \ (3\ \underline{4}\ \ \underline{6} \\ & \underline{9\qquad} \\ 6\ \underline{4}) & 297 \\ & \underline{256\quad} \\ 68\ \underline{6}) & 4116 \\ & \underline{4116} \\ & 0 \end{array}$$

34×2 = 68

For this type of questions, it is better to check from options in the exam.

• Key Points on Finding Square Root:

1. A number ending with 2, 3, 7, 8 cannot be a perfect square. The last digit of any perfect square must be any one among 0, 1, 4, 5, 6, 9.

2. A number ending with odd number of zeros can never be a perfect square. e.g.: 1000, 2000 etc.

3. The difference between squares of two consecutive numbers is always an odd number.

e.g.: $4^2–3^2 = 16–9 = 7$ (odd).

4. Finding square root of a decimal fraction:

First eliminate the decimal point by dividing and multiplying with even powers of 10 then find the square root of both numerator and denominator separately and then you can conclude the square root.

e.g.: Find the square root of 1190.25.

$$\sqrt{1190.25}=\sqrt{\frac{1190.25}{10^2}\times10^2}=\frac{\sqrt{119025}}{10^2}=\frac{345}{10}=34.5$$

• Simplification: In simplification we are supposed to follow the order which is essentially demanded by Mathematics and given by a common note of remembrance as VBODMAS.

V – Vinculum (bar $\bar{x}$), B – Bracket ( ) { },
O – of, D – Division (÷), M – Multiplication (×),
A – Addition (+), S – Subtraction (–).

Use of Algebraic Identities: The following algebraic identities will be useful in simplification.

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a – b)^2 = a^2 – 2ab + b^2$
3. $(a + b)^2 + (a – b)^2 = 2(a^2 + b^2)$
4. $(a + b)^2 – (a – b)^2 = 4ab$
5. $a^2 – b^2 = (a + b)\ (a – b)$
6. $(a + b)^3 = a^3 + 3a^2b +3ab^2 + b^3 = a^3 + b^3 + 3ab(a + b)$
7. $(a – b)^3 = a^3 – 3a^2b + 3ab^2 – b^3 = a^3 – b^3 – 3ab(a – b)$
8. $a^3 + b^3 = (a + b)\ (a^2 – ab + b^2)$
9. $a^3 – b^3 = (a – b)\ (a^2 + ab + b^2)$
8. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
10. $a^3+b^3+c^3– 3abc = (a + b + c)(a^2 + b^2 + c^2 – ab – bc – ca)$
11. $a^4 – b^4 = (a^2 + b^2)(a^2 – b^2)$

• Number of Divisors of a Composite Number

If N is a composite number of the form N= $a^p\, b^q\, c^r$. . . . where *a*, *b*, *c* are primes, then the number of divisors of N is given by $(p+1)\ (q+1)\ (r+1)$ . . . . .

e.g.: Let the number be 600.

$$\begin{array}{r|l} 2 & 600 \\ 2 & 300 \\ 2 & 150 \\ 3 & 75 \\ 5 & 25 \\ & 5 \end{array}$$

$600 = 2^3 \times 3^1 \times 5^2$

∴ Number of divisors of 600 = (3+1)(1+1)(2+1) = 24.

In these 24 divisors 1 and the number itself are also included. So, number of divisors of 600 excluding 1 and its self is 24 – 2 = 22.

• Sum of Divisors of a Composite Number :

If *N* is a composite number of the form $a^p\, b^q\, c^r$ . . . . Where *a*, *b*, *c* are primes, then the sum of the divisors, S_N is given by $S_N=\frac{(a^{p+1}-1)(b^{q+1}-1)(c^{r+1}-1)}{(a-1)(b-1)(c-1)}$

e.g.: Let the number be 600. $600 = 2^3 \times 3^1 \times 5^2$

Sum of the divisors $S_N=\frac{(2^{3+1}-1)(3^{1+1}-1)(5^{2+1}-1)}{(2-1)(3-1)(5-1)}$

$$\Rightarrow\frac{(16-1)(9-1)(125-1)}{(1)(2)(4)}\Rightarrow\frac{(15)\times(8)\times(124)}{(1)\times(2)\times(4)}=1860$$

- **Important Key Points:**

1) Sum of natural numbers from 1 to n = $\frac{n(n+1)}{2}$.

2) Sum of squares of first n natural numbers = $\frac{n(n+1)(2n+1)}{6}$.

3) Sum of cubes of first n natural numbers $\left[\frac{n(n+1)}{2}\right]^2$

4) Number of odd numbers from 1 to n = $\frac{\text{Last Odd Number}+1}{2}$.

5) Number of even numbers from 1 to n = $\frac{\text{Last Even Number}}{2}$.

6) Sum of even numbers from 1 to n is $k(k+1)$, where k indicates number of even numbers from 1 to n.

e.g.: Sum of even no from 1 to 80 = 40(40+1) = 1640.

Here from 1 to 80 there exists 40 even numbers.

7) Sum of odd numbers from 1 to $n = k^2$, where k is equal to number of odd numbers from 1 to n.

e.g.: Sum of odd numbers from 1 to 60 is $(30)^2 = 900$.

30 odd natural numbers exist from 1 to 60.

8) Sum of the squares of first 'n' even natural numbers = $\frac{2}{3}(n)(n+1)(2n+1)$.

9) Sum of squares of first 'n' odd natural numbers is $\frac{n(2n+1)(2n-1)}{3}$.

10) Sum of any 5 consecutive whole numbers will always be divisible by 5.

e.g.: (3 + 4 + 5 + 6 + 7) = 25 is divisible by 5.

11) XY – YX; The difference between a two digit number and its reverse is divisible 9.

e.g.: Let the two numbers be 95 and 59. Here 59 is reverse of 95. Now 95 – 59 = 36 (which is divisible by 9).

12) Products: odd × odd = odd;
odd × even = even;
even × even = even;

13) $n! = n(n-1)(n-2)(n-3)\ldots\ldots(3)(2)(1)$.

e.g.: 6! = 6 × 5 × 4 × 3 × 2 × 1 = 720.

The product of any 'r' consecutive integers is divisible by (r!)

14) Finding the units digit of the numbers like $(252)^{54}$.

Here the units digit of 252 is 2 and the index is 54. We know that $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$. Here units digit is repeated after each 4 indices. So divide 54 by 4 to get the remainder. Here the remainder is 2.

Hence the last digit in $(252)^{54}$ is same as 2^2 *i.e.* 4.

CONCEPTUAL EXAMPLES

1) Find the sum of first 20 multiples of 12.

a) 1830 b) 2520 c) 3494 d) None

Explanation: Sum of first 20 multiples of 12 are

= (12×1) + (12×2) + (12×3) + ... + (12×19) + (12×20).

= 12 (1+2+3+ . . . . . + 20)

Use the formula: $\frac{n(n+1)}{2} \Rightarrow \frac{12\times(20\times21)}{2} = 2520$.

Ask doubt with Question Id: 1647

2) The smallest number which when added to 4, the sum is exactly divisible by 24, 36, 48 and 60 is:

a) 700 b) 716 c) 720 d) 730

Explanation:

2	24	36	48	60
2	12	18	24	30
3	6	9	12	15
2	2	3	4	5
	1	3	2	5

∴ LCM of 24, 36, 48, 60 = 2 × 2 × 3 × 2 × 3 × 2 × 5 = 720.

∴ Required number = 716

Ask doubt with Question Id: 1648

3) Mr. Srinivas saves one coin of ₹5 on first day of the week, three coins of ₹5 on the second day of the week. Five coins of ₹5 on third day and so on. How much money will he has at the end of the week?

a) 78 b) 125 c) 245 d) 289

Explanation: Number of ₹5 coins with him at the end of week = 5 × (1 + 3 + 5 + 7 + 9 + 11 + 13)

= 5 × (sum of first 7 odd numbers)

(By using the formula discussed earlier).

∴ Sum of all numbers = $5 \times 7^2 = 245$.

Ask doubt with Question Id: 1649

4) Number of divisors of 22050 except 1 and itself is.

a) 24 b) 28 c) 36 d) 52

Explanation:

2	22050
3	11025
3	3675
5	1225
5	245
7	49
	7

$\Rightarrow 22050 = 2^1 \times 3^2 \times 5^2 \times 7^2$

Using the formula discussed earlier,

∴ Number of divisors = (1+1) (2+1) (2+1) (2+1) = 54

∴ Number of divisors except 1 and itself = 54 – 2 = 52.

Ask doubt with Question Id: 1650

5) $\sqrt{12+\sqrt{12+\sqrt{12+.......\ \infty \text{ terms}}}}=?$

a) 2 b) 3 c) 4 d) 5

Explanation: Let the given expression = x

Then, we can write $\sqrt{12+x}=x \Rightarrow 12+x=x^2$

$\therefore x^2 - x - 12 = 0 \Rightarrow (x-4)(x+3) = 0$

So, $x = +4$ (x cannot be negative since $\sqrt{12}=3.46$).

Ask doubt with Question Id: 1651

6) A Mango seller saves two coins of ₹2 on first day of the week, four coins of ₹2 on the second day of the week. Six coins of ₹2 on third day and so on. The total amount saved by him at the end of the week is

a) 246 b) 112 c) 88 d) None

Explanation: Number of ₹2 coins saved by him at the end of the week = 2×(2+4+6+8+10+12+14)

Sum of first seven even numbers = $k(k+1)$ = 7×8 = 56

where k = number of even numbers.

Sum of the money with him = 56×2 = ₹112.

Ask doubt with Question Id: 1652

7) The sum of all the odd numbers starting from 1 and ending at the greatest number of three digits is.

a) 500 b) 5000 c) 2500 d) 250000

Explanation: Greatest number of three digits = 999.

There are 500 odd numbers from 1 to 999.

$\therefore$ Sum of first 500 odd numbers = $(500)^2$ = 250000.

Ask doubt with Question Id: 1653

8) What is the number whose eleventh part multiplied by its fifth part gives 2695.

a) 385 b) 434 c) 560 d) 583

Explanation:

Let x be the required number. Then $\frac{x}{11}\times\frac{x}{5}=2695$

$x^2 = 11\times5\times2695 \Rightarrow x^2 = 11\times5\times5\times7\times7\times11 \Rightarrow x = 5\times7\times11 = 385$

Alternate Method: To solve by options.

Ask doubt with Question Id: 1655

9) What least number must be added to the least number of six digits so that the resulting number may be a perfect square.

a) 283 b) 344 c) 489 d) 523

Explanation: The least number of six digits = 100000

3	10 00 00	317
	9	
61	100	
	61	
627	3900	
	4389	
	– 489	

Least number to be added = 489.

Alternative method is to solve by options.

Ask doubt with Question Id: 1654

10) What is the least number of cut pieces of equal length that can be cut out of two lengths 10 meters 857 millimeters and 15 meters 87 millimeters.

a) 78 b) 184 c) 232 d) None

Explanation: Here, you need to find the HCF. Because, to get least number of equal cut pieces you should cut as big as possible. So, HCF of 10857 and 15087 = 141. (This is the size of each cut piece). Then the number of cut pieces = (10857 + 15087) ÷ 141 = 184.

Ask doubt with Question Id: 1656

11) A certain number when successively divided by 3, 4 and 5 leaves remainders 1, 2 and 3 respectively. When the same number be divided by 4, what is the remainder?

a) 3 b) 4 c) 5 d) 6

Explanation: Let p be the certain number and q, r, s are successive quotients upon successive division. Given,

$\frac{p}{3}=q$, Remainder 1; $\Rightarrow p = 3q+1$ .... (I)

$\frac{q}{4}=r$, remainder 2; $\Rightarrow q = 4r+2$ .... (II)

$\frac{r}{5}=s$, Remainder 3; $\Rightarrow r = 5s+3$ .... (III)

From (I) and (II), $p = 3(4r+2)+1 \Rightarrow p = 12r+7$ .... (IV)

From (IV) and (III), $p = 12(5s+3)+7 \Rightarrow p = 60s+43$.

Now question asked for remainder if p is divided by 4.

i.e. $\frac{60s+43}{4}$.

Observe carefully when $60s$ is divided by 4, it leaves remainder 0, but when 43 is divided by 4 it leaves remainder 3.

So when $60s+43$ is divided by 4 it leaves remainder 3.

Alternate Method-1:

So, a number when successfully divided by 3, 4, 5 leaves remainder 1, 2, 3 would be of the form = $3[4(5n+3)+2]+1$ = $60n+43$.

Now, if $60n+43$ is divided by 4, the remainder is 3.

• **Alternate Method-2: Formula Approach**.

As discussed earlier, when there are three divisors d_1, d_2, d_3 and three remainders r_1, r_2, r_3 the complete remainder is given by $d_1 d_2 r_3 + d_1 r_2 + r_1$.

By applying above formula, we get,

(3)(4)(3) + (3)(2) + (1) = 36 + 6 + 1 = 43.

When 43 is divided by 4 the remainder is 3.

Ask doubt with Question Id: 1657

12) Find the greatest number of four digits which is perfectly divisible by 3 and when divided by 5, 7 and 9 leaves a remainder 3 in each case.

a) 9985 b) 9960 c) 9768 d) 9660

Explanation: LCM of 3, 5, 7, 9 = 315.

Greatest number of four digits which is divisible by 315 is 9765. The required number = 9765 + 3 = 9768.

Ask doubt with Question Id: 1658

13) Find greatest number of four digits which when increased by 3568 is exactly divisible by 6, 8, 12, 20.

a) 9992 b) 9785 c) 9840 d) None

Explanation: Greatest number of 4 digits = 9999.

9999 + 3568 = 13567. LCM of 6, 8, 12 and 20 is 120.

$$\begin{array}{l} 120)13567(113 \\ \quad \underline{13560} \\ \quad\quad 7 \end{array}$$

The remainder is 7.

$\therefore$ Required number of 4 digits = 9999 – 7 = 9992

i.e. 9992 is the number to which if 3568 is added, then it is exactly divisible by 6, 8, 12, 20.

Ask doubt with Question Id: 1659

14) Find the greatest number which is such that when 794, 858 and 1351 are divided by it, the remainders are all same.

a) 35 b) 21 c) 14 d) 1

Explanation: Given, the remainders are same *i.e.* differences of that numbers are exactly divisible.

Hence, you have to find HCF (x–y, y–z, z–x).

858–794 = 64; 1351–794 = 557; 1351–858 = 493.

HCF of (64, 557, 493) = 1.

Ask doubt with Question Id: 1660

15) Find the greatest number of five digits to which if 7143 is added, the final number becomes exactly divisible by 18, 24, 30, 32 and 36.

a) 99846 b) 99682 c) 99417 d) None

Explanation: LCM of 18, 24, 30, 32 and 36 is 1440.

99999 + 7143 = 107142.

Dividing 107142 by 1440, the remainder is 582.

$\therefore$ Required number = 99999 – 582 = 99417

Alternate Method: Cross check with options.

Ask doubt with Question Id: 1662

16) Two numbers are in the ratio of 11:13. If the HCF of these numbers is 19, determine those numbers.

a) 304, 369 b) 209, 247 c) 182, 199 d) None

Explanation: Let the numbers be $11x$ and $13x$.

Since the HCF of given numbers is 19 which indicates that 19 is the common factor of these two numbers. Hence, it is obvious that value of x is 19.

$\therefore$ The numbers are 209 and 247 respectively.

Ask doubt with Question Id: 1665

17) Find the least number which when divided by 6, 9, 14, 21 and 24 leaves 1, 4, 9, 16 and 19 as remainders respectively, but when divided by 17 leaves no remainder.

a) 2425 b) 1895 c) 1003 d) 944

Explanation: LCM of 6, 9, 14, 21 and 24 is 504.

Required number = (504 k–5) which is divisible by 17 for the least value of k.

$(504k - 5) = (493k + 11k - 5) = (17\times29)k + (11k-5)$

Let $k = 1 \Rightarrow (11\times1 - 5) = 7$ (not divisible by 17)

Let $k = 2 \Rightarrow (11\times2 - 5) = 17$ (divisible by 17)

$\therefore (17\times29)k + (11k-5)$ is exactly divisible by 17 for $k = 2$.

$\therefore$ Required Number = $(504)k - 5 = (504\times2) - 5 = 1003$.

Ask doubt with Question Id: 1663

18) I collected some money by raising subscription for opening a society. If the whole amount collected by 720 currency notes of ₹1000 denomination and each person subscribed as many rupees as twice the number of subscribers. Then find the number of subscribers.

a) 500 b) 550 c) 600 d) 650

Explanation: Total amount collected =720×1000 =720000.

Let there be x subscribers so that each paid ₹ $2x$.

Total amount collected = (Number of subscribers) × (Amount paid by each subscriber).

$x\times2x = 720000 \Rightarrow 2(x^2) = 720000 \Rightarrow x^2 = 360000 \Rightarrow x = 600$

Ask doubt with Question Id: 1664

19) 3 bells commence tolling together and toll at intervals of 4, 7 and 14 seconds respectively. At which of the following time they might toll together?

a) 30 *sec* b) 78 *sec* c) 84 *sec* d) 92 *sec*

Explanation: Calculate LCM for time of tolling together.

LCM of 4, 7, 14 = 28 *sec*.

These 3 balls toll together after every 28 *sec*.

$\therefore$ By checking with options, 84 is divisible by 28.

$\therefore$ They will toll together at 84^{th} *sec*.

Ask doubt with Question Id: 7699

20) What is the HCF of the fractions $\frac{6}{10}, \frac{9}{24}, \frac{15}{20}$.

a) $\frac{1}{120}$ b) $\frac{4}{120}$ c) $\frac{120}{3}$ d) $\frac{3}{120}$

Explanation: HCF of fraction = $\dfrac{\text{HCF of numerator}}{\text{LCM of denominator}}$

$$= \frac{\text{HCF}(6,9,15)}{\text{LCM}(10,24,20)} = \frac{3}{120}$$

Ask doubt with Question Id: 7700

EXERCISE

1) Find the greatest number of five digits which is a perfect square.
a) 99225 b) 99856 c) Both a, b d) None

2) Simplify $\frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2}\text{ of }\frac{16}{3}+\left\{11-\left(3-\left(\frac{5}{4}-\frac{5}{8}\right)\right)\right\}\right]$
a) $-\frac{11}{120}$ b) $-\frac{21}{120}$ c) $-\frac{31}{120}$ d) None

3) Find the LCM of the fractions $\frac{108}{375},\frac{42}{25},\frac{54}{55}$.
a) $\frac{756}{5}$ b) $\frac{326}{5}$ c) $\frac{434}{5}$ d) $\frac{282}{5}$

4) Sum of three numbers is 132. First number is twice the second and third number is $\frac{1}{3}$ of the first. Find the second number.
a) 12 b) 24 c) 36 d) 42

5) If $\sqrt{1+x}=\frac{13}{12}$ then the value of x is.
a) $\frac{9}{144}$ b) $\frac{16}{144}$ c) $\frac{25}{144}$ d) $\frac{36}{144}$

6) Five bells begin to toll together and they toll at an interval of 36, 45, 72, 81 and 108 seconds. After what interval of time they will keep on tolling together?
a) 3240 *sec* b) 3080 *sec* c) 3140 *sec* d) 3200 *sec*

7) The least perfect square number which is exactly divisible by 4, 6, 8, 10 or 12 is
a) 9260 b) 7921 c) 5625 d) 3600

8) Each student in a class contributed as many paise as the number of students in the class, the teacher contributed ₹13, the total collection is of ₹ 49. How many students were there in the class?
a) 48 b) 60 c) 72 d) None

9) The sum of square of two numbers is 80 and the square of their difference is 36. The product of the two numbers is.
a) 22 b) 34 c) 42 d) 51

10) Find the greatest number that will divide 148, 246 and 623 leaving remainders 4, 6 and 11 respectively.
a) 11 b) 12 c) 13 d) 14

11) Find the least number which when divided by 36, 48 and 64 leaves the remainders 25, 37 and 53 respectively
a) 656 b) 563 c) 565 d) 657

12) $\sqrt{3\sqrt{3\sqrt{3\sqrt{3\sqrt{3}}}}}=?$
a) $3^{\frac{31}{64}}$ b) $3^{\frac{31}{32}}$ c) $3^{\frac{1}{64}}$ d) None

13) The HCF of two numbers is 16 and their LCM is 160. If one number is 32, then other number is.
a) 48 b) 80 c) 96 d) 112

14) Find the size of the largest square slabs which can be paved on the floor of a room 5 meters 44 *cm* long and 3 meters 74 *cm* broad.
a) 56 b) 42 c) 38 d) 34

15) Least number that must be added to 8492 such that the resulting number may be divisible by 72 is.
a) 68 b) 25 c) 11 d) 4

16) The LCM of two numbers is 1950 and their HCF is 65. If one of the number is 195, find the other number.
a) 398 b) 650 c) 792 d) None

17) Find the greatest number that will divide 532, 894 and 1003 leaving remainders 22, 44 and 68 respectively.
a) 85 b) 105 c) 90 d) 95

18) A biscuit dealer has 378 kgs, 434 kgs and 582 kgs of three different qualities of biscuits. He wants it all to be packed into boxes of equal size without mixing. Find the capacity of the largest possible box.
a) 5 kg b) 3 kg c) 2 kg d) 1 kg

19) Find the least number which when divided by 35 leaves remainder 25, when divided by 25 leaves remainder 15, when divided by 15 leaves remainder 5.
a) 420 b) 515 c) 435 d) 518

20) Find the least number which when increased by 4 is divisible by 21, 25, 27 and 35.
a) 4721 b) 4725 c) 4758 d) 2418

21) The product of two numbers is 211428 and their LCM is 3356. Find their GCM?
a) 72 b) 48 c) 36 d) 63

22) Find the least number for which when 5046 is divided or multiplied, becomes a perfect square.
a) 25 b) 15 c) 10 d) 6

23) Find the smallest number between 450 to 550 which is exactly divisible by 7, 8 and 14.
a) 454 b) 482 c) 504 d) 546

24) Three bells ring at an interval of 10, 12 and 14 seconds respectively. They ring together at 11:00 then at what time they ring together again.
a) 12 hours 12 *min* 12 sec b) 11 hours 7 *min*
c) 11 hours 35 *min* d) 10 hours 45 *min*

25) Sum of 4 consecutive natural numbers each divisible by 5 is 130. What is the greatest number?
a) 35 b) 40 c) 45 d) 50

26) The smallest number on being divided by 3, 4, 6, 10 and 16 leaves 9 as remainder in each case but is completely divisible by 9. What is that number?
a) 720 b) 729 c) 846 d) None

27) Two numbers 2035 and 2880 when divided by a certain number of three digits, leaves the same remainder. Find the number.

a) 271 b) 293 c) 169 d) 421

28) If a boy saves ₹ 1 on day-1, ₹2 on day-2, ₹3 on day-3 and so on. Then in how many days will he have ₹36?

a) 5 b) 6 c) 7 d) 8

29) The last digit of a number 49825# is missing. It is also given that the number is divisible by 8. Find the digit at unit's place.

a) 4 b) 6 c) 8 d) 0

EXPLANATIONS

1) The greatest number of 5 digits = 99999.

3	9 99 99	3 1 6
	9	
61	0 99	
	61	
626	3899	
	3756	
	143	

$\therefore$ Required number = 99999 – 143 = 99856.

Ask doubt with Question Id: 1667

2) Applying VBODMAS rule,

$$= \frac{17}{2} - \left[\frac{16}{5} \div \frac{9}{2} \text{ of } \frac{16}{3} + \left\{11 - \left(3 - \frac{5}{8}\right)\right\}\right]$$

$$= \frac{17}{2} - \left[\frac{16}{5} \div \frac{9}{2} \text{ of } \frac{16}{3} + \left\{11 - \frac{19}{8}\right\}\right]$$

$$= \frac{17}{2} - \left[\frac{16}{5} \div \frac{9}{2} \text{ of } \frac{16}{3} + \frac{69}{8}\right] = \frac{17}{2} - \left[\frac{16}{5} \div \frac{9}{2} \times \frac{16}{3} + \frac{69}{8}\right] =$$

$$\frac{17}{2} - \left[\frac{16}{5} \div \frac{24}{1} + \frac{69}{8}\right] = \frac{17}{2} - \left[\frac{16}{120} + \frac{69}{8}\right]$$

$$= \frac{17}{2} - \left[\frac{16+1035}{120}\right] = \frac{17}{2} - \frac{1051}{120} = \frac{1020-1051}{120} = \frac{-31}{120}$$

Ask doubt with Question Id: 1668

3) $\frac{108}{375}$ can be minimized to $\frac{36}{125}$.

$$\text{LCM} = \frac{\text{LCM of } 36, 42, 54}{\text{HCF of } 125, 25, 55} = \frac{756}{5}.$$

Ask doubt with Question Id: 1669

4) Let the second number be $3x$, so that the first number is $6x$ and the third number is $2x$.

$\therefore 6x + 3x + 2x = 132 \Rightarrow x = 12.$

Second number = $3x = 3\times12 = 36$.

Ask doubt with Question Id: 1670

5) $1 + x = \frac{169}{144} \Rightarrow x = \frac{169}{144} - 1 = \frac{25}{144}$

Ask doubt with Question Id: 1671

6) The interval of time is the LCM of the numbers.

2	36, 45, 72, 81, 108
3	18, 45, 72, 81, 54
2	6, 15, 24, 27, 18
3	3, 15, 12, 27, 9
3	1, 5, 4, 9, 3
	1, 5, 4, 3, 1

Ask doubt with Question Id: 1672

7) LCM of 4, 6, 8, 10, 12 = 120.

120 can be written as 2×2×2×3×5

To make it a perfect square, you have to multiply by 2×3×5. If you can see in the factors that 2×2×2×3×5 = 120 can not make a perfect square until we multiply it by 2 to make 2×2×2×2 and by 3 to make 3×3 and by 5 to make 5×5. Now all the numbers are squares.

i.e. $4^2 \times 3^2 \times 5^2 = (4\times3\times5)^2 = 60^2 = 3600.$

Ask doubt with Question Id: 1673

8) Let x be the number of students so that each contributed x paise.

Contribution of the students = 49–13 = ₹36 = 3600 paise.

$\Rightarrow x^2 = 3600 \Rightarrow x = 60.$

$\therefore$ Number of students in the class is 60.

Ask doubt with Question Id: 1674

9) Let the number be x and y, it is required to find $x \times y$.

$x^2 + y^2 = 80$ and $(x - y)^2 = 36$

Now $(x - y)^2 = (x^2 + y^2) - 2xy$

$2xy = (x^2 + y^2) - (x - y)^2 = 80 - 36 = 44$ then $xy = 22$.

Ask doubt with Question Id: 1675

10) Required number = HCF (148–4), (246–6), (623 – 11)

= HCF of 144, 240 and 612 = 12.

Ask doubt with Question Id: 1676

11) Since (36 – 25) = (48 – 37) = (64 – 53) = 11

$\therefore$ Required smallest number = (LCM of 36, 48, 64)–11

= 576 – 11 = 565.

Ask doubt with Question Id: 1677

12) $\sqrt{3\sqrt{3\sqrt{3\sqrt{3\sqrt{3}}}}} \Rightarrow \sqrt{3\sqrt{3\sqrt{3\sqrt{3.3^{\frac{1}{2}}}}}}$

$\Rightarrow \sqrt{3\sqrt{3\sqrt{3\sqrt{3^{\frac{3}{2}}}}}} \Rightarrow \sqrt{3\sqrt{3\sqrt{3.3^{\frac{3}{2}\times\frac{1}{2}}}}} \Rightarrow \sqrt{3\sqrt{3\sqrt{3.3^{\frac{3}{4}}}}}$

$\Rightarrow \sqrt{3\sqrt{3\sqrt{3^{\frac{7}{4}}}}} \Rightarrow \sqrt{3\sqrt{3.3^{\frac{7}{8}}}} \Rightarrow \sqrt{3\sqrt{3^{\frac{15}{8}}}} \Rightarrow \sqrt{3.3^{\frac{15}{16}}}$

$\Rightarrow \sqrt{3^{\frac{31}{16}}} \Rightarrow 3^{\frac{31}{32}}$

Ask doubt with Question Id: 1678

13) Product of numbers = HCF × LCM

$32 \times K = 16 \times 160 \Rightarrow K = 80.$

Ask doubt with Question Id: 1679

14) 5 meters 44 *cm* = 544 *cm;*
3 meters 74 *cm* = 374 *cm*
The side of the square slab = HCF of 544, 374 = 34.
Ask doubt with Question Id: 1680
15) Divide 8492 by 72, the remainder is 68.
∴ Least number to be added = 72 – 68 = 4.
Ask doubt with Question Id: 1681
16) $\frac{\text{HCF}\times\text{LCM}}{\text{Given number}}=\frac{65\times1950}{195}=650$
Ask doubt with Question Id: 1682
17) 532–22 = 510; 894–44 = 850; 1003–68 = 935;
HCF of 510 and 850 is 170. HCF of 170 and 935 is 85.
Ask doubt with Question Id: 1683
18) The capacity of the largest possible box =
HCF (378, 434, 582) = 2.
Ask doubt with Question Id: 1684
19) Here 35–25 = 25–15 = 15–5 = 10
Required number = (LCM of 35, 25, 15) – 10.
= 525 – 10 = 515.
Ask doubt with Question Id: 1685
20) LCM of 21, 25, 27, 35 = 4725
∴ Required number = 4725 – 4 = 4721.
Ask doubt with Question Id: 1686
21) GCM × LCM = Product of the two numbers

$$\text{GCM}=\frac{211428}{3356}=63$$

Ask doubt with Question Id: 1687
22) 5046 = 6 × 29 × 29.
Hence 5046 must be multiplied or divided by 6 to make it a perfect square. If you multiply by 6 it becomes $(6\times29)^2$ which is a perfect square (or) if you divide by 6 it becomes $(29)^2$ which is also a perfect square.
Ask doubt with Question Id: 1688
23) We have to find the least common multiple of 7, 8, 14 that lies between 450 and 550.

2	7, 8, 14
7	7, 4, 7
	1, 4, 1

LCM of 7, 8, 14 = 56.
∴ Required number =504 which is exactly divisible by 56.
Ask doubt with Question Id: 1690
24)

2	10, 12, 14
	5, 6, 7

∴ LCM = 2 × 5 × 6 × 7 = 420 *sec* = 7 minutes
i.e. They ring together again at 11 hours 7 *min.*
Ask doubt with Question Id: 1691

25) Let the 4 consecutive numbers divisible by 5 are x, $x+5$, $x+10$, $x+15$.
∴ $x + (x+5) + (x+10) + (x+15) = 130 \Rightarrow x = 25$
Largest number = $(x+15)$ = 25 + 15 = 40.
Ask doubt with Question Id: 7701
26) LCM of 3, 4, 6, 10 and 16 = 240.
Hence the smallest number which when divided by 3, 4, 6, 10 and 16 leaves 9 as remainder in each case will be 240 + 9 = 249.
But 249 is not completely divisible by 9.
Hence on considering (240×2+9), (240×3+9) . . . . . . ,
By trial and error we conclude that 240×3+9 = 729 is the required number.
Ask doubt with Question Id: 1666
27) There is a rule: When two numbers divided by a third number leaves the same remainder, the difference of the two numbers is also divisible by the third number.
Difference = 2880 – 2035 = 845 = 5×169.
Hence, 169 is the required number of three digits.
Ask doubt with Question Id: 1661
28) ₹ 1 + ₹ 2 + ₹ 3 + ... + ₹ n = 36

$$\frac{n(n+1)}{2}=36 \Rightarrow n^2+n=72 \Rightarrow n^2+n-72=0$$

$$n^2+9n-8n-72=0 \Rightarrow (n+9)(n-8)=0$$

$$n=-9 \text{ (or) } n=8$$

Days cannot be negative, hence n = 8.
Ask doubt with Question Id: 7702
29) A number is divisible by 8, if the number formed by last 3 digits of it is divisible by 8.
Last 3 digits of given number = 25#. From the given options only 256 is perfectly divisible by 8.
Ask doubt with Question Id: 7703

RATIO – PROPORTION

CONCEPTS

Ratio: A ratio is the relation between two quantities which is expressed by a fraction.

• The ratio of the number *'a'* to the number *'b'* is written as $\frac{a}{b}$ (or) $a : b$ or a to b

e.g.: The ratio of 5 hours to 3 hours can be written as $\frac{5}{3}(\text{or})5:3$.

• The ratio is always a comparison between the quantities of same kind or of same units.

For example, you cannot form the ratio between 5 hours and 3 days. Because the two numbers are expressed in different units. Hence, convert 3 days to hours.

i.e. 3 days = 72 hours. Thus the proper form of this ratio is $\frac{5}{72}$ (or) 5:72.

• Two quantities which are being compared ($a : b$) are called its terms. The first term (a) is called *antecedent* and second term (b) is called *consequent.*

• The ratio of two quantities is always an abstract number (without any units).

• If the terms of a ratio are multiplied or divided by the same quantity the value of the ratio remains unaltered.

e.g.: The ratio $a : b$ is same as $Ma : Mb$.

Proportion: Equality of two ratios is called proportion. Consider the two ratios, $a : b$ and $c : d$, then proportion is written as,

$a : b :: c : d$ (or) $a : b = c : d$ (or) $\frac{a}{b}=\frac{c}{d}$

Here *a, b, c, d* are called *Terms. a, d* are called *Extremes* (end terms) and *b, c* are called *Means* (middle terms).

e.g.: Since the ratio $4:20$ (or) $\frac{4}{20}$ is equal to the ratio $1:5(\text{or})\frac{1}{5}$ we may write the proportion as 4 : 20 :: 1 : 5 or 4 : 20 = 1 : 5 or $\frac{4}{20}=\frac{1}{5}$

• In a proportion, product of *means* (middle terms) is equal to product of *extremes* (end terms).

i.e. $ad=bc$ or $\frac{a}{b}=\frac{c}{d}$.

Key Notes: If a and b are two quantities, then

1) Duplicate ratio of $a:b=a^2:b^2$

2) Sub-duplicate ratio of $a:b=\sqrt{a}:\sqrt{b}$

3) Triplicate ratio of $a:b=a^3:b^3$

4) Sub-triplicate ratio $a:b=\sqrt[3]{a}:\sqrt[3]{b}$

5) Inverse or reciprocal ratio of $a:b=\frac{1}{a}:\frac{1}{b}$

6) Third proportional to a and b is $\frac{b^2}{a}$.

7) If $a : b = x : y$ and $b : c = p : q$, then

a) $a:c=\frac{x\times p}{y\times q}$

b) $a : b : c = px : py : qy$

8) Compound Ratio of ($a : b$), ($c : d$), ($e : f$) is $\frac{a}{b}\times\frac{c}{d}\times\frac{e}{f}$.

9) The ratio in which two kinds of substances must be mixed together one at ₹ x per kg and another at ₹ y per kg, so that the mixture may cost ₹ n per kg. The ratio is $\frac{n-y}{x-n}$.

10) Let the incomes of two persons be in the ratio of $a : b$ and their expenditure be in the ratio of $x : y$ and if the savings of each person is ₹ n then income of each is ₹$\frac{an(y-x)}{ay-bx}$ and ₹$\frac{bn(y-x)}{ay-bx}$ respectively.

11) In a mixture the ratio of milk and water is $a : b$. In this mixture another n liters of water is added, then the ratio of milk and water in the resulting mixture became $a : m$. Then, the quantity of milk in the original mixture = $\frac{an}{m-b}$ and the quantity of water in the original mixture = $\frac{bn}{m-b}$

12) In a mixture of n liters, the ratio of milk and water is $x : y$. If another m liters of water is added to the mixture, the ratio of milk and water in the resulting mixture = $xn:(yn+mx+my)$

13) If four numbers a, b, c and d are given then

a) $\frac{ad-bc}{(b+c)-(a+d)}$ should be added to each of these numbers so that the resulting numbers may be proportional.

b) $\frac{ad-bc}{(a+d)-(b+c)}$ should be subtracted from each of these numbers so that the resulting numbers may be proportional.

CHAIN RULE / VARIATION

CONCEPTS

What is Chain Rule or Variation:

Variations deal with, how one quantity changes with respect to one or more other quantities. Basically there are two types of variations: Direct variation and Indirect variation.

Direct Variation: Suppose that a painter charges ₹100 to paint a room. The below table shows the relationship between the number of rooms painted and the cost of the total job for 1 through 5 rooms.

Number of Rooms	Cost of the Job
1	₹ 100
2	₹ 200
3	₹ 300
4	₹ 400
5	₹ 500

From the above table we observe that as the number of rooms *increase*, cost of the job also *increases* and vice versa. There is a *direct variation* between these two quantities. It means these two quantities are *directly proportional* to each other.

• If the two quantities 'x' and 'y' are directly proportional to each other, then $x = k\,y$ (or) $\frac{x_1}{x_2} = \frac{y_1}{y_2}$.

e.g.: If 5 computers costs ₹ 275, how much would 18 computers cost?

Explanation: More number of computers : More cost
Less number of computers : Less cost

The two quantities, *computers* and *cost* are directly proportional to each other.

Computers	Cost
5 (x_1)	275 (y_1)
18 (x_2)	? (y_2)

$$\frac{x_1}{x_2} = \frac{y_1}{y_2} \Rightarrow \frac{5}{18} = \frac{275}{x} \Rightarrow x = 990.$$

(or) $x = k\,y \Rightarrow 5 = 275\,k \Rightarrow k = \frac{1}{55}$

$$18 = k\,y \Rightarrow 18 = \frac{1}{55}y \Rightarrow y = 990.$$

Indirect Variation: When two variables or quantities change in opposite directions, you have inverse variation.

The below table shows the relationship between the number of persons and number of days required to complete a work.

Persons	Days
1	120
2	60
3	20
4	5
5	1

If the number of persons *increase*, the days required to complete the work will *decrease*. There is an *indirect variation* between these two quantities. It means these two quantities are *inversely proportional* to each other.

• If the two quantities x, y are indirectly proportional to each other, then $x = \frac{k}{y}$ (or) $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

e.g.: There are 6 workers to paint a house. They typically paint the house in 8 hours. If 4 workers are not came to work today, how long will it take the remaining workers to paint the house.

Explanation: If there are *more workers*, it takes *less days* to complete the work. These two quantities are indirectly proportional each other.

Workers	Hours
6 (x_1)	8 (y_1)
2 (x_2)	? (y_2)

$$\frac{x_1}{x_2} = \frac{y_2}{y_1} \Rightarrow \frac{6}{2} = \frac{x}{8} \Rightarrow x = 24.$$

Combined Variation:

It involves both direct and indirect variation.

• If 'x' varies directly with 'y' and indirectly with 'z', then the general form of the combined variation is

$$x = k\frac{y}{z} \quad \text{or} \quad \frac{x_1}{x_2} = \frac{y_1}{y_2} \times \frac{z_2}{z_1}.$$

e.g.: If 300 men can complete a work in 16 days, how many men would do $\frac{1}{5}$ of the work in 15 days?

Explanation:

Men	Work	Days
300 (x_1)	1 (y_1)	16 (z_1)
x (x_2)	$\frac{1}{5}$ (y_2)	15 (z_2)

Compare *men* with *work* and *days*.

More men can do *more work*. (*Direct Variation*)

If there are *more men*, it takes *less days* to complete the work. (*Indirect Variation*)

Hence, It is a *Combined Variation*: $\frac{x_1}{x_2} = \frac{y_1}{y_2} \times \frac{z_2}{z_1}$.

$$\frac{300}{x} = \frac{1}{1/5} \times \frac{15}{16} \Rightarrow \frac{4}{x} = \frac{1}{16} \Rightarrow x = 64.$$

CONCEPTUAL EXAMPLES

1) 20 men complete one-fourth of a piece of work in 10 days. How many more men should be employed to finish the remaining work in 15 more days?

a) 32 b) 40 c) 20 d) 12

Explanation:

Work done= $\frac{1}{4}$; Work to be done = $1-\frac{1}{4}=\frac{3}{4}$

Let the required number of men = x

Work	Days	Men
$\frac{1}{4}$	10	20
$\frac{3}{4}$	15	x

Compare *men* with *work* and *days*.

Men and *work* are directly proportional.

Men and *days* are indirectly proportional.

It is a combined variation. *i.e.* $\frac{20}{x}=\frac{1/4}{3/4}\times\frac{15}{10}$ $\Rightarrow x = 40$

Total 40 men. Given that, 20 men are already employed, hence 20 more men are required.

Ask doubt with Question Id: 1355

2) If 5 examiners can examine a certain number of answer books in 6 days by working 4 hours a day; for how many hours a day would 6 examiners have to work in order to examine thrice the number of answer books in 10 days?

a) 4 b) 6 c) 8 d) 10

Explanation:

Books	Days	Examiners	Hours
1	6	5	4
3	10	6	x

Compare *hours* with *Books*, *Days* and *Examiners.*

If there are more *Books,* it takes more *hours* to correct them (*Direct Variation*).

If there are more *Examiners*, it takes less *hours* for them to complete the correction. (*Indirect variation*)

If there are more *Days*, less *hours* per day are required to complete the work. (*Indirect Variation*)

$\frac{4}{x}=\frac{1}{3}\times\frac{10}{6}\times\frac{6}{5}$ $\Rightarrow x = 6.$

Alternative method:

Let the number of answer books = 5×4×6.

Examiner hours days =120

Thrice = 120×3 = 360

$\therefore 360 = 6\times10\times x \Rightarrow$ $x=\frac{360}{60}=6$

Ask doubt with Question Id: 1356

3) 3 men and 4 women earn ₹ 264 in 8 days. 2 men and 3 women earn ₹ 184 in the same specified time. In how many days 6 men and 7 women will earn ₹ 315?

a) 4 b) 5 c) 6 d) 7

Explanation: The time of their earning is same.

$\therefore$ The ratio of both these terms will be equal.

$\frac{3\text{men}+4\text{ women}}{2\text{men}+3\text{ women}}=\frac{264}{184}\Rightarrow 3\text{ men}=7\text{ women}$

3 men + 4 women = 11 women

6 men + 7 women = 21 women

Women	Earning	Days
11	264	8
21	315	x

Days ↔ Women (Indirect Variation)

Days ↔ Earning (Direct Variation)

$\frac{8}{x}=\frac{21}{11}\times\frac{264}{315}\Rightarrow x=5$

Ask doubt with Question Id: 1357

EXERCISE

1) If the cost of 46 apples is ₹ 391. Then find the cost of 7 dozen of apples ?

a) ₹714 b) ₹821 c) ₹687 d) ₹736 e) ₹724

2) If 8 men working 9 hours per day can complete a work in 32 days. Then 12 men working 8 hours per day, require how many days to complete the work?

a) 18 b) 22 c) 36 d) 24 e) 28

3) 25 workers construct 25 houses in 25 days. Find one worker construct one house in how many days?

a) 12.5 b) 25 c) 5 d) 1 e) 50

4) 6 men or 9 women can do a piece of work in 30 days. Then find in how many days 10 men and 12 women can do the work?

a) 9 b) 16 c) 12 d) 15 e) 10

5) 12 clerks can clear 600 files in 20 days. Then how many clerks are required to complete 400 files in 10 days?

a) 14 b) 12 c) 16 d) 15 e) 18

6) The cost of 12 cartons of each weighs 4½ kg of chocolates is ₹10800. Then find the cost of 20 chocolate cartons weighing 5 kg each?

a) ₹19600 b) ₹18400 c) ₹16800 d) ₹20000 e) ₹18000

7) 4 men can reap 40 hectares in 12 days, then how many hectares, 24 men reap in 20 days?

a) 400 hectares b) 360 hectares c) 320 hectares d) 380 hectares e) 420 hectares

8) 8 rail engines consume 12 tons of coal when each engine is running 6 hours a day. 12 rail engines running at 9 hours a day will consume how many tons of coal?

a) 27 b) 20 c) 25 d) 30 e) 50

PERCENTAGES

CONCEPTS

A percentage is a way of expressing a number as a fraction of 100. The word 'per cent' or 'percentage' means for every one hundred. In other words, it gives rate of a parameter per hundred. It is denoted by the symbol %.

e.g.: 30% means 30 out of one hundred or $\frac{30}{100}$.

Key Notes:

• To convert a percent into a fraction, divide by 100.

e.g.: $20\% = \frac{20}{100} = \frac{1}{5}$

• To convert a fraction into a percent, multiply by 100.

e.g.: $\frac{3}{4} = \frac{3}{4} \times 100 = 75\%$

• To write a decimal as a percent we move the decimal point two places to the right and put the % sign.

e.g.: $0.35 = \frac{35}{100} = 35\%$

• Conversely to write a percent as a decimal, we drop the % sign and insert or move the decimal point two places to the left.

e.g.: 43% = 0.43; 12% = 0.12.

Calculating a Percentage:

$$Percentage = \left(\frac{Value}{Total}\right) \times 100.$$

For example, if you obtained 18 marks out of 25 marks, what was your percentage of marks?

Explanation: Total marks = 25. Marks obtained = 18.

∴ Percentage of marks obtained = $\frac{18}{25} \times 100 = 72\%$.

Calculating Percentage Increase or Decrease:

• **% Increase :**

New value = (1+ Increase %) × (Original Value)

• **% Decrease :**

New value = (1–Decrease %) × (Original Value)

e.g.: If a book costs ₹ 80 and few months later it was offered at a 30% discount. How much does the book cost now?

Explanation:

New Amount = $\left(1 - \frac{30}{100}\right) \times 80 = 0.70 \times 80 = 56$

• **Calculating Percent Change:**

Percentage change refers to the relative percent change of an increase or decrease in the original amount.

$$Percent = \frac{Change}{Original\,Value} \times 100$$

e.g.: If a book costs ₹80 and few months later it was offered at a price of ₹64. What was the discount percentage on that book?

Explanation: Change = 80–64 = 16. Original Value = 80.

Discount Percentage = $\frac{16}{80} \times 100 = \frac{1}{5} \times 100 = 20\%$

Calculating Successive Percentages:

• If a number is successively increased by x% and y% then a single equivalent increase in that number will be $\left(x + y + \frac{xy}{100}\right)\%$.

e.g.: The price of an article is successively increased by 10% and 20%. What is the overall percent increase in the price of the article.

Explanation:

$$₹100 \xrightarrow{\text{10\% Increase}} ₹110 \xrightarrow{\text{20\% Increase}} ₹132$$

Overall 32% Increase

(or) By using formula:

$$= \left(x + y + \frac{xy}{100}\right)\% = \left(10 + 20 + \frac{(10)(20)}{100}\right)\% = 30 + 2 = 32\%.$$

• If there's an increase and a decrease, in that case, the decrease will be considered a negative value.

e.g.: If there is an increase of 20% and then a decrease of 10% on the price of a commodity, the successive percentage will be

$$20 + (-10) + \frac{20 \times (-10)}{100} = 20 - 10 - 2 = 8\% \text{ increase.}$$

• In case of discounts, the value of discount percentages will be considered negative.

e.g.: If a shop keeper give 20% and 10% discounts on a festival day, the final discount given by shopkeeper is

$$(-20) + (-10) + \frac{(-20)(-10)}{100} = \text{-100+25 = 75\% discount.}$$

• If there are three discounts as x%, y% and z% then first find the total discount of x% and y% and using it find the total discount with z%.

• If the price of commodity increases by x%, the percentage should a family reduce its consumption so as not to increase the expenditure on the commodity = $\frac{x}{100 + x} \times 100$.

• If the price of commodity decreases by x%, the percentage should a family increase its consumption so as not to decrease the expenditure on the commodity = $\frac{x}{100 - x} \times 100$.

• Let the present population of a town is P. If it increases at the rate of R% per annum, then:

Population after 'n' years = $P\left(1+\frac{R}{100}\right)^n$.

Population 'n' years ago = $\frac{P}{\left(1+\frac{R}{100}\right)^n}$.

Note:

1. There are no units for percentage.

2. 0.2 of a work means 20% of the work, vice-versa.
50% of work means 0.5 of a work.

3. If A is 20% more than B means, A=120 if B=100.

4. If A is 20% less than B means, A=80 if B=100.

5. Take 100 as standard value, it will be easy to perform calculations on 100.

6. For most of the percentage questions, assume any value and solve. No need to apply formulas, common sense is enough.

Shortcut ways to calculate percentages:

1. To calculate 10% of any number, just move one decimal place to the left.

e.g.: 10% of 150 = 15.0 = 15.
40% of 150 = (4×10%) of 150 = 4×15 = 60.

2. To calculate 1% of any number, just move two decimal places to the left. (*i.e.* 1% of 150 = 1.50)

3. 5% = half of 10%. (*i.e.* 5% of 150 = $\frac{15}{2}$ = 7.5)

4. 15% of a number = 10% + 5% of the number.

5. 20% of a number = 2×10% or divide the given number by 5.

e.g.: 20% of 150 = 2×15 = 30 (or) $\frac{150}{5}=30$.

6. 25% of a number = Divide the given number by 4.

e.g.: 25% of 160 = $\frac{160}{4}=40$.

7. 50% of a number = Divide the given number by 2.

8. 75% of a number=(50%+25%) or $\frac{3}{4}$ of given number

e.g.: 75% of 160 = 80 + 40 = 120 (or) $\frac{3}{4}\times160=120$.

Similarly you can calculate percentage value for any value very quickly.

CONCEPTUAL EXAMPLES

1) A reduction of 25% in the tax resulted in increase of 30% in the daily attendance in a theater. The total daily collection will be?

a) $2\frac{1}{2}$ % more b) Same c) 5% less d) $2\frac{1}{2}$% less

Explanation: Let cost of ticket be ₹ 100.
Let the attendance be 100 members.
Total daily collection = ₹ 100 × 100 = ₹ 10000
Ticket rate is reduced by 25% and attendance increased by 30%.
So, new ticket rate = 75 and attendance = 130.
Total daily collection = ₹ 75 × 130 = ₹ 9750
Decrease in collection = 10000 – 9750 = 250

$$\text{Percentage Decrease} = \frac{250}{10000}\times100 = 2.5 = 2\frac{1}{2}\%$$

Option-d is the correct answer.

Ask doubt with Question Id: 1383

2) The population of a town is 30000. During first year the population increased by 15%. During second year the population increased by 10%. During third year the population increased by 10%. Find the population after 3 years.

a) 32800 b) 41745 c) 54895 d) None

Explanation: The population after 3 years

$$= 30000\left(1+\frac{15}{100}\right)\left(1+\frac{10}{100}\right)\left(1+\frac{10}{100}\right)$$

$$= 30000\times\frac{115}{100}\times\frac{110}{100}\times\frac{110}{100} = 41745$$

Ask doubt with Question Id: 1385

3) The population of a city increased by 20% in the first year and decreased by 25% in the second year. If the present population is 54000, population before two years is.

a) 55000 b) 57500 c) 60000 d) 62500

Explanation: Let the population before two years = x.
Present population = 54000.

$$\frac{120}{100}\times\frac{75}{100}\times x = 54000 \Rightarrow \frac{54000\times10}{9} = 60000$$

Ask doubt with Question Id: 1384

4) In an examination a student who secured 28% marks failed by 60 marks and another student who secured 32% marks got 8 marks more than necessary to pass. What is the percentage of marks required to pass?

a) 42.5 b) 31.53 c) 28.5 d) 15

TIME AND WORK

CONCEPTS

1) If a person completes a piece of work in 'n' days, then work done by that person in one day = $\frac{1}{n}^{th}$ part of the work.

2) If a person completes $\frac{1}{n}^{th}$ part of the work in one day, then the person will take 'n' days to complete the work.

3) The total work to be done is usually considered as one unit.

4) If M_1 persons can do W_1 work in D_1 days and M_2 persons can do W_2 work in D_2 days then

$$M_1 D_1 W_2 = M_2 D_2 W_1.$$

5) If M_1 persons can do W_1 work in D_1 days working T_1 hours per day and M_2 persons can do W_2 work in D_2 days working T_2 hours per day then

$$M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_1.$$

6) If A can do a piece of work in 'x' days and B can do it in 'y' days then A and B working together will do the same work in $\frac{xy}{(x+y)}$ days.

7) If A, B and C can do a piece of work in x, y and z days respectively then all of them working together can finish the work in $\frac{xyz}{(xy+yz+zx)}$ days.

8) If A is thrice as good a workman as B then,
Ratio of work done by A and B = 3 : 1.
Ratio of times taken by A and B to finish a work =1 : 3.

9) If A is 'k' times efficient than B and is therefore able to finish a work in 'n' days less than B, then

a) A and B working together can finish the work in $\frac{kn}{k^2-1}$ days.

b) A working alone can finish the work in $\frac{n}{k-1}$ *days.*

c) B working alone can finish the work in $\frac{kn}{k-1}$ *days.*

10) If A, working alone takes a days more than A and B working together. B alone takes b days more than A and B working together. Then the number of days taken by A and B working together to finish the job is $\sqrt{ab}$.

CONCEPTUAL EXAMPLES

1) A is twice as good a workman as B and takes 10 days less to do a piece of work than B takes. Find the time in which B alone can complete the work.

a) 22 days b) 25 days c) 23 days d) 20 days

Explanation: Let B alone takes 'x' days to complete the work. A is twice as good workman as B.

It means A takes $\frac{x}{2}$ days to complete the work.

From the given information we can write $x-\frac{x}{2}=10$

$$\Rightarrow \frac{2x-x}{2}=10 \Rightarrow \frac{x}{2}=10 \Rightarrow x=20.$$

Alternate Method: Using Formula.
Here, $k = 2$ and $n = 10$

$\therefore$ Time taken by B working alone to complete the work= $\frac{kn}{k-1}$ days $\Rightarrow \frac{2\times10}{2-1}=20$ days.

Ask doubt with Question Id: 1179

2) 25 men can reap a field in 20 days. When should 15 men leave the work, if the whole field is to be reaped in 37½ days after they leave the work.

a) 5 days b) 4 days c) 3 days d) 4½ days

Explanation: 25 men can reap the field in 20 days. ⇒ 1 man can reap that field in 25×20 *i.e.* 500 days.

Let 15 men leave the work after x days so that remaining 10 men can complete the work in 37½ days. It means 25 men have worked for x days and 10 men have worked for 37½ days.

$$\therefore\ 25x+10\times37\frac{1}{2}=500 \Rightarrow 25x=500-375=125 \text{ (or) } x=5$$

$\therefore$ 15 men must leave the work after 5 days.

Ask doubt with Question Id: 1180

3) A man is paid ₹30 for each day he works, and forfeits ₹5 for each day he is idle. At the end of 60 days he gets ₹50. Then, he was idle for _______ days.

a) 20 b) 25 c) 30 d) 50

Explanation: Suppose, the man was idle for x days.

$\therefore 30(60 - x) - 5x = 50 \Rightarrow x = 50$

Ask doubt with Question Id: 1181

4) 12 men or 15 women can do a work in 20 days. In how many days 7 men and 5 women would complete the work?

a) 21.8 b) 22.8 c) 25.3 d) 29

Explanation: ***or*** means either men or women.
and means both men and women.

12 men **or** 15 women ⇒ 12 men **=** 15 women
⇒ 4 men = 5 women.

CLOCKS

CONCEPTS

1) 60 minute space traces an angle of 360^0.

$\therefore$ 1 minute space traverses an angle of 6^0.

2) In 1 hour:

Minute hand traverses 60 minute space or 360^0.

Hour hand traverses 5 minute space or 30^0.

3) The minute hand travels 90^0 in 15 minutes.

4) The hands of the clock are in straight line when they coincide (or) when they are opposite to each other.

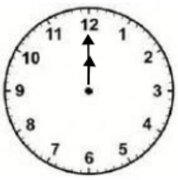
5) The hands of the clock are perpendicular to each other for 22 times in 12 hours and for 44 times in day.

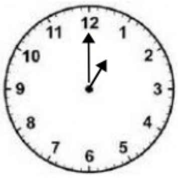
6) The hands of the clock are opposite to each other for 11 times in 12 hours and 22 times in a day.

7) The hands of the clock coincides with each other for 11 times in 12 hours and 22 times per day.

8) The hands of the clock are 44 times in a straight line per day.

9) 55 minute spaces are gained by minute hand in 60 minutes period.





To find how many minute spaces it has actually gained, let us assume a standard point where the both minute hand and hour hand coincides. After 60 minutes, minute hand moves 60 minute spaces and hour hand moves 5 minute spaces. Now there are 55 minute spaces between minute hand and hour hand. So we can say in 60 minutes of time, minute hand leads/gains hours hand by 55 minute spaces.

10) Angle Concept: To find angle between hour hand and minute hand use the below formula.

$\theta = \frac{11}{2}m - 30h \quad \left(if \frac{11}{2} > 30h\right)$ **(or)**

$\theta = 30h - \frac{11}{2}m \quad \left(if\ 30h > \frac{11}{2}\right)$

e.g.: At what time between 2 O'clock and 3 O'clock the hands of the clock be together.

Explanation: At 2 O'clock the minute hand is at 12 and hour hand is at 2. They are 10 minute spaces apart. To be together, minute hand must gain 10 minute spaces over hour hand.

55 minutes are gained in 60 minutes.

10 minutes are gained in x minutes.

i.e. $x = \frac{10 \times 60}{55.} = 10\frac{10}{11}$ minutes after 2 O'clock the two hands of a clock will be together.

Alternate Method: Hands of the clock are together. It means the angle between minute hand and hour hand is zero.

$\theta = \frac{11}{2}m - 30h \quad \Rightarrow \quad \frac{11}{2}m - (30 \times 2) = 0$

$\frac{11}{2}m = 60 \quad \Rightarrow \quad m = \frac{120}{11} = 10\frac{10}{11}$

e.g.: At what time between 2 O'clock and 3 O'clock the hands of the clock are opposite to each other.

a) $3\left(\frac{6}{11}\right)$ past 2 O'clock b) $43\left(\frac{7}{11}\right)$ past 2 O'clock

c) $56\left(\frac{8}{11}\right)$ past 2 O'clock d) $64\left(\frac{9}{11}\right)$ past 2 O'clock

Explanation:

To coincide minutes hand with the hour hand, first it should trace 10 minute spaces. And then the hands of the clocks to be opposite to each other minute hand should trace 30 minute spaces *i.e.* totally it should gain 10 + 30 = 40 minute spaces to be opposite to hour hand.

55 minutes are gained in 60 minutes.

40 minutes are gained in x minutes.

$x = 40 \times \left(\frac{60}{55}\right) = 43\left(\frac{7}{11}\right)$

Hence the hands of the clock will be opposite to each other at $43\left(\frac{7}{11}\right)$ past 2 O'clock.

Therefore, option-b is correct.

- **When clock is too fast, too slow:**

1) If a clock indicates 6 hours 10 mins when the correct time is 6, it is said that the clock is 10 minutes too fast.

2) If it indicates 6.40 when the correct time is 7, it is said to be 20 minutes too slow.

e.g.: My watch, which gains uniformly, is 2 minutes behind when shown at noon on Sunday. And it is 4 *min* 48 seconds fast at 2 pm on the following Sunday. When was it correct?

Explanation: From Sunday noon to the following Sunday at 2 pm there are 7 days 2 hours. (or) 170 hours.

The watch gains $\left(2 + 4\frac{4}{5}\right) = 6\frac{4}{5}\,min$ in 170 hours.

To show the correct time, the clock has to gain 2 minutes initially.

PROFIT AND LOSS

CONCEPTS

Cost Price (CP) is the price at which an article is bought.
Selling Price (SP) is the price at which an article is sold.
Marked Price (MP) or List Price is the price marked on the article. For example, a vendor buys 1kg of mangoes for ₹50. He labeled the price as ₹80. But sold for ₹ 70. Here CP = ₹50, MP = ₹80, SP = ₹70.

The expenses incurred on transportation, maintenance, packaging, advertisement etc. are considered as *overhead*. These *overheads* and the *profit* when added to the *cost price* determine the *selling price.*

Profit or Gain: Profit is made when the selling price is greater than the cost price.

Profit = SP – CP ; **Profit % =** $\dfrac{\textbf{Profit}}{\textbf{Cost Price}}\times 100$

Considering the same example given above,

Profit = 70–50 = ₹20. Profit % = $\dfrac{20}{50}\times 100 = 40\%$

Loss: Loss is made when the cost price is greater than the selling price.

Loss = CP – SP ; **Loss % =** $\dfrac{\textbf{Loss}}{\textbf{Cost Price}}\times 100$

• Profit or Loss is calculated on cost price only.
Discount is always calculated on the marked price.

Discount = MP–SP; **Discount%=** $\dfrac{\textbf{Discount}}{\textbf{MP}}\times 100$

Consider the same example given above,

Discount = 80–70 =10; Discount% = $\dfrac{10}{80}\times 100 = 12.5\%$

• To calculate Gain, Loss, Selling Price and Cost Price directly use the formula,

$$\textbf{SP} = \frac{(100 \pm \textbf{Gain or Loss})\times \textbf{CP}}{100}$$

Use + sign for profit and – sign for loss.

Example: Cost Price of an article is ₹70. At what price it should be sold in order to gain 20%?

$$SP = \frac{(100+20)\times 70}{100} = \frac{120\times 70}{100} = 12\times 7 = 84$$

• If a man purchased m articles for ₹p and sold n articles for ₹ q. Then how much profit or loss does he make?

$$\begin{array}{ccc} m & & ₹\,p \\ & \times & \\ n & & ₹\,q \end{array}$$

Profit or Loss % = $\dfrac{mq - np}{np}\times 100$

Example: A merchant purchased 7 watches for ₹500 and sold 5 watches for ₹400. What is loss or gain percent?

Explanation:

$$\frac{7\times 400 - 5\times 500}{5\times 500}\times 100 = \frac{2800-2500}{2500}\times 100 = \frac{300}{25} = 12$$

• By selling an article for ₹ P, a merchant would gain or loss x%. The price at which he sell it to gain or loss y% is

$SP = P\left(\dfrac{100\pm y}{100\pm x}\right)$. (+ sign for gain; − sign for loss)

Example: By selling a furniture for ₹180 a merchant will loss 10%. At what price must he sell to gain 20%.

Explanation: $SP = 180\times\left(\dfrac{100+20}{100-10}\right) = 240.$

• When a man buys two things on equal price each and in those things one is sold at a profit of x% and another is sold at a loss of x%, then there will be no loss or no gain percent.

Example: A merchant purchased a watch and a bag for ₹100 each. But he sold the watch at a profit of 20% and bag at a loss of 20%. What is his loss or gain percentage?

Explanation:

	CP		SP
Watch -	₹100	+ 20% Profit =	₹120
Bag -	₹100	– 20% Loss =	₹ 80
	₹ 200		₹ 200

Cost price = Selling Price. Hence, no gain or no loss.

• By selling two articles at the same price a merchant incurs x% loss on the first article and x% gain on the second article. In such a case there is always a loss.

Loss = $\dfrac{2\times SP}{\left(\dfrac{100}{x}\right)^2 - 1}$

Example: By selling a watch and a bag at ₹100 each a merchant incurred a loss of 20% on watch and gain of 20% on bag. What is his loss or gain percentage?

Explanation:

	SP		CP
Watch	₹ 100 (20% Loss on CP)	=	₹ 125
Bag	₹ 100 (20% Profit on CP)	=	₹ 83.33
	₹ 200		₹ 208.33

Here, CP > SP. Hence, Loss = $\dfrac{8.33}{208.33}\times 100 = 3.9\%$

(or) Using Formula: Loss = $\dfrac{2\times 100}{\left(\dfrac{100}{20}\right)^2 - 1} = \dfrac{200}{24} = 8.33.$

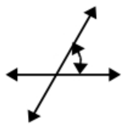
Then Cost Price = 200 + 8.33 = 208.33

And Loss Percentage = $\dfrac{8.33}{208.33}\times 100 = 3.9\%$

BASIC GEOMETRY

CONCEPTS

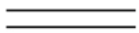
Angle: When two non-parallel and co-planar lines (lines in the same plane) intersect, at the point of intersection the measure of rotational displacement is called an angle.



Types of Angles: If θ is an angle such that
1) If $\theta = 0°$ then θ is *zero angle.*
2) If $0° < \theta < 90°$ then θ is called an *acute angle.*
3) If $\theta = 90°$ then θ is *right angle.*
4) If $\theta > 90^0$ then θ is *obtuse angle.*
5) If $\theta = 180°$ then θ is called a *straight angle.*
6) If $180° < \theta < 360°$ then θ is called *reflex angle.*
7) If $\theta = 360°$ then θ is called *complete angle.*

Parallel and Non-Parallel lines:

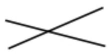
1) Two lines are said to be parallel lines if they are co-planar (in the same plane) and non intersecting.



The point of intersection of parallel lines is at infinite places which is not real.

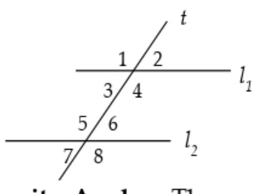
2) The angle between parallel lines is undefined, or it can be either 0° or 180° or any multiple of 180°.

3) Two lines are said to be non parallel (inclined lines) if they are co-planar and intersect at a real point.



The point of intersection of inclined lines is real.

Transversal: A line that intersects two parallel lines is called a *transversal.* Suppose l_1 , l_2 are two parallel lines and '*t*' is a transversal, then we will have eight angles as shown in figure.



• **Vertical Opposite Angles:** The angles ∠1 ∠4, ∠2 ∠3, ∠5 ∠8, ∠6 ∠7 pair wise are called pairs of *vertical angles.* The corresponding pairs of vertical angles are always equal *i.e.* ∠1=∠4, ∠2=∠3, ∠5=∠8, ∠6=∠7.

• **Corresponding Angles:** The angles ∠1 ∠5, ∠2 ∠6, ∠3 ∠7, ∠4 ∠8 pair wise are called *corresponding angles.* The pairs of corresponding angles are always equal. *i.e.* ∠1=∠5, ∠2=∠6, ∠3=∠7, ∠4=∠8.

• **Alternate Interior Angles:** The angles ∠3 ∠6, ∠4 ∠5 are called pairs of *alternate interior angles.*
The corresponding pairs of alternate angles are equal.
i.e. ∠3=∠6, ∠4=∠5

• **Alternate Exterior Angles:** The angles ∠1 ∠8, ∠2 ∠7 are called pairs of *alternate exterior angles.* ∠1=∠8, ∠2=∠7.

• **Complementary Angles:** Two angles whose sum is 90° are called *complementary angles.*

• **Supplementary Angles:** Two angles whose sum is 180° are called *supplementary angles.*

POLYGONS

• A closed plane figure made up of several line segments that are joined together is called a Polygon.

• If all the sides of a polygon are equal then it is called Regular Polygon.
Regular polygons are both equiangular and equilateral.
Equiangular = all angles are equal.
Equilateral = all sides are the same length.

Exterior angle: The angle subtended by a side of the regular polygon at the vertex outside.
Sum of the exterior angles of any polygon = 360°.
Each exterior angle (regular polygon) = $\frac{360}{n}$.
(where 'n' is the number of sides in a polygon).

Interior angle:
Sum of the interior angles of a polygon = $(n–2)×180°$.
Each interior angle of a regular polygon = $\frac{180(n-2)}{n}$.

• The number of diagonals in a polygon = $\frac{n(n-3)}{2}$.

• The number of triangles (when you draw all the diagonals from one vertex) in a polygon = $(n–2)$.

Polygon Names:

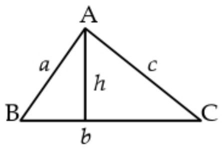
Sides	**Name**
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
10	Decagon

Special Triangles:
Equilateral, Isosceles, Scalene, Right Angled, Acute, Obtuse.

Special Quadrilateral:
Square, Rhombus, Parallelogram, Rectangle, Trapezoid.

TRIANGLE

1) Sum of the three angles in a triangle is always 180°**.**
i.e. $\angle 1 + \angle 2 + \angle 3 = 180°$.



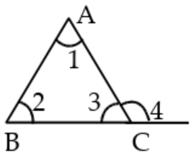
2) In a triangle sum of the lengths of any two sides is greater than the third side.
i.e. AB+BC>AC; AB+AC>BC; BC+CA>AB;

3) Area of the $\Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times b \times h$.

Perimeter (P) of $\Delta ABC = a + b + c$.

4) The side opposite to greatest angle is greatest and the side opposite to smallest angle is smallest.

5) The exterior angle is equal to the sum of the other two interior opposite angles. *i.e.* $\angle 4 = \angle 1 + \angle 2$.

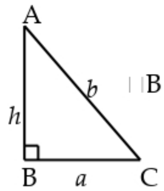


Types of Triangles:

• **Right Angled Triangle:** One angle is 90°.

Perimeter = $a + b + h$.

Area = $\frac{1}{2} \times a \times h$.



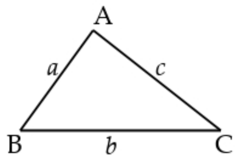
$\angle B = 90°$

• **Scalene Triangle:** All angles are different and all sides have different length.

Perimeter = $a + b + c$

Area = $\sqrt{s(s-a)(s-b)(s-c)}$;

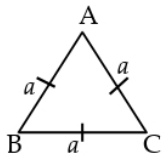
where $s = \frac{a+b+c}{2}$.



• **Equilateral Triangle:** Every angle is equal (*i.e.* 60°). Every side is equal in length.

Perimeter = $3a$.

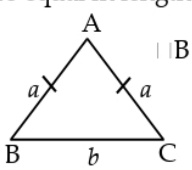
Area = $\frac{\sqrt{3}}{4} \times a^2$



• **Isosceles Triangle:** Two angles of the triangle are equal. Two sides of the triangle are equal in length.

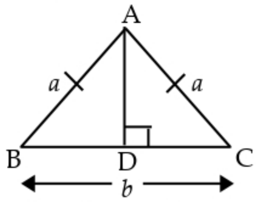
Perimeter = $2b + a$

Area = $\frac{b}{4}\sqrt{4a^2 - b^2}$

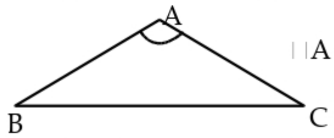


$\angle B = \angle C$

The perpendicular from the vertex to the base line (the height) in an isosceles triangle divides the triangle into two equal right angled triangles.

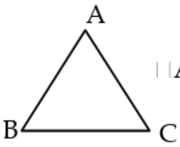


• **Obtuse Triangle:** One angle is greater than 90°. The longest side is opposite to the largest angle.



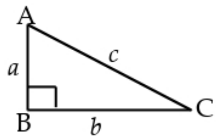
$\angle A > 90°$

• **Acute Triangle:** One angle is less than 90°.



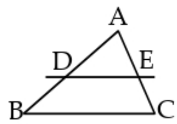
$\angle A < 90°$; $\angle B < 90°$; $\angle C < 90°$

Pythagoras theorem: In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



$a^2 + b^2 = c^2$ $\angle B = 90°$.

6) The line joining mid point of two sides of a triangle is always parallel to the 3rd side and it is half of the 3rd side.

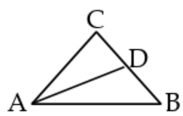


i.e. DE || BC and $DE = \frac{1}{2} BC$

7) Basic Proportionality Theorem: If a line is drawn parallel to one side of a triangle, then it divides other two sides in the same ratio.

If DE || BC then $\frac{AD}{DB} = \frac{AE}{EC}$

8) Median of Triangle: A line joining the mid point of a side to the opposite vertex is called Median of a triangle.



9) Property of Median: In a ΔABC, if 'AD' is the median then it divides ΔABC into two equal parts *i.e.* $\Delta ADB = \Delta ADC$.

PERMUTATIONS AND COMBINATIONS

CONCEPTS

• **Fundamental Principal of Multiplication:**

In general if some procedure can be performed in n_1 different ways, and if, following this procedure, a second procedure can be performed in n_2 different ways, and if, following this second procedure, a third procedure can be performed in n_3 different ways, and so fourth then the number of ways the procedure can be performed in the order indicated is the product of $n_1 . n_2 . n_3......$

e.g.: A letter lock consists of 5 rings each marked with 10 different letters. What is the maximum number of unsuccessful attempts to open the lock.

Explanation: Each ring is marked with 10 different letters. Hence each ring has 10 positions.

Thus, the total number of attempts that can be made to open the lock is $10 \times 10 \times 10 \times 10 \times 10 = 10^5$.

Out of these, there must be one attempt in which the lock will open.

$\therefore$ Total number of unsuccessful attempts = $10^5 - 1$.

•**Fundamental Principle of Addition:**

If there are two operations such that they can be performed independently in m and n ways respectively, then either of the two operations can be performed in $(m+n)$ ways.

•**Factorial:** The product of first 'n' natural numbers is called the 'n'-factorial and is denoted by $n!$

$n! = 1.2.3.4 .... (n-2).(n-1).n$

Example: $4! = 1.2.3.4 = 24$, $5! = 1.2.3.4.5 = 120$,
$5! = 5.4! = 5.24 = 120$, $6! = 6.5! = 6.120 = 720$.

Note: 1) $0! = 1$

2) The product of 'r' consecutive positive integers is divisible by $r!$

3) $(kn)!$ Is divisible by $(n!)k$ for all k is a positive constant.

4) The product of $2n!$ consecutive positive integers is equal to $2(n!)$.

PERMUTATIONS

•**Permutation:** An arrangement of any $r \le n$ of these objects in a given order is called an r–permutation or a permutation of the 'n' objects taken 'r' at a time.

Example: Consider the set of letters a, b, c, and d. Then

(i) *bdca, dcba* and *acdb* are permutations of the 4 letters taken all at time.

(ii) *bad, adb, cbd* and *bca* are permutations of the 4 letters taken 3 at a time.

(iii) *ad, cb, da* and *bd* are permutations of the 4 letters taken 2 at a time.

The number of permutations of 'n' objects taken 'r' at a time will be denoted by $P(n, r)$.

Before we derive the general formula for $P(n, r)$ we consider a special case. Find the number of permutations of 7 objects, say a, b, c, d, e, f, g taken three at a time. In other words, find the number of 'three letter words' with distinct letters that can be formed from the above seven letters.

Let the general three letters word be represented by three boxes.

Now the first letter can be chosen in 7 different ways; following this, the second letter can be chosen in 6 different ways; and, the last letter can be chosen in 5 different ways. Write each number in its appropriate box as follows:

7	6	5

Thus by the fundamental principle of counting there are 7.6.5=210 possible three letter words without repetitions from the seven letters. (or) There are 210 permutations of 7 objects taken 3 at a time.

i.e. $P(7, 3) = 210$.

The derivation of the formula for $P(n, r)$ follows the procedure in the preceding example:

The first element in an r-permutation of n-objects can be chosen in 'n' different ways; following this, the second element in the permutation can be chosen in $(n-1)$ ways; and, the third element in the permutation can be chosen in $(n-2)$ ways. Continuing in this manner, we have that the r^{th} (last) element in the r–permutation can be chosen in $n-(r-1) = n-r+1$ ways.

Thus $P(n, r) = n(n-1)(n-2) \ldots (n-r+1) = \dfrac{n!}{(n-r)!}$

The second part of the formula follows from the fact that $n(n-1)(n-2) \ldots\ldots (n-r+1) =$

$$\frac{n(n-1)(n-2)\cdots\cdots(n-r+1)\cdot(n-r)!}{(n-r!)} = \frac{n!}{(n-r)!}$$

$\therefore$ A formula for the number of possible permutations of 'r' objects from a set of 'n' is $P(n, r)$ or ${}^{n}p_{r} = \dfrac{\boldsymbol{n!}}{\boldsymbol{(n-r)!}}$

In the special case that $r = n$, we have $P(n, n) = n(n-1)(n-2). \ldots\ldots 3.2.1 = \boldsymbol{n!}$ (in other words there are $n!$ permutations of 'n' objects taken all at a time).

PROBABILITY

CONCEPTS

•**Random Experiment:** Probability is the study of random or non deterministic experiments. If the dice is tossed in the air, then it is certain that the dice will come down, but is not certain that, say a 3 will appear.

Definition: A *random experiment* is an experiment whose result would not be predicted but the list of possible outcomes are known. The unpredicted outcomes could not be taken under random experiments. The result of random experiments may not be predicted exactly but the result must be with in the list of predicted outputs.

Example:

1) Tossing a fair coin.
2) Rolling a dice is a random experiment, since its results could not be predicted in any trial.
3) Selection of a plastic component and verification of its compliance.
4) Life time of a computer.
5) Number of calls to a communication system during a fixed length interval of time.

•**Outcome:** The result of a random experiment will be called an outcome.

Example:

1) Tossing a coin. The result is either Head(H) or Tail(T).
2) In an experiment of throwing a six-faced dice. The possible outcomes are 1, 2, 3, 4, 5 and 6.

•**Sample Space:** The set of all possible outcomes of some given experiment is called *sample space.* A particular outcome, *i.e.* an element in that set is called a *sample point* or *sample*.

Example:

1) Toss a dice and observe the number that appears on top. Then the sample space consists of the six possible numbers: S = {1, 2, 3, 4, 5, 6}
2) Toss a coin 2 times and observe the sequence of heads (H) and tails (T) that appears. Then the sample space S consists of four elements: S = {HH, HT, TH, TT}

Note: Shortcut: Tossing a coin 2 times is same as tossing 2 coins at a time.

S = {H, T} × {H, T}

S = {HH, HT, TH, TT}

3) Toss a coin until a head appears and then count the number of times the coin was tossed. The sample space of this experiment is S = {1, 2, 3, ... ∞}. Here ∞ refers to the case when a head never appears and so the coin is tossed an infinite number of times. This is an example of a sample space which is countably infinite.

•**Events:** An event A is a set of outcomes or, in other words, a subset of the sample space S.

Example: If A random experiment is associated with what is the day today. It may be from Sunday to Saturday. If today is Friday and Friday belongs to the sample space S = {Sun, Mon, Tue, Wed, Thu, Fri, Sat}.

Different Types of Events:

•**Simple or Elementary Events:** An event with only one sample point is called *simple* or *elementary event.*

In an experiment of tossing three coins at a time, the event 'A' is that all coins turns up with heads consists of only one point HHH. Then 'A' is a simple event.

As a matter of fact each outcome of an experiment is a simple event.

•**Complimentary Event:** An event $\bar{A}$ (or A^1) is said to be *complementary* to an event 'A' in sample space 'S' consists of all those points which are not in 'A'.

Example: In tossing a coin three times, sample space S consists of eight points.

S = {HHH, HHT, HTH, THH, HTT, TTH, THT, TTT}

The event 'A' is such that there should be no heads in the sample point is {TTT}. Then the event $\bar{A}$ (or A^1) complementary to 'A' is that there exists at least one head in the sample space *i.e.* (HHH), (HHT), (HTH), (THH), (HTT), (TTH), (THT).

•**Equal Events:** Two events A and B are said to be *equal* if $A \subset B$ and $B \subset A$. This statement implies that all the points of A are also the points of B and vice-versa.

Example: Let sample space S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Let A be the event defined as *'even number'* and event B is defined as *'multiplies of 2'*.

Then A = {2, 4, 6, 8, 10}; B = {2, 4, 6, 8, 10} here every point in A is also a point in B and vice-versa. Therefore events A and B are said to be equal events.

•**Transitivity of events:** If A, B and C are 3 events such that $A \subset B$ and $B \subset C$ it implies that $A \subset C$ such a property of events is known s *transitivity* of events.

Let the sample space S = {1, 2,. . . . , 100}.

Event A be the *'even numbers'*: A = {2, 4, 6, 8,.....,100}

Event B be the *'multiples of 4'* : B = {4, 8, 12,.......,100}

Event C be the *'multiples of 8'*: C = {8, 16, 24,.....,100}

Event point in C is also point in B and event point in B is also point in A but not vice–versa *i.e.* $A \supset B \supset C$.

•**Compound event:** An event which is not simple or elementary is called a *compound event.* Every compound event can be uniquely represented by the union of a set of elementary events.

DATA SUFFICIENCY

CONCEPTS

Purpose of Data sufficiency:

Here the examiners intention is to check the student's capability in decision making. One can agree that the decision making is the sense of checking whether the data is sufficient or not.

Nature of Questions: You will be given a question followed with the two statements.

You don't need to solve the question. You just have to judge whether given two statements have enough information to solve the question.

CONCEPTUAL EXAMPLES

Each of the questions below consist of a question and two statements numbered I and II. You have to decide whether the data provided in the statements are sufficient to answer the given question. Read both the statements and give answer as

a: If the data in statement-I alone is sufficient and the data in statement-II alone is not sufficient to answer the question.

b: If the data in statement-II alone is sufficient and the data in statement-I alone is not sufficient to answer the question.

c: If the data either in statement-I or in statement-II alone are sufficient to answer the question.

d: If the data either in statement-I and II together are not sufficient to answer the questions. And some more data needed.

e: If the data in both statement-I and II together are necessary to answer the question.

1) What is the average of p, q and r?

I. r is 25. **II.** $p + q$ is 20.

Explanation: To find the average, we need values of p, q, r. From the given two statements values of p, q, r are known. Hence, we require both the statement-I and II to answer the given question. Hence, option-*e* is correct.

Ask doubt with Question Id: 5503

2) Who is youngest among Raju, Vamsi and Rajni?

I. Raju is one year elder to Vamsi.

II. Vamsi age is average age of Raju and Rajni.

Explanation:

From statement-II, Vamsi's age is between the ages of Raju and Rajni.

From statement-I, Raju is one year elder to Vamsi. It means Rajni will be one year younger to Vamsi.

$\therefore$ From both the statements, we can say, Rajni is youngest among the three. Hence, option-*e* is correct.

Ask doubt with Question Id: 5504

3) What is the value of x?

I. $x^2+2x-3=0$ **II.** $x^2+4x-5=0$

Explanation: From statement-I, $x^2+2x-3=0$

$x^2+3x-x-3=0 \Rightarrow x(x+3)-1(x+3)=0 \Rightarrow x = 1$ or -3

$\therefore$ From statement-I alone we can't say exact value of x.

From statement-II, $x^2+4x-5=0 \Rightarrow x^2+5x-x-5=0$

$x(x+5)-1(x+5)=0$ *i.e.* $x = 1$ or -5.

$\therefore$ From statement-I and II, we conclude, $x = 1$.

As both the statements together are required to answer the given question, option-e is correct.

Ask doubt with Question Id: 5505

4) Find the area of the square?

I. The side of the square is 7 cm

II. The circumference of the square is 28 cm

Explanation: $Area = (side)^2$

From statement-I, we know the value of *side*. Therefore *area* can be found.

From statement-II, circumference *i.e.* $4(side)=28$.

From this we can find the value of *side*. As a result *area* can also be found.

Here, either of the statements-I or II alone are sufficient to answer the given question. Hence, option-c is correct.

Ask doubt with Question Id: 5506

5) What is the cost price of the chair?

I. The selling price of the chair is ₹324 at profit of 8%.

II. The profit is 12%.

Explanation:

From statement-I, $CP = \frac{100}{100+8} \times 324 = ₹300$

$\therefore$ Statement-I alone is sufficient to answer.

Statement-II does not have the enough information to solve the given question. Hence, option-a is correct.

Ask doubt with Question Id: 5507

6) Who is tallest?

I. C is eldest.

II. A is shortest and B is youngest but taller than C.

Explanation:

Statement-I alone is not sufficient to answer.

From statement-II, A is shortest. And B is taller than C. It means B is taller than A and C. *i.e.* only statement -II is sufficient to answer the question.

Hence, option-b is correct.

Ask doubt with Question Id: 5508

7) Is $r > s$?

I. $r > t$ **II.** $at > ar$, $a < 0$.

Explanation: Statement-I and II gave information about t and r. But not s. So, it is not possible to say whether $r > s$ or not because of insufficient information

DATA INTERPRETATION

CONCEPTS

The information related to any event given in the form of graphs, tables, charts etc is termed as data. The methodology of interpreting data to get the information is known as data interpretation.

Mathematical identities which we use in data interpretation are given below.

To solve the problems on data interpretation, you need to be thorough in 'Percentages', 'Ratios' and 'Averages' chapters.

Percentage: Proportions with the base 100 are known as percentages (%).

For example, $\frac{x}{y}\times 100$ is in percentage form.

e.g.: If the ratio of boys to total number of students in a college is $\frac{1015}{4060}$. This can be written in a percentage form as $\frac{1015}{4060}\times 100 = 25\%$.

To find by how much percent $'x'$ is more or less than y (or over y) when compared to y is given as

$$\text{Required Percentage} = \frac{\text{Value of X} - \text{Value of Y}}{\text{Value of Y}}\times 100$$

Observe that the denominator contains the value with which the comparison is made.

In the above formula, if numerator is positive, then there is percentage growth. If numerator is negative, then there is a decline in the percentage.

Ratio: In the simplest possible form, ratio is a quotient or the numerical quantity obtained by dividing one figure by the other figure of same units.

TABULAR DATA INTERPRETATION

In this type of questions a table with data as well as a set of questions on the same data is given to you. You need to analyze the table data and answer the given questions.

Example: Study the following table carefully and answer the questions that follow.

Table: Percentage of marks scored by students in SSC

Marks percentage	Girls	Boys
>75	25	12
60-75	15	12
50-59	10	23
35-49	5	2
< 35	3	1

1) Give the total percentage of Girls who wrote SSC examination from that School.

a) 25% b) 54% c) 23% d) 58%

2) Give the percentage of students who scored distinction (> 75).

a) 43% b) 34.25% c) 24.85% d) 40%

3) Give fail percentage of students in SSC examination.

a) 1% b) 2% c) 4% d) 8%

4) Give pass percentage of boys in SSC examination.

a) 90% b) 88% c) 98% d) 99%

5) Give the percentage of students who scored more than 60% in the SSC examination.

a) 25% b) 59.3% c) 22.2% d) 50%

Explanation:

1)b; Total no.of girls appeared for SSC Examination = 25 + 15 + 10 + 5 + 3 = 58.

Total no.of students appeared for SSC examination = 58 + 50 = 108.

$\therefore$ Percentage of girls who wrote SSC Examination =

$\frac{58}{108}\times 100 = 53.7 = 54\%$ (approximately)

2)b; No.of students who scored distinction = 25 +12 =37.

$\therefore$ Percentage of students who scored distinction =

$\frac{37}{108}\times 100 = 34.25\%$

3)c; Total no.of students failed in SSC examination = 4.

$\therefore$ Fail % = $\frac{4}{108}\times 100 = 3.7 = 4\%$ (approximately)

4)c; No.of boys passed in the examination = 49.

$\therefore$ Boys pass percentage = $\frac{49}{50}\times 100 = 98\%$

5)b; No.of students who scored more than 60% = 64.

$\therefore$ Percentage of students who scored more than 60% =

$\frac{64}{108}\times 100 = 59.26\% = 59.3\%$ (approximately)

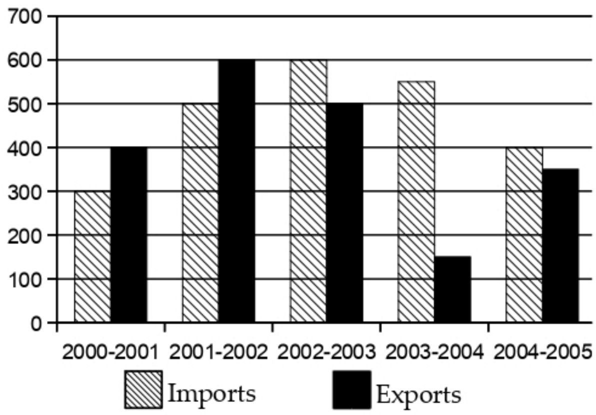
Example: Production of cars by different companies in the span of 1980-2005 given. Interpret the data to answer the questions given below.

Company	Production of cars in thousands				
	1980-85	1985-90	1990-95	1995-00	2000- 05
Maruthi	12.5	15.0	16.2	18.0	22
Hindustan Motors	10.4	11.1	11.5	11.5	12
Hyundai Motors	12	14.3	16.2	17.8	18.9
Ford	14.4	14.1	13.2	18.1	25.3
General Motors	19.2	13.8	13.5	14.1	15.8

BAR GRAPHS

Bar graphs normally comprise X-axis, Y-axis and bars. X and Y-axes represent the data. And bars represent the trend of data with respect X and Y-axes. In this type of questions, data is given in the form of bar graphs. You need to analyze the bars with respect to X and Y-axes to answer the given questions.

Example: Imports and exports of a country from 2000 - 2001 to 2004 -2005.



1) In which of the following year the gap between import and export was maximum.

a) 2001-02 b) 2002-03 c) 2003-04 d) 2004-05

2) In which of the following year the gap between imports and exports was minimum.

a) 2002-2003 b) 2003-2004 c) 2004-2005 d) none

3) Exports in 2001–2002 was approximately how many times that of the year 2003–2004.

a) 2 b) 3 c) 4 d) 5

4) Give the ratio between the number of years in which exports is greater than imports and import is greater than exports.

a) 3 : 2 b) 2 : 3 c) 3 : 1 d) 1 : 3

5. Difference between average of imports and exports is

a) 100 b) 90 c) 80 d) 70

Explanations:

1)c; From the graph, gap between import and export was maximum in 2003-2004.

2)c; From the graph, gap between imports and exports is minimum in 2004-2005 = 400–350 = 50 crore.

3)c; Exports of the year 2001–2002 = 600

Exports of the year 2003–2004 = 150

∴ Exports of 2001–2002 is 4 times greater than that of 2003–2004.

4)b; In 2 years *i.e.* 2000-2001 and 2001-2002 exports are greater than imports.

In 3 years *i.e.* 2002-2003, 2003-2004, 2004-2005 imports are greater than exports.

5)d; Average of imports during 2000-2005 =

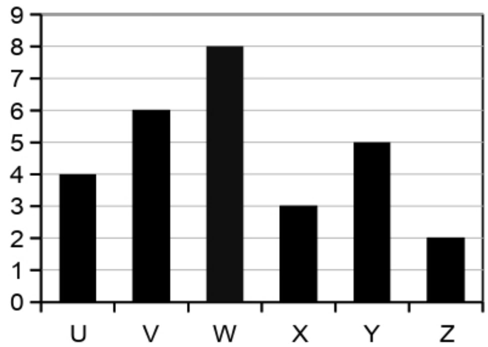
$$\frac{300+500+600+550+400}{5}=\frac{2350}{5}=470$$

Average of exports during 2000-2005 =

$$\frac{400+600+500+150+350}{5}=\frac{2000}{5}=400$$

∴ Difference = 470 – 400 = 70.

Example: Turnover in crores of six companies (U, V, W, X, Y and Z) are given.



1) Which company's turn over is highest?

a) U b) V c) W d) X

2) What is the percentage of turn over of the company-X over the turn over of the company-V?

a) 25% b) 50% c) 75% d) 100%

3) Give the difference of average turnovers of first three companies and last three companies.

a) 3.33 b) 6.66 c) 2.67 d) 1.85

4) Give the percentage contribution of turnover of W in the overall turnover of all the companies.

a) 12% b) 50% c) 40% d) 29%

5) Difference of average percentage contribution of turnovers of companies U, V and X, Y is.

a) 1% b) 2% c) 3% d) 4%

Explanations:

1)c; It is clear from the graph that turn over of company W is highest *i.e.* 8 crores.

2)b; Turnover of company X = 3 crores

Turnover of company V = 6 crores

∴ Percentage of turn over of X over V= $\frac{3}{6}\times100=50\%$

3)a; Average turn over of first three companies

$$\frac{4+6+8}{3}=\frac{18}{3}=6$$

Average turn over of last three companies

$$\frac{3+5+2}{3}=\frac{10}{3}=3.33$$

∴ Difference = 6.00 – 3.33 = 2.67

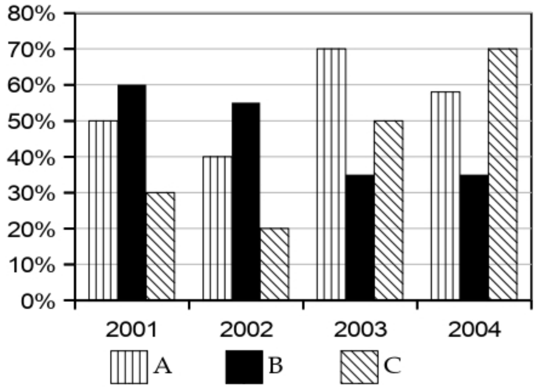
MIXED DIAGRAMS

CONCEPTS

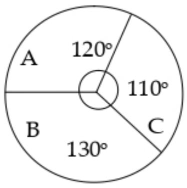
In this type of data interpretation, data will be given in the form of two or more diagrams. The combination of the diagrams can be a bar diagram and a pie chart (or) a line graph and a table diagram (or) a pie chart and line graph.

A) Study the following graphs carefully to answer the questions given below it.

Readers of newspapers in percentages in 3 different cities A, B and C over the years.



Total population of 3 crores in 3 cities is represented in the following diagram.



1) In 2002 in the city B how many people were reading a news paper in lakhs ?

a) 108.333 b) 59.5883 c) 48.7499 d) 38.9421

2) According to the data in city B what is the difference between minimum number of newspaper readers in a particular year and maximum number of newspaper readers in a particular year (*approximately*)?

a) 34 b) 31 c) 29 d) 27

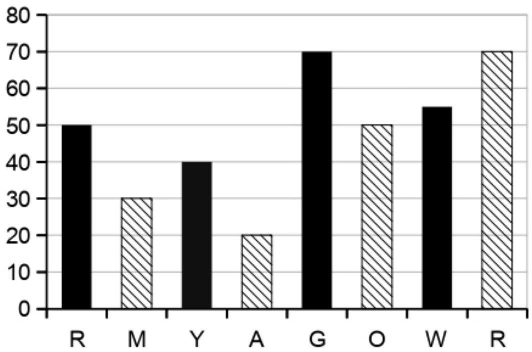
3) What is the sum of populations of city A those who don't read any newspapers in all the 4 years (in lakhs)?

a) 220 b) 200 c) 180 d) 160

4) In the 2 years in which same and maximum percentage of readership is maintained in the cities A and C. What is the decrease in readership in the city A?

a) 5 lakhs b) 10 lakhs c) 20 lakhs d) 30 lakhs

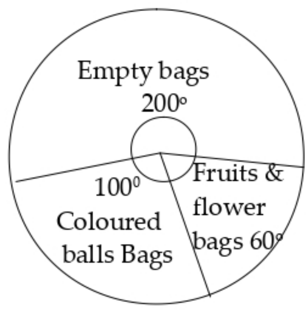
B) Study the following graphs carefully to answer the questions given below it. (Use most approximate figures, if necessary).



Different Bags containing colored (red, yellow, green, white) balls

Different Bags containing fruits and Flowers (Mango, Apple, Orange, Rose)

Percentage of bags (empty, fruits and Flowers, colored balls) available in every house is given in the following Pie chart.



There are 3 go-downs namely AB, BC, CE which have *n* bags in different days of the week as shown below.